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Source: *The British Journal for the Philosophy of Science*, Sep., 1986, Vol. 37, No. 3 (Sep., 1986), pp. 263-278

Published by: Oxford University Press on behalf of The British Society for the Philosophy of Science

Stable URL: https://www.jstor.org/stable/687287

Brit. J. Phil. Sci. 37 (1985), 263–278 *Printed in Great Britain*

Twelve Questions about Keynes's Concept of Weight

by L. JONATHAN COHEN

INTRODUCTION: KEYNES'S CONCEPT OF WEIGHT

In chapter VI of his [1921] Keynes treats the probability of H on E as a property of the argument from E to H. The probability depends for its value on the balance between the favourableness and unfavourableness of the evidence that E states in relation to H. But he considers that there may be another respect in which some kind of quantitative comparison between arguments is possible. 'This comparison turns,' he says, 'upon a balance, not between the favourable and the unfavourable evidence, but between the *absolute* amounts of relevant knowledge and relevant ignorance. As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence; but *something* seems to have increased in either case,—we have a more substantial basis upon which to rest our conclusion.' Keynes expresses this

Received 7 May 1985

by saying that an accession of relevant evidence increases what he calls the 'weight' of an argument. Thus the weight of an argument is independent of the correctness or incorrectness with which such-or-such a probability is assigned to the argument and is not necessarily determined by the probable error of the argument's conclusion (where that conclusion assigns a value to a magnitude). Keynes says that, metaphorically, the weight of the argument from E to H measures the *sum* of the favourable and unfavourable evidence that E states for H, and the probability measures the *difference*. But he does not suggest any method by which weights might be measured and in fact admits that often one cannot even compare the weights of different arguments. He thinks that, 'in deciding on a course of action, it seems plausible to suppose that we ought to take account of the weight as well as the probability of different expectations'. But he finds it difficult to think of any clear example of this, and he does not feel sure that the theory of evidential weight has much practical significance.

In this paper I shall raise twelve questions about weight, in Keynes's sense; and by answering those questions I shall try to show not only why a theory of evidential weight is needed, but also what form it should take if Keynes's seminal intuition is to be preserved. I shall not, however, use Keynes's own symbolism. Keynes used the letters 'h', 'h_1', 'h_2', *etc.* for the premisses of an argument, not its conclusion—which is confusing to a contemporary reader. So I shall use 'E', 'E_1', 'E_2', *etc.* for premisses and 'H', 'H_1', 'H_2', *etc.* for conclusions. Also he abbreviated 'the probability of the argument from h to a' as 'a/h', whereas I shall use instead the more conventional formula '$p(H/E)$' for the probability of the argument from E to H. But I shall follow Keynes in speaking sometimes of the weight of the argument from, say, E to H, sometimes of the weight of the evidence E for H, and sometimes of the weight of the probability of H on E. This flexibility is harmless so long as one remembers that by speaking of the weight of the probability of H on E Keynes does not intend to treat weight as a property of certain propositions such as of the proposition $p(H/E) = n$. On his view it is possible to know the weight of a probability without knowing its value, or to know its value without knowing its weight.

1 HOW DOES WEIGHT COME TO MATTER?

In calculating the premium that a client should pay for a life insurance policy maturing at age 65 a company that wanted to minimise the risk of a loss on this class of business would ideally determine the probability of the client's death before the age of 65 on the basis of all the relevant facts, *i.e.* of all the facts that affect the probability one way or the other. In practice it may well be uneconomic to enquire too closely into the client's health, ancestry and life-style, and some of the relevant facts (*e.g.* about grandparents' medical histories) may be quite unobtainable. But at least the company needs to know the client's sex, say, and whether he or she is at present ill or has a particularly dangerous job or hobby. Since people who have a reason to fear early death are particularly likely to want life insurance protection in the interests of their dependents, a company risks bankruptcy if it does not take such reasons into account in its calculation of premiums to be charged. Equally it risks losing business to other companies if it does not offer appropriate reductions in premiums to clients who show obvious prospects of exceptional longevity. In other words the company must in each case have an appropriate weight of evidence for the probability of survival to 65 that is accepted as the basis for calculating an economic premium. Since an individual may have many relevant features that constitute items of favourable or unfavourable evidence about his survival probabilities, a reliable determination of these probabilities will need to be based upon a sufficiently weighty combination of evidential features. And just the same concern for the weight of relevant evidence would be needed if, instead of trying to *predict* whether their client will survive to the age of 65, the company were trying rather to *explain* why he has done so.

