

# Rebooting the new evidence scholarship

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## Abstract

The new evidence scholarship addresses three distinct approaches: legal probabilism, Bayesian decision theory and relative plausibility theory. Each has major insights to offer, but none seems satisfactory as it stands. This paper proposes that relative plausibility theory be modified in two substantial ways. The first is by defining its key concept of plausibility, hitherto treated as primitive, by generalising the standard axioms of probability. The second is by complementing the descriptive component of the theory with a normative decision theory adapted to legal process. Because this version of decision theory is based on plausibilities rather than probabilities, it generates plausibilistic expectations as outputs. Because these outputs are comparable, they function as relative plausibilities. Hence the resulting framework is an extension of relative plausibility theory, but it retains deep ties to legal probabilism, through the proposed definition of plausibility, and to Bayesian decision theory, through the normative use of decision theory.

## Keywords

Bayesian decision theory, legal probabilism, plausibility, probability, relative plausibility

## 1. Introduction

The new evidence scholarship is no longer new, but it may be due for renewal. Professor Richard Lempert (1986: 440) characterised the movement as a ‘third wave’ of evidence scholars. The first wave included systematisers like Wigmore, Maguire, McCormick and Morgan; the second wave began with the enactment of the Federal Rules of Evidence by the American Congress in 1975; and the third wave seeks ‘to build on or criticise mathematical models as modes of proof or as a means of understanding trial processes’.<sup>1</sup> Hence the new evidence scholarship was developed by two disparate groups: a pro group, disposed to build on mathematical models; and a con group, critical of the juridical use of these models.

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1. Lempert’s description of the new evidence scholarship was part of a presentation to the Symposium on Probability and Inference in the Law of Evidence held at Boston University in April 1986, where he noted that evidence scholarship is changing its focus from the articulation of rules to the process of proof. Symposium papers were published in *Boston University Law Review* 66 (1986: 377–952) and in Tillers and Green (1988).

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The pro group includes both legal probabilists and Bayesian decision theorists. Legal probabilists recommend reliance on probabilities in juridical decision making. They propose that probabilities be employed to clarify standards of proof, for example. The standard of proof in most civil cases, ‘the balance of probabilities’ in the United Kingdom or ‘the preponderance of the evidence’ in the United States, can be defined as a probability  $> .5$  (Kaye, 1982); the standard in exceptionally serious civil cases, ‘clear and convincing evidence’, might be interpreted as a probability around .75 (Schauer and Zeckhauser, 1996: 34); and the standard in criminal cases, ‘beyond a reasonable doubt’, could be glossed as a probability of .90 or .95 (Laudan, 2006: 56).<sup>2</sup> Probabilities could also be used to determine the probative value of evidence (Finkelstein and Fairley, 1970; Friedman, 1986; Kaye, 1986). Like probabilists, Bayesian decision theorists emphasise probabilities, which register beliefs, but they also insist on utilities, which express desires. They recommend combining both kinds of input in order to ensure that juridical decisions maximise expected utility. However, some decision theorists modify this basic idea by aiming at minimal expected regret (Kaplan, 1968), maximal expected desirability (Cullison, 1969) or minimal expected loss (Kaye, 1988, 1999).

Opposed to the pro attitudes of both legal probabilists and Bayesian decision theorists are scholars who criticise mathematical models of evidence. Prominent among them are advocates of relative plausibility theory (Allen, 1986, 1991, 1994).<sup>3</sup> These scholars insist that legal fact finders do not reason as legal probabilists and Bayesian decision theorists say they should; rather, fact finders first consider plaintiff’s and defendant’s explanations of disputed events and then compare these explanations in light of the evidence, their background knowledge and the relevant standard of proof (Allen and Pardo, 2019: 17). The comparison results in an inference to the best explanation (Pardo and Allen, 2008).<sup>4</sup> Relative plausibility theory is consistent with empirical work in cognitive psychology, notably the story model of juror decision making (Pennington and Hastie, 1986, 1991), which claims that jurors impose a narrative structure on information received at trial and base their decisions on these narratives. However, relative plausibility theorists stress that their theory is distinct from the story model, which ‘does not provide an explanation of standards of proof and other features of the proof process’ (Allen and Pardo, 2019: 17, n. 86). In addition, some explanations may not count as stories at all; statistical explanations, for example, are not stories in the usual sense of the term.

To sum up: a case of assault, for example, might be decided from three apparently divergent points of view. A legal probabilist would find the posterior probability of the plaintiff’s account and determine whether it meets the relevant standard of proof. In light of this standard, a Bayesian decision theorist would weigh the expected utility of finding for the plaintiff against the expected utility of finding for the defendant. A relative plausibilist would compare the plausibility of the plaintiff’s account and the plausibility of the defendant’s account and find for the plaintiff only if her account meets the relevant standard of proof.

Despite sharp divergences among the currents of new evidence scholarship, muted calls for reconciliation occasionally surface. Professor Friedman, who is a Bayesian decision theorist, claims that relative plausibility theory and Bayesian decision theory are ‘perfectly consistent’ (Friedman, 2001: 2045); and Professors Allen and Pardo, who are advocates of relative plausibility theory, note ‘there is no reason to see explanatory [relative-plausibilistic] and Bayesian [decision-theoretic] approaches as necessarily incompatible’ (Pardo and Allen, 2008: 253; cf. Allen, 1994: 607). More recently, Professors Biedermann and

2. Even trained judges differ in their numerical interpretations of the criminal standard, though they generally vary between .75 and .90 (Posner, 1999: 1506).

3. Because relative plausibility theory has roots in the tradition of naturalised epistemology (Allen and Leiter, 2001), it would be misleading to consider it simply as a critical response to the pro currents of new evidence scholarship. But since the focus of this paper is new evidence scholarship, its treatment of relative plausibility theory will be circumscribed accordingly.

4. Professor Allen (2008: 327) admits that his sense of ‘best explanation’ may not be what philosophers mean by the term, but ‘the central point is the explanation-based nature of juridical proof’. Ribeiro (2018) proposes a different approach to inference to the best explanation.

Vuille (2019: 18–20) affirm the compatibility of relative plausibility and their probabilistic approach.<sup>5</sup> Similarly, Professor Gelbach (2019: 169) describes relative plausibility as a ‘friendly amendment to the best version of the probabilistic paradigm’, and others have expressed comparable views.<sup>6</sup> I concur with the tenor of these comments, but I believe that the conditions for reconciling these approaches are not yet in place.

The purpose of this paper is to suggest how these conditions might be met. To contextualise the exposition of these conditions, we should first notice an ambiguity in the main strands of new evidence scholarship. Each can be taken as descriptive, as an account of how fact finders actually reason; or as normative, as a proposal about how fact finders ought to reason. The focus of this inquiry is normative. The normative focus is not meant to downplay the importance of the descriptive issues, which is undeniable. Rather, it is an attempt to configure a standard that could be used to evaluate actual instances of judicial decision. Given that end, descriptive questions are significant but secondary.

The paper unfolds as follows. Section 2 critiques the new evidence scholarship, raising difficulties for each of its three main strands: legal probabilism, Bayesian decision theory and relative plausibility theory. The remainder of the paper attempts to synthesise positive features of each of these strands. Although the paper emphasises relative plausibility theory, it also advocates two additions to this theory. The first of these additions is proposed in Section 3.1; the second, in Section 4. Possible objections to these proposals are addressed in Section 5. Finally, the claim that relative plausibility theory and certain features of legal probabilism and Bayesian decision theory are mutually consistent is summed up in Section 6.

