Burdens of Proof - Sample Chapter

Marcello Di Bello and Rafal Urbaniak

SAMPLE CHAPTER PLAN

In rethinking the sample chapter, we should perhaps stick to a simpler structure, trying to offer a more focused and compelling argument. Right now I think we have too many possible accounts under consideration, and the structure is not very tight or cohesive. It feels more like a literature review, especially the first few sections.

So here is how I proposed we do it:

1. Begin by stating the simplest probabilistic account based on a threshold for the posterior probability of guilt/liability. The threshold can be variable or not. Add brief description of decision-theoretic ways to fix the threshold. (Perhaps here we can also talk about intervals of posterior probabilities or imprecise probabilities.)
2. Formulate two common theoretical difficulties against ths posterior probability threshold view: (a) naked statistical evidence and (b) conjuction. (We should state these difficilties before we get into alternative probabilistic accounts, or else the reader might wonder why so many different variants are offerred of probabilistic accounts).

   R: Yes. That's what I thought.

   We might also want to add a third difficulty: (c) the problem of priors (if priors cannot be agreed upon then the posterior probability threshold is not functionally operative). Dahlman I think has quite a bit of stuff on the problem of priors.
3. As a first response to the difficulties, articulate the likelihood ratio account. This is the account I favor in my mind paper. Kaplow seems to do something similar. So does Sullivan. So it's a popular view, worth discusing in its own right. You say that Cheng account is one particular variant of this account, so we can talk about Cheng here, as well.
4. Examine how the likelihood ratio account fares against the two/three difficulties above. One could make an argument (not necessarily a correct one) that the likelihood ratio account can address all the two/three difficulties. So we should say why one might think so, even thought the argument will ultimately fail. I think this will help grab the reader's attention. This is what I have in mind:

   4a: the LR approach solves the naked stat problem because LR=1 (Cheng, Sullivan) or L1=unknown (Di Bello).

   4b: the LR approach solves the conjuction problem because – well this is Dawid's point that we will have to make sense of the best we can

   4c: the LR approach solves the priors problem b/c LR do not have priors.
5. Next, poke holes in the likelihood ratio account:

   against 4a: you do not believeLR=1 or LR=unknown , so we should talk about this

   against 4b: this is your cool argument against Dawid

   against 4c: do you believe the arguemt in 4c? we should talk about this

   In general, we will have to talk to see where we stand. As of now, I tentatively believe that the likelihood ratio account can solve (a) and (c), and you seem to disagree with that. Even if I am right, the account is still not good enough becaue it cannot solve (b).
6. Articulate (or just sketch?) a better probabilistic account overall. Use Bayesian networks, narratives, etc. I am not sure if this should be another paper. That will depend on how much we'll have to say here.

Contents

1 Introduction

After the evidence has been presented, examined and cross-examined at trial, trained judges or lay jurors must reach a decision. In many countries, the decision criterion is defined by law and consists of a standard of proof, also called the burden of persuasion. So long as the evidence against the defendant meets the requisite proof standard, the defendant should be found liable.

In criminal proceedings, the governing standard is 'proof beyond a reasonable doubt.' If the decision makers are persuaded beyond a reasonable doubt that the defendant is guilty, they should convict, or else they should acquit. In civil cases, the standard is typically 'preponderance of the evidence.' The latter is less demanding than the former, so the same body of evidence may meet the preponderance standard, but not meet the beyond a reasonable doubt standard. A vivid example of this difference is the 1995 trial of O.J. Simpson, who was charged with the murder of his wife. He was acquitted of the criminal charges, but when the family of the victim brought a lawsuit against him, they prevailed. O.J. Simpson did not kill his wife according to the beyond a reasonable doubt standard, but he did according to the preponderance standard. An intermediate standard, called 'clear and convincing evidence', is sometimes used for civil proceedings in which the decision is particularly weighty, for example, a decision whether someone should be committed to a hospital facility.

Not sure if it is clear what you mean by this.

How to define standards of proof—and whether they should be even defined in the first place—remains contentious (Diamond, 1990; Horowitz & Kirkpatrick, 1996; Laudan, 2006; Newman, 1993; Walen, 2015). Judicial opinions offer different, sometimes conflicting, paraphrases of what these standards mean. The meaning of 'proof beyond a reasonable doubt' is the most controversial. It has been equated with 'moral certainty' or 'abiding conviction' (Commonwealth v. Webster, 59 Mass. 295, 320, 1850) or with 'proof of such a convincing character that a reasonable person would not hesitate to rely and act upon it in the most important of his own affairs' (US Federal Jury Practice and Instructions, 12.10, at 354, 4th ed. 1987). But courts have also cautioned that there is no need to define the term because 'jurors know what is reasonable and are quite familiar with the meaning of doubt' and attempts to define it only 'muddy the water' (U.S. v. Glass, 846 F.2d 386, 1988).

To further complicate things, differences between countries and legal traditions exist. The tripartite distinction of proof standards—beyond a reasonable doubt; preponderance; clear and convincing evidence—is common in Anglo-american jurisprudence. It is not universal, however. Different countries may use different standards. France, for example, uses the standard of 'intimate conviction' for both civil and criminal proceedings. Judges deciding cases 'must search their conscience in good faith and silently and thoughtfully ask themselves what impression the evidence given against the accused and the defence's arguments have made upon them' (French Code of Criminal Procedure, art. 353). German law is similar. Germany's Code of Civil Procedure, Sec. 286, states that 'it is for the court to decide, based on its personal conviction, whether a factual claim is indeed true or not.'

While there are inevitable differences between legal traditions, the question of how strong the evidence should be to warrant a finding of civil or criminal liability has universal appeal. Any system of adjudication whose decisions are informed by evidence will confront this question in one way or another. Not all legal systems will explicitly formulate standards of proof for trial decisions. Some legal systems may specify rules about how evidence should be weighed without formulating decision criteria such as standards of proof. But even without an explicit proof standards, the triers of facts, judges or jurors, will have to decide whether the evidence is sufficient to deem the defendant legally liable.

need to revise this once done.

We will not survey the extensive legal literature and case law about proof standards. We will instead examine whether or not probability theory can bring conceptual clarity to an otherwise heterogeneous legal doctrine. This chapter outlines different probabilistic approaches, formulates the most common challenges against them, and offers a number of responses from the perspective of legal probabilism. The legal and philosophical literature has focused on the theoretical and analytical challanges. We will do the same here. We will focus on two key theoretical challanges that have galvanized the philosophical literature: the problem of naked statistical evidence and the conjunction paradox. One reason to choose these two in particular is that it would be desirable to be able to handle basic conceptual difficulties before turning to more complex issues or attempting to implement probabilistic standards of proof in trial proceedings.

2 Probability thresholds

Imagine you are a trier of fact, say a judge or a juror, who is expected to make a decision about the guilt of a defendant who faces criminal charges. The defendant denies the accusation. The prosecution presented evidence to support its accusation, and the defense had the opportunity to offer counterevidence. As a trier of fact, you are confronted with the question, does the totality of the evidence presented at trial, all things considered, warrant a conviction. For instance, the question you are confronted with might be: does the evidence prove guilt beyond a reasonable doubt?

2.1 The basic idea

Legal probabilists have proposed to interpret proof beyond a reasonable doubt as the requirement that the defendant's probability of guilt, given the evidence presented at trial, meet a certain threshold (see Bernoulli, 1713; Dekay, 1996; Kaplan, 1968; Kaye, 1979a; Laplace, 1814; Laudan, 2006). In other words, so long as the guilt of the defendant is established with a sufficiently high probability, say 95%, guilt is proven beyond a reasonable doubt and the defendant should be convicted. If the probability of guilt does not reach the requisite threshold, the defendant should be acquitted. This intepretation can be spelled out more formally by means of conditional probabilities. That is, a body of evidence E establishes guilt G beyond a reasonable doubt if and only if $\mathsf{P}(G|E)$ is above a threshold. From this perspective, a conviction is justified whenever guilt is sufficiently probable given the evidence.

This interpretation is, in many respects, plausible. From a legal standpoint, the requirement that guilt be established with high probability, still short of 100%, accords with the principle that proof beyond a reasonable doubt is the most stringent standard but does not require—as the Supreme Court of Canada put it—'proof to an absolute certainty' and thus 'it is not proof beyond any doubt' (R v Lifchus, 1997, 3 SCR 320, 335). The plausibility of a probabilistic intepretation is further attested by the fact that such an intepretation is tacitly assumed in empirical studies about people's understanding of proof beyond a reasonable doubt (Dhami, Lundrigan, & Mueller-Johnson, 2015). This research examines how high decision-makers set the bar for convictions, say at 80% or 90% probability, but does not question the assumption that standards of proof function as probabilistic thresholds of some kind.

This comment somehow disturbs the flow, think about making this nicer.

Reliance on probability is even more explicit in the standard 'preponderance of the evidence'—also called 'balance of probabilities'—which governs decisions in civil disputes. This standard can be interpreted as the requirement that the plaintiff—the party making the complaint against the defendant in a civil case—establish its version of the facts with greater than 50% probability. The 50% threshold, as opposed to a more stringent threshold of 95% for criminal cases, reflects the fact that preponderance is less demanding than proof beyond a reasonable doubt. The intermediate standard 'clear and convincing evidence' is more stringent than the preponderance standard but not as stringent as the beyond a reasonable doubt standard. Since it lies in between the other two, it can be interpreted as the requirement that the plaintiff establishes her versions of the facts with, say, 75-80% probability.

2.2 Mixed reactions from legal practitioners

When appellate courts have examined the question whether standards of proof can be quantified using probabilities, they have often answered in the negative. One of the clearest opposition to quantification was formulated by Germany's Supreme Court, the Federal Court of Justice, in the case of Anna Anderson who claimed to be a descendant of the Tsar family. In 1967, the Regional Court of Hamburg ruled that Anderson failed to present sufficient evidence to establish that she was Grand Duchess Anastasia Nikolayevna, the youngest daughter of Tsar Nicholas II, who allegedly escaped the murder of the Tsar family by the Bolsheviks in 1918. (In fact, DNA testing later demonstrated that Anna Anderson had no relationship with the Tsar family.) Anderson appealed to Germany's Federal Court, complaining that the Regional Court had set too demanding a proof standard. Siding with the lower court, the Federal Court made clear that '[t]he law does not presuppose a belief free of all doubts', thus recognizing the inevitable fallibility of trial decisions. The Court warned, however, that it would be 'wrong' to think that a trial decision could rest on 'a probability bordering on certainty' (Federal Court of Justice, February 17, 1970; III ZR 139/67).

This decision is all the more interesting as it applies to a civil case. It semes as though the German court did not think trial decisions could rest on a probability, not even in a civil case. Turnign from civil to criminal cases, Buchak (2014) has argued that an attribution of criminal culpability is an ascription of blame which requires a full belief in someone's guilt. One is left wondering, however. If a high probability of guilt short of 100% isn't enough but absolute certainty cannot be required either, how else could the standard of proof be met? The question becomes more pressing in civil cases if we replace 'guilt' with 'civil liability'. Anticipating this worry, Germany's Federal Court in the Anderson case endorsed a conception of proof standards that acknowledged the inveitable fallibility of trial decisions while at the same time maintaining the need for certainty. The Federal Court wrote that a judge's decision must satisfy 'a degree of certainty which is useful for practical life and which makes the doubts silent without completely excluding them' (Federal Court of Justice, February 17, 1970; III ZR 139/67).

The words of Germany's Federal Court echo dillemmas that bedeviled early theorists of probability and evidence law. When Jacob Bernoulli—one of the pionerres of probability theory—discusses the requirement for a criminal conviction in his *Ars Conjectandi* (1713), he writes that ‘it might be determined whether 99/100 of probability suffices or whether 999/1000 is required’ (part IV). This is one of the earliest suggestions that the criminal standard of proof be equated to a threshold probability of guilt. A few decades later, the Italian legal penologist Cesare Beccaria in his celebrated treatise *On Crimes and Punishments* (1764) remarks that the certainty needed to convict is ‘nothing but a probability, though a probability of such a sort to be called certainty’ (chapter 14). This suggestive yet—admittedly—quite elusive remark indicates that the standard of decision in criminal trials should be a blend of probability and certainty. But what this blend of probability and certainity should amount to is unclear. At best, it leads back to the unhelpful paraphrases of proof beyond a reasonable doubt such as ‘moral certainity’ or ‘abiding conviction.’

However, it would be wrong to assume that all legal practitioners resist a probabilistic interpretation of standards of proof. Some actually find it quite plausible, even obvious. For example, here is Justice Harlan of the United State Supreme Court:

> . . . in a judicial proceeding in which there is a dispute about the facts of some earlier event, the factfinder cannot acquire unassailably accurate knowledge of what happened. Instead, all the factfinder can acquire is a belief of what probably happened. The intensity of this belief – the degree to which a factfinder is convinced that a given act actually occurred – can, of course, vary. In this regard, a standard of proof represents an attempt to instruct the factfinder concerning the degree of confidence our society thinks he should have in the correctness of factual conclusions for a particular type of adjudication. In re Winship, 397 U.S. 358, 370 (1970).[1]

Following this methodological premise, Justice Harlan explicitly endorses a probabilistic interpretation of standards of proof, using the expression ‘degree of confidence’ instead of ‘probability’:

> Although the phrases ’preponderance of the evidence’ and ’proof beyond a reasonable doubt’ are quantitatively imprecise, they do communicate to the finder of fact different notions concerning the degree of confidence he is expected to have in the correctness of his factual conclusions.

2.3 Practical worries

The remarks by Jastice Harlan notwithstanding, legal practioners seem in general quite opposed to quantifying standards of proof probabiistically. This resistance has many causes. One key factor is certainly the conviction that a probabilistic intepretation of proof standards is unrealistic insofar as its implementation would face unsurmountable challenges. How are the probabilities—say the probability of someone’s guilt—going to be quantified probabilistically? How will the triers of facts apply probabilistic thresholds? Should the application of the threshold be automatic—that is, if the evidence is above the requisite threshold, find against the defedant (say, convict in a criminal trial) and otherwise find for the defedant (say, acquit)? The challenge, in general, is how probabilistic thresholds can be operationalized as part of trial decisions. This is by no means clear. Judges and jurors do not weigh evidence in an explicitly probabilistic manner. They do not seem to explicitly use probability thresholds ato guide their decisions.

I’m not sure if this talk of being automatic isn’t too hasty, think about rephrasing.

The probabilistic interpretion of proof standards can be broken down into two separate claims, what we might call the ‘quantification claim’ and the ‘threshold claim’. In a criminal trial, these claims would look as follows:

> Quantification Claim: a probabilistic quantification of the defendant’s guilt can be given through an appropriate weighing of all the evidence available (that is, of all the evidence against, and of all the evidencein defense of, the accused).
>
> Threshold Claim: an appropriately high threshold guilt probability, say 95%, should be the decision criterion for criminal convictions.

Those worried about implementation might reason as follows. If guilt cannot be quantified probabilistically—for example, in terms of the conditional probability of G given the total evidence E—no probabilistic threshold could ever be used as a decision criterion. Since the quantification claim is unfeasible and the threshold claim rests on the quantification claim, the threshold claim should be

[1] This is a landmark decision by the United States Supreme Court establishing that the beyond a reasonable doubt standard must be applied to both adults and juvenile defendants.

rejected.

One way to answer this objection is to bite the bullet. Legal probabilists can admit that probabilistic thresholds constitute a revisionist theory. If they are to be implemented in trial proceedings, they will require changes. Jurors and judges will have to become familiar with probabilistic ideas. They will have to evaluate the strength of the evidence numerically, even for evidence that is not, on its face, quantitative in nature. But this response will simply increase
the resistance toward a probabilistic intepretation of proof standards. After all, the likelihood of success of such a program of radical reform of trial proceedings is uncertain. Fortunately, there is a less radical way to respond.

2.4 Idealization

Legal probabilists can admit they are not—at least, not yet—engaged with implementation or trial reform. In fact, the quantification claim can be interpreted in at least two different ways. One interpretation is that a quantification of guilt—understood as an actual reasoning process—can be fairly effectively carried out by the fact-finders. The quantification claim can also be understood as an idealization or a regulative ideal. For instance, the authors of a book on probabilistic inference in forensic science write:

> the . . . [probabilistic] formalism should primarily be considered as an aid to structure and guide one's inferences under uncertainty, rather than a way to reach precise numerical assessments' (p. xv) (CITE TARONI).

Even from a probabilist standpoint, the quantification of guilt can well be an idealization which has, primarily, a heuristic role.

Just as the quantification claim can be interpreted in two different ways, the same can be said of the threshold claim. For one thing, we can interpret it as describing an effective decision procedure, as though the fact-finders were required to mechanically convict whenever the defendant's probability of guilt happened to meet the desired probabilistic threshold. But there is a second, and less mechanistic, interpretation of the threshold claim. On the second interpretation, the threshold claim would only describe a way to understand, or theorize about, the standard of proof or the rule of decision. The second interpretation of the threshold claim—which fits well with the 'idealization interpretation' of the quantification claim—is less likely to cause outrage.

Lawrence Tribe, in his famous 1971 article 'Trial by Mathematics', expresses disdain for a trial process that were mechanically governed by numbers and probabilities. He claims that under this scenario judges and jurors would forget their humanizing function. He writes:

> Guided and perhaps *intimidated by the seeming inexorability of numbers*, induced by the persuasive force of formulas and the precision of decimal points to perceive themselves as performing a largely mechanical and automatic role, *few jurors ... could be relied upon to recall, let alone to perform, [their] humanizing function.* (CITE TRIBE)

But this worry does not apply if we interpret the threshold claim in a non-mechanistic way. This is the interpretation we shall adopt for the purpose of this chapter. To avoid setting the bar for legal probabilism too high, we will not be concerned with practical issues that would arise if we wanted to deploy a probabilistic threshold directly. We will grant that, at least for now, successful deployments of such thresholds are not viable. For the time being, probabilistic thresholds are best understood as offerring an theoretical, analytical model of trial decisions. The fact that this theorethical model cannot be easily operationalized does not mean the model is pointless. There are multiple ways in which such a model, even if unfit for direct deployment in trial proceedings, can offer insight into trial decision-making.

2.5 Minimizing expected costs

Here is an illustration of the analytic power of the probabilistic interpretation of proof standards. Standards of proof are usually ranked from the least demanding (such as preponderance of the evidece) to the most demanding (such as proof beyond a reasonable doubt). But why think this way? Can we give a principled justification for the existence of multiple standards and their the ranking? A common argument is that more is at stake in a criminal trial than in a civil trial. A mistaken conviction will injustly deprive the defedant of basic liberties or even life. Instaed, a mistaken decision in a civil trial would not encroach upon someone's basic liberties since decisions in civil trials are mostly about imposing monetary compensation. This argument can be made precise by pairing probability thresholds with expected utility theory, a well-establish paradigm of rational decision-making used in psychology and economic theory. At its simplest, decision theory based on the maximization of expected utility states that between a number of alternative courses of action, the one with the highest expected utility (or with the lowest expected cost) should be preferred. The decision-theoretic framework is very general and can be applied to a variety of situations, including civil or criminal trials.

To see how this works, note that trial decisions can be factually erroneous in two ways. A trial decision can be a false positive—i.e. a decision to hold the defendant liable (to convict, in a criminal case) even though the defedant committed no wrong (or committed no crime). A trial decision can also be a false negative—i.e. a decision not to hold the defendant liable (or to acquit, in a criminal case) even though the defendant did commit the wrong (or committed the crime). Let $\mathsf{cost}(CI)$ and $\mathsf{cost}(AG)$ be the costs associated with the two decisional errors that can be made in a criminal trial, convicting an innocent (CI) and acquitting a guilty defendant (AG). Let $\mathsf{P}(G|E)$ and $\mathsf{P}(I|E)$ be the guilt probability and the innocence probability estimated on the basis of the evidence presented at trial. Given a simple decision-theoretic model, a conviction should be preferred to an acquittal whenever the expected cost resulting from a mistaken conviction—namely, $\mathsf{P}(I|E) \cdot \mathsf{cost}(CI)$—is lower than the expected cost resulting from a mistaken acquittal—namely, $\mathsf{P}(G|E) \cdot \mathsf{cost}(AG)$. That is,

$$\text{convict provided} \quad \frac{\mathsf{cost}(CI)}{\mathsf{cost}(AG)} < \frac{\mathsf{P}(G|E)}{\mathsf{P}(I|E)}.$$

For the inequality to hold,[2] the ratio of posterior probabilities $\frac{\mathsf{P}(G|E)}{\mathsf{P}(I|E)}$ should exceed the cost ratio $\frac{cost(CI)}{cost(AG)}$. So long as the costs can be quantified, the probability threshold can be determined. For example, consider a cost ratio of nine according to which a mistaken conviction is nine times as costly as a mistaken acquittal. The corresponding probability threshold will be 90%. On this reading, in order to meet the standard of proof beyond a reasonable doubt, the prosecution should provide evidence that establishes the defendant's guilt with at least 90% probability, or in formulas, $\mathsf{P}(G|E) > 90\%$. The higher the cost ratio, the higher the requisite threshold. The lower the cost ratio, the lower the requiste threshold. For example, if the cost ratio is 99, the threshold would be as high as 99%, but if the cost ratio is 2, the threshold would only be 75%.

The same line of argument applies to civil cases. Let a false attribution of liability FL be a decision to find the defendant liable when the defendant committed no civil wrong (analogous to the conviction of an innocent in a criminal case). Let a false attribution of non-liability FNL be a decision not to find the defendant liable when the defendant did commit the civil wrong (analogous to the acquittal a factually guilty defendant in a criminal case). Let $\mathsf{P}(L|E)$ and $\mathsf{P}(NL|E)$ be the liability probability and the non-liability probability guven the total evidence presented at trial. So long as the objective is to minimize the costs of erroneous decisions, the rule of decision would be as follows:

$$\text{find the defendant civilly liable provided} \quad \frac{cost(FL)}{cost(PN)} < \frac{\mathsf{P}(L|E)}{\mathsf{P}(NL|E)}.$$[3]

If the cost ratio $\frac{cost(FP)}{cost(PN)}$ is set to 1, the threshold for liability judgments should equal 50%, a common intepretation of the preponderance standard in civil cases. This means that $\mathsf{P}(L|E)$ should be at least 50% for a defedant to be found civilly liable.

The difference between proof standards in civil and criminal cases lies in the different cost ratios. The cost ratio in civil cases, $\frac{cost(FP)}{cost(PN)}$, is typically lower than the cost ratio in criminal cases, $\frac{cost(CI)}{cost(AG)}$, because a false positive in a criminal trial (a mistaken conviction) is considered a much graver error than a false positive in a civil trial (a mistaken attribution of civil liability). This difference in the costs ratio can have a consequentialist or a retributivist justification (Walen, 2015). From the consequentialist perspective, the loss of personal freedom or even life can be considered a greater loss than paying an undue monetary compensation. From a retributivist perspective, the moral wrong that results from the mistaken conviction of an innocent person can be regarded as more egregious than the moral wrong that results from the mistaken attribution of civil liability. This difference in consequences or moral wrongs can be captured by a higher cost ratio in criminal than civil cases, $\frac{cost(FP)}{cost(PN)}$.

[2] This follows from $\mathsf{P}(I|E) \cdot \mathsf{cost}(CI) < \mathsf{P}(G|E) \cdot \mathsf{cost}(AG)$; see (**???**). This model assumes, simplistically, that correct decisions do not bring any positive utility. More complex models are also possible, but the basic idea is the same; see (Dekay, 1996).

[3] This follows from $\mathsf{P}(NL|E) \cdot cost(FP) < \mathsf{P}(L|E) \cdot cost(FNL)$

While courts are often resistant in allowing a numerical intepretation of proof standards, they sometimes make remarks that—at least, at the theoretical level—agree with a probabilistic analysis of standards of proof. Justice Harlan of the United Suprem Court draws a clear differencce in the costs between criminal and civil litigation:

> In a civil suit between two private parties for money damages, for example, we view it as no more serious in general for there to be an erroneous verdict in the defendant's favor than for there to be an erroneous verdict in the plaintiff's favor ... In a criminal case, on the other hand, we do not view the social disutility of convicting an innocent man as equivalent to the disutility of acquitting someone who is guilty (In Re Winship, 397 U. S. 358, 371).

