Rethinking legal probabilism

Rafał Urbaniak

1 Scientific goal

As many miscarriages of justice indicate, scientific evidence is easily misinterpreted in court. This happens partially due to miscommunication between the parties involved, partially due to the usual probabilistic fallacies, but also because incorporating scientific evidence in the context of a whole case can be really hard. While probabilistic tools for piecemeal evaluation of scientific evidence and spotting probabilistic fallacies in legal contexts are quite well developed,the construction of a more general probabilistic model of incorporating such evidence in a wider context of a whole case, probabilistic explication of and theorizing about evidence evaluation and legal decision standards, remain a challenge. Legal probabilism (LP), for our purpose, is the view that this challenge can and should be met. This project intends to contribute to further development of this enterprise in a philosophically motivated manner.

say sth about repli-cability crisis in forensic sciences at some point?

The assessment of evidence in the court of law can be viewed from at least three perspectives: as an interplay of arguments, as an assessment of probabilities involved, or as an interaction of competing narrations. Each perspective presents an account of legal reasoning (Di Bello & Verheij, 2018; van Eemeren & Verheij, 2017). Individually, each of these strains has been investigated. The probabilistic approach is the most developed one but LP is still underdeveloped —to a large extent this is so in light of various lines of criticism developed by the representatives of the other strains.

The **goal of this project** is to contribute to the **development of legal probabilism** by formulating its variant that accomodates important **insights provided by its critics**. A crucial point of criticism is that the fact-finding process should be conceptualized as **a competition of narrations**. I plan to develop methods that allow the probabilist to take this perspective, and explain how such methods allow the legal probabilist to address various objections present in the literature. The key idea is that once **narrations are represented as bayesian networks, various criteria on, features of and operations on narrations can be explicated in terms of corresponding properties of and operations on bayesian networks**. Further, the hypothesis is that such an improved framework will facilitate addressing key objections raised against LP.

The conceptual developments will be accompanied by technical accounts. **R** code capturing the technical features developed will be made available to the reader. Thus, the output will be a **unifying extended probabilistic model embracing key aspects of the narrative and argumentative approaches, susceptible to AI implementation.** The methods employed include: Bayesian statistical methods (including Bayesian approach to higher-order probability), imprecise probabilities, and Bayesian networks.

Contents

2 Significance

(state of the art, justification for tackling a specific scientific problem, justification for the pioneering nature of the project, the impact of the project results on the development of the research field and scientific discipline);

2.1 State of the art

2.1.1 Legal probabilisim

One of the functions of the trial is to resolve disputes about questions of facts. Did the defendant rob the bank? Who is the father of the child? Did this drug cause birth defects? To answer such questions, the litigants will present evidence of different kinds: eyewitness testimonies, DNA matches, epidemiological studies, etc. The evidence presented will often be in conflict with other evidence. In a bank robbery case, for example, the prosecution may present eyewitness testimony that the defendant was seen driving a truck near the bank a few minutes after the robbery took place. The defense may respond that no traces were found at the crime scene that would match the defendant. The fact-finders, judges or lay jurors, should address these conflicts by assessing and weighing the evidence, and on that basis reach a final decision. This is a difficult task. The evidence presented at trial can be complex and open to multiple interpretations, and even when it is assessed carefully, it may still lead to an incorrect verdict. How should judges and jurors respond to this uncertainty?

From among the three perspectives mentioned in the beginning, the probablistic approach will be my point of departure, for the following key reasons:

- The project is to be informed by and reflect on the actual practice of legal evidence evaluation, and much of scientific evidence in such contexts has probabilistic form.
- Probabilistic tools are fairly well-developed both for applications and within formal epistemology, reaching a state of fruition which should inspire deeper reflection.
- Statistical computing tools for such methods are available, which makes programming development and preliminary computational and data-driven evaluation of the ideas to be defended a viable enterprise.

