"Till at last there remain nothing"

Atkinson, David; Peijnenburg, Jeanne

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Hume's *Treatise* 1.4.1 in contemporary perspective

David Atkinson[1] · **Jeanne Peijnenburg**[1]

Abstract
In *A Treatise of Human Nature*, David Hume presents an argument according to which all knowledge reduces to probability, and all probability reduces to nothing. Many have criticized this argument, while others find nothing wrong with it. In this paper we explain that the argument is invalid as it stands, but for different reasons than have been hitherto acknowledged. Once the argument is repaired, it becomes clear that there is indeed something that reduces to nothing, but it is something other than what, according to many, Hume had in mind. Thus two views emerge of what exactly it is that reduces. We surmise that Hume failed to distinguish the two, because he lacked the formal means to differentiate between a rendering of his argument that is in accordance with the probability calculus, and one that is not.

1 Introduction

The section 'Of scepticism with regard to reason' in David Hume's *A Treatise of Human Nature* (Book 1, Part 4, Section 1) continues to play on the philosopher's mind. After an apparent lull, William Morris attracted renewed attention to it in 1989:

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✉ David Atkinson
d.atkinson@rug.nl

Jeanne Peijnenburg
jeanne.peijnenburg@rug.nl

1   Faculty of Philosophy, University of Groningen, Oude Boteringestraat 52, 9712 GL Groningen, The Netherlands

If we ever are to understand Hume's view of the role of reason, … we should first figure out how to integrate 'Of scepticism with regard to reason' into the picture.[1]

In response to Morris's appeal many interpretations were advanced, which greatly improved and deepened our understanding of this "least understood" passage in Hume's writings (ibid.).

'Of scepticism with regard to reason' aims to show the absurd situation that will result if we rely purely on reason, ignoring habit, custom, and the sensitive part of our natures. As David Owen has pointed out, the text consists of three arguments, two negative and one positive; the positive argument moreover incorporates the conclusion of the section as a whole.[2] In this paper we concentrate on the second negative argument, which is the one that has received the most attention. The majority of the philosophers who studied this argument think it is incorrect, but there are also quite a few who believe there is nothing wrong with it. We shall argue that the argument is invalid as it stands, but for different reasons than have so far been acknowledged. However it can be repaired, and then it becomes clear what is the problem: the argument is open to two reconstructions, one that is in accordance with the probability calculus and one that is not. We surmise that Hume failed to distinguish between the two, a failure that, we argue, is inadvertently revealed by David Owen's reading of the argument.

Before going into the details, let us briefly recall all three arguments. The first negative argument encompasses the idea that "knowledge degenerates into probability" (T 1.4.1.1) or that "all knowledge resolves itself into probability" (T 1.4.1.4).[3] The upshot is that we can never be certain of knowing a proposition p, even if it is the case that we indeed know p. For example, we might have correctly carried out a long calculation and in that sense be said to know the outcome, but at the same time be unsure about whether we were really correct: we do not know that we know.[4] In this case we only believe with a certain probability that we know we have carried out the calculation correctly.

The second negative argument has been given various names: 'Hume's probability argument', the 'probability reduces to nothing argument', the 'iterative probability argument', or the 'regression argument'.[5] Basically it is like the first negative argument, but now applied to probability judgements, including the probability judgement that we made about whether we carried out the calculation correctly. We cannot be certain that the latter judgement is correct; at best we can believe it with a certain probability, so that we have a second-order probability judgement about a first-order probability judgement. But of course we cannot be certain of the second-order probability judge-

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[1] Morris (1989, p. 58). Slightly earlier the section in question had also been discussed in Fogelin (1983) and Fogelin (1985), albeit critically.

[2] Owen (1999), Owen (2004).

[3] *A Treatise of Human Nature* (Hume 1739/2006). References to this work are given by 'T', followed by four numbers, which indicate Book, Part, Section, and paragraph as in the volume edited by D.F. Norton and M.J. Norton, Oxford University Press, 2006 (first published 2000).

[4] In Cartesian terms: we lack transparent clear and distinct ideas. Nowadays one would call this a breakdown of the *KK*-principle, according to which knowing p implies knowing that one knows p, in symbols: $Kp \rightarrow KKp$.

[5] DeWitt (1985), Lolordo (2000, p. 433), Loeb (2010, p. 327), Meeker (2000), Fogelin (2009, p. 43).

ment either; at best we can only have a third-order judgement about the second-order judgement, and so on. According to Hume this series of higher and higher-order probability judgements will reduce the first probability, until, if the regress is infinitely long, the first probability will vanish completely: "at last there remain nothing of the original probability, however great we may suppose it to have been, and however small the diminution by every new uncertainty" (T 1.4.1.6).

Together the two negative arguments entail "a total extinction of belief and evidence" (T 1.4.1.6). This may be a welcome conclusion for "those sceptics, who hold that all is uncertain, and that our judgment is not in *any* thing possest of *any* measures of truth and falsehood" (T 1.4.1.7), but Hume hastens to say that he is no part of "that fantastic sect" (T 1.4.1.8). He recalls that we *do* have beliefs, both in philosophy and in daily life. From this he however does not draw the conclusion that the negative arguments are fallacious; on the contrary, although the arguments are "entirely superfluous" (T 1.4.1.7), Hume indicates that one "can find no error" in them (T 1.4.1.8).