To underscore the differences in the cost ratios, Harlan cites an earlier decision of the United Stated Court:

> [t]here is always in litigation a margin of error ..., representing error in factfinding, which both parties must take into account ... [w]here one party has at stake an interest of transcending value – as a criminal defendant his liberty – ... this margin of error is *reduced* as to him by the process of placing on the other party [i.e. the prosecutor] the standard of ... persuading the factfinder at the conclusion of the trial of his guilt beyond a reasonable doubt (357 U.S. 513, 525-26).

Claims aboutcost ratios, their magnitiuted and differences between criminal and civil cases, can of of course be contested. Some have argued, for example, that the standard of proof in criminal cases should in fact be lower than 90%, and they have done so by offerring a different assessment of the cost ratio (CITE LAUDAN). We will not examine this debate here. Rather, the point for now is that probabilistic thresholds, when paired with expected utility theory, provide an analytical framework for a meaningful debate about the different degrees of stringency necessary for decision criteria—i.e. legal proof standards—in civil or criminal trials. In later chapters, we will examine more in detail how a probability-based analytical framework acan help to theorize about the values that should inform trial decisions, such as the minimization of expected costs, the maximization of truth and accuracy, and the fair allocation of the risk of of error. (REFER HERE TO THESE LATER CHAPTERS)

3 Theoretical challenges

Let's take stock. We briefly examine difficulties in implementation for probabilistic standards of proof and set thsoe aside. But even if probabilistic thresholds are used solely as analytical tools, legal probabilist are not yet out of the woods. Even if the practical problems can be addressed or set aside, theoretical difficulties do remain. We will focus on two in particular: naked statistical evidence, or proof paradoxes, and the difficulty about conjunction, also called the conjunction paradox.

3.1 The problem of priors

3.2 Naked statistical evidence

Suppose one hundred, identically dressed prisoners are out in a yard during recreation. Suddenly, ninety-nine of them assault and kill the guard on duty. We know that this is what happened from a video recording, but we do not know the identity of the ninety-nine killers. After the fact, a prisoner is picked at random and tried. Since he is one of the prisoners who were in the yard, the probability of his guilt would be 99%. But despite the high probability, many have the intuition that this is not enough to establish guilt beyond a reasonable doubt (**???**, **???**, **???**; Ho, 2008). Hypothetical scenarios of this sort suggest that a high probability of guilt, while perhaps necessary, is not sufficient to establish guilt beyond a reasonable doubt.

A similar hypothetical can constructed for civil cases. Suppose a bus company, Blue-Bus, operates 90% of the buses in town on a certain day, while Red-Bus only 10%. That day a bus injures a pedestrian. Although the buses of the two companies can be easily recognized because they are respectively painted blue and red, the pedestrian who was injured cannot remember the color of the bus involved in the accident. No other witness was around. Still, given the statistics about the market shares of the two companies, it is 90% probable that a Blue-Bus bus was involved in the accident. This is a high probability, well above the 50% threshold. Yet the 90% probability that a Blue-Bus bus was involved in the accident would seem—at least intuitively—insufficient for a judgment of liability against Blue-Bus.

This intuition challenges the idea that the preponderance standard in civil cases only requires that the plaintiffi establish the facts with a probability greater than 50%.

MENTION OTHER EXAMPLES OF NAKED STASTICAL EVIDENCE HERE?

Confronted with these hyptheticals, legal probabilists could push back. Hypotheticals such as these heavily rely on intuitive judgments, for example, that the high probability of the prisoners's guilt in the scenario above does not amount to proof beyond a reasonable doubt. But suppose we changed the numbers and imagined there were one thousand prisoners of whom nine hundred and ninety-nine killed the guard. The guilt probability of a prisoner picked at random would be 99.9%. Even in this situation, many would insist that guilt has not been proven beyond a reasonable doubt despite the extremely high probability of guilt. But others might say that when the guilt probability reaches such extreme values, values as high as 99.9% or higher, people's intuitive resistance to convicting should subside (Roth, 2010).

A more general problem is that intuitions in such hypothetical scenarios are removed from real cases and thus are unreliable as a guide to theorize about the standard of proof (**???**, **???**; Lempert, 1986).

Another reason to be suspicious of these hypotheticals is that they seem to track biases in human reasoning. Say an eyewitness was present during the accident and testified that a Blue-Bus bus was involved. Intuitively, the testimony would be considered enough to rule against Blue-Bus, at least provided the witness survived cross-examination. We exhibit, in other words, an intuitive preference for judgments of liability based on testimonial evidence compared to judgments based on statistical evidence. This preference has been experimentally verified (**???**; Niedermeier, Kerr, & Messeé, 1999, Arkes, Shoots-Reinhard, & Mayes (2012)) and seems to exist beyond the law (Ebert, Smith, & Durbach, 2018; Friedman & Turri, 2015; Sykes & Johnson, 1999). But the latter are no more prone to error than the former, and in fact, they may well be less prone to error. So are we really justified in exhibiting this intuitive preference for eyewitness testimony?

These reservations notwhithstanding, the puzzles about naked statistical evidence cannot be easily dismissed. Puzzles about statistical evidence in legal proof have been around for a while (**???**, **???**; Kaye, 1979b; Thomson, 1986), and philosophers and legal scholars have shown a renewed interest in naked statistical evidence and the puzzles that it raises in both criminal and civil cases (**???**, **???**, **???**; Di Bello, 2019a; Enoch, Spectre, & Fisher, 2012; Ho, 2008; Moss, 2018; Nunn, 2015; Pardo, 2018; Pritchard, 2015: Blome-Tillmann (2015); Pundik, 2017; Roth, 2010; Smith, 2018; Stein, 2005; Wasserman, 1991). Given the growing interest in the topic, legal probabilism cannot be a defensible theoretical position without offering a story about naked statistical evidence.

3.3 Conjuction paradox

The *Difficulty About Conjunction* (DAC) proceeds as follows. Say we focus on a civil suit where a plaintiff is required to prove their case on the balance of probability, which for the sake of argument we construe as passing the 0.5 probability threshold.[4] Suppose the plaintiff's claim to be proven based on total evidence E is composed of two elements, A and B, independent conditionally on E.[5] The question is, what exactly is the plaintiff supposed to establish? It seems we have two possible readings:

Requirement 1	$\mathsf{P}(A \wedge B \vert E) > 0.5$
Requirement 2	$\mathsf{P}(A \vert E) > 0.5$ and $\mathsf{P}(B \vert E) > 0.5$

Requirement 1 says that the plaintiff should show that their *whole* claim is more likely than its negation. There are strong intuitions that this is what they should do. But the problem is, this requirement is not equivalent to **Requirement 2**. In fact, if we need $\mathsf{P}(A \wedge B|E) = \mathsf{P}(A|E) \times \mathsf{P}(B|E) > 0.5$ (the identity being justified by the independence assumption), satisfying **Requirement 2** is not sufficient for this purpose. For instance, if $\mathsf{P}(A|E) = \mathsf{P}(B|E) = 0.51$, $\mathsf{P}(A|E) \times \mathsf{P}(B|E) \approx 0.26$, and so the plaintiff's claim as a whole still fails to be established. This means that requiring the proof of $A \wedge B$ on the balance of probability puts an importantly higher requirement on the separate probabilities of the conjuncts.

Moreover, what is required exactly for one of them depends on what has been achieved for the other. If I already established that $\mathsf{P}(A|E) = 0.8$, I need $\mathsf{P}(B|E) \geq 0.635$ to end up with $\mathsf{P}(A \wedge B|E) \geq 0.51$. If, however, $\mathsf{P}(A|E) = 0.6$, I need $\mathsf{P}(B|E) \geq 0.85$ to reach the same threshold. This would mean that

[4] This is a natural choice given that the plaintiff is supposed to show that their claim is more probable than the defendant's. The assumption is not essential. DAC can be deployed against any $\neq 1$ guilt probability threshold.

[5] These assumptions, again, are not too essential. In fact, the difficulties become more severe as the number of elements grows, and, extreme cases aside, do not tend to disappear if the elements are dependent.

standards of proof for a given claim could vary depending on how well a different claim has been argued for and on whether it is a part of a more complex claim that one is defending, and this does not seem very intuitive. At least, this goes strongly against the equal treatment requirement mentioned already in the introduction.

Should we then abandon **Requirement 1** and remain content with **Requirement 2**? [Cohen1977The-probable-an: 66] convincingly argues that we should not. Not evaluating a complex civil case as a whole is the opposite of what the courts themselves normally do. There are good reasons to think that every common law system subscribes to a sort of conjunction principle, which states that if A and B are established on the balance of probabilities, then so is $A \wedge B$.

So, on one hand, if we take our decision standard from **Requirement 2**, our acceptance standard will not involve closure under conjunction, and might lead to conviction in cases where $\mathsf{P}(G|E)$ is quite low, just because G is a conjunction of elements which separately satisfy the standard of proof – and this seems unintuitive. On the other hand, following Cohen, if we take our decision standard from **Requirement 1**, we will put seemingly unnecessarily high requirements sensitive to fairly contingent and irrelevant facts on the prosecution, and treat various elements to be proven unevenly. Neither seems desirable.

4 Likelihood thresholds

4.1 The likelihood strategy

The most natural probabilistic interpretation of proof standards imposes a threshold on posterior probabilisties. For example, in criminal cases, the requirement is usually formulated as follows: guilt is proven beyond a reasonable doubt provoded $\mathsf{P}(G|E)$ is above a suitable threshold, say 95%. The threshold will be lower in civil trials. This interpretation is quite flexibile. We should think of it as a family of inteoretation rather than a interpretation.

The claim that the defendant is guilty can be replaced by a more fine-grained hypothesis, call it H_p, the hypothesis put foward by the prosecutor, for example, hypothesis that the defendant killed the victim with a firearm while bulglarizing the victim's apartment. H_p can be any hypothesis which, if true, would entail the defendanat is guilty (according to the governing law). Hypothesis H_p is a more precise description of what happaned that establishes, if true, the defendant's guilt. In defining proof standards, instead of saying that $\mathsf{P}(G|E)$ should be above a threshold, a probabilistic interpretation could read: guilt is proven beyond a reasonable doubt provided $\mathsf{P}(H_d|E)$ is above a threshold.

Here is another possible variation. Say the defense offers an alternative hypothesis about what happened, call H_d. This may be more common in civil than criminal trial. At any rate, the standard of proof can be defined comparatively as follows. Given a body of evidence E and two competing hypotheses H_p and H_d, the probability $\mathsf{P}(H_p|E)$ should be significantly higher than $\mathsf{P}(H_d|E)$, or in other words, $\frac{\mathsf{P}(H_p|E)}{\mathsf{P}(H_d|E)}$ should above a suitably high threshold. If the threshold is for example 2, $\mathsf{P}(H_p|E)$ should be two times $\mathsf{P}(H_d|E)$. Note that H_p and H_d need not be one the negation of the other If they are one the negation of the other, for example, G and I, then $\frac{\mathsf{P}(G|E)}{\mathsf{P}(I|E)} > 2$ implies that $\mathsf{P}(G|E) > 75\%$.

What is common to these variations is that they set a threshold that is based, in one way or another, on the posterior probability given the evidence, such as $\mathsf{P}(G|E)$, $\mathsf{P}(H_p|E)$, $\frac{\mathsf{P}(H_p|E)}{\mathsf{P}(H_d|E)}$ or $\frac{\mathsf{P}(G|E)}{\mathsf{P}(I|E)}$. But focusing on posterior probabilities is not the only approach that legal probabilists can pursue. By Bayes' theorem, the following holds, using G and I as competing hypotheses:

$$\frac{\mathsf{P}(G|E)}{\mathsf{P}(I|E)} = \frac{\mathsf{P}(E|G)}{\mathsf{P}(E|I)} \times \frac{\mathsf{P}(G)}{\mathsf{P}(I)},$$

or using H_p and H_d as competing hypotheses,

$$\frac{\mathsf{P}(H_p|E)}{\mathsf{P}(H_d|E)} = \frac{\mathsf{P}(E|H_p)}{\mathsf{P}(E|H_d)} \times \frac{\mathsf{P}(H_p))\Pr(H_d}{,}$$

or in words

$$\textit{posterior odds} = \textit{likelihood ratio} \times \textit{prior odds}.$$

A difficult problem is to assign numbers to the prior probabiliteis such as $\mathsf{P}(G)$ or $\mathsf{P}(H_p)$, or priors odds such as $\frac{\mathsf{P}(G)}{\mathsf{P}(I)}$ or $\frac{\mathsf{P}(H_p)}{\mathsf{P}(H_d)}$.

DISCUSS DIFFICULTIES ABOUT ASSIGNING PRIORS! WHERE? CAN WE USE IMPRECISE PROBABILKITIES T TALK ABOUT PRIORS – I.E. LOW PRIORS = TOTAL IGNORANCE = VERY IMPRECISE (LARGE INTERVAL) PRIORS? THE PROBLME WITH THIS WOULD BE THAT THERE IS NO UPDSATING POSSIBLE. ALL UPDATING WOULD STILL GET BACK TO THE STARTING POINT. DO YOU HAVE AN ANSWER TO THAT? WOULD BE INTERETSING TO DISCUSS THIS!

Given these difficulties, both practical and theoretical, one option is to dispense with priors altogether. This is not implausible. Legal disputes in both criminal and civil trials should be decided on the basis of the evidence presented by the litigants. But it is the likelihood ratio – not the prior ratio – that offers the best measure of the overall strength of the evidece presented. So it is all too natural to focus on likekihood ratios and leave the priors out of the picture. If this is the right, the question is, how would a probabilistic interpretation of standards of proof based on the likelihood rato look like? At its simplest, this stratgey will look as follows. Recall our discussion of expected utility theory:

$$\text{convict provided} \frac{\mathsf{cost}(CI)}{\mathsf{cost}(AG)} < \frac{\mathsf{P}(H_p|E)}{\mathsf{P}(H_d|E)},$$

which is equivalent to

$$\text{convict provided} \frac{\mathsf{cost}(CI)}{\mathsf{cost}(AG)} < \frac{\mathsf{P}(E|H_p)}{\mathsf{P}(E|H_d)} \times \frac{\mathsf{P}(H_p))\Pr(H_d}{.}$$

By rearraing the terms,

$$\text{convict provided} \frac{\mathsf{P}(E|H_p)}{\mathsf{P}(E|H_d)} > \frac{\mathsf{P}(H_d))\Pr(H_p}{\times} \frac{\mathsf{cost}(CI)}{\mathsf{cost}(AG)}.$$

Then, on this intepretation, the likelihood ratio should be above a suitable threshold that is a function of the cost ratio and the prior ratio. The outstanding question is how this threshold is to be determined.

4.2 Cheng

Here is one way to think about the decision thresholds in terms of likelihoods, stemming from (Cheng, 2012). The idea is to conceptualize juridical decisions in analogy to statistical hypothesis testing. We have two hypotheses under consideration: defendant's H_Δ and plaintiff's H_Π, and we are to pick one: D_Δ stands for the decision for H_Δ and D_Π is the decision that H_Π. On this approach, rather than directly evaluating the probability of H_Π given the evidence and comparing it to a threshold, we compare the support that the evidence provides for these hypotheses, and decide for the one for which the evidence provides better support.

Cheng motivates this approach by the following considerations. Suppose that if the decision is correct, no costs result, but incorrect decisions have their price. Let us say that if the defendant is right and we find against them, the cost is c_1, and if the plaintiff is right and we find against them, the cost is c_2:

		Decision	
		D_Δ	D_Π
Truth	H_Δ	0	c_1
	H_Π	c_2	0

Intuitively, it seems that we want a decision rule which minimizes the expected cost. Say that given our total evidence E the relevant conditional probabilities are:

$$p_\Delta = \mathsf{P}(H_\Delta|E)$$
$$p_\Pi = \mathsf{P}(H_\Pi|E)$$

The expected costs for deciding that H_Δ and H_Π, respectively, are:

$$\begin{aligned} E(D_\Delta) &= p_\Delta 0 + p_\Pi c_2 = c_2 p_\Pi \\ E(D_\Pi) &= p_\Delta c_1 + p_\Pi 0 = c_1 p_\Delta \end{aligned}$$

For this reason, on these assumptions, we would like to choose H_Π just in case $E(D_\Pi) < E(D_\Delta)$. This condition is equivalent to:

$$\begin{aligned} c_1 p_\Delta &< c_2 p_\Pi \\ c_1 &< \frac{c_2 p_\Pi}{p_\Delta} \\ \frac{c_1}{c_2} &< \frac{p_\Pi}{p_\Delta} \end{aligned} \tag{1}$$

Cheng (2012) (1261) insists:

> At the same time, in a civil trial, the legal system expresses no preference between finding erroneously for the plaintiff (false positives) and finding erroneously for the defendant (false negatives). The costs c_1 and c_2 are thus equal...

If we grant this assumption, $c_1 = c_2$, (1) reduces to:

$$\begin{aligned} 1 &< \frac{p_\Pi}{p_\Delta} \\ p_\Pi &> p_\Delta \end{aligned} \tag{2}$$

That is, in standard civil litigation we are to find for the plaintiff just in case H_Π is more probable given the evidence than H_Δ, which seems plausible.[6] Let's call this decision standard **Relative Legal Probabilism (RLP)**.[7]

Here is a slightly different perspective, due to Dawid (1987), that also suggests that juridical decisions should be likelihood-based. The focus is on witnesses for the sake of simplicity. Imagine the plaintiff produces two independent witnesses: W_A attesting to A, and W_B attesting to B. Say the witnesses are regarded as 70% reliable and A and B are probabilistically independent, so we infer $\mathsf{P}(A) = \mathsf{P}(B) = 0.7$ and $\mathsf{P}(A \wedge B) = 0.7^2 = 0.49$.

But, Dawid argues, this is misleading, because to reach this result we misrepresented the reliability of the witnesses: 70% reliability of a witness, he continues, does not mean that if the witness testifies that A, we should believe that $\mathsf{P}(A) = 0.7$. To see his point, consider two potential testimonies:

A_1	The sun rose today.
A_2	The sun moved backwards through the sky today.

Intuitively, after hearing them, we would still take $\mathsf{P}(A_1)$ to be close to 1 and $\mathsf{P}(A_2)$ to be close to 0, because we already have fairly strong convictions about the issues at hand. In general, how we should revise our beliefs in light of a testimony depends not only on the reliability of the witness, but also on our prior convictions.[8] And this is as it should be: as indicated by Bayes' Theorem, one and the same testimony with different priors might lead to different posterior probabilities.

So far so good. But how should we represent evidence (or testimony) strength then? Well, one pretty standard way to go is to focus on how much it contributes to the change in our beliefs in a way independent of any particular choice of prior beliefs. Let a be the event that the witness testified that A. It is useful to think about the problem in terms of *odds, conditional odds (O)* and *likelihood ratios (LR)*:

[6] Notice that this instruction is somewhat more general than the usual suggestion of the preponderance standard in civil litigation, according to which the court should find for the plaintiff just in case $\mathsf{P}(H_\Pi|E) > 0.5$. This threshold, however, results from (2) if it so happens that H_Δ is $\neg H_\Pi$, that is, if the defendant's claim is simply the negation of the plaintiff's thesis. By no means, Cheng argues, this is always the case.

[7] We were not aware of any particular name for Cheng's model so we came up with this one. We're not particularly attached to it, and it is not standard terminology.

[8] An issue that Dawid does not bring up is the interplay between our priors and our assessment of the reliability of the witnesses. Clearly, our posterior assessment of the credibility of the witness who testified A_2 will be lower than that of the other witness.

$$O(A) = \frac{\mathsf{P}(A)}{\mathsf{P}(\neg A)}$$
$$O(A|a) = \frac{\mathsf{P}(A|a)}{\mathsf{P}(\neg A|a)}$$
$$LR(a|A) = \frac{\mathsf{P}(a|A)}{\mathsf{P}(a|\neg A)}.$$

Suppose our prior beliefs and background knowledge, before hearing a testimony, are captured by the prior probability measure $\mathsf{P}_{prior}(\cdot)$, and the only thing that we learn is a. We're interested in what our *posterior* probability measure, $\mathsf{P}_{posterior}(\cdot)$, and posterior odds should then be. If we're to proceed with Bayesian updating, we should have:

$$\frac{\mathsf{P}_{posterior}(A)}{\mathsf{P}_{posterior}(\neg A)} = \frac{\mathsf{P}_{prior}(A|a)}{\mathsf{P}_{prior}(\neg A|a)} = \frac{\mathsf{P}_{prior}(a|A)}{\mathsf{P}_{prior}(a|\neg A)} \times \frac{\mathsf{P}_{prior}(A)}{\mathsf{P}_{prior}(\neg A)}$$

that is,

$$O_{posterior}(A) = O_{prior}(A|a) = \underbrace{LR_{prior}(a|A)}_{\text{conditional likelihood ratio}} \times O_{prior}(A) \tag{3}$$

The conditional likelihood ratio seems to be a much more direct measure of the value of a, independent of our priors regarding A itself. In general, the posterior probability of an event will equal to the witness's reliability in the sense introduced above only if the prior is $1/2$.[9]

Quite independently, a similar approach to juridical decisions has been proposed by Kaplow (2014) – we'll call it **decision-theoretic legal probabilism (DTLP)**. It turns out that Cheng's suggestion is a particular case of this more general approach. Let $LR(E) = \mathsf{P}(E|H_\Pi)/\mathsf{P}(E|H_\Delta)$. In whole generality, DTLP invites us to convict just in case $LR(E) > LR^\star$, where $LR^\star$ is some critical value of the likelihood ratio.

Say we want to formulate the usual preponderance rule: convict iff $\mathsf{P}(H_\Pi|E) > 0.5$, that is, iff $\frac{\mathsf{P}(H_\Pi|E)}{\mathsf{P}(H_\Delta|E)} > 1$. By Bayes' Theorem we have:

$$\frac{\mathsf{P}(H_\Pi|E)}{\mathsf{P}(H_\Delta|E)} = \frac{\mathsf{P}(H_\Pi)}{\mathsf{P}(H_\Delta)} \times \frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)} > 1 \Leftrightarrow$$
$$\Leftrightarrow \frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)} > \frac{\mathsf{P}(H_\Delta)}{\mathsf{P}(H_\Pi)}$$

So, as expected, $LR^\star$ is not unique and depends on priors. Analogous reformulations are available for thresholds other than 0.5.

[9] Dawid gives no general argument, but it is not too hard to give one. Let $rel(a) = \mathsf{P}(a|A) = \mathsf{P}(\neg a|\neg A)$. We have in the background $\mathsf{P}(a|\neg A) = 1 - \mathsf{P}(\neg a|\neg A) = 1 - rel(a)$. We want to find the condition under which $\mathsf{P}(A|a) = \mathsf{P}(a|A)$. Set $\mathsf{P}(A) = p$ and start with Bayes' Theorem and the law of total probability, and go from there:

$$\begin{aligned}
\mathsf{P}(A|a) &= \mathsf{P}(a|A)\\
\frac{\mathsf{P}(a|A)p}{\mathsf{P}(a|A)p + \mathsf{P}(a|\neg A)(1-p)} &= \mathsf{P}(a|A)\\
\mathsf{P}(a|A)p &= \mathsf{P}(a|A)[\mathsf{P}(a|A)p + \mathsf{P}(a|\neg A)(1-p)]\\
p &= \mathsf{P}(a|A)p + \mathsf{P}(a|\neg A) - \mathsf{P}(a|\neg A)p\\
p &= rel(a)p + 1 - rel(a) - (1 - rel(a))p\\
p &= rel(a)p + 1 - rel(a) - p + rel(a)p\\
2p &= 2rel(a)p + 1 - rel(a)\\
2p - 2rel(a)p &= 1 - rel(a)\\
2p(1 - rel(a)) &= 1 - rel(a)\\
2p &= 1
\end{aligned}$$

First we multiplied both sides by the denominator. Then we divided both sides by $\mathsf{P}(a|A)$ and multiplied on the right side. Then we used our background notation and information. Next, we manipulated the right-hand side algebraically and moved $-p$ to the left-hand side. Move $2rel(a)p$ to the left and manipulate the result algebraically to get to the last line.

