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On Keynes's conception of the weight of evidence

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ABSTRACT

Various modern decision theories seek to capture the intuition behind Keynes's conception of evidential weight. Keynes was nevertheless hesitant about the practical relevance of weight in the process of rational decision making because of the 'stopping problem' of finding a rational principle to decide where to stop the process of acquiring information in forming a probability judgment before making a decision. This paper discusses the relevance of the stopping problem by way of an inquiry into the nature, properties and implications for rational decision making of Keynes's conception of evidential weight. It is argued that in practical choice situations the decision maker often decides where to stop the process of acquiring information by following Keynes's advice to consider the degree of completeness of the available information before making a decision. This method implies that the decision maker is able to arrive at an assessment of the dimension of what may be called her 'relevant ignorance'. By considering some examples of how the acquisition of new evidence may affect the decision maker's behaviour, it is argued that it is in fact possible to talk reasonably about relevant ignorance, or what are sometimes called 'unknown unknowns', and that this concept might explain a range of human behaviours. While this concept does not provide a rational principle to solve the stopping problem, it does provide a method of inquiry for dealing with a number of paradoxes not solvable within the Bayesian approach.

1. Introduction

Until quite recently, decision theorists have largely ignored the ideas advanced by J. M. Keynes in his 1921 *A Treatise on Probability* (henceforth TP; Keynes, 1973). Perhaps the most important reason for this neglect has been the influence of the subjective conception of probability stemming from the work of Ramsey (1926) and De Finetti (1937), which is at odds with the objective conception proposed by Keynes. Indeed, it is fair to say that the Subjective Expected Utility (SEU) model proposed by Savage (1954), which incorporates the subjective theory of probability, has dominated the theory of choice under uncertainty over the last 60 years almost to the exclusion of other approaches.

Nevertheless, over the last 20 years or so, a small but increasing number of decision theorists have questioned the SEU model, both on the basis of empirical violations and philosophical arguments. Some of the points made in this literature are strongly reminiscent of the TP, and in particular Keynes's conception of the 'weight of evidence'. Keynes's distinction between '*probability*, representing the balance of evidence in favour of a particular proposition and the *weight of evidence*, representing the quantity of evidence supporting that balance' (Fox and Tversky, 1995, p. 585) has been widely used to explain some of the limits of the SEU model (Anand, 1991; Camerer and Weber, 1992; Curley and Yates, 1989; Ellsberg,

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2001; Einhorn and Hogarth, 1985; Fox and Tversky, 1995; Gärdenfors and Sahlin, 1988; Kelsey, 1994; Tversky, 1997). The seminal contribution here is Ellsberg's (1961) early critique of the SEU model, which is built around an example that is virtually identical to Keynes's two colour urn example (Runde, 1994a; Fox and Tversky, 1995; Feduzi, 2007) and which inspired a significant research program over the last 30 years.[1] Further, various modern decision theories are explicitly related to, or inspired by, the intuition behind Keynes's conception of evidential weight (Anand, 1991; Brady, 1993; Einhorn and Hogarth, 1985; Ellsberg, 2001; Fox and Tversky, 1995; Gärdenfors and Sahlin, 1988; Kelsey, 1994; Tversky, 1997).

However, despite this revival of interest in the theory of evidential weight, Keynes himself was actually quite hesitant about its relevance to rational decision making. Various reasons have given for this hesitancy (Carabelli, 1988; Cohen, 1986a; Cottrell, 1993; Ellsberg, 2001), but I will argue that the most convincing is related to the 'stopping problem', i.e. the problem of finding a rational principle to decide where to stop the process of acquiring information in forming a probability judgment before making a decision. Keynes was not able to overcome the stopping problem, and this raises the question of whether it is sufficiently serious to undermine the relevance of the theory of evidential weight. Some scholars argued that 'the question of when to stop gathering information is a pragmatic one. . .' (Kyburg, 1970, p. 169), and it is not worth answering. However, I maintain that a discussion of the stopping problem is interesting since (1) it leads to an understanding of the nature and properties of the concept of evidential weight, and (2) it throws light on a number of paradoxes which cannot be solved using the standard Bayesian model.

The purposes of this paper are two: first, to discuss the relevance of the stopping problem for the theory of evidential weight, and second, to analyse its implications for rational choice theory.

I start with an account of Keynes's logical theory of probability and evidential weight, and go on to discuss the consequences of the stopping problem for the theory of evidential weight in particular. It turns out that the absence of a rational principle by which to establish whether the available evidence is 'sufficient' to inform a decision does indeed limit the relevance of the theory of evidential weight in the context of a logical theory of probability, as it might lead the decision-maker to permanent indecision and inaction.

Nevertheless, I will argue that in 'practical' choice situations the decision maker often decides where to stop the process of acquiring information by following Keynes's advice to consider the weight of evidence as a measure of the *degree of completeness* of the information upon which a probability is based. As pointed out by Runde (1990), defining the evidential weight along these lines seems to require that the decision maker is able to form estimates of the magnitude of her relevant ignorance. One of the aims of what follows is to provide support for this observation by showing that it is indeed possible to talk reasonably about relevant ignorance or what are sometimes called 'unknown unknowns' and that a rational decision maker should take her relevant ignorance into consideration when making a decision.

But I will also argue that, due to the subjective nature of the decision maker's assessment of her relevant ignorance, the weight of evidence represents a highly subjective measure. To see this, I will provide an example of how the acquisition of new evidence may affect the decision maker's behaviour. It turns out that if we assume that the decision maker's confidence in a probability judgement is related to her personal assessment of the degree of completeness of the information upon which that probability is based, the following might occur: first, the decision maker's degree of confidence in her forecast may undergo drastic changes during the process of acquiring information;[2] second, changes in the decision maker's awareness of the dimension of her ignorance may produce either overconfidence or underconfidence in her probability judgements and the forecasts made on their basis; third, different people, on the basis of the same evidence, might hold different degrees of confidence in their forecasts; and finally, different people, confronting the same decision problem, might feel confident that the evidence acquired is enough to inform the decision at different stages of the learning process.

Keynes's advice that the decision maker should address questions of evidential completeness does not provide a rational principle to solve the stopping problem, as the decision about what constitutes a 'sufficient' amount of information in any situation will always be highly arbitrary. Nevertheless, the notion of the degree of completeness of the information upon which a probability judgment is based does help to explain a number of economic phenomena. Moreover, it provides a method of inquiry for dealing with some paradoxes not solvable within the Bayesian approach. Support for this last claim is provided by discussing the importance of Keynes's concept of evidential weight in legal settings. It is argued that courts of law often establish whether the available evidence is enough to convict by relying on weight-like considerations rather than Bayesian maximal conditionalization.

2. Keynes's *A Treatise on Probability*

2.1. Probability

The distinctive feature of the TP is that it analyses probability as a logical relation between a set of evidential propositions and a conclusion. If E is a set of evidential premises and H is the conclusion of an argument, then $p = H/E$ is the degree of rational belief that the probability relation between E and H justifies. All probabilities are conditional on a set of evidential premises, that is the probability of a certain proposition H is always relative to the actual or hypothetical body of knowledge

[1] See Camerer and Weber (1992) for a review of the literature generated by Ellsberg's QJE paper.

[2] For an early discussion of this issue, see Runde (1991).

stated in E. The acquisition of a new piece of evidence E_1 gives thus rise to a new probability relation $H/E\&E_1$, but does not affect the validity of the previous probability relation between E and H.