Accoringly, the view in focus of this research is legal probabilism (LP)—an ongoing research program that comprises a variety of claims about evidence assessment and decision-making at trial. At its simplest, it comprises two core tenets: first, that the evidence presented at trial can be assessed, weighed and combined by means of probability theory; and second, that legal decision rules, such as proof beyond a reasonable doubt in criminal cases, can be explicated in probabilistic terms.

In the Middle Ages, before the advent of probability theory, there existed an informal mathematics of legal evidence (Wigmore, 1901). Formalistic procedures fixed the number of witnesses required to establish a claim. Lawyers would list ways in which items of evidence could be added or subtracted to weaken or strengthen one's case. This formalistic system fell into disrepute as the Enlightenment principle of 'free proof' gained wide acceptance (Damaška, 1995). Concurrently, the development of probability theory brought forth a new approach to weighing evidence and making decisions under uncertainty. The early theorists of probability in the 17th and 18th century were as much interested in games of chance as they were interested in the uncertainty of trial decisions (Daston, 1988; Franklin, 2001; Hacking, 1975). Bernoulli (1713) was one of the first to formulate probabilistic rules for combining different pieces of evidence in legal cases and assessing to what extent they supported a claim of interest. He was also one of the first to suggest that decision rules at trial could be understood as probability thresholds.

Bernoulli's prescient insights attained greater popularity in the 20th century amidst the law and economics movement (Becker, 1968; Calabresi, 1961; Posner, 1973). In a seminal article, Finkelstein & Fairley (1970) gave one of the first systematic analyses of how probability theory, and Bayes' theorem in particular, can help to weigh evidence at trial. Lempert (1977) was one of the first to rely on probability theory, specifically likelihood ratios, for assessing the relevance of evidence. Such contributions fueled what has been called the New Evidence Scholarship, a rigorous way of studying the process of legal proof at trial (Lempert, 1986).

2.1.2 Skeptical voices and challenges

In response to such developments, Tribe (1971) attacked what he called 'trial by mathematics'. His critique ranged from listing well-known cases of misuse or probabilities in legal contexts and practical difficulties in assessing the probability of someone's criminal or civil liability to the dehumanization of trial decisions. After Tribe, many have criticized legal probabilism on a variety of grounds, both theoretical and practical, arguing that probabilistic models are either inadequate or unhelpful (Brilmayer, 1986; Cohen, 1986; Dant, 1988, Allen (1986); Underwood, 1977).

After the discovery of DNA fingerprinting in the eighties, many legal probabilists focused on how probability theory could be used to quantify the strength of a DNA match under various circumstances (Kaye, 1986, 2010; Koehler, 1996; National Research Council, 1992; Robertson & Vignaux, 1995).

Some legal scholars and practitioners have voiced their support for legal probabilism explicitly (Tillers & Gottfried, 2007). Yet skepticism about mathematical and quantitative models of legal evidence is still widespread among prominent legal scholars and practitioners (see, for example, Allen & Pardo, 2007). Even among legal probabilists, few would think it possible to quantify precisely the probability of someone's guilt or civil liability. In response, probabilistists such as Taroni, Biedermann, Bozza, Garbolino, & Aitken (2014) suggest that the probabilistic formalism is still useful as an aid to structure and guide one's inferences under uncertainty, rather than a way to reach precise numerical assessments.

Conceptually, the probabilistic approach togehter with decision-theoretic considerations, can be used to theorize about the standard of proof and its properties. But for this project to be successfull, a proper probabilist explication of such a standard needs to be agreed upon. Imagine you are a trier of fact in a legal proceeding in which the defendant's guilt is identified as equivalent to a certain factual statement G and that somehow you succeeded in properly evaluating $\mathsf{P}(G|E)$—-the probability of G given the total evidence presented to you, E One question that arises in such a situation is: when should you decide against the defendant? when is the evidence good enough? What we are after here is a condition Ψ, formulated in (primarily) probabilistic (and perhaps decision-theoretic) terms, such that the trier of fact, at least ideally, should accept any relevant claim A (including G) just in case $\Psi(A,E)$. One straightforward attempt might be to say: convict if $\mathsf{P}(G|E)$ is above a certain threshold, otherwise acquit.