The negative arguments sketch the grotesque scenario that would appear if our reasoning were left to its own devices. Reason would then annihilate itself and the beliefs it produces would "terminate in a total suspence of judgment" (T 1.4.1.8). Fortunately, this unnatural scenario will never come to pass—and that is the gist of the positive argument. This argument makes us understand that reasoning is "*deriv'd from nothing but custom*", and that belief is "*more properly an act of the sensitive, than of the cogitative part of our natures*" (T 1.4.1.8). Thus "nature breaks the force of all sceptical arguments in time, and keeps them from having any considerable influence on the understanding" (T 1.4.1.12). In its entirety, 'Of scepticism with regard to reason' offers a reductio: it aims to demonstrate that a regress of subjective probability judgements leads to an absurdity.[6]

Especially the negative arguments in Section 1.4.1 have been subjected to many different interpretations; and although these interpretations have significantly improved our understanding, there is as of yet no agreement on how to deal with the arguments. One of the disagreements concerns the question whether formal tools, notably from modern probability theory, can serve to clarify the matter. Some scholars think they can, for example Fred Wilson, Richard DeWitt, Robert Fogelin, Kevin Meeker, and in the early days Popkin, Quine, Von Wright, and Peirce.[7] Others however are strongly against using formal devices. Thus David Owen writes: "[W]hat Hume is concerned with in these negative arguments is not some formal assignment of probability, calculated according to the calculus".[8]

Both formalists and anti-formalists, as we will call them for convenience, rely on their own reasonings. Thus anti-formalists tend to point out that a formal interpreta-

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[6] *Cf.* Meeker (2013, p. 55).

[7] Wilson (1983), Wilson (1985), DeWitt (1985), Fogelin (1985), Meeker (2000), Popkin (1951), Quine (1946), Von Wright (1941), Peirce (1905).

[8] Owen (2015, p. 116). Don Garrett is of the same opinion (*cf.* Garrett 2000; Garrett 2004; Garrett 2006). See also Gower (1991). The disagreement about whether Hume's arguments can be fruitfully evaluated with the help of probability theory is not restricted to his regress argument in 1.4.1. A similar dispute applies for example to his argument about miracles. For an in-depth use of formal tools to clarify Hume's assertions about the credence to be given to a testimony that a miracle has occurred, see Ahmed (2015).

tion leads to glaring inconsistencies in Hume's text, and to the unlikely conclusion that Hume, beside defending a sceptical position, also embraces a form of dogmatism. Formalists, on the other hand, bring to mind Hume's talk about degrees of probability and his suggestion that these can be measured. They explain that formal probability theory was very much in vogue during Hume's lifetime: Jacob Bernoulli's *Ars Conjectandi* had just been published, and Hume was a contemporary of Thomas Bayes (1702–1761), whose famous essay on probability was posthumously published by Hume's knowledgeable friend Richard Price.[9] Although Hume himself may not have been particularly well-versed in eighteenth century probability theory, he knew that such a theory existed, and that some of his contemporaries were making significant contributions to it.[10]

In this paper we will give a formal analysis of the second negative argument; however, as we shall see, our approach differs essentially from that of the formalists mentioned above. We begin in Sect. 2 by citing the regress argument as it appears in *Treatise* 1.4.1. In Sect. 3 we illustrate the argument by giving an example, which we then analyze in the standard formalist manner of Peirce, Von Wright, Quine, Popkin, DeWitt, Wilson, Fogelin and others. Although the approaches of these scholars differ in details, they all explicitly or implicitly assume that the regress argument is based on a simple multiplication of probabilities. This assumption, which in a sense can indeed be read in Hume's text, is actually incorrect. It leads to a dilemma (explained in Sect. 4) and the way out is to realize that the assumption violates the probability calculus (Sect. 5). In Sect. 6 we explain what happens after we have reconstructed the regress argument in a way that does agree with the calculus, i.e. by means of a *sum* of products rather than a single product of probabilities. We show that then there is indeed something that diminishes and finally fades away, but it is something other than what Hume presumably had in mind. Thus, as we explain in Sect. 7, two different views emerge as to what exactly it is that diminishes in the iterative probability argument. We surmise that Hume failed to differentiate between them, since he lacked the means to discriminate between two formalizations of his argument, one that is in accordance with the probability calculus and one that is not. This may have contributed to the many confusions surrounding *Treatise* 1.4.1.

2 Probability reduces to nothing

Here is Hume's regress argument as it appears in 1.4.1:

"In every judgment … we ought always to correct the first judgment, derived from the nature of the object, by another judgment, deriv'd from the nature of the understanding. … As demonstration is subject to the controul of probability, so is probability liable to a new correction by a reflex act of the mind, wherein

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[9] Hacking (1975), Daston (1988).

[10] Richard Price had convinced Hume that some part of Hume's reasoning was inconclusive (Peach 1980). We are not sure which part this is, but Price explicitly criticizes Hume's regress argument in *A Review of the Principal Questions in Morals*, incidentally without using formal tools. Since the regress argument in 1.4.1 is among the arguments from the *Treatise* that are not repeated in Hume's later writings, David Raynor has suggested that Price, perhaps in early conversations, convinced Hume that this piece of reasoning is incorrect (Raynor 1981).

the nature of our understanding, and our reasoning from the first probability, become our objects. (T 1.4.1.5)

Having thus found in every probability, beside the original uncertainty inherent in the subject, a new uncertainty deriv'd from the weakness of that faculty, which judges, and having adjusted these two together, we are oblig'd by our reason to add a new doubt deriv'd from the possibility of error in the estimation we make of the truth and fidelity of our faculties. This is a doubt, which immediately occurs to us, and of which, if we wou'd closely pursue our reason, we cannot avoid giving a decision. But this decision, tho' it should be favourable to our preceding judgment, being founded only on probability, must weaken still further our first evidence, and must itself be weaken'd by a fourth doubt of the same kind, and so on *in infinitum*; till at last there remain nothing of the original probability, however great we may suppose it to have been, and however small the diminution by every new uncertainty. No finite object can subsist under a decrease repeated *in infinitum*; and even the vastest quantity, which can enter human imagination, must in this manner be reduc'd to nothing." (T 1.4.1.6)

At first sight the argument might seem perfectly reasonable: doubting the probability you attach to a proposition, and then casting doubt on your doubt, and so on, would appear to diminish the probability with which you believed the proposition in the first place. Nevertheless something odd appears to be going on, although it is unclear what exactly it is. Thus many scholars have railed against the argument, but few made clear what is formally wrong with it.[11]

In order to see what exactly is wrong with the argument, we will first in Sect. 3 concentrate on a particular example. In line with Hume's text, this example is taken from the "demonstrative sciences" (T 1.4.1.1) and we shall analyze it in the standard formalist manner of Von Wright, Quine, Popkin and others. As intimated above, this analysis in fact contains a fundamental formal mistake that, remarkably enough, was not noted by any of the formalists. The mistake involves the erroneous use of a product of probabilities, but a full explanation of it will be postponed until Sect. 5. We therefore ask the attentive reader to patiently bear with us as we go through the standard formalist reconstruction of Hume's argument, and not be perturbed by an occasional offending formula.

