



HIGHER-ORDER LEGAL PROBABILISM

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(SIMPLE) LEGAL PROBABILISM

“Legal probabilism is a research program that relies on probability theory to analyze, model and improve the evaluation of evidence and the process of decision-making in trial proceedings”

1. Assess and evaluate evidence (sometimes multiple pieces of evidence as well as conflicting evidence)

- *Formal tools used:* likelihood ratios, Bayes' factor, sensitivity analysis, Bayesian networks, etc.

2. Trial decision-making

- *Formal tools used:* decision thresholds, maximizing expected utility, etc.

3. Reduce risks of error and distribute risks fairly

- *little discussed in the literature*

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Higher-order legal probabilism relies on the same formal tools of simple legal probabilism, with the addition of distributions over suitable probability values

A DEBATE AMONG FORENSIC SCIENTISTS

- How should experts present likelihood ratios for match evidence to the triers of facts?
- Single number?
- Range, intervals?
- Distribution?

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Special issue on measuring and reporting the precision of forensic likelihood ratios: Introduction to the debate★

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THE PLAN

1. Reconstruct the two sides of the debate *charitably*
2. Take a stance — *spoiler*: we think a single number likelihood ratio isn't good enough
3. Sketch higher-order legal probabilism

Disclaimer: We are philosophers interested in how evidence should be evaluated using probability theory, so we are looking at this debate from the sidelines

1. THE DEBATE

A STANDARD EXAMPLE

- Match evidence: the genetic profile at the crime scene matches the genetic profile of the defendant
- Prosecution hypothesis (source hypothesis): the defendant and the person who left the traces at the scene are the same person



THE NEED TO QUANTIFY UNCERTAINTY

- Match evidence supports the source hypothesis to some extent, but never without doubt



ROYALL'S THREE QUESTIONS

1. What does the evidence say?
2. What should one believe?
3. What should one do?

Statistical Evidence

A likelihood paradigm

Richard Royall



CHAPMAN & HALL/CRC

EXPERT V. TRIERS OF FACTS

- The task of the expert is to address the first question: *what does the evidence say?*
- The triers of fact will
 - form a belief about the hypothesis (second question) and then
 - make a decision (third question)



THE PROBLEM

- **How should the expert explain to the triers of facts the extent to which match evidence supports the hypothesis?**

A. This question is partly about the psychology of the triers of facts and the communication between experts and lay people

B. The question also raises a theoretical problem

This talk focuses on the theoretical problem, setting aside psychology and communication

- ***Consensus:***

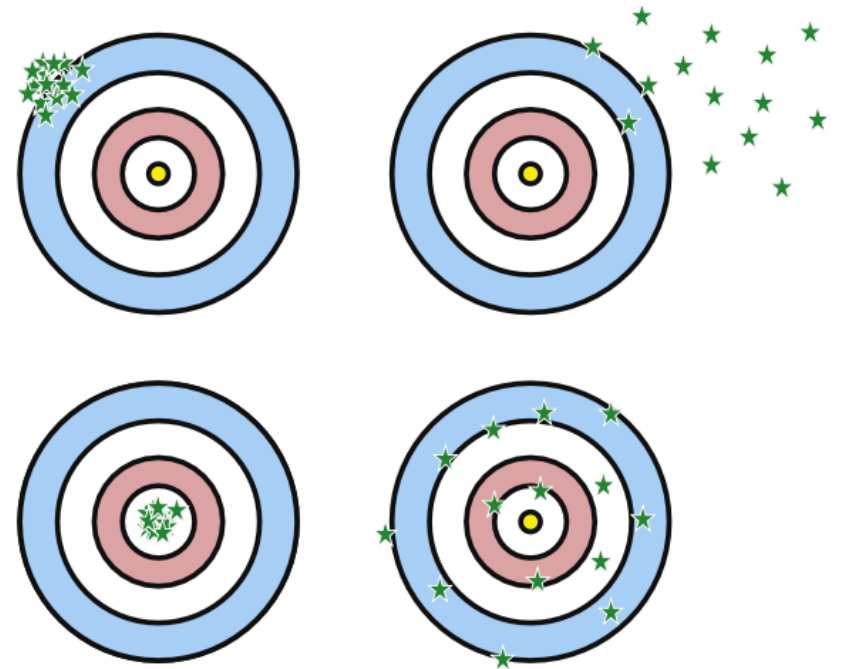
the likelihood ratio is a good way to quantify the uncertainty of how match evidence supports the source hypothesis

- ***What is debated:***

should the expert present one single numerical value for the likelihood ratio, a range of values or a distribution over possible values of the likelihood ratio?

ESTIMATING LIKELIHOOD RATIOS?

- The debate is sometimes framed as though there is a “true” likelihood ratio to be estimated, just like there is a true parameter (the target) that we are estimating
- Others respond that the likelihood ratio isn’t a parameter to be estimated



- This way of framing the debate leads to a stalemate
- *Let's move beyond it*

LIKELIHOOD RATIO: SIMPLE FORMULATION

- Likelihood ratio
(p =prosecution, d =defense):

$$\frac{Pr(Match \mid H_p)}{Pr(Match \mid H_d)}$$

- Simplifying a bit, the likelihood ratio is

$$1/\theta$$

where θ is the probability that a random person would have the matching profile of interest

- **Question:** how should we assess θ ?