Perhaps the most difficult conceptual challenge to such probabilistic explications—at least, one that has galvanized philosophical attention in recent years—comes from the paradoxes of legal proof or puzzles of naked statistical evidence. In a number of seminal papers, Nesson (1979), Cohen (1981), and Thomson (1986) formulated scenarios in which, even if the probability of guilt or civil liability, based on the available evidence, is particularly high, a verdict against the defendant seems unwarranted.

A variant of such a scenario—the gatecrasher paradox—goes as follows. Suppose our guilt threshold is high, say at 0.99. Consider the situation in which 1000 fans enter a football stadium, and 991 of them avoid paying for their tickets. A random spectator is tried for not paying. The probability that the spectator under trial did not pay exceeds 0.99. Yet,intuitively, a spectator cannot be considered liable on the sole basis of the number of people who did and did not pay.

Another problem with the proposal is the so-called difficulty about conjunction. It arises, because intuitively there should be no difference between the trier's acceptance of A and B separately, and her acceptance of their conjunction, $A \wedge B$, that is, that $\Psi(A,E)$ and $\Psi(B,E)$ just in case $\Psi(A \wedge B,E)$. If $\Psi(H,E)$ just the threshold criterion, requiring that $\mathsf{P}(H|E)$ be sufficiently high, Ψ in general fails to satisfy this equivalence.

Arguably, these scenarios underscore a theoretical difficulty with probabilistic accounts of legal standards of proof. Many articles have been written on the topic, initially by legal scholars. In the last decade, philosophers have also joined the debate—for critical surveys see Redmayne (2008), Gardiner (2018) and Pardo (2019). Crucially, even fairly recent proposals to mend the situation (Dawid, 1987, Cheng (2012), Kaplow (2014)) on the part of the legal probabilist have failed (Urbaniak, 2019 contains a detailed analysis).

At least *prima facie*, then, it seems that some conditions other than high posterior probability of liability have to be satisfied for the decision to penalize (or find liable) to be justified. Accordingly, various informal notions have been claimed to be essential for a proper explication of judiciary decision standards (Haack, 2014; Wells, 1992). For instance, evidence is claimed to be insufficient for conviction if it is not *sensitive* to the issue at hand: if it remained the same even if the accused was innocent (Enoch & Fisher, 2015). Or, to look at another approach, evidence is claimed to be insufficient for conviction if it doesn't *normically support* it: if—given the same evidence—no explanation would be needed even if the accused was innocent (Smith, 2017). A legal probabilist needs either to show that these notions are unnecessary or inadequate for the purpose at hand, or to indicate how they can be explicated in probabilistic terms.

2.1.3 The narrative approach

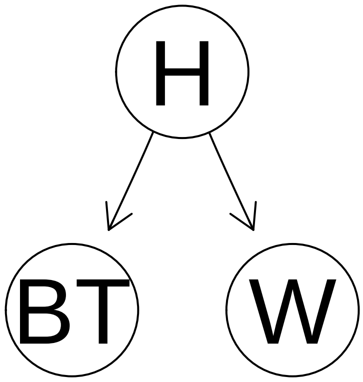
More recently, alternative frameworks for modeling evidential reasoning and decision-making at trial have been proposed. They are based on inference to the best explanation (Allen, 2010; Pardo & Allen, 2008), narratives and stories (Allen, 1986, 2010; Allen & Leiter, 2001; Clermont, 2015; Pardo, 2018; Pennington & Hastie, 1991a), and argumentation theory (Bex, 2011; Gordon, Prakken, & Walton, 2007; Walton, 2002). Those who favor a conciliatory stance have combined legal probabilism with other frameworks, offering preliminary sketches of hybrid theories (Urbaniak, 2018; Verheij, 2014).