On this approach, probabilities are epistemic, as they are regarded as a property of the way in which individuals think about the world. If interpreted as degrees of belief, then probabilities are subjective to the extent that information (and reasoning powers) differs between individuals. They are not, however, subjective in the sense that probabilities are independent of the individuals' opinions. Given a set E of evidential premises and a conclusion H, the probability $p = H/E$ is *objective* and corresponds to the degree of belief it is *rational* for an individual to hold.[3] If E makes H certain, then the conclusion follows directly from the premises and $p = 1$; if the relation between H and E is contradictory, then $p = 0$. In the intermediate cases between these two extremes, in which E provides some but not conclusive grounds for believing (or disbelieving) H, then p lies somewhere in the interval [0,1].

Degrees of belief, however, can be measured numerically only when it is possible either to apply the 'Principle of Indifference' or to estimate statistical frequencies. In terms of the Principle of Indifference, if each of an exhaustive and mutually exclusive list of indivisible hypotheses H_i ($i = 1, 2\ldots n$) is judged to be equiprobable relative to E, then $p(H_i/E) = 1/n$ for each i (Keynes, 1973, chapters IV and XV). On the frequency view, the probability of an event H is p if the relative frequency of H in a large number of repeated trials performed under identical conditions tends to p (Keynes, 1973, chapter VIII). In many cases, the necessary conditions to apply either the principle of indifference or the frequency approach are not met, and 'no exercise of the practical judgement is possible, by which a numerical value can actually be given to the probability. . .' (Keynes, 1973, p. 29).

Although not all probability relations yield numerical values of p, Keynes holds that they may sometimes be amenable to binary comparisons of the form $H_1/E_1 \geq H_2/E_2$ (the symbols >, ≥ and = denote the qualitative probability relations 'more probable than', 'at least as probable as' and 'as probable as'); moreover, he demonstrates how, from probability comparisons already given, it may be possible to derive further probability comparisons.[4] Keynes discusses in particular two kinds of comparison: comparisons between different hypotheses relative to the same evidence and comparisons between the same hypothesis relative to different evidence. These comparisons may be schematized as follows:

Judgements of preference	$H_1/E > H_2/E$
Judgement of indifference	$H_1/E = H_2/E$
Judgements of relevance	$H/(E\&E_1) > H/E$ or $H/E > H/(E\&E_1)$
Judgments of irrelevance	$H/(E\&E_1) = H/E$

Judgments of preference or indifference are relatively straightforward. Getting to grips with the definition of relevance and irrelevance, however, requires a consideration of the notion of the 'weight of the evidence'.

2.2. Weight of evidence

When an individual is making a judgment under uncertainty, it is often assumed that she should try to acquire all the information she can (Locke's maxim) and to use all of the information at her disposal to inform her decision (Bernoulli's maxim). Obviously, not all of the information that may be available need be relevant to determining the probability of a certain proposition H. Accordingly, Keynes proposes a general criterion of relevance which states that a new piece of evidence E_1 is relevant to H, if $H/E\&E_1 > H/E$, or $H/E\&E_1 < H/E$.[5]

Nevertheless, Keynes recognizes that this definition is open to the objection that it may be possible to find cases in which the addition of E_1 to the premises E does not change the probability p but is nevertheless relevant. He refers to this possibility by saying: '. . .we must regard evidence as relevant, part of which is favourable and part unfavourable, even if taken as a whole, it leaves the probability unchanged' (Keynes, 1973, p. 78). A few paragraph earlier Keynes had stated:

> As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence; but *something* seems to have increased in either case, – we have more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the *weight* of argument (Keynes, 1973, p. 77).

This leads Keynes to a more general definition of relevance: 'If we are to be able to treat 'weight' and 'relevance' as correlative terms. . .to say that a new piece of evidence is 'relevant' is the same thing as to say that it increases the 'weight' of the argument' (Keynes, 1973, p. 78). Or alternatively: 'One argument has more *weight* than another if it is based on a greater

[3] The last two decades have seen considerable discussion of the ontological status of Keynes's probability relations and whether Keynes abandoned his logical interpretation of probability in the light of Ramsey's (1926) scepticism about the existence of logical probability relations (e.g. Bateman, 1987; Gillies, 2006; O'Donnell, 1989; Winslow, 1989). As this matter would take us too far from the concerns of the present paper, and as Keynes's logic of comparative probability is also compatible with more subjective interpretations of what Keynes called the probability relation (Runde, 1994b), I will not pursue it any further here.

[4] Nevertheless, Keynes insists that some pairs of probability relations may not even be comparable in qualitative terms (Keynes, 1973, p. 29). In some cases, it may in fact not be possible 'to say that the degree of our rational belief is either equal to, greater than, or less than the degree of our belief in another' (Keynes, 1973, p. 37). In symbols, it is not true that for all H_1, H_2, E_1 and E_2, that either $H_1/E_1 \geq H_2/E_2$ or $H_1/E_1 \leq H_2/E_2$. Furthermore, Keynes maintains that is sometimes impossible to establish even if a proposition is more probable than, less probable than, or as likely as, its negation.

[5] This definition of relevance is consistent with the standard definition of relevance adopted by the subjective theory. For a discussion of the notion of relevance in the SEU model, see, for instance, Gärdenfors and Sahlin (1988).

amount of relevant evidence. . .The weight to speak metaphorically, measures the *sum* of the favourable and unfavourable evidence, the probability measures the *difference*' (Keynes, 1973, p. 84).

Keynes thus seems to be thinking about the weight of arguments as a measure of the absolute amount of relevant knowledge expressed in the evidential premises of a probability relation.[6] If by evidence E we mean the same thing as the relevant knowledge K in respect of some proposition, an assumption that Keynes appears to make,[7] and V represents the weight, then we can write:

$$V(H/E) = K \tag{A}$$

Although in the logical approach, the probability of a certain proposition H always depends on some evidence E, there may be choice situations in which the evidence available is very small. Keynes refers to the probability of such arguments as *a priori probability* and regards the attached evidential weight as being at its lowest level. Starting with a minimum weight, corresponding to *a priori probabilities*, the accretion of relevant evidence will raise the weight of an argument, whist the probability might either rise or fall (Keynes, 1973, p. 78).

Keynes then moves on to explain why the neglect of the importance of the concept of evidential weight by the theory of mathematical expectation leads to a number of difficulties in the application of probability to conduct. In fact, Keynes sees the theory of evidential weight as providing a critique of Locke's and Bernoulli's maxim:

> Bernoulli's maxim, that in reckoning a probability we must take into account all the information which we have, even when reinforced by Locke's maxim that we must get all the information we can, does not completely seem to meet the case. If, for one alternative, the available information is necessarily small, that does not seem to be a consideration which ought to be left out of the account altogether (Keynes, 1973, pp. 345–346).

In other words, Keynes is suggesting that whatever the balance between favourable and unfavourable evidence is, the decision maker should always consider the amount of evidence upon which that balance is based before making a decision. Keynes in fact asks himself: 'if two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge?' (Keynes, 1973, p. 345). And, in order to explain the relevance of this question, he provides a pedagogical example of drawing a white ball from two different urns:

> . . .in the first case we know that the urn contains black and white in equal proportions; in the second case the proportion of each colour is unknown, and each ball is as likely to be black as white. It is evident that in either case the probability of drawing a white ball is 1/2, but that the weight of the argument in favour of this conclusion is greater in the first case (Keynes, 1973, p. 82).