## 2. Challenging the new evidence scholarship

Both pro and con currents of the new evidence scholarship have run into difficulties. Legal probabilism has been attacked along multiple fronts: ‘trial by mathematics’ may decrease the likelihood of accurate outcomes (Tribe, 1971); probabilism, if linked to the frequentist interpretation of probability, faces an unresolved problem of reference classes (Allen, 2017: 136; Allen and Pardo, 2007); and high probability may be insufficient for judgments of liability (Redmayne, 2008). Bayesian decision theory’s juridical uses have been censured for demeaning defendants’ individuality and autonomy (Wasserman, 1991) and for ignoring base rates and systematic errors in probability judgments (Allen and Leiter, 2001: 1503–1506). Relative plausibility theory has also come under fire, being charged with difficulties in aggregating evidence (Friedman, 1992: 93, n. 40); ignoring the possibility of a third story in addition to those of plaintiff and defendant (Nance, 2001: 1575–1583; 2019: 156–157); and encouraging an unacceptably high proportion of innocent but falsely convicted defendants (Laudan, 2007: 7).

The literature on these assorted charges is labyrinthine, but I intend to trace a relatively straight line through it in order to motivate the paper’s proposals. I will first state briefly what I take to be the central shortcomings of legal probabilism, Bayesian decision theory and relative plausibility theory in turn. I will then suggest how it might be possible to remedy these shortcomings.

### 2.1. Legal probabilism

The term ‘legal probabilism’ is doubly ambiguous. It can refer to legal reliance on the mathematical form of probability, exemplified by the probability calculus. But this Pascalian form of probability, which emphasises enumerative induction, can be contrasted with Baconian probability, which relies on eliminative induction (Cohen, 1977).<sup>7</sup> Since the problems about probability that figure in this study emerge

5. This compatibility thesis is further developed in Biedermann et al. (2020).

6. Professors Di Bello (2019a: 163–164), Spottswood (2019: 79), Sullivan (2019: 100–101) and Wittlin (2019: 189) stress the compatibility of probabilism and relative plausibility. Analogously, Professor Cheng (2013: 1258) downplays the contrast between probabilism and the story model.

7. Schum (1994, ch 5) attempts to integrate Pascalian and Baconian probability in an interdisciplinary approach to evidence.

primarily as problems of enumerative induction, the term ‘legal probabilism’ will be used here in its Pascalian sense.

The term ‘legal probabilism’ can also refer to relatively simple or more sophisticated appeals to probability. To illustrate the relatively simple usage, consider a much-discussed hypothetical in the debate over ‘naked statistical evidence’ (Kaye, 1980, 1982). Is a plaintiff who was injured at night by a bus but who cannot further identify the bus entitled to win judgment against the Blue Bus Company, which owns and operates 80% of the buses in town?<sup>8</sup> Although an affirmative response has been cited (Nesson, 1985: 1378–1379) as an example of a highly probable but unacceptable verdict, legal probabilism in its simple form would support this verdict. Since the probability that the offending bus belonged to Blue Bus is .8, unvarnished legal probabilism would find that this probability meets the civil standard of proof.

A more sophisticated form of legal probabilism would likely disagree. Suppose we consider hypotheticals of this sort in the light of Bayes’ theorem. Though the theorem can be stated in different ways, the odds form will suffice for our purposes. Where  $h$  is a hypothesis,  $e$  is the evidence,  $p$  is a probability measure, and expressions of the form  $p(\cdot|\cdot)$  represent conditional probabilities, the theorem is stated by Equation (1).

$$\frac{p(h|e)}{p(-h|e)} = \frac{p(h)}{p(-h)} \times \frac{p(e|h)}{p(e|-h)} \quad (1)$$

The ratio on the left is known as the posterior odds; the product’s left-hand factor gives the prior odds; and the product’s right-hand factor is the likelihood ratio.

Professor Di Bello (2019b) draws on this form of Bayes’ theorem to argue that high probability of guilt may be insufficient to convict. He introduces a hyperprobable hypothetical in which 99 of 100 unidentified prisoners collectively kill a guard, and the posterior odds of guilt for a randomly selected prisoner are taken to be 99:1. The argument relies on the concept of probative value, which is measured by the likelihood ratio. The evidence has probative value if, and only if, the likelihood ratio is greater than 1; that is, the probability of the evidence if the accused is guilty must be greater than the probability of the evidence if the accused is not guilty (Di Bello, 2019b: 1049). Equation (1) shows that the likelihood ratio can be calculated provided the posterior odds and the prior odds are given. But even when posterior odds are given, as in Di Bello’s hypothetical, the prior odds can remain unspecified due to gaps in background information. In those cases, then, the likelihood ratio is unclear. But if the likelihood ratio is unclear, then the probative value of the evidence is unclear. Hence high probability of guilt, reflected in the posterior odds, may provide little or no reason to convict.<sup>9</sup>

The same kind of analysis can be used to explain real cases where naked statistical evidence has been found sufficient to convict. Di Bello (2019b: 1073–1076) argues that an analysis in terms of probative value can justify conviction on the basis of a DNA match alone. For example, an Ohio stranger-rape case resulted in conviction even though there was ‘literally no other evidence’ (*State v Hunter*, 861 N.E.2d 898 (Ohio App. Ct. 2006) at 898) than a cold-hit DNA match.<sup>10</sup> Here the probability of the evidence of a DNA match given a hypothesis of guilt is extremely high, while the probability of a DNA match given a hypothesis of non-guilt is quite low.<sup>11</sup> Hence the likelihood ratio is greater than 1, and the evidence has clear probative value.

8. The hypothetical is based on *Betty Smith v Rapid Transit, Inc.*, 317 Mass. 469, 58 N.E.2d 754 (1945), where the trial court found for the bus company and the Massachusetts Supreme Court affirmed.

9. Urbaniak et al. (2020) respond to Di Bello (2019b), arguing in part that the risk of mistaken conviction based on naked statistical evidence can be treated like the risk of mistaken conviction based on any other sort of evidence.

10. Some courts have convicted on the strength of DNA evidence even when there is contrary evidence (Song et al., 2009: 23). Just how strong the DNA evidence should be is treated in Roth (2010).

11. Thomson (1986: 203–206) presents a complementary line of argumentation, arguing that statistical evidence in the bus hypotheticals fails to establish an appropriate causal connection.

In sum, a more sophisticated form of legal probabilism can correct the relatively simple form that would find Blue Bus liable due to its 80% ownership share and convict a randomly selected prisoner on the basis of a 99% participation rate. But sophisticated legal probabilism has its own problems. I will mention just two.

The first problem is how to utilise the numerical givens in high-probability hypotheticals and their real-life counterparts. The numerical givens are not probabilities, though they can serve as the basis of probabilities.<sup>12</sup> But probabilities of what kind? Prior probabilities, posterior probabilities or evidential probabilities? In discussing the paradox of the gatecrasher (Cohen, 1977: 75), Professor Kaye (1979: 75) employs the given that 501 of 1000 rodeo attendees did not pay as the basis of a prior probability of .501 that a random attendee did not pay.<sup>13</sup> Professor Di Bello (2019b: 1046) utilises the given that 99 of 100 prisoners participated in killing the guard as the basis of a posterior probability of .99 that a random prisoner is guilty. I would like to float a third possibility, which is to treat the numerical given as the basis of evidential probability: the probability of the evidence. This third possibility will not be defended here; I will merely present it as an alternative—and then a problem—for sophisticated legal probabilism.