UNCERTAINTY ABOUT θ

- Probability θ is the output of a genetic model paired with the frequency data available
 - There is uncertainty about the model itself
 - Data available can be more or less extensive
- θ is like the bias of a coin
 - A physical model of the coin along with frequency data about heads and tails are needed for assessing the bias of the coin

MODELING UNCERTAINTY ABOUT θ

- To model the uncertainty about θ , assume a distribution of values over θ , denote it by $\pi(\theta)$

- θ is treated as a random variable

- Likelihood ratio becomes:

$$\frac{1}{\int \theta \pi(\theta) d\theta}$$

In the denominator we are integrating over the different values of θ

TWO OPTIONS FOR THE EXPERT

1. Present a single number likelihood ratio as:

$$\frac{1}{\int \theta \pi(\theta) d\theta}$$

by integrating over the possible values of θ

- This is an expectation, an average value of all the possible values of the likelihood ratio

2. Preset the full distribution of possible likelihood ratios, following the distribution $\pi(\theta)$ of values of θ

2. AGAINST SINGLE NUMBER LIKELIHOOD RATIOS

- **Does the expert who presents an expectation, an average – a single number likelihood ratio – robs the triers of fact of important information that could make a difference for how they form beliefs and make decisions?**

- **Yes, because an expectation isn't always sufficiently informative to guide belief formation and action**

ANALOGY 1 – FARMING

- In a certain region, it never rains the average amount each year: it either rains above average or below average
- A farmer who relied on rainfall average for planning their annual irrigation needs would be worse off than a farmer who relied on rainfall distribution over the years



Moral: an expectation can remove information useful for belief formation and decision-making

ANALOGY 2 – BETTING

You wonder whether a coin will land heads.

Scenario 1: You know the coin was picked at random from a bag in which half of the coins have a bias of .9 for heads and half have a .9 bias for tails

Scenario 2: You know the coin was picked at random from a bag in which all coins have a bias of .5



- In expectation, your coin should land heads with .5 probability in both scenarios

Is there any difference between the two scenario as far as your “betting behavior” goes?

MAYBE IT MAKES NO DIFFERENCE?

- The shape of your uncertainty about the outcome of the coin toss is quite different in the two scenarios
- Still, one could argue that if you are engaging in a bet, all that matters is the probability the coin will land heads, which is the same in both scenarios

How good is this argument?

THERE IS A DIFFERENCE

If every coin has a known bias of 0.5, there is nothing else you should do in terms of seeking further information

If half coins have a bias of 0.9 for heads and the other a bias of 0.9 for tails, you should want to know which of the two batches of coin you got

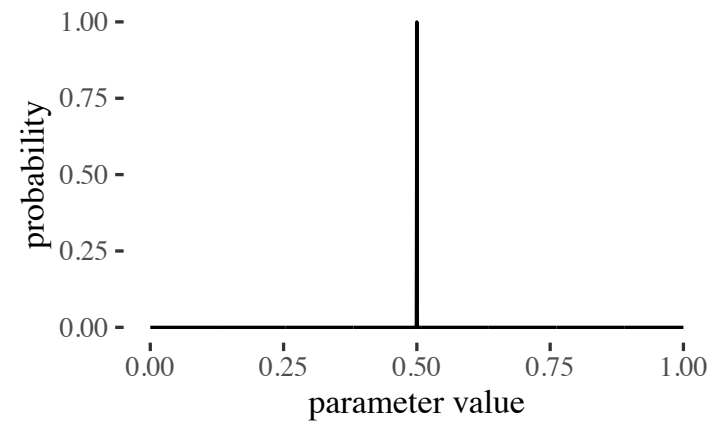
- You have a reason to look for more evidence

- If, as a bettor, you were given simply the “point” information that, in expectation, the coin will land heads with 0.5 you would be deprived of crucial information

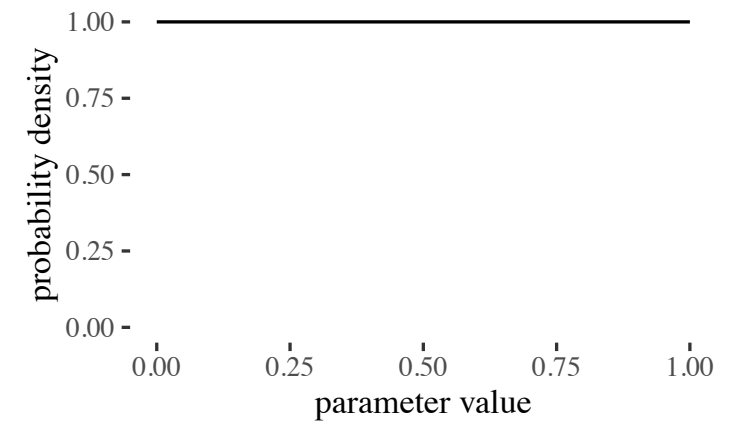
MORE GENERALLY

- Fair coin
- Unknown bias
- Two biases (balanced)
- Two biases (unbalanced)

