

Unanticipated Possibilities for Legal Probabilism

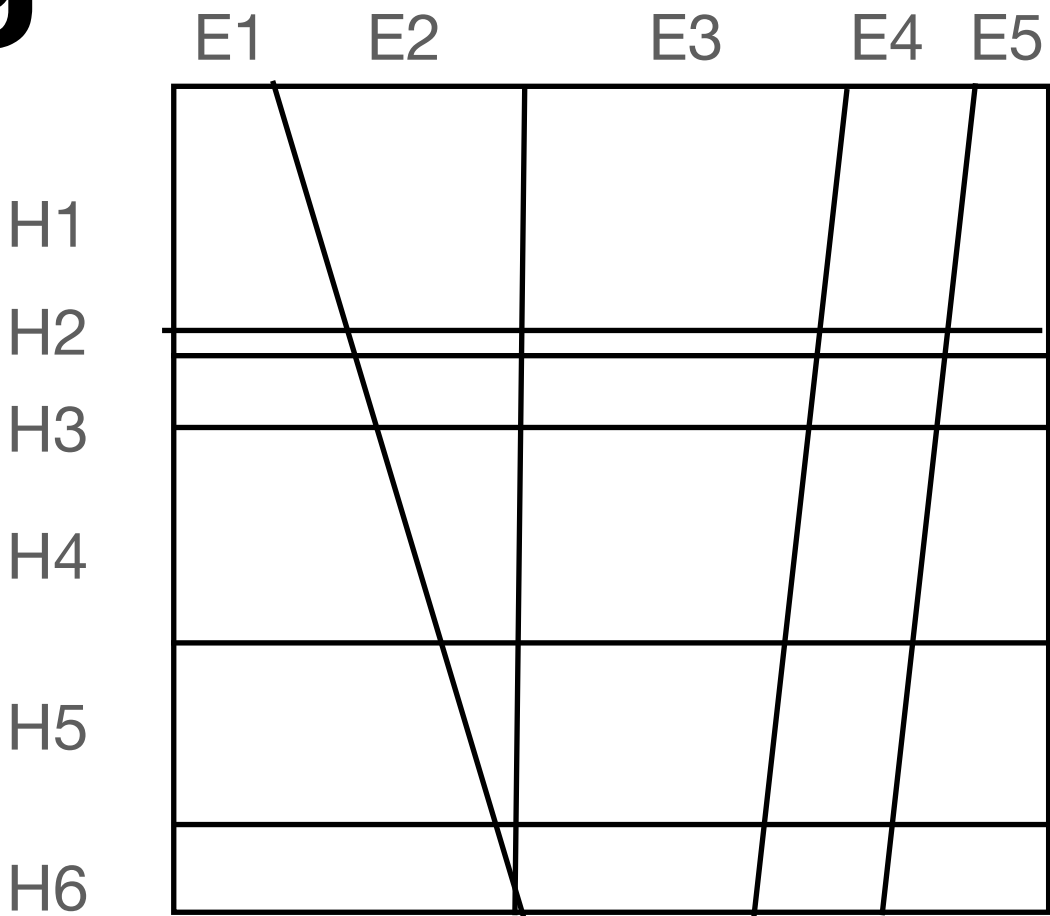
**Marcello Di Bello
Arizona State University
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Michele Taruffo Girona Evidence Week - May 27, 2022

Simple Bayesian Learning

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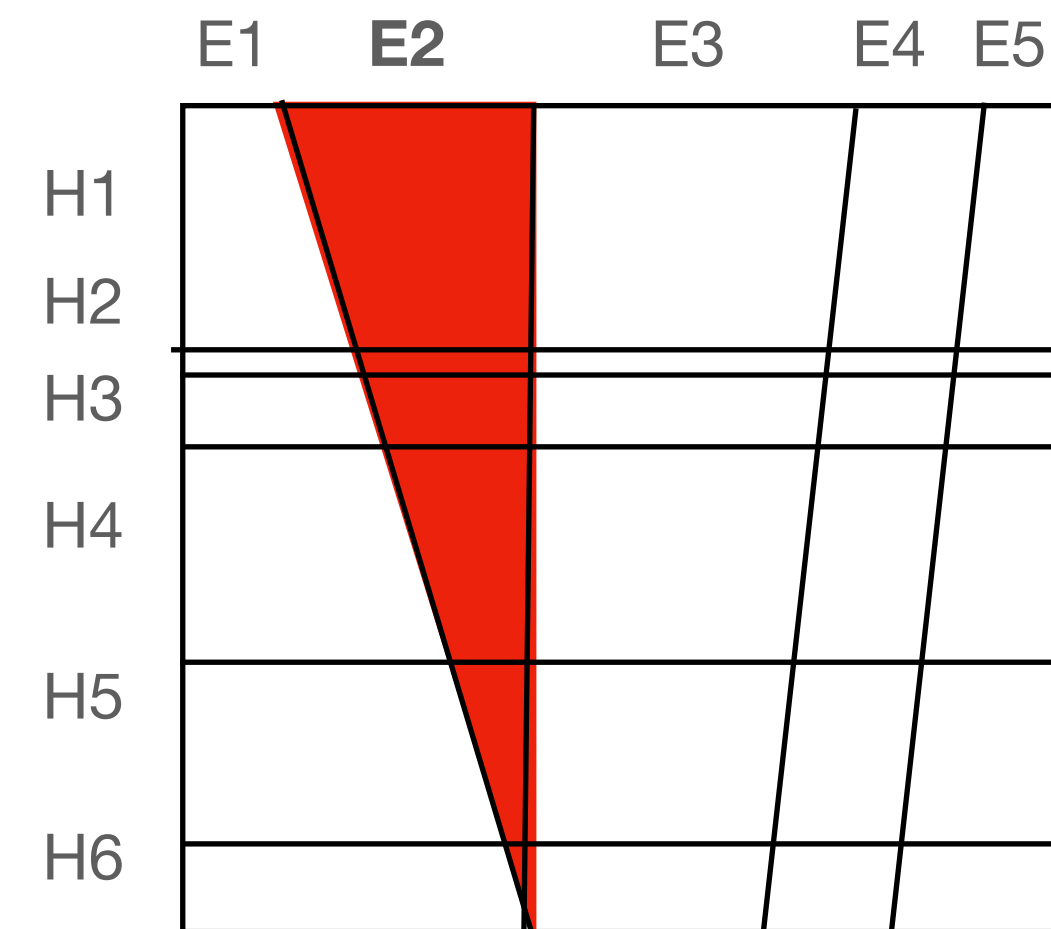
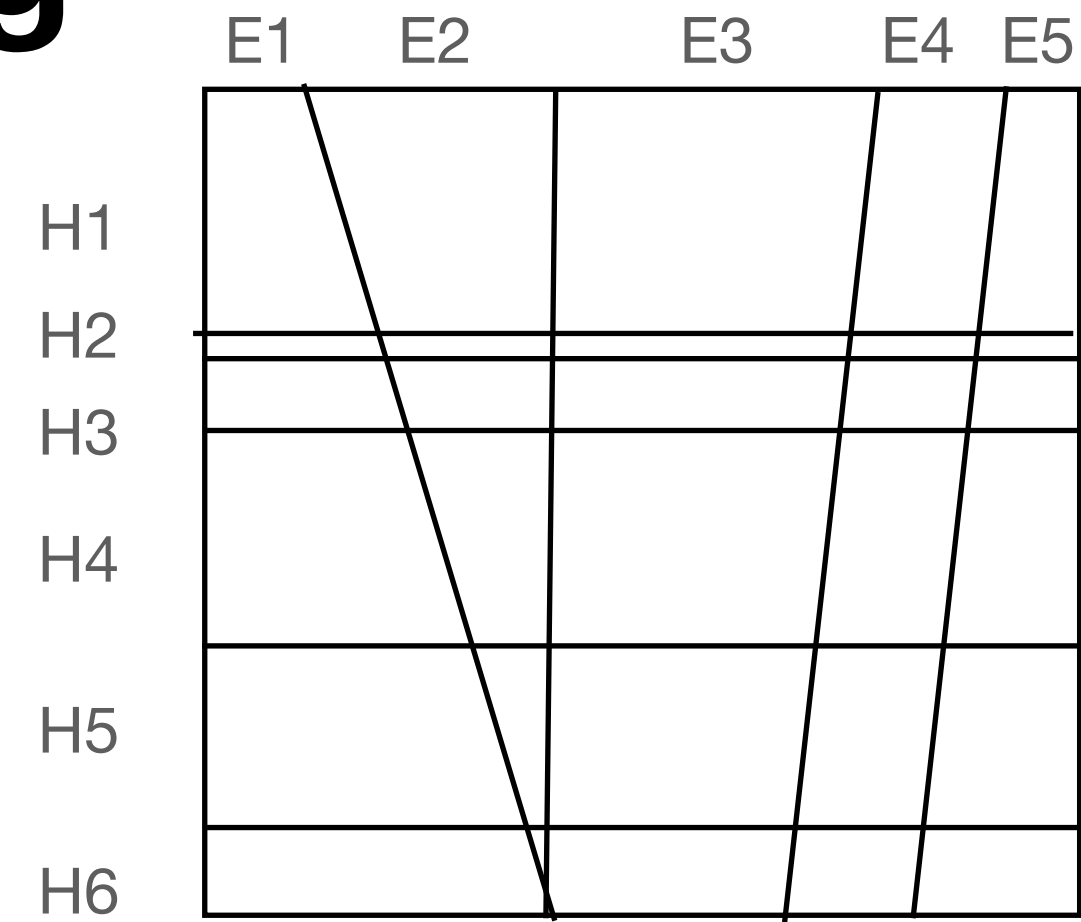
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Area approximates probability.



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You learn that the possibilities corresponding to **E2** are true. Everything else is ruled out.

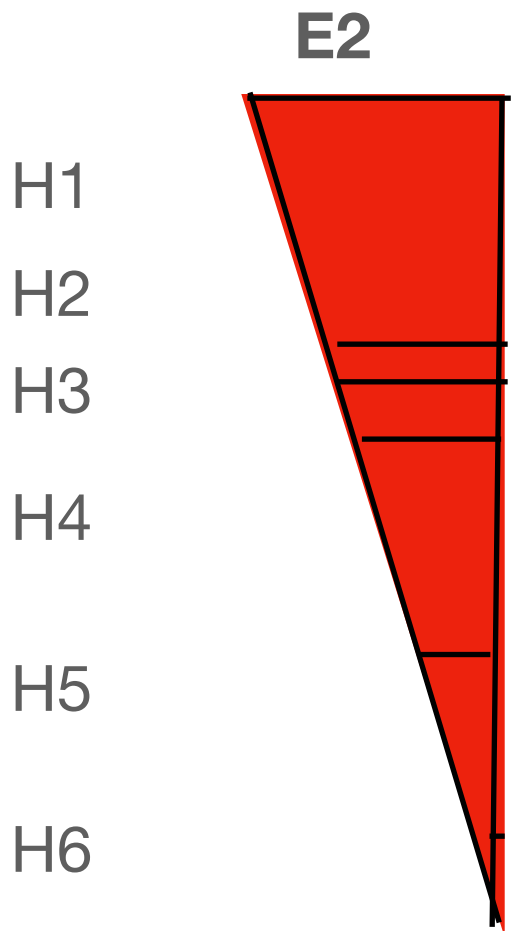
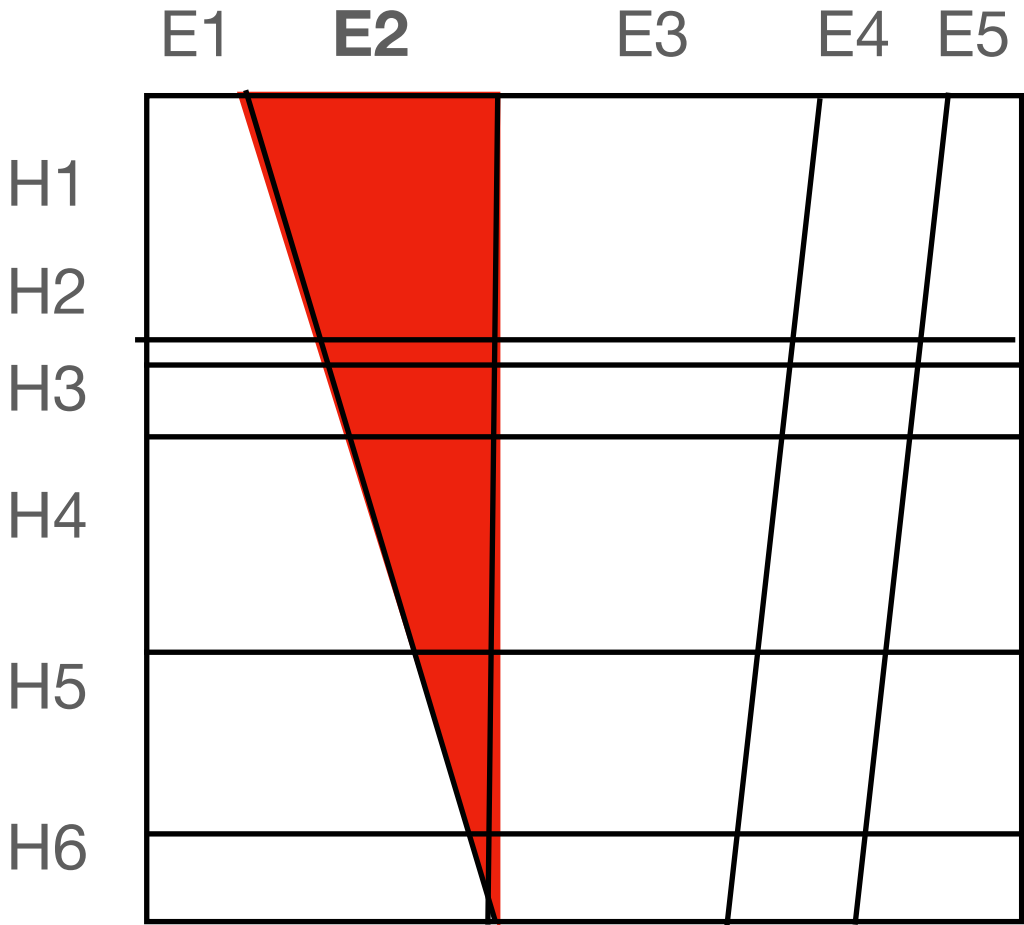
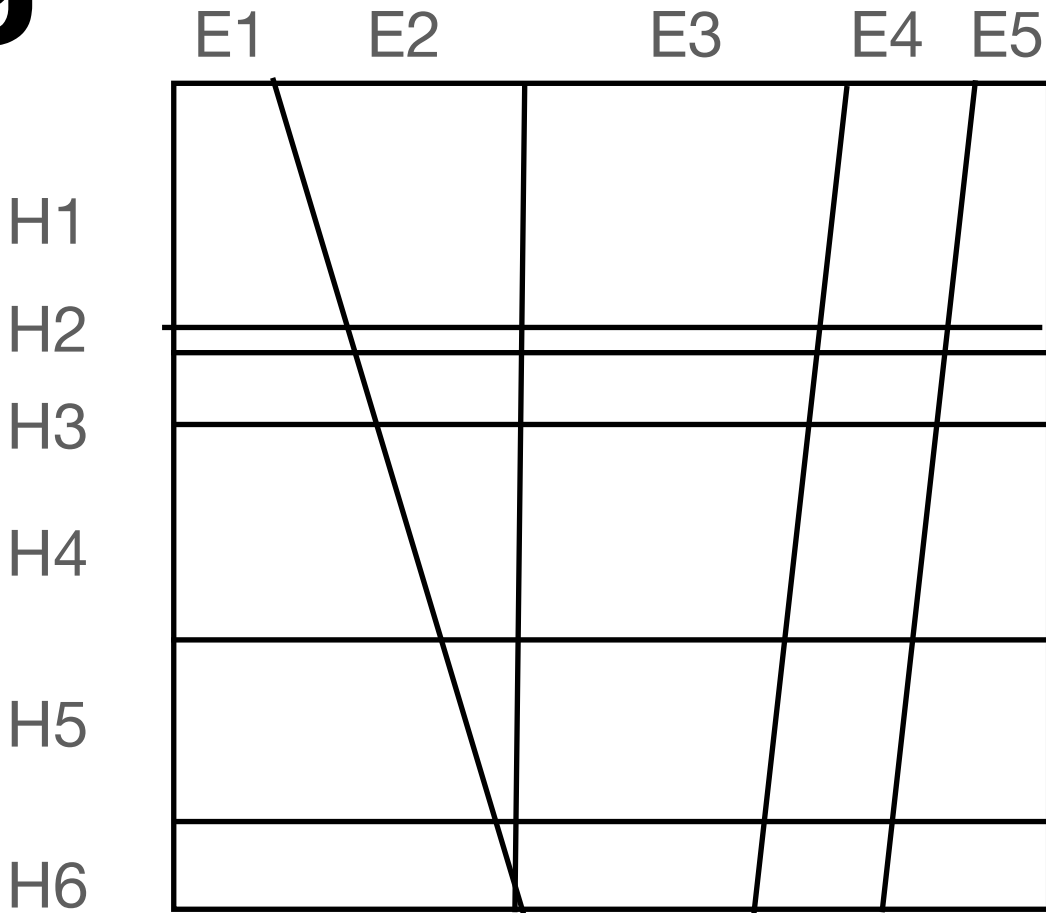