To illustrate this third possibility, let us return to the 99 prisoners who collectively killed the guard. If we posit a prior probability of .5 that a randomly selected prisoner participated in the killing, the prior odds would be equal to 1: .5/.5. In addition, we might consider the 99% participation rate as the basis of evidential probability. Note that Di Bello (2019b: 1050) cautions that the likelihood ratio in this case cannot be assumed to equal 1, for the 99% participation rate is sensitive to counterfactual test. That is, if the prisoner participated in the killing, the rate would be 100%; but if the prisoner did not participate, the rate would be 98%. Consequently, if the participation rate is treated as the basis of evidential probability, the probability of the evidence given that the prisoner participated in the killing is 1, while the probability of the evidence given that the prisoner did not participate is .98. The likelihood ratio would then be  $1/.98$ . Using the odds form of Bayes' theorem, we could then calculate the posterior odds of guilt as  $.5/.5 \times 1/.98 = 50/49$ . Posterior odds of 50:49, which we have obtained by treating the participation rate as the basis of evidential probability, are a far cry from posterior odds of 99:1, obtained from taking the participation rate as the basis of posterior odds. Posterior odds of 50:49 would clearly not meet the criminal standard of proof, and this would confirm any intuition that the 99% participation rate would be insufficient for conviction.<sup>14</sup>

As stated, I will not attempt to defend the option of treating the numerical given as the basis of evidential probability. Rather, the two previous paragraphs suggest two points. First, they illustrate that a numerical given can serve as the basis of evidential probability as well as the basis of prior probability and posterior probability. Second, these alternatives pose a problem for legal probabilism that, as far as I know, has not been frontally addressed in the literature. Given these three alternatives, is there a general strategy that should be followed for numerical givens? Or should one alternative be chosen over the others on a case-by-case basis? If a case-by-case strategy is preferable, what are the criteria for determining whether a numerical given grounds a prior probability, a posterior probability or an evidential probability?

Hyperprobable hypotheticals may seem to be slender reeds on which to base any legal inference. But legal probabilism has a second problem that arises for real cases as well as hypotheticals. To introduce this problem, we first note its presence in other forms of probabilism. For example, probabilism in science, which can be summarised as the imperative to accept the most probable of a set of rival hypotheses, has been much discussed in the philosophy of science. Scientific probabilism is widely

12. The numerical given is 'just a piece of information; it is not a probability, which is a statement of a level of confidence in a given proposition' (Friedman, 2018: 1593–1594).

13. Cohen (1981) and Kaye (1981) continue the debate over the gatecrasher.

14. People seem to distrust naked statistical evidence. Empirical research on the so-called Wells effect (Wells, 1992) has shown that people are less likely to impute liability on the basis of naked statistical evidence than on evidence such as eyewitness testimony.

regarded as unsatisfactory (e.g., Bar-Hillel, 1968: 122–123; Hintikka, 1968: 144; Huber, 2008: 92; Kuipers, 2000: 93, 207; Maher, 1993: 208). If high probability is all that matters, scientists might play it safe and simply affirm the evidence on which the rival hypotheses are based (Levi, 1967: 57). But this would abort the quest for information that drives scientific hypotheses in the first place. Scientists strive for cognitive gain, but this requires cognitive risk: the risk of obtaining misinformation rather than information. Hence there are actually two cognitive aims: high probability and increased information.<sup>15</sup> Accepting a hypothesis, then, ‘involves a trade-off’ of these two ‘competing considerations’ (Maher, 1993: 140). High probability is not enough.

This insight can be adapted to sophisticated forms of legal probabilism. Most accounts of the goals of a judicial system emphasise accuracy in fact finding. The US Supreme Court, for example, affirms that ‘The basic purpose of a trial is the determination of truth’ (*Tehan v US ex rel. Shott*, 382 U.S. 406 (1966) at 416). For participants in comparable systems, this basic purpose motivates the judicial emphasis on high probability. Nevertheless, most accounts of judicial goals are not limited to truth determination. The Federal Rules of Evidence aim at ‘ascertaining the truth and securing a just determination’ (Federal Rule of Evidence 102). Indeed, Federal Rule 403 authorises a balancing of the interests in truth and justice, stipulating that relevant evidence may be excluded if its probative value is ‘substantially outweighed’ by a danger of ‘unfair prejudice, confusing the issues, misleading the jury, undue delay, wasting time or needlessly presenting cumulative evidence’. Other accounts posit other non-alethic goals (some are cited below in Section 3.2.1). But even if these non-alethic goals depend in some way on truth, they are not reducible to it. Hence the same normative point holds for all: High probability may be necessary for conviction, but it cannot be sufficient. Even sophisticated forms of legal probabilism are incomplete at best.

## 2.2. Bayesian decision theory

Some forms of decision theory require that probabilities of relevant states of the world be based on objective frequencies (e.g., von Neumann and Morgenstern, 1953). By contrast, Bayesian decision theory permits probabilities to express the decision maker’s degrees of belief in propositions. In principle, then, Bayesian decision theory could be applied by a fact finder facing a decision about liability or guilt in a court of law.

Bayesian decision theory sorts decisions into two classes: decisions under ignorance, which are not informed by probabilities of states, and decisions under risk, which are. For decisions under risk, Bayesian decision theory recommends choosing acts that maximise expected utility. Suppose that we are faced with a choice among a finite number of acts  $a_1, a_2, \dots, a_n$ . Say that a finite number of states of the world  $s_1, s_2, \dots, s_n$  are relevant to this choice and that a probability measure  $p$  assigns the state probabilities  $p(s_1), p(s_2), \dots, p(s_n)$ . In addition, choice of one of these acts when exactly one of  $s_1, s_2, \dots, s_n$  holds would produce one of the outcomes  $o_1, o_2, \dots, o_n$ , respectively. Let a utility function  $u$  specify the utilities  $u(o_1), u(o_2), \dots, u(o_n)$  of these outcomes. Then the expected utility  $EU$  of choosing act  $a$  can be calculated using Equation (2).

$$EU(a) = \sum_{j=1}^n p(s_j) u(o_j). \quad (2)$$

Bayesian decision theory is applicable in principle to any kind of decision. It is therefore applicable in particular to cognitive decisions such as the choice of a hypothesis, theory or axiom. Viewed through a Bayesian lens, an inquirer might be said to decide by choosing one cognitive option over another. The outcome of this act depends on the relevant states of the world. If the states are as the inquirer posits, the

15. Cf. William James (1897: 17) on ‘our first and great commandments as would-be knowers’: to know the truth and to avoid error.



outcome is information; but if the states are not as the inquirer posits, the outcome is misinformation instead.<sup>16</sup> Explanation and further details of this point of view are elaborated below in Section 3.2.3.

The cognitive aims of high probability and increased information were identified in Section 2.1. One of Bayesian decision theory's strengths is its ability to respond directly to both of these aims. Equation (2) specifies a summation of products composed of a probability factor and a utility factor. When the equation is applied to cognitive decisions, the probability factor registers probabilities of relevant states, which are consulted for the sake of cognitive security. The utility factor indicates the utilities of information outcomes, which express the interest in cognitive gain. Hence the expected utility of choosing a hypothesis represents a synthesis of probability (the higher, the better) and information (the more, the better). Compared to legal probabilism, Bayesian decision theory's response to these two cognitive goals is clearly more comprehensive.

I will assume initially that a fact finder's choice of one version of the facts among others presented at trial is comparable to the choice of a hypothesis. Though this assumption will be relaxed in Section 4 below, here it will allow us to consider the possibility that a fact finder's choice of a version of the facts is a cognitive decision. The fact finder acts by choosing one hypothesis, one version of the facts, over others. The outcome of this act depends on the relevant states of the world. If the states are as the fact finder hypothesises, the outcome is information; but if the states are not as the fact finder hypothesises, the outcome is misinformation instead.

Then how should a fact finder choose a hypothesis? Bayesian decision theorists claim that decision makers of any stripe should choose the act that maximises expected utility. Hence inquirers in general and legal fact finders in particular should act to maximise expected utility. Bayesian decision theorists do not claim that decision makers actually think about maximising expected utility or even that they should think about maximising expected utility. Instead, they claim that rational decisions are made *as if* the decision maker were maximising expected utility (Gilboa, 2009: 95; Halpern, 2003: 166).<sup>17</sup>

The claim about as-if maximising is crucial, for applications of Equation (2) require exceptional numerical inputs. Probability measures returning point-valued state probabilities and utility functions yielding point-valued outcome utilities are indispensable. While ingenious procedures for eliciting probabilities (Anscombe and Aumann, 1963; Ramsey, 1931) and utilities (Farquhar, 1984; Gilboa, 2009: 87, 131) have been devised, these procedures may not yield the precision required (Savage, 1971: 795; 1972: 59). In the best of cases, they require considerable time and expertise to be employed. Hence the requisite precision may be attainable in these best cases, but they are the exception rather than the rule.