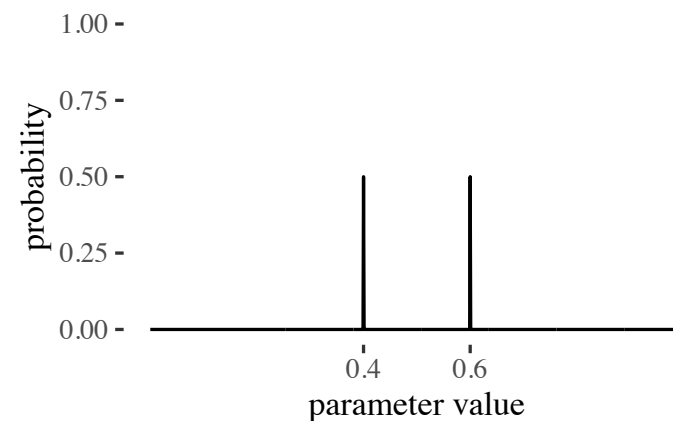
Fair coin



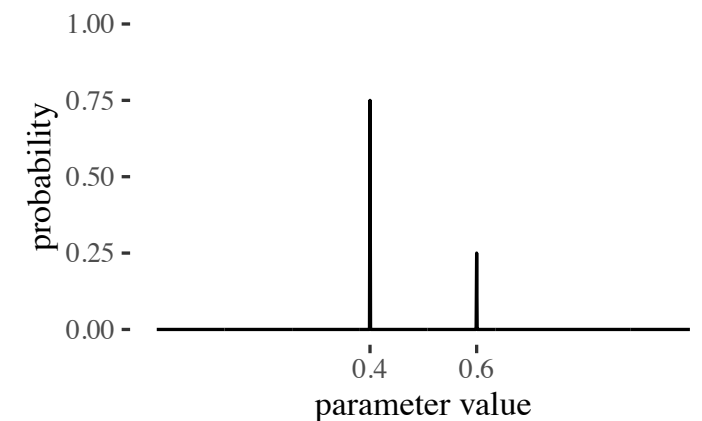
Unknown bias



Two biases (insufficient reason)



Two unbalanced biases



The first three scenarios require different responses or decisions, even though—in expectation—they are indistinguishable

SIMILARLY, IN THE TRIAL CONTEXT

Suppose, in expectation, θ is small:

Scenario 1: the database used is small, so the distribution $\pi(\theta)$ of values of θ is spread out

Scenario 2: the database used is large, so $\pi(\theta)$ is concentrated around the average value

(θ is genotype probability)

- *The two cases are different. If θ is small but its distribution spread out, this is a good reason for the triers of fact to seek more information or give more weight to other incriminating evidence*

3. HIGHER-ORDER LEGAL PROBABILISM

TWO ITEMS OF MATCH EVIDENCE

Match evidence 1: Hair found at the crime scene matches the defendant's hair

LR1 (source hypothesis)= 1/.025

Match evidence 2: The defendant owns a dog whose fur matches the dog fur found in a carpet wrapped around one of the bodies

