

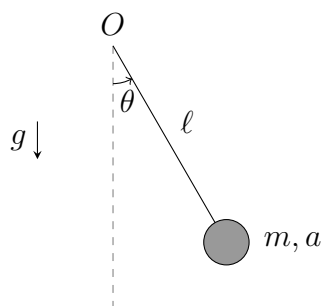
## Homework 8

Due date: someday, sometime!

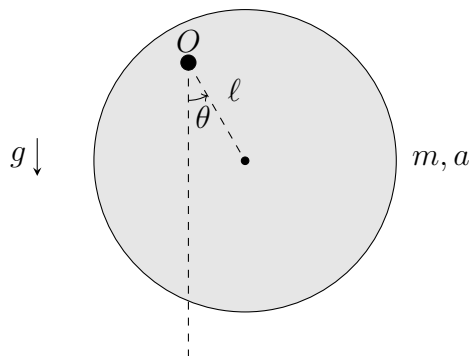
Submit on NYU Brightspace.

**Exercise 1.** [35 pts] Consider a spherical pendulum of mass  $m$  and radius  $a$  connected to an anchor with a weight-less rod of length  $\ell$  (cf. Figure below). Note that the dimensions of the sphere matter (not a simple point particle) and the total distance between the center of the sphere to the anchor point is  $a + \ell$ .

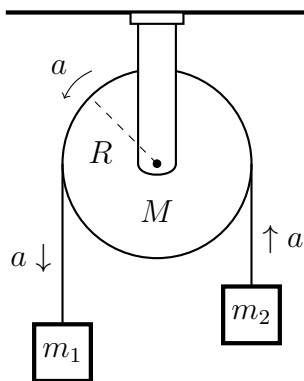
In class, we have calculated  $I_0$ , the moment of inertia for a sphere about an axis passing through its center of mass. Use the parallel axis theorem to find the moment of inertia about the anchor point. The only torque on the sphere around the anchor point is due to gravity. Use this torque and the definition of angular momentum to find the equation of motion for this pendulum. Show that the obtained differential equation simplifies to that of a simple pendulum (with a point mass) as  $a/\ell \rightarrow 0$ .



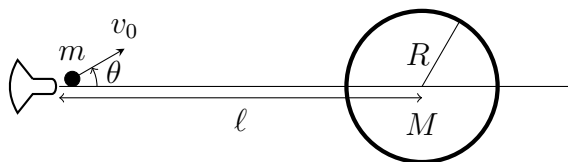
**Exercise 2.** [30 pts] Consider a physical pendulum as shown in figure below. A thin disk of mass  $m$  and radius  $a$  is anchored at point  $O$  to an axis ( $z$ , perpendicular to the plane) at a distance  $\ell$  away from its center of mass. (Imagine that a thin stick has pierced the disk at point  $O$ .) The disk can freely rotate about the anchor point (no friction). Neglecting the mass(area) loss due to the anchor, derive the differential equation of motion for the disk.



**Exercise 3.** [35 pts] Consider a pulley system as shown in figure below. Assume that the rope is weightless and it does not slip on the pulley, and the effect of the massive pulley with mass  $M$  and radius  $R$  is **NOT** negligible. Find the acceleration  $a$  of the arrangement. Start by writing the equations of motion for  $m_1$  and  $m_2$ , and the pulley. Hint: you might find the yo-yo problem helpful!



**Exercise 4.** [will not be graded] A spaceship hangs motionless in space at a distance  $\ell$  from the center of a planet of mass  $M$  and radius  $R$ . The ship fires an instrument package of mass  $m$  with initial speed  $v_0$  as shown in the sketch below. Find a formula for the angle  $\theta$  for which the package will graze the surface of the planet. Ignore the gravitational pull from the spaceship. As a consistency check, discuss if your formula gives reasonable results when there is no gravitational pull from the planet.



**notes:** Show all of the steps of your work. Solutions without details of the work and interpretation of the results will not receive full credits. You need to justify all of your assumptions. Unless otherwise specified, no result can be directly taken from the notes or elsewhere.