Philosophers sometimes invoke, in this kind of connection, Carnap's ([1950], p. 211) requirement of total evidence. For a conditional probability judgement to be applied as the major premiss of a prediction or explanation, they say, it must be based on all the relevant evidence available. Or, if they prefer a relative-frequency theory of probability to a logical-relation one, they may invoke Hempel's ([1965], p. 397) requirement of maximal specificity for the reference class. But in practice such requirements can rarely, if ever, be satisfied. Even to know just the *available* evidence (physical, meteorological, geological, astrophysical, epidemiological, socio-political, *etc.*) bearing on a person's survival to 65 one would have to work away indefinitely, since what is not available to-day might, with sufficient effort, be made available to-morrow. And almost certainly, if one were thinking in terms of relative frequencies, one would soon be reduced to a reference-class of one member—the person himself—so that no statistical data could be compiled. What is important in practice, therefore, is for such a probability to have as much weight as is permitted by the methodology of estimation, by pertinent economic constraints, by the current state of scientific knowledge, by the nature of the subject-matter and by any other limits to enquiry. So at least comparative judgements of weight have to be made, and it may be worth while investigating whether the weights of different probabilities can also be ranked or measured. But we can afford to take rather less interest in theoretical ideals like the requirement of total evidence. In the real world we are occupied with what is better and what is worse, not with what is perfect. (Of course, Bayesian conditionalisation may be seen in Keynesian terms as a device for increasing weight. But it too does not issue in explicit comparisons, rankings or measurements of weight.)

An illuminating way to look at the matter is this. Suppose that the estimated probability that a person will survive to age 65, on the evidence that he or she is a lorry-driver, is 0.8. This is a generalised judgement of conditional probability, in the sense that it does not assert anything about a particular person. From it we can derive its instantiation for a particular person only on the assumption that our reference to that person does not add to or subtract from the evidence on which the probability is conditional. (Indeed without that assumption we could hardly derive its instantiation—as we can[1]—in counterfactual cases, *i.e.* even in the case of a person who is not in fact a lorry-driver.) But on the unconditional issue no such assumption is possible. If we want to infer the *unconditional* survival prospects of a particular person—say, Mr Smith—we cannot avoid the need to allow for the fact that Mr Smith may be specially circumstanced in some relevant way: lorry-driving may be his work, but hang-gliding may be his hobby. So inferability to a singular judgement of unconditional probability—to the judgement, say, that the probability of Mr Smith's surviving to 65, is 0.8—must depend on how much of the relevant facts about Mr Smith is included in the premisses. That is, this inferability varies directly with the weight of the evidence. If $p(H/E) = n$, then the weight of $p(H/E)$ determines the strength of our entitlement to infer from E to $p(H) = n$.

At any rate it is convenient to speak thus in the present context. But strictly speaking, and to avoid any possibility of confusion, we should note that, because of the considerations just mentioned, there is some equivocation here between '$p(H) = n$' and '$p(H/E) = n$'. In the latter—the singularised judgement of conditional probability—each occurrence of the referring expression 'Mr Smith' may be replaced *salva veritate* by an occurrence of, say 'Mr Brown', whereas in the singularised judgement of unconditional probability that is not so. Correspondingly the value of this $p(H)$ is not related to the value of this $p(H/E)$ by Bayes' theorem. For, if '$p(H) = n$' had instead been used here to state a prior probability relative to '$p(H/E) = n$', we should have had to grant the former the same substitutivity entitlements as the latter. Indeed philosophers who use the expression 'statistical syllogism' to name an inference of the kind in question, from 'E' and '$p(H/E) = n$' to '$p(H) = n$', are speaking rather misleadingly. Genuine syllogisms, whether valid or invalid, do not equivocate thus between premiss and conclusion.

2 WHY DID KEYNES NOT APPRECIATE HOW WEIGHT MATTERS?

When we view the problem in the above terms it is easy enough to see not only how weight matters but also why Keynes was unable to appreciate how

[1] The difference between counterfactualisable and non-counterfactualisable probabilities is discussed in (Cohen [1986], §17–19). A generalised conditional probability that is non-counterfactualisable has zero weight, since its value is the outcome of an accidental relationship and affords no basis for inference, in a particular case, to the value of an unconditional probability.

it matters. Because he held that probability should be thought of as a logical relation (and because he apparently had no conception of deducibility from the null premiss), he could hardly allow a place for judgements of unconditional probability other than as mere ellipses of 'ordinary speech' ([1921], p. 7). *A fortiori* he could hardly appreciate the existence of the problem of how to grade our entitlement to detach an unconditional probability from a conditional one. It is scarcely surprising therefore that Keynes admitted to doubting whether the theory of weight had any practical significance, since by his very treatment of probability as a function of ordered pairs of propositions he had cut himself off from the possibility of articulating the nature of this significance.