LR2 (source hypothesis)= 1/.025

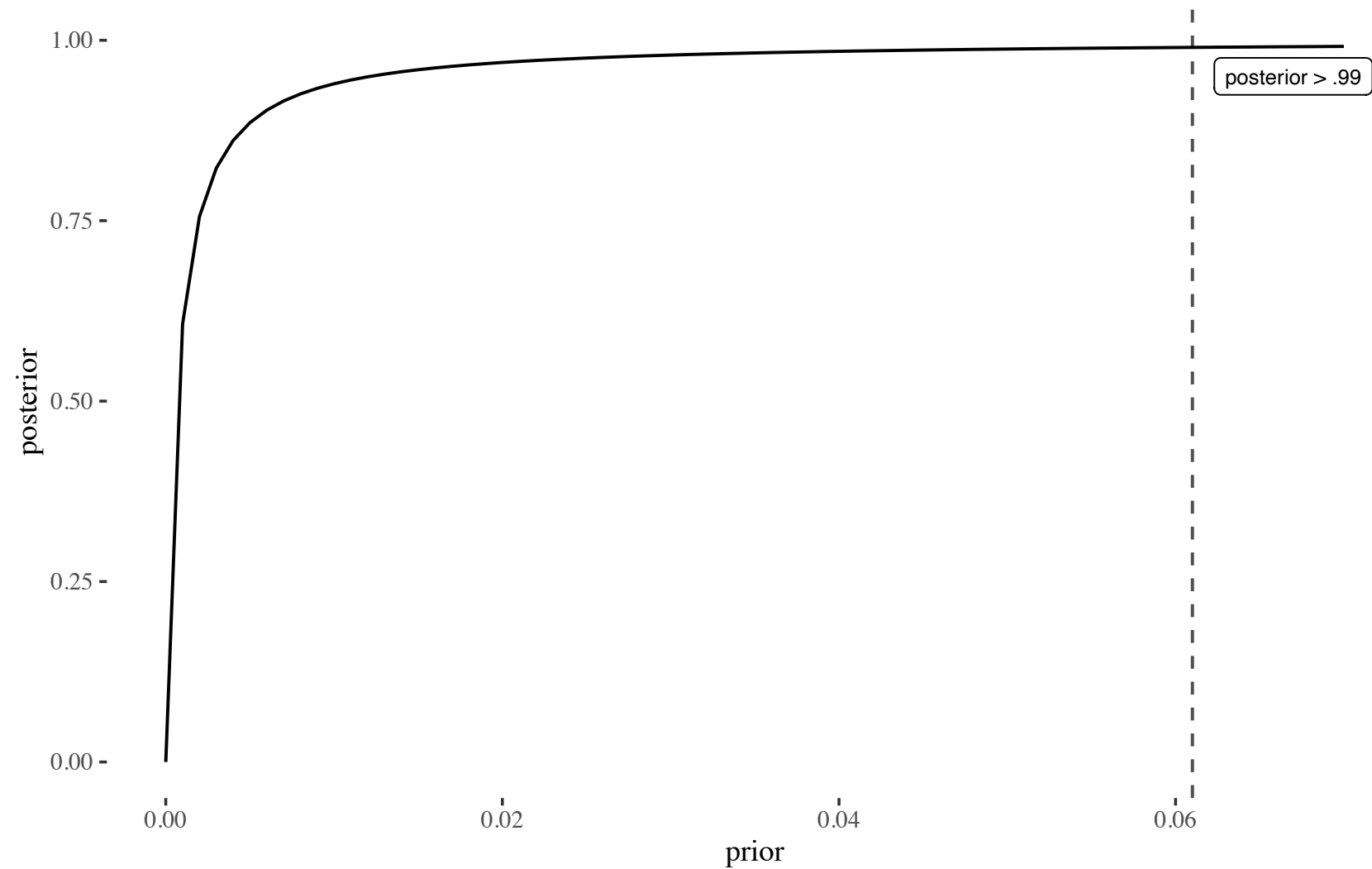
Simple Legal Probabilism

- *The two items of match evidence are combined by multiplying the likelihood ratios*
- $LR1 \times LR2 = 6.5 \times 10^{-4}$

SENSITIVITY ANALYSIS: JOINT EVIDENCE

Prior vs. posterior, based on point estimates

Joint evidence: dog & hair

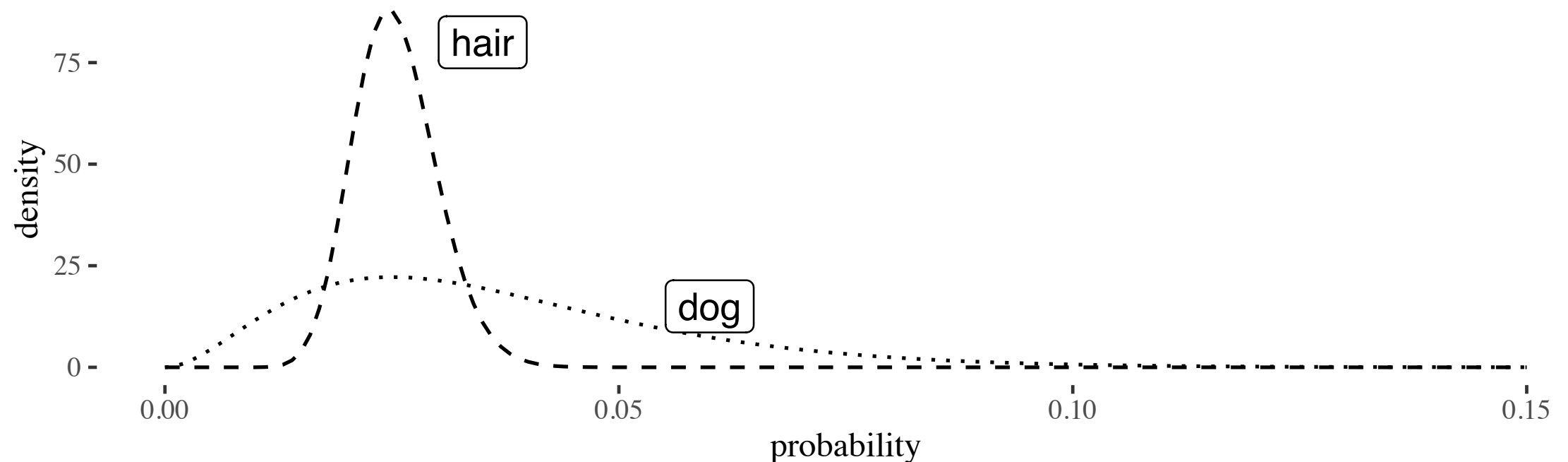


TAKING INTO ACCOUNT DIFFERENCE SAMPLE SIZES

The random match probability for the *hair evidence* is based on 29 matches in a database of size 1148 (so $29/1149=0.025$)