3 The standard formalist approach

Imagine Maurice, a mathematician who has just performed the laborious addition of the first 1000 natural numbers: $S = 1+2+3+\cdots+1000$ by hand. Maurice has concluded that

$$A : \text{The sum } S \text{ is equal to } 500,500.$$

Although A is correct, Maurice can never be entirely sure of that—there is always the possibility that he made a mistake. Or as Hume puts it, "[i]n all demonstrative sciences,

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[11] David Stove famously commented "the argument is generally recognized as being the worst ever conceived by a man of genius", without actually pointing to a formal mistake (Stove 1965, p. 174, note 12). Mikael Karlsson simply called the argument "a morass" (Karlsson 1990, p. 126), thereby echoing Fogelin's words (Fogelin 1985, p. 16; Fogelin 1992, p. 11).

the rules are certain and infallible; but when we apply them our fallible and uncertain faculties are very apt to depart from them, and fall into error" (1.4.1.1). Maurice, realizing that his application of the "certain and infallible" rules is prone to error, can therefore only conclude with a certain probability that A is true (T 1.4.1.1). Of course, if his colleagues independently arrive at the same result, then he may become more confident that he applied the rules correctly (T 1.4.1.2). The salient point however is that he can never be certain that he has not made a mistake: the scepticism described in 1.4.1 is aimed at our *credence* in mathematical statements like A, not at the statements themselves.

Let us suppose that Maurice trusts his calculational ability to at least 75%. In other words, he believes:

B: The probability that A is true is at least $^3/_4$,

where 'the probability' means Maurice's credence. Of course, even if B is true, that does not guarantee that Maurice's calculation of S was correct. We will write the conditional probability that A is true, on the assumption that B is true, as

$$P^1(A|B),$$

where the superscript '1' indicates a first-order probability.[12] Maurice has no idea what the precise value of this conditional probability is, but he does believe (because he believes B) that it must lie in the interval between ¾ and 1. If Maurice had a full belief in B, his estimate of the unconditional probability of B would have been one, i.e. $P^1(B) = 1$. In that case

$$P^1(A) \;=\; P^1(A|B)P^1(B) \tag{1}$$

would boil down to $P^1(A) = P^1(A|B)$. For the purposes of the exposition we assume the latter probability to have some definite value in the interval [¾, 1]; this assumption is however not necessary as the argument also works when imprecise values are used (*cf*. footnote 31).

However, Maurice does not believe B in full. Following Hume 1.4.1.6, he entertains second-order worries about his first-order judgement: he doubts that B is true.[13] Let us suppose that Maurice's belief in B also ranges between ¾ and 1, so he believes:

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[12] Strictly speaking, we are here dealing with a second-order probability (since B itself already contains the word 'probability'), but for convenience we use P^1 and call it a first-order probability. Recall that all the probabilities are credences. In particular, it is not the case that A has an objective chance, and that Maurice has a degree of belief in that chance. After all, A is a mathematical statement and is thus either true or false; it does not make sense to attach an objective chance to A. However, it does make sense to say that Maurice has a particular credence in A's truth. This does not mean, as one reviewer thought, that Maurice "is unsure about his own state of mind". Maurice knows what he thinks (he knows the contents of his thoughts), he just does not know whether what he thinks is true. The regress is about *degrees* of belief, not about *contents* of belief.

[13] That Hume's argument involved second-order probabilities is acknowledged not only by formalists, but also by scholars who shun a formal approach. See for example Garrett (2015, pp. 224 and 236).

C: The probability that B is true is at least $3/4$,

where again 'probability' means Maurice's credence.[14] If Maurice had a full belief in C, his estimate of the unconditional probability of would have been one: $P^2(C) = 1$, where the superscript '2' indicates a probability of the second order. In that case

$$P^2(B) = P^2(B|C)P^2(C) \qquad (2)$$

would simplify to $P^2(B) = P^2(B|C)$. Following Hume's lead, Maurice now has to revise his estimate of the probability that the sum S is equal to 500,500 as follows:

$$P^2(A) = P^2(A|B)P^2(B) = P^2(A|B)P^2(B|C). \qquad (3)$$

Thus Maurice's credence in A has been reduced from the first-order value (1) to the second-order value (3).[15]

This reasoning is based on the assumption that Maurice fully believes C, but of course he does not. He has doubts about C, and in fact believes:

D: The probability that C is true is at least $3/4$.

If he had a full belief in D, so that $P^3(D) = 1$, then according to the same reasoning $P^3(C) = P^3(C|D)$, so that $P^3(B) = P^3(B|C)P^3(C|D)$, and finally

$$P^3(A) = P^3(A|B)P^3(B|C)P^3(C|D). \qquad (4)$$

But the fact is that he does not fully believe D, and so on. It is evident that, after an infinite number of doubtings, the indefinitely revised probability that his calculation is correct is

$$P^*(A) = P^*(A|B)\,P^*(B|C)\,P^*(C|D)\,P^*(D|E)\,\ldots \qquad (5)$$

where P^* is the limit of P^n as n goes to infinity, and it represents Maurice's definitive credence in A. Since (5) confronts us with an infinite product of factors all less than one, it is claimed that this yields zero for $P^*(A)$: the belief in A has been "reduc'd to nothing", as Hume puts it. Or in the words of Richard Popkin, which capture the formalists' position well: "Since [the] probabilities are smaller than 1, the product is smaller than either of them. … This process of introducing new probabilities … can go on ad infinitum, and thus, the probability that we could ever recognize … that a particular piece of reasoning was correct, approached to zero."[16]

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[14] Had Maurice's credence been *exactly* (rather than at least) ¾, then his second-order credence that his first-order credence was ¾ would have been 0 (since the measure of a point is zero), and there would then have been no chain of Humean doubtings.