It is clear that, even if on the basis of some kind of indifference argument the probability of drawing a white ball from the two urns may be given to be the same,[8] knowledge of the exact proportions of black and white balls in the first urn provides 'a more substantial basis' upon which to infer the probability. Although weight and confidence are not the same thing, it seems intuitive to regard increases in weight as leading to increases in confidence placed in the probability as a guide to conduct. That is to say, we will be more confident that our forecast is an appropriate guide to action in the case of the probability with higher evidential weight rather than in the case of *a priori probability*. If this is indeed the case, then the decision maker should choose her course of action by taking into consideration both the probability judgments and the evidential weight upon which these judgments are based.

Keynes accordingly proposes a theory of action where the decision maker takes into consideration, in the process of rational decision making, risk, probability and weight.[9] In chapter XXVI, he proposes the following conventional coefficient c of weight and risk, that is, a general rule to combine both coefficient of risk and weight, and the probability:

$$c = \frac{2pw}{(1+q)(1+w)} \tag{1}$$

where p and q assume the conventional meanings and w measures the evidential weight (Keynes, 1973, p. 348).

3. Keynes's hesitancy about the theory of evidential weight

As the material summarized above suggests, Keynes did quite a good job in his embryonic chapter on evidential weight. The intuition underlying the concept is plausible and widely invoked in the decision theory literature questioning the SEU model.[10]

[6] As pointed out by Runde (1990), Keynes provides three definitions of evidential weight in the TP, which I will discuss later in the paper.

[7] In the TP, Keynes does not distinguish between 'evidence', 'knowledge' and 'information' (Levi, 1967; Fioretti, 2001).

[8] The indifference argument in fact asserts that 'if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an *equal* probability' (Keynes, 1973, p. 45).

[9] Keynes's definition of risk is similar to that of variance. In a gamble which pays the amount A with probability p and nothing with probability $(1 - p)$, risk is $R = p(1 - p)A$ (Keynes, 1973, p. 348).

[10] In their review article on uncertainty and ambiguity, Camerer and Weber open the section dedicated to the challenge to the SEU model as follows: 'Keynes (1921) drew the distinction between the *implications* of evidence – the likelihood judgment that evidence implies – and the *weight* of evidence, or

Nevertheless, Keynes was quite skeptical about the relevance of the concept of evidential weight, opening chapter VI of the TP on 'the weight of arguments' with the remark that: 'The question to be raised in this chapter is somewhat novel; after much consideration I remain uncertain as to how much importance to attach to it' (Keynes, 1973, p. 77). The same sentiment is reiterated at the end of the book: 'The question appears to me to be highly perplexing, and it is difficult to say much that is useful about it' (Keynes, 1973, p. 345).

All of this begs the question why Keynes was so hesitant about the relevance of the theory of evidential weight. Keynes's interpreters are largely divided on this point. While Carabelli regards Keynes's remarks as '. . .isolated instances of a prudential attitude' (Carabelli, 1988, pp. 58–59), Ellsberg (2001) suggests that Keynes's hesitancy was due to his inability to develop appropriate decision criteria for guiding action. Cottrell (1993) and Cohen (1986a) go even further by arguing that Keynes's attitude to evidential weight is a symptom of an internal difficulty in the logical theory of probability he was proposing.

Whilst I believe that Keynes's hesitancy is an important matter that deserves serious consideration – and thus also the above-mentioned attempts to explain it – there appears to be one incontrovertible fact that should not be overlooked. In chapter VI (paragraph 7), Keynes explicitly explains that he is hesitant about the practical relevance of the theory of evidential weight because he thinks it is difficult to give it a significant role in the process of inductive reasoning.[11] After having admitted his reservations about the practical relevance of the concept of evidential weight, he qualifies the nature of his concerns. Specifically, Keynes asks if 'we ought to make the weight of our arguments as great as possible by getting all the information we can' (Keynes, 1973, p. 83), remarks that '[i]t is difficult to see, however, to what point the strengthening of an argument's weight by increasing the evidence ought to be pushed' (Keynes, 1973, p. 83), and continues by saying:

> When our knowledge is slight but capable of increase, the course of action, which will, relative to such knowledge, probably produce the greatest amount of good, will often consist in the acquisition of more knowledge. But there clearly comes a point when it is no longer worthwhile to spend trouble, before acting, in the acquisition of further information, and there is no evident principle by which to determine *how far* we ought to carry our maxim of strengthening the weight of our argument (Keynes, 1973, p. 83, emphasis in original).

In other words: (1) Keynes believes that, before making a decision, it is reasonable to increase the amount of available information, that is to strengthen the weight of an argument; but (2) he regards the absence of a principle which determines where to stop the process of acquiring information as a possible objection against the use of the concept of evidential weight. I will refer to this problem, which I regard as the primary source of Keynes's hesitancy about weight, as the 'stopping problem'.

Keynes's emphasis on the relevance of the stopping problem is not surprising as, at the time of the TP, he was a neo-Platonist who conceived probabilities as real objects (Bateman, 1996; Runde, 1994b). If the probability of a conclusion H always depends on some evidence E, and the probability of a conclusion H given the evidence E is fixed objectively, it is a natural next step to ask how much evidence E is *objectively* enough to reckon the probability of a conclusion H.

The absence of a rational (objective) criterion of this sort might have significant consequences for the relevance of the theory of evidential weight. As discussed in the previous section, according to Keynes, the accession of new evidence always increases the weight of argument. If $E_1, E_2 \ldots E_N$ are pieces of evidence relevant to H/E, it then follows that:

$$V(H/E) < V(H/E\&E_1) < V(H/E\&E_1\&E_2)\ldots < V(H/E\&E_1\&E_2\&\ldots E_N)$$

The 'maxim' of strengthening the evidential weight before making a decision thus implies that, in absence of a criterion that establishes where to stop the process of acquiring information, the decision maker should continue the learning process until all the relevant evidence is acquired, that is until the weight is equal to $V(H/E\&E_1\&E_2\&\ldots E_N)$. Leaving aside practical considerations about the opportunity of pushing the process of acquiring information this far, it is clear that two situations might hold:

1. If the relevant information is infinite, then the theory of evidential weight would lead to the absurd conclusion that the decision maker will never stop the learning process.
2. If the relevant information is finite, once the decision maker had acquired all of it, she would still not be able to establish whether the evidence is sufficient to inform a decision.[12]

In both situations, Keynes's theory of evidential weight would then lead to permanent indecision and inaction. According to Keynes, the main problem with making the theory of evidential weight relevant in the process of rational decision making thus lies in finding a rational principle by which to establish whether the available evidence is 'sufficient' to inform a decision.

the confidence in assessed likelihood. Keynes wondered whether a single probability number could express both dimensions of evidence' (Camerer and Weber, 1992, p. 327, emphasis in original).

[11] Levi advances a similar argument in his *1967 Gambling with Truth*. Nevertheless, Levi's analysis of the problem at hand is quite different from mine. Whilst a discussion of these differences is beyond the scope of this paper, I will point out some of the differences between my position and Levi's as I proceed.

[12] It is clear that this consideration affects the strength of Keynes's critique of Bernoulli's and Locke's maxim discussed in the previous section: if the decision-maker does not have a criterion by which to establish how much information is enough, how can she determine if the available information is too small to inform a decision?

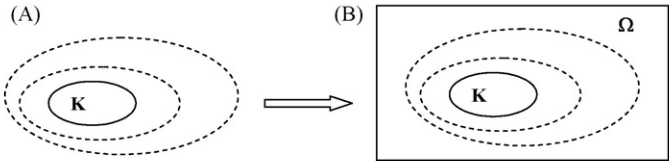

Fig. 1. Evidential weight and the stopping problem.