Simple Bayesian Learning

Algebra of propositions is fixed: E1, E2....., H1, H2, and Boolean combinations.
Area approximates probability.

You learn that the possibilities corresponding to **E2** are true. Everything else is ruled out.

The probabilities of the hypothesis are then adjusted accordingly



Unanticipated Possibilities

Original
Possibilities

	E1	E2	E3	E4	E5
H1					
H2					
H3					
H4					
H5					
H6					

Unanticipated Possibilities

Original
Possibilities

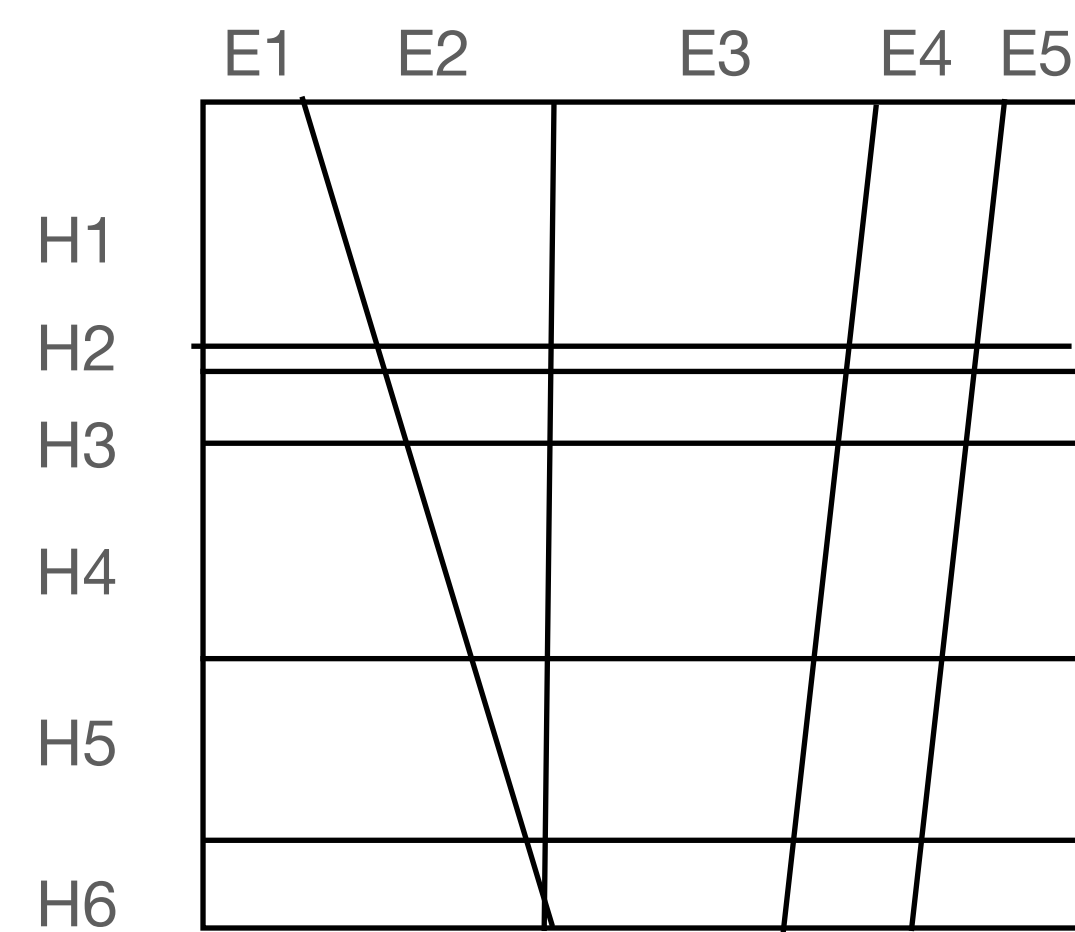
	E1	E2	E3	E4	E5
H1					
H2					
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H4					
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H6					

Refinement

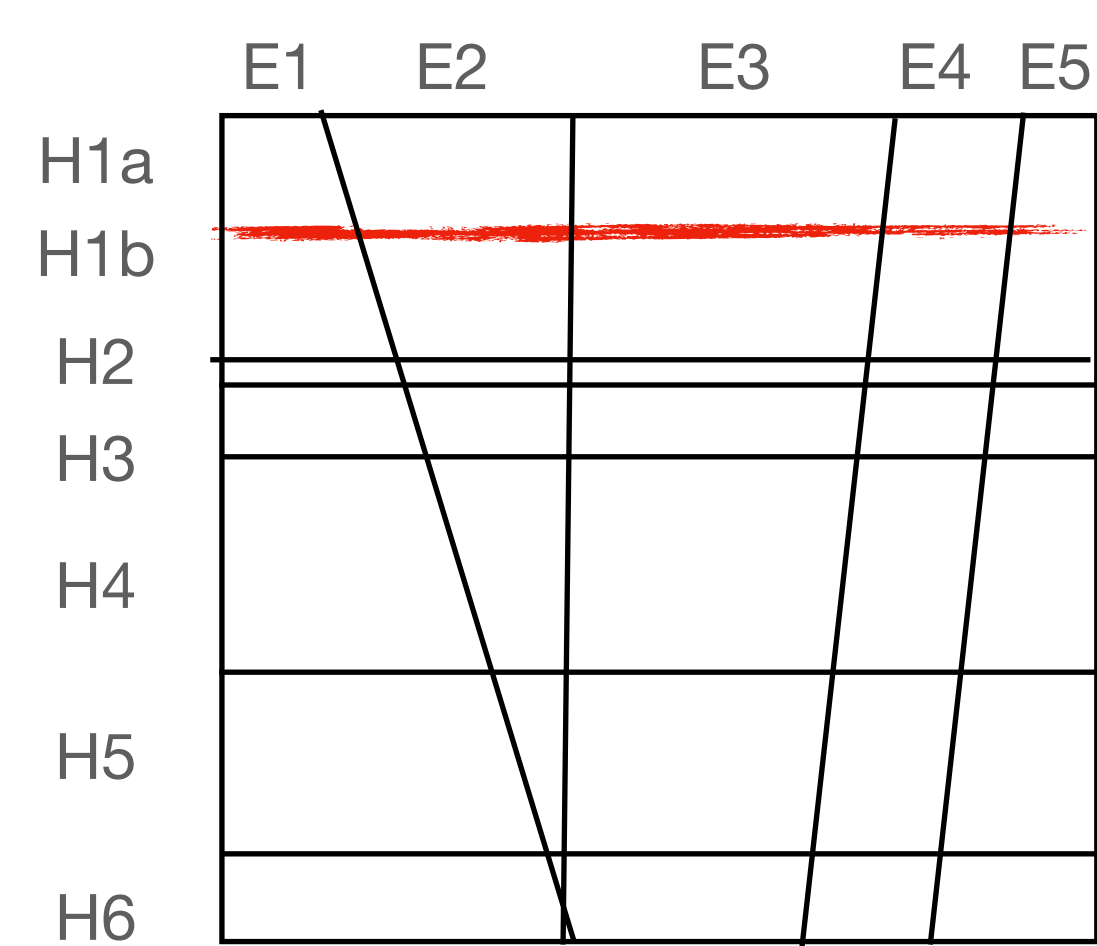
	E1	E2	E3	E4	E5
H1a					
H1b					
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Unanticipated Possibilities

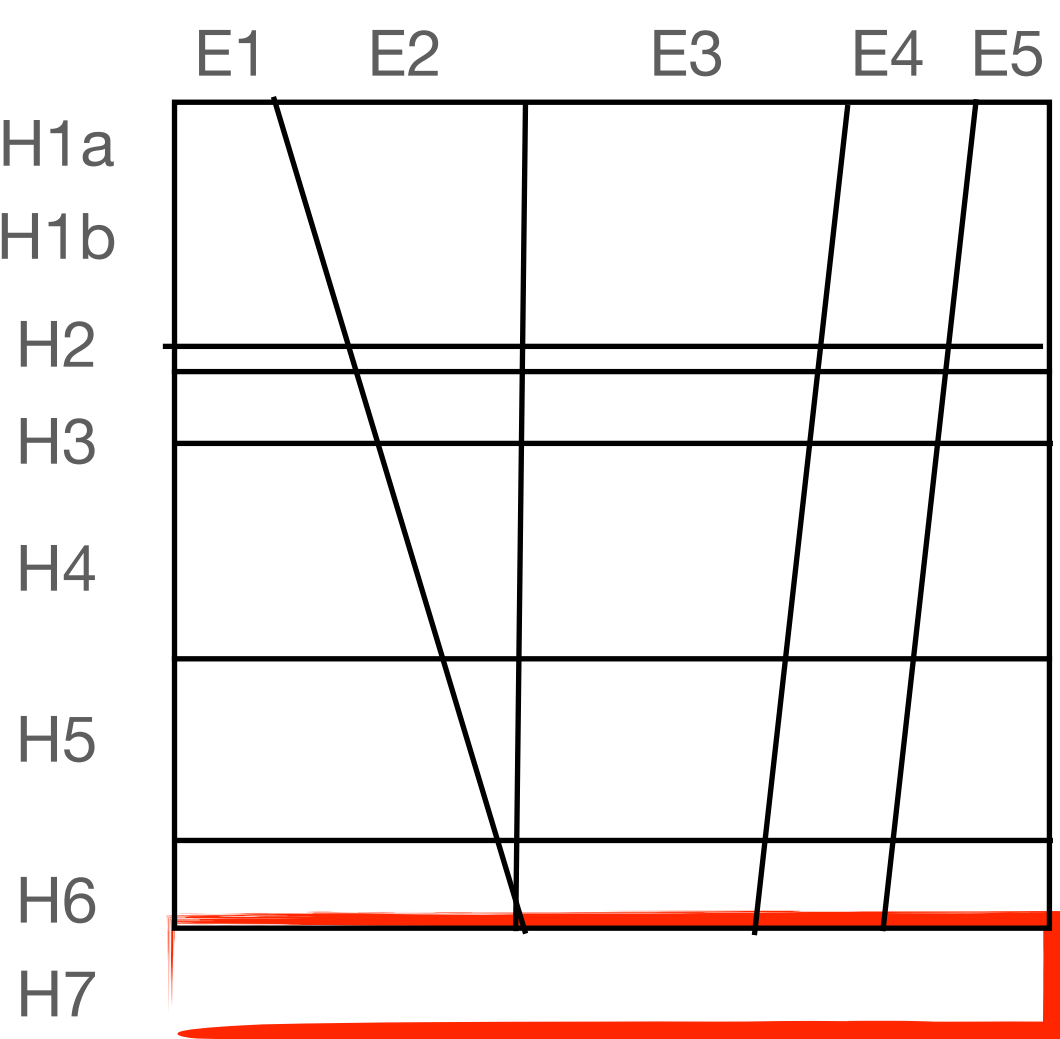
Original
Possibilities



Refinement



Expansion



Unanticipated Possibilities

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Possibilities

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Refinement

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H1b					
H2					
H3					
H4					
H5					
H6					

Expansion

	E1	E2	E3	E4	E5
H1a					
H1b					
H2					
H3					
H4					
H5					
H6					
H7					

Bayesian updating cannot handle cases of *refinement* or *expansion*. Both these situations add new propositions that were not in the original algebra.