Kaplow's point is not that we can reformulate threshold decision rules in terms of priors-sensitive likelihood ratio thresholds. Rather, he insists, when we make a decision, we should factor in its consequences. Let G represent potential gain from correct conviction, and L stand for the potential loss resulting from mistaken conviction. Taking them into account, Kaplow suggests, we should convict if and only if:

$$\mathsf{P}(H_\Pi|E) \times G > \mathsf{P}(H_\Delta|E) \times L \tag{4}$$

Now, (4) is equivalent to:

$$\begin{aligned}
\frac{\mathsf{P}(H_\Pi|E)}{\mathsf{P}(H_\Delta|E)} &> \frac{L}{G} \\
\frac{\mathsf{P}(H_\Pi)}{\mathsf{P}(H_\Delta)} \times \frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)} &> \frac{L}{G} \\
\frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)} &> \frac{\mathsf{P}(H_\Delta)}{\mathsf{P}(H_\Pi)} \times \frac{L}{G} \\
LR(E) &> \frac{\mathsf{P}(H_\Delta)}{\mathsf{P}(H_\Pi)} \times \frac{L}{G} \qquad (5)
\end{aligned}$$

This is the general format of Kaplow's decision standard.

4.3 Likelihood and DAC

But how does our preference for the likelihood ratio as a measure of evidence strength relate to DAC? Let's go through Dawid's reasoning.

A sensible way to probabilistically interpret the 70% reliability of a witness who testifies that A is to take it to consist in the fact that the probability of a positive testimony if A is the case, just as the probability of a negative testimony (that is, testimony that A is false) if A isn't the case, is 0.7:[10]

$$\mathsf{P}_{prior}(a|A) = \mathsf{P}_{prior}(\neg a|\neg A) = 0.7.$$

$\mathsf{P}_{prior}(a|\neg A) = 1 - \mathsf{P}_{prior}(\neg a|\neg A) = 0.3$, and so the same information is encoded in the appropriate likelihood ratio:

$$LR_{prior}(a|A) = \frac{\mathsf{P}_{prior}(a|A)}{\mathsf{P}_{prior}(a|\neg A)} = \frac{0.7}{0.3}$$

Let's say that *a provides (positive) support* for A in case

$$O_{posterior}(A) = O_{prior}(A|a) > O_{prior}(A)$$

that is, a testimony a supports A just in case the posterior odds of A given a are greater than the prior odds of A (this happens just in case $\mathsf{P}_{posterior}(A) > \mathsf{P}_{prior}(A)$). By (3), this will be the case if and only if $LR_{prior}(a|A) > 1$.

One question that Dawid addresses is this: assuming reliability of witnesses 0.7, and assuming that a and b, taken separately, provide positive support for their respective claims, does it follow that $a \wedge b$ provides positive support for $A \wedge B$?

Assuming the independence of the witnesses, this will hold in non-degenerate cases that do not involve extreme probabilities, on the assumption of independence of a and b conditional on all combinations: $A \wedge B$, $A \wedge \neg B$, $\neg A \wedge B$ and $\neg A \wedge \neg B$.[11],~[12]

Let us see why the above claim holds. The calculations are my reconstruction and are not due to Dawid. The reader might be annoyed with me working out the mundane details of Dawid's claims, but

[10] In general setting, these are called the *sensitivity* and *specificity* of a test (respectively), and they don't have to be equal. For instance, a degenerate test for an illness which always responds positively, diagnoses everyone as ill, and so has sensitivity 1, but specificity 0.

[11] Dawid only talks about the independence of witnesses without reference to conditional independence. Conditional independence does not follow from independence, and it is the former that is needed here (also, four non-equivalent different versions of it).

[12] In terms of notation and derivation in the optional content that will follow, the claim holds if and only if $28 > 28p_{11} - 12p_{00}$. This inequality is not true for all admissible values of p_{11} and p_{00}. If $p_{11} = 1$ and $p_{00} = 0$, the sides are equal. However, this is a rather degenerate example. Normally, we are interested in cases where $p_{11} < 1$. And indeed, on this assumption, the inequality holds.

it turns out that in the case of Dawid's strategy, the devil is in the details. The independence of witnesses gives us:

$$
\begin{aligned}
\mathsf{P}(a \wedge b|A \wedge B) &= 0.7^2 = 0.49\\
\mathsf{P}(a \wedge b|A \wedge \neg B) &= 0.7 \times 0.3 = 0.21\\
\mathsf{P}(a \wedge b|\neg A \wedge B) &= 0.3 \times 0.7 = 0.21\\
\mathsf{P}(a \wedge b|\neg A \wedge \neg B) &= 0.3 \times 0.3 = 0.09
\end{aligned}
$$

Without assuming A and B to be independent, let the probabilities of $A \wedge B$, $\neg A \wedge B$, $A \wedge \neg B$, $\neg A \wedge \neg B$ be $p_{11}, p_{01}, p_{10}, p_{00}$. First, let's see what $\mathsf{P}(a \wedge b)$ boils down to.

By the law of total probability we have:

$$
\begin{aligned}
\mathsf{P}(a \wedge b) = {} & \mathsf{P}(a \wedge b|A \wedge B)\mathsf{P}(A \wedge B) + \\
& + \mathsf{P}(a \wedge b|A \wedge \neg B)\mathsf{P}(A \wedge \neg B)\\
& + \mathsf{P}(a \wedge b|\neg A \wedge B)\mathsf{P}(\neg A \wedge B) + \\
& + \mathsf{P}(a \wedge b|\neg A \wedge \neg B)\mathsf{P}(\neg A \wedge \neg B)
\end{aligned} \tag{6}
$$

which, when we substitute our values and constants, results in:

$$= 0.49p_{11} + 0.21(p_{10} + p_{01}) + 0.09p_{00}$$

Now, note that because p_{ii}s add up to one, we have $p_{10} + p_{01} = 1 - p_{00} - p_{11}$. Let us continue.

$$
\begin{aligned}
&= 0.49p_{11} + 0.21(1 - p_{00} - p_{11}) + 0.09p_{00}\\
&= 0.21 + 0.28p_{11} - 0.12p_{00}
\end{aligned}
$$

Next, we ask what the posterior of $A \wedge B$ given $a \wedge b$ is (in the last line, we also multiply the numerator and the denominator by 100).

$$
\begin{aligned}
\mathsf{P}(A \wedge B|a \wedge b) &= \frac{\mathsf{P}(a \wedge b|A \wedge B)\mathsf{P}(A \wedge B)}{\mathsf{P}(a \wedge b)}\\
&= \frac{49p_{11}}{21 + 28p_{11} - 12p_{00}}
\end{aligned}
$$

In this particular case, then, our question whether $\mathsf{P}(A \wedge B|a \wedge b) > \mathsf{P}(A \wedge B)$ boils down to asking whether

$$\frac{49p_{11}}{21 + 28p_{11} - 12p_{00}} > p_{11}$$

that is, whether $28 > 28p_{11} - 12p_{00}$ (just divide both sides by p_{11}, multiply by the denominator, and manipulate algebraically).

Dawid continues working with particular choices of values and provides neither a general statement of the fact that the above considerations instantiate nor a proof of it. In the middle of the paper he says:

> Even under prior dependence, the combined support is always positive, in the sense that the posterior probability of the case always exceeds its prior probability. . . When the problem is analysed carefully, the 'paradox' evaporates [pp. 95-7]

where he still means the case with the particular values that he has given, but he seems to suggest that the claim generalizes to a large array of cases.

The paper does not contain a precise statement making the conditions required explicit and, *a fortriori*, does not contain a proof of it. Given the example above and Dawid's informal reading, let us develop a more precise statement of the claim and a proof thereof.

Fact 1. *Suppose that $rel(a), rel(b) > 0.5$ and witnesses are independent conditional on all Boolean combinations of A and B (in a sense to be specified), and that none of the Boolean combinations of A and B has an extreme probability (of 0 or 1). It follows that* $\mathsf{P}(A \wedge B|a \wedge b) > \mathsf{P}(A \wedge B)$. *(Independence of A and B is not required.)*

Roughly, the theorem says that if independent and reliable witnesses provide positive support of their separate claims, their joint testimony provides positive support of the conjunction of their claims.

Let us see why the claim holds. First, we introduce an abbreviation for witness reliability:

$$\begin{aligned}\mathbf{a} &= rel(a) = \mathsf{P}(a|A) = \mathsf{P}(\neg a|\neg A) > 0.5\\ \mathbf{b} &= rel(b) = \mathsf{P}(b|B) = \mathsf{P}(\neg b|\neg A) > 0.5\end{aligned}$$

Our independence assumption means:

$$\begin{aligned}\mathsf{P}(a \wedge b|A \wedge B) &= \mathbf{ab}\\ \mathsf{P}(a \wedge b|A \wedge \neg B) &= \mathbf{a}(\mathbf{1}-\mathbf{b})\\ \mathsf{P}(a \wedge b|\neg A \wedge B) &= (\mathbf{1}-\mathbf{a})\mathbf{b}\\ \mathsf{P}(a \wedge b|\neg A \wedge \neg B) &= (\mathbf{1}-\mathbf{a})(\mathbf{1}-\mathbf{b})\end{aligned}$$

Abbreviate the probabilities the way we already did:

$$\begin{array}{ll}\mathsf{P}(A \wedge B) = p_{11} & \mathsf{P}(A \wedge \neg B) = p_{10}\\ \mathsf{P}(\neg A \wedge B) = p_{01} & \mathsf{P}(\neg A \wedge \neg B) = p_{00}\end{array}$$

Our assumptions entail $0 \neq p_{ij} \neq 1$ for $i, j \in \{0, 1\}$ and:

$$p_{11} + p_{10} + p_{01} + p_{00} = 1 \tag{7}$$

So, we can use this with (6) to get:

$$\begin{aligned}\mathsf{P}(a \wedge b) &= \mathbf{ab}p_{11} + \mathbf{a}(\mathbf{1}-\mathbf{b})p_{10} + (\mathbf{1}-\mathbf{a})\mathbf{b}p_{01} + (\mathbf{1}-\mathbf{a})(\mathbf{1}-\mathbf{b})p_{00}\\ &= p_{11}\mathbf{ab} + p_{10}(\mathbf{a}-\mathbf{ab}) + p_{01}(\mathbf{b}-\mathbf{ab}) + p_{00}(1-\mathbf{b}-\mathbf{a}+\mathbf{ab})\end{aligned} \tag{8}$$

Let's now work out what the posterior of $A \wedge B$ will be, starting with an application of the Bayes' Theorem:

$$\begin{aligned}\mathsf{P}(A \wedge B|a \wedge b) &= \frac{\mathsf{P}(a \wedge b|A \wedge B)\mathsf{P}(A \wedge B)}{\mathsf{P}(a \wedge b)}\\ &= \frac{\mathbf{ab}p_{11}}{p_{11}\mathbf{ab} + p_{10}(\mathbf{a}-\mathbf{ab}) + p_{01}(\mathbf{b}-\mathbf{ab}) + p_{00}(1-\mathbf{b}-\mathbf{a}+\mathbf{ab})}\end{aligned} \tag{9}$$

To answer our question we therefore have to compare the content of (9) to p_{11} and our claim holds just in case:

$$\frac{\mathbf{ab}p_{11}}{p_{11}\mathbf{ab} + p_{10}(\mathbf{a}-\mathbf{ab}) + p_{01}(\mathbf{b}-\mathbf{ab}) + p_{00}(1-\mathbf{b}-\mathbf{a}+\mathbf{ab})} > p_{11}$$

$$\frac{\mathbf{ab}}{p_{11}\mathbf{ab} + p_{10}(\mathbf{a}-\mathbf{ab}) + p_{01}(\mathbf{b}-\mathbf{ab}) + p_{00}(1-\mathbf{b}-\mathbf{a}+\mathbf{ab})} > 1$$

$$p_{11}\mathbf{ab} + p_{10}(\mathbf{a}-\mathbf{ab}) + p_{01}(\mathbf{b}-\mathbf{ab}) + p_{00}(1-\mathbf{b}-\mathbf{a}+\mathbf{ab}) < \mathbf{ab} \tag{10}$$

Proving (10) is therefore our goal for now. This is achieved by the following reasoning:[13]

[13] Thanks to Pawel Pawlowski for working on this proof with me.

1.	$\mathbf{b} > 0.5,\ \mathbf{a} > 0.5$	assumption
2.	$2\mathbf{b} > 1,\ 2\mathbf{a} > 1$	from 1.
3.	$2\mathbf{ab} > \mathbf{a},\ 2\mathbf{ab} > \mathbf{b}$	multiplying by **a** and **b** respectively
4.	$p_{10}2\mathbf{ab} > p_{10}\mathbf{a},\quad p_{01}2\mathbf{ab} > p_{01}\mathbf{b}$	multiplying by p_{10} and p_{01} respectively
5.	$p_{10}2\mathbf{ab} + p_{01}2\mathbf{ab} > p_{10}\mathbf{a} + p_{01}\mathbf{b}$	adding by sides, 3., 4.
6.	$1 - \mathbf{b} - \mathbf{a} < 0$	from 1.
7.	$p_{00}(1 - \mathbf{b} - \mathbf{a}) < 0$	From 6., because $p_{00} > 0$
8.	$p_{10}2\mathbf{ab} + p_{01}2\mathbf{ab} > p_{10}\mathbf{a} + p_{01}\mathbf{b} + p_{00}(1 - \mathbf{b} - \mathbf{a})$	from 5. and 7.
9.	$p_{10}\mathbf{ab} + p_{10}\mathbf{ab} + p_{01}\mathbf{ab} + p_{01}\mathbf{ab} + p_{00}\mathbf{ab} - p_{00}\mathbf{ab} > p_{10}\mathbf{a} + p_{01}\mathbf{b} + p_{00}(1 - \mathbf{b} - \mathbf{a})$	8., rewriting left-hand side
10.	$p_{10}\mathbf{ab} + p_{01}\mathbf{ab} + p_{00}\mathbf{ab} > -p_{10}\mathbf{ab} - p_{01}\mathbf{ab} + p_{00}\mathbf{ab} + p_{10}\mathbf{a} + p_{01}\mathbf{b} + p_{00}(1 - \mathbf{b} - \mathbf{a})$	9., moving from left to right
11.	$\mathbf{ab}(p_{10} + p_{01} + p_{00}) > p_{10}(\mathbf{a} - \mathbf{ab}) + p_{01}(\mathbf{b} - \mathbf{ab}) + p_{00}(1 - \mathbf{b} - \mathbf{a} + \mathbf{ab})$	10., algebraic manipulation
12.	$\mathbf{ab}(1 - p_{11}) > p_{10}(\mathbf{a} - \mathbf{ab}) + p_{01}(\mathbf{b} - \mathbf{ab}) + p_{00}(1 - \mathbf{b} - \mathbf{a} + \mathbf{ab})$	11. and equation (7)
13.	$\mathbf{ab} - \mathbf{ab}p_{11} > p_{10}(\mathbf{a} - \mathbf{ab}) + p_{01}(\mathbf{b} - \mathbf{ab}) + p_{00}(1 - \mathbf{b} - \mathbf{a} + \mathbf{ab})$	12., algebraic manipulation
14.	$\mathbf{ab} > \mathbf{ab}p_{11} + p_{10}(\mathbf{a} - \mathbf{ab}) + p_{01}(\mathbf{b} - \mathbf{ab}) + p_{00}(1 - \mathbf{b} - \mathbf{a} + \mathbf{ab})$	13., moving from left to right

%\end{adjustbox}

The last line is what we have been after.

OPTIONAL CONTENT ENDS

Now that we have as a theorem an explication of what Dawid informally suggested, let's see whether it helps the probabilist handling of DAC.

4.4 Kaplow

On RLP, at least in certain cases, the decision rule leads us to (14), which tells us to decide the case based on whether the likelihood ratio is greater than 1.

[14] While Kaplow did not discuss DAC or the gatecrasher paradox, it is only fair to evaluate Kaplow's proposal from the perspective of these difficulties.

Add here stuff from Marcello's Mind paper about the prisoner hypothetical. Then, discuss Rafal's critique of the likelihood ratio threshold and see where we end up.

4.5 p-value (Cheng?)

5 Challenges (again)

5.1 Likelihood ratio and the problem of the priors

5.2 Dawid's likelihood strategy doesn't help

Recall that DAC was a problem posed for the decision standard proposed by TLP, and the real question is how the information resulting from Fact 1 can help to avoid that problem. Dawid does not mention any decision standard, and so addresses quite a different question, and so it is not clear that 'theparadox' evaporates", as Dawid suggests.

What Dawid correctly suggests (and we establish in general as Fact 1) is that the support of the conjunction by two witnesses will be positive as soon as their separate support for the conjuncts is positive. That is, that the posterior of the conjunction will be higher that its prior. But the critic of probabilism never denied that the conjunction of testimonies might raise the probability of the conjunction if the testimonies taken separately support the conjuncts taken separately. Such a critic can still insist that Fact 1 does nothing to alleviate her concern. After all, at least *prima facie* it still might be the case that:

- the posterior probabilities of the conjuncts are above a given threshold,
- the posterior probability of the conjunction is higher than the prior probability of the conjunction,
- the posterior probability of the conjunction is still below the threshold.

That is, Fact 1 does not entail that once the conjuncts satisfy a decision standard, so does the conjunction.

At some point, Dawid makes a general claim that is somewhat stronger than the one already cited:

> When the problem is analysed carefully, the 'paradox' evaporates: suitably measured, the support supplied by the conjunction of several independent testimonies exceeds that supplied by any of its constituents.
> [p. 97]

[14] Again, the name of the view is by no means standard, it is just a term I coined to refer to various types of legal probabilism in a fairly uniform manner.

This is quite a different claim from the content of Fact 1, because previously the joint probability was claimed only to increase as compared to the prior, and here it is claimed to increase above the level of the separate increases provided by separate testimonies. Regarding this issue Dawid elaborates (we still use the p_{ij}-notation that we've already introduced):

> "More generally, let $\mathsf{P}(a|A)/\mathsf{P}(a|\neg A) = \lambda$, $\mathsf{P}(b|B)/\mathsf{P}(b|\neg B) = \mu$, with $\lambda, \mu > 0.7$, as might arise, for example, when there are several available testimonies. If the witnesses are independent, then
>
> $$\mathsf{P}(A \wedge B|a \wedge b) = \lambda\mu p_{11}/(\lambda\mu p_{11} + \lambda p_{10} + \mu p_{01} + p_{00})$$
>
> which increases with each of λ and μ, and is never less than the larger of $\lambda p_{11}/(1 - p_{11} + \lambda p_{11}), \mu p_{11}/(1 - p_{11}1 + \mu p_{11})$, the posterior probabilities appropriate to the individual testimonies." [p. 95]

This claim, however, is false.

Optional Content Starts

Let us see why. The quoted passage is a bit dense. It contains four claims for which no arguments are given in the paper. The first three are listed below as (11), the fourth is that if the conditions in (11) hold, $\mathsf{P}(A \wedge B|a \wedge b) > max(\mathsf{P}(A|a), \mathsf{P}(B|b))$. Notice that $\lambda = LR(a|A)$ and $\mu = LR(b|B)$. Suppose the first three claims hold, that is:

$$\begin{aligned}
\mathsf{P}(A \wedge B|a \wedge b) &= \lambda\mu p_{11}/(\lambda\mu p_{11} + \lambda p_{10} + \mu p_{01} + p_{00}) \qquad (11)\\
\mathsf{P}(A|a) &= \frac{\lambda p_{11}}{1 - p_{11} + \lambda p_{11}}\\
\mathsf{P}(B|b) &= \frac{\mu p_{11}}{1 - p_{11} + \mu p_{11}}
\end{aligned}$$

Is it really the case that $\mathsf{P}(A \wedge B|a \wedge b) > \mathsf{P}(A|a), \mathsf{P}(B|b)$? It does not seem so. Let $\mathbf{a} = \mathbf{b} = 0.6$, $pr = \langle p_{11}, p_{10}, p_{01}, p_{00}\rangle = \langle 0.1, 0.7, 0.1, 0.1\rangle$. Then, $\lambda = \mu = 1.5 > 0.7$ so the assumption is satisfied. Then we have $\mathsf{P}(A) = p_{11} + p_{10} = 0.8$, $\mathsf{P}(B) = p_{11} + p_{01} = 0.2$. We can also easily compute $\mathsf{P}(a) = \mathbf{a}\mathsf{P}(A) + (1 - \mathbf{a})\mathsf{P}(\neg A) = 0.56$ and $\mathsf{P}(b) = \mathbf{b}\mathsf{P}(B) + (1 - \mathbf{b})\mathsf{P}(\neg B) = 0.44$. Yet:

$$\begin{aligned}
\mathsf{P}(A|a) &= \frac{\mathsf{P}(a|A)\mathsf{P}(A)}{\mathsf{P}(a)} = \frac{0.6 \times 0.8}{0.6 \times 0.8 + 0.4 \times 0.2} \approx 0.8571\\
\mathsf{P}(B|b) &= \frac{\mathsf{P}(b|B)\mathsf{P}(B)}{\mathsf{P}(b)} = \frac{0.6 \times 0.2}{0.6 \times 0.2 + 0.4 \times 0.8} \approx 0.272\\
\mathsf{P}(A \wedge B|a \wedge b) &= \frac{\mathsf{P}(a \wedge b|A \wedge B)\mathsf{P}(A \wedge B)}{\begin{array}{l}\mathsf{P}(a \wedge b|A \wedge B)\mathsf{P}(A \wedge B) + \mathsf{P}(a \wedge b|A \wedge \neg B)\mathsf{P}(A \wedge \neg B) + \\ \quad + \mathsf{P}(a \wedge b|\neg A \wedge B)\mathsf{P}(\neg A \wedge B) + \mathsf{P}(a \wedge b|\neg A \wedge \neg B)\mathsf{P}(\neg A \wedge \neg B)\end{array}}\\
&= \frac{\mathbf{ab}p_{11}}{\mathbf{ab}p_{11} + \mathbf{a}(1 - \mathbf{b})p_{10} + (1 - \mathbf{a})\mathbf{b}p_{01} + (1 - \mathbf{a})(1 - \mathbf{b})p_{00}} \approx 0.147
\end{aligned}$$

The posterior probability of $A \wedge B$ is not only lower than the larger of the individual posteriors, but also lower than any of them!