Nevertheless it is very much to Keynes's credit that he did not suppress his various intuitions about the nature of weight. And perhaps these intuitions were strengthened by the fact that, like most people, he could not altogether rid himself of the intuitive idea that probability can also be conceived as the relative frequency with which one set shares its membership with another, while he recognised that the weight of a relative frequency can be increased by a relevant partitioning of the reference class ([1921], p. 27). Unfortunately, however, the relative-frequency theory of probability has a parallel difficulty in articulating just how weight matters. Just as the logical relation theory seems not to assign probabilities to single propositions (as distinct from ordered pairs of propositions), so too the relative frequency theory, seems not to assign them to single sets (as distinct from ordered pairs of sets).

3 IS WEIGHT INCREASED BY EACH ADDITION OF RELEVANT EVIDENCE?

In the standard, probability-theoretic sense of 'relevant' E_2 is relevant to $p(H/E_1)$ if and only if $p(H/E_1\&E_2) \neq p(H/E_1)$. But, if the weight of an argument is to continue to grow with each increment of relevant evidence, it must clearly also be taken to grow even with the addition of a certain kind of irrelevant evidence. This is because one might add the evidence $E_2\&E_3$ where E_2 alters the probability of the argument in one direction exactly as much as E_3 alters it in the other. In such a case the addition of E_2 on its own would increase the weight of the argument first, and then the addition of E_3 to the evidence would increase the weight yet further. So, since it should presumably make no difference whether E_2 and E_3 are added successively or conjunctively, the addition of the conjunction $E_2\&E_3$ must also increase the weight of $p(H/E_1)$ even though it is not—in the standard sense—a relevant piece of evidence. Keynes therefore defined a proposition as 'relevant' to $p(H/E_1)$ in this connection if and only if it entails a proposition E_2 such that $p(H/E_1\&E_2) \neq p(H/E_1)$. But it would obviously be simpler to retain the normal probability-theoretic sense of 'relevant', and then to say—using Keynes's functor '$V(\ldots/---)$' for 'the weight of the probability of..., given

---'—that $V(H/E_1\&E_2) > V(H/E_1)$ if and only if E_2 entails a proposition that is relevant to $p(H/E_1)$.

Unfortunately this will not quite do as it stands. There is a further difficulty, which apparently Keynes did not see. According to classical logic any proposition E_2 you like, however irrelevant to $p(H/E)$, nevertheless entails the disjunction $E_2 \vee H$, and $E_2 \vee H$ is certainly relevant to $p(H/E_1)$ because $p(E_2 \vee H/H\&E_1) = 1$ and so by Bayes's theorem

$$p(H/E_1\&(E_2 \vee H)) = \frac{p(H/E_1) \times p(E_2 \vee H/H\&E_1)}{p(E_2 \vee H/E_1)} = \frac{p(H/E_1)}{p(E_2 \vee H/E_1)}$$

where $p(E_2 \vee H/E_1) > 0$. So Keynes seems to have trivialised the concept of weight by allowing any proposition to increase the weight of any argument. In order to avoid such trivialisation we need to tighten the conditions under which $V(H/E_1\&E_2) > V(H/E_1)$. We need to say that this inequality holds if and only if E_2 entails a proposition E_3 that is relevant to $p(H/E_1)$, where no proposition E_4 occurs in E_3 (or in any equivalent of E_3) such that E_2 entails E_4 and, without affecting the relevance of E_3 to $p(H/E_1)$, E_4 can be replaced in E_3 (or in some equivalent of E_3) by a proposition that has no relevance to $p(H/E_1)$. And we can also say that under just these same conditions E_2 will give at least as much weight to $p(H/E_1)$ as E_3 does.

4 DO ARGUMENTS INHERIT WEIGHT VIA THE ENTAILMENTS OF THEIR CONCLUSIONS?

We have seen in 3 how entailment between evidential propositions affects weight. The question also arises whether entailment between conclusions carries with it any necessary consequences for assessments of weight. But the answer must be that it does not. For we know that, where H_1 entails H_2, E_2 may be relevant to H_2 on E_1 and yet not relevant to H_1 on E_1, or to H_1 on E_1 and yet not relevant to H_2 on E_1 ([Carnap], 1950, pp. 348–97). On the other hand it is certainly reasonable to assume, so far as weight is a grading of inferability, that the weight of an argument is unaffected when any proposition in its premisses or conclusion is replaced by another proposition that is necessarily equivalent to it.