[15] Maurice's move here is actually incorrect, as are his subsequent moves; as we intimated, we shall address these mistakes in Sect. 5.

[16] Popkin (1951, p. 390). Popkin seems to have had a simpler version of (5) in mind, namely one involving a product of unconditional probabilities rather than conditional ones: $P(A)P(B)P(C)P(D)\ldots$. For the purposes

4 A dilemma

The above standard formalists' reconstruction of Hume's regress argument has been criticized in two different ways (neither of which coincides with our criticism, as we will see in the next section). First, it has been argued that the reconstruction, although presenting us with a valid argument, does not reflect the actual argument which Hume was making. Second, it has been claimed that the reconstruction does reflect Hume's actual argument, but shows that this argument is invalid. The first line of criticism is favoured by anti-formalists, the second by formalists.

David Owen can be seen as a representative of the first group. In a very interesting chapter on *Treatise* 1.4.1, he argues that Hume's regress argument cannot be interpreted as "the apparent mathematical truism that a probability of less than one will continually decrease as it is successively multiplied by numbers less than one".[17] He points out that such an interpretation has unHumean consequences, for if "we end up … believing *p* with zero probability", then we would "have a belief with probability one in not-*p*". Owen remarks that this is clearly not what Hume intended and concludes: "The point of Hume's argument is 'the total extinction of belief and evidence' … It is a sceptical argument, not the argument of a negative dogmatist."[18]

By calling the argument that comes out of the reconstruction a "mathematical truism", Owen implies that it is valid. But in fact it is not. Simple as a successive multiplication of numbers less than one may be, it is not a "mathematical truism" that it always yields zero. This takes us to the second way in which the reconstruction in the previous section has been criticized. Although *usually* it is true that a product of factors all less than one tends to zero in the limit, this is *not always* the case. Quite a few formalists commenting on the regress argument in 1.4.1 have noticed this. They draw attention to the fact that a product such as (5) can converge to a non-zero number, and they criticise Hume for failing to see this. An early such formal author was Charles Sanders Peirce.[19] Another one was Willard Van Orman Quine, who gives an abstruse example.[20] Simpler cases can be found in Richard DeWitt.[21] The fact is also stressed by Richard Fogelin in early work.[22]

Footnote 16 continued

of our paper the distinction between (5) and this simpler version is irrelevant. Other authors who argued in this vein are Peirce (1905), Von Wright (1941, p. 153; *cf.* p. 223); Quine (1946, pp. 50–51); Wilson (1983, p. 123), DeWitt (1985, p. 130), Fogelin (1985, p. 16). That is not not to say that all these writers use formula (5), but their reasoning is in accordance with assuming something like (5).

[17] Owen (2015, p. 114).

[18] Owen (2015, p. 114). *Cf.* Owen (1999, Chapter 8); Ainslie (2015, pp. 21–22).

[19] Pollock and Agler (2015). These authors quote Peirce on the matter and give references to the relevant papers.

[20] Quine (1946/2008, pp. 50–51). Thanks to Sander Verhaegh for drawing our attention to Quine's lectures on Hume, which paved the way for this paper.

[21] DeWitt (1985, p. 131).

[22] Fogelin (1985, p. 174, note 4). Later, however, Fogelin moves away from his criticism: "That criticism, though fair enough, now strikes me as shallow in conceding too much to Hume's argument." (Fogelin 2009, p. 54, note 4).

Peirce, Quine, DeWitt and Fogelin all are right to observe that sometimes (5) can converge to a non-zero number. Yet their observation is beside the point. In stressing that the product in (5) might not be equal to zero, the authors refer to a very special case, one that only occurs under exceptional circumstances (a necessary condition for this special case to occur is that the higher probability factors should get increasingly closer to one, and there is no indication whatsoever that Hume had this condition in mind). Moreover, their observation ignores what is at the heart of Hume's argument. Whatever exactly Hume is trying to express in his iterative probability argument, he is emphasizing that a continual *diminution* takes place. There is *something* that, in the end, becomes nothing at all. To point out that a product may converge to a positive number and leave it at that may be too simple.[23]

We now find ourselves beset by a dilemma. If we follow Owen and some of his fellow anti-formalists, then we must assume that Hume cannot have meant that (5) yields zero, for this would have turned his argument into that of a negative dogmatist.[24] But assuming with formalists like Peirce and Quine that (5) yields a number other than zero leaves it unclear why Hume writes that "at last there remain nothing" of the original probability, P(*A*).

A possible reaction to this dilemma is to blame the formal approach, and to conclude that the probability calculus simply is inappropriate for evaluating Hume's argument. Such a conclusion has indeed be drawn by several Hume scholars. David Owen, Don Garrett, Barry Gower, Annette Baier, Antonia Lolordo, to mention only a few, have all more or less explicitly advocated a non-formal reading of Hume's argument. In the current case, however, we think that the use of formal tools is not out of place. First, since Hume talks about probabilities and their continual decrease, invoking probability theory seems not unnatural if we aim to assess what he says. Second and more importantly, the rejection of all formal tools when it comes to evaluating the regress argument in 1.4.1 deprives us of a worthwhile instrument to make sense of a key concept in the argument, namely the idea of a continuous diminution culminating in nought. As we will explain in Sect. 6, a correct application of probability theory makes it clear that indeed there is something that is "reduc'd to nothing"; it is just that what is annihilated is something other than what has to date been thought. But first, in Sect. 5, we show why the formulas (3)–(5), intuitive as they perhaps may seem, are actually incorrect.