This problem led him to conclude the paragraph by admitting: 'A little reflection will probably convince the reader that this is a very confusing problem' (Keynes, 1973, p. 84).

4. On a relative conception of evidential weight

The absence of a rational principle to solve the stopping problem does indeed limit the relevance of the theory of evidential weight in the context of a logical theory of probability and thus provides a convincing explanation of Keynes's hesitancy. Nevertheless, as it seems possible to disconnect many elements of Keynes's theory from the ontology of Platonic logical probability relations (Runde, 1994b), I will move on to discuss whether the stopping problem actually undermines the practical relevance of the theory of evidential weight.[13]

I will start by considering an apparently unrelated aspect of Keynes's work, namely Keynes's shift from an absolute to a relative conception of evidential weight. As discussed by Runde (1990), in chapter XXVI of the TP on 'The application of probability to conduct', Keynes provides a second definition of evidential weight by claiming that the decision maker should consider 'the degree of completeness of the information upon which a probability is based. . .in making practical decisions' (Keynes, 1973, p. 345).[14] The relationship between this second definition of evidential weight and the stopping problem can be easily grasped by considering the following example. If I tell you that 'I have covered twenty miles', you cannot say if I have come very far in my journey. But if I tell you my final destination, you can tell if I am at the beginning or at the end of my journey, or if I am almost there. In the same way, the absolute amount of evidence E one has already acquired (first definition of weight) does not reveal *how far* one has come in the learning process (Fig. 1, situation A); but if one knows how much information is relevant to the proposition H, one can say whether the evidence acquired so far is *relatively* 'scanty', 'complete' or simply 'sufficient' to make a decision (Fig. 1, situation B). This is because no evidence is itself 'scanty' or 'complete' in the same way as no place can be intrinsically distant.[15]

In other words, if it were possible to follow Keynes's advice and determine the degree of completeness of the information upon which a probability is based in making a decision, we might try to escape the stopping problem by shifting from an absolute to a relative conception of weight.[16]

Leaving aside, for the moment, considerations about the actual effectiveness of this shift to solve the problem at hand, the main issue here is to establish whether it is actually possible to make sense of the notion of evidential weight as a relative concept. Defining the evidential weight as the degree of completeness of the information implies that the decision maker is able to identify evidence that at present she does not know but which is relevant for the proposition H, that is, her 'relevant ignorance'. The point can be easily grasped by introducing some definitions. Let Ω be the set of all evidence which is relevant for estimating the probability of a proposition H. Let K be the set of relevant evidence (knowledge) possessed by the decision maker; in other words, let K represent the relevant evidence which has already been acquired by the agent ($K \subseteq \Omega$). Since all the evidence in Ω is relevant, and the decision maker knows only part of it (K), then the set theoretic difference I of Ω and K ($I = \Omega \backslash K$) represents all relevant evidence that the decision maker does not presently know, that is her 'relevant ignorance' (obviously, $\Omega = K \cup I$). If so, then, as pointed out by Runde (1990), Keynes's definition of weight as the degree of completeness

[13] Keynes's hesitancy about the possibility of finding an objective principle to solve the stopping problem deserves further consideration. It might be argued that, in artificial choice situations involving random drawings from urns, where the weight of evidence may be simply interpreted as an expression of the amount of the information provided by the sample, it might be possible to solve the stopping problem by relying on statistical criteria for determining the optimal dimension of the sample (Feduzi, 2007). Nevertheless, such standard statistical criteria do not seem to satisfy Keynes's requirements for an objective principle as they also depend – to some extent – on subjective parameters such as the level of confidence chosen or the maximum error admitted.

[14] Keynes's second definition of weight has been largely ignored in the relevant literature. Most interpreters, both economists in the Keynesian tradition as well as decision theorists, focus only on the weight of evidence as a measure of the absolute amount of relevant knowledge expressed in the evidential premises of the probability relation (Einhorn and Hogarth, 1985; Ellsberg, 2001; Gärdenfors and Sahlin, 1988; Kelsey, 1994). A few scholars have recognised the other definition of weight and concluded that this was just a sign of Keynes being inconsistent (Cottrell, 1993; Runde, 1990, 1994a).

[15] As readers familiar with the TP will know, Keynes discussed the absolute/relative nature of the probability relation in these terms.

[16] Levi, ignoring Keynes's own shift from an absolute to a relative conception of weight, explicitly suggests that Keynes should have avoided the stopping problem by viewing the 'weight of evidence not as a measure of the absolute amount of relevant evidence but as an index of the sufficiency of available evidence' (Levi, 1967, p. 142). The weight of evidence 'would then be viewed as of high value when no further evidence is needed and would fall away from that high value as the demand for new evidence increases. The problem still remains of determining the conditions under which new evidence is needed and when it is pointless; this remains, as Keynes says, a confusing problem. But some partial headway might be made by avoiding Keynes' question-begging assumption that the issue is one of absolute amounts of relevant evidence' (Levi, 1967, p.142).

of relevant information may be written as:

$$V(H/E) = \frac{K}{K+I} \tag{B}$$

This view is confirmed by Keynes who suggests that the evidential weight might be described the balance of 'the *absolute* amounts of relevant knowledge and of relevant ignorance' on which a probability is based (Keynes, 1973, p. 77).[17] But if this is the case, the problem with a relative definition of weight lies in providing a logical explanation of what relevant ignorance is; the principal question is whether it is in fact possible to talk reasonably about relevant ignorance, that is, of knowing something about the extent of our ignorance. Unfortunately, Keynes does not provide an answer to this question in the TP, since he did not clearly define the concept of relevant ignorance.[18] Nevertheless, in recent years, Keynes's interpreters have advanced some conjectures about what he meant by relevant ignorance. For instance, decision theorists such as Camerer and Weber (1992) claim that Keynes's conception of 'the weight of evidence can be defined as the amount of available information relative to the amount of *conceivable* information', where the gap between the two 'is the amount of missing information' (Camerer and Weber, 1992, p. 331, emphasis added). If this is the case, then relevant ignorance might represent the decision maker's subjective assessment of possible information that she does not presently know (missing information), but which if it were known, she believes (*conceives*) it would be relevant for the decisional problem (independently from the fact that this information is actually relevant or not for the problem at hand).

Economists interested in the philosophical underpinning of Keynes's economics such as Runde (1990, 1991) go further by claiming that relevant ignorance might represent the decision maker's subjective assessment of possible information that she does not presently know, but which if it were known, would be *actually* relevant for the decisional problem. In other words, the idea is that 'in practice we do often know of, or at least are able to identify, factors of which we are to a large extent ignorant, but which are relevant to our probability estimates' (Runde, 1990, p. 282). Moreover, we might be aware of the possibility that better information could be available or that there are relevant factors that we have omitted altogether (Runde, 1990). Relevant ignorance might thus be considered as 'some measure of our apprehension of the extent of our ignorance about evidence which we *know* to be relevant to some conclusion' (Runde, 1991, p. 132, emphasis added).

Both interpretations thus suggest that Keynes's shift from an absolute to a relative conception of evidential weight is meaningful and that the decision maker should take into account, in the process of rational decision making, her personal assessment of her relevant ignorance. Nevertheless, these interpretations presuppose a sophisticated and self-aware decision maker, who has the capacity to recognize the possibility of her own ignorance. It is thus necessary to 'unpack' the decision maker's epistemic condition;[19] let us consider the following possible four situations:[20]

1. The decision maker knows all the available evidence relevant to some conclusion and knows that she knows all of it.
2. The decision maker does not know some of the evidence relevant to some conclusion and knows that this is the case.
3. The decision maker does not know some part of the evidence relevant to some conclusion, does not know that she does not know this part of the evidence, but knows that there may be some part of the evidence that she does not know.
4. The decision maker does not know a part of the evidence relevant to some conclusion, does not know that she does not know this part of the evidence, and does not know that she might not know some relevant evidence.