So what went wrong in Dawid's calculations in (11)? Well, the first formula is correct. However, let us take a look at what the second one says (the problem with the third one is pretty much the same):

$$\mathsf{P}(A|a) = \frac{\frac{\mathsf{P}(a|A)}{\mathsf{P}(\neg a|A)} \times \mathsf{P}(A \wedge B)}{\mathsf{P}(\neg(A \wedge B)) + \frac{\mathsf{P}(a|A)}{\mathsf{P}(\neg a|A)} \times \mathsf{P}(A \wedge B)}$$

Quite surprisingly, in Dawid's formula for $\mathsf{P}(A|a)$, the probability of $A \wedge B$ plays a role. To see that it should not take any B that excludes A and the formula will lead to the conclusion that *always* $\mathsf{P}(A|a)$ is undefined. The problem with Dawid's formula is that instead of $p_{11} = \mathsf{P}(A \wedge B)$ he should have used $\mathsf{P}(A) = p_{11} + p_{10}$, in which case the formula would rather say this:

$$\begin{aligned}
\mathsf{P}(A|a) &= \frac{\frac{\mathsf{P}(a|A)}{\mathsf{P}(\neg a|A)} \times \mathsf{P}(A)}{\mathsf{P}(\neg A) + \frac{\mathsf{P}(a|A)}{\mathsf{P}(\neg a|A)} \times \mathsf{P}(A)} \\
&= \frac{\frac{\mathsf{P}(a|A)\mathsf{P}(A)}{\mathsf{P}(\neg a|A)}}{\frac{\mathsf{P}(\neg a|A)\mathsf{P}(\neg A)}{\mathsf{P}(\neg a|A)} + \frac{\mathsf{P}(a|A)\mathsf{P}(A)}{\mathsf{P}(\neg a|A)}} \\
&= \frac{\mathsf{P}(a|A)\mathsf{P}(A)}{\mathsf{P}(\neg a|A)\mathsf{P}(\neg A) + \mathsf{P}(a|A)\mathsf{P}(A)}
\end{aligned}$$

Now, on the assumption that witness' sensitivity is equal to their specificity, we have $\mathsf{P}(a|\neg A) = \mathsf{P}(\neg a|A)$ and can substitute this in the denominator:

$$= \frac{\mathsf{P}(a|A)\mathsf{P}(A)}{\mathsf{P}(a|\neg A)\mathsf{P}(\neg A) + \mathsf{P}(a|A)\mathsf{P}(A)}$$

and this would be a formulation of Bayes' theorem. And indeed with $\mathsf{P}(A) = p_{11} + p_{10}$ the formula works (albeit its adequacy rests on the identity of $\mathsf{P}(a|\neg A)$ and $\mathsf{P}(\neg a|A)$), and yields the result that we already obtained:

$$\begin{aligned}
\mathsf{P}(A|a) &= \frac{\lambda(p_{11} + p_{10})}{1 - (p_{11} + p_{10}) + \lambda(p_{11} + p_{10})} \\
&= \frac{1.5 \times 0.8}{1 - 0.8 + 1.5 \times 0.8} \approx 0.8571
\end{aligned}$$

The situation cannot be much improved by taking **a** and **b** to be high. For instance, if they're both 0.9 and $pr = \langle 0.1, 0.7, 0.1, 0.1\rangle$, the posterior of A is ≈ 0.972, the posterior of B is ≈ 0.692, and yet the joint posterior of $A \wedge B$ is 0.525.

The situation cannot also be improved by saying that at least if the threshold is 0.5, then as soon as **a** and **b** are above 0.7 (and, *a fortriori*, so are λ and μ), the individual posteriors being above 0.5 entails the joint posterior being above 0.5 as well. For instance, for $\mathbf{a} = 0.7$ and $\mathbf{b} = 0.9$ with $pr = \langle 0.1, 0.3, 0.5, 0.1\rangle$, the individual posteriors of A and B are ≈ 0.608 and ≈ 0.931 respectively, while the joint posterior of $A \wedge B$ is ≈ 0.283.

Optional Content Ends

The situation cannot be improved by saying that what was meant was rather that the joint likelihood is going to be at least as high as the maximum of the individual likelihoods, because quite the opposite is the case: the joint likelihood is going to be lower than any of the individual ones.

Optional Content Starts

Let us make sure this is the case. We have:

$$\begin{aligned}
LR(a|A) &= \frac{\mathsf{P}(a|A)}{\mathsf{P}(a|\neg A)} \\
&= \frac{\mathsf{P}(a|A)}{\mathsf{P}(\neg a|A)} \\
&= \frac{\mathbf{a}}{\mathbf{1-a}}.
\end{aligned}$$

where the substitution in the denominator is legitimate only because witness' sensitivity is identical to their specificity.

With the joint likelihood, the reasoning is just a bit more tricky. We will need to know what $\mathsf{P}(a \wedge b|\neg(A \wedge B))$ is. There are three disjoint possible conditions in which the condition holds: $A \wedge \neg B, \neg A \wedge B$, and $\neg A \wedge \neg B$. The probabilities of $a \wedge b$ in these three scenarios are respectively $\mathbf{a}(\mathbf{1-b}), (\mathbf{1-a})\mathbf{b}, (\mathbf{1-a})(\mathbf{1-b})$ (again, the assumption of independence is important), and so on the assumption $\neg(A \wedge B)$ the probability of $a \wedge b$ is:

$$\begin{aligned}
\mathsf{P}(a \wedge b | \neg(A \wedge B)) &= \mathbf{a}(1-\mathbf{b}) + (1-\mathbf{a})\mathbf{b} + (1-\mathbf{a})(1-\mathbf{b}) \\
&= \mathbf{a}(1-\mathbf{b}) + (1-\mathbf{a})(\mathbf{b}+1-\mathbf{b}) \\
&= \mathbf{a}(1-\mathbf{b}) + (1-\mathbf{a}) \\
&= \mathbf{a} - \mathbf{ab} + 1 - \mathbf{a} = 1 - \mathbf{ab}
\end{aligned}$$

So, on the assumption of witness independence, we have:

$$\begin{aligned}
LR(a \wedge b | A \wedge B) &= \frac{\mathsf{P}(a \wedge b | A \wedge B)}{\mathsf{P}(a \wedge b | \neg(A \wedge B))} \\
&= \frac{\mathbf{ab}}{\mathbf{1-ab}}
\end{aligned}$$

With $0 < \mathbf{a}, \mathbf{b} < 1$ we have $\mathbf{ab} < \mathbf{a}$, $1 - \mathbf{ab} > 1 - \mathbf{a}$, and consequently:

$$\frac{\mathbf{ab}}{\mathbf{1-ab}} < \frac{\mathbf{a}}{\mathbf{1-a}}$$

which means that the joint likelihood is going to be lower than any of the individual ones.

Optional Content Ends

Fact 1 is so far the most optimistic reading of the claim that if witnesses are independent and fairly reliable, their testimonies are going to provide positive support for the conjunction,\footnote{And this is the reading that Dawid in passing suggests: "the combined support is always positive, in the sense that the posterior probability of the case always exceeds its prior probability." (Dawid, 1987: 95) and any stronger reading of Dawid's suggestions fails. But Fact 1 is not too exciting when it comes to answering the original DAC. The original question focused on the adjudication model according to which the deciding agents are to evaluate the posterior probability of the whole case conditional on all evidence, and to convict if it is above a certain threshold. The problem, generally, is that it might be the case that the pieces of evidence for particular elements of the claim can have high likelihood and posterior probabilities of particular elements can be above the threshold while the posterior joint probability will still fail to meet the threshold. The fact that the joint posterior will be higher than the joint prior does not help much. For instance, if $\mathbf{a} = \mathbf{b} = 0.7$, $pr = \langle 0.1, 0.5, 0.3, 0.1 \rangle$, the posterior of A is ≈ 0.777, the posterior of B is ≈ 0.608 and the joint posterior is ≈ 0.216 (yes, it is higher than the joint prior $= 0.1$, but this does not help the conjunction to satisfy the decision standard).

To see the extent to which Dawid's strategy is helpful here, perhaps the following analogy might be useful.

Imagine it is winter, the heating does not work in my office and I am quite cold. I pick up the phone and call maintenance. A rather cheerful fellow picks up the phone. I tell him what my problem is, and he reacts:

— Oh, don't worry.

— What do you mean? It's cold in here!

— No no, everything is fine, don't worry.

— It's not fine! I'm cold here!

— Look, sir, my notion of it being warm in your office is that the building provides some improvement to what the situation would be if it wasn't there. And you agree that you're definitely warmer than you'd be if your desk was standing outside, don't you? Your, so to speak, posterior warmth is higher than your prior warmth, right?

Dawid's discussion is in the vein of the above conversation. In response to a problem with the adjudication model under consideration Dawid simply invites us to abandon thinking in terms of it and to abandon requirements crucial for the model. Instead, he puts forward a fairly weak notion of support (analogous to a fairly weak sense of the building providing improvement), according to which, assuming witnesses are fairly reliable, if separate fairly reliable witnesses provide positive support to the conjuncts, then their joint testimony provides positive support for the conjunction.

As far as our assessment of the original adjudication model and dealing with DAC, this leaves us hanging. Yes, if we abandon the model, DAC does not worry us anymore. But should we? And if we do, what should we change it to, if we do not want to be banished from the paradise of probabilistic methods?

Having said this, let me emphasize that Dawid's paper is important in the development of the debate, since it shifts focus on the likelihood ratios, which for various reasons are much better measures of evidential support provided by particular pieces of evidence than mere posterior probabilities.

Before we move to another attempt at a probabilistic formulation of the decision standard, let us introduce the other hero of our story: the gatecrasher paradox. It is against DAC and this paradox that the next model will be judged.

Optional Content Starts

In fact, Cohen replied to Dawid's paper (Cohen, 1988). His reply, however, does not have much to do with the workings of Dawid's strategy, and is rather unusual. Cohen's first point is that the calculations of posteriors require odds about unique events, whose meaning is usually given in terms of potential wagers – and the key criticism here is that in practice such wagers cannot be decided. This is not a convincing criticism, because the betting-odds interpretations of subjective probability do not require that on each occasion the bet should really be practically decidable. It rather invites one to imagine a possible situation in which the truth could be found out and asks: how much would we bet on a certain claim in such a situation? In some cases, this assumption is false, but there is nothing in principle wrong with thinking about the consequences of false assumptions.

Second, Cohen says that Dawid's argument works only for testimonial evidence, not for other types thereof. But this claim is simply false – just because Dawid used testimonial evidence as an example that he worked through it by no means follows that the approach cannot be extended. After all, as long as we can talk about sensitivity and specificity of a given piece of evidence, everything that Dawid said about testimonies can be repeated *mutatis mutandis*.

Third, Cohen complaints that Dawid in his example worked with rather high priors, which according to Cohen would be too high to correspond to the presumption of innocence. This also is not a very successful rejoinder. Cohen picked his priors in the example for the ease of calculations, and the reasoning can be run with lower priors. Moreover, instead of discussing the conjunction problem, Cohen brings in quite a different problem: how to probabilistically model the presumption of innocence, and what priors of guilt should be appropriate? This, indeed, is an important problem; but it does not have much to do with DAC, and should be discussed separately.

5.3 Problems with Cheng's relative likelihood

How is RLP supposed to handle DAC? Consider an imaginary case, used by Cheng to discuss this issue. In it, the plaintiff claims that the defendant was speeding (S) and that the crash caused her neck injury (C). Thus, H_Π is $S \wedge C$. Suppose that given total evidence E, the conjuncts, taken separately, meet the decision standard of RLP:

$$\frac{\mathsf{P}(S|E)}{\mathsf{P}(\neg S|E)} > 1 \qquad\qquad \frac{\mathsf{P}(C|E)}{\mathsf{P}(\neg C|E)} > 1$$

The question, clearly, is whether $\frac{\mathsf{P}(S \wedge C|E)}{H_\Delta|E} > 1$. But to answer it, we have to decide what H_Δ is. This is the point where Cheng's remark that H_Δ isn't normally simply $\neg H_\Pi$. Instead, he insists, there are three alternative defense scenarios: $H_{\Delta_1} = S \wedge \neg C$, $H_{\Delta_2} = \neg S \wedge C$, and $H_{\Delta_3} = \neg S \wedge \neg C$. How does H_Π compare to each of them? Cheng (assuming independence) argues:

$$\begin{aligned} \frac{\mathsf{P}(S \wedge C|E)}{\mathsf{P}(S \wedge \neg C|E)} &= \frac{\mathsf{P}(S|E)\mathsf{P}(C|E)}{\mathsf{P}(S|E)\mathsf{P}(\neg C|E)} = \frac{\mathsf{P}(C|E)}{\mathsf{P}(\neg C|E)} > 1 \\ \frac{\mathsf{P}(S \wedge C|E)}{\mathsf{P}(\neg S \wedge C|E)} &= \frac{\mathsf{P}(S|E)\mathsf{P}(C|E)}{\mathsf{P}(\neg S|E)\mathsf{P}(C|E)} = \frac{\mathsf{P}(S|E)}{\mathsf{P}(\neg S|E)} > 1 \\ \frac{\mathsf{P}(S \wedge C|E)}{\mathsf{P}(\neg S \wedge \neg C|E)} &= \frac{\mathsf{P}(S|E)\mathsf{P}(C|E)}{\mathsf{P}(\neg S|E)\mathsf{P}(\neg C|E)} > 1 \end{aligned} \tag{12}$$

It seems that whatever the defense story is, it is less plausible than the plaintiff's claim. So, at least in this case, whenever elements of a plaintiff's claim satisfy the decision standard proposed by RLP, then so does their conjunction.

Similarly, RLP is claimed to handle the gatecrasher paradox. It is useful to think about the problem in terms of odds and likelikoods, where the *prior odds* (before evidence E) of H_Π as compared to H_Δ, are $\frac{\mathsf{P}(H_\Pi)}{\mathsf{P}(H_\Delta)}$, the posterior odds of H_Δ given E are $\frac{\mathsf{P}(H_\Pi|E)}{\mathsf{P}(H_\Delta|E)}$, and the corresponding likelihood ratio is $\frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)}$.

Now, with this notation the *odds form of Bayes' Theorem* tells us that the posterior odds equal the likelihood ratio multiplied by prior odds:

$$\frac{\mathsf{P}(H_\Pi|E)}{\mathsf{P}(H_\Delta|E)} = \frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)} \times \frac{\mathsf{P}(H_\Pi)}{\mathsf{P}(H_\Delta)}$$

[@cheng2012reconceptualizing: 1267] insists that in civil trials the prior probabilities should be equal. Granted this assumption, prior odds are 1, and we have:

$$\frac{\mathsf{P}(H_\Pi|E)}{\mathsf{P}(H_\Delta|E)} = \frac{\mathsf{P}(E|H_\Pi)}{\mathsf{P}(E|H_\Delta)} \tag{13}$$

This means that our original task of establishing that the left-hand side is greater than 1 now reduces to establishing that so is the right-hand side, which means that RLP tells us to convict just in case:

$$\mathsf{P}(E|H_\Pi) > \mathsf{P}(E|H_\Delta) \tag{14}$$

Thus, (14) tells us to convict just in case $LR(E) > 1$.

Now, in the case of the gatecrasher paradox, our evidence is statistical. In our variant E=`991 out of 1000 spectators gatecrashed''. Now pick a random spectator, call him Tom, and let $H_\Pi$=`Tom gatecrashed." (Cheng, 2012: 1270) insists:

> But whether the audience member is a lawful patron or a gatecrasher does not change the probability of observing the evidence presented.

So, on his view, in such a case, $\mathsf{P}(E|H_\Pi) = \mathsf{P}(E|H_\Delta)$, the posterior odds are, by (13), equal to 1, and conviction is unjustified.

There are various issues with how RLP has been deployed to resolve the difficulties that CLP and TLP run into.

First of all, to move from (1) to (2), Cheng assumes that the costs of wrongful decision is the same, be it conviction or acquittal. This is by no means obvious. If a poor elderly lady sues a large company for serious health damage that it supposedly caused, leaving her penniless if the company is liable is definitely not on a par with mistakenly making the company lose a small percent of their funds. Even in cases where such costs are equal, careful consideration and separate argument is needed. If, for instance, $c_1 = 5c_2$, we are to convict just in case $5 < \frac{p_\Pi}{p_\Delta}$. This limits the applicability of Cheng's reasoning about DAC, because his reasoning, if correct (and I will argue that it is not correct later on), yields only the result that the relevant posterior odds are greater than 1, not that they are greater than 5. The difficulty, however, will not have much impact on Cheng's solution of the gatecrasher paradox, as long as $c_1 \leq c_2$. This is because his reasoning, if correct (and I will argue that it is not correct later on), establishes that the relevant posterior odds are below 1, and so below any higher threshold as well.

Secondly, Cheng's resolution of DAC uses another suspicious assumption. For (12) to be acceptable we need to assume that the following pairs of events are independent conditionally on E: $\langle S, C\rangle$, $\langle S, \neg C\rangle$, $\langle \neg S, C\rangle$, $\langle \neg S, \neg C\rangle$. Otherwise, Cheng would not be able to replace conditional probabilities of corresponding conjunctions with the result of multiplication of conditional probabilities of the conjuncts. But it is far from obvious that speeding and neck injury are independent. If, for instance, the evidence makes it certain that if the car was not speeding, the neck injury was not caused by the accident, $\mathsf{P}(\neg S \wedge C|E) = 0$, despite the fact that $\mathsf{P}(\neg S|E)\mathsf{P}(C|E)$ does not have to be 0!

Without independence, the best that we can get, say for the first line of (12), is:

$$\begin{aligned}\mathsf{P}(S \wedge C|E) &= \mathsf{P}(C|E)\mathsf{P}(S|C \wedge E)\\ \mathsf{P}(S \wedge \neg C|E) &= \mathsf{P}(\neg C|E)\mathsf{P}(S|\neg C \wedge E)\end{aligned}$$

and even if we know that $\mathsf{P}(C|E) > \mathsf{P}(\neg C|E)$, this tells us nothing about the comparison of $\mathsf{P}(S \wedge C|E)$ and $\mathsf{P}(S \wedge \neg C|E)$, because the remaining factors can make up for the former inequality.

Perhaps even more importantly, much of the heavy lifting here is done by the strategic splitting of the defense line into multiple scenarios. The result is rather paradoxical. For suppose $\mathsf{P}(H_\Pi|E) = 0.37$ and the probability of each of the defense lines given E is 0.21. This means that H_Π wins with each of the scenarios, so, according to RLP, we should find for the plaintiff. On the other hand, how eager are we to convict once we notice that given the evidence, the accusation is rather false, because $\mathsf{P}(\neg H_\Pi|E) = 0.63$?

The problem generalizes. If, as here, we individualize scenarios by boolean combinations of elements of a case, the more elements there are, into more scenarios $\neg H_\Pi$ needs to be divided. This normally would lead to the probability of each of them being even lower (because now $\mathsf{P}(\neg H_\Pi)$ needs to be "split" between more different scenarios). So, if we take this approach seriously, the more elements a case has, the more at disadvantage the defense is. This is clearly undesirable.

In the process of solving the gatecrasher paradox, to reach (13), Cheng makes another controversial assumption: that the prior odds should be one, that is, that before any evidence specific to the case is obtained, $\mathsf{P}(H_\Pi) = \mathsf{P}(H_\Delta)$. One problem with this assumption is that it is not clear how to square this with how Cheng handles DAC. For there, he insisted we need to consider *three different* defense scenarios, which we marked as $H_{\Delta_1}, H_{\Delta_2}$ and H_{Δ_3}. Now, do we take Cheng's suggestion to be that we should have

$$\mathsf{P}(H_\Pi) = \mathsf{P}(H_{\Delta_1}) = \mathsf{P}(H_{\Delta_2}) = \mathsf{P}(H_{\Delta_3})?$$

Given that the scenarios are jointly exhaustive and pairwise exclusive this would mean that each of them should have prior probability 0.25 and, in principle that the prior probability of guilt can be made lower simply by the addition of elements under consideration. This conclusion seems suboptimal.

If, on the other hand, we read Cheng as saying that we should have $\mathsf{P}(H_\Pi) = \mathsf{P}(\neg H_\Pi)$, the side-effect is that even a slightest evidence in support of H_Π will make the posterior probability of H_Π larger than that of $\neg H_\Pi$, and so the plaintiff can win their case way too easily. Worse still, if $\mathsf{P}(\neg H_\Pi)$ is to be divided between multiple defense scenarios against which H_Π is to be compared, then as soon as this division proceeds in a non-extreme fashion, the prior of each defense scenario will be lower than the prior of H_Π, and so from the perspective of RLP, the plaintiff does not have to do anything to win (as long as the defense does not provide absolving evidence), because his case is won without any evidence already!

Finally, let us play along and assume that in the gatecrasher scenario the conviction is justified just in case (14) holds. Cheng insists that it does not, because $\mathsf{P}(E|H_\Pi) = \mathsf{P}(E|H_\Delta)$. This supposedly captures the intuition that whether Tom paid has no impact on the statistics that we have.

But this is not obvious. Here is one way to think about this. Tom either paid the entrance fee or did not. Consider these two options, assuming nothing else about the case changes. If he did pay, then he is among the 9 innocent spectators. But this means that if he had not paid, there would have been 992 gatecrashers, and so E would be false (because it says there was 991 of them). If, on the other hand, Tom in reality did not pay (and so is among the 991 gatecrashers), then had he paid, there would have been only 990 gatecrashers and E would have been false, again!

So whether conviction is justified and what the relevant ratios are depends on whether Tom really paid. Cheng's criterion (14) results in the conclusion that Tom should be penalized if and only if he did not pay. But this does not help us much when it comes to handling the paradox, because the reason why we needed to rely on E was exactly that we did not know whether Tom paid.

If you are not buying into the above argument, here is another way to state the problem. Say your priors are $\mathsf{P}(E) = e$, $\mathsf{P}(H_\Pi) = \pi$. By Bayes' Theorem we have:

$$\begin{aligned}\mathsf{P}(E|H_\Pi) &= \frac{\mathsf{P}(H_\Pi|E)e}{\pi}\\ \mathsf{P}(E|H_\Delta) &= \frac{\mathsf{P}(H_\Delta|E)e}{1-\pi}\end{aligned}$$

Assuming our posteriors are taken from the statistical evidence, we have $\mathsf{P}(H_\Pi|E) = 0.991$ and $\mathsf{P}(H_\Delta|E) = 0.009$. So we have:

$$
\begin{aligned}
LR(E) &= \frac{\mathsf{P}(H_{\Pi}|E)e}{\pi} \times \frac{1-\pi}{\mathsf{P}(H_{\Delta}|E)e} \\
&= \frac{\mathsf{P}(H_{\Pi}|E) - \mathsf{P}(H_{\Pi}|E)\pi}{\mathsf{P}(H_{\Delta}|E)\pi} \\
&= \frac{0.991 - 0.991\pi}{0.009\pi}
\end{aligned}
\tag{15}
$$

and $LR(E)$ will be > 1 as soon as $\pi < 0.991$. This means that contrary to what Cheng suggested, in any situation in which the prior probability of guilt is less than the posterior probability of guilt, RLP tells us to convict. This, however, does not seem desirable.

5.4 Problem's with Kaplow's stuff

Kaplow does not discuss the conceptual difficulties that we are concerned with, but this will not stop us from asking whether DTLP can handle them (and answering to the negative). Let us start with DAC.

Say we consider two claims, A and B. Is it generally the case that if they separately satisfy the decision rule, then so does $A \wedge B$? That is, do the assumptions:

$$
\begin{aligned}
\frac{\mathsf{P}(E|A)}{\mathsf{P}(E|\neg A)} &> \frac{\mathsf{P}(\neg A)}{\mathsf{P}(A)} \times \frac{L}{G} \\
\frac{\mathsf{P}(E|B)}{\mathsf{P}(E|\neg B)} &> \frac{\mathsf{P}(\neg B)}{\mathsf{P}(B)} \times \frac{L}{G}
\end{aligned}
$$

entail

$$
\frac{\mathsf{P}(E|A \wedge B)}{\mathsf{P}(E|\neg(A \wedge B))} > \frac{\mathsf{P}(\neg(A \wedge B))}{\mathsf{P}(A \wedge B)} \times \frac{L}{G}?
$$

Alas, the answer is negative.

Optional Content Starts

This can be seen from the following example. Suppose a random digit from 0-9 is drawn; we do not know the result; we are told that the result is < 7 ($E =$'the result is < 7'), and we are to decide whether to accept the following claims:

A	the result is < 5.
B	the result is an even number.
$A \wedge B$	the result is an even number < 5.