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[23] Richard DeWitt remarks that, if one prohibits the higher factors from exceeding some arbitrary value very close to one, say 0.999999, then Hume's assertion about the vanishing of all infinite products is correct (DeWitt 1985, p. 132). Fogelin makes an equivalent point when he observes that Hume could rescue his claim by saying "that there is some finite degree or probability below which the chance or error can never fall" (Fogelin 2009, p. 17). DeWitt and Fogelin are correct, but, as they acknowledge themselves, the cases they describe are even more exceptional, and therefore ill-suited as reconstructions of what Hume may have meant.

[24] Unless, of course, Hume did not realize that "believing *p* with zero probability" implies "having a belief with probability one in not-*p*". Owen does not seem to consider this possibility.

5 A sum rather than a multiplication

Consider again the example of the addition of the first 1000 natural numbers: $S = 1+2+3+\cdots+1000$. We argued Sect. 3 that the first-order probability that A is true is $P^1(A)=P^1(A|B)P^1(B)$. However, this reasoning ignores an important possibility that must be taken into account when determining the probability that A is true. This is the possibility that A is true, even if B is false, that is, even if it is not true that the probability that all the intermediate calculations were correct is at least ¾. After all, Maurice might have made two mistakes that cancel one another out, or committed some other fortuitous blunder that does not change the final answer. Indeed, he might be so prone to such errors that the probability that all his calculations were correct is less than ¾, even though the result actually turns out to be the right answer. We symbolize this situation by the conditional probability $P^1(A|\neg B)$, which in general will not be zero. The correct expression for $P^1(A)$ is not (1) but rather

$$P^1(A) \; = \; P^1(A|B)\,P^1(B) \; + \; P^1(A|\neg B)\,P^1(\neg B), \tag{1$'$}$$

where a second term has been added; (1$'$) is an instantiation of the rule of total probability. Now if $P^1(B)$ is equal to one, $P^1(\neg B)$ is zero, and then the second term in (1$'$) makes no numerical contribution; in that case $P^1(A)=P^1(A|B)$, as in (1), which 'fortuitously' gives the correct answer for $P^1(A)$. This luck will not persist in higher orders of Humean doubt, however, as we will see.

At the second order of doubting, we again use the rule of total probability, writing $P^2(A)=P^2(A|B)\,P^2(B)+P^2(A|\neg B)\,P^2(\neg B)$, and instead of (2) we write

$$P^2(B) \; = \; P^2(B|C)\,P^2(C) \; + \; P^2(B|\neg C)\,P^2(\neg C). \tag{2$'$}$$

If $P^2(C)=1$, this reduces to $P^2(B)=P^2(B|C)$, but then $P^2(\neg B)=1-P^2(B|C)$, which is not zero. Therefore the second term, $P^2(A|\neg B)\,P^2(\neg B)$, in the expression for $P^2(A)$ is not zero, and (3) is definitely incorrect. No luck at this level of doubt, nor at any higher levels!

What happens if we repeat these transformations, in particular infinitely many times? In fact it can be proven that an infinite iteration of the rule of total probability usually converges, not to zero, but to a unique and well-defined number between one and zero.[25] Which number that is depends on the values of the probabilities in the chain. We will not give the general proof here, all the more so since we do not have to work out the details in order to see that $P^*(A)$ is not zero: this outcome can already be grasped without exhaustive calculations as follows.

Rather than saying that $P^*(A)$ is a function of an infinite sequence of doubts (where we take into account that these doubts may be well or ill founded), we could also say

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[25] For a general proof, see Atkinson and Peijnenburg (2017, pp. 194–195). A simpler version of the proof, applying to the uniform case only, goes as follows. $P^*(A_n)=\beta+\gamma\,P^*(A_{n+1})$, where A_0 stands for A, A_1 for B, A_2 for C, and so on. Here $\gamma=\alpha-\beta$ where α and β are the two conditional probabilities, assumed to be the same from step to step (the uniformity assumption). By iterating this rule one finds that $P^*(A_0)=\beta+\beta\,\gamma+\beta\,\gamma^2+\cdots+\beta\,\gamma^n+\gamma^{n+1}\,P^*(A_{n+1})$. Since $|\gamma|<1$ and $P^*(A_{n+1})<1$, it follows that the infinite series is convergent, and that $P^*(A_0)$ is in fact equal to β times a geometric series, with the value $\beta/(1-\gamma)$.

that P*(*A*) is determined by P*(*B*), and that P*(*B*) rather than P*(*A*) is a function of an infinite sequence of doubts. Now whatever the value of P*(*B*) is, it is a probability value, so it must be a number between zero and one. Let us first consider these two extreme cases: if P*(*B*) = 1, so that P*(¬*B*) = 0, then P*(*A*) = P*(*A*|*B*); but if P*(*B*) = 0, then P*(¬*B*) = 1, and in that case P*(*A*) = P*(*A*|¬*B*). Whatever the value of P*(*B*) is, P*(*A*) must lie between P*(*A*|*B*) and P*(*A*|¬*B*).[26] If neither of these two conditional probabilities is zero, P*(*A*) cannot be zero.

The conclusion therefore is that P*(*A*) is not zero, no matter what the infinite sequence of higher and higher-order corrections to P*(*B*) might be. We have arrived at this conclusion without the exceptional measures that formalists like Peirce, Quine, DeWitt and others saw fit to take. They have criticized Hume's regress argument not because it lacks a second term and thereby violates the rule of total probability, but because in exceptional cases formula (5) is not equal to zero. Even though the latter is true, as a criticism of Hume it is beside the point, as we have seen.[27]

However, if P*(*A*) is not equal to zero, what to do with the "continual diminution" that Hume is talking about? Is there something that is "reduc'd to nothing" in the long run? In Sect. 6 we explain that there is, but that it is different from what Hume (at least in the opinion of most interpretors) had in mind.