Situation (1) represents the kind of choice situation usually analysed by standard decision theory. The decision maker knows that she possesses a correct and exhaustive list of possible states of the world; she does not know which state of the world is going to occur, but she knows that her subjective assessment of likelihoods of these events is based on all the evidence which is relevant to establish the case.[21] Situation (4), paradoxically, is very similar to situation (1). The decision maker acts 'as if' she actually possessed a correct and exhaustive list of states of the world and her subjective assessment of likelihoods of these events was based on all the evidence which is relevant to establish the case. However, since her representation of the choice situation might be inadequate, she might be 'genuinely' surprised by the discovery of relevant evidence which was not previously considered or by the occurrence of a state of the world that was not contemplated in the list. New information can thus show (ex post) that she was fully 'unaware' (ex ante) of some of the features of the choice situation; she might thus simply discover that the model she assumed (without any doubts) as a good description of reality was actually wrong.

[17] If we use again the symbol I to denote Keynes's conception of relevant ignorance, then we might write $V(H/E) = K/I$ (Runde, 1990).

[18] Keynes nevertheless gave an important role to the concept of relevant ignorance in economic writings he produced while working on the TP. For instance, in his 1910 *New Quarterly* article on Great Britain's Foreign Investments, Keynes states: '. . .The amount of risk to any investor principally depends, in fact, upon *the degree of his ignorance* respecting the circumstances and prospects of the investment he is considering' (Keynes, 1971, p. 46; emphasis added).

[19] For recent discussion of the decision-maker's epistemic conditions, see, for instance, Dekel et al. (1998) and Modica and Rustichini (1999).

[20] Obviously, I am leaving aside the 'certainty' case, where the decision-maker has perfect knowledge of the truth or falsity of a given proposition.

[21] This corresponds to Savage's remark about what a state of the world is: 'a description of the world, leaving no *relevant* aspect undescribed' (Savage, 1954, p. 9, emphasis added). In this framework, the decision-maker has no ignorance about the state space and the subjective beliefs are fully reliable.

In my view, both choice situations (1) and (4) are largely irrelevant in real world choice situations.[22] Surprise is an unavoidable fact of life, and the assumption that the decision maker acts 'as if' she 'anticipated' or 'believed to anticipate' every eventuality that might befall her appears to be very unrealistic; in most choice situations, people are in fact not sure about their own ability to imagine and think through an exhaustive list of states of the world. Moreover, individuals are usually aware that their probability estimates are based upon partial or highly incomplete information; the decision maker's subjective degree of beliefs cannot thus represent both the relative likelihoods of events and the amount, type, and reliability of the information underlying those likelihoods. In other words, ignorance about the state space and/or about part of the evidence that is relevant to make probability judgments is a standard (ex ante) epistemic condition.

Situations (2) and (3) are thus more interesting and somehow correspond to Runde's and, Camerer's and Weber's original insights. Situation (2) describes a choice situation in which the decision maker is aware that her evidence is incomplete, but she can recognize/describe factors of which she is to a large extent ignorant but that she knows to be relevant to her probability estimates. Obviously, the ability to individuate these factors varies from case to case. Sometimes, the choice situation is clearly defined and the decision maker can describe the main features of her ignorance. This usually happens in circumstances in which relevant ignorance represents an objective concept (for instance, choice situations involving random drawings from urns). In other cases, choice situations are very complex and even though the decision maker is able to recognize the main features of her ignorance, she cannot provide a description of them. This might happen because, in these circumstances, relevant ignorance represents a highly subjective concept.

Of course, in many cases the decision maker cannot recognize the main features of her ignorance. The decision maker is frequently unable even to imagine factors that could affect the probability of an event. However, I claim that she is *always* aware of the possibility that there might be relevant factors that she could have omitted altogether. Situation (3) thus represents a choice situation in which the decision maker is not able to recognize relevant factors of which she is ignorant, but she is 'aware of the possibility of being surprised'; she does not 'have in mind' how she is going to be surprised, but she knows that this eventuality is likely to happen. In contrast to situation (4), 'surprise' is here a possible (ex ante) state of the world.

Situations (2) and (3), I believe, represent the most common decision maker's epistemic states, since they correspond to the idea that people act on the assumption that they have only a coarse description of the world.

5. Weight of evidence and the stopping problem

Given that it is reasonable to speak in terms of relevant ignorance, let us now consider the possibility of *overcoming* the stopping problem in practical choice situations by relying on a relative conception of evidential weight. In what follows, I will refer to 'practical' choice situations as decision situations where no particular assumption is made about the information available to the decision maker. In particular, nothing is assumed about the characteristics of the agent learning process or the representativeness of the evidential sample, which may therefore contain any amount of bias.[23] Consider the following example.

Suppose that our decision maker, call her Joan, is conducting literature based research on the topic of decision theory under ambiguity and that she is trying to establish who is the most quoted author in that literature. Let Ω be the set of all N papers (pieces of evidence $E_1, E_2 \ldots E_N$) on the topic of decision theory under ambiguity which are relevant to establish Joan's case; this information is objectively relevant for the problem and exists independently of the decision maker's knowledge. For the sake of simplicity, assume that Ω is closed and that it is not expanding (that is, nobody else is going to publish a new work on the subject in the near future). Let K be a subset of Ω containing the relevant papers ($E_1, E_2 \ldots E_M$, with $M < N$) that Joan has read so far ($K \subseteq \Omega$). Since all papers in Ω are relevant, and the decision maker knows only part of it (K), then the set theoretic difference I_O of Ω and K ($I_O = \Omega \backslash K$) represents all relevant papers that the decision maker does not presently know, that is, her 'objective relevant ignorance' (obviously, $\Omega = K \cup I_O$). Finally, let S be the set of papers that Joan 'believes to exist' in this field of research and $I_S = S \backslash K$ (the difference between the sets S and K) be a measure of her belief about the existence of relevant papers that she has yet to encounter. In other words, I_S represents her subjective assessment of possible existing information that she does not presently have, but which if it were known, she believes would be relevant for establishing who the most cited author is, that is her personal relevant ignorance.[24] Joan's assessment of weight as the degree of completeness of relevant information, which represents here a subjective measure, might thus be written as:

$$V(H/E_1 \& E_2 \ldots \& E_M) = \frac{K}{K + I_S} \tag{2}$$

This is clearly a choice situation of kind (2): the decision maker does not know part of the relevant evidence, but she knows that she does not know it. She can identify factors of which she is to a large extent ignorant, but she cannot provide

[22] Both choice situations rely on the idea of a *naïve* decision-maker acting in the kind of 'small world' described by Savage (1954).

[23] I will however assume that each additional piece of evidence is 'atomistic' in the sense that it is independent in its impact on the judgement of probability being made and does not disturb the veracity of the body of information acquired prior to it. While this assumption is an idealisation, it makes it easier to understand the concept of relevant ignorance in practical settings.

[24] The set S, of course, may be larger than, equal to or smaller than Ω, depending on her personal estimate of the size of I_S.