Consequently, Bayesian decision theory presents formidable problems of application. Some of these problems can be sidestepped by insisting on the as-if view of maximising expected utility, but others persist. Consider two standard uses of Bayesian decision theory. One features a decision theorist who analyses or evaluates a decision. If the decision theorist claims that the decision maker merely acted as if she were maximising expected utility, then the decision theorist needs to specify the standard that the decision maker has met. The standard is the maximal act, and identifying it requires Equation (2). Alternatively, a decision maker may deliberate about what act to perform.<sup>18</sup> If the decision maker consciously attempts to maximise expected utility, then she too must identify the maximal act, and she too must grapple with Equation (2). In both situations, then, the numerical exigencies of Equation (2) remain.<sup>19</sup>

Both situations have legal counterparts. A fact finder's decision might be analysed or evaluated in terms of maximising expected utility, and a fact finder might attempt to deliberate by maximising expected utility. But legal discourse seldom supplies reliable probabilities and utilities; Equation (2) is rarely in

16. What is information? Some accounts are statistical (Shannon, 1948); others are semantic (Hintikka and Pietarinen, 1966); still others are pragmatic (Levi, 1984). Section 3.2.3 below relies on a pragmatic account.

17. Friedman (2018: 1589–1590) makes an analogous point about legal probabilism.

18. That idea that decision theory can be used as an aid to deliberation goes back at least to Jeffrey (1965).

19. Interval versions of decision theory such as those of Kyburg (1979) and Kaplan (1996) soften the problem, but they do not solve it.

sight and even more rarely applied. Rather, legal discourse is awash in qualitative descriptions of states ('likely', 'unconfirmed') and outcomes ('equitable', 'unacceptable'). These descriptions are the raw materials available to legal fact finders, and they must either work with them or close up shop.

### 2.3. Relative plausibility theory

By contrast with the difficulties of Bayesian decision theory in securing the numerical inputs required for Equation (2), relative plausibility theory has the advantage of emphasising qualitative judgment. Professor Allen (1991: 381, 417, 422) has pointed out that litigation normally requires an ordinal approach to proof. The empirical evidence that legal fact finders impose a narrative structure on the evidence presented at trial was noted already in Section 1. According to relative plausibility theory, fact finders compare the plausibility of the plaintiff's narrative or explanation to that of the defendant, and they base their decision on these relative plausibilities. There is usually no question of sharp probabilities and utilities. A narrative or explanation is simply judged more or less plausible against the backdrop formed by its competitors.

Despite this crucial advantage, relative plausibility theory appears to have two notable limitations. The first is that the theory's fundamental concept of plausibility is undefined, treated explicitly as a primitive (Allen, 1994: 612, n. 28). Second, as we will see below, relative plausibility shares the one-sidedness of legal probabilism. In the interest of cognitive security, legal probabilism emphasises high probability and relative plausibility theory stresses high plausibility, but both understate the competing cognitive imperative identified in Section 2.1: to increase information.

## 3. Renewing relative plausibility

Despite the limitations just mentioned, it seems to me that there is a great deal that is right about relative plausibility theory. So much so, in fact, that I want to look for ways to overcome its limitations. The search for these ways develops in three stages. Section 3.1 proposes a way of defining plausibility instead of treating it as primitive. Section 3.2 then makes a case for a certain incompleteness in relative plausibility theory. Finally, Section 4 asks whether the theory's incompleteness can be remedied, and if so, how. The overall project is to divest relative plausibility theory of its disadvantages while retaining its ordinal advantages.

### 3.1. Plausibility as primitive

As noted in Section 2.3, plausibility is treated as a primitive in relative plausibility theory. However, in a fortuitous coincidence, the concept of plausibility has been axiomatised in terms that correlate well with relative plausibility theory. The axiomatisation, which is due to Professors Halpern (2003: 51) and Chu and Halpern (2004: 209–210; 2008: 7), is a generalisation of the standard axioms of probability. Let  $\top$  be a nonnumeric limit representing maximum plausibility and  $\perp$  be a nonnumeric limit representing minimum plausibility. Then a plausibility measure  $\pi$  can be defined for propositions  $q$  and  $r$  as follows:<sup>20</sup>

P1. If  $q$  is logically false,  $\pi(q) = \perp$ .

P2. If  $q$  is logically true,  $\pi(q) = \top$ .

P3. If  $q$  implies  $r$ ,  $\pi(q) \leq \pi(r)$ .<sup>21</sup>

20. The axioms presented here are propositional variants of Halpern's and Chu and Halpern's set-theoretical versions.

21. Finite additivity is sometimes considered a condition for functions to count as measures. But Halpern uses the term 'plausibility measure' even though plausibilities are not generally additive, and this paper follows suit.



P1 and P2 simply generalise the probability calculus' minimum and maximum probabilities. P3 is the monotonicity condition that generalises the Kolmogorov axiom on finite additivity (Halpern, 2003: 54). Stated for mutually exclusive  $q$  and  $r$ , the axiom stipulates that  $p(q \text{ or } r) = p(q) + p(r)$ . The rationale for P3 can be glimpsed easily from the point of view of possible-world semantics. When  $q$  implies  $r$ , the possible worlds in which  $q$  is true are (proper) subsets of the possible worlds in which  $r$  is true. For example, ' $q$  and  $r$ ' implies ' $q$  or  $r$ '. Now ' $q$  and  $r$ ' is true under just one set of truth values (one possible world), while ' $q$  or  $r$ ' is true under three sets of truth values (three possible worlds). The possible world in which ' $q$  and  $r$ ' is true is a proper subset of the possible worlds in which ' $q$  or  $r$ ' is true. Consequently, ' $q$  and  $r$ ' is harder to make true, so less plausible; and ' $q$  or  $r$ ' is easier to make true, so more plausible. Thus the plausibility of ' $q$  and  $r$ ' is less than or equal to the plausibility of ' $q$  or  $r$ '.

So defined, plausibility is the most general of current modes of representing uncertainty. Probability measures, Dempster–Shafer belief functions, possibility measures and ranking functions all turn out to be special cases of plausibility measures (Halpern, 2003: ch 2). In fact, the axiomatisation permits plausibility values to be attributed to members of any partially ordered set. Hence plausibility values *can* be quantitative, but they can also be qualitative, such as 'highly likely' or 'doubtful'.

Qualitative plausibilities can be straightforwardly adapted to legal fact finding. According to Allen and Pardo (2019: 3, n. 7), fact finders evaluate explanations presented by parties at trial using criteria such as consistency, coverage, simplicity, coherence, consilience and fit with background knowledge. Based on these criteria, a fact finder's overall evaluation of an explanation can be expressed by a qualitative plausibility as just defined. This plausibility—consistent with Allen and Leiter's (2001: 1527–1528) views on comparative plausibility—can then be compared to the plausibility of a competing explanation in determining the fact finder's verdict. Nonetheless, plausibility values alone would be insufficient for a defensible verdict, as Section 3.2 contends.

The major task of this section is to generalise probability as plausibility, but we can also carry out two further generalisations that will be needed below. The first is the generalisation of utility as desirability. Since 'utility' can connote hedonistic forms of utilitarianism, I will use Jeffrey's (1983: 21) more general term 'desirability' instead.<sup>22</sup> To complement our plausibility measure  $\pi$  for states of the world, we can speak of a desirability function  $\delta$  to express the desirability of outcomes.

The second generalisation is of expected utility as plausibilistic expectation. Because expected utility is probabilistic expectation, the generalisation of probability as plausibility requires a corresponding generalisation of expected utility as plausibilistic expectation. Plausibilistic expectation can be calculated by relying on an equation analogous to Equation (2) (Welch, 2013: 327; 2014: 64). Where  $\oplus$  is the operation of quasi-summation over the Cartesian products  $\otimes$  of state plausibilities  $\pi$  and outcome desirabilities  $\delta$ , the plausibilistic expectation  $PE$  of an act  $a$  can be calculated by Equation (3).<sup>23</sup>

$$PE(a) = \bigoplus_{j=1}^n \pi(s_j) \otimes \delta(o_j) \quad (3)$$

The generalisations of utility as desirability and of expected utility as plausibilistic expectation will prove useful in Section 4 below.