Suppose that $L = G$ (this is for simplicity only — nothing hinges on this, counterexamples for when this condition fails are analogous). First, notice that A and B taken separately satisfy (5). $\mathsf{P}(A) = \mathsf{P}(\neg A) = 0.5$, $\mathsf{P}(\neg A)/\mathsf{P}(A) = 1$ $\mathsf{P}(E|A) = 1$, $\mathsf{P}(E|\neg A) = 0.4$. (5) tells us to check:

$$
\begin{aligned}
\frac{\mathsf{P}(E|A)}{\mathsf{P}(E|\neg A)} &> \frac{L}{G} \times \frac{\mathsf{P}(\neg A)}{\mathsf{P}(A)} \\
\frac{1}{0.4} &> 1
\end{aligned}
$$

so, following DTLP, we should accept A.
For analogous reasons, we should also accept B. $\mathsf{P}(B) = \mathsf{P}(\neg B) = 0.5$, $\mathsf{P}(\neg B)/\mathsf{P}(B) = 1$ $\mathsf{P}(E|B) = 0.8$, $\mathsf{P}(E|\neg B) = 0.6$, so we need to check that indeed:

$$
\begin{aligned}
\frac{\mathsf{P}(E|B)}{\mathsf{P}(E|\neg B)} &> \frac{L}{G} \times \frac{\mathsf{P}(\neg B)}{\mathsf{P}(B)} \\
\frac{0.8}{0.6} &> 1
\end{aligned}
$$

But now, $\mathsf{P}(A \wedge B) = 0.3$, $\mathsf{P}(\neg(A \wedge B)) = 0.7$, $\mathsf{P}(\neg(A \wedge B))/\mathsf{P}(A \wedge B) = 2\frac{1}{3}$, $\mathsf{P}(E|A \wedge B) = 1$, $\mathsf{P}(E|\neg(A \wedge B)) = 4/7$ and it is false that:

$$\frac{\mathsf{P}(E|A \wedge B)}{\mathsf{P}(E|\neg(A \wedge B))} > \frac{L}{G} \times \frac{\mathsf{P}(\neg(A \wedge B))}{\mathsf{P}(A \wedge B)}$$
$$\frac{7}{4} > \frac{7}{3}$$

The example was easy, but the conjuncts are probabilistically dependent. One might ask: are there counterexamples that involve claims which are probabilistically independent?[15]

Consider an experiment in which someone tosses a six-sided die twice. Let the result of the first toss be X and the result of the second one Y. Your evidence is that the results of both tosses are greater than one ($E =: X > 1 \wedge Y > 1$). Now, let A say that $X < 5$ and B say that $Y < 5$.

The prior probability of A is $2/3$ and the prior probability of $\neg A$ is $1/3$ and so $\frac{\mathsf{P}(\neg A)}{\mathsf{P}(A)} = 0.5$. Further, $\mathsf{P}(E|A) = 0.625$, $\mathsf{P}(E|\neg A) = 5/6$ and so $\frac{\mathsf{P}(E|A)}{\mathsf{P}(E|\neg A)} = 0.75$ Clearly, $0.75 > 0.5$, so A satisfies the decision standard. Since the situation with B is symmetric, so does B.

Now, $\mathsf{P}(A \wedge B) = (2/3)^2 = 4/9$ and $\mathsf{P}(\neg(A \wedge B)) = 5/9$. So $\frac{\mathsf{P}(\neg(A \wedge B))}{\mathsf{P}(A \wedge B)} = 5/4$. Out of 16 outcomes for which $A \wedge B$ holds, E holds in 9, so $\mathsf{P}(E|A \wedge B) = 9/16$. Out of 20 remaining outcomes for which $A \wedge B$ fails, E holds in 16, so $\mathsf{P}(E|\neg(A \wedge B)) = 4/5$. Thus, $\frac{\mathsf{P}(E|A \wedge B)}{\mathsf{P}(E|\neg(A \wedge B))} = 45/64 < 5/4$, so the conjunction does not satisfy the decision standard.

Optional Content Ends

Let us turn to the gatecrasher paradox.

Suppose $L = G$ and recall our abbreviations: $\mathsf{P}(E) = e$, $\mathsf{P}(H_\Pi) = \pi$. DTLP tells us to convict just in case:

$$LR(E) > \frac{1-\pi}{\pi}$$

From (15) we already now that

$$LR(E) = \frac{0.991 - 0.991\pi}{0.009\pi}$$

so we need to see whether there are any $0 < \pi < 1$ for which

$$\frac{0.991 - 0.991\pi}{0.009\pi} > \frac{1-\pi}{\pi}$$

Multiply both sides first by 009π and then by π:

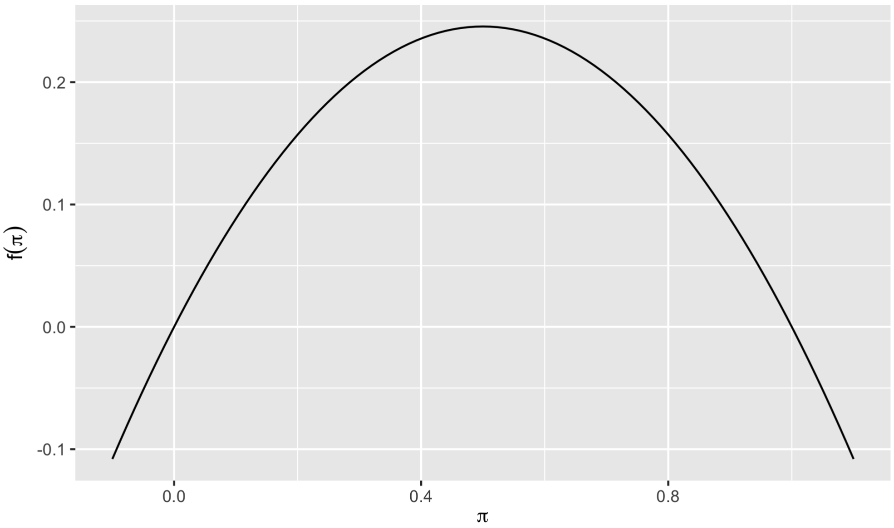
$$0.991\pi - 0.991\pi^2 > 0.09\pi - 0.009\pi^2$$

Simplify and call the resulting function f:

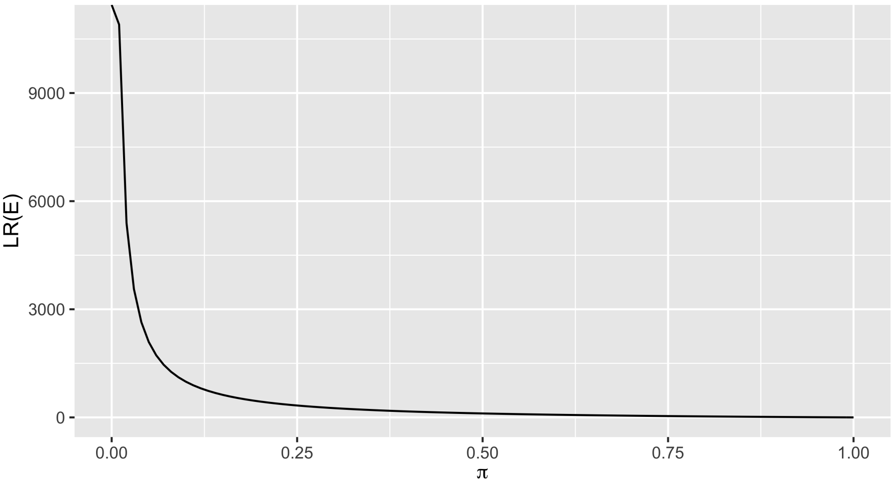
$$f(\pi) = -0.982\pi^2 + 0.982\pi > 0$$

The above condition is satisfied for any $0 < \pi < 1$ (f has two zeros: $\pi = 0$ and $\pi = 1$). Here is a plot of f:

[15] Thanks to Alicja Kowalewska for pressing me on this.



Similarly, $LR(E) > 1$ for any $0 < \pi < 1$. Here is a plot of $LR(E)$ against π:



Notice that $LR(E)$ does not go below 1. This means that for $L = G$ in the gatecrasher scenario DTLP wold tell us to convict for any prior probability of guilt $\pi \neq 0, 1$.

One might ask: is the conclusion very sensitive to the choice of L and G? The answer is, not too much.

Optional Content Starts

How sensitive is our analysis to the choice of L/G? Well, $LR(E)$ does not change at all, only the threshold moves. For instance, if $L/G = 4$, instead of f we end up with

$$f'(\pi) = -0.955\pi^2 + 0.955\pi > 0$$

and the function still takes positive values on the interval $(0, 1)$. In fact, the decision won't change until L/G increases to ≈ 111. Denote L/G as ρ, and let us start with the general decision standard, plugging in our calculations for $LR(E)$:

$$
\begin{aligned}
LR(E) &> \frac{\mathsf{P}(H_{\Delta})}{\mathsf{P}(H_{\Pi})}\rho \\
LR(E) &> \frac{1-\pi}{\pi}\rho \\
\frac{0.991-0.991\pi}{0.009\pi} &> \frac{1-\pi}{\pi}\rho \\
\frac{0.991-0.991\pi}{0.009\pi}\frac{\pi}{1-\pi} &> \rho \\
\frac{0.991\pi-0.991\pi^2}{0.009\pi-0.009\pi^2} &> \rho \\
\frac{\pi(0.991-0.991\pi)}{\pi(0.009-0.009\pi)} &> \rho \\
\frac{0.991-0.991\pi}{0.009-0.009\pi} &> \rho \\
\frac{0.991(1-\pi)}{0.009(1-\pi)} &> \rho \\
\frac{0.991}{0.009} &> \rho \\
110.1111 &> \rho
\end{aligned}
$$

Optional Content Ends

So, we conclude, in usual circumstances, DTLP does not handle the gatecrasher paradox.

6 Probabilistic Thresholds Revised

6.1 Likelihood ratios and naked statistical evidence

6.2 Conjunction paradox and Bayesian networks

7 STUFF FROM SEP

7.1 Something to use in motivations, early in the chapter?

The fallacies considered so far—base rate, prosecutor's, defense attorney's fallacy—show how the posterior probability can be misjudged, upwards or downwards.
The posterior probability of a hypothesis given the evidence %(whose correct assessment depends on the prior probability) should not be confused with the strength (or probative value, weight) of the evidence in favor of the hypothesis. To a rough approximation, the strength of an item of evidence reflects its impact on the posterior probability given a prior probability. Suppose the prior probability of H is extremely low, say $\mathsf{P}(()H) = 0.01\%$, but taking evidence E into account brings this probability up to 35%, that is, $\mathsf{P}(()H|E) = 35\%$. This is a dramatic upward shift. Even though the posterior probability of H given E is not very high, E strongly favors H. Conversely, suppose the prior probability of H is extremely high, say $\mathsf{P}(()H) = 99.9\%$, but taking evidence E into account brings this probability down to 75%, that is, $\mathsf{P}(()H|E) = 75\%$. This is a dramatic downward shift. Even though the posterior probability of H given E is still quite high, E speaks against H.

%Consider the blood stain example from **??**. The posterior probability given the match turned out to be an unimpressive 17% (assuming a 1% prior probability). But this does not mean the match was weak incriminating evidence. After taking it into account, the posterior probability rose from 1% (the stipulated prior) to 17% (the posterior). The match strongly favored the claim that the defendant was the source of the traces, but was not not strong enough to make it very likely. %Similarly, in the Collins case, the posterior probability jumped from the $1/6\ million$ prior to 70% after taking the match into account, still not enough for a conviction but a remarkable increase nonetheless.

7.2 Material on Bayes factor, LR and some examples

One measure of the strength of evidence is the Bayesian factor $\mathsf{P}(()E|H)/\mathsf{P}(()E)$. %The Bayesian factor is a probabilistic measure of the extent to which the evidence, regardless of the absolute posterior probability, supports or does not support the hypothesis. This is an intuitively plausible measure of evidential strength. Note that by Bayes' theorem % %

$$\mathsf{P}(()H|E) = \mathit{BayesianFactor}(H,E) \times \mathsf{P}(()H),$$

% and thus the Bayesian factor is greater than one if and only if the posterior probability $\mathsf{P}(()H|E)$ is higher than the prior probability $\mathsf{P}(()H)$. %$\mathsf{P}(()H) < \mathsf{P}(()H|E)$, . %So E *positively* supports H whenever the Bayesian factor is greater than one. The greater the Bayesian factor (for values above one), the greater the upward shift from prior to posterior probability, the more strongly E positively supports H. %The posterior probability of H given E could still be low even if the Bayesian factor is significantly above one. %Conversely, %again by Bayes' theorem, %the probability of H given E is lower than the probability of H %, $\mathsf{P}(()H) > \mathsf{P}(()H|E)$, %if and only if the Bayesian factor is less than one. %So E *negatively* supports H whenever the Bayesian factor is less than one. Conversely, the smaller the Bayesian factor (for values below one), the greater the downward shift from prior to posterior probability, the more strongly E negativey supports H. If $\mathsf{P}(()H) = \mathsf{P}(()H|E)$ the evidence has no impact, upwards or downwards, on the prior probability of H. %So long as the Bayesian factor is greater than one, the evidence supports the hypothesis. If it is negative, the evidence negatively supports the hypothesis. If it equals one, the evidence is irrelevant.

%There are important differences between the likelihood %ratio and the Bayesian factor as measures of the strength of evidence. %The likelihood ratio is alike the Bayesian factor $\mathsf{P}(()E|H)/\mathsf{P}(()E)$. They are both measures of the extent to which the evidence favors (or does not favor) a hypothesis of interest.

The Bayesian factor $\mathsf{P}(()E|H)/\mathsf{P}(()E)$ is an absolute measure of E's support toward H since it compares the probability of E under hypothesis H against the probability of E in general. The denominator is calculated following the law of total probability: %

$$\mathsf{P}(()E) = \mathsf{P}(()E|H)\mathsf{P}(()H) + \mathsf{P}(()E|\neg H)\mathsf{P}(()\neg H).$$

% The catch-all alternative hypothesis $\neg H$ can be replaced by a more fine-grained set of alternatives, say $H_1, H_2, \dots H_k$, provided H and its alternatives cover the entire space of possibilities. The law of total probability would then read: %

$$\mathsf{P}(()E) = \mathsf{P}(()E|H)\mathsf{P}(()H) + \sum_{i=1}^{k} \mathsf{P}(()E|H_i)\mathsf{P}(()H_i).$$

%, just like H and its negation.

Some might worry that assessing the strength of evidence in this way would impose too great a cognitive burden, since it would require sifting through the entire space of possibilities, a seemingly impossibly task. %This is is certainly true if $\mathsf{P}(()E)$ is calculated by applying the law of total probability. %But the law of total probability %does not mandate how $\mathsf{P}(()E)$ should be assessed. Rather, it places a coherence constraint on probability assignments. But this need not be the case. In some cases, it might be possible to assess $\mathsf{P}(()E)$ by considering a more manageable set of alternative hypotheses, say, only those hypotheses the litigants disagree about. So long as the hypotheses are justifiably believed to be mutually exclusive and jointly exhaustive, the reasoning goes through. Suppose H is the hypothesis put forward by the prosecutor and H_1 and H_2 are the only alternative hypotheses that the defense deems plausible. If neither side finds $H_3, H_4, \dots, H_k$ worthy of consideration, the prior probability of these hypotheses can be conveniently set to zero. The law of total probability would then simplify to a more manageable formula: %

$$\mathsf{P}(()E) = \mathsf{P}(()E|H)\mathsf{P}(()H) + \mathsf{P}(()E|H_1)\mathsf{P}(()H_1) + \mathsf{P}(()E|H_2)\mathsf{P}(()H_2).$$

% In this way, the litigants would no longer need to sift through the entire space of possibilities, but only through those that they deemed worth considering.

%, just like H and its negation. %

%This need to always be the case, however. %In fact, what one is epistemically required to do and why is not determined by the formula. Sometimes, direct assessment of the prior is easier than reaching it by LOTP, sometimes not. Moreover, it is never the case that to calculate $\mathsf{P}(()E)$ one needs to apply LOTP with respect to all possible lists of exclusive and exhaustive hypotheses, and considering just one sensible list for which the appropraite priors and conditional probabilities are available might be not such a huge burden. Crucialy, LOTP %and requires that $Pr(E)$ depends on weighted sum of $Pr(E|H)$ for evepossible hypotheses. This does not mean that, in each and every case, the asessnment of %$\mathsf{P}(()E)$ by looking the entire space of all possible hypothesis. Sometimes is may even be possible to assess $\mathsf{P}(()E)$ upfront. %Equally well, you can start with establishing $\mathsf{P}(()E)$ first—in such a case, LOTP will constraint the interaction of your conditional probabilities of type $\mathsf{P}(()E|H_i)$ and $\mathsf{P}(()H_i)$.

The task of assessing the strength of evidence can be simplified even further. Instead of deploying the Bayesian factor, the strength of evidence can be assessed by means of the likelihood ratio, a comparative measure of whether evidence E supports a hypothesis H more than a competing hypothesis H', in symbols, $\mathsf{P}(()E|H)/\mathsf{P}(()E|H')$. %a measure of the probability of the evidence under two hypotheses, H and H', . The likelihood ratio is a comparative measure of whether the evidence E supports a hypothesis H more than a competing hypothesis H', where the two hypotheses need not cover the entire space of possibilities. If the evidence supports H more than H', the ratio would be above one, and if the evidence supports H' more than H, the ratio would be below one. The greater the likelihood ratio (for values above one), the stronger the evidence in favor of H as contrasted with H'. The smaller the likelihood ratio (for values below one), the stronger the evidence in favor of the competing hypothesis H' as contrasted with H. The likelihood ratio is a simpler and more workable measure than the Bayesian factor, since it does not require one to think about the probability of the evidence in general, namely $\mathsf{P}(()E)$. This apparent simplicity, however, can often give rise to errors in the assessment of the evidence, especially if the two hypotheses are not chosen carefully (more on this below in **??**).

% %The likelihood ratio is used in the odd version of %Bayes's theorem (see earlier in **??**): %

Experts sometimes testify by offering the likelihood ratio as a measure of the strength of the evidence. An expert, for instance, may testify that the blood-staining on the jacket of the defendant is ten times more likely to be seen if the wearer of the jacket hit the victim (prosecutor's hypothesis) rather than if he did not (defense's hypothesis) (**?**, p. 38). %A similar probabilistic measure of the strength of the evidence is the Bayesian factor (see discussion in~**??**). Experts are typically advised not to comment on the posterior odds given the evidence. The relationship between likelihood ratio $\mathsf{P}(()E|H)/\mathsf{P}(()E|H')$ and posterior odds $\mathsf{P}(()H|E)/\mathsf{P}(()H'|E)$ is apparent in the odds version of Bayes' theorem (see earlier in~**??**): %

$$\frac{\mathsf{P}(()H|E)}{\mathsf{P}(()H'|E)} = \frac{\mathsf{P}(()E|H)}{\mathsf{P}(()E|H')} \times \frac{\mathsf{P}(()H)}{\mathsf{P}(()H')}.$$

% If the likelihood ratio is greater (lower) than one, the posterior odds will be greater (lower) than the prior odds of H. The likelihood ratio, then, is a measure of the upward or downward impact of the evidence on the odds of two hypotheses H and H'. %just like the Bayesian factor is a measure of upward or downward impact of the evidence on the probability of a hypothesis H. % %The difference is that the Bayesian factor require that E be assessed in relation to an exhaustive space of hypotheses, while %The likelihood ratio only requires that E be assessed relative to two %hypotheses which need not cover the entire space. % As Bayes's theorem makes clear, an assessment of the posterior odds will require a judgment about the prior odds, and the latter lies beyond the competence of an expert. A prominent forensic scientist recommends that experts 'not trespass on the province of the jury %by commenting directly on the accused's guilt or innocence, . . . and should generally confine their testimony to presenting the likelihood of their evidence under competing propositions' (**?**, p. 42).

R: removed the following bit in light of my addition about LOTP and in light of my addition about the prior

A potential competitor of the likelihood ratio as a measure of evidentiary strength is an even simpler notion, the probability $\mathsf{P}(()E|H)$. It is tempting to think that, whenever $\mathsf{P}(()E|H)$ is low, E should be strong evidence against H. %and in favor of an alternative hypothesis. Consider an example by **?**. In a child abuse case, the prosecutor offers evidence that a couple's child rocks and that only 3% of non-abused children rock, P(()child rocks|no abuse) = 3%. If it is unlikely that a non-abused child would rock, the fact that this child rocks might seem strong evidence of abuse. But this reading of the 3% figure is mistaken. It could well be that 3% of abused children rock, P(()child rocks|abuse) = 3%. %The two probabilities, P(()child rocks|abuse) and P(()child rocks|¬abuse), need not add up to 100%, and If rocking is unlikely under either hypothesis—which means the likelihood ratio P(()child rocks|abuse)/P(()child rocks|no abuse) equals one—rocking cannot count as evidence of abuse. Thus, in order to avoid exaggerations of the evidence, it is best to assess the evidence by means of the likelihood ratio rather the probability of the evidence given a hypothesis (**???**).

%This observation applies generally to all forms of evidence, inclusive of DNA evidence, although it might not always make a practical difference. %For suppose an expert testifies that the crime traces genetically %match the defendant and that the random match probability is extremely low, say 1 in 100 million. Is the match strong evidence that the defendant is the source of the traces? The random match probability—often interpreted as the probability that someone who is not the source would coincidentally match, P(()match|¬source)—is a common measure of the strength of a DNA match. The lower this probability, the more strongly incriminating the match. This is sensible because a low random match probability suggests it is unlikely two people could share the same DNA profile. Yet, strictly speaking, a match is strong evidence that the defendant is the source only if %the probability that the person who left the traces (the 'source') would match is significantly greater than the probability that a random person (someone who is not the source) would also match, or in other words, %Formally, if %P(()DNA match|¬source) is low, the DNA match is strong incriminating evidence only if P(()DNA match|source) is much higher. %only if %the likelihood ratio P(()DNA match|source)/P(()DNA match|¬source) is significantly greater than one. A low random match probability just means that the denominator is low. If the numerator is equally low, the match would be worthless evidence. % % This conceptual point, however, often does not make a difference in practice. The probability that someone who is the source would match, P(()match|source), should be high so long as the test has a low false negative rate. %Assuming P(()match|source) is high, % That P(()match|¬source) is low should be enough to ensure that the likelihood ratio is significantly above one. For practical purposes, then, a suitably low random match probability does count as strong incriminating evidence. But the conceptual point still stands.

7.3 Cold-hit

I would seriously consider having an extensive section illustrating the utility of LR with the case of cold hits, including the details that were commented out in SEP, I'm pasting this here, we'll discuss it tomorrow.

7.4 Cold-hit DNA matches

To better appreciate the theoretical virtues of likelihood ratios, it is instructive to look at a case study, DNA evidence, focusing in particular on so-called cold-hit matches. DNA evidence is one of the most widely used forms of quantitative evidence currently available.
It may be used to corroborate other evidence in a case or as the primary incriminating evidence. For example, suppose different investigative leads point to an individual, Mark Smith, as the perpetrator. The investigators also find several traces at the crime scene left by the perpetrator. Laboratory analyses show that the genetic profile associated with the traces matches Smith. In this scenario, the DNA match corroborates the other evidence against Smith. In contrast, suppose the police has no other investigative lead except the traces left at the crime scene. Hoping to find the perpetrator, the police run the genetic profile associated with the traces through a database of profiles and find a match, a so-called *cold-hit*. %—the individual with the matching profile can face trial and possibly a conviction. %When it is presented as evidence of guilt, a DNA match will often supplement other evidence, such as evidence from police investigation. %Consider, for example, the California murder case of Diana Sylvester. % There is a rich scholarly debate about cold-hit matches and how to correctly assess their probative value. Cold-hit DNA matches have been the focus of intense discussion in recent years. Since in cold-hit cases there is little or no other evidence, cold-hit matches are often the primary item of evidence against the defendant. Some believe that this circumstance weakens the match. Others disagree. This debate illustrates how probability theory—in particular, the likelihood ratio—can help to assess the strength of evidence at trial. What follows examines some of the main arguments.

%There is a rich scholarly debate about DNA matches and how to correctly assess their probative value. %This observation applies generally to all forms of evidence, inclusive of DNA evidence, although it might not always make a practical difference.