5 CAN ARGUMENTS IN CO-ORDINATE TERMS BE COMPARED FOR WEIGHT?

The question now arises whether any comparisons of weight can be drawn between $p(H_1/E_1)$ and $p(H_2/E_2)$ when *no* entailments hold in either direction between E_1 and E_2. It is tempting to claim, for example, that $p(H_1/E_1)$ and $p(H_2/E_2)$ should be said to have the same weight when each pair of propositions ascribes the same pair of predicates though to different individuals, and Keynes does claim this ([1921], p. 73). After all, the weight of a probabilificatory argument ought surely to remain invariant under uniform exchanges of the individuals to which it refers, much as the validity of a causal or logical argument remains invariant under these conditions. But the same point is perhaps more accurately made by treating weight as being *primarily* a property of generalised conditional probabilities, and *derivatively* of their substitution-instances. Indeed, the weight of the evidence is the same for a generalised conditional probability as for each of its substitution-instances because the relevance of the evidence is the same (on the assumption that our reference to the instantiating individual does not add to or subtract from the evidence on which the probability is conditional).

It is also possible to extend the theory of weight by assuming that the weight of an argument is unaffected when each occurrence of a particular predicate P_1 within its premisses or conclusion is replaced by an occurrence of another predicate P_2, just so long as P_1 and P_2 are both members of the same family of co-ordinate but mutually inconsistent predicates. A predicate might thus derive its weight-increasing potential for a given argument from the relevance to that argument of another predicate in the same family, or from its relevance to an argument formulated in terms of predicates that belong to the same families as the predicates in the given argument.

What would be the justification for this? First, it seems reasonable that, when we have conducted an enquiry to find out whether a candidate for life insurance runs exceptional health risks in his leisure-time activity, we should be able to add the results of the enquiry to the premisses of the argument for his survival to 65 in such a way as to give that argument the same increment of weight independently of whether the probability of the argument is affected by this (as might be the case if his only hobby were discovered to be hang-gliding) or not affected (as might be the case if his only hobby were discovered to be stamp-collecting). After all, whichever the outcome the detachment of a probability for the argument's conclusion is now protected in relation to that issue, and the primary purpose of assessing weight is, as we have seen, in 1, to evaluate such detachability. So though Keynes's test gives us no authority for the idea, it is difficult to avoid supposing that a viable theory of weight should allow invariance of weight under changes of a premiss's predicate within the same family: if one member of the family increases the weight of an argument to a particular conclusion, each of the others does also, and if one argument to a particular conclusion with premisses $E_1, E_2 \ldots E_n$ has its weight increased by a certain additional premiss then so does every other argument (to the same conclusion) that differs only by replacing a predicate in one of $E_1, E_2, \ldots, E_n$ by a term co-ordinate with it.

Secondly, if the value of the probability $p(H_1/E_1)$ is known to be affected by the addition of E_2 to the premisses, a change is thereby made in the constraints that are known to affect the value of any probability

$p(H_2/E_1\&H_2)$ where H_2 is inconsistent with H_1, because we know mathematically that $p(H_2/E_1\&E_2) \leqslant 1 - p(H_1/E_1\&E_2)$. So it is reasonable to suppose that E_2 increases the weight of $p(H_2/E_1)$ just as much as it increases that of $p(H_1/E_1)$. Keynes speaks of this only in the case in which H_2 is the contradictory of H_1, where he accepts that $V(H/E) = V(\bar{H}/E)$. But the principle at issue is a more general one, extending to families of mutually exclusive predicates as well as to pairs of contradictory ones.

We see thus that the same principle is at work in regard to both the premisses and the conclusion of an argument. The weight of the argument is unaffected by the substitution of one predicate for another within the same family of co-ordinate but mutually exclusive predicates.

6 DOES WEIGHT HAVE ANY LIMITING CASES?

If a necessarily true proposition is added to an argument's premisses (whatever they may be) it cannot change the probability of the argument. Hence it adds no weight to an argument, and if *all* the premisses of an argument are necessarily true, the argument has minimal weight.

On the other hand, according to classical logic, if a necessarily false proposition is added to an argument's premisses, it entails every proposition and *a fortiori* therefore it entails every proposition that is relevant to the conclusion. So on this basis, if the premisses of an argument contain a contradiction, the argument has maximal weight.

If an argument has a necessarily true conclusion, it has a probability of 1 whatever the premisses. So no additional premiss can affect its probability, and it must therefore be regarded as already having maximal weight. (Correspondingly, if H is necessarily true, we are fully entitled to detach $p(H) = 1$ from $p(H/E) = 1$ whatever E may be.)