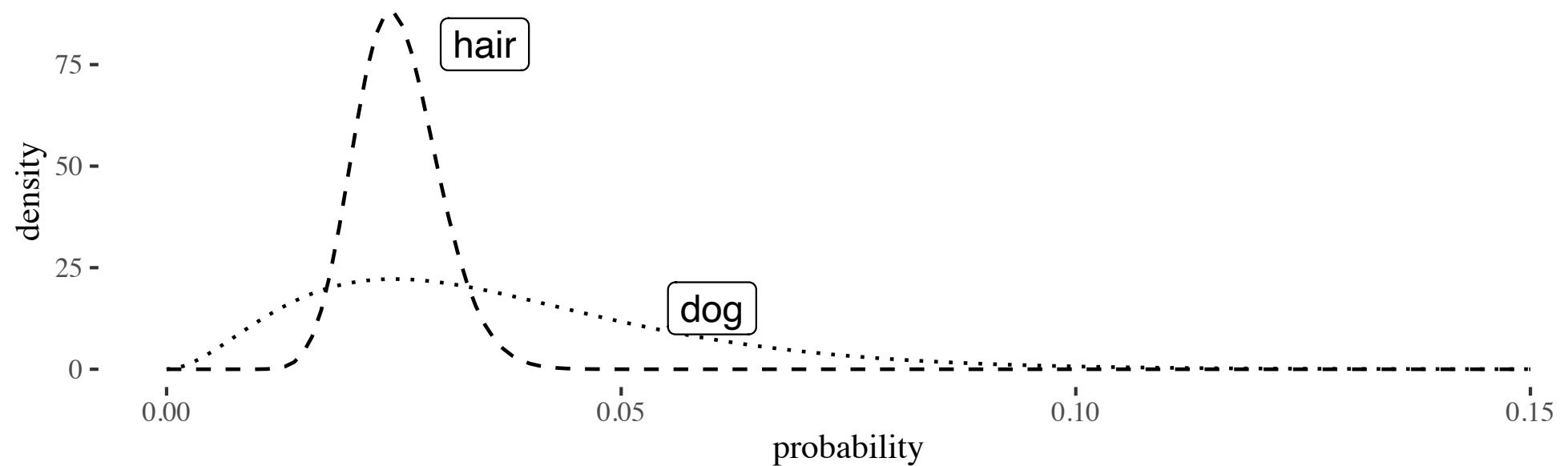
The random match probability for the *dog evidence* is based on finding 2 matches in a database of size 78 (so $1/78=0.025$).

Conditional densities for individual items of evidence if the source hypothesis is false