### 3.2. Relative plausibility as incomplete

Section 2.1 has already noted that science operates with the twin cognitive aims of high probability and increased information. Professors Allen and Pardo (2019: 13–14) recognise that probabilistic and relative-plausibilistic accounts share 'the same end: assessing the likelihood of the disputed facts'. Hence legal probabilism and relative plausibility emphasise high probability and high plausibility, respectively.

22. Cullison (1969) also opts for 'desirability' over 'utility'.

23. Quasi-summation is the addition-like operation defined in Welch (2014: 63).

Yet there is reason to think that both are incomplete, for both understate the need for cognitive risk—the risk of misinformation—in order to gain information. That this is so for relative plausibility in particular is the theme of this section.

There is a preliminary caveat. Relative plausibility theory's 'primary aim is to understand the general nature of juridical proof. . . . This task is empirical: what is the best explanation of the data, where "the data" are observations of how the American legal system structures proof at trial?' (Allen and Pardo, 2019: 4). Hence relative plausibility theory is primarily descriptive. But Section 1 noted that the focus of this inquiry is normative. To engage relative plausibility theory in this context, it is therefore necessary to address its secondary aim: to consider 'whether the empirically true is normatively appropriate, in light of the goals of the legal system' (Allen and Pardo, 2019: 4). Our question, then, is whether relative plausibility theory is normatively appropriate. I will argue that it is not—not in the radical sense that it is inappropriate, but in the revisionist sense that it is incomplete.

The argument in support of this thesis proceeds through three stages. Since the issue of normative appropriateness cannot be clarified without attention to the goals of the legal system, these are identified in Section 3.2.1. In light of these goals, two normative issues are then introduced. The appropriateness of the instructions given to fact finders occupies Section 3.2.2, and the appropriateness of the fact finders' response to these instructions is the subject of Section 3.2.3.

**3.2.1. The goals of the legal system.** To make some progress on the question of normative appropriateness, we need to attend to the goals of the legal system. Fundamentally, there are three ways of understanding these goals: cognitive, where fact finders' decisions are motivated by the search for truth alone; partly cognitive, where fact finders have multiple goals, some cognitive and some not; and noncognitive, where fact finders pursue cognitive outcomes indirectly or not at all. Let us look briefly at each in turn.

The cognitive view is probably the best-known of the three. An instance was cited already in Section 2.1: 'The basic purpose of a trial is the determination of truth' (*Tehan v U.S. ex rel. Shott*, 382 U.S. 406 (1966) at 416). This traditional focus on truth continues to be affirmed by some legal scholars (Nance, 2007: 163; Park and Saks, 2006: 1030–1031).<sup>24</sup> As Professors Allen and Stein (2013: 567) put it, 'the coin of the legal realm is truth'.<sup>25</sup>

Partly cognitive approaches include Professor Twining's (2006: 83) ideal type of the rationalist tradition in law, which treats the pursuit of truth as a means to securing justice under the law. Bentham, who is undeniably a major figure in this tradition, identified both direct and collateral ends of adjective law. The direct end is 'rectitude of decision', which can be summarised as 'the correct application of valid laws (presumed to be consonant with utility) to true facts' (Twining, 2006: 41), while the collateral ends include minimising 'vexation, expense and delay'. More recent partly cognitive views emphasise the ends of preserving order and doing justice (Denning, 1952: 9); party participation, individualisation and accuracy of result (Thompson, 1991); trial costs and accuracy (Posner, 1999: 1485); fairness in allocating risk of error together with cost-efficiency (Stein, 2005: 217–218); error reduction, proper error distribution and protecting the rights of the accused (Laudan, 2006: 1–2); truth and justice (Ho, 2008); and overall error reduction as well as desirable error distribution (Di Bello, 2016).

Noncognitive views are exemplified by claims like the following: lawsuits are meant to smooth over social conflicts (Cullison, 1969: 573; Tribe, 1971: 1376); the law should promote wealth maximisation (Posner, 1979, 1980, 1981);<sup>26</sup> trials aim to produce socially acceptable rather than true verdicts (Nesson,

24. In addition, Professor Laudan (2006: 4) carries out the 'thought experiment' of treating truth finding as 'the predominant concern in any criminal proceeding', but he considers it 'doubtful' that truth finding predominates in actual criminal proceedings.

25. However, Professor Allen (2010: 211; Allen and Leiter, 2001: 1500) adds elsewhere that truth determination is not the only goal of the legal system. This view is discussed below.

26. In later work, Professor Posner (1985, 1990, 1995) recognised that the principle of wealth maximisation is an incomplete guide to social decision making.

1985); rational agents should minimise expected loss (Kaye, 1988: 175–176; 1999); the point of a trial is justice, which may, but need not, entail correct results (Lempert, 2001: 1630–1631); trials attempt to influence individual behaviour by dispensing incentives (Sanchirico, 2001), particularly incentives for ex ante socially desirable behaviour (Kaplow, 2012). These alternatives can be proposed as normative or descriptive monisms, that is, as overriding objectives for legal process.

Despite the insights contributed by these noncognitive views, I suggest that they are largely speculative. Cognitive and partly cognitive approaches appear better suited to common-law legal systems. Fortunately, the relevant considerations for cognitive and partly cognitive specifications of the goals of the legal system are not fundamentally different. But for the sake of specificity, let us consider one partly cognitive view in particular: that of relative plausibility theory. According to this theory, truth determination is not the only goal of the legal system (Allen and Leiter, 2001: 1500; Allen, 2010: 211), and legal standards of proof respond to ‘two related, primary considerations: accuracy and allocating the risk of error between the parties’ (Allen and Pardo, 2019: 7). Accuracy reflects the traditional goal of truth; and since no legal system can be presumed to be free of error, risk of error should be distributed fairly among the parties at trial. Because the plurality of goals makes partly cognitive views complex, it will be convenient to work initially with the cognitive goal of accuracy in the remainder of Section 3. The results will then be merged with the goal of risk allocation in the following Section 4.

**3.2.2. *The fact finder’s instructions.*** In both jury and bench trials, fact finders receive instructions. In a jury trial, the judge instructs the jury. In a bench trial, the legal system can be said to instruct the judge.

When a trial concerns a civil matter, fact finders are instructed to answer the question ‘Does the balance of probabilities (or the preponderance of the evidence) show that the defendant is liable?’ But when a trial treats a criminal matter, the instruction is to answer the question ‘Does the evidence show that the defendant is guilty beyond a reasonable doubt?’ These instructions can be put in a generic form that covers both the civil and criminal cases. This generic instruction is to answer the question ‘Does the evidence show that the plaintiff’s explanation meets the relevant standard of proof?’

Why should fact finders heed this directive? The reason is straightforward. Fact finders should attempt to answer this question because they have accepted a role in the legal system. The legal system requires fact finders to evaluate an ampliative inference, an inference that goes beyond the evidence to venture something new.<sup>27</sup> In a civil case, premises based on the evidence aim to support a conclusion that the defendant is liable. In a criminal case, premises based on the evidence aim to support a conclusion that the defendant is guilty. Fact finders must evaluate the conclusion that the defendant is liable or that the defendant is guilty relative to a plausibility scale, however implicit and ill-defined, whose critical points are the qualitative notions of ‘more plausible than not’ (for civil cases) and ‘beyond a reasonable doubt’ (for criminal cases). Only if the plausibility of the inference’s conclusion reaches the relevant critical point should fact finders favour the pro-plaintiff hypothesis.