Another important point of criticism of LP is that legal proceedings are back-and-forth between opposing parties in which cross-examination is of crucial importance, reasoning goes not only evidence-to-hypothesis, but also hypotheses-to-evidence (Allen & Pardo, 2007; Wells, 1992) in a way that seems analogous to inference to the best explanation (Dant, 1988), which notoriously is claimed to not be susceptible to probabilistic analysis (Lipton, 2004). An informal philosophical account inspired by such considerations—The **No Plausible Alternative Story (NPAS)** theory (Allen, 2010)—is that the courtroom is a confrontation of competing narrations (Ho, 2008; Wagenaar, Van Koppen, & Crombag, 1993) offered by the sides, and the narrative to be selected should be the most plausible one. The view is conceptually plausible (Di Bello, 2013), and finds support in psychological evidence (Pennington & Hastie, 1991b, 1992).

It would be a great advantage of legal probabilism if it could model phenomena captured by the narrative approach. But how is the legal probabilist to make sense of them? From her perspective, the key disadvantage of NPAS is that it abandons the rich toolbox of probabilistic methods and takes the key notion of plausibility to be a primitive notion which should be understood only intuitively.

2.1.4 Bayesian networks as a tool for legal probabilism

The idea that Bayesian networks can be used for probabilistic reasoning in legal fact-finding started gaining traction in late eighties and early nineties (Edwards, 1991), and it found its way to nowadays standard textbooks on the topic (Fenton & Neil, 2018; Taroni et al., 2014).
A Bayesian network comprises two components: first, a directed acyclic graph of relations of dependence (represented by arrows) between variables (represented by nodes); second, conditional probability tables. Consider the graphical component first. The graph is acyclic because the arrows connecting the nodes do not form loops. As an illustration, let H be the claim that the suspect committed the murder, BT the presence of a blood type B match with a crime scene stain, and W the fact that an eyewitness observed the suspect near the scene around the time of the crime. The graphical component of the Bayesian network would look like this:



The *ancestors* of a node X are all those nodes from which we can reach X by following the arrows going forwards. The *parents* of a node X are those for which we can do this in one step. The *descendants* of X are all which can be reached from X by following the arrows going forward. The *children* are those for which we can do this in one step. In the example, H is the parent (and ancestor) of both $ W$ and BT, which are its children (and descendants). There are no non-parent ancestors or non-children descendants.

The variables, which are represented by nodes and are connected by arrows, stand in relation of probabilistic dependence. To describe these relations, the graphical model is accompanied by conditional probability tables. For parentless nodes such as H, the tables specify the prior probabilities of all their possible states. Assuming H stands for a binary random variable, with two possible states, the prior probabilities could be:

	Prior
H=murder	.01
H=no.murder	.99

The .01 figure for H=murder rests on the assumption that, absent any incriminating evidence, the defendant is unlikely to be guilty. For children nodes, the tables specify their conditional probability given combinations of their parents' states. If the variables are binary, an assignment of values for them could be:

	H=murder	H=no.murder
W=seen	.7	.4
W=not.seen	.3	.6

	H=murder	H=no.murder
BT=match	1	.063
BT=no.match	0	.937

According to the tables above, even if the defendant is not the culprit, the eyewitness testimony would still incriminate him with probability of .4, while the blood evidence with probability equal to only .063. The blood type frequency estimate is realistic (Lucy, 2013), and so are the conditional probabilities for the eyewitness identification. As expected, eyewitness testimony is assumed to be less trustworthy than blood match evidence (Urbaniak, Kowalewska, Janda, & Dziurosz-Serafinowicz, 2020; but for complications about assessing eyewitness testimony, see Wixted & Wells, 2017).