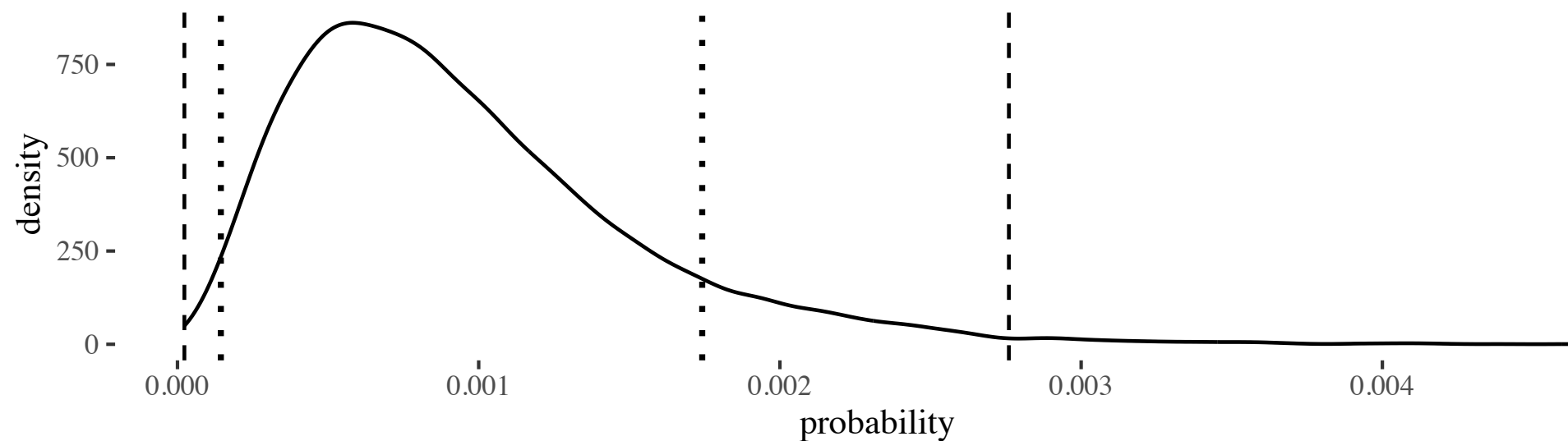


JOINT DENSITY

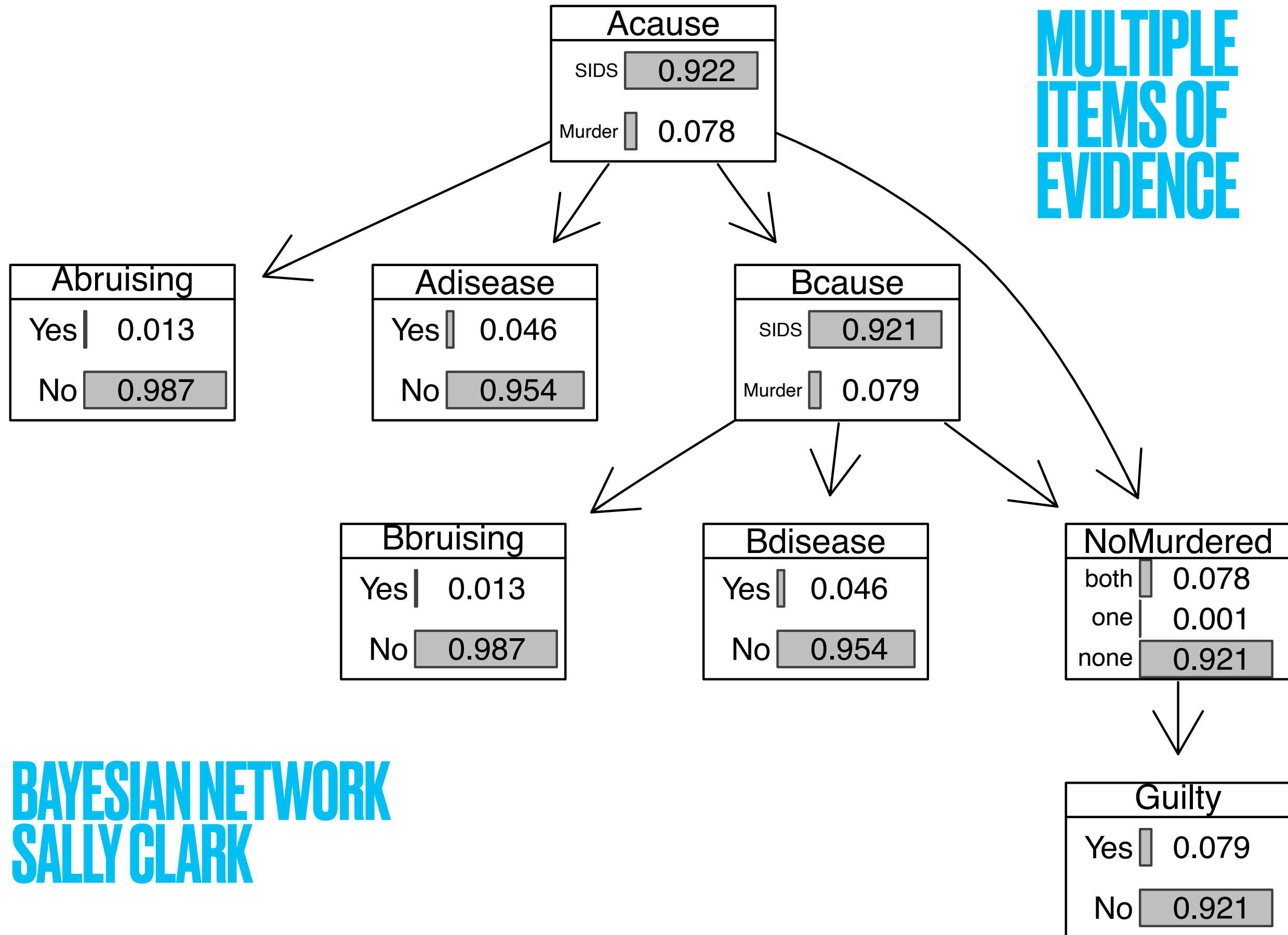
Conditional densities for individual items of evidence if the source hypothesis is false



Conditional density for joint evidence
(with .99 and .9 HPDIs)

