Homework 3

Due date: someday, sometime!
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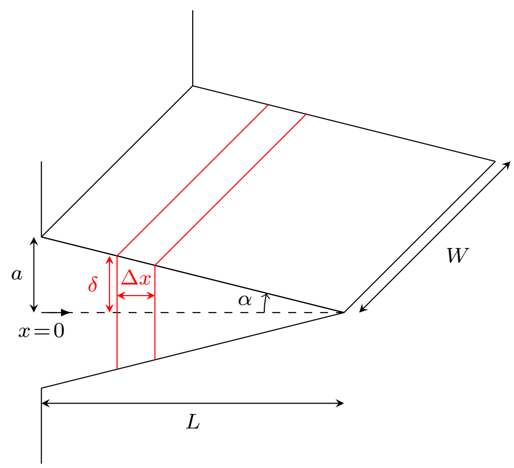
Exercise 1. [50 pts] Consider a straight rectangular heat fin illustrated below. Assuming steady-state conditions, write a thermal energy balance for the elemental volume of width Δx (from x to $x + \Delta x$) demarcated by the red solid lines. Then, let $\Delta x \to 0$ to find the governing ordinary differential equation.



You need to account for the unidirectional heat conduction within the fin, with flux $q_x = -k dT/dx$, and convection to the ambient, with flux $h(T - T_\infty)$. The sides of the fin (the two triangles with base $2a$ and height L) are insulated, and thus, does not contribute to the convection to ambient. Here, the symbols stand for thermal conductivity, k (assume constant); temperature of the fin, T; ambient temperature, T_∞; horizontal distance from the base, x; and the convective heat transfer coefficient, h.

Next, change variable $y = 2\sqrt{\gamma(L - x)}$ where $\gamma = hL/(ak\cos(\alpha))$, and $\theta = T - T_\infty$, and write the governing equation in terms of y and θ. You will reach to the modified Bessel equation of order zero, which should look like

$$y^2 \frac{d^2\theta}{dy^2} + y\frac{d\theta}{dy} - y^2\theta = 0.$$

General solution to the above Bessel equation is

$$\theta = C_1 I_0(y) + C_2 K_0(y),$$

where $I_0(y)$ and $K_0(y)$ are the modified Bessel functions of the first and second kinds. For the boundary conditions, let $T = T_0$ at the base ($x = 0$) and enforce a finite temperature at $x = L$. Find the coefficients C_1 and C_2, and express the final answer for θ in terms of x and Bessel functions. **Hint:** $K_0(y)$ blows up ($\to \infty$) at $y = 0$.

Exercise 2. [50 pts] Consider a heated sphere of radius R, with constant surface temperature T_R, suspended in a large, motionless (no velocity), body of fluid at $T = T_\infty$. Assuming steady state conditions, find the temperature distribution within the fluid as a function of r, distance from the center of the sphere. Start by writing a heat energy balance for a spherical shell between r and $r+\Delta r$ for an arbitrary $r > R$. Let $\Delta r \to 0$ and find the governing differential equation. Use the boundary conditions $T(R) = T_R$ and $T \to T_\infty$ as $r \to \infty$ to find the constants, and subsequently, the temperature distribution.

Next, equate the conduction heat flux at the surface of the sphere to the total heat transfer to the ambient:

$$\left(-k\frac{dT}{dr}\right)_{r=R} = h(T_R - T_\infty).$$

Having the temperature distribution $T(r)$, solve the above equation for h, the heat transfer coefficient. Then show that the dimensionless heat transfer coefficient, known as the Nusselt number, Nu, is

$$\mathrm{Nu} = \frac{hD}{k} = 2,$$

in which D is the sphere diameter.