Similarly if an argument has a necessarily false conclusion, it has a probability of 0 on any premiss, and again no additional premiss can affect its probability. So it too has maximal weight (as, indeed, we could also show by combining the principle $V(H/E) = V(\bar{H}/E)$ from **5** with the principle established in the previous paragraph).

Also, if the premisses of an argument already entail its conclusion, or if they already contradict its conclusion, no additional premiss can affect its probability. So under these conditions too the argument has maximal weight.

7 IF ONE PREMISS IS OF GREATER RELEVANCE THAN ANOTHER, DOES IT ADD MORE WEIGHT?

The question here—not discussed by Keynes—is this. Suppose $p(H/E_1\&E_2)$ differs more from $p(H/E_1)$ than does $p(H/E_1\&E_3)$. We can then say that E_2 has a relevance to $p(H/E_1)$ that is of greater extent than E_3

has. Is the weight of $p(H/E_1\&E_2)$ therefore greater than that of $p(H/E_1\&E_3)$, or not? There are some reasons for saying that it is, but stronger reasons for saying that it isn't.

On the one hand it seems at first sight intuitively implausible to hold that the probability of a male person's surviving to age 65 has just as much weight as the probability that a person with a dangerous hobby will survive to age 65. If time or other resources for enquiry were in short supply, would it not be much more important in assessing a life insurance premium to determine whether the person concerned had a dangerous hobby than to determine what the person's sex was? And, in general, if we consider the kind of purpose for which comparisons of weight are needed, it looks at first as though the extent of a new premiss's relevance ought to enter into comparisons of incremental weight. If we want to detach a value for the probability of H, we seem better off starting with a value for $p(H/E_1\&E_2)$ than with one for $p(H/E_1\&E_3)$ if E_2 is more relevant to $p(H/E_1)$ than E_3 is.

On the other hand this way of assessing weight soon plunges into paradox. Suppose a set of evidential items $E_1, E_2, \ldots, E_{100}$ in regard to a hypothesised conclusion H. Suppose too that quite a lot of these items, on their own, ground low probabilities in favour of H, quite a lot ground high probabilities in favour of H, and quite a lot ground intermediate probabilities at varying levels. One way of ordering these items would be to begin with those highly in favour of H, then proceed with those slightly less in favour and so on down, ending up with those highly in favour of $\bar{H}$. In such a carefully graduated order the extent of the relevance of each new piece of evidence, after the first, would tend to be small. So if the weight of the argument were to be affected by the extent of the relevance of each incremental piece of evidence, as well as by the number of those pieces, the additional effect on the overall weight would be minimal. But, if instead the evidential premisses were ordered so as to alternate as violently as possible between favourable and unfavourable items, the overall effect on the weight would be very different, if extent of relevance was allowed to affect the issue at each incremental step. Hence, if we allow the extent of an added premiss's relevance to affect the cumulative weight of an argument, we could end up with different weights for an argument to the same conclusion from logically equivalent premisses, just because we calculated the weights on the basis of different orderings for the addition of new premisses. This is certainly inconsistent with what was said about the substitutivity of equivalents in **4**, and seems unacceptably paradoxical. The total weight of the evidence for a conclusion ought to be independent of the order in which different evidential items are stated. For some people the psychological effect may vary with the order of statement, but a competent reasoner ought to be able to discount such effects. So I conclude that in a theory developing Keynes's seminal idea any weight added to an argument by a new premiss should stem solely from the fact that this premiss entails a relevant proposition and not at all from the extent of that relevance.

8 IS WEIGHT DETERMINED BY RELATED PROBABILITIES?

The term 'weight' has been used by philosophers to cover one or other of a variety of probability-related measures. Thus Reichenbach ([1949], p. 465) held that, if we know the limit towards which the relative frequency of a certain kind of outcome tends within a sequence of events, then this value can be regarded as 'the weight of an individual posit concerning an unknown element of the sequence'. Or, in other words, 'the weight may be identified with the probability of the single case'. Clearly this is not the sense in which Keynes was using the term in chapter VI of his [1921], since he emphasised that an argument of high weight is not, as such, 'more likely to be right' than one of low weight. It is easy to find arguments that have high probability but low weight, or low probability but high weight. (Elsewhere in his [1921], however, Keynes sometimes uses the term 'weight' more loosely: *e.g.* p. 218.)