Simple Bayesian Learning Cannot Model Learning in the Courtroom

Simple Bayesian Learning Cannot Model Learning in the Courtroom

...factual propositions are not set in stone prior to the trial...the litigation process is dynamic, not static, and frequently new theories will emerge. When they do, the probability space must be reconfigured.

It is thus pointless to configure the probability space ... prior to receiving all the evidence.

*RJ Allen (2017), "The nature of juridical proof: Probability as a tool in plausible reasoning",
Int. J. Evidence & Proof*

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Refinement

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The probability ratio between basic propositions (=those without Boolean connectives) in the original algebra is constant through refinement and expansion. If A and B are basic propositions, then

$$P_0(A)/P_0(B) = P_+(A)/P_+(B)$$

Reverse Bayesianism

Original
Possibilities

	E1	E2	E3	E4	E5
H1					
H2					
H3					
H4					
H5					
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Refinement

	E1	E2	E3	E4	E5
H1a					
H1b					
H2					
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H4					
H5					
H6					

Expansion

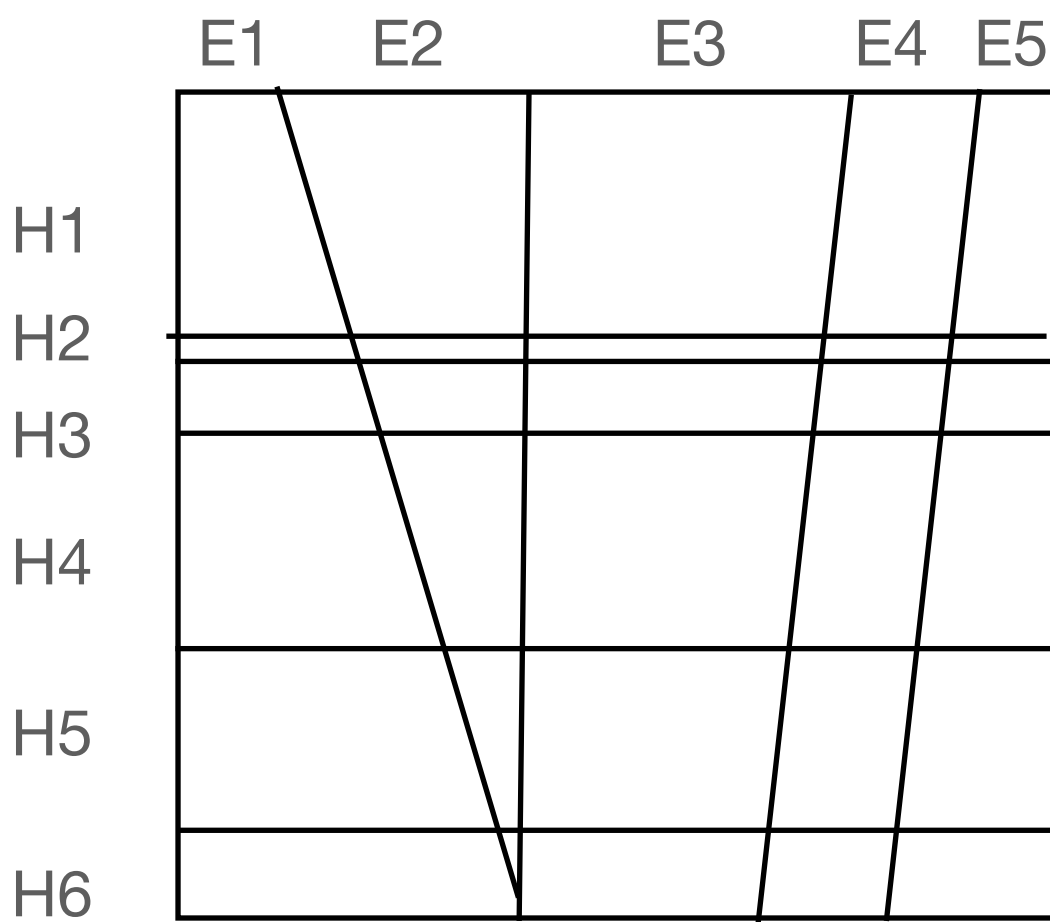
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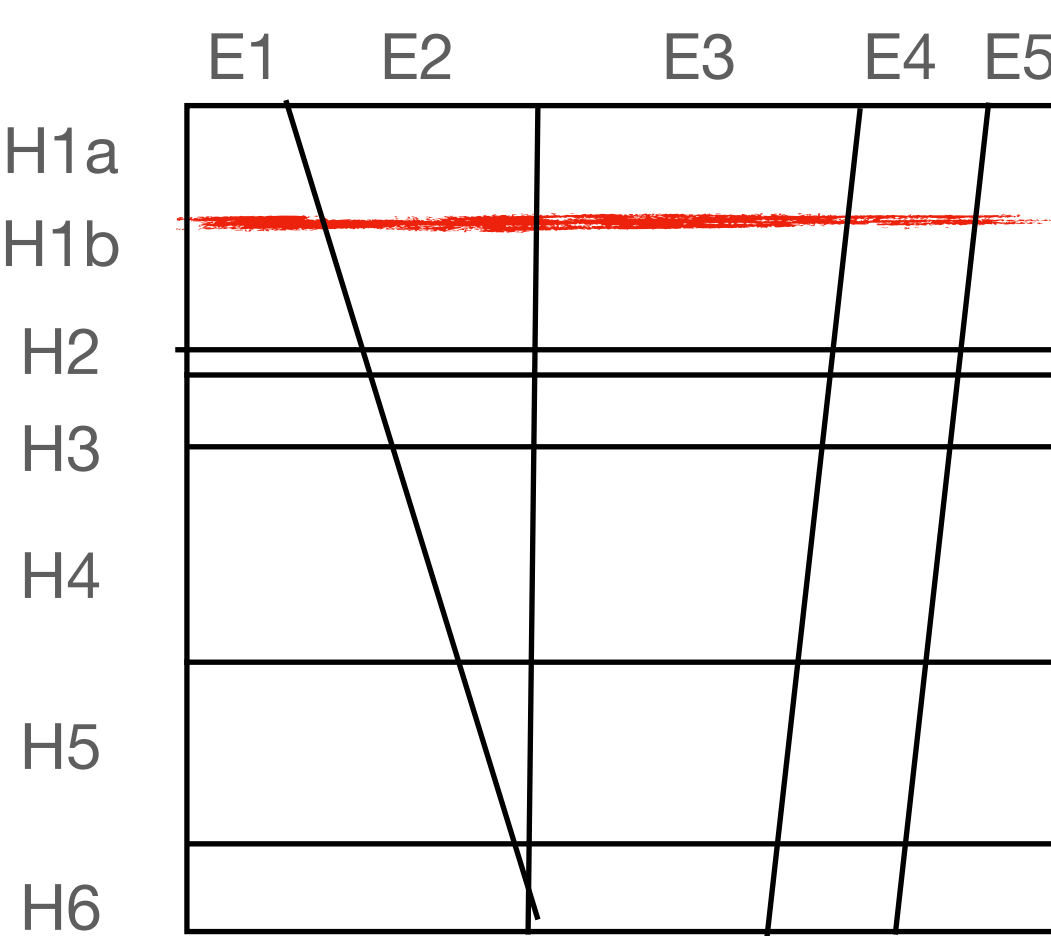
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Reverse Bayesianism

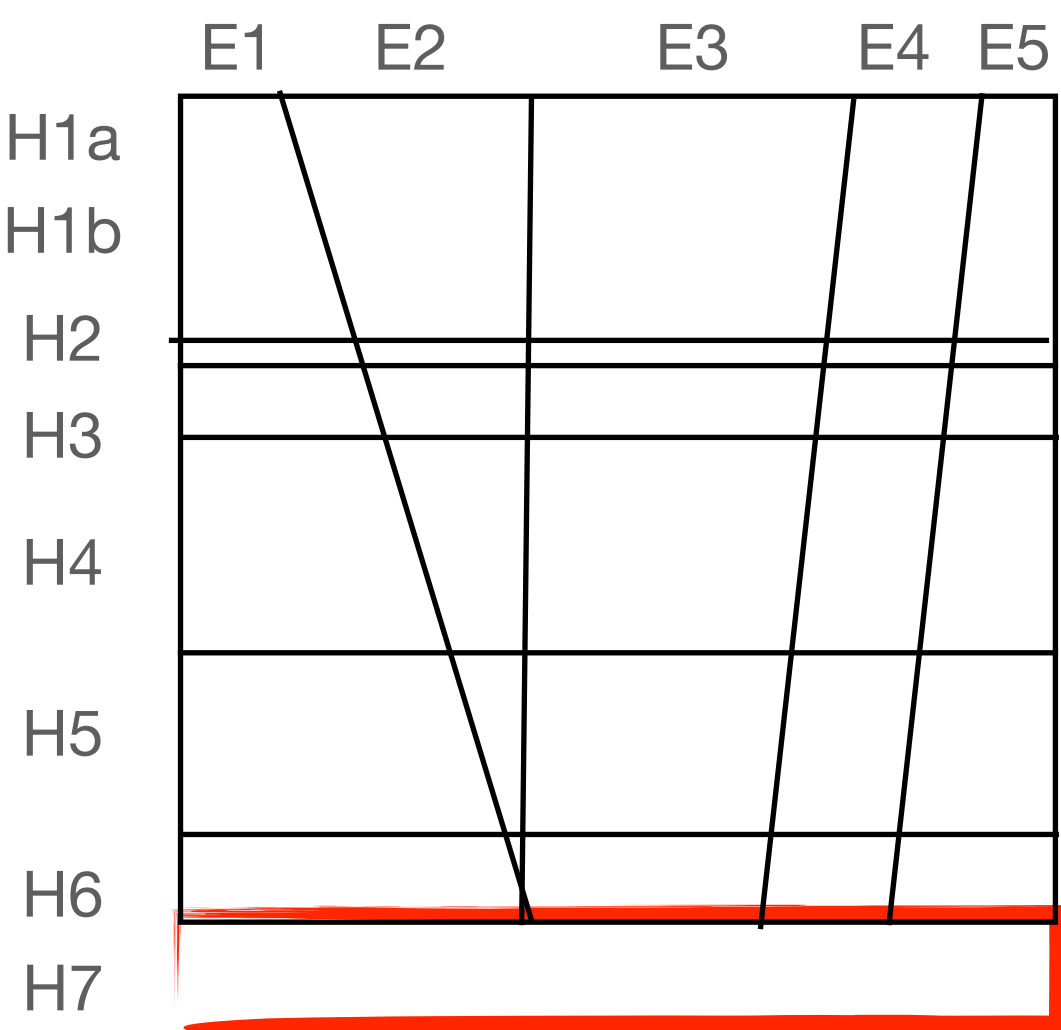
Original
Possibilities



Refinement



Expansion



The probability ratio between basic propositions (=those without Boolean connectives) in the original algebra is constant through refinement and expansion. If A and B are basic propositions, then

$$P_0(A)/P_0(B) = P_+(A)/P_+(B)$$

Reverse Bayesianism is an attempt to define a bridge between old and new probability function, so that as much of the old probability function is retained

A Counterexample to Reverse Bayesianism

Steele & Stefánsson (2021), Belief Revision for Growing Awareness, *Mind* 130 (520):1207–1232

A Counterexample to Reverse Bayesianism

“Suppose you happen to see your partner enter your best friend’s house on an evening when your partner had told you she would have to work late. At that point, you become convinced that your partner and best friend are having an affair, as opposed to their being warm friends or mere acquaintances.

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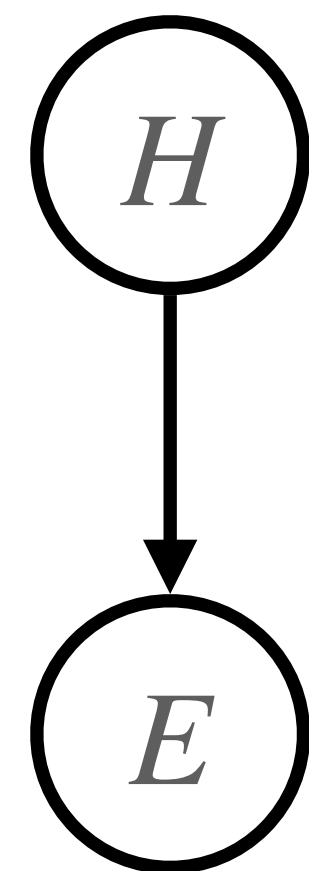
“Suppose you happen to see your partner enter your best friend’s house on an evening when your partner had told you she would have to work late. At that point, you become convinced that your partner and best friend are having an affair, as opposed to their being warm friends or mere acquaintances.