Hence the legal system imposes two goals. One goal is high plausibility, since any finding for the plaintiff requires an inference from the evidence to a highly plausible conclusion. The other goal is increased information, since any conclusion that the defendant is liable or that the defendant committed a crime results from an ampliative inference whose conclusion overreaches the evidence.<sup>28</sup> In pursuit of these goals, the legal system instructs fact finders to answer the question ‘Does the evidence show that the plaintiff’s explanation meets the relevant standard of proof?’ In order to play their part in the attainment of these goals, fact finders should attempt to answer this question.

27. In deductive inference, when premises imply a conclusion, the possible worlds in which the premises are true are (proper) subsets of the possible worlds in which the conclusion is true (Section 3.1). But the inductive situation is different: the possible worlds in which the premises are true are not subsets of the possible worlds in which the conclusion is true. Instead, there are possible worlds in which the conclusion is false even though the premises are true. In such cases, to infer that the conclusion is true is to go beyond the evidence of the premises. In this sense, the inference is ampliative.

28. The possibility of additional goals is treated below in Section 4.

**Table 1.** A fact finder’s choice.

	$s_p$	$s_d$
choose $h_p$	$i_p$	$-i_d$
choose $h_d$	$-i_p$	$i_d$

Now relative plausibility theory identifies the goals of the legal system as ‘accuracy and allocating the risk of error between the parties’ (Allen and Pardo, 2019: 7). The emphasis on accuracy responds to the legal system’s goal of high plausibility. But relative plausibility theory merely presupposes the legal system’s goal of increased information; it takes the need for ampliative inference for granted. Viewed normatively, however, fact finders’ scrutiny of rival hypotheses cannot be motivated by the goal of high plausibility alone. If high plausibility were all that matters, the evidence would serve far better than a risky inference to liability or guilt. But the pro-plaintiff hypothesis that the defendant is liable or that the defendant is guilty is an ampliative inference, and the cognitive risk involved can only be justified by reference to the goal of increased information. I suggest, then, that relative plausibility theory’s implicit recognition of this second goal should be made explicit.

**3.2.3. The fact finder’s response.** The second normative issue is an individual fact finder’s response to the instructions detailed in Section 3.2.2. Suppose that a fact finder infers, based on the evidence, that the defendant in a civil trial is liable or that the defendant in a criminal trial is guilty. Is this an appropriate response?

To evaluate this inference from a purely cognitive point of view, consider an insight due to Hintikka and Pietarinen (1966) about an inquirer’s choice between a hypothesis  $h$  and its contradictory  $-h$ .

If  $h$  is true, the utility of his [the theorist’s] decision is the valid information he has gained . . . . If  $h$  is false, it is natural to say that his disutility or loss is measured by the information he lost because of his wrong choice between  $h$  and  $-h$ , i.e. by the information he would have gained if he had accepted  $-h$  instead of  $h$ . (Hintikka and Pietarinen, 1966: 107–108; cf. Hintikka, 1970: 16)

This take on cognitive choice has been developed by a number of people, including Levi (1967, 1984), Hintikka (1983), Maher (1993) and Welch (2011).

The idea that information gains and losses can be outcomes of cognitive choice is applicable in legal contexts. If a fact finder favours the plaintiff’s version of the facts and this version is true, then the outcome is information about the disputed events. But if the fact finder opts for the plaintiff’s version of the facts when the defendant’s version is true, then the outcome is loss of the information that would have been obtained by favouring the defendant’s version instead. Like other outcomes, these cognitive outcomes have desirabilities, which are positive for information gains and negative for information losses.

For a criminal trial, the situation can be represented by Table 1, where  $h_p$  is a fact finder’s hypothesis of plaintiff veridicality;  $h_d$  is the same fact finder’s hypothesis of defendant veridicality;  $s_p$  is a possible state of the world in which the defendant committed the crime;  $s_d$  is a possible state in which the defendant did not commit the crime;  $i_p$  is the information outcome of choosing  $h_p$  when  $s_p$  holds; and  $i_d$  is the information outcome of choosing  $h_d$  when  $s_d$  holds.

How to understand the fact finder’s deliberation about these hypotheses depends greatly on whether our aim is descriptive or normative. If it is descriptive, we must ultimately do some psychology, whether individual or social. We may discover that a fact finder opted for the defendant’s hypothesis because she found it overwhelmingly plausible; or because she wanted to help the defence attorney; or because she was motivated by racial bias. A descriptive point of view must be open to all these possibilities. But if our aim is normative, some of these deliberations will be acceptable, and some will not. How could we separate the acceptable from the unacceptable deliberations?

Answering this question requires attention to the nature of information. Information can be understood as reduction of uncertainty about states of the world (cf. Hintikka, 1970: 264). A standard measure of the information  $i$  conveyed by a statement is the ratio of the number  $m$  of options inconsistent with the statement to the total number  $n$  of options:  $i = m/n$  (Levi, 1967: 74). Given four points of the compass, the statement 'The lake is north' is  $3/2 = 1.5$  times more informative than the statement 'The lake is north or east'. The ratio  $3/2$  presupposes that the lake is in fact north, which makes both statements true.<sup>29</sup> This way of quantifying information coincides with our sense that a precise truth is more informative than a platitude, for the precise truth is inconsistent with many states of the world, while the platitude is inconsistent with few.

However, this way of conceptualising information may appear to give an unfair advantage to one of the parties at trial. Consider a plaintiff who argues that the butler did it, not the cook or the gardener, while the defendant maintains that the cook or the gardener did it, but not the butler. Here choice of the plaintiff's account appears to be more informative if true, since it eliminates two of three options; and choice of the defendant's response seems to be less informative if true, because it eliminates just one of the three options. This would appear to give an unfair advantage to the plaintiff on no other ground than greater specificity.

The appearance is deceiving, however. Information in general is best understood as pragmatic (Levi, 1984), whereby information can vary with context. For example, the true statement that there is a grocery store down the street may be informative if you do not know this but uninformative if you do. Now the legal contexts for information are highly specific. In ordinary civil trials, the question addressed is: Does the balance of probabilities (or the preponderance of the evidence) show that the defendant is liable? In criminal trials, the question is: Does the evidence show that the defendant is guilty beyond a reasonable doubt? For both questions, there are just two potentially informative answers: yes and no. ('I don't know' is not an informative answer.) Hence a 'yes' answer is inconsistent with one of two possible options, and a 'no' answer is inconsistent with one of two possible options. So even if one party's version of the facts is quite specific and the other party's is less so, the search for information is contextualised by the questions just cited. The context ensures that the pro-plaintiff explanation and the pro-defendant explanation are potentially informative to the same degree, since each eliminates one of two possible options.

Suppose now that relative plausibility's descriptive claim is correct: fact finders focus on relative plausibility. But why should they do so? The preceding points explain why. We have seen that the legal system imposes two desiderata: high plausibility and increased information. We have also seen that the pro-plaintiff and pro-defendant hypotheses are potentially informative to the same degree. Consequently, if one hypothesis turns out to be more plausible than the other, this would break the information tie. But if fact finders do not attend to relative plausibility, neither party can prevail on cognitive grounds. This is why fact finders ought to focus on relative plausibility.

Looked at in normative context, then, the issue is not 'Are the states of the world posited by the plaintiff's hypothesis more plausible than those posited by the defendant's hypothesis?' Rather, the issue is 'Given that the plaintiff's hypothesis and the defendant's hypothesis are potentially informative to the same degree, are the states of the world posited by the plaintiff's hypothesis more plausible than those posited by the defendant's hypothesis?' The starring role of tiebreaker is played by relative plausibility, and this tends to mask the more discrete role of increased information. But the drive for more information is equally constitutive of the decision problem. As we have seen in Section 3.2.2, relative plausibility theory presupposes this drive in instructions given to fact finders, but it does not affirm it. Similarly, it presupposes the normative role of this drive in fact finders' responses to their instructions, but it does not affirm it. This is then a second sense in which relative plausibility theory is incomplete.

29. Whether data must be truthful in order to count as information is controversial. Floridi (2004, 2005) upholds the affirmative; Fetzer (2004), the negative. I favour the affirmative.