7.4.0.1 Random match v. database match

Suppose an expert testifies that the crime traces genetically match the defendant and that the random match probability is extremely low, say 1 in 100 million. %Is the match strong evidence that

the defendant is the source of the traces? The random match probability—often interpreted as the probability that someone who is not the source would coincidentally match, $\mathsf{P}(()\mathsf{match}|\neg\mathsf{source})$—is a common measure of the strength of a DNA match. The lower this probability, the more strongly incriminating the match. %The rationale here is that a low random match probability suggests that it is unlikely that two people would share the same DNA profile. But, strictly speaking, a match is strong evidence that the defendant is the source only if %the probability that the person who left the traces (the 'source') would match is significantly greater than the probability that a random person (someone who is not the source) would also match, or in other words, %Formally, if %$\mathsf{P}(()\mathsf{DNA\ match}|\neg\mathsf{source})$ is low, the DNA match is strong incriminating evidence only if $\mathsf{P}(()\mathsf{DNA\ match}|\mathsf{source})$ is much higher. %only if the likelihood ratio ${}^{\mathsf{P}(()\mathsf{DNA\ match}|\mathsf{source})}/_{\mathsf{P}(()\mathsf{DNA\ match}|\neg\mathsf{source})}$ is significantly greater than one. A low random match probability just means that the denominator is low, and if the numerator is equally low, the match would effectively be worthless evidence. % This point, albeit conceptually correct, often does not make a difference in practice.
%Assuming $\mathsf{P}(()\mathsf{match}|\mathsf{source})$ is high, That the random match probability is low—that is, $\mathsf{P}(()\mathsf{match}|\neg\mathsf{source})$ is low—should be enough to ensure that the likelihood ratio is significantly above one. After all, the probability that the individual who is the source would match, $\mathsf{P}(()\mathsf{match}|\mathsf{source})$, should be high so long as the test has a low false negative rate. For practical purposes, then, a low random match probability does count as strong incriminating evidence.

When it comes to cold-hit matches, however, further complications emerge. %The random match probability is no longer %an acceptable measure of the evidentiary value of the DNA match. To see what is at stake, the Puckett case can serve as an illustration. In 2008, John Puckett was identified through a database search of 338,000 profiles. He was the only individual in the database who matched the traces collected from Diana Sylvester, a victim of rape in 1972. The expert witness testified that % this particular pattern of alleles is (conservatively) expected to Puckett's genetic profile should occur randomly among Caucasian men with a frequency of 1 in 1.1 million. This would seem strong evidence against Puckett. %The ansere would be affirmativewould be the case with an ordinary DNA match? But the DNA expert for the defense, Bicka Barlow, pointed out that %this was a cold-hit case. besides the cold-hit match the evidence against Puckett was slim. It included Puckett's previous rape convictions, a *modus operandi* common to this and previous crimes, and the fact that Puckett was in the area. Barlow argued that the correct assessment of the cold-hit match required to multiply 1/1.1 million by the size of the database. Call the result of this multiplication the *database match probability*. Multiplying 1/1.1 million by 338,000 gives a database match probability of roughly 1/3, a rather unimpressive figure. % a less impressive number than 1 in 1.1 million. %%%If the random mat chrobability is 1/n and the database has size k, the multiplication would yield $1/n \times k$. %The result of this multiplication is the %the Database Match Probability. %According to this calculation, it was no longer very unlikely that a person from the database would match. If someone in the database could match with a probability as high as 1/3, the cold-hit match should no longer count as strong evidence against Puckett. At least, this was Barlow's argument.

%Barlow's assessment of the cold-hit match is by no means uncontroversial. Some argue it is correct and others that it is mistaken.

%

7.4.0.2 NRC II recommendations and their problems

Barlow followed a 1996 report by the National Research Council called NRC II % (**?**). This report was preceded by an earlier report on DNA evidence called NRC I (**?**). NRC II recommended that in cold-hit cases the random match probability %(RMP) should be multiplied by the size of the database, yielding the database match probability. %, precisely what Barlow did. %(DMP). % %The NRC formed the Committee on DNA Technology in Forensic Science, which issued its first report, NRC I, in 1992 . In that report they advised against using cold hit results as evidence, and insisted that only the frequencies related to loci not used in the original identification should be presented at trial. % %This recommendation has been criticized by many because it underestimates the value of cold-hit matches. Presumably, %As a consequence, the larger the size of the data, the higher the database match probability, the lower the strength of the match. This correction was meant to guard against the heightened risk of mistaken matches for the innocent people in the database. %As seen from the debate surrounding the Diana Sylvester case, one factor that can increase the confusion is that when we focus on the frequency of matches found in large databases, extremely low Random Match Probability

R: moved stuff to fn to avoid ambiguity.

might seem in stark contrast with fairly high frequency of matches found. % For instance, the Arizona Department of Public Safety searched for matching profiles in a database comprising 65,000 individuals. The search found 122 pairs of people whose DNA partially matched at 9 out of 13 loci; 20 pairs people who matched at 10 loci; and one pair of people who matches at 12 loci. So it is not that unlikely to find two people in a database who share the same genetic profiles. This argument was actually used by John Puckett's defense attorney in the Diana Sylvester case. % %NRC II offered two arguments in support of its recommendation, both of which have been criticized by **?**. %We'll only look briefly at the key issues. %One argument (let's call it the *frequentist* argument) had to do with Database Match Probability (DMP). The committee compared a database trawl to multiple hypothesis testing, which is not a good practice in light of classical statistical methods. NRC II used an analogy. %NRC explained the idea in terms of coin tosses: If you toss several different coins at once and all show heads on the first attempt, this seems strong evidence that the coins are biased. If, however, you repeat this experiment sufficiently many times, it is almost certain that at some point all coins will land heads. %you will encounter a toss in which all coins show heads. This outcome should not count as evidence that the coins are biased. According to NRC II, repeating the coin toss experiment multiple times is analogous to trying to find a match by searching through a database of profiles. As the size of the database increases, %and the number of attempts at finding a match also increases, it is more likely that someone in the database who had nothing to do with the crime would match. % %Let γ be the Random Match Probability (RMP). % % %

7.4.0.3 Shortcomings of the database match probability

It is unclear how the analogy with coin tossing translates to cold-hit cases (**?**). Searching a larger database no doubt increases the probability of finding a match at some point. But judges and jurors should not be concerned with %the probability of the statement %The NRC II recommendation pertained to the probability of the statement `At least one of the profiles in the database %of size d would randomly match the crime sample.' %Call it the general match hypothesis. %This statement has probability $(d+1)\times p$, where p is the random match probability and $d+1$ is the database size. \todo{Is this right? You have d+1. Why?} % %are interested in whether a particular defendant is the source of the crime traces. %They are concerned with the statement`The profile of the particular defendant on trial would randomly match the crime sample.'Call this the particular match hypothesis. %and the probability of at least one false positive. %So what is of interest is the probability of the particular, not the general match hypothesis. % This is in line with the general idea that one should update on total available evidence: the claim that the defendant's profile match entails but is not entailed by the claim that at least one of the database profiles match, and so it is the former and not the latter that the fact-finders should rely on. % They should be concerned with %the probability of the statement 'The profile of the defendant on trial would randomly match the crime sample.' The probability of finding a match between the defendant and the crime sample does not increase because other people in the database are tested. In fact, suppose everyone in the world is recorded in the database. A unique cold-hit match would then be extremely strong evidence of guilt since everybody would be excluded as a suspect except one matching individual. Instead, if the random match probability is multiplied by the size of the database, the probative value of the match in this case should be quite low. This is counter-intuitive.%

%Even without a world database, the NRC II proposal remains problematic since it sets up a way for defendant to arbitrarily weaken the weight of cold-hit DNA matches. It is enough to make more tests against more profiles in more databases. Even if all the additional profiles are excluded (intuitively, pointing even more clearly to the defendant as the perpetrator), the NRC II recommendation would require to devalue the cold-hit match even further. % [16] %Multiplication by database size would be appropriate if the matches excluded each other and thus were not independent. Suppose I toss

[16] The Database Match Probability is not a real probability either. Suppose a given profile frequency is 1/10 and one searches for this profile in a database of size 10. Does the probability of a match equal $1/10 \times 10 = 1$? Surely not. Assuming the matches are independence, this probability should be

$$\begin{aligned} 1 - \mathsf{P}(()\text{no match}) &= 1 - (9/10)^{10} \\ &= 1 - 0.3486784 \approx 0.65. \end{aligned}$$

a die, and the database contains $n =$ three *different* numbers: $1, 2$ and 3. Then, for each element of the database, the probability p of each particular match is $1/6$, and the probability of *a* match is $1/6 + 1/6 + 1/6 = 1/6 \times 3 = n \times p = 1/2$.}

%I could use the addition between in such a situation each match excludes other matches. I could still use $1 - \mathsf{P}(()\text{no match})$ to calculate the same probability, but I can't calculate $\mathsf{P}(()\text{no match})$ by taking it to be $5/6 \times 5/6 \times 5/6 = 5/6^3 \approx 0.58$, because the multiplication here requires independence, which is missing (if my die result is not a 3, it's more likely to be one of the other numbers).}

%A database that contained everybody however, is %a far-fetched possibility. At least, the NRC II recommendation could very well apply to more limited databases. Leaving coin tossing aside, another analogy has been used to argue that the evidentiary value of a cold-hit match should be weakened. The analogy is between searching for a match in a database and multiple hypothesis testing, which is a dubious research practice. In classical hypothesis testing, if the probability of type I error in a single test of a hypothesis is 5%, this probability will increase by testing the same hypothesis multiple times. The database match probability—the argument goes—would correct for the increased risk of type I error. This analogy with multiple testing, however, is misplaced. As **?** points out, multiple testing consists in testing the *same* hypothesis multiple times against new evidence. In cold-hit cases, there is no such multiple testing. %there is no multiple hypothesis testing involved here. %Rather that the hypothesis about the suspect being formulated after the test, Rather, multiple hypotheses—each concerning a different individual in the database—are tested only once and then excluded if a negative match occurs. %before the search multiple individual hypotheses about each of potential suspects in the database are formulated, and each of them is tested once during the search. From this perspective, the hypothesis that the defendant is the source was one of the many hypotheses subject to testing. The cold-hit match supports that hypothesis and rules out the others.

Perhaps, there is a better analogy here (**?**, p. 950). Imagine a biased coin whose physical appearance is indistinguishable from the fair coins in a piggy bank. The biased coin is the perpetrator and the piggy bank is the database containing innocent people. After the biased coin is thrown into the bank with the other coins, someone picks out a handful of coins at random and flips each of them twenty times. Each coin lands heads approximately ten times—except for one coin, which lands heads on all twenty flips. The fact that other coins seem unbiased makes the claim that this one is biased
better supported since at least one coin in the bank must be biased. % % Contrary to NRC II, **?** argue that if potential suspects in the database are excluded as sources, this should increase, not decrease, the probability that the defendant who matches the crime traces is the source. A cold-hit match, then, is stronger and not weaker evidence of guilty than ordinary DNA matches.

7.4.0.4 The likelihood ratio of cold-hit matches

%NRC II made another recommendation. They recommended that in cold-hit cases the likelihood ratio R associated with the DNA match should be divided by the size of the database $d + 1$. This recommendation, too, is questionable. Suppose R is not too high, say because the identified profile is common since the crime scene DNA is degraded and only a few markers could be used. Then, $d + 1$ can be greater than R, so $R/(d + 1) < 1$. The match would then be exculpatory, a very counter-intuitive result. The recommendation seems mistaken on more general grounds, as well. If the defendant on trial is the source, the probability that he would match is practically 1. If he is not, the probability that he would still match equals the random match probability. Neither of these probabilities change because other suspects have been tested in the database search. In fact, if potential suspects are excluded as potential sources,this should increases, not decrease, the probability that the defendant who matches the crime traces is the source.

A more principled way to assess cold-hit matches, one based on the likelihood ratio, exists. The proposal draws from the literature on the so-called *island problem*, studied by **?**, **?**, and **?**. Let the prosecutor's hypothesis H_p be `The suspect is the source of the crime traces' and the defense's hypothesis $H_d$ be`The suspect is not the source of the crime traces'. Let E be the DNA match between the crime stain and the suspect (included in the database) and D the information that no one among the $N - 1$ profiles in the database matches the crime stain. The likelihood ratio associated with E and D should be (**??**): %

$$V = \frac{\mathsf{P}(()E, D|H_p)}{\mathsf{P}(()E, D|H_d)}.$$

% Since $\mathsf{P}(()A \wedge B) = \mathsf{P}(()A|B)\mathsf{P}(()B)$, for any statement A and B, this ratio can be written as %

$$V = \frac{\mathsf{P}(()E|H_p, D)}{\mathsf{P}(()E|H_d, D)} \times \frac{\mathsf{P}(()D|H_p)}{\mathsf{P}(()D|H_d)}.$$

% The first ratio $\mathsf{P}(()E|H_p,D)/\mathsf{P}(()E|H_d,D)$ is roughly $1/\gamma$, where γ is the random match probability. The second ratio $\mathsf{P}(()D|H_p)/\mathsf{P}(()D|H_d)$— call it the *database search ratio*—requires some more work. Consider first the denominator $\mathsf{P}(()D|H_d)$. If the suspect is not the source (H_d), someone else is, either someone who is in the database or someone not in the database. Let S stand for 'The source is someone in the database.' By the law of total probability, %

$$\mathsf{P}(()D|H_d) = \mathsf{P}(()D|S, H_d)\mathsf{P}(()S|H_d) + \mathsf{P}(()D|\neg S, H_d)\mathsf{P}(()\neg S|H_d).$$

% If the source is someone in the database (S) and the suspect is not the source (H_d), it is very unlikely that no one in the database would match (D). %the probability of D that no one other than the suspect would match can be set to zero. So $\mathsf{P}(()D|S, H_d) \approx 0$. %The above equality therefore simplifies to %

$$\mathsf{P}(()D|H_d) = \mathsf{P}(()D|\neg S, H_d)\mathsf{P}(()\neg S$$

The database search ratio would therefore be: %

$$\frac{\mathsf{P}(()D|H_p)}{\mathsf{P}(()D|H_d)} = \frac{\mathsf{P}(()D|H_p)}{\mathsf{P}(()D|\neg S, H_d)\mathsf{P}(()\neg S|H_d)}.$$

% Note that $\mathsf{P}(()D|H_p) = \mathsf{P}(()D|\neg S, H_d)$ because whether the suspect is the source (H_p) or not (H_d) does not affect whether there is a match in a database that does not contain the source ($\neg S$).% %and let —the probability that no person in the database other than the suspect would match (D), assuming the suspect was in fact the source—be ψ_{N-1}. %Notice that $\mathsf{P}(()D|\neg S, H_d)$ % is the probability that no one other than the suspect matches in the database that does not contain the real source, if the suspect is not the source. But , so this conditional probability can also be estimated as ψ_{N-1}. [17] Let $\mathsf{P}(()S|H_d) = \varphi$. The database search ratio then would reduce to %

$$\frac{\mathsf{P}(()D|H_p)}{\mathsf{P}(()D|H_d)} = \frac{1}{1-\varphi}.$$

% As the database gets larger, φ increases and the database search ratio also increases. This ratio equals one only if no one in the database could be the source, that is, $\varphi = 0$.

Since the likelihood ratio V of the cold-hit match results by multiplying the likelihood ratio of the DNA match and the database search ratio, V will always be greater than the mere likelihood ratio of the match (except for the unrealistic case in which $\varphi = 0$) . Thus, a cold-hit DNA match should count as stronger evidence than a DNA match of a previously identified suspect. **?** study different database search strategies and consider the possibility that information about the match is itself uncertain, but the general point remains. Under reasonable assumptions, ignoring the database search would give a conservative assessment of the evidentiary strength of the cold-hit match.[18]

This proposal is able to accommodate different competing intuitions. First, consider the intuition that as the size of the database grows, it is more likely that someone in the database would match. This

[17] If the prior probability that the perpetrator is in the database was high, the calculations would need to be different. But normally, this prior isn't too high.

[18] **?**, with slightly different assumptions, derived the formula $R \times [1 + mD/N]$, where $R = 1/\gamma$, D is the database size, N the number of people in population not in database, and m is an optional multiplier reflecting how much more likely persons in the database are though to be the source when compared to the rest of the population. The expression cannot be less than γ. If no other profile has been tested, $D = 0$ and LR is simply the regular DNA match LR. If N is zero, that is, everyone in population is in the database, the result is infinitely large.

intuition is captured by the fact that φ increases proportionally to the size of the database even though this increase does not imply that the evidential value of the cold-hit match should decrease. Second, there is intuitive resistance in basing a conviction on a cold-hit match, although this resistance is less strong in case of an ordinary match (more on this later in Section **??**). This preference for convictions based on an ordinary DNA match seems in tension with the claim that a cold-hit match is stronger evidence of guilt than an ordinary match. There is a way to reconcile both sides, however. The key is to keep in mind that the evidentiary strength—measured by the likelihood ratio—should not be confused with the posterior probability of guilt given the evidence. %Even if a cold-hit match is stronger evidence of guilty, this fact does not imply that the posterior probability of the defendant's guilt should be higher. If the cold-hit match is the only evidence of guilt, the posterior probability of guilt may well be lower compared to cases in which other evidence, such as investigative leads, supplements the DNA match. This lower posterior probability would justify the intuitive resistance towards convictions in cold-hit cases despite the stronger probative value of cold-hit matches in themselves.

%Think about the following scenario: first, you identified the suspect by some means. Then, it turned out his DNA profile matches the crime scene stain. Fine. Now, imagine further database search for a database not containing this suspect finds no matches. Would you think that this information supports the guilt hypothesis? If your answer is yes, then you do have the intuition that the lack of matches with other people (whose profiles, in this particular case, happen to be in a database) strengthens the evidence.

%What about the intuition that we'd be uneasy about a conviction based solely on a cold hit match, much less than about a DNA-match based conviction of a previously and independently identified suspect? We think so because now we're thinking about the posterior probability given total evidence.

As the preceding discussion shows, the likelihood ratio is a fruitful conceptual framework for assessing the strength of the evidence, even in complex cases such as cold-hits. One major difficulty, however, is the choice of the hypotheses H and H' that should be compared. Generally speaking, the hypotheses should in some sense compete with one another—say, in a criminal trial, H is the hypothesis put forward by the prosecution and H' is the hypothesis put forward by the defense. % Presumably, the two hypotheses should be something that the two parties disagree about. But this minimal constraint offers too little guidance and leaves open the possibility for manipulations and misinterpretations of the evidence. What follows outlines some of the main arguments in the literature on this topic.

slightly modified the last sentence

7.4.0.5 Ad hoc hypotheses and Barry George

Consider a stylized DNA evidence case. Suppose the prosecutor puts forward the hypothesis that the suspect left the traces found at the crime scene. This hypothesis is well supported by laboratory analyses showing that the defendant genetically matches the traces. The defense, however, responds by putting forward the following *ad hoc* hypothesis: 'The crime stain was left by some unknown person who happened to have the same genotype as the suspect.' Since the probability of the DNA match given either hypothesis is 1, the likelihood ratio equals 1 (**?**). The problem generalizes. For any item of evidence and any given prosecutor's hypothesis H, there is an *ad hoc* competing hypothesis H^* such that $\mathsf{P}(()E|H)/\mathsf{P}(()E|H^*) = 1$. Hypothesis H^* is a just-so hypothesis, one that is selected only because it explains the evidence just as well as hypothesis H does (**?**). % If no further constraints are placed on the choice of the competing hypotheses—it would seem—no evidence could ever incriminate a defendant. This is unsettling. %But this conclusion need not be so damning in practice. Judges and jurors, however, will often recognize *ad hoc* hypotheses for what they are—artificial theories that should not be taken seriously. Perhaps, the reasonable expectations of the participants in a trial will suffice to constrain the choice of hypotheses in just the right way. At the same time, real cases can be quite complex, and it is not always obvious whether a certain choice of competing hypotheses, which are not obviously *ad hoc*, is legitimate or not.

%

7.4.0.6 Barry George

%Even when the competing hypotheses are not obviously *ad hoc*, %the absence of a clear rationale for their choice %may create confusions in the assessment of the evidence.

A notable example is R. v. Barry George (2007 EWCA Crim 2722). Barry George was accused of murdering TV celebrity Jill Dando. % %\begin{center} %\begin{tabular}{lp{12cm}} % E &
A single particle of firearm residue %(FDR) was found one year later in George's coat pocket and

it matched the residue from the crime scene. This was the key incriminating evidence against him. %\end{tabular} %\end{center} % %The defense argued that, since it was only one particle, there must have been contamination. The experts for the prosecution, however, testified that it was not unusual that a single particle would be found on the person who fired the gun. George was convicted, and his first appeal was unsuccessful. % After the first appeal, Dr. Evett from the Forensic Science Service worried that the evidence had not been properly assessed at trial. The jurors were presented with the conditional probability %P(()residue|H_d) of finding the firearm residue in George's coat given the defense hypothesis %H_d that George *did not* fire the gun. This probability was estimated to be quite low, indicating that the evidence spoke against the defense's hypothesis. But the jurors were not presented with the conditional probability %P(()residue|H_p) of finding the same evidence given the prosecutor's hypothesis %H_p that George *did* fire the gun that shot Dando. % %\begin{center} % \begin{tabular}{lp{12cm}} %H_d & BG did not fire the gun that shot JD.\ %H_p & BG fired the gun that shot JD. %\end{tabular} %\end{center} % An expert witness, Mr. Keeley, was asked to provide both conditional probabilities and estimated them to be 1/100, which indicated that the firearm residue had no probative value. %After new guidelines for reporting low level FDR in 2006, the FSS re-assessed the evidence and concluded that it was irrelevant. George appealed again in 2007, and relying on Keely's estimates, won the appeal. %

At first, this case seems a good illustration of how likelihood ratios help to correctly asses the value of the evidence presented at trial. But this reading of the case would be overly optimistic. In fact, a close study of the trial transcript shows that Keeley's choice of hypotheses lacked coherence and the likelihood ratio based on them was therefore meaningless (**?**). % % For instance, Mr Keeley is reported to have said: % %\begin{quote} % It was necessary to balance the likelihood that the particle came from a gun fired by the appellant and the likelihood that it came from some other source. Both were unlikely but both were possible. %\end{quote} %% On one occasion, Keeley compared the hypothesis that the particle found in George's pocket came from a gun fired by George himself, and the alternative hypothesis that the particle came from another source. %At the same time, Keeley said that the prior probabilities of both hypotheses should be low, which is mathematically impossible if they were exhaustive and exclusive.

On another occasion, Keeley took the prosecutor's hypothesis to be The particle found in George's pocket came from the gun that killed Dando'. But the conditional probability of the evidence given this hypothesis should not be low. It should actually be one. %\dots %\begin{quote} %\dots Mr Keeley gave to us, this was an equally unlikely event, whether it had come from the cartridge that killed Miss Dando, or from some innocent source. %\end{quote} % %\noindent Note that here the persecution hypothesis is taken to be: \emph{the particle found in BG's pocket came from the gun that killed JD}. Now, E is a logical consequence of this hypothesis, and so this likelihood should be 1. In some other contexts, Keeley took the defense hypothesis to beThe particle on George's pocket was inserted by contamination', but again, the conditional probability of the evidence given this hypothesis should be one. The most charitable reading of the trial transcript suggests that the expert had in mind the hypotheses George was the man who shot Dando' andThe integrity of George's coat was corrupted'. But %these hypotheses are neither exhaustive nor exclusive, and Keeley gave no clear criterion for why these hypotheses should be compared in the likelihood ratio (see **?**, for further details).

7.4.0.7 Exclusive and exhaustive?

The confusion in the Barry George case is attributable to the absence of clear rules for choosing the hypotheses in the likelihood ratio. One such rule could be: pick competing hypotheses that are exclusive (they cannot be both true) and exhaustive (they cannot be both false). In this way, the parties would not be able to pick *ad hoc* hypotheses and skew the assessment of the evidence in their own favor. % %One of the competing hypotheses in the likelihood ratio does not have to be the negation of the other. In some cases, they could both be false (hence, not exhaustive), and in some they could both be true (hence, not mutually exclusive).