6 The contribution diminishes

Consider again how Hume words his regress argument:

> Let our first belief be never so strong, it must infallibly perish by passing thro' so many examinations, of which each diminishes somewhat of its force and vigour. When I reflect on the natural fallibility of my judgment, I have less confidence in my opinions, than when I only consider the objects concerning which I reason. (T 1.4.1.6)

If the chain is infinite,

> all the rules of logic require a continual diminution, and at last a total extinction of belief and evidence. (ibid.)

We have seen that it is problematic to interpret this as entailing that our original belief in a proposition indefinitely diminishes as the chain of probability judgements over

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[26] Since *P**(*A*) is a linear, and *a fortiori* a monotone function of *P**(*B*).

[27] The formalists' focus on the fact that (5) does not always yield zero might explain why they did not see that (5) is actually invalid. That is, they noticed that Hume was presumably thinking of some multiplication like (5), but rather than realizing that (5) is the wrong formula, they pointed out that an infinite product of factors all less than one can be positive. Another explanation of why formalists failed to see that we are actually dealing with a sum rather than a product, is that the rule of total probability may have been less obvious than it is today. Note that the formalists published their work several decades ago, when probability theory was not so widely used in philosophy as it is today. Also, even philosophers like Bertrand Russell sometimes forgot about the rule of total probability; *cf.* his mistake in *Human Knowledge. Its Scope and Limits* of 1948, which was pointed out to him by Hans Reichenbach in a letter of March 28, 1949 (Reichenbach and Cohen 1978, pp. 405–411). In a reply to Reichenbach on April 22, 1949, Russell acknowledged the error. In daily reasoning the error also occurs, and may be connected to the confirmation bias fallacy (thanks to Ulrike Hahn for this suggestion); however, as far as we know no research has been done on this matter.

probability judgements lengthens. That is, P*(*A*) is not simply a product of terms like those in Sect. 2, but is a sum of terms, each of which is positive. Then what is it that decreases?

Kevin Meeker has suggested that it is the epistemic *justification* of the belief. In his view, the original belief in a proposition becomes less and less likely, in the sense that its justification reduces as the chain gets longer. If one assumes, as Meeker says most epistemologists do, that justification is necessary for knowledge, then "Hume's argument reveals that we lack knowledge because all our beliefs lack justification".[28] David Owen and Don Garrett have contested this view. According to them it is the *retention* of the belief that is at stake. As the chain lengthens, beliefs will cease to be beliefs and turn into sterile ideas without any force or vivacity:

> The point is not that a belief, with full force and vigor, is seen to be unjustified; rather, it is that because the force and vigor continually decrease, the idea seems in danger of ceasing to be a belief at all.[29]

And:

> When Hume worries about 'a total extinction of belief and evidence' …, the concern is not justification, but quite literally the extinction of belief. … Hume's concern here is the retention, not the justification, of belief.[30]

Both camps, dissimilar as they may be, concur in assuming that the diminution refers to properties of the original belief. According to Meeker, what weakens and finally dies out is the *justification* of the original belief. According to Owen and Garrett, it is the belief's *force* and *vigour* that decreases, putting the *retention* and ultimately the very *existence* of the original belief at risk.

However neither Meeker's view nor that of Owen and Garrett sits well with the observations that we made above on the basis of the rule of total probability. There we saw that P*(*A*), supported by an infinite chain, can take on any value between one and zero, including quite a large one. But if the probability of *A* converges to a value that is large, then it seems strange to say that the justification of the belief in *A* decreases or that the force with which *A* is believed weakens until it dies out completely.

We have seen that some authors solve this problem by eschewing a formal reading of the text altogether. We think there is no need to do so. For once the correct formalism is in place, including the second term, it becomes clear that indeed something dwindles away to nothing as the chain gets longer and longer. However it is neither the justification nor the force of the belief in *A*. Rather, it is the *contribution* of the successive links in the chain to the value of P*(*A*), which itself may very well be high. In order to understand this, let us go back to our example.

At the *n*th order of doubting, the probability that S = 500,500 can be written as

$$P^n(A) \;=\; P^n(A|B)\,P^n(B) \;+\; P^n(A|\neg B)\,P^n(\neg B), \tag{1''}$$

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[28] Meeker (2000, p. 228); *cf*. Meeker (1998) and Meeker (2013).

[29] Owen (2015, p. 113).

[30] Owen (2015, p. 116). Ibid., 120. See also Owen (1999, Chapter 8), and Garrett (1997).

where $P^n(A|B)$ is the nth-order probability that $S = 500,500$, given that the probability that Maurice's calculations were correct is at least ¾. Let us suppose, as an illustration, that this conditional probability is 0.9.[31] The conditional probability $P^n(A|\neg B)$ is the probability that $S = 500,500$, if the probability that his calculations were correct is *less* than ¾. This conditional probability will be lower than $P^n(A|B)$, but it will generally not be zero. Let us suppose this conditional probability to be 0.5.[32] Thus (1″) becomes

$$P^n(A) \;=\; 0.9\,P^n(B) \;+\; 0.5\,P^n(\neg B). \tag{6}$$

Had Maurice been sure that B is true, then $P^1(B)$ would have been equal to unity, and $P^1(A)$ would have been 0.9. However, he has doubts about the veracity of B, and calculates

$$P^2(B) \;=\; P^2(B|C)\,P^2(C) \;+\; P^2(B|\neg C)\,P^2(\neg C), \tag{2′}$$

under the assumption that $P^2(C)$ is one. For the sake of our illustration, we suppose that the conditional probabilities in (2′) are the same as those in (6)—but see footnote 32. Thus (2′) becomes

$$P^2(B) \;=\; 0.9\,P^2(C) \;+\; 0.5\,P^2(\neg C). \tag{7}$$

With $P^2(C) = 1$, we have $P^2(B) = 0.9$. The latter implies that the second-order probability of A will be less than the first-order probability:

$$P^2(A) \;=\; 0.9 \times 0.9 \;+\; 0.5 \times 0.1 \;=\; 0.86. \tag{8}$$

At the third level, $P^3(D) = 1$, $P^3(C) = 0.9$, $P^3(B) = 0.86$, and finally $P^3(A) = 0.844$. It might seem that we are getting nowhere in our attempt to find a definitive value for the subjective probability that $S = 500,500$. However nothing could be further from the truth. Table 1 not only shows how the higher-order probabilities of A decrease as the order increases, but also that they approach a definite value greater than zero. It can indeed be proved that, in the limit, $P^*(A)$ is equal to 5/6.

