Homework 1

Due date: someday, sometime!
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Exercise 1. [50 pts] We wish to use the Buckingham π theorem to estimate the drag force on an airplane cruising at 400 km/hr in standard air (atmospheric pressure) using a scaled 1 : 10 down model (geometrical dimensions of the model are 0.1 of that of the prototype). Let the air speed in the wind tunnel be 400 km/hr as well. Assuming the same air temperature for the model and prototype, determine the required pressure in the tunnel, and the drag on the prototype corresponding to 1 N on the model.

Hints:

(i) The drag force, F, is expected to depend on the following parameters: V (wind speed), μ (viscosity), ρ (density), ℓ (length of the airplane), and w (wing span of the airplane).

(ii) Assume that the air viscosity does not depend on the air pressure.

(iii) Use Ideal Gas Law $pMW = \rho RT$ to relate the air density to pressure. (p: pressure, R: universal gas constant, MW: molecular weight of air (constant), T: absolute temperature)

Exercise 2. [50 pts] From dimensional analysis, we know that the drag coefficient C_D of a smooth sphere moving in a fluid is a function of Reynolds number:

$$C_D = f(\mathrm{Re}) = f\left(\frac{\rho V D}{\mu}\right),$$

where ρ and μ are the density and viscosity of the fluid (e.g., air), D is the diameter of the sphere, and V is its velocity. The drag force can then be expressed as

$$F = \tfrac{1}{2} C_D \rho A V^2$$

with A as the projected area. Note that you can derive the Stokes drag force formula (valid when $\mathrm{Re} \lesssim 1$) from the above equation by inserting $C_D = 24/\mathrm{Re}$.

Now consider two raindrops, one with diameter 2 mm and another one with diameter 4 mm. Which one falls faster in the air and by how much?

Hints:

(i) Assume that the drops fall at their 'terminal' velocities, which is the velocity at which the drag force and gravitational forces cancel each other: $F = \frac{1}{6}\pi D^3(\rho_s - \rho)g$, where $\frac{1}{6}\pi D^3$ is the volume of the droplet, ρ_s is its density (≈ 1000 kg/m^3), and $g \approx 9.8$ m/s^2 is the gravitational acceleration. Also, for the air, use $\rho = 1$ kg/m^3 and $\mu = 2\times10^{-5}$ Pa $\cdot$ s.

(ii) Initially assume that the drops fall in the Stokes regime (i.e., $\mathrm{Re} \lesssim 1$) and use the Stokes drag formula. After finding the terminal velocity calculate the Re number to verify your initial assumption.

(iii) In case the Stokes regime is not valid: for high Reynolds number $10^3 \lesssim \mathrm{Re} \lesssim 10^5$, one can assume that C_D is constant and not a function of Re. Use $C_D \approx 0.4$ and redo the above process. Verify if the Reynolds number is within the range.

Exercise 3. [10+10+10+10 pts] Heat transfer flux from a surface at temperature T_s to an environment at temperature T_∞ is given by $h(T_s - T_\infty)$, where h is the heat transfer coefficient with dimensions energy/(time $\cdot$ area $\cdot$ temperature). For a sphere suspended in a still air, the heat transfer coefficient depends on the sphere diameter, D, and conductivity of the air, k, (with dimensions energy/(time $\cdot$ length $\cdot$ temperature)). Hence, one can write $h = f(k, D)$.

(i) List all of the parameters and their dimensions. Hint: energy $\doteq$ force $\cdot$ length.

(ii) Find the number of *reference* dimensions, and subsequently, the number of dimensionless groups.

(iii) Find the dimensionless groups using the method described in class.

(iv) Interpret the result.