The three probability tables above are all that is needed to define the probability distribution. The tables do not specify probabilistic dependencies between nodes that are not in a relation of child/parent, such as BT and W. Since there is no arrow between them, nodes BT and W are assumed to be independent conditional on H, that is, $\mathsf{P}(()W|H) = \mathsf{P}(()W|H \wedge BT)$. This fact represents, as part of the structure of the network, the independence between eyewitness testimony and blood evidence. A generalization of this fact is the so-called Markov condition. While the Bayesian network above—comprising a directed acyclic graph along with probability tables—is simple, a correct intuitive assessment of the probability of the hypothesis given the evidence is already challenging. The reader is invited to try to estimate intuitively the probability that the defendant committed the murder (H=murder) given the following states of the evidence:

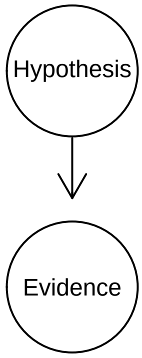
- The suspect's blood type matches the crime stain but information about the witness is unavailable.
- The suspect's blood type matches the crime stain but the witness says they did not see the suspect near the crime scene.
- The suspect's blood type matches the crime stain and the witness says they saw the suspect near the crime scene.

Already at this level of complexity, calculations by hand become cumbersome. In contrast, software for Bayesian networks (see, for example, the **R** package **bnlearn** developed by Marco Scutari and described in Scutari & Denis, 2015) will easily give the following results:

	H=murder
BT=match,W=?	.138
BT=match,W=not.seen	.074
BT=match, W=seen	.219

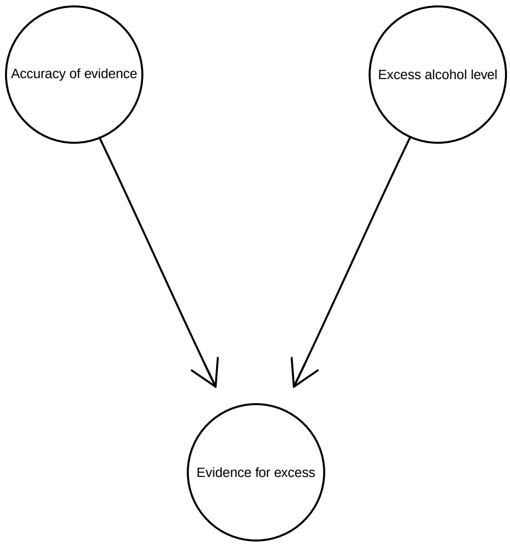
Perhaps surprisingly the posterior probability of H is about .22 even when both pieces of evidence are incriminating (BT=match, W=seen).

Simple graphical patterns (called *idioms*) often recur while modeling the relationships between evidence and hypotheses. Complex graphical models can be created by combining these basic patters in a modular way. Discussion of general methods for Bayesian network constructions can be found in (Neil, Fenton, & Nielson, 2000), (Hepler, Dawid, & Leucari, 2007) and general idioms are discussed in (Fenton, Neil, & Lagnado, 2013). As an example, consider the *evidence idiom* is the most basic graphical representation of the relation between a hypothesis and a piece of evidence:



This directed graph suggests that the direction of influence—which could, but need not, be interpreted as causal influence—goes from hypothesis to evidence (though the probabilistic dependence goes both ways). The hypothesis node and the evidence node can be binary variables, such as "The defendant was the source of the crime scene traces" (hypothesis) and "The defendant genetically matches the crime traces" (evidence). But the variables need not be binary. The hypothesis node might take values from the range of $1-40$, say the distance in meters from which the gun was shot, and the evidence node might be a continuous variable representing the density of gun shot residues (Taroni et al., 2014).

As an example of a more complex idiom, called the *evidence accuracy idiom*, consists of two arrows going into the evidence node [Bovens & Hartmann (2004); friedman1974]. One incoming arrow comes from the hypothesis node and the other from the accuracy node. This idiom can be used to model, say, an alcohol test:



The directions of the arrows indicate that the accuracy of the evidence (accuracy node) and the alcohol level (hypothesis node) influence the outcome of the test (evidence node). The graphical model represents different sources of uncertainty. The uncertainty associated with the sensitivity and specificity of the test—that is, the probability that the tests reports excessive alcohol level when the level is excessive (sensitivity) and the probability that the test reports normal alcohol level when the level is normal (specificity)—is captured by the arrow going from the hypothesis node (Excess alcohol level) to the evidence node (Evidence for excess). Other sources of uncertainty comprise the possibility that the police officer lied about the test report or the possibility that the driver took medications which then affected the alcohol level. These possibilities can be taken into consideration by adding an accuracy node (or multiple accuracy nodes, if each factor is kept separate from the other).