**Table 2.** Cognitive choice: The basic binary array.

Case	Plausibility	Desirability
1	$\pi(s_1) < \pi(s_2)$	$\delta(o_1) < \delta(o_2)$
2	$<$	$>$
3	$<$	$=$
4	$>$	$<$
5	$>$	$>$
6	$>$	$=$
7	$=$	$<$
8	$=$	$>$
9	$=$	$=$

4. Toward normative completeness

Section 3.2 has not argued that relative plausibility theory is wrong-headed. But it has argued that the normative dimension of the theory is incomplete. Might there be a remedy? This section argues for the affirmative.

Professor Friedman (2001: 2044) remarks that Bayesian decision theory strikes him as ‘a virtually inevitable tool’, for ‘[t]he determination of a lawsuit is a classic instance of a decision made under uncertainty’. Bayesian decision theory is a powerful as well as an inevitable tool, and this section proposes that it can be made more powerful still. Section 3.1 reported an axiomatisation of plausibility in terms general enough to admit plausibility values such as ‘unlikely’ and ‘probably true’. Note that this axiomatisation does not amount to abandoning the concept of probability, for probabilities are special kinds of plausibilities; hence there are quantitative as well as qualitative plausibilities. Similarly, there are quantitative as well as qualitative desirabilities. Even though desirabilities are often expressed in numerical terms, qualitative desirabilities such as ‘good’ and ‘terrible’ are admissible as well (Halpern, 2003: 165; Chu and Halpern, 2008: 4–5). By drawing on these generalisations, Bayesian decision theory can be extended to work with qualitative as well as quantitative inputs.

Let us first consider a cognitive choice by an inquirer, who could—but need not—be a legal fact finder. The inquirer faces a choice between two hypotheses:  $h_1$  and  $h_2$ . If this inquirer can rank the plausibilities of the relevant states and the desirabilities of the corresponding outcomes in the comparative terms of less than ( $<$ ), equal to ( $=$ ) or greater than ( $>$ ), then she has what amounts to a plausibility function  $\pi$  and a desirability function  $\delta$ . Together, these functions can generate nine possible cases. These cases form the basic binary array of Table 2. Case 1, for example, is constituted by two comparisons. The first is the plausibility comparison of states  $s_1$  and  $s_2$ , where  $s_1$  is posited by  $h_1$  and  $s_2$  is posited by  $h_2$ . The second is the desirability comparison of outcomes  $o_1$  and  $o_2$ , where  $o_1$  results from the choice of  $h_1$  when  $s_1$  holds, and  $o_2$  results from the choice of  $h_2$  when  $s_2$  holds. To minimise clutter, the other eight cases are identified only by their distinguishing relations, but their relata can be filled in mechanically by simply copying  $\pi(s_1)$ ,  $\pi(s_2)$ ,  $\delta(o_1)$  and  $\delta(o_2)$  to the same positions they occupy in Case 1.

Plausibilistic expectations for these cases can be calculated by relying on Equation (3), and we will promptly see a legal instance. But note for the moment that the resolutions of Cases 1, 3, 5, 6, 7 and 8 are evident at a glance. Case 1, for example, offers a plausibility advantage for  $h_2$  and a desirability advantage for  $h_2$ ; hence the obvious decision-theoretic choice is  $h_2$ . Case 9 differs slightly, for both plausibility considerations and desirability considerations back the two hypotheses equally. But this is an embarrassment of riches, for there are two good solutions instead of one—in other words, a tie. The only cases that stand out are Cases 2 and 4, where plausibility considerations support one hypothesis and desirability considerations support the other. Here we recall that Bayesian decision theory relies on Equation (2), which treats probability and utility on a par. Since we are working with an expanded decision-theoretic framework that generalises probability as plausibility and utility as desirability, this framework transforms the parity of probability and utility into the parity of plausibility and desirability



**Table 3.** Cognitive choice: The basic binary array with resolutions.

Case	Plausibility	Desirability	Resolution
1	$\pi(s_1) < \pi(s_2)$	$\delta(o_1) < \delta(o_2)$	$h_2$
2	$<$	$>$	no decision
3	$<$	$=$	$h_2$
4	$>$	$<$	no decision
5	$>$	$>$	$h_1$
6	$>$	$=$	$h_1$
7	$=$	$<$	$h_2$
8	$=$	$>$	$h_1$
9	$=$	$=$	$h_1$ or $h_2$

**Table 4.** A legal instance.

	$\pi(s_p) = l$	$\pi(s_d) = L$
$h_p$	$\delta(o_p) = D$	$\delta(-o_d) = -D$
$h_d$	$\delta(-o_p) = -D$	$\delta(o_d) = D$

in Equation (3). As a result, there is no comparative resolution to Cases 2 and 4—unless, of course, exceptional circumstances permit either plausibility or desirability to count more than the other.<sup>30</sup> The basic array of cases and their resolutions are summarised in Table 3.

Let us now consider a legal instance. Suppose that a juror in a civil trial must choose between a pro-plaintiff hypothesis  $h_p$  and a pro-defendant hypotheis  $h_d$ .  $h_p$  posits states of the world  $s_p$ , while  $h_d$  posits alternatives states  $s_d$ . Suppose also that the juror considers the states posited by  $h_p$  to be more plausible than the states posited by  $h_d$ . Hence  $\pi(s_p) = L$  and  $\pi(s_d) = l$ , where  $L > l$ .<sup>31</sup> Also, as we noted in Section 3.2.3, the question ‘Does the balance of probabilities (or the preponderance of the evidence) show that the defendant is liable?’ ensures that choice of  $h_p$  and choice of  $h_d$  are potentially informative to the same degree. Hence the desirability of the information outcome of choosing  $h_p$  when  $s_p$  holds is equal to the desirability of the information outcome of choosing  $h_d$  when  $s_d$  holds:  $\delta(o_p) = \delta(o_d) = D$ . Consequently, the choice between  $h_p$  and  $h_d$  can be represented by Table 4.

Equation (3) of Section 3.1 indicates that the plausibilistic expectation of choosing  $h_p$  is

$$PE(h_p) = lD \oplus L-D,$$

while the plausibilistic expectation of choosing  $h_d$  is

$$PE(h_d) = l-D \oplus LD.$$

Since  $l < L$ , the plausibilistic expectation of choosing  $h_p$  is negative while that of choosing  $h_d$  is positive. Hence  $h_d$  is the better decision-theoretic choice.

For the purposes of judicial inquiry, Table 3 can be immediately simplified by exploiting the previous point that choices of pro-plaintiff and pro-defendant hypotheses are potentially informative to the same degree. Consequently, the only cases that are really instantiated in judicial process are Cases 3, 6 and 9, which feature potentially equal information outcomes and correspondingly equal desirabilities. In judicial contexts, then, Table 3 can be reduced to Cases 3, 6 and 9. The results are shown in Table 5.

30. Welch (2019) analyses a major economic decision in which the Federal Reserve Board appears to have weighted desirability of outcomes over plausibility of states.

31. A more obvious choice for a plausibility value would be ‘ $p$ ’, but ‘ $p$ ’ is already in use as a probability measure. I have therefore used the second letter of ‘plausibility’—think ‘ $l$ ’ for ‘likely’.

**Table 5.** Cognitive choice: The simplified binary array with resolutions.

Case	Plausibility	Desirability	Resolution
3	<	=	$h_d$
6	>	=	$h_p$
9	=	=	$h_p$ or $h_d$

**Table 6.** Partly cognitive choice: Civil trials.

Case	Plausibility	Desirability	Resolution
3	<	=	$h_d$
6	>	=	$h_p$
9	=	=	$h_d$

Tables 4 and 5 are based on the assumption that the fact finder’s decision is purely cognitive; that is, the only relevant outcomes are information lost and information gained, and the only desirabilities are cognitive in nature. But the goals of relative plausibility theory identified in Sections 3.2.1 and 3.2.2 are only partly cognitive: accuracy, which is cognitive, and fair risk allocation, which is not. From the point of view of relative plausibility theory, therefore, Tables 4 and 5 list interim results.