You discuss your suspicion with another friend of yours, who points out that perhaps they were meeting to plan a surprise party to celebrate your upcoming birthday—a possibility that you had not even entertained. Becoming aware of this possible explanation for your partner’s behaviour makes you doubt that she is having an affair with your friend, relative, for instance, to their being warm friends.”

Steele & Stefánsson (2021), Belief Revision for Growing Awareness, *Mind* 130 (520):1207–1232

Bayesian Network Representation

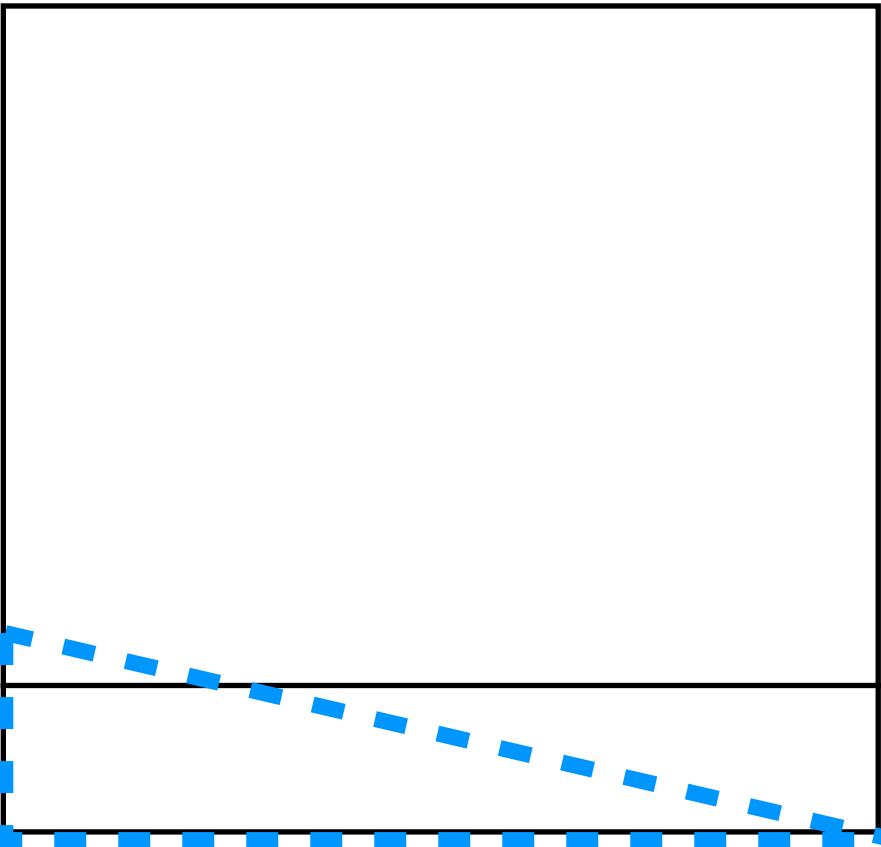
Bayesian Network Representation



$$P_0(E | H = fr) = .05$$

$$P_0(E | H = aff) = .7$$

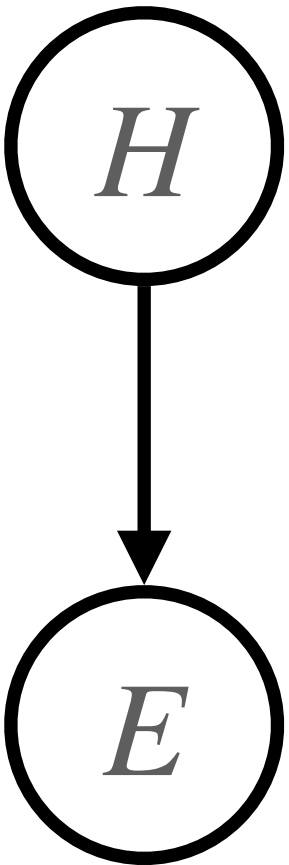
E



Friends or acquaintances

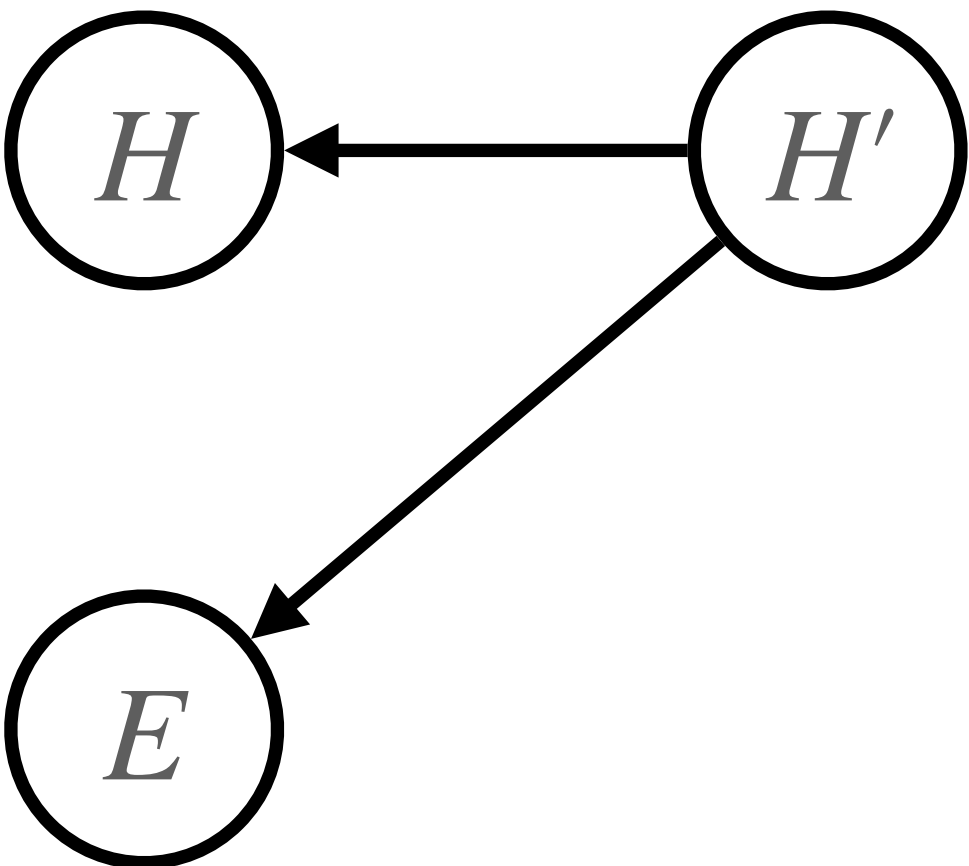
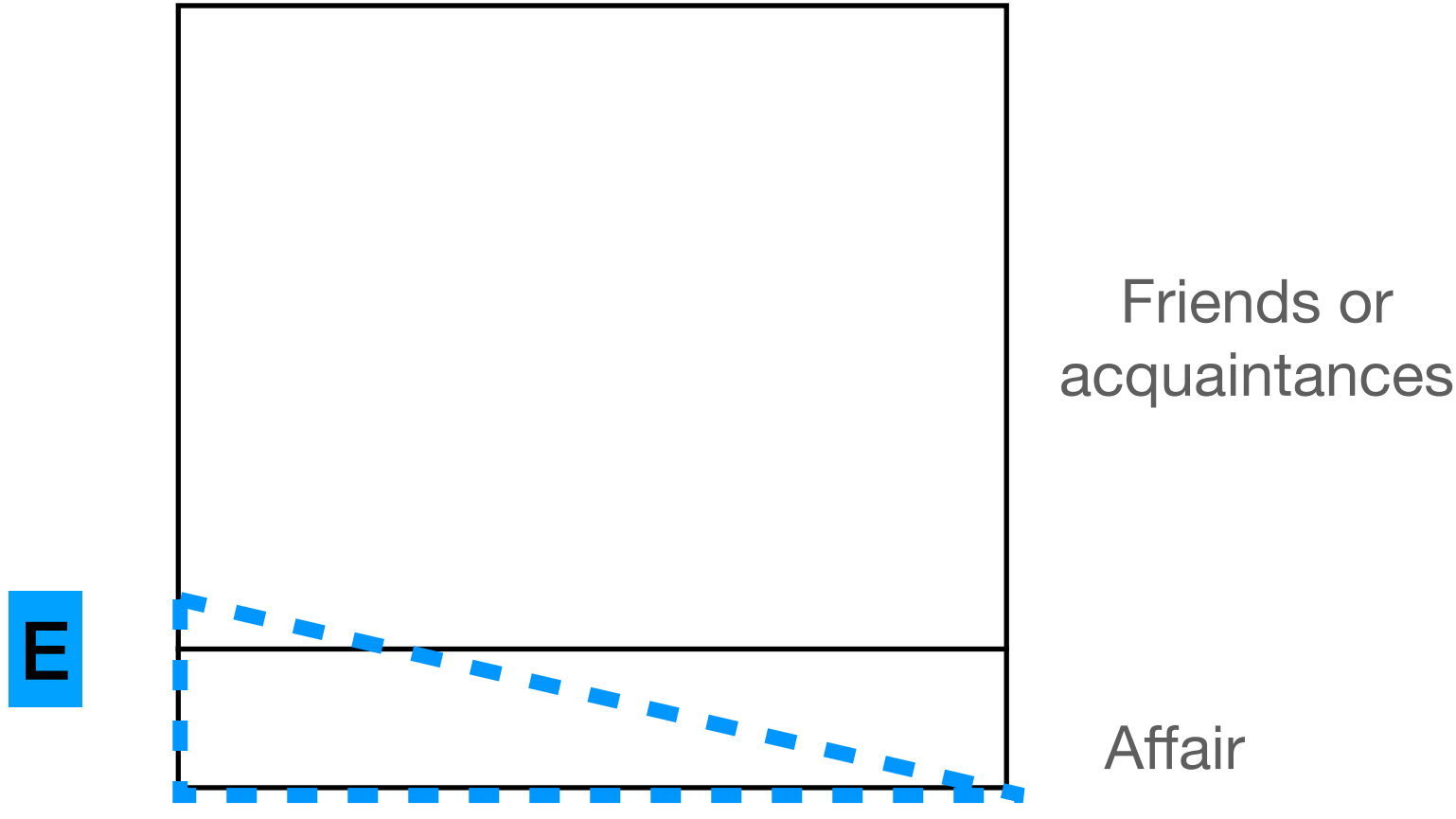
Affair