Again, Good [1968] has discussed weight in the sense in which the weight of evidence concerning H that is provided by E, given G, is equal to log $\{p(E/HG) \div p(E/\bar{H}G)\}$. But the quantity of evidence relevant to a certain argument is independent of the probability of the evidence given the conclusion. A great quantity of evidence might have been collected in a murder trial, with most of it tending to incriminate the accused, but it might also include an unshakable alibi. In such a case the evidence available might have relatively low probability, given the innocence of the accused, but it would have a heavy Keynesian weight. Similarly, that a person died before the age of 80 given that he died before the age of 8 has maximum probability, but the evidential fact that he died before the age of 80 gives a rather small increment of weight to any argument that he died before the age of 8.

It is sometimes suggested that, if the probability that a person assigns to his belief may be quantified in terms of the odds at which he would accept a bet on its truth, given specified evidence, then the weight of that evidence may be taken to be reflected in the amount that he is prepared to bet: he may be expected to be willing to put a larger sum at risk when there is more evidence from which to estimate appropriate odds. But other considerations also may affect our attitude towards the size of a bet. Suppose that there is a great deal of evidence, and that this evidence suggests the appropriateness of very long odds. Would you really be willing to risk losing just as large a part of your fortune then as you would risk losing if the odds, on the same evidence, were much shorter?

It is also sometimes suggested that the weight of an argument may be taken to vary inversely with the mathematical expectation of gain from a search for further relevant evidence. But this suggestion is open to at least two cogent objections. First, in order to avoid begging the question the gain talked about must presumably be in some non-epistemic kind of utility. And this raises familiar problems about the evaluation of epistemic functions by reference to non-epistemic criteria, as discussed in *e.g.* Levi's [1984].

Secondly what are we to say when, for example, a vital eye-witness has died without ever disclosing what he saw? The expectation of any kind of gain from further research in that direction may then be zero, but the weight of the evidence about what actually happened is not increased because of the missing data. This is because the weight of the evidence obtained is being assessed by comparison with the supposed totality of relevant facts, not with the supposed totality of discoverable relevant facts. So, even if we had all the available evidence, our argument might still not have maximal weight. Sometimes, for example, the prosecution cannot prove guilt beyond reasonable doubt even though someone must have committed the crime in question.

9 CAN ONE ARGUMENT BE COMPARED FOR WEIGHT WITH ANOTHER IF ITS TERMS NEITHER ENTAIL NOR ARE CO-ORDINATE WITH THE OTHER'S TERMS AND NEITHER ARGUMENT IS A LIMITING-CASE?

We have seen that arguments may be compared with one another for weight where certain entailment relations hold between their premisses (as in **3**), or where their terms are co-ordinate and mutually exclusive (as in **5**), and that weight is maximised or minimised in certain special cases (**7**), but that weight is not determined by the sizes of associated probabilities (**8**). The question now arises whether any other principles bear on the determination of weight. In particular, since the extent of a premiss's relevance does not affect its incremental weight (**6**), should we treat this as a licence to suppose, by what I shall call 'the principle of equipollence', that the members of different families of predicates enhance the weight of an argument equally when they enter relevantly into its premisses? For example, in **7** we have seen reason to reject the view that the probability that a person with a dangerous hobby will survive to age 65 has greater weight than the probability that a male person will survive to age 65. But does it therefore follow that these two probabilities should be attributed the same weight? Is the inequality to be rejected because no comparisons of this kind are possible, or just because the true comparison is one of equality? I shall argue for the former thesis, or—more exactly—for the thesis that comparisons of this kind are possible only on the basis of unacceptable assumptions.

Consider the two predicates 'has a dangerous hobby' and 'has a dangerous hobby and a weak heart'. We obviously cannot accept (as the principle of equipollence would seem to require) that the weight of evidence is unaffected by which of these two predicates is ascribed in the evidential proposition, since having a weak heart is relevant to whether a person survives to age 65 even on the condition that he or she is a lorry driver and has a dangerous hobby. So either we have to reject the principle of equipollence altogether, or we must restrict its application to primitive predicates in some appropriately tailored language-system. But the latter kind of move would introduce a substantial element of linguistic convention into the assessment of weight. The weight of an argument would depend not just on facts about probabilistic relevance but also on which predicates were chosen as primitive and therefore as having no non-trivial entailments. If, for example, 'has a dangerous hobby' and 'is male' were both treated as primitive predicates, the probability that a person who has a dangerous hobby will survive to age 65 would be assigned the same weight as the probability that a male person will survive to age 65; but, if one of the two predicates were treated as primitive and the other not, the two probabilities would have different weights. So, unless there is reason in a particular area of enquiry to suppose that the primitiveness or non-primitiveness of a predicate is unambiguously determined by the facts rather than by convention, it looks as though the principle of equipollence cannot be rescued.