To look at Table 5 from the ampler perspective of relative plausibility theory, we need to consider the relative priority of accuracy and risk allocation. In common law countries, the main mechanisms for managing the relative priority of these goals are standards of proof. In ordinary civil trials, the standard of proof aims to allocate the risk of error equally between plaintiff and defendant.<sup>32</sup> In criminal trials, however, the standard of proof minimises the risk of error for the defendant. This reflects the traditional judgment that innocence found guilty is worse than guilt found innocent. Famously, Blackstone’s (1765–1769: 352) ratio was ‘better that ten guilty persons escape, than that one innocent suffer’.

Let us consider how the cases in Table 5 would be resolved in both civil and criminal proceedings. For ordinary civil trials, the standard of proof is the balance of probabilities in the United Kingdom and the preponderance of the evidence in the United States. Hence Case 3 would continue to be resolved in favour of the defendant and Case 6 in favour of the plaintiff. But Case 9 would be different. Looked at from the purely cognitive perspective of Table 5, the pro-plaintiff and pro-defendant hypotheses are potentially informative to the same degree. But we are assuming a partly cognitive approach that also pursues fair risk allocation, where this is understood to impose the burden of proof on the plaintiff. Accordingly, if the plaintiff can prove no more than equal plausibility, there is no proof and the pro-defendant hypothesis prevails. The results for the three cases are summarised in Table 6.

We can also consider these three cases as exemplified in criminal proceedings, where the standard is proof beyond a reasonable doubt. Case 3 continues to be resolved in favour of the defendant, but Case 6 is different. It divides into two subcases:

- (a) The plausibility  $L$  of the states posited by the pro-plaintiff hypothesis is greater than the plausibility  $I$  of the states posited by the pro-defendant hypothesis, but the plaintiff’s evidence does not remove all reasonable doubt. The plaintiff has not met the standard of proof, so the verdict favours the defendant.

32. Stein (2005: 216–217) justifies the civil standard by appeal to a principle of equality.

**Table 7.** Partly cognitive choice: Criminal trials.

Case	Plausibility	Desirability	Resolution
3	<	=	$h_d$
6a	$L > I$	=	$h_d$
6b	$L+ > I$	=	$h_p$
9	=	=	$h_d$

- (b) The plausibility  $L+$  of the states posited by the pro-plaintiff hypothesis is so much greater than the plausibility  $I$  of the states posited by the pro-defendant hypothesis that the plaintiff’s evidence removes all reasonable doubt. Hence the plaintiff has met the standard of proof, and the verdict favours the plaintiff.

Finally, Case 9 is resolved in favour of the defendant. Even though the pro-plaintiff and pro-defendant hypotheses are equally plausible, the plaintiff fails to meet the standard of proof beyond a reasonable doubt. The results for all three cases are summarised in Table 7.

The civil results in Table 6 and the criminal results in Table 7 are hardly surprising. In fact, they are exactly what we would expect. Hence the advantage of the foregoing analysis is not that it gives us new results; the advantage is that it supplies a normative foundation for the results that we know we should get. The proposed foundation synthesises key advantages of each of the main strands of new evidence scholarship: legal probabilism, Bayesian decision theory and relative plausibility theory. These advantages are summarised in the concluding Section 6.

5. Potential objections

This penultimate section of the paper addresses two potential objections to its proposals. The first objection picks out a limitation of the comparative decision theory described in Section 4. This form of decision theory is limited to binary decisions: choices between two alternatives. Of course, this limitation is unproblematic for most legal fact finding, since there is an explanation based on the plaintiff’s version of the facts and an explanation based on the defendant’s version. But it is possible that a litigant might present more than one version of the facts. In a criminal trial, for example, the defence counsel might present a disjunctive defence: either  $A$  happened or  $B$  happened, but either way the defendant is not guilty.

A disjunctive defence is no real obstacle, however. The fact finder simply proceeds in stages: first, by comparing the pro-plaintiff explanation with the pro-defendant explanation  $A$ ; then by comparing the pro-plaintiff explanation with the pro-defendant explanation  $B$ . If both comparisons favour the plaintiff’s explanation beyond a reasonable doubt, then a verdict of guilty should follow; but if the plaintiff’s explanation fails to meet the reasonable doubt standard in at least one comparison, then the verdict should be not guilty (cf. Allen, 1994: 609).

The second objection is also directed at the comparative decision theory of Section 4. The objection is that fact finders do not deliberate, and cannot in general be expected to deliberate, using the decision-theoretic vocabulary of plausibility and desirability.

The response to this objection is to grant the point. Although a fact finder *could* use this vocabulary to deliberate, ordinary jurors are presumably thinking of the jury instructions imparted by the judge. These instructions require jurors to concentrate on the evidence presented by plaintiff and defendant in light of the relevant standard of proof. But, as noted in Section 1, the focus of this inquiry is normative. Hence the decision-theoretic framework of Section 4 is not an attempt to describe fact finders’ decision making. Rather, it is an attempt to facilitate the evaluation of fact finders’ decision making. As noted in Section 2.2, a Bayesian evaluation of a decision can show that a decision is rational provided it proceeds *as if*

informed by certain probability and utility functions.<sup>33</sup> Similarly, in our generalised decision theory, a fact finder's decision can be shown rational when carried out *as if* based on the kind of plausibility and desirability functions described in Section 4.<sup>34</sup>

## 6. Conclusion

The new evidence scholarship is carried out primarily by representatives of three distinct approaches: legal probabilism, Bayesian decision theory and relative plausibility theory. Each has major insights to offer. Probabilism provides its emphasis on cognitive security; Bayesian decision theory, when adapted to cognitive decisions, contributes the complementary stress on cognitive gain; and relative plausibility theory supplies an account of how legal fact finders actually make decisions. But none of these approaches seems satisfactory as it stands. Probabilism and relative plausibility theory suffer from one-sidedness in their focus on cognitive security (high probability and high plausibility, respectively). When applied to cognitive decisions, Bayesian decision theory supplies the missing emphasis on gaining information through assuming cognitive risk, but its applicability to legal fact finding is undercut by its demanding numerical requirements.

Hence this paper proposes a renewal of new evidence scholarship. It calls for a synthesis of selected features from its three main strands. In the proposed synthesis, the concept of plausibility plays a pivotal role. Because this concept is obtained by generalising the standard axioms for probability, the proposal retains deep ties to legal probabilism. In addition, the proposal is decision-theoretic, obtained by selectively generalising the Bayesian approach to cognitive decisions. Finally, the proposal is fully consistent with the descriptive component of relative plausibility theory, but it suggests two additions: a definition of the primitive concept of plausibility, and comparative decision-theoretic norms for fact finders' instructions and fact finders' responses to these instructions.

The paper also attempts to advance claims, examples of which were cited in Section 1, that decision theory and relative plausibility theory are mutually compatible. It does this by generalising the categories of probability and utility so that non-numeric as well as numeric values are admissible. This permits the markedly imperfect information characteristic of legal decision making to serve as inputs for a comparative form of decision theory. When this form of decision theory is supplied with qualitative inputs, it generates plausibilistic expectations of a qualitative sort. Because they are comparable, they function as relative plausibilities; that is, they can be applied to an explanation at trial to characterise its plausibility relative to competing explanations. Hence the resulting framework is both relative-plausibilistic and decision-theoretic. It retains the emphasis on cognitive security from relative plausibility theory, and it holds with decision theory that cognitive security needs to be balanced against cognitive gain. The result is a comparative version of decision theory that is, I submit, fully applicable to the normative dimension of legal decision making.

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33. The decision-theoretic rationality referred to here does not guarantee that the probability (or plausibility) and desirability inputs are rational. It simply accepts these inputs and produces a rational output on their basis.

34. Applications of these functions are special cases of a simple heuristic known as tallying (Welch 2019).

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