Bayesian Network Representation

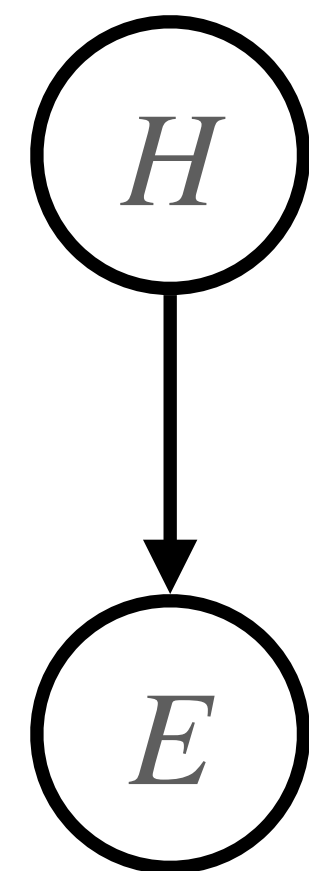


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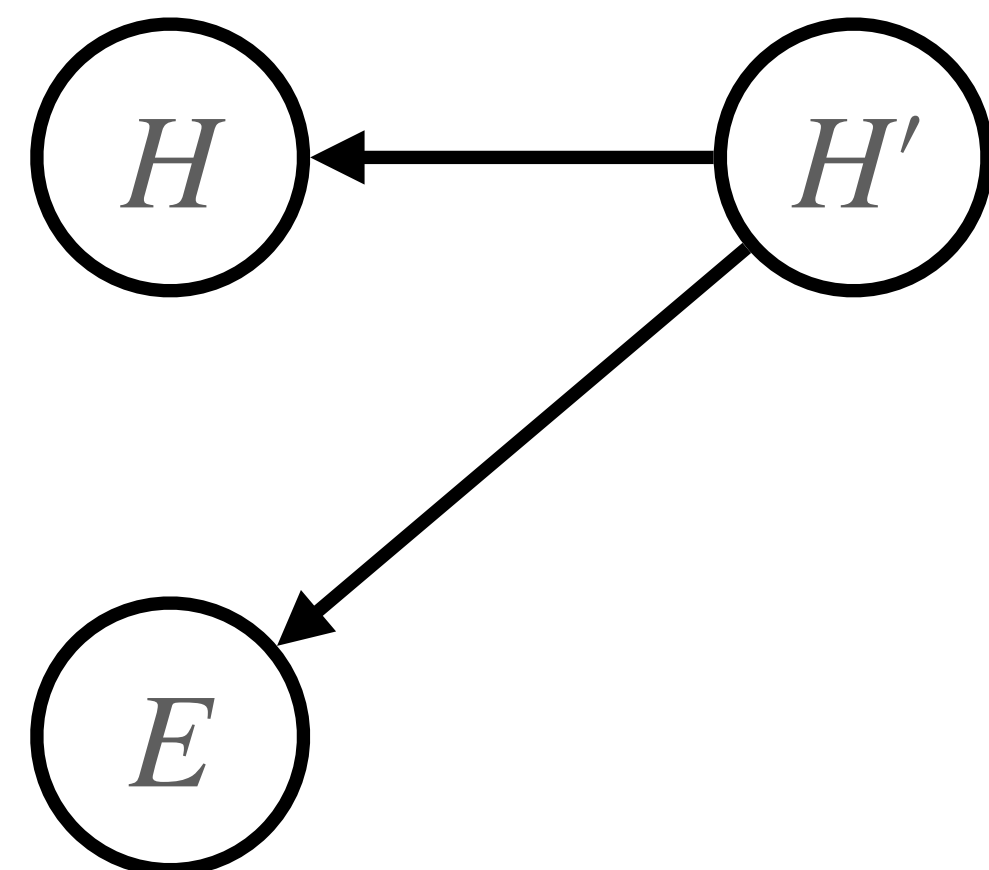
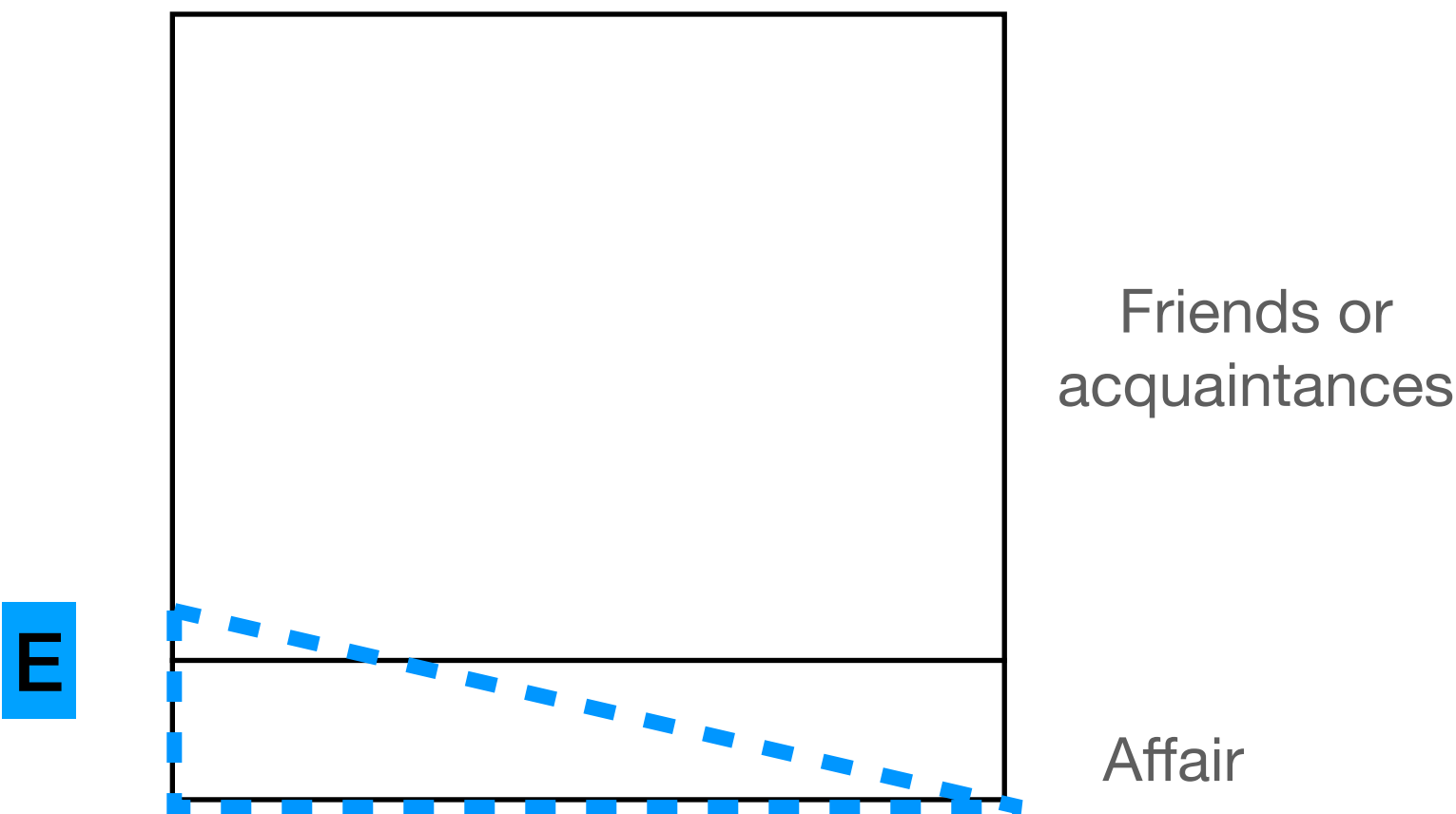
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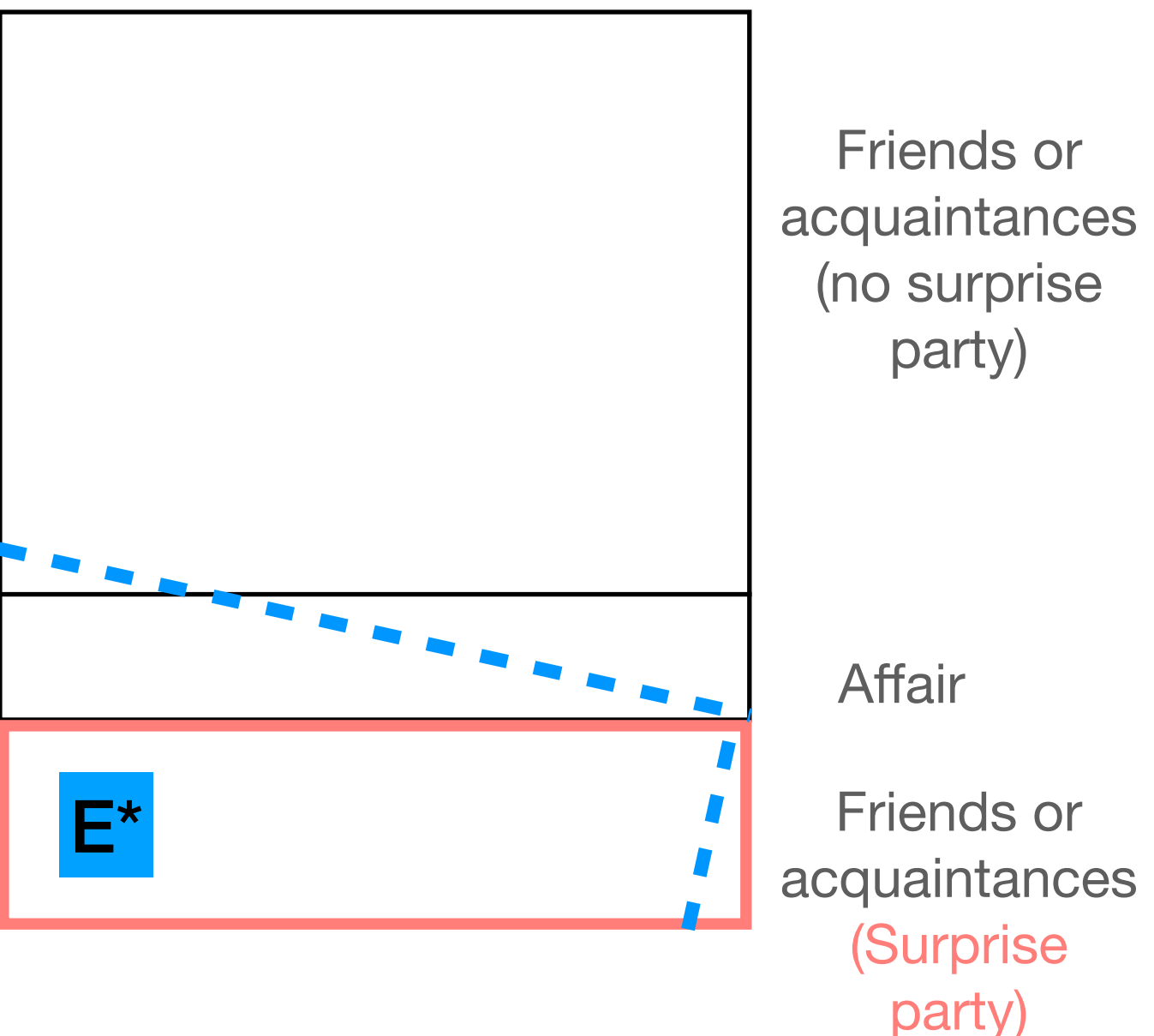
Bayesian Network Representation



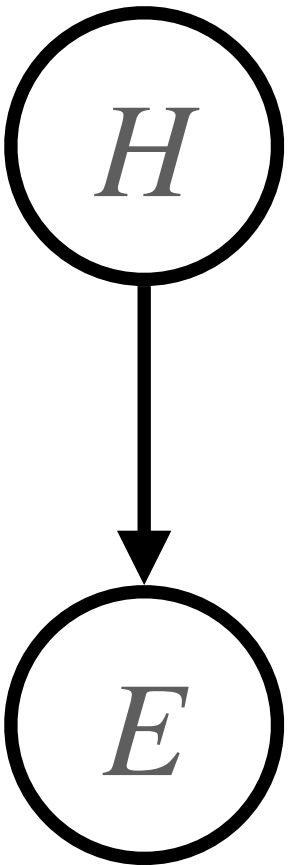
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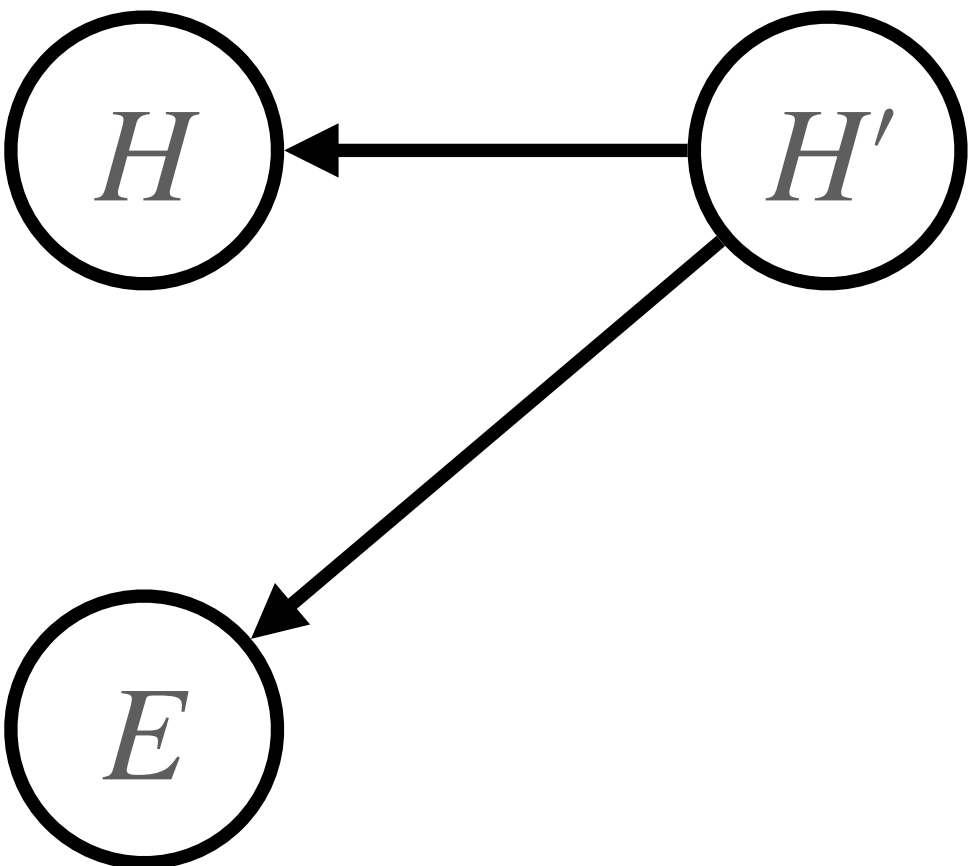
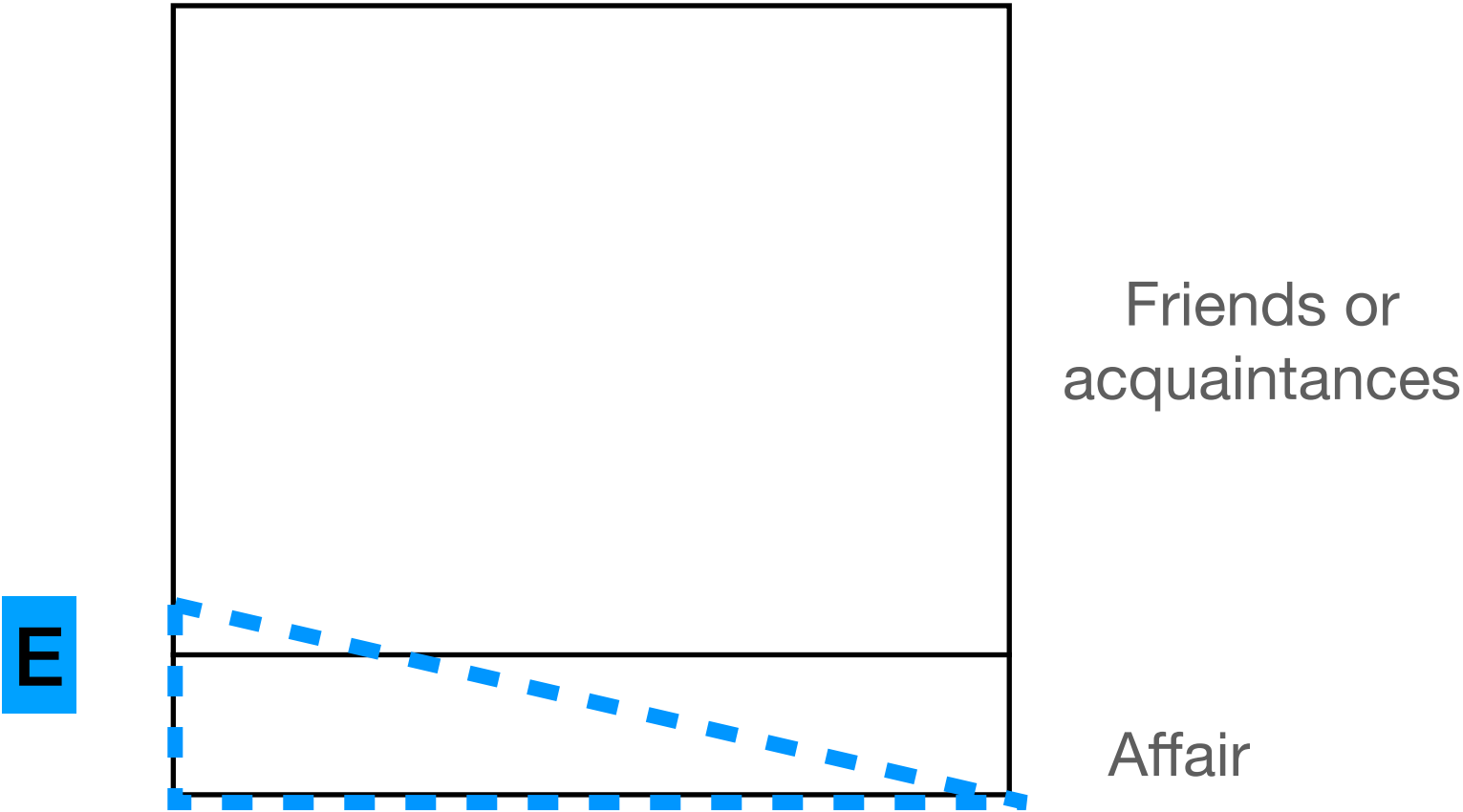
$$P_0(E | H = fr) = .05 = P_+(E | H' = fr \wedge \neg supr)$$
$$P_0(E | H = aff) = .7 = P_+(E | H' = aff \wedge \neg supr)$$
$$P_+(E | H' = supr) = .99$$