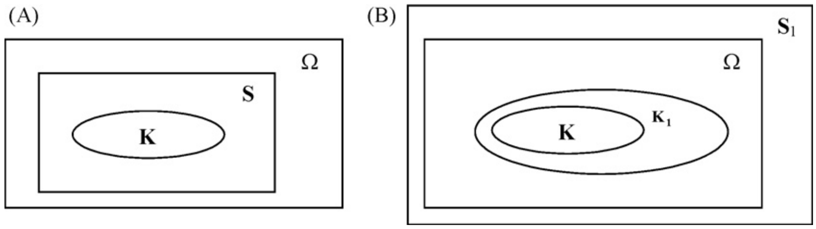

Fig. 2. Weight of evidence and the stopping problem in practical choice situations.

a clear description of them. Relevant ignorance represents a subjective variable here, as she has to make a conjecture about how many articles and authors exist in the field.[25]

Obviously, she can update her personal assessment on the existence of relevant papers that she has yet to encounter by acquiring information. Let us say that, having a background in economics, Joan started her research by collecting articles published in economics journals and that, after having picked up a sample of M papers, she recognizes Daniel Ellsberg as one of the main authors. At that stage, Joan's problem is to estimate the probability of the proposition *H*: 'Daniel Ellsberg is the most highly cited author in the ambiguity literature'. Following Bernoulli's maxim, Joan uses all the information at her disposal to form the evidential premises from which she infers the probability relation. Obviously, she does not know where she actually is in the process of acquiring information because she does not know how many relevant articles and authors exist in the field (by which I mean that she does not know how far or close she is from complete knowledge of the objective relevant set of information Ω). However, on the basis of the literature already collected, she is quite confident that her knowledge is *sufficient* to establish her case. In other words, she thinks that, eventually, the papers that she has yet to read would not significantly affect the probability of *H*. Joan expresses her opinion by saying: 'I think I have read enough to identify the most cited author'.

For instance, we might assume that (1) Joan believes that, on the basis of *K*, the probability of the proposition *H* is high, say close to 0.7. And that (2), having read a considerable amount of articles (M) and on the basis of I_S, she also thinks that the weight of evidence $V(H/E_1\&E_2...\&E_M)$ is quite high, say 0.8, but not at its maximum as she is aware that she might not have read all of the relevant literature yet.

In terms of sets, Joan thinks that *S* is not much larger than *K* and that *S* is smaller than Ω. On the basis of *K*, Joan feels that she has not read all the relevant papers ($K \subset S$) but also thinks that the papers that she has yet to read would not make that much difference to the probability of *H*. Indeed, from an objective point of view, she underestimates the amount of papers that she has not yet encountered ($S \subset \Omega$). This case is represented by Fig. 2A.

Since this kind of eventuality appears to be quite common, I would suggest that many people make (implicitly or explicitly) some form of assumption about the dimension of their ignorance or, analogously, about the degree of completeness of their information. And I would also suggest that they rely on this assumption to establish whether the information at their disposal is enough to inform a decision.[26] In other words, in many practical choice situations, the stopping problem is solved by comparing/contrasting the amount of available evidence with the total amount of conceivable relevant evidence.

This pragmatic approach to solve the problem at hand might nevertheless be misleading, given the subjective nature of the relative conception of evidential weight in practical choice situations. Suppose, for instance, that after her first estimation of $p(H/E_1\&E_2...\&E_M)$, Joan continues collecting literature and comes across a new article (E_{M+1}). After some consideration she realizes that the article is relevant for her research. The author of the article refers in the text to some scholars Joan has not heard of before. Moreover, Joan has not read some of the articles indicated in the bibliography and she is aware that any of these articles *could* potentially refer to other relevant papers in their bibliographies (and the same could happen for articles in the bibliographies of those papers and so forth). Since some of these articles are published in journals belonging to a different discipline (let us say, psychology rather than economics), the new article suggests the existence of an ambiguity literature in a field Joan was completely unaware of. What are the consequences of this discovery for Joan's decision problem?

[25] Whist Keynes's conception of evidential weight entails both issues of quality and quantity of information (Runde, 1990; Fioretti, 2001), this example is explicitly designed to give centre stage to problems of the quantity of information. Nevertheless, the decision-maker's ignorance about the number of authors relevant to the field also reflects some information quality considerations. In a choice situation involving random drawings from urns, this would correspond to the decision-maker's ignorance about how many different colours can be found in the urn (on this point, see Fioretti, 2001).

[26] Whilst questions of evidential completeness are usually linked to the concept of *uncertainty perception*, it is often argued that the decision maker's confidence in the state of expectations also depends on the concept of *uncertainty aversion* (Dequech, 1999), that is the decision-maker's willingness to face or to avoid uncertainty. An anonymous referee suggested that uncertainty aversion might be responsible for determining how much evidence the decision-makers holds sufficient to inform a decision. I do recognize that uncertainty aversion is relevant to the problem at hand. It seems fair to say that the more averse to uncertainty a decision-maker is, the higher is the degree of completeness of the information which she might hold necessary to inform a decision. For instance, in artificial choice situations involving random drawings from urns, where the weight of evidence may be simply interpreted as an expression of the amount of the information provided by the sample and relevant ignorance represents an objective variable, uncertainty aversion might influence the optimal dimension of the sample by affecting the value of subjective parameters such as the level of confidence chosen or the maximum error admitted.

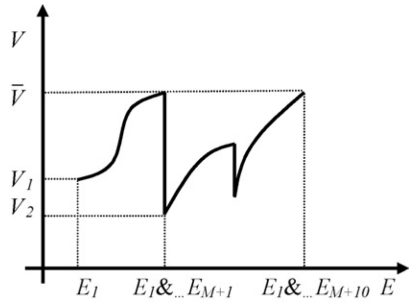

Fig. 3. Weight of evidence and a hypothetical learning process.

In the first place, due to the discovery of other scholars writing on the same topic, the probability of the proposition H will presumably decrease. The absolute amount of relevant knowledge has increased with the acquisition of the new article: the set of relevant knowledge K_1 is greater than K (a new piece of evidence E_{M+1} has been added to the evidential premises) and, consequently, the difference between Ω and K_1 is now smaller than the difference between Ω and K. In other words, since the size of the set Ω has not changed at all, and is independent of Joan's beliefs, the decision maker's relevant ignorance has objectively decreased with the acquisition of new evidence ($I_{O1} = \Omega \backslash K_1 < I_O = \Omega \backslash K$). But what can we say about the decision maker's personal assessment of her relevant ignorance and the weight of evidence attached to the probability relation?

In the spirit of Popper (1968),[27] I believe that the answer to this question depends on the impact the new article has on Joan's personal perception of her relevant ignorance, that is on her belief about the existence of relevant papers that she has yet to encounter. The discovery of the new article has in fact shown to Joan that her subjective assessment of amount of relevant ignorance was wrong, that is she was underestimating the dimension of I_S. But the main issue here is how much significance Joan attaches to the article found and thus how much her estimate of her relevant ignorance I_S has changed. If Joan attaches great importance to the article found, and to the relative potential bibliography, her assessment of the extent of her (subjective) relevant ignorance can sensibly expand. In extreme cases, if Joan overestimates the importance of the article just found, the set S could sensibly expand, possibly becoming even larger than Ω (Fig. 2B). If this is the case, while $p(H/E_1\&E_2\ldots\&E_{M+1})$ will presumably decrease, $V(H/E_1\&E_2\ldots\&E_{M+1})$ will fall. The situation might be represented as follows:

$$V(H/E_1\&E_2\ldots\&E_{M+1}) = \frac{K_1 \uparrow}{K_1 + I_{S1} \uparrow\uparrow} \downarrow\downarrow \tag{3}$$

This situation suggests that the evidence previously available to Joan was actually not sufficient to identify the most cited author. If Joan had stopped the process of acquiring information on the basis of her previous estimation of her relevant ignorance, her decision would have been made overestimating the degree of completeness upon which her probability judgment was based. This is not the end of the story, however. Obviously, Joan might in fact continue the process of acquiring information and proceed to discover that she had overestimated the amount of relevant papers published in psychology journals; this eventuality would then lead her to revise again her personal assessment of the extent of the relevant ignorance and the weight of evidence. And these significant revisions might occur several times during the learning process, leading to drastic changes of the evidential weight, as represented in Fig. 3.