It would certainly be very convenient if each of the primitive predicates that might enter relevantly into the premisses of any argument for a certain conclusion added an equal weight and all the relevant predicates that added least weight were primitive. The addition of a premiss containing any one such predicate could then be supposed to add one unit of weight to the argument. But this convenient state of affairs is most unlikely to obtain. The primitiveness or non-primitiveness of a predicate is a relatively *a priori* property of it that cannot usefully be made to depend on facts about relevance to a particular argument for a particular conclusion. There is a fairly obvious reason for this. An issue that has no relevance for one argument may have considerable relevance for another, whether the arguments be for the same conclusion or for different ones, and the structure of a language—including the primitiveness or non-primitiveness of certain predicates—cannot usefully be geared to one particular series of arguments for one particular conclusion. If it were so geared, the strategy would be self-defeating because the range of comparisons permissible within the language would be excessively narrow.

10 CAN WEIGHT BE RANKED AS WELL AS COMPARED?

If the principle of equipollence is indefensible, there is no natural unit of weight and the prospects of any non-arbitrary system for *measuring* weight are very poor. But that does not exclude the possibility of having a principled system for ranking it, at least in relation to arguments about a given subject-matter, *i.e.* about conclusions that involve predicates of a given family. Such a system would require that the weight of any two arguments about the given subject-matter should be comparable, that superiority in weight should be a transitive relation, and that a level of least weight should be recognisable. And we should have such a system if we had an ordering for a certain set of families of evidential predicates and concerned ourselves only with arguments from premisses that contain just predicates belonging to the first family, or just those predicates plus predicates belonging to the second family, or just predicates from each of the first three families, and so on cumulatively. Moreover, when our list of such families was finite and determinate, we would also have a recognisable level of maximum weight and could tell how far short of it a particular argument fell. But how are predicate-families to be ordered appropriately?

We have to bear in mind that the primary purpose of the weight-judging enterprise is to grade our entitlement to detach a value n for the probability of H, when given premisses E and $p(H/E) = n$. And we have to bear in mind also that the time or other resources available for enquiry may be limited and that it is desirable for the probability of H that we detach to be as close as is possible within those limits to H's true probability. Indeed, because it may be the case that very few individuals in the population available for sampling possess certain complex combinations of characteristics, it may be impracticable to determine a probability that has more than a certain level of weight. For reasons such as these it will be important to give priority in the ordering of predicate-families to those families that contain at least one predicate which is highly relevant in relation to the accepted prior probability of at least one conclusion in the given field. This is where the intuition about extent of relevance that was discounted in **6** comes into its own. The predicate-family containing a predicate of greater relevance to some conclusion in the given field than any other predicate-family contains should be placed first in the ordering, and so on down. But, of course, two or more predicate-families might tie at any stage, and if so we should either have to resort to some arbitrary criterion of priority in their case—say, an alphabetical one—or better, if we want to avoid any element of arbitrariness in constructing our system of ranking, we should combine such a set of two or more tying predicate-families into a single predicate-family that contains every possible combination of the predicates belonging to the tying predicate-families. With our predicate-families thus ordered we should not only be able to rank the weights of any arguments from premisses of the kind described in the previous paragraph: we should also be able to ensure that, even if the argument was not based on a conjunction of *all* potentially relevant premises and so its weight was not maximal, then at least it would be based on a conjunction of the most relevant premisses so that the probability detached was as near to the true one as it could be for that number of premisses.

Moreover, if on the basis of the same ordered set of predicate-families we can rank evidential weight for *conclusions* containing predicates that belong to two or more different families, these weight-rankings are obviously going to be comparable with one another. For example, if E adds weight to the probability that Mr Smith will survive to 65, given that he's a lorry-driver, then presumably E adds the same weight to the probability that Mr Smith will survive to 66 given that he's a lorry-driver, even though surviving to 65 and surviving to 66 are not incompatible with one another.

11 IS IT WORTH WHILE KNOWING THE WEIGHT OF AN ARGUMENT WITHOUT KNOWING ITS PROBABILITY?

Keynes defines his concept of weight in such a way that it is possible to know the relative weight of an argument without knowing its probability. Yet the point of the weight of an argument, as we have seen in 1 above, is in order to be able to grade our entitlement to detach an unconditional probability from a conditional one. We need to know the relative value of $V(H/E)$ in order to grade our entitlement to infer $p(H) = n$ from the conjunction of E with $p(H/E) = n$. So, associated with the weight of an argument from E to H, there is also a value n such that we are directed by $p(H/E) = n$ towards the assignment of n to $p(H)$ when E is given. The weight of the argument grades our entitlement to proceed in that direction. Hence, though the size of the weight and the value of the probability are independent of one another, we need to know the direction towards which the argument is headed if we are to be able to use our knowledge of its weight.