Bayesian Network Representation

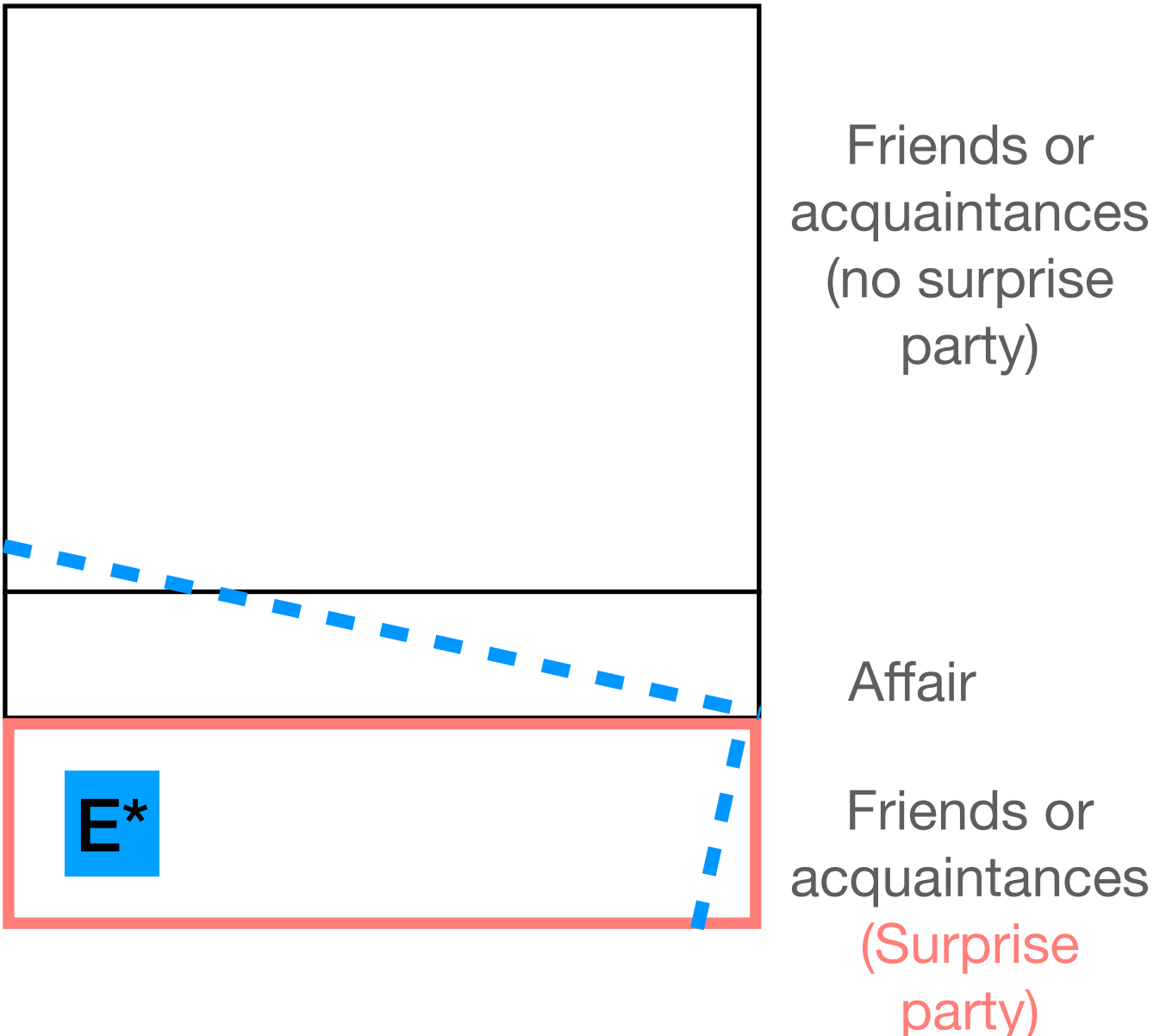


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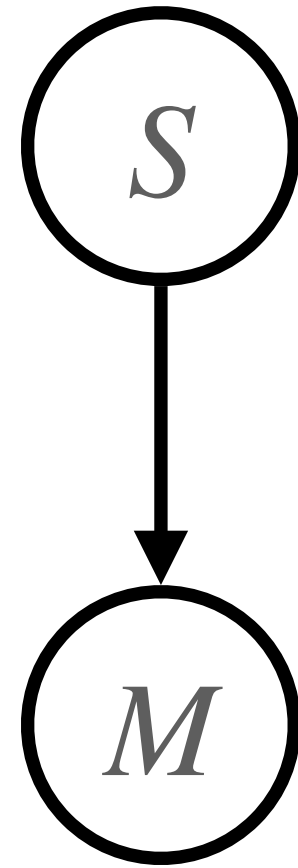
$$P_0(E | H = fr) = .05 = P_+(E | H' = fr \wedge \neg supr)$$
$$P_0(E | H = aff) = .7 = P_+(E | H' = aff \wedge \neg supr)$$
$$P_+(E | H' = supr) = .99$$

$$P_+(H = fr | H' = fr) = 1; P_+(H = fr | H' = aff) = 0$$
$$P_+(H = aff | H' = fr) = 0; P_+(H = aff | H' = aff) = 1$$
$$P_+(H = fr | H' = supr) = 1; P_+(H = aff | H' = supr) = 0;$$



“Downstream” Refinement

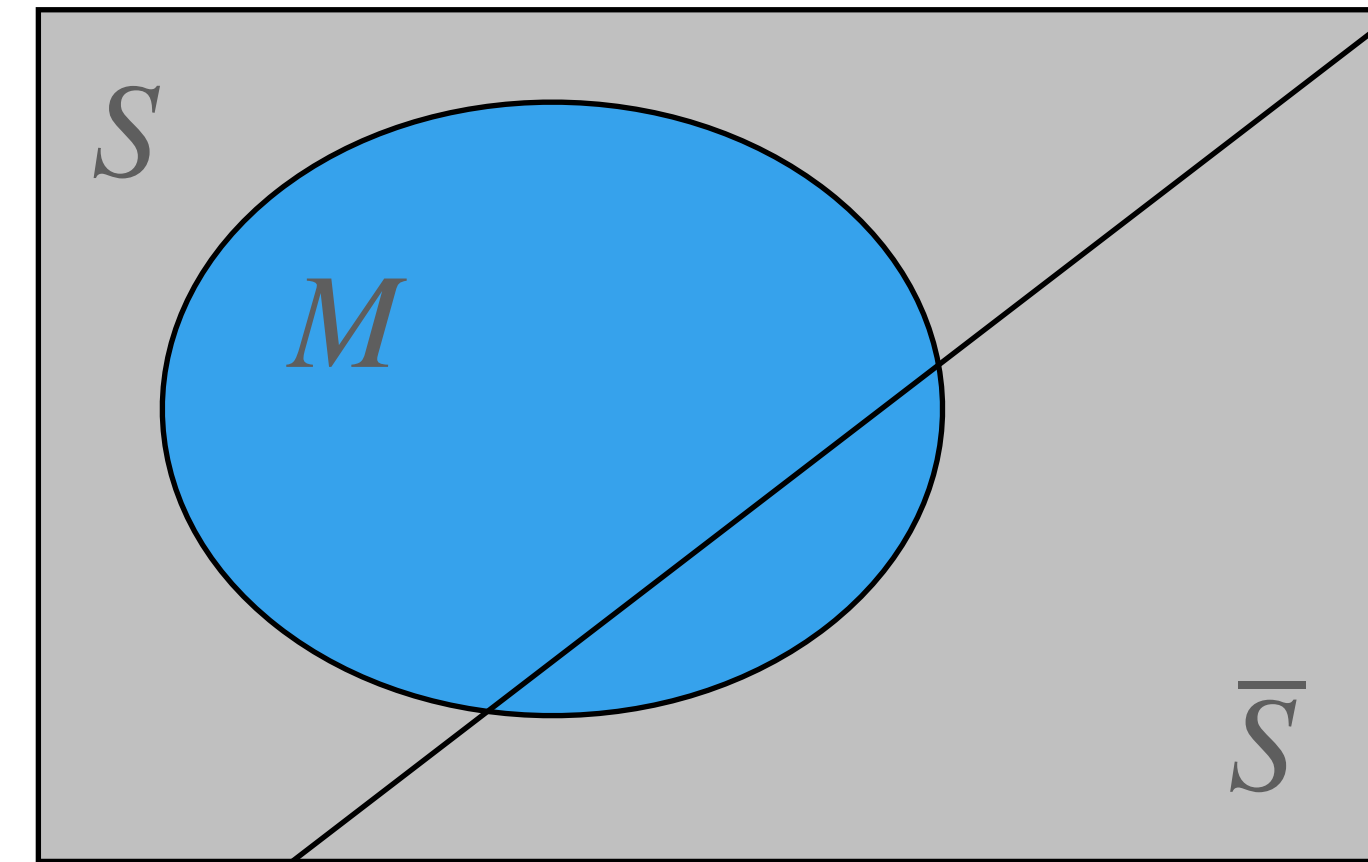
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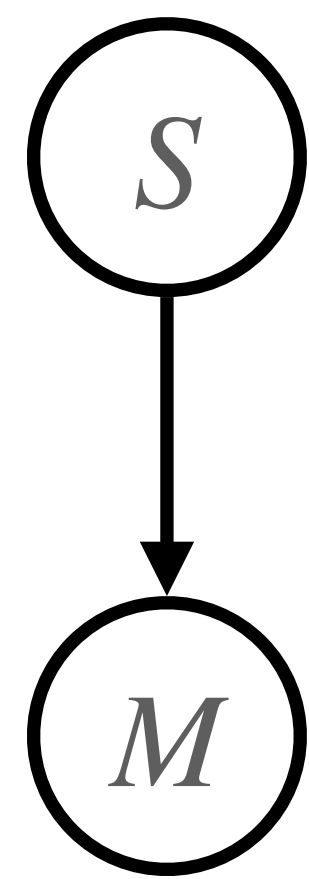
$$P_0(S)$$