% Besides blocking partisan interpretations of the evidence, there are other principled reasons to follow the exclusive-and-exhaustive rule, specifically, the fact that when the hypotheses are not exclusive or exhaustive, the likelihood ratio delivers counterintuitive results. %andconfusions in the assessment of the strength of the evidence often arise. % %but it is not obvious that the hypotheses selected should always be exclusive and exhaustive. %Whether this rule would be a good guiding principle, however, is not clear-cut. %

If two competing hypotheses %H_p and H_d are not mutually exclusive, it is possible that %the evidence supports them to an equal extent % they both make the evidence equally likely (the likelihood ratio is one), and yet the posterior probabilities of the hypotheses given the evidence are higher than their prior probabilities. For instance, let H_p stand for The defendant is guilty' and H_d forThe defendant was not at the crime scene'. Both hypotheses might be true. %Say the prior probability of both is 50%. Let E stand for Ten minutes before the crime took place the defendant---seen at a different location--- was overheard on the phone saying \emph{go ahead and kill him}.' %E supports both H_p and H_d, and It is conceivable that the likelihood ratio should equal one in this context, yet the posterior probabilities of each hypothesis, given E, should be higher than the prior probability. So, intuitively, the evidence should positively support each hypothesis, contrary to what the likelihood ratio would suggest. % Further, when the two competing hypotheses are not exhaustive, the likelihood ratio may once again clash with our intuitions. %The likelihood ratio might then equal one even though the evidence lowers their posterior probability. For example, suppose Fred and Bill attempted to rob a man. The victim resisted, was struck on the head and died. Say H_p stand forFred struck the fatal blow' and H_d stand for Bill struck the fatal blow.' The hypotheses are not exhaustive. A missing hypothesis isThe man did not die from the blow.' Suppose E is the information that the victim had a heart attack six months earlier. The likelihood ratio $\mathsf{P}(()E|H_p)/\mathsf{P}(()E|H_d)$ equals one since $\mathsf{P}(()E|H_p) = \mathsf{P}(()E|H_d)$. Yet E reduces the probability of both H_p and H_d. So, in this case, the evidence should negatively support each hypothesis, contrary to what the likelihood ratio suggests.

This was wrong. Fixed.

Despite these problems, however, always relying on exclusive and exhaustive hypotheses is not without complications either. For consider an expert who decides to formulate the defense hypothesis by negating the prosecution hypothesis, say, the defendant did not hit the victim in the head.' This choice of defense hypothesis can be unhelpful in assessing the evidence because the required probabilities are hard to estimate. For instance, what is the probability that the suspect would carry such and such blood stain if he did not hit the victim in the head? This depends on whether he was present at the scene, what he was doing at the time and many other circumstances. %Similarly, in a rape case, it is hard to estimate the probability of the matching evidence if the suspect did not have the intercourse with the victim. Instead, what is considered is the hypothesis that someone else, unrelated to the suspect, had intercourse with the victim. As \citet{evett2000MoreHierarchyPropositions} point out, the choice of a particular hypothesis to be used in the evaluation of the strength of the evidence %of, say, the lack of semen in a rape case, will depend on contextual factors. %Sometimes it will be \emph{intercourse did not take place}, sometimes it will be \emph{the intercourse took place, but the complainant used a vagina douche}, or sometimes \emph{another sexual act took place}. More often than not, the hypotheses chosen will not be mutually exclusive. % Comparing exclusive and exhaustive hypotheses can also be unhelpful for jurors or judges making a decision at trial. %is not required for the application of Bayes' Theorem, is not the usual practice, and could lead to a In a paternity case, for example, % in which the results of DNA profiling are to be evaluated, the expert should not compare the hypothesesThe accused is the father of the child' and its negation, but rather, The accused is the father of the child' andThe father of the child is a man unrelated to the putative father' (**?**). The choice of the latter pair of competing hypotheses is preferable. Even though the relatives of the accused are potential fathers, considering such a far-fetched possibility would make the assessment of the evidence more difficult than needed %Similarly, for a given DNA mixture, strictly speaking for any n there is a hypothesis that covers the suspect and n other unkown sources, but for sufficiently large n these are usually discarded as unrealistic. % %At the same time, if the defense hypothesis is too specific, *ad hoc* and entails the evidence, it won't be of much use. For example, take `The crime stain was left by some unknown person who happened to have the same genotype as the suspect.' %The probability of a DNA match given this hypothesis would be 1. %But usually the probability of the DNA match given the prosecution's hypothesis, say`The crime stain was left by the suspect,' is also 1. This would result in a rather uninformative likelihood ratio of 1 %For example, consider the prosecution hypothesis The probability of a match given this hypothesis is practically 1.% Another feature of such specific explanations is that it's hard to reasonably estimate their prior probability, and so hard to use them in arguments between opposing sides. (**?**).

7.4.0.8 Variability

Any choice of competing hypotheses lies between two extremes. For one thing, exclusive and exhaustive hypotheses guard against arbitrary comparisons and ensure a more objective assessment of the evidence. The drawback is that exhaustive and exclusive hypothesis cover the entire space of possibilities, and sifting through this space is cognitively unfeasible. So, in this respect, comparing more circumscribed hypotheses is preferable. The danger of doing so, however, is slipping into arbitrariness as likelihood ratios heavily depend on the hypotheses that are compared. The more latitude in the choice of the hypotheses, the more variable the likelihood ratio as a measure of evidentiary value. %It is hard not to view this variability as a sign of arbitrariness. %% R; this last sentence was too much

Here is a particularly troubling %example phenomenon. % Competing hypotheses can concern any factual dispute, from minute details such as whether the cloth used to suffocate the victim was red or blue, to ultimate questions such as whether the defendant stabbed the victim.

changed, because this wasn't really an example.

As it turns out, the likelihood ratio varies across hypotheses formulated at different levels of granularity: offense, activity and source level hypotheses (on this distinction, see earlier in **??**). It is even possible that, at the source level, the likelihood ratio favors one side, say the prosecution, but at the offence level, the likelihood ratio favors the other side, say the defense, even though the hypotheses at the two levels are quite similar. Further, a likelihood ratio that equals 1 when source level hypotheses are compared may tip in favor of one side or the other when offence level hypotheses are compared (**?**). This variability makes the likelihood ratio a seemingly arbitrary—and easily manipulable—measure of evidentiary value. This is not to say that it is unhelpful. Expert witnesses often rely on the likelihood ratio when they assess the probative value of many forms of evidence (**?**).

% The likelihood ratio can be misleading, but this risk is mitigated when its assessment is accompanied by a careful discussion of a number of issues, such as: which hypotheses are being compared; how they are formulated; their level of granularity (that is, source, activity and offense level); why the hypotheses are (or are not) exclusive and exhaustive; why other hypotheses are ruled out as unworthy of consideration.

%the choice of the hypotheses to be compared, their level, their being exclusive and exhaustive (or not), the formulation of the evidential statement, and the reasons not to use other hypotheses in the evidence evaluation.

% %M: Great, nice summary. I changed a few things here and there

Revised this paragraph a bit, take a look again.

7.5 Even more stuff on cold hits commented out completely at the end of the long SEP entry

%

7.6 Cold-hit DNA matches

%When it is presented as evidence of guilt, a DNA match will often supplement other evidence, such as eyewitness testimony or evidence from police investigation. At times, however, the DNA match will be the only evidence against the defendant. This happens when the police has no other investigative lead except the traces left at the crime scene. The genetic profile associated with the traces can be run through a database of profiles. If the trawl yields a match—a cold-hit—the individual with the matching profile could face trial and possibly a conviction. % %Consider, for example, the California murder case of Diana Sylvester. In 2008, John Puckett was identified through a database search of 338,000 profiles. He was the only individual in the database who matched the traces collected from Diana Sylvester, a victim of rape in 1972. %unique match to a semen DNA profile collected from a 1972 victim. %According to an expert witness, % this particular pattern of alleles is (conservatively) expected to %Puckett's genetic profile %should occur randomly among Caucasian men with a frequency of 1 in 1.1 million. This is the Random Match Probability. Although 1 in 1.1 million should not be confused with the probability of Puckett's innocence (see **??** for details), the small figure indicates it is very unlikely that a random person unrelated to the crime would match. The match is therefore strong evidence of Puckett's guilt. During the pretrial hearing, however, Bicka Barlow, the DNA expert for the defense, pointed out that %this was a cold-hit case. %no evidence tied Puckett to the crime other than the cold-hit match, Puckett's previous rape convictions and the fact that he was in the area at the time of the murder. In order to correctly assess the probative value of the cold-hit match, Barlow argued, the random match probability should %be multiplied by the size of the database. Call the result of this multiplication the *database match probability*. In Puckett's case, the multiplication of 1/1.1 million by 338,000 resulted in %a database match probability of roughly 1/3. % a less impressive number than 1 in 1.1 million. %%%If the random mat chrobability is 1/n and the database has size k, the multiplication would yield $1/n \times k$. %The result of this multiplication is the %the Database Match Probability. %According to this calculation, it was no longer very unlikely that a person from the database would match. %If someone in the database like Puckett could match with a probability as high as 1/3, the cold-hit DNA match was no longer strong evidence of guilt. At least, this was Barlow's argument. % %This interpretation of cold-hit matches is by no means controversial. Many argue it is mistaken. % %There is a rich scholarly debate about cold-hit matches and how to correctly assess their probative value. This section examines this debate and how probability theory can help address it.

%Suppose a single crime stain has been submitted to a DNA typing, and the DNA profile of an independently apprehended suspect has been obtained. If the profiles match, what is the evidential value of this match? %which would change the odds of an innocent person being a match in Puckett's case from 1 in 1.1 million to ≈ 1/3. The example illustrates that it is rather easy to be confused about the value of a cold hit DNA evidence. How should RMP and DMP be used, if they are to be used at all?

%

7.7 NRC II recommendations and their problems

%Barlow's testimony was in agreement with a 1996 report by the National Research Council, often called NRC II (**?**). This report should be distinguished from an earlier one called NRC I (**?**). NRC II recommended that in cold-hit cases the random match probability %(RMP) %should be multiplied by the size of the database, yielding the database match probability. %, precisely what Barlow did. %(DMP). % %The NRC formed the Committee on DNA Technology in Forensic Science, which issued its first report, NRC I, in 1992 . In that report they advised against using cold hit results as evidence, and insisted that only the frequencies related to loci not used in the original identification should be presented at trial. % %This recommendation has been criticized by many because it underestimates the value of cold-hit matches. Presumably, %As a consequence, the larger the size of the data, the higher the database match probability, the lower the strength of the match. % This correction is meant to guard against the heightened risk of mistaken matches for people in the database. As the size of the database increases, %and the number of attempts at finding a match also increases, % it is more likely that someone in the database who had nothing to do with the crime could mistakenly match. %As seen from the debate surrounding the Diana Sylvester case, one factor that can increase the confusion is that when we focus on the frequency of matches found in large databases, extremely low Random Match Probability might seem in stark contrast with fairly high frequency of matches found. % For instance, the Arizona Department of Public Safety searched for matching profiles in a database comprising 65,000 individuals. The search found 122 pairs of people whose DNA partially matched at 9 out of 13 loci; 20 pairs people who matched at 10 loci; and one pair of people who matches at 12 loci. So it is not that unlikely to find two people in a database who share the same genetic profiles. This argument was actually used by John Puckett's defense attorney in the Diana Sylvester case. % %NRC II offered two arguments in support of its recommendation, both of which have been criticized by **?**. %We'll only look briefly at the key issues. %One argument (let's call it the *frequentist* argument) had to do with Database Match Probability (DMP). The committee compared a database trawl to multiple hypothesis testing, which is not a good practice in light of classical statistical methods. %To make this point, NRC II used an analogy. %NRC explained the idea in terms of coin tosses: %If you toss several different coins at once and all show heads on the first attempt, this seems strong evidence against the hypothesis that the coins are fair. If, however, you repeat this experiment sufficiently many times, it is almost certain that at some point all coins will land heads. %you will encounter a toss in which all coins show heads. This outcome should not count as evidence that the coins are biased. % %Let γ be the Random Match Probability (RMP). % %

%It is unclear, however, how the analogy translates to cold-hit cases. Searching a larger database no doubt increases the probability of finding a match at some point. But judges and jurors should not be concerned with %the probability of the statement %The NRC II recommendation pertained to the probability of the statement %`At least one of the profiles in the database %of size d would randomly match the crime sample.' %Call it the general match hypothesis. %This statement has probability $(d+1)\times p$, where p is the random match probability and $d+1$ is the database size. \todo{Is this right? You have d+1. Why?} % %are interested in whether a particular defendant is the source of the crime traces. %They are concerned with the statement`The profile of the particular defendant on trial would randomly match the crime sample.'Call this the particular match hypothesis. %and the probability of at least one false positive. %So what is of interest is the probability of the particular, not the general match hypothesis. % This is in line with the general idea that one should update on total available evidence: the claim that the defendant's profile match entails but is not entailed by the claim that at least one of the database profiles match, and so it is the former and not the latter that the fact-finders should rely on. % %Rather, they should be concerned with %the probability of the statement 'The profile of the defendant on trial would randomly match the crime sample.' The probability of finding a match between the particular defendant and the crime sample does not increase because other people in the database are tested. In fact, suppose everyone in the world is recorded in the database. In this case, a unique cold-hit match would be extremely strong evidence of guilt since everybody is excluded as a suspect except one matching individual. But if the random match probability is multiplied by the size of the database, the probative value of the match should be quite low. This is counter-intuitive.
%Even without a world database, the NRC II proposal remains problematic since it sets up a way for defendant to arbitrarily weaken the weight of cold-hit DNA matches. It is enough to make more tests against more profiles in more databases. Even if all the additional profiles are excluded (intuitively, pointing even more clearly to the defendant as the perpetrator), the NRC II recommendation would require to devalue the cold-hit match even further. %

%NRC II was concerned with the fact that in cold-hit cases the identification of a particular defendant occurs after testing several individuals. This concern has to do with the data-dependency of one's hypothesis. But, as **?** points out, the problem of data-dependency arises when the same hypothesis is tested multiple times against new evidence. In cold-hit cases, however, there is no multiple testing. %there is no multiple hypothesis testing involved here. %Rather that the hypothesis about the suspect being formulated after the test, %Rather, multiple hypotheses—each concerning a different individual in the database—are tested only once and are excluded by a negative match. %before the search multiple individual hypotheses about each of potential suspects in the database are formulated, and each of them is tested once during the search. %From this perspective, the hypothesis that the defendant is the source was one of the many hypotheses subject to testing. The cold-hit match supports that hypothesis and rules out the others.

%Perhaps, there is a better analogy here %(**?**, p. 950). Imagine a coin that is known to be biased but whose physical appearance makes it indistinguishable from the fair coins in a piggy bank. The biased coin is the perpetrator and the %piggy bank is the database containing innocent people. %After the biased coin is thrown into the bank with the other coins, someone picks out a handful of coins at random and flips each of them twenty times. Each coin lands heads approximately ten times—except for one coin, which lands heads on all twenty flips. The fact that other coins seem unbiased makes the claim that this one is biased better supported since at least one coin in the bank must be biased.% %\footnote{Notice that the Database Match Probability is not really a probability. Just to take a simple example, suppose a given profile frequency is $ %1/10$ and you search for this particular profile in a database of size 10.

Does the probability of a match equal $1/10 \times 10 = 1$? No. Assuming the independence of no match for the members of the database, it is

$$\begin{aligned} 1 - \mathsf{P}(()\text{no match}) &= 1 - (9/10)^{10} \\ &= 1 - 0.3486784 \approx 0.65 \end{aligned}$$

%Multiplication by database size would make sense if we thought of it as addition of individual match probabilities, *provided matches exclude each other* (and so, are not independent). Suppose I toss a die, and my database contains n = three *different* numbers: $1, 2$ and 3. Then, for each element of the database, the probability p of each particular match is $1/6$, and the probability of *a* match is $1/6 + 1/6 + 1/6 = 1/6 \times 3 = n \times p = 1/2$. I could use the addition between in such a situation each match excludes other matches. I could still use $1 - \mathsf{P}(()\text{no match})$ to calculate the same probability, but I can't calculate $\mathsf{P}(()\text{no match})$ by taking it to be $5/6 \times 5/6 \times 5/6 = 5/6^3 \approx 0.58$, because the multiplication here requires independence, which is missing (if my die result is not a 3, it's more likely to be one of the other numbers).} % %Contrary to NRC II, **?** argue that if potential suspects in the database are excluded as sources, this should increase, not decrease, the probability that the defendant who matches the crime traces is the source. A cold-hit match, then, is stronger and not weaker evidence of guilty than ordinary DNA matches.

% %This can be made clear by relying on the likelihood ratio as a measure of the evidential strength of the cold-hit DNA match. If the defendant on trial is the source, the probability that he would match is practically 1. If he is not, the probability that he would still match equals the random match probability. Neither of these probabilities change because other suspects have been tested in the database search. In fact, if potential suspects are excluded as potential sources, %this should increases, not decrease, the probability that the defendant who matches the crime traces is the source.[19] % %Since the focus should not be on the particular, not the general match hypothesis, the coin tossing analogy does not work. %In a DNA trawl you're not repeating an experiment to reject a general hypothesis. Rather, you consider a particular hypothesis (*the defendant is the source of the material*) and test once for *the defendant's profile matches the crime sample*, and once for each claim of type *person i in the database does not have a matching profile*.

%moved this bit to a footnote

%NRC II made another recommendation. They recommended that in cold-hit cases the likelihood ratio R associated with the DNA match should be divided by $d + 1$. Their first recommendation was about a correction of the random match probability (that is, to multiply it by the size of the database), and this second recommendation is about the likelihood ratio (that is, to divide it by $d + 1$). This has counterintuitive consequences. Suppose R is not too high, say because the identified profile is common since the crime scene DNA is degraded and only a few markers could be used. Then, $d + 1$ can be greater than R, so $R/(d + 1) < 1$. The match would be exculpatory, a very counter-intuitive result.

%Moreover, NRC II would entail that if the police pick a random person from the database, test their profile and it is a match (call this *lucky strike*)and don't test other members of the database, this evidence — if we follow NRC II — should be stronger than a unique hit in a systematic database trawl, which also does not seem too convincing. % %Interestingly, the frequentist argument focused on a different hypothesis than the likelihood one. The former assessed the probability that at least one profile in the database would match the crime sample by chance, the latter focused on the hypothesis that exactly one profile would match. %In the case of frequency, multiplication by $D + 1$ results in a conservative and imprecise estimate, while in the case of likelihood, division by $D + 1$ is supposed to give a precise calculation of the likelihood of a cold hit. %

% %emphasized that, in cold-hit cases, the likelihood ratio should taken into account that the suspect is identified only after the search not before. If this fact is not taken into account the likelihood ratio would be data dependent. % %who argued that the hypotheses proposed by the critics of NRC II are data dependent, because the suspect is identified only after the search (see Subsection **??** for further details). %

%Commented this out, perhaps, this is not needed

%(3 CT 669, 675; 5 CT 1093.)11

[19] Of course, a classical statistician might refuse to rely on Bayesian reasoning, and rely on classical hypothesis testing instead. There are various good reasons not to do so, but we'll just point out that in such a case they simply have nothing to say about the posterior probability of the identity claim.

% How do we square the supposedly extremely low DNA profile frequency with the seemingly high match frequency in empirical studies on existing databases? The key difference is between the probability of a particular previously selected profile matching one in the database, and the probability of the database containing *some* match. % [MARCELLO'S COMMENTS: YES, YOU MADE THIS POINT EARLIER WHEN YOU CRITICIZED THE USE OF DMP, SO I DON'T THINK THIS SHOULD BE REPEATED HERE. MAYBE YOU CAN JUST CLARIFY WHAT YOU SAID EARLIER WHEN YOU DISTINGUISHED DMP FROM RMP?]

% Continuing with the die example, say I pick number one as my `suspect,'' and my database has $n=5$ with individual`profile frequency in the population" of $1/6$. On the assumption that the database is randomly drawn from the population (with replacement), the probability of one of the numbers in the database being 1 is $\approx 1 - 0.6 = 0.4$ (notice this time the database can contain repetitions and the sampling was random and so we can't use addition to calculate P(()no match with 1)). % [MARCELLO'S COMMENT: I REALLY LOST YOU HERE. NOW YOU SEEM TO BE ADDRESSING A DIFFERENT PROBLEM THAN COLD-HIT MATCHES.]

% In contrast, there are 6^5 possible states of the database, and $6 \times 5 \times 4 \times 3 \times 2$ of them contain no repetitions, so P(()no match) is ≈ 0.093 and the probability of *a* matching pair being present in the database is ≈ 0.91. Intuitively, the difference arises because now we're searching through $\binom{5}{2} = 10$ different pairs looking for any of the 6 possible profile matches, instead of searching through 5 profiles looking exactly for one profile.[20] % [MARCELLO'S COMMENTS: OK, SO HERE YOU SEEM TO BE ADDRESSING ANOTHER PROBLEM, SUCH AS BIRTHDAY PROBLEM AND RELATED. I WONDER IF THIS IS ANOTHER PROBLEM, NOT IMMEDIATELY HAVING TO DO WITH COLD-HIT DNA MATCHES. CAN THIS GO INTO A SEPARATE SUBSECTION? THAT MIGHT HELP ORIENT THE READER AND NOT MAKE THIS ONE SUBSECTION SO LONG.]

%

7.8 Sensitivity of LR to hypothesis choice and NRC II

%This approach has also been defended defended by %**?** who points out that since the suspect is identified after the databases search, the hypothesis is formulated *ad hoc*. Without the correction, then, the likelihood ratio would be data-dependent.

%Recalling the discussion of sensitivity of the likelihood ratio to the choice of hypotheses, let's take a look at an interesting attempt to defend NRC II by **?**. He insists that hypotheses such as *JS was one of the crime scene donors* are evidence-dependent in the case of database search, "since we had no way of knowing prior to the search that Smith would be the person that matched" (p. 672). Instead, Stockmarr insists, we should evaluate LR using hypotheses that can be formulated prior to the search, such as *the true perpetrator is among the suspects identified from the database*. And indeed, the likelihood of this hypothesis is as NRC II suggests, k/np, where k is the number of matching profiles, n the database size, and p the random match probability (see **?**, for a derivation).

%Dawid, in a discussion with Stockmarr (**?**) points out that Stockmarr's hypotheses, while not depending on the result of the search, depend on the data themselves (because they change with the database size). More importantly, he also indicates that Stockmarr's hypotheses are composite and the assessment of LR therefore requires additional assumptions about the priors. Once these are used with Stocmarr's own LR, the posterior is the same as the one obtained using the methods proposed by the critics of NCR II. The phenomenon is a particular case of the one we already discussed, because Stockmarr's hypothesis and the one originally used in the database search problem are equivalent conditional on the evidence. This turns out to be another example of how LR on its own might be insufficiently informative, especially when it is unclear what the involved hypotheses are.[21]

%

7.9 Resolving the Database Search Problem

%So we know not to use DMP. But the question remains: what's the relation between the value of a DNA match in a cold hit case as compared to a DNA match for a previously and independently identified

[20] This is a reformulation of the well-known birthday problem, it's just we discussed it using smaller numbers to make the considerations more intuitively compelling.

[21] See however, Stockmarr's own reply (*ibidem*).

suspect? After all we need to have a justified stance on cold hit evidence, and there seems to be no hope of reaching one without the tools provided by legal probabilism.

%Various versions of probabilistic calculations have been developed, depending on what idealizations and additional complexities are being considered. %In order to better understand how to properly assess a cold-hit match, it is instructive to consider a simple scenario, the *island problem*, studied by **?**, **?**, and **?**.

M: Nowhere do you talk about the island problem. Did you delete something here?