12 WHAT IS THE CONNECTION BETWEEN KEYNESIAN WEIGHT AND BACONIAN LEGISIMILITUDE?

Elsewhere, in a development of Francis Bacon's seminal ideas about inductive reasoning (Cohen [1970] and [1977]) I have argued for a method of ranking the reliability of any generalised conditional (or of any of its substitution-instances) within a particular field of factual enquiry by reference to the complexity of the controlled experiments that it survives. I call this 'the method of relevant variables'; and I call the parameter that it grades 'legisimilitude' (Cohen [1980a] and [1985]) *i.e.* proximity to being a natural law. Experiments of different degrees of complexity are (if properly insulated from external factors) like simulations of possible worlds that differ from one another in the variety of combinations of inductively relevant circumstances that they contain, and generalisations are shown to possess greater and greater legisimilitude as they are shown to hold good over varieties of possible worlds that are more and more richly stocked with combinations of inductively relevant circumstances. One can show too that these rankings of legisimilitude constrain one another in accordance with the principles of a modal logic that generalise on the Lewis-Barcan system S4.[1] It follows that universally quantified conditionals (or their

[1] There are isomorphisms also with the Levi-Shackle theory of potential surprise: see (Cohen [1980b], pp. 64–66 and p. 171).

substitution-instances) which are qualified in such a way as to withstand falsification by any experiment of a certain level of complexity may be attributed an appropriate level of legisimilitude by the same system of ranking.

The various lines of reasoning that I have developed in the past in order to justify this system of ranking Baconian legisimilitude and the lines of reasoning developed in **3–9** above in regard to the comparing or ranking of Keynesian weight are quite independent of one another. The Baconian scheme was defended in a context of concern with generalisations that are rooted in causality and it treats of generalisations about probabilities only as a special case. The Keynesian scheme has been defended in a context of concern with probabilistic relevance and only a limiting case of it (where the probability involved is 1) instantiates deterministic generalisation. But both lines of reasoning converge on precisely the same underlying structure. The method of ranking weight that was defended in **9–10** is an application of the method of relevant variables, and the various logical constraints on the comparison or assignment of weight that were defended in **3–6** are all derivable within the logical syntax of legisimilitude.[1] More exactly, on a proper reckoning the Keynesian weight of $p(H/E)$, where $p(H/E) = n$, should turn out equal to the Baconian legisimilitude of $E \rightarrow p(H) = n$.

The importance of this fact seems to me to be that, even if your intuitions about causality are insufficiently powerful to drive you down the Baconian road, you may well find that your intuitions about probability will drive you down the Keynesian road to the same destination as that to which the Baconian road would have led you. Moreover it should be clearer now how Baconian (*i.e.* weight-orientated) modes of reasoning are not intrinsically in any kind of conflict with probabilistic ones but can serve to complement them.

Finally it is worth considering Keynes's theory of weight in relation to his proposal of a probabilistic mode of assessment for Baconian induction, *i.e.* for inductive support that depends on the variety of relevant evidence. When Keynes talked about such induction in Part III of his [1921] he had in mind the supporting of deterministic generalisations like 'All A are B', which he calls 'universal inductions', rather than the supporting of probabilistic ones like 'All A have an n per cent probability of being B', which he calls 'inductive correlations' or 'statistical inductions'. If he had thought more about the latter in connection with his theory of weight, he could have used it to provide an appropriate mode of assessment for their support and he might then have sought to extend the theory of weight so as to cover deterministic generalisations also and produce a general theory of Baconian legisimilitude. But in fact when he briefly ([1921], pp. 409–12) has

[1] The development of this logical syntax was begun in Cohen ([1970], pp. 216–37) and continued in Cohen ([1977], pp. 229–40): compare theorem-schema 710 there with **3** here, 248 with **4**, 357 and 613 with **5**, and 703, 704, 707 and 728 with **6**.

regard to considerations of weight in connection with statistical inductions he does not use the term 'weight' at all or refer back to his earlier discussion of the subject.[1]

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[1] I am grateful to the Editor for helpful comments on an earlier draft, and also to participants in a discussion at the Conference on Cambridge Philosophy of Science, 1850–1950, held in Bologna, Italy, during May 30–June 1, 1985.