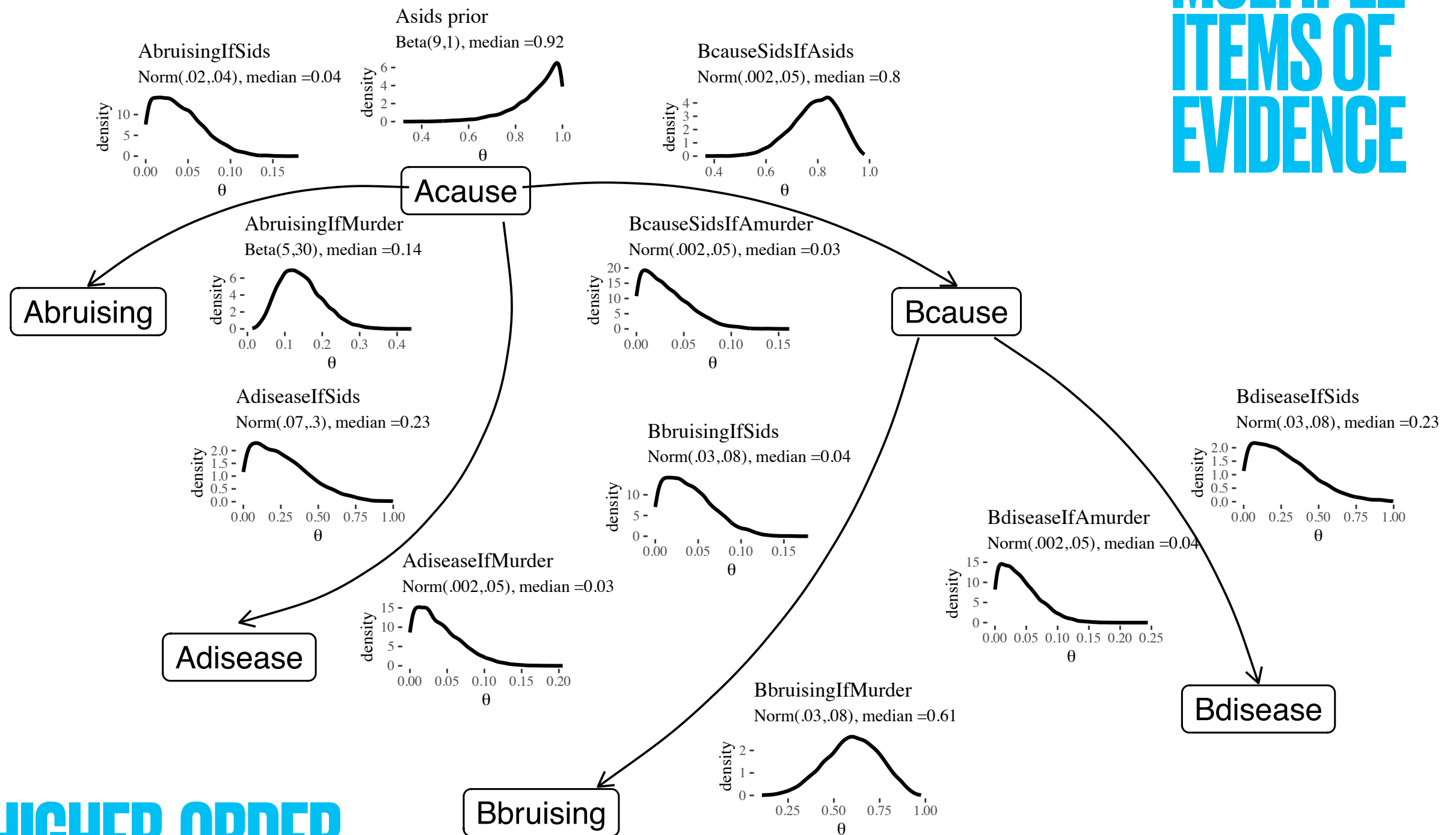


MULTIPLE
ITEMS OF
EVIDENCE



BAYESIAN NETWORK
SALLY CLARK

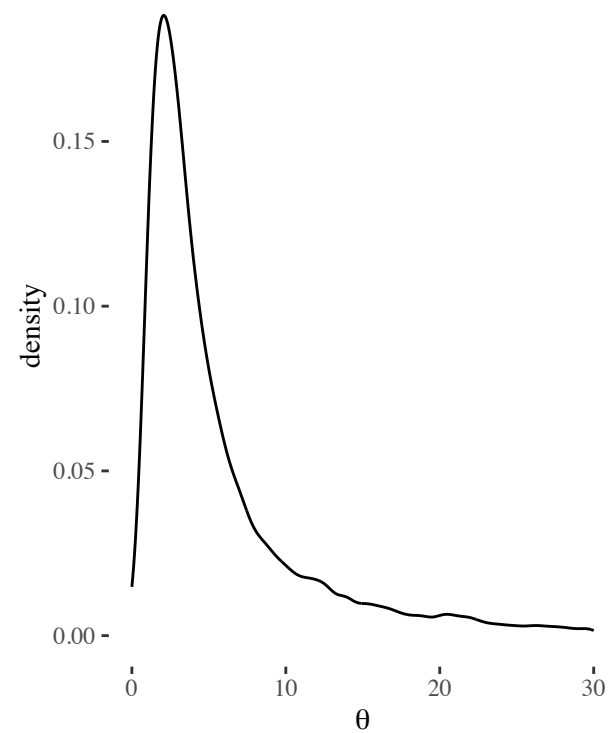
MULTIPLE ITEMS OF EVIDENCE



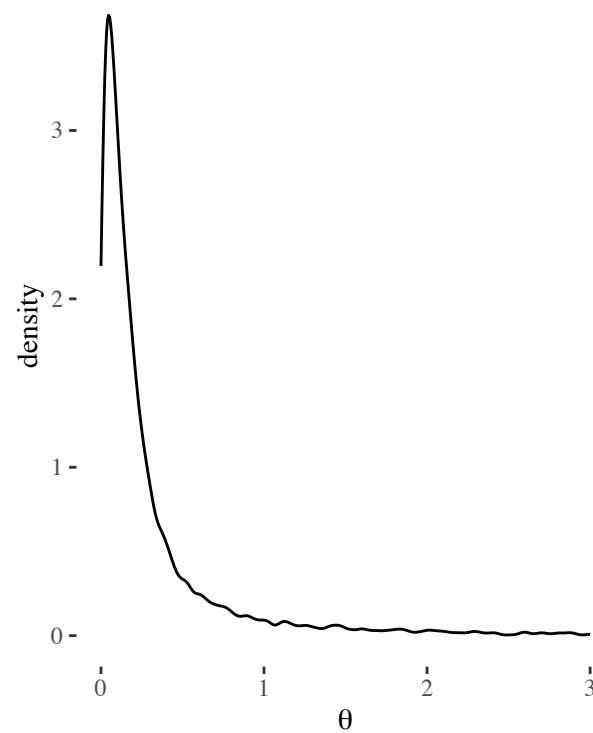
HIGHER-ORDER BAYESIAN NETWORK SALLY CLARK

DISTRIBUTIONS OVER

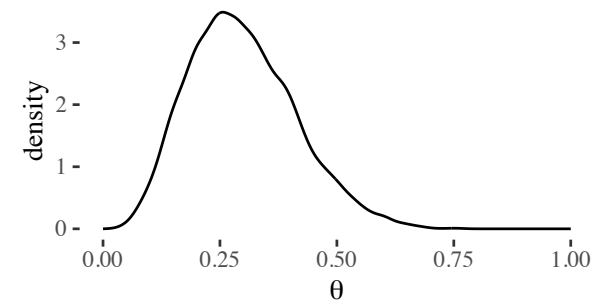
LR: Abruising
median = 3.89, 89%HPDI = 0.4–19



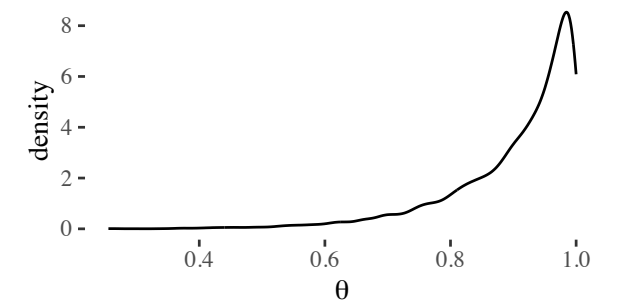
LR: Adisease
median = 0.15, 89%HPDI = 0–0.85



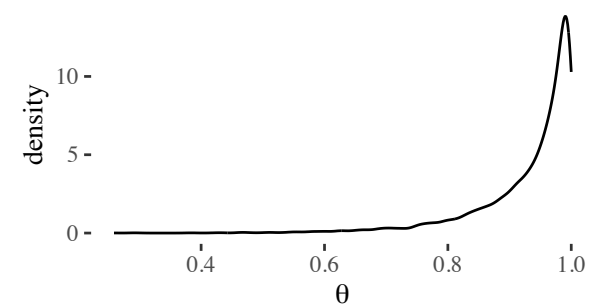
GuiltPrior
median = 0.29, 89%HPDI = 0.12–0.47



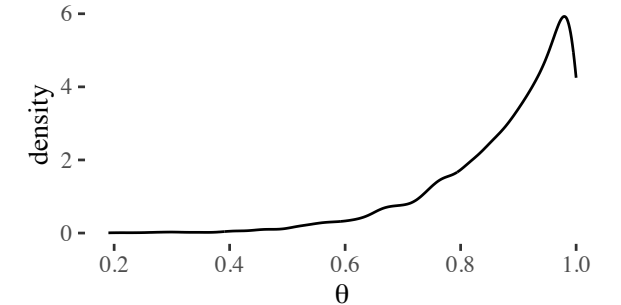
GuiltABbruising
median = 0.94, 89%HPDI = 0.77–1



GuiltABbruisingNoDisease
median = 0.96, 89%HPDI = 0.83–1



GuiltABbruisingDiseaseA
median = 0.91, 89%HPDI = 0.72–1



LIKELIHOOD RATIOS

POSTERIOR PROBABILITIES

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CAVEAT

Higher-order legal probabilism is

- More informally rich and better equipped to guide evidence evaluation and decision-making than simple legal probabilism
- Computationally feasible

Still, for practical purposes, simple legal probabilism can be preferred

STILL TO BE DONE

Legal probabilism focuses on



- 1. evidence assessment**
- 2. decision making**
- 3. accuracy and fairness**

More work to be done on



1. How higher-order probabilism can inform trial decisions



2. Whether higher-order probabilism makes decision more accurate and fair

Thank you!

Questions and comments welcome:

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