$$P_0(M|S) = 1$$

$$P_0(M|\neg S) = RMP$$



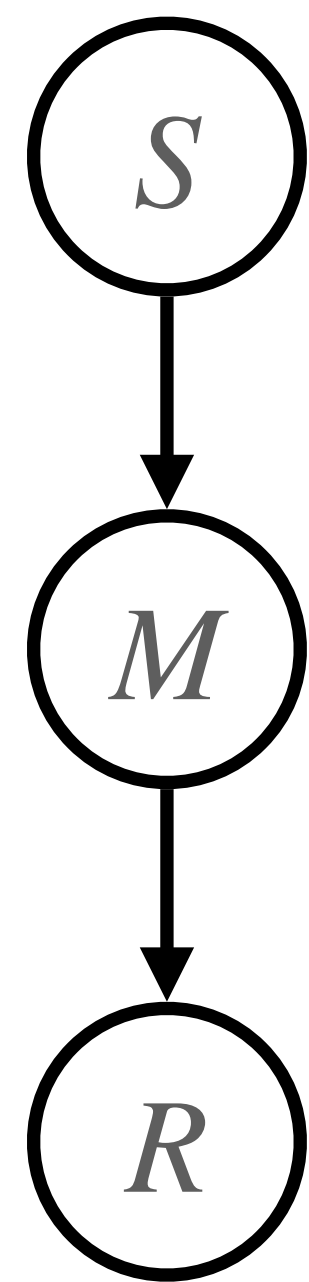
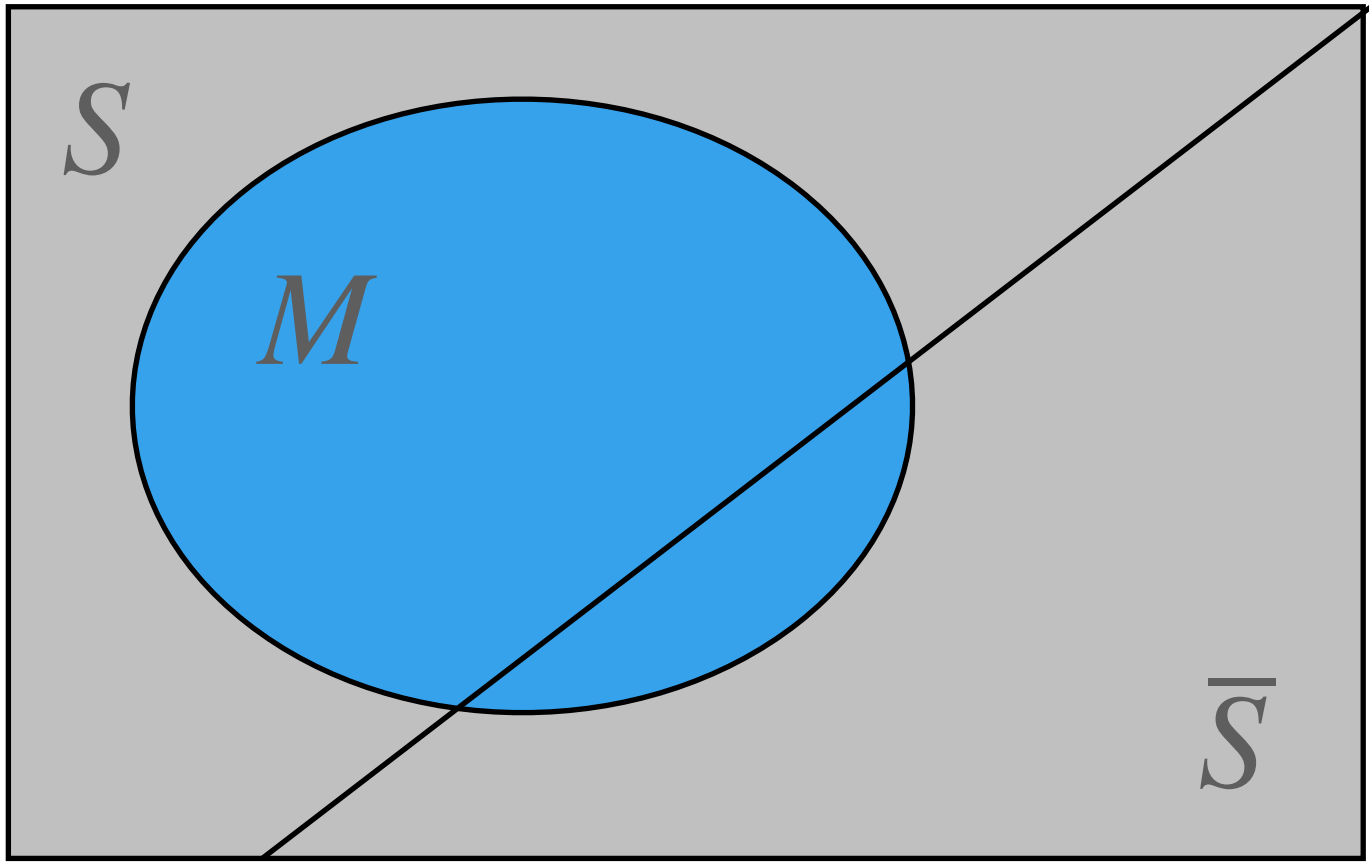
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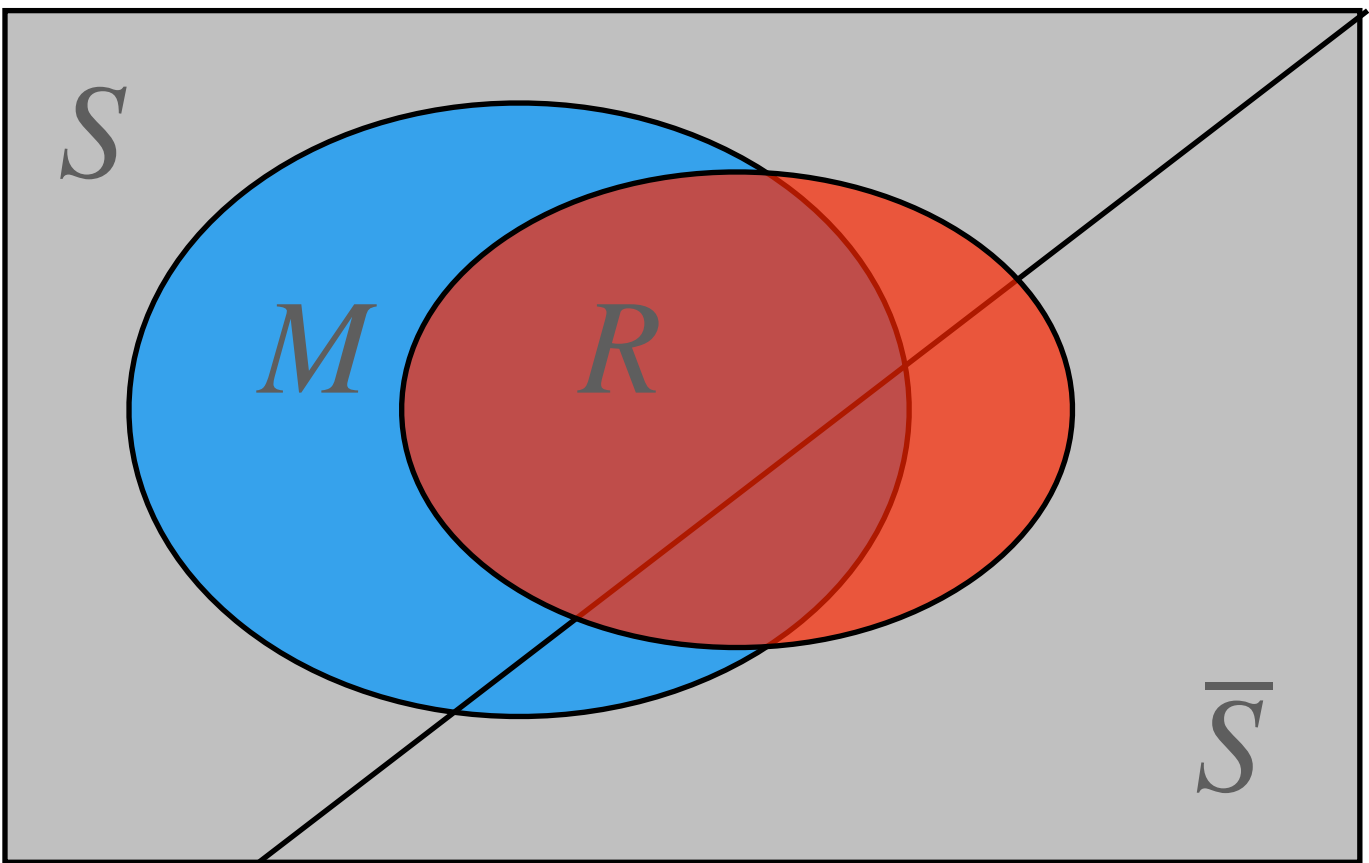
$$P_0(S) = P_+(S)$$

$$P_0(M|S) = P_+(M|S) = 1$$

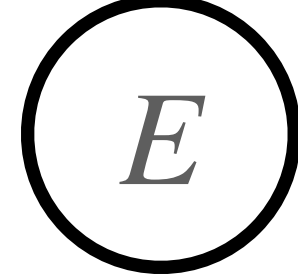
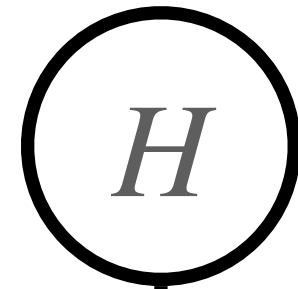
$$P_0(M|\neg S) = P_+(M|\neg S) = RMP$$