Somehow paradoxically, Joan might believe she has achieved the same degree of completeness of the available information at different times, when she is in possession of different amounts of relevant evidence ($E_1\&E_2\ldots\&E_{M+1}$ and $E_1\&E_2\ldots\&E_{M+10}$ in the above figure). Moreover, if we regard the level of weight $\bar{V}$ as sufficient to inform a decision, then there might be several points during the learning process when she might consider it rational to stop the process of acquiring information.

All of this suggests that, due to the subjective nature of the decision maker's assessment of her relevant ignorance, the method of comparing/contrasting the amount of available evidence with the total amount of conceivable relevant evidence does not provide a rational principle to solve the stopping problem. The reason for this is that the judgement of the amount of information that suffices to inform a decision is a highly personal one and will to this extent always be highly arbitrary.

Nevertheless, the above account also suggests that the decision maker's subjective assessment of relevant ignorance might play an important role in the process of rational decision making and may explain a number of related phenomena.[28] If we regard confidence and weight as correlative terms, it might explain:

[27] As noted by Popper, 'The more we learn about the world, and the deeper our learning, the more conscious, specific, and articulate will be our knowledge of what we do not know, our knowledge of our ignorance. For this, indeed, is the main source of our ignorance – the fact that our knowledge can be only finite, while our ignorance must necessarily be infinite' (Popper, 1968, p. 28).

[28] Obviously, the argument presented here does not rule out the possibility of explaining the same phenomena by appealing to other elements determining the state of expectations (Dequech, 1999) or to more irrational sides of human behaviour (Winslow, 1986). If we follow 'Keynes in using the term "irrational" to describe beliefs which are held for *inadequate* reasons' (Winslow, 1986, p. 551, fn. 8) and recognize that rational beliefs in economic affairs are often unattainable, we have to admit that the foundations of expectations are often conventional or instinctive rather than rooted in rational calculus.

1. Why the decision maker's confidence in her forecasts might undergo drastic change – along with her beliefs – during the learning process.
2. Why the decision maker's can be either underconfident or overconfident in her forecasts. Following Camerer and Weber (1992), a new piece of evidence might in fact make the decision maker aware that the amount of evidence that she did not know but she *conceived* as relevant was wrong; the decision maker might in fact have overestimated the actual amount of missing information and thus realize (ex post) that the weight of evidence upon which the probability was based (ex ante) was actually greater than she maintained (underconfidence). Or, as suggested by Runde (1990, 1991) the decision maker might find that she had underestimated the amount of her relevant ignorance: a new piece of evidence could make her aware that there are more possible alternatives than she had previously imagined and thereby cause a fall in the evidential weight (a situation in which she was formerly overconfident). This might happen because '. . .we may have enlarged our perception of relevant ignorance. . .' (Runde, 1990, p. 283).[29]
3. Why different people, on the basis of the same evidence, might have different levels of confidence in their forecasts. On this view, different people, on the basis of the same set of evidence E, are in fact entitled to attach different weights $V(H/E)$ to the same conclusion H.
4. Why different people, confronting the same decisional problem, might feel confident that the evidence acquired is enough to inform the decision at different stages of the learning process.[30]

6. Weight of evidence, Bayesianism and the courtroom example

Bayesian decision theorists might claim that the stopping problem is never solved along the lines just described, that is by comparing/contrasting the amount of available evidence with the total amount of conceivable relevant evidence. They might argue that Bayesian conditionalization can be simply seen as a device for increasing the evidential weight and it is thus possible to refer to standard statistical criteria to solve the stopping problem.[31] These criteria however seem to be of little help in 'practical' choice situations. Let's consider the much discussed 'blue bus case':[32]

> While driving late at night on a dark, two-laned road, a person confronts an oncoming bus speeding down the centerline of the road in the opposite direction. In the glare of the headlights, the person sees that the vehicle is a bus, but cannot otherwise identify it. He swerves to avoid a collision, and his car hits a tree. The bus speeds past without stopping. The injured person later sues the Blue Bus Company. He proves, in addition to the facts stated above, that the Blue Bus Company owns and operates 80% of the buses that run on the road where the accident occurred. Can he win? (Nesson, 1985, pp. 1378–1379).

Although the logic of the Bayesian decision theory holds that the plaintiff should win because, on the basis of the evidence, he has shown much more probably than not, that a bus owned by the Blue Bus Company ran him off the road, most scholars have suggested that the judge should grant a direct verdict to the defendant: 'in this case, and others like it, the plaintiff will lose; in fact the case is unlikely even to reach the jury' (Nesson, 1985, p. 1379).[33] The main point here is that even if the available evidence strongly supports the probability that a blue bus was involved, the judge hearing only this piece of evidence would find for the defendant because the evidence is lacking (Callen, 1986; Jackson, 1996). The dismissal of the case is not an indication that, on the evidence that has already been admitted, the probability that the plaintiff is correct is less than 50%. It simply indicates that the evidence is insufficient to make a decision (analogous to refusing to bet in any realistic betting example).[34]

A remarkable example of the relevance of this possibility is provided by Scots Law. In that legal system, even if all the admissible evidence is believed, the case will still fail unless the said evidence is sufficient in law to entitle the court to

[29] In the above example, overconfidence and underconfidence are transient states of confidence defined in subjective terms, as they emerge from a comparison between the decision-maker's personal assessment of evidential completeness at different stages of the learning process. Determining overconfidence and underconfidence in objective terms would require a comparison between the decision-maker's assessment of evidential completeness and the objective evidential completeness at the same point in time. This comparison arises methodological problems similar, but not identical, to those emerging in the studies on calibration of subjective probabilities (see, for instance, Lichtenstein et al., 1982).

[30] These phenomena are even more likely in situations in which the decision-maker is 'aware of the possibly of being surprised' but she is not able to identify/recognize relevant factors of which she is ignorant.

[31] For a discussion of issues related to the stopping problem in Bayesian analysis see, for instance, Pham-Gia and Turkkan (1992).

[32] The blue bus case is based on *Smith v Rapid Transit Inc.*, 317, Mass. 469, 58 N.E. 2d 754 (1945). The example has generated a considerable discussion on the way inferences should be drawn from a mass of evidence in legal settings. See, for instance, the proceedings of the Boston University Law Review Symposium on 'Probability and Inference' (1986) and the proceedings of the Symposium on 'Decision and Inference in Litigation' under the auspices of the Jacob Burns Institute of Legal Studies (1991).

[33] Note that the civil proof standard of a preponderance of the evidence is one-half and therefore, from a strictly probabilistic point of view, recovery should be allowed (Brilmayer, 1986). Against this, many scholars maintain that the judge should grant a direct verdict to the defendant even in variations of this example where the blue bus company owns more than 80% of the buses in town (Jackson, 1996).