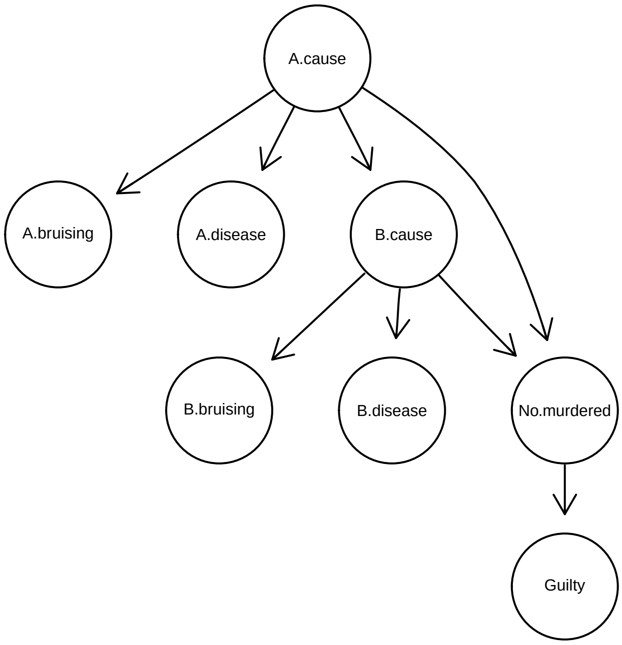
The key poin there is that large steps have been made towards the development of BN-related tools for evidence evaluation. However, so far, most of them have to do with presentation and evaluation of various pieces of evidence, not with the development of a more general model to facilitate more general probabilistic reflection on legal decision standards.

2.2 Pioneering nature of the project

2.2.1 Key elements of the approach

For the reasons already mentioned, Bayesian Networks and probabilistic methods will be in the focus of this project. It is quite clear that BNs are useful tool when it comes to piecemeal modeling and evaluation of scientific evidence in court. The question is, whether they can be useful for modeling whole cases and casting light on both the conceptual challenges that we already mention and for incorporating the points made by the representatives of other strains of research, most crucially, the NPAS.

Attempts have been made to use Bayesian networks to weigh and assess complex bodies of evidence consisting of multiple components. On one hand, we have serious reconstructions of real complex cases. Kadane & Schum (2011) made one the first attempts to model an entire criminal case, Sacco & Vanzetti from 1920, using probabilistic graphs. More recently, Fenton & Neil (2018) constructed a Bayesian network for the famous Sally Clark case:



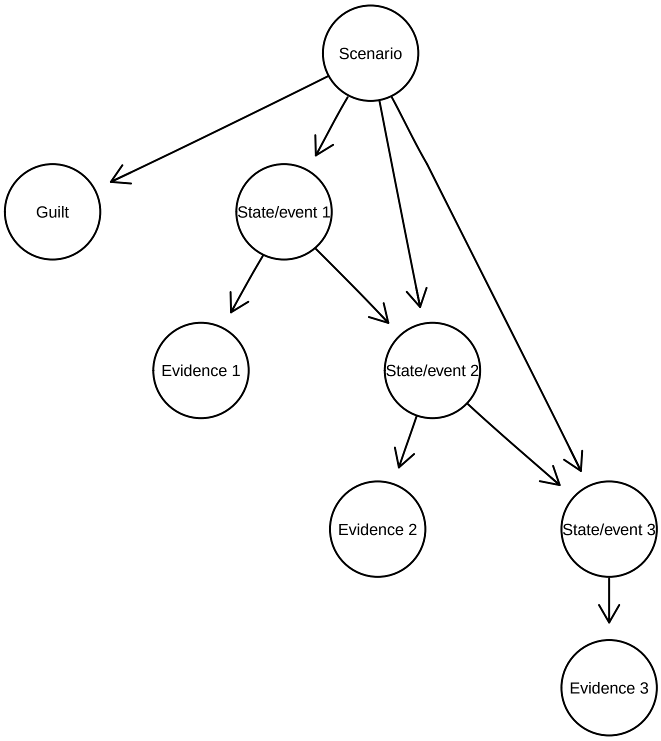
The arrows depict relationships of influence between variables. Whether Sally Clark's sons, call them *A* and *B*, died by SIDS or murder (A.cause and B.cause) influences whether signs of disease (A.disease and B.disease) and bruising (A.bruising and B.bruising) were present. Since son A died first, whether A was murdered or died by SIDS (A.cause) influences how son B died (B.cause). How the sons died determines how many sons were murdered (No.murdered), and how many sons were murdered decides whether Sally Clark is guilty (guilty).

