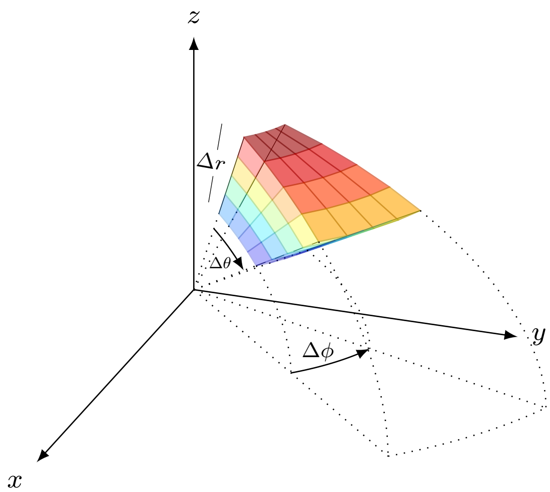
# Homework 2

Due date: someday, sometime!
Submit on NYU Brightspace.

**Spherical Coordinates:** The spherical coordinate system uses a distance $r$ from the origin along with two angles $\theta$ and $\phi$ to uniquely locate a point in space. An elemental volume in spherical coordinates is illustrated below:




The radial distance $r$ is measured from the origin; the angle $\phi \in [0, 2\pi)$ varies in the $xy$ plane and is zero on the $x$ axis; the angle $\theta \in [0, \pi]$ measures the deviation from the $z$ axis. The projected radius onto the $xy$ plane is $r\sin(\theta)$. The radii of curvature for angles $\theta$ and $\phi$ are $r$ and $r\sin(\theta)$, respectively. Therefore, we have the following formula for the volume of the element, $V$, and area in different directions (treating the elemental volume as a rectangular box for small $\Delta r, \Delta\theta, \Delta\phi$):

$$
\begin{aligned}
V &= (\Delta r) \cdot (r\Delta\theta) \cdot (r\sin(\theta)\Delta\phi) = r^2\sin(\theta)\Delta r\Delta\theta\Delta\phi, \\
A_r &= (r\Delta\theta) \cdot (r\sin(\theta)\Delta\phi) = r^2\sin(\theta)\Delta\theta\Delta\phi, \\
A_\theta &= (\Delta r) \cdot (r\sin(\theta)\Delta\phi) = r\sin(\theta)\Delta r\Delta\phi, \\
A_\phi &= (\Delta r) \cdot (r\Delta\theta) = r\Delta r\Delta\theta.
\end{aligned}
$$

Here, $A_r,\ A_\theta,\ A_\phi$ denote the area in the $r,\ \theta,\ \phi$ directions.

**Exercise 1.** [50 pts] The mass flux (mass per unit area per unit time) can be expressed as

$$\rho \mathbf{u} = \rho u_r \hat{e}_r + \rho u_\theta \hat{e}_\theta + \rho u_\phi \hat{e}_\phi,$$

where $\rho$ is density of the material and $\mathbf{u}$ is the vector of velocity with scalar components $u_r$, $u_\theta$, $u_\phi$ in $r$, $\theta$, $\phi$ directions, respectively.

Write a shell balance for the elemental volume to derive the differential form of the continuity equation in spherical coordinates. Next, simplify your answer for constant $\rho$.

**Exercise 2.** [50 pts] The heat flux (thermal energy per unit area per unit time) can be expressed as

$$\rho \mathbf{u} C_p T - k \nabla T,$$

where the new parameters $C_p$, $k$, $T$ are the specific heat capacity, thermal conductivity, and temperature, respectively, and the gradient operator in spherical coordinates is

$$\nabla T = \frac{\partial T}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{e}_\theta + \frac{1}{r\sin(\theta)}\frac{\partial T}{\partial \phi}\hat{e}_\phi.$$

Write a shell balance (with a heat source $s$) for the elemental volume to derive the differential form of the heat equation in spherical coordinates. Next, simplify your answer for constant $\rho$, $C_p$, $k$.