   Our table goes counter to Hume's contention that the probability of the original belief always goes to zero. In addition, and this is the important point, it shows that there is something that *does* diminish to zero as the chain increases. This is not the justification of the belief in A, as Meeker thought, nor its vigour, as Owen and Garrett have claimed. Rather it is the *effect* of higher-order doubtings on Maurice's credence

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[31] Attaching a precise numerical value to this conditional probability raises of course various philosophical problems, but this is done purely for illustrative purposes. Our argument can be easily extended to the case where intervals, rather than precise values, are assumed.

[32] The numbers 0.9 and 0.5 only serve as examples. We could have chosen pretty well any numbers for the conditional probabilities, since our argument is insensitive to the values that are selected. In particular it is not essential that all the conditional probabilities in the chain have the same value: our argument also works if the numbers are not uniform from link to link. Note that, given the meaning of B, C, etc., we would expect that $P^n(A|B)$ etc. are greater than ¾, and that $P^n(A|\neg B)$ etc. are less than ¾.

Table 1 Decreasing higher-order probabilities of A

Order n of probability	1	2	3	5	10	Infinite number
nth order proby. of A	0.9	0.86	0.844	0.835	0.83334	$0.83333\ldots = 5/6$

Table 2 Increasing higher-order probabilities of A

Order n of probability	1	2	3	5	10	Infinite number
nth order proby. of A	0.55	0.575	0.588	0.597	0.5999	$0.6 = 3/5$

in *A*. The higher the doubt is, that is the more intermediate doubtings there are, the smaller is its incremental effect on the final value of *P(A)*.

The details can be read off from Table 1. There we see that at the first level the probability is 0.9; but when we go to second order the value of the probability is reduced to 0.86, which means that the second-order (negative) correction is 0.04. At the third order, the value goes down still further to 0.844, so the third order contributes a correction of $0.86–0.844 = 0.016$ to the second-order value. The combined effect of the sixth to the tenth orders, as can be seen from the table, produces a correction of less than two parts in a thousand.

In Table 1 the higher-order values of the probability become smaller as the number of doubtings grows, but this is not invariably so. Whether they do depends on the values of the conditional and unconditional probabilities. In the construction of Table 1 we set $P^n(A|B)$, $P^n(B|C)$, $P^n(C|D)$, and so on, all equal to 0.9; $P^n(A|\neg B)$, $P^n(B|\neg C)$, $P^n(C|\neg D)$, and so on, were all 0.5, while $P^1(B)$, $P^2(C)$, $P^3(D)$, and so on, were equal to one. If however we choose as values 0.8, 0.3 and 0.5 respectively (where the last reflects the idea that Maurice initially thinks he might be just as well right as wrong), then this would lead to Table 2.

Here the probability goes up rather than down as the number of doubtings increases. Moreover, as in Table 1, the value of $P^*(A)$ is a well-defined number, and this is what usually happens.[33] Although these probabilities increase as the order increases, the *contribution* of the higher orders to the final value of the probability of *A* still decreases, as can be seen from Table 2. Further it can be proved that this value, after an infinite number of doubtings, does not depend at all on whether we set $P^1(B)$, $P^2(C)$, $P^3(D)$, and so on, equal to a half or to one: it is a function solely of the conditional probabilities.

Both tables illustrate that the probability of the original belief not only fails to go to zero, but generally approaches a positive number that is unique and well-defined. Moreover, they show that there is something that invariably diminishes as the chain of doubtings increases, namely the *effect* of higher-order doubtings on the probability

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[33] The only situation in which this does not happen is when the (nonuniform) conditional probabilities in the chain rapidly approach 1, that is, when they are close to material implications. We will not give the proofs, since again the point can be readily grasped at an intuitive level.

that the sum S is equal to 500,500. The further away a level is from *A*, that is the more intermediate doubtings there are, the smaller is its influence on the final value of the probability of *A*. Moreover the tables show that the approach to the limit can be rather rapid. This should remove any feeling of uneasiness that one might have about drawing conclusions from reasoning that goes on forever. In line with Hume's claim that the diminution already occurs in a finite sequence of doubting, the tables show that we do not need to go all the way to infinity in order to see the effect that we have been talking about: a few steps are enough to suggest that a regress of higher and higher-order probabilities converges to a non-zero value.[34] Also, a few steps suffice to indicate that the significance of the higher orders diminishes as their number increases. In this sense, then, it is correct that something goes to zero in the process. It is just not the probability of *A*, or the justification of the belief in *A*; nor is it the force or vigour of that belief. Rather it is the contribution to the final non-zero value by the successive doubts.

7 Two views

In his insightful paper on 'Of scepticism with regard to reason', David Owen writes:

> As the number of intermediate ideas increases and the chain of reasoning becomes longer, the relationship between the ideas at each end of the chain of ideas becomes more indirect and the certainty of the conclusion is lessened.[35]

Owen may well be right that this is what Hume had envisaged. If so, however, then it is clear that Hume fails to discriminate between a valid and an invalid version of his argument. For modern probability theory teaches us that the first part of Owen's sentence hits the mark, but the second part is false. It is indeed the case that, as the chain becomes longer, the relationship between the ideas at each end of the chain becomes less direct. It is however not so that the certainty of the conclusion is lessened. No matter how long the chain is, the conclusion can still be almost certain, and moreover be believed with great force and vivacity. If the above quotation by Owen correctly represents Hume's view, which in our opinion is very likely, then it reveals that Hume failed to distinguish between a diminishing influence on the probability of the conclusion and a diminishing probability of the conclusion itself.

