Homework 7

Due date: someday, sometime!
Submit on NYU Brightspace.

Exercise 1. Consider a particle of mass m and electric charge q, initially at rest (zero velocity, $v(0) = 0$), placed in a unidirectional potential $\psi(x) = -E_0 qx \sin(\omega t)$. Assume that the particle experiences a spatially uniform electric force given by (for this one-dimensional system) as $F_E = -\partial\psi/\partial x$, and a nonlinear drag force in the from $F_D = \delta v^3$.

(i) Use the Newton's second law to write the equation of motion for the particle's velocity.

(ii) Take $\tilde{t} = \omega t$ and $\tilde{v} = v/v_0$ (with v_0 to be determined) as the normalized, dimensionless, time and velocity. Substitute into the equation of motion and find v_0, so that there remains only one dimensionless group and it is the coefficient of $\tilde{v}^3$.

(iii) Solve the obtained dimensionless equation of motion for $\tilde{v}(t)$ using a regular first-order perturbation method: $\tilde{v}(t) = \tilde{v}^{(0)}(t) + \epsilon\tilde{v}^{(1)}(t) + O(\epsilon^2)$. Clearly, and with details, show how you find the zeroth-order and first-order systems, and the solution procedure. You might need the trigonometric identities $\cos^3(t) = \frac{1}{4}(\cos(3t) + 3\cos(t))$ and $\cos^2(t) = \frac{1}{2}(\cos(2t) + 1)$.

(iv) For $\epsilon = 0.05$, solve the problem using Euler's method with $\Delta\tilde{t} = 0.01\gamma$ and a total time of 10γ, where $\gamma = 2\pi$ is the dimensionless period. Plot the solution $\tilde{v}$ obtained using the regular perturbation and the one from the Euler's method versus $\tilde{t}/\gamma$ in one figure. How do these two solutions compare? Discuss. Hint: If done properly, you should see from the Euler's method that the particle's velocity oscillates and after a few cycles, reaches to a harmonic solution with zero time-average, i.e., the particle should not move on average over a cycle when the oscillation is harmonic.

(v) Solve the equation of motion using two-timing. Let $\tau = \tilde{t}$ and $T = \epsilon\tilde{t}$ be the fast and slow times. Investigate what caused the problems, if any, in the regular perturbation theory. Perhaps some terms on the right hand side of the first-order system? Get rid of them by a suitable choice of the involved constant (or a relation between the constant and its derivative). Hint: at some point you encounter the integral $\int (2x^3 + 3x)^{-1} dx$ which you can evaluate by partial fractions. Plot the two-timing and numerical $\tilde{v}$ versus $\tilde{t}/\gamma$ in one figure. Use the same parameters as the ones in part (iv). Discuss the new results.

notes: Show all of the steps of your work. Solutions without details of the work and interpretation of the results will not receive full credits. No result can be directly taken from the notes or elsewhere. Submit your code (commented) as well.