[34] In *Smith v Rapid Transit Inc*, the Massachusetts Supreme Court explicitly explained that this dismissal of the case did not depend on probability considerations. As noted by Lempert (1988), the Massachusetts Supreme Court justified its verdict by quoting from an earlier Massachusetts case: '[it is] not enough that mathematically the chances somewhat favor a proposition to be proved; for example, the fact that colored automobiles made in the current year outnumber black ones would not warrant a finding that an undescribed automobile of the current year is colored and not black. . .' Sargent v. Massachusetts Accident Co., 307 Mass. 246, 250, 29 N.E. 2d 825, 827 (1940).

consider the essential allegations proved. It is in fact possible for a party to produce the only evidence in a case and still find this evidence insufficient in law to justify a decision in its favour. If there is insufficient evidence in law, then the case should be withdrawn from the tribunal of fact before any question as to the quality of the evidence can arise (Sheldon, 1996). This might happen in criminal cases when a judge rules that there is 'no case to answer' or where a trial judge refuses to allow a particular point to go to the jury because insufficient evidence has been led to justify it. Although in the event of 'no case to answer', the law does not say which form the jury's verdict should take, 'lack of a case to answer is a question which may be viewed as the *locus classicus* of "not proven"' (Gebbie et al., 1999).[35]

The debate surrounding the 'blue bus case' is clearly connected to the discussion advanced in the present paper in two important respects. Firstly, it is not difficult to see that the 'blue bus case' evokes Keynes's early critique of Bernoulli's and Locke's maxims and highlights the importance of weight like considerations. In this case, the jury has collected as much information as it can (Locke's maxim) and in calculating the probability of the accused being guilty, took into account all the available information (Bernoulli's maxim). But even if the balance between favourable and unfavourable evidence implies a guilty verdict, the jury cannot leave 'out of account' the consideration that 'the available information' upon which this balance is based is 'necessary small' (Keynes, 1973, p. 345). Secondly, the 'blue bus case' shows that, in practical choice situations, Bayesian maximal conditionalization is not a valid criterion to establish how much evidence is enough to inform a decision: a given amount of available evidence might be sufficient according to Bayesian's rules but, at the same time, also highly incomplete. By claiming that 'the information available is lacking', the jury seems in fact to assume the existence of a gap (missing information/relevant ignorance) between the amount of evidence available and the amount of *conceivable* relevant evidence, and that such a gap renders the available evidence insufficient to support the plaintiff's case. The jury's judgment seems thus to rely on considerations about the degree of completeness of the information upon which the probability is based, that is, Keynes's second definition of weight.

Given the importance of questions of evidential coverage in legal settings, it is not surprising that scholars such as Cohen (1977, 1986b) and a number of his followers (see, for instance, Brilmayer, 1986; Schum, 1994; Anderson et al., 2005) have shown a deep interest in using Keynes's conception of evidential weight to transcend the limits of the Bayesian approach.[36] Cohen claims that it is impossible for the judge to 'avoid using, implicitly or explicitly, an assessment of the completeness of the facts before the court' (Cohen, 1986b, p. 639) and that Bayes's rule simply overlooks the problem of how complete is the evidential coverage of matters believed to be relevant in the inference at hand. He therefore proposed a method of assessing the inductive support for a hypothesis in legal settings based on Keynes's notion of evidential weight.

Whilst Cohen's work has the merit of making explicit the importance of the completeness of evidential coverage in inductive reasoning, it does not address the question of finding a rational criterion to evaluate the sufficiency of the available evidence. He speaks about a 'reasonable completeness of the evidence' (Cohen, 1986b, p. 649), but never clarifies what he means by this. This phenomenon, I believe, led scholars such as Shafer (1986) to dismiss Cohen's emphasis on the importance of evidential completeness by claiming that it is completeness relative to established ground rules for the acquisition of evidence that is needed.

But the main point here is indeed to find rational criteria to establish these ground rules. Perhaps, the relevance of this claim might be grasped by looking again at the Scots Criminal Law. The problem of establishing how much evidence is enough to inform a decision in the Scots Law is solved by stating that there must be corroboration – that is, there must be *at least* two sources of evidence to prove every fact that is essential for the Crown to prove before the court is entitled to convict. For instance, the identification of the accused person who committed the crime is not proven unless there are two separate sources of evidence to demonstrate that it was that person (Brown, 2002). If the only evidence in support of a case is the uncorroborated testimony of one witness, it is the duty of the judge to direct the jury that the proof is not sufficient in point of law:

> By the law of Scotland no person can be convicted of a crime or of a statutory offence. . .unless there is evidence of at least two witnesses implicating the person accused or charged with the commission of the crime or offence with which he is charged (as quoted by Wilkinson, 1986, p. 204).

It is clear that the established ground rules here state that two pieces of evidence are enough to convict, and that the judge has to check if the available evidence is complete relative to these ground rules. But the main point is how these ground rules have been set up. I have no doubt that, in many situations, pragmatism is the ultimate response, but when we come to crucial decisions further reflection is needed.

7. Conclusion

Keynes was hesitant about the practical relevance of the theory of evidential weight in the process of rational decision making because of the stopping problem, i.e. the problem of finding a rational principle to decide where to stop the process

[35] In the Scots Law there are in fact three verdicts – guilty, not guilty, and not-proven. The not-proven verdict is used quite often by the Scottish courts: one-third of all jury acquittals and one-fifth of all non-jury trials acquittals are the product of a not proven verdict (Duff, 1999). Camerer and Weber (1992) have explicitly drawn a parallel between Keynes's conception of evidential weight and the Scottish's 'not proven' verdict.

[36] For an introduction to the literature generated by Cohen's work, see Jackson (1996).

of acquiring information in forming a probability judgment before making a decision. Scholars often claim that we should not bother with this problem since this is a pragmatic question that depends on a number of contingent facts such as the methodology of estimation, economic constraints, and other limits to enquiry. I have argued instead that a discussion of the stopping problem is important since it sheds light on the role of the decision maker's subjective assessment of relevant ignorance in the process of rational decision making.

Through some examples, I considered how the acquisition of new evidence may affect the decision maker's subjective assessment of relevant ignorance and therefore of evidential weight. I argued that if we regard weight and confidence as correlative terms, we can explain the following phenomena: that the decision maker's degree of confidence in her forecast may undergo drastic changes during the process of acquiring information; that changes in the decision maker's awareness of the dimension of her ignorance may produce either overconfidence or underconfidence in her forecast; that, different people, on the basis of the same evidence, might hold different degrees of confidence in their forecasts; and finally, that different people, confronting the same decision problem, might feel confident that the evidence acquired is enough to inform the decision at different stages of the learning process.

The first three of these findings are consistent with existing explanations of modern economic phenomena such as the instability of agents' beliefs and expectations (Runde, 1991; Rosser, 2001; Fontana and Gerrard, 2004). However, the last finding, which highlights the absence of a rational criterion to solve the stopping problem, has received little attention. I believe this issue deserves further consideration since it might help us understand why different people, with the same evidence and similar beliefs, show different propensities to act. The arbitrary nature of the stopping rules might thus help explaining different economic behaviour including Keynes's ideas of the entrepreneurs' "animal spirits" and the agent's demand for liquid assets. Moreover, Keynes's advice that we should address questions of evidential completeness may provide a route for dealing with some paradoxes that emerge in legal settings not solvable within the Bayesian approach. Courts of law, in fact, often establish whether the available evidence is enough to convict by relying on weight-like considerations rather than Bayesian maximal conditionalization.

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