$$P_+(R|M) = TPP$$

$$P_+(R|\neg M) = FPP$$



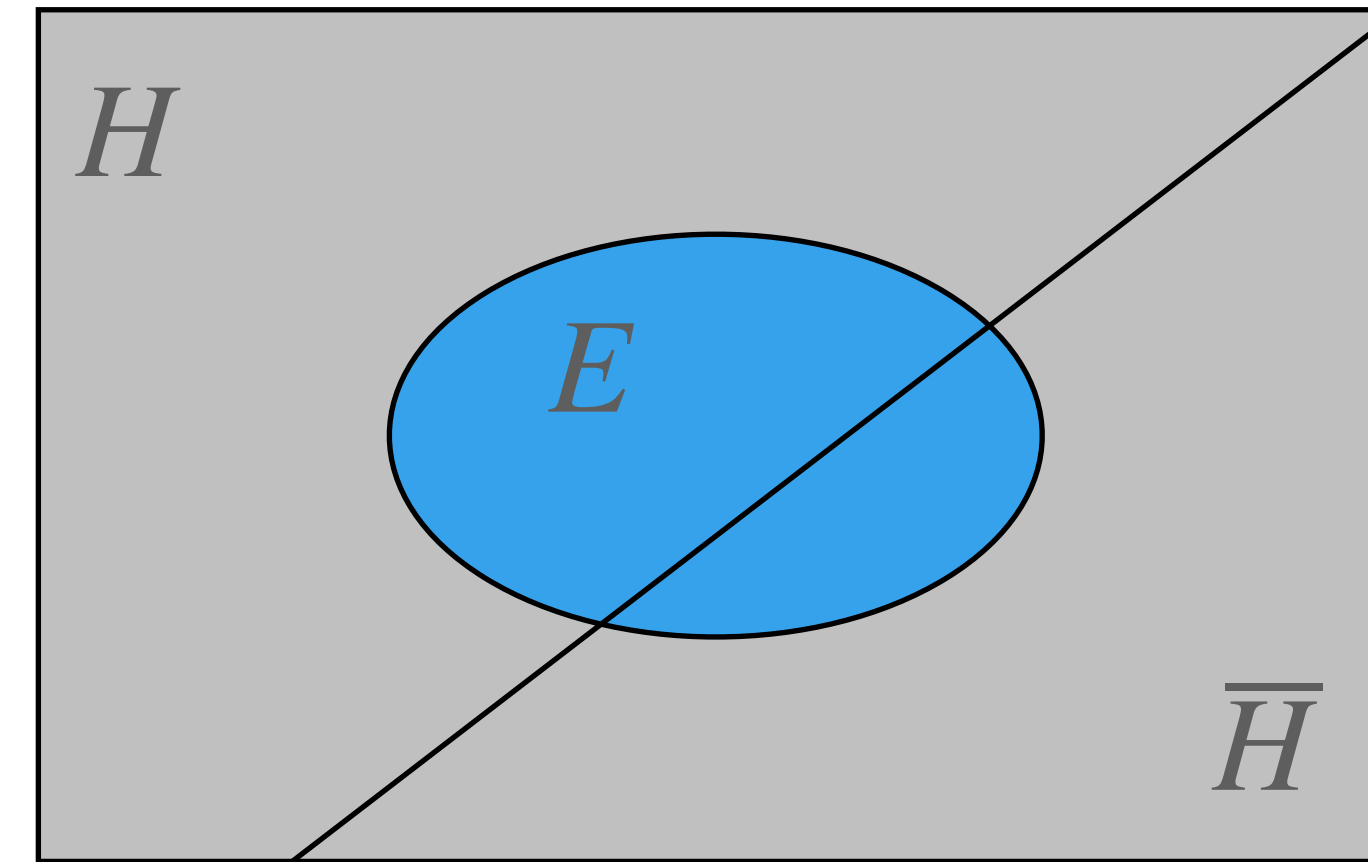
“Upstream” Refinement



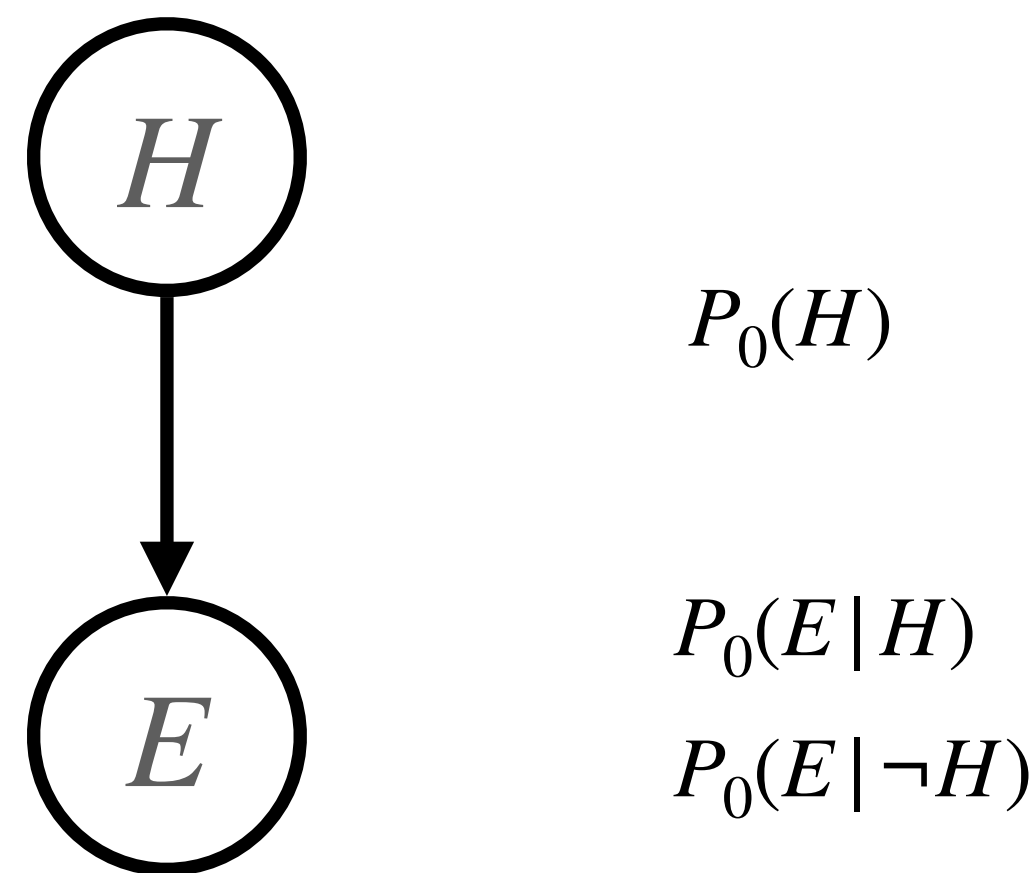
$$P_0(H)$$

$$P_0(E|H)$$

$$P_0(E|\neg H)$$



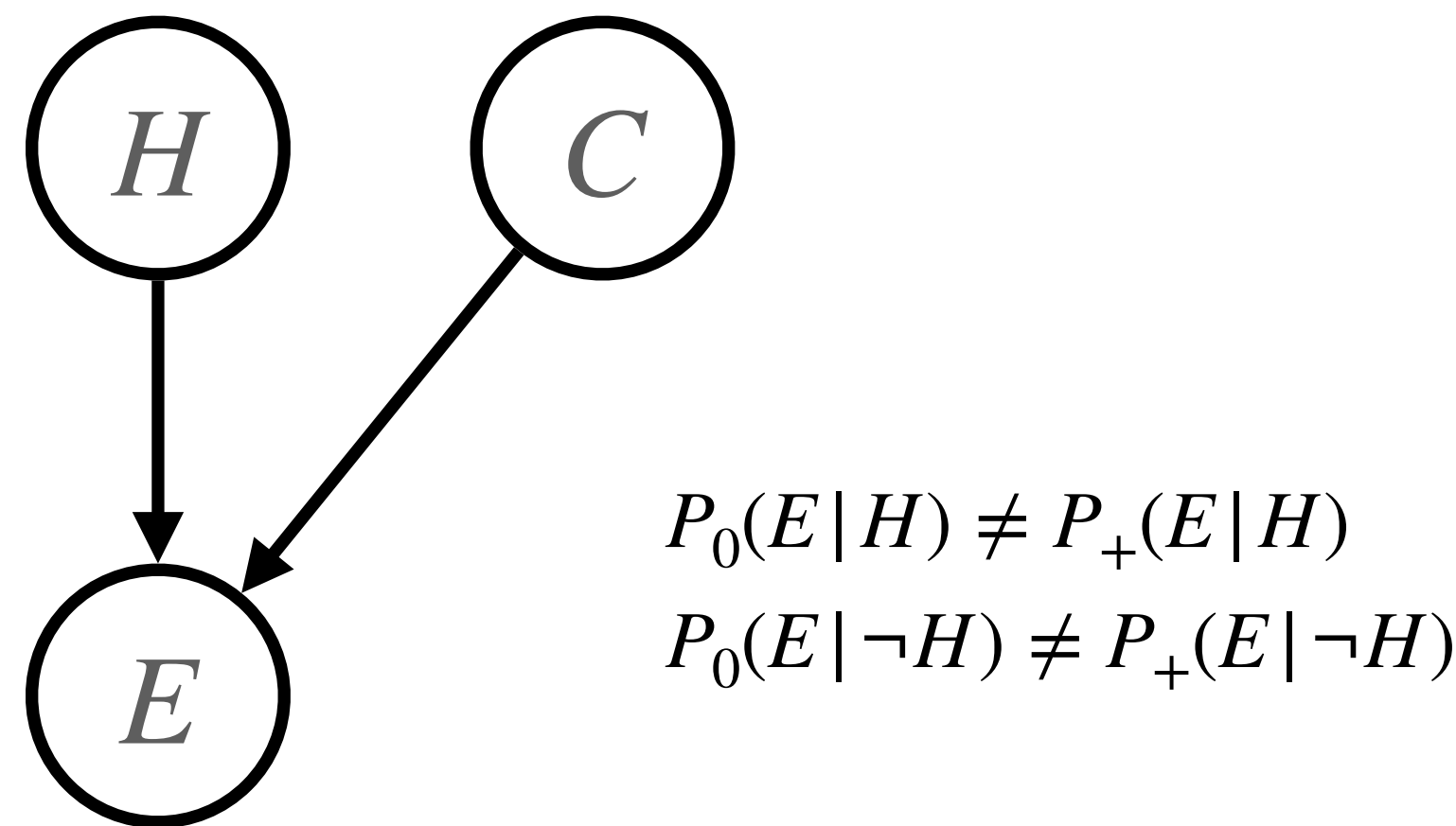
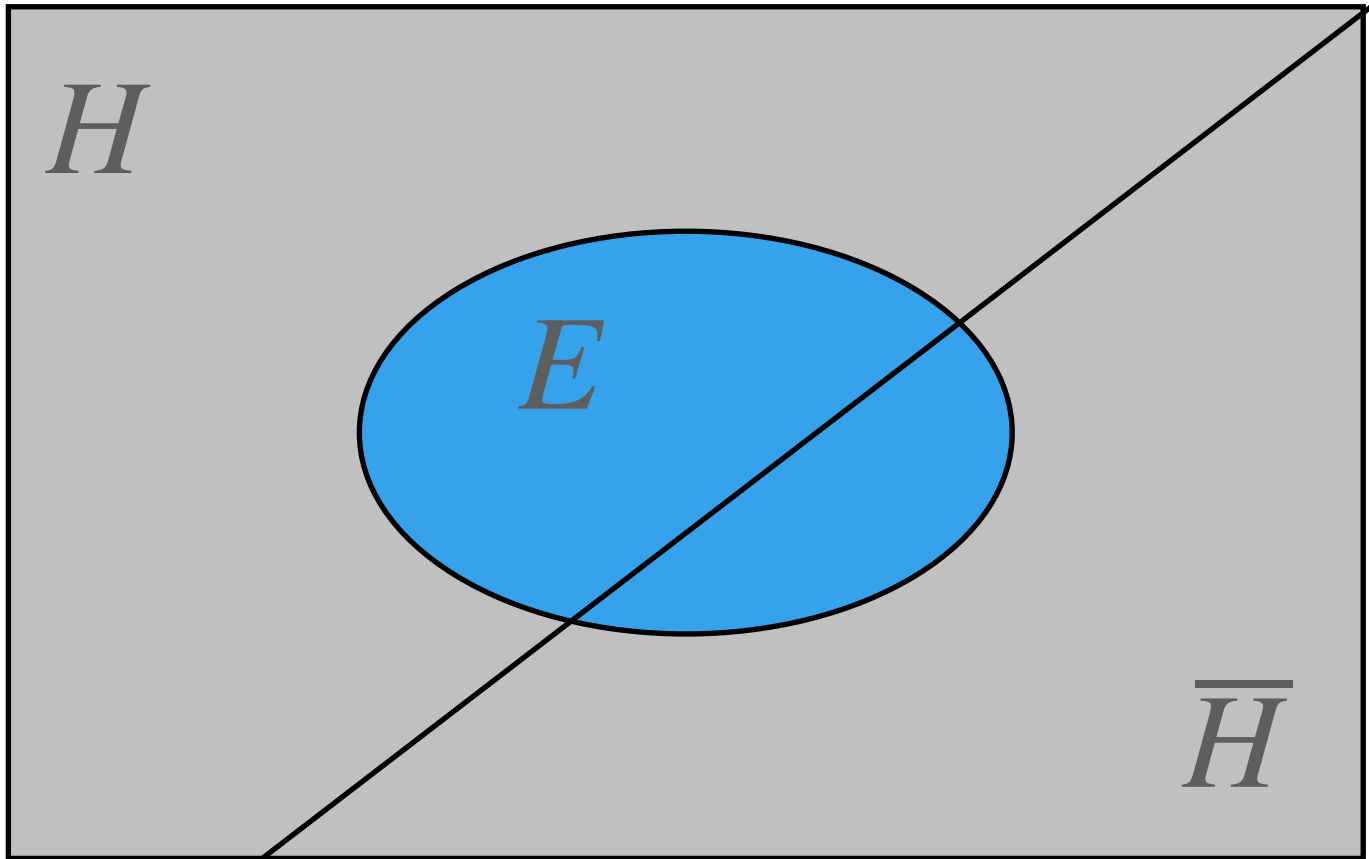
“Upstream” Refinement



$$P_0(H)$$

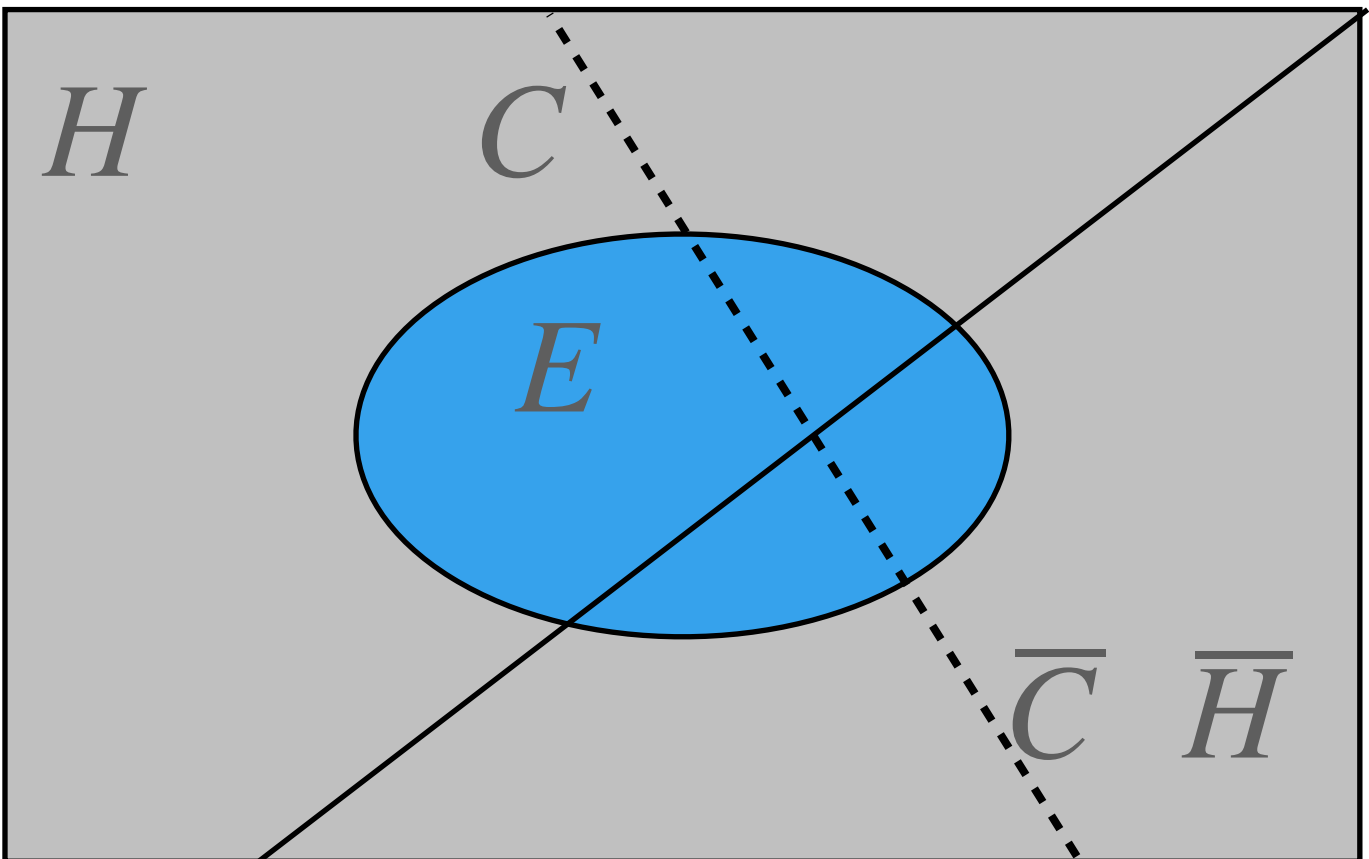
$$P_0(E|H)$$

$$P_0(E|\neg H)$$



$$P_0(E|H) \neq P_+(E|H)$$

$$P_0(E|\neg H) \neq P_+(E|\neg H)$$



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Advantages of Using Bayesian Networks

- Previous two examples are both cases of refinement, but they have a different “causal” structure
- Thus, the information that can be retained is different in the two cases
- Since reverse Bayesianism is oblivious to the additional causal structure, it would treat both cases the same
- Thanks to Bayesian networks, it is clear that Reverse Bayesianism fails in “upstream” refinement but succeeds in “downstream” refinement

Thank you!