According to the calculation by **?**, the prior probability of Guilty = Yes should be .0789. After taking into account the incriminating evidence presented at trial, such as that there were signs of bruising but no signs of a preexisting disease affecting the children, the posterior probabilities are as follows:

Evidence (cumulative)	P(()Clark guilty)
A bruising	.2887
A no signs of disease	.3093
B bruising	.6913
B no signs of disease	.7019

The incriminating evidence, combined, brings the probability of guilt from .0789 to .7019. This is a significant increase, but not quite enough for a conviction. If one wishes to perform sensitivity analysis by modifying some of the probabilities, this can be easily done. During the appeal trial, new evidence was discovered, in particular, evidence that son A was affected by a disease. Once this evidence is taken into account, the probability of guilt drops to .00459 (and if signs of disease were also present on B, the guilt probability would drop even further to .0009). For a general discussion on how to elicit probabilities, see (Renooij, 2001) and (Gaag, Renooij, Witteman, Aleman, & Taal, 2013).

On the other hand, we have more general methodological reflection on the use of BNs for modeling whole cases. The main idea is that once all the pieces of evidence and claims are represented as nodes, one should use the *scenario idiom* to model complex hypotheses, consisting of a sequence of events organized in space and time: a scenario (Vlek, Prakken, Renooij, & Verheij, 2014). A graphical model that uses the scenario idiom would consist of the following components: first, nodes for the states and events in the scenario, with each node linked to the supporting evidence; second, a separate scenario node that has states and events as its children; finally, a node corresponding to the ultimate hypothesis as a child of the scenario node. The graphical model would look like this:



The scenario node unifies the different events and states.
Because of this unifying role, increasing the probability of one part of the scenario (say State/event 2) will also increase the probability of the other parts (State/event 1 and State/event 3). This is intended to capture the idea that the different components of a scenario form an interconnected sequence of events.[1], Bex (2011), Bex (2015) and Verheij (2017). See also the survey by Di Bello & Verheij (2018). Dawid & Mortera (2018) give a treatment of scenarios in terms of Bayesian networks. Lacave & Díez (2002)} show how Bayesian Network can be used to construct explanations.

Some preliminary moves towards such an explication have been made in (Urbaniak, 2018). However, as (Neil, Fenton, Lagnado, & Gill, 2019) correctly points out that the paper `fails to offer a convincing and operational means to structure and compare competing narratives.`

Initial philosophical analysis of the approach has been performed, (Di Bello, 2013) pioneering a probabilistic understanding of narrations.

add more about Marcello

2.2.2 Impact

3 Work plan

(general work plan, specific research goals, results of preliminary research, risk analysis);

4 Methodology

(underlying scientific methodology, methods, techniques and research tools, methods of results analysis, equipment and devices to be used in research);

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[1] A discussion of modelling crime scenarios by means of graphical devices mixed with probabilities can be also found in the work of @shen2007ScenariodrivenDecisionSupporta

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