There are more indications in Owen's text that Hume failed to make this distinction. For example, Owen states that the negative arguments in Hume's text "threaten to lessen the degree of force and vivacity characteristic of our beliefs" and show

> how our degree of confidence in our beliefs might lessen on reflection.[36]

This appears to refer to the diminution of the original belief, P(*A*), which we have deemed incorrect, in the sense that it violates the probability calculus. But Owen also takes Hume as saying:

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[34] Of course, we need a mathematical proof to demonstrate that what we are actually observing is a firm fact rather than a fluctuation. But this has been provided.

[35] Owen (2015, p. 120).

[36] Owen (1999, pp. 175, 196).

as the number of steps required to reduce the probability of our judgements to zero approaches infinity, so its *influence* on our beliefs gets vanishingly small.[37]

If this means that the influence of *each successive step* reduces (rather than that the influence of *all the steps together* lessens), then it *is* in line with the calculus. On the other hand, however, when Owen writes "In Hume's case, … length and complexity lessen the certainty with which the conclusion of an argument is held", this is again incorrect.[38] For, as we have seen, a conclusion can be almost certain, even if it follows from a long chain of reasoning.

There are thus two views on what exactly diminishes in an argument based on iterated probabilities, a correct and an incorrect one. According to the latter, the thing that diminishes is the probability, or the justification, or the vivacity of the original belief. According to the former, what diminishes is the *effect* that successive links in the chain have. The more remote a link is, the smaller is its contribution to the probability or justification, which may however continue to be close to one.

Which of these two views did Hume have in mind? The fact that a Hume authority like Owen does not distinguish the one from the other, and apparently without being aware of it goes back and forth between them, suggests that Hume himself failed to make the distinction. This failure is of course understandable, given the rather rudimentary nature of probability theory in the eighteenth century. True, knowledge of the calculus is not really required to make the distinction, but it certainly simplifies the job. Be that as it may, the distinction between the two views is of major importance for Hume's regress argument: on the one view the argument is formally valid, whereas on the other it is not.

The distinction in question also sheds light on a recent discussion about the meaning of the term 'evidence' as it occurs in 'Of scepticism with regard to reason'. What is this evidence of which Hume says that it is weakened until at last it becomes totally extinct? Should we interpret it in a straightforwardly epistemological way, i.e. as 'evidential grounds'? Or is it a more subjective notion, as Garrett and Owen have argued, corresponding to "psychological 'evidentness', not the external basis of such evidentness"?[39] David Owen aptly pointed out that difficulties would arise if we were to choose the first option:

> How could a subsequent judgment affect the original evidence, when 'evidence' is treated as 'evidential grounds'? It is not as if the original evidence turns out to be false or misleading: it is just that it ceases to have the effect on it that it originally did. What is weakened is my confidence or degree of belief in the the first judgment[40].

Kevin Meeker, on the other hand, is quick to note that confidence only weakens in reaction to the supposed weakening of evidential grounds:

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[37] Ibid., 195, our italics.

[38] Owen (2015, p. 132, note 38).

[39] Garrett (2015, p. 225).

[40] Owen (2004, p. 15). *Cf.* Garrett (1997).

> … if the evidential grounds are *not* weakened, then it is puzzling why our *confidence* should be weakened. In other words, why does the *evidentness* falter if it is not because we see the lack of supporting evidential grounds?[41]

As seen from the correct view on what exactly diminishes, both parties have a point in criticizing the other. Interpreting 'evidence' as subjective "confidence" or as "degree of belief in the first judgment" leads to problems; but it is no less troublesome to construe the term as the more objective 'evidential grounds'. The former interpretation implies, in our example, that what weakens and ultimately vanishes is our belief in the proposition that S equals 500,500 (*A*). The latter implies that what weakens, and ultimately vanishes, are the higher-order probabilities. Both implications are incorrect, as we have seen. There is no reason why we should not be very confident that S equals 500,500, and there is no reason why the higher-order probabilities should not have large values. What weakens and finally vanishes are the corrections that the higher-order probabilities make to the degree of our belief in *A*.

8 Summary and conclusion

According to Hume's iterative probability argument in *Treatise* 1.4.1, a sequence of subjective probability judgements over subjective probability judgements will, if continued ad infinitum, reduce the original belief to nothing. Among the many controversies surrounding this argument, there is the disagreement about the appropriate method for its evaluation. Should we or should we not use formal tools, in particular those drawn from probability theory?

In this paper we have to a certain extent sided with the formalists, as we have called them for convenience. 'To a certain extent', because our path is also very different from theirs. Some formalists assumed the argument to be valid, on the grounds that a product of probability values all less than one tends to zero. Other formalists have however observed that there are exceptions to this rule: under some circumstances such a product gives a number greater than zero; we have argued that their observation is correct, but irrelevant for the matter at hand.

Our approach differs from both kinds of formalists. In trying to determine how probable a proposition is, given some evidence, we should also take into account what the probability of the proposition would be if the evidence were false. This leads us to the rule of total probability, which shows that we are not dealing with a product of factors, as both groups incorrectly assume, but rather with a sum of terms, each of which is a product of factors. If we iterate the rule of total probability infinitely many times, we see that it generally yields a probability that is not zero, contrary to what the standard formalists maintain. Moreover the iteration reveals that there is something else that continually decreases until it finally dies out, namely the effect of higher-order doubtings on the original probability.

There are thus two different views about what exactly it is that vanishes in the long run. Each of them corresponds to a different formalization of the iterative probability argument. According to the incorrect standard formalization that we gave in Sect. 3, it is

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[41] Meeker (2000, p. 225).

the probability of the target that diminishes. According to the corrected formalization in Sect. 5, what lessens are the contributions of the successive higher-order probabilities.

The two different views of what vanishes could in principle be distinguished without using formal tools, but the job becomes much easier once we have availed ourselves of contemporary probability theory. Moreover we can now see that the one view is in accordance with the calculus whereas the other is not. For the eighteenth century Hume, the latter was out of reach.

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