%%Let H_p stand for `The suspect is the source of the crime traces' and $H_d$ for`The suspect is not the source of the crime traces'. Let evidence E be the DNA match between the crime stain and the suspect (included in the database) and D the information that no one among the $N-1$ profiles in the database matches the crime stain. The likelihood ratio associated with E and D should be (**??**): %

$$V = \frac{\mathsf{P}(()E, D|H_p)}{\mathsf{P}(()E, D|H_d)}$$

% %Since $\mathsf{P}(()A \wedge B) = \mathsf{P}(()A|B)\mathsf{P}(()B)$, for any statement A and B, this ratio can be written as %

$$V = \frac{\mathsf{P}(()E, |H_p, D)}{\mathsf{P}(()E|H_d, D)} \times \frac{\mathsf{P}(()D|H_p)}{\mathsf{P}(()D|H_d)}$$

% %The ratio $\mathsf{P}(()E,|H_p,D)/\mathsf{P}(()E|H_d,D)$ is roughly $1/\gamma$, where γ is the random match probability. The ratio $\mathsf{P}(()D|H_p)/\mathsf{P}(()D|H_d)$ is called the *database search ratio*. To determine this ratio, consider first the denominator $\mathsf{P}(()D|H_d)$. If the suspect is not the source, someone else is, either someone who is in the database or someone not in the database. Let S stand for ‘The source is someone in the database.’ By the law of total probability, %

$$\mathsf{P}(()D|H_d) = \mathsf{P}(()D|S, H_d)\mathsf{P}(()S|H_d) + \mathsf{P}(()D|\neg S,$$

% %Note that if the source is someone in the database (S) and the suspect is not the source (H_d), it is nearly impossible that no one in the database would match (D). %the probability of D that no one other than the suspect would match can be set to zero. % So $\mathsf{P}(()D|S, H_d) \approx 0$. The above equation %therefore simplifies to: %

Is this right? I elaborated what you had. Please check!

$$\mathsf{P}(()D|H_d) = \mathsf{P}(()D|\neg S, H_d)\mathsf{P}(()\neg S|H_d)$$

% %Let $\mathsf{P}(()D|H_p)$ %be ψ_{N-1}. %Notice that $\mathsf{P}(()D|\neg S, H_d)$ %is the probability of a random match in a database of size $N-1$ not containing the the real source, and can also be estimated as ψ_{N-1}. If $\mathsf{P}(()S) = \varphi$, then the second ratio in our calculation of V therefore reduces to: %

I don’t follow the reasoning. Can you clarify? Why do these two probabilities equal the same thing?

$$\frac{\psi_{N-1}}{\psi_{N-1}(1-\varphi)} = \frac{1}{1-\varphi}$$

% %As the database gets larger, φ increases and the database search ratio also increases. This ratio equals one only if no one in the database could be the source, that is, $\varphi = 0$.

%Since in order to obtain the likelihood ratio V, the likelihood ratio of the DNA match is multiplied by the database search ratio, the result will alwaysbe greater than the mere likelihood ratio of the match (except for the unrealistic case in which $\varphi = 0$) . Thus, a cold-hit DNA match should count as stronger evidence than a DNA match of a previously identified suspect. **?** study variations of the database search problem, with different search strategies and including the possibility that information about the match is itself uncertain, but the general point remains. Under reasonable assumptions, ignoring the search errs on the side of caution.[22]

[22] **?**, with slightly different assumptions, derived the formula $R \times [1 + mD/N]$, where $R = 1/\gamma$, D is the database size, N the number of people in population not in database, and m is an optional multiplier reflecting how much more likely persons in the database are though to be the source when compared to the rest of the population. The expression cannot be less than γ. If no other profile has been tested, $D = 0$ and LR is simply the regular DNA match LR. If N is zero, that is, everyone in population is in the database, the result is infinitely large.

%Think about the following scenario: first, you identified the suspect by some means. Then, it turned out his DNA profile matches the crime scene stain. Fine. Now, imagine further database search for a database not containing this suspect finds no matches. Would you think that this information supports the guilt hypothesis? If your answer is yes, then you do have the intuition that the lack of matches with other people (whose profiles, in this particular case, happen to be in a database) strengthens the evidence.

%The fact that, compared to an ordinary DNA match, a cold-hit DNA match is stronger evidence of guilty does not imply that the posterior probability of the defendant's guilt should be higher in cold-hit cases. The other evidence used to first identify the suspect, jointly with the DNA match, might result in a higher posterior probability of guilt, even if the likelihood ratio of the cold-hit match is higher. This different in the posterior probability of guilt accounts for the intuition that in cold-hit cases we should resist conviction so long as the evidence only consists of a DNA match. The resistance toward conviction should be less strong in other cases so long as the DNA match is supplemented by other incriminating evidence.

%What about the intuition that we'd be uneasy about a conviction based solely on a cold hit match, much less than about a DNA-match based conviction of a previously and independently identified suspect? We think so because now we're thinking about the posterior probability given total evidence.

7.10 Levels of Hypotheses

Consider using the stuff that was commented out! I left percentage signs.

Difficulties in assessing probabilities go hand in hand with the choice of the hypotheses of interest. To some approximation, hypotheses can be divided into three levels: offence, activity, and source level hypotheses. At the offence level, the issue is one of guilt or innocence, as in the statement `Smith intentionally attacked the victim with a knife'. At the activity level, hypotheses do not include information about intent but simply describe what happened and what those involved did or did not do. An example of activity level hypothesis is`Smith bled at the scene.'Finally, source level hypotheses describe the source of the traces, such as 'Smith left the stains at the crime scene,' without specifying how the traces got there. Overlooking differences in hypothesis level can lead to serious confusions.

To illustrate, consider a case in which a DNA match is the primary incriminating evidence. In testifying about the DNA match at trial, experts will often assess the probability that a random person, unrelated to the crime, would coincidentally match the crime stain profile %For a survey of developments and complications of this model, see (**?**). The random match probability is often an impressively low number, say 1 in 100 million or lower, at least excluding the possibility that relatives or identical twins would coincidentally match (**?**).

%This low probability indicates that it is extremely unlikely that a random person would match.

%The match should count as strong evidence against the defendant % See %This presentation is similar to classical hypothesis testing. In such a context one starts by setting up a chance hypothesis, often called the null H_0, and then calculates the probability of the evidence (or data obtained) being such as it is or more extreme on the assumption that H_0 holds. If this probability is small (conventionally, below 0.01 or 0.05), this counts as evidence against %H_0. But note that just because $\mathsf{P}(()E|H_0)$ is small, it does not follow that, for any alternative hypothesis, H_* $\mathsf{P}(()E|H_*)$ is large. The latter probability could be equally low.
% %A DNA profile consists of pairs of alleles at several loci. Individual allele probabilities are used to calculate the expected frequency of a given profile Γ in a relevant population, so that we obtain the genotype probability γ. Assuming the so-called Hardy-Weinberg equilibrium, γ can be obtained by multiplying the allele probabilities. Very roughly, the probability of a match at any particular locus is around $1/10$, and so the probability of a match on all 20 loci used by the FBI CODIS system should be $(1/10)^{20}$. % %Such calculations, however, are a bit simplistic. One factor is that given the crime stain, we already know that at least one population member has profile Γ, and the observation of a gene increases the chance of another of the same type. Actual calculations are quite complicated, (see **?**), as we'll discuss in the section on BNs for DNA evidence evaluation. % %In legal context, γ — the so-called Random Match Probability (RMP), that is, the probability that a random person from the population matches the crime stain profile — is taken to be the probability that the suspect is a match if in fact he is innocent and is usually estimated as the frequency of a given profile in the relevant population. % % is not the probability of innocence (see **??** for details), it % %And yet, in what way the match

counts as evidence against the %defendant is not straightforward. %Here it is paramount to avoid a number of confusions. % % %First, the random match probability is not the posterior probability of innocence since $\mathsf{P}(()\mathsf{match}|\mathsf{innocence})$ should not be confused with $\mathsf{P}(()\mathsf{innocence}|\mathsf{match})$. %The random match probability speaks to the former, not the latter probability. %To confuse the two would be to commit the prosecutor's fallacy (see **??** for details). %Second, %Since the random match probability low probability indicates that it is extremely unlikely that a random person would match. It is tempting to equate the random match probability to $\mathsf{P}(()\mathsf{match}|\mathsf{innocence})$ and together with the prior $\mathsf{P}(()\mathsf{innocence})$ use Bayes' theorem to calculate the posterior probability of innocence $\mathsf{P}(()\mathsf{innocence}|\mathsf{match})$. But this would be a mistake. % Applying Bayes' theorem is of course recommended and helps to avoid the prosecutor's fallacy, the conflation of $\mathsf{P}(()\mathsf{innocence}|\mathsf{match})$ and $\mathsf{P}(()\mathsf{match}|\mathsf{innocence})$. %(see **??** for details). The problem lies elsewhere. Equating the random match probability with $\mathsf{P}(()\mathsf{match}|\mathsf{innocence})$ overlooks the difference between offense, activity and source level hypothesis. It is hasty to assume that, in one way or another, a DNA match can speak directly to the question of guilt or innocence. Even if the suspect actually left the genetic material at the scene—source level proposition—the match does not establish guilt. %The match could be the result of a laboratory error. Or Even if the defendant did visit the scene and came into contact with the victim, it does not follow that he committed the crime he was accused of. %So even if the random match probability is usually very low, it does not follow that the posterior probability of innocence is near the random match probability given by the expert. %It is true, however, that in many circumstances the random match probability and the posterior probability of innocence given a match would both be very low.

Few forms of evidence can speak directly to offense level hypotheses. Circumstantial evidence that is more amenable to a probabilistic quantification, such as DNA matches and other trace evidence, does not. Eyewitness testimony may speak more directly to offense level hypotheses, but it is also less easily amenable to a probabilistic quantification. This makes it difficult to assign probabilities to offense level hypotheses. Experts are usually not supposed to comment directly on offense level hypotheses, but they often comment on activity level and source level hypotheses. In moving from source to activity level, however, additional sources of uncertainty come into play. %(**?**). % The assessment of activity level hypotheses depends on additional variables other than those on which the assessment of source level hypotheses depends. %For example, is this the kind of staining that would occur if the suspect kicked the victim in the head? For example, the probability of finding such and such quantity of matching glass if the suspect smashed the window depends on how the window was smashed, when it was smashed, and what the suspect did after the action. Another problem arises due to recent improvements in DNA profiling technology (**?**). Since today investigators are able to obtain profiles from minimal amounts of genetic material, transfer probabilities become more difficult to assess as more opportunities of transfer arise. If small traces such as dust speckles can be used as evidence, the possibility that the traces were brought to the scene accidentally becomes more likely. For this reason, moving beyond source level hypotheses requires a close collaboration between scientists, investigators and attorneys (see **?**, for a discussion). %The hypotheses themselves are up for revision as evidence is obtained or facts about what happened are accepted (**?**).

% % % %Since the likelihood ratio might change with the choice of the hypotheses, the clarity on the choice of the hypotheses, and their level in particular, is crucial. For instance, % % %

7.11 The two-stain problem

Also, worth a discussion as an illustration

A case study that further illustrates both advatanges and limitations of the likelihood ratio as a measure of evidentiary strength is the two-stain problem, originally formulated by **?**. The key limitation is due to the combination of two circumstances: first, that likelihood ratios vary depending on the choice of hypotheses being compared; second, that it is not always clear which hypotheses should be compared. To illustrate what is at stake, what follows begins with Evett's original version of the two-stain problem (which does not pose any challenge to the likelihood ratio) and then turns to a more complex version (which suggests that likelihood ratios, in and of themselves, are insufficiently informative).

7.11.0.1 Evett's two-stain problem

Suppose two stains from two different sources were left at the crime scene, and the suspect's blood matches one of them. More precisely, the two items of evidence are as follows: %

E_1 The blood stains at the crime scene are of types γ_1 and γ_2 of estimated frequencies q_1 and q_2 respectively.

E_2 The suspect's blood type is γ_1.

% Let the first hypothesis be that the suspect was one of the two men who committed the crime and the second hypothesis the negation of the first. %

H_p The suspect was one of the two men who committed the crime.

H_d The suspect was not one of the two men who committed the crime.

% **?** shows that the likelihood ratio of the match relative to these two hypotheses is $1/2q_1$ where q_1 is the estimated frequency of the characteristics of the first stain. Surprisingly, the likelihood ratio does not depend on the frequency associated with the second stain.

To understand Evett's argument, consider first the likelihood ratio:

$$\frac{\mathsf{P}(()E_1 \wedge E_2|H_p)}{\mathsf{P}(()E_1 \wedge E_2|H_d)} = \frac{\mathsf{P}(()E_1|E_2 \wedge H_p)}{\mathsf{P}(()E_1|E_2 \wedge H_d)} \times \frac{\mathsf{P}(()E_2|H_p)}{\mathsf{P}(()E_2|H_d)}.$$

Notice that the suspect's blood type as reported in E_2 is independent of whether or not he participated in the crime, that is, $\mathsf{P}(()E_2|H_p) = \mathsf{P}(()E_2|H_d)$. So the likelihood reduces to:

$$\frac{\mathsf{P}(()E_1 \wedge E_2|H_p)}{\mathsf{P}(()E_1 \wedge E_2|H_d)} = \frac{\mathsf{P}(()E_1|E_2 \wedge H_p)}{\mathsf{P}(()E_1|E_2 \wedge H_d).}$$

The numerator $\mathsf{P}(()E_1|E_2 \wedge H_p)$ is the probability that one of the stains is γ_1 and the other γ_2 given that the suspect is guilty and has profile γ_1. The probability that one of the stains is γ_1 is simply 1, and assuming blood type does not affect someone's propensity to commit a crime, the probability that the second stain is γ_2 equals its relative frequency in the population, q_2. So the numerator is $1 \times q_2 = q_2$. % Next, consider the denominator $\mathsf{P}(()E_1|E_2 \wedge H_d)$. If H_d is true, %the fact that the suspect has profile γ_1 is irrelevant for the crime scene profiles. the crime was committed by two randomly selected men with profiles γ_1 and γ_2. %who can be seen as two random samples from the general population as far as their blood profiles are concerned. There are two ways of picking two men with such profiles (γ_1, γ_2 and γ_2, γ_1), each having probability $q_1 q_2$. So the denominator equals $2q_1 q$. By putting numerator and denominator together, %

$$\frac{q_2}{2q_1 q_2} = \frac{1}{2q_1}.$$

% which completes the argument. In general, if there are n bloodstains of different phenotypes, the likelihood ratio is $1/nq_1$, or in other words, the likelihood ratio depends on the number of stains but not on the frequency of the other characteristics.

7.11.0.2 A more complex two-stain problem

Consider now a more complex two-stain scenario. Suppose a crime was committed by two people, who left two stains at the crime scene: one on a pillow and another on a sheet. John Smith, who was arrested for a different reason, genetically matches the DNA on the pillow, but not the one on the sheet. What likelihood ratio should we assign to the DNA match in question? **?** argue that there are three plausible pairs of hypotheses associated with numerically different likelihood ratios (see their paper for the derivations). The three options are listed below, where R is the random match probability of Smith's genetic profile and δ the prior probability that Smith was one of the crime scene donors. %

H_p	H_d	LR
Smith was one of the crime scene donors.	Smith was not one of the crime scene donors.	$R/2$
Smith was the pillow stain donor.	Smith was not one of the crime scene donors.	R
Smith was the pillow stain donor.	Smith was not the pillow stain donor.	$R(2-\delta)/2(1-\delta)$

Two facts are worth noting here. First, even though the likelihood ratios associated with the hypotheses in the table above are numerically different, the hypotheses are in fact equivalent conditional on the evidence. After all, Smith was one of the crime scene donors just in case he was the pillow stain donor, because he is excluded as the stain sheet donor. Smith was not one of the crime scene donors just in

case he was not the pillow stain donor, because he is excluded as the sheet stain donor.
% %Second, the example illustrates that sometimes the likelihood ratio is sensitive to the prior probability (after all, δ occurs in the third likelihood ratio in the table). % In addition, even though the likelihood ratios are numerically different, their posterior probabilities given the evidence are the same. To see why, note that the prior odds of the three H_p's in the table should be written in terms of δ. Following **?**, the prior odds of the first hypothesis in the table are $\nicefrac{\delta}{1-\delta}$. The prior odds of the second hypothesis are $\nicefrac{(\delta/2)}{(1-\delta)}$. The prior odds of the third hypothesis are $\nicefrac{(\delta/2)}{(1-(\delta/2))}$. In each case, the posterior odds — the result of multiplying the prior odds by the likelihood ratio — are the same: $R \times \nicefrac{\delta}{2(1-\delta)}$. So despite differences in the likelihood ratio, the posterior odds of equivalent hypotheses are the same so long as the priors are appropriately related (this point holds generally).

? cautions that the equivalence of hypotheses, conditional on the evidence, does not imply that they can all be presented in court. He argues that the only natural hypothesis for the two-stain problem is that Smith is guilty as charged. **?** reply that focusing on the guilt hypothesis is beyond the competence of expert witnesses who should rather select pairs of hypotheses on which they are competent to comment. Some such pairs of hypotheses, however, will not be exclusive and exhaustive. When this happens, as seen earlier, the selection of hypotheses is prone to arbitrariness. To avoid this problem, **?** recommend that the likelihood ratio should be accompanied by a tabular account of how a choice of prior odds (or prior probabilities) will impact the posterior odds, for a sensible range of priors (for a general discussion of this strategy called sensitivity analysis, see earlier discussion in **??**). In this way, the impact of the likelihood ratio is made clear, no matter the hypotheses chosen. This strategy concedes that likelihood ratios, in and of themselves, are insufficiently informative, and that they should be combined with other information, such as a range of priors, to allow for an adequate assessment of the evidence.[23]

% and prior odds needed to calculate LR in cases in which LR depends on priors are available. % %\footnote{Further discussion of the phenomenon is quite interesting. %

7.11.0.3 Likelihood ratio variability in cold-hit cases

%The sensitivity of the likelihood ratio to the choice of hypotheses is not confined to the two-stain problem or alike scenarios.

A similar conclusion holds for DNA matches in cold-hit cases. When the suspect is identified through a database search of different profiles, **?** and **?** have argued that the likelihood ratio of the match %— which usually equals $1/\gamma$ where γ is the random match probability— should be adjusted by the database search ratio (see earlier in 7.4 for details). This proposal tacitly assumes that the hypothesis of interest is something like `the defendant is the true source of the crime traces.' %This assumption is eminently plausible but not uncontroversial. %The National Research Council (NRC II) recommended in 1996 that that the likelihood ratio of the match 1/γ be divided by the size of the database. In defending this proposal, In contrast, \cite{stockmarr1999LikelihoodRatiosEvaluating}, who defends the NRC II 1996 recommendation, argues that the likelihood ratio of the match in cold-hit cases should be divided by the size of the database. He points out that the hypothesis`the defendant is the true source of the crime traces' is evidence-dependent because the investigators had no way of knowing prior to the search that anyone in particular would match. % (p. 672). Accordingly, Stockmarr believes we should evaluate the likelihood ratio using hypotheses that can be formulated prior to the database search, such as 'The true source of the crime traces is among the suspects in the database.' The likelihood associated with this hypothesis and its negation is $\nicefrac{k}{np}$, where k is the number of matching profiles, n the database size, and p the random match probability (see **?**, for a derivation).

In response to this argument, Dawid points out that even though Stockmarr's hypothesis does not depend on the result of the search, it still depends on the data themselves (because it changes with the database size) (**?**). Dawid also points out that Stockmarr's hypothesis is composite and thus the assessment of the likelihood ratio requires additional assumptions about the priors. Once these assumptions are made clear, the posterior of Stockmarr's hypothesis is the same as that of other hypotheses. %obtained using others methods.

[23] The reference class problem is lurking in the background. **?** argues that, in order to calculate the probability of a match given the evidence, the class of possible culprits should be identified, and different choices of such a class might lead to different likelihood ratios. On the problem of priors, see **??**. On the reference class problem, see **??**.

This phenomenon is a particular case of what we discussed earlier since the hypothesis 'The true source is among the suspects in the database' (Stockmarr's) and the hypothesis 'The defendant is the true source of the crime traces' (Taroni's) are equivalent conditional on the evidence. % % This is another example of how likelihood ratios on their own might be insufficiently informative to allow for an adequate assessment of the evidence.

I changed this is a bit. Is this correct?

Yes, there was a minor confusing one passage before, fixed.

7.12 LR & relevance, small-town murder etc.

The U.S. Federal Rules of Evidence define relevant evidence as one that has 'any tendency to make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without the evidence' (rule 401). This definition is formulated in a probabilistic language. Legal probabilists interpret it by relying on the likelihood ratio, a standard probabilistic measure of evidential relevance (**?????**). The likelihood ratio (initially introduced in **??** and more extensively in Section **??**) is the probability of observing the evidence given that the prosecutor's or plaintiff's hypothesis is true, divided by the probability of observing the same evidence given that the defense's hypothesis is true.

Let E be the evidence, H the prosecutor's or plaintiff's hypothesis, and H' the defense's hypothesis. The likelihood ratio %$LR(E,H,H')$ is defined as follows: %

$$LR(E,H,H') = \frac{P(E|H)}{P(E|H')}$$

% On this interpretation, relevance depends on the choice of the competing hypotheses. %H_p and H_d are used as examples, but other competing hypotheses H and H' could also be used. %When there are no ambiguities, $LR(E,H_p,H_d)$ will be shortened into the less cumbersome $LR(E)$. % A piece of evidence is relevant—in relation to a pair of hypotheses H and H'—provided the likelihood ratio $LR(E,H,H')$ is different from one and irrelevant otherwise. For example, the bloody knife found in the suspect's home is relevant evidence in favor of the prosecutor's hypothesis because we think it is far more likely to find such evidence if the suspect committed the crime (prosecutor's hypothesis) than if he didn't (defense's hypothesis) %(**?**, 7) (**?**). In general, for values greater than one, $LR(E,H,H') > 1$, the evidence supports the prosecutor's or plaintiff's hypothesis H, and for values below one, $LR(E,H,H') < 1$, the evidence supports the defense's hypothesis H'. If the evidence is equally likely under either hypothesis, $LR(E,H,H') = 1$, the evidence is considered irrelevant.

7.13 The Small Town Murder objection

This account of relevance has been challenged by cases in which the evidence is intuitively relevant and yet its likelihood ratio, arguably, equals one. Here is one of them: %One such case is *Small Town Murder*

> *Small Town Murder:* A person accused of murder in a small town was seen driving to the small town at a time prior to the murder. The prosecution's theory is that he was driving there to commit the murder. The defense theory is an alibi: he was driving to the town because his mother lives there to visit her. The probability of this evidence if he is guilty equals that if he is innocent, and thus the likelihood ratio is 1 . . . Yet, every judge in every trial courtroom of the country would admit it [as relevant evidence]. (The difficulty has been formulated by Ronald Allen, see the discussion in **?**)

Counterexamples of this sort abound. Suppose a prisoner and two guards had an altercation because the prisoner refused to return a food tray. The prisoner had not received a package sent to him by his family and kept the tray in protest. According to the defense, the prisoner was attacked by the guards, but according to the prosecution, he attacked the guards. The information about the package sent to the prisoner and the withholding of the tray fails to favor either version of the facts, yet it is relevant evidence (**?**). %Counterexamples of this sort abound.

- In response to an eyewitness testimony the defendant claims that his identical twin is the culprit. The testimony is unable to favor any of the two options and yet is considered relevant.
- Suppose the evidence at issue is that a fight occurred and the only dispute is over who started it.
- Or suppose the defendant was stopped because of speeding three minutes after an aborted bank robbery and 1/2 a mile away from the site. The prosecution says this is evidence of guilt: it shows

the defendant was escaping. The defense responds that this is evidence of innocence: no bank robber would speed and attract attention.

- Or, in a murder case, the defendant is the victim's son. Is that relevant to show he's guilty? Is it relevant to show he's innocent? The answer seems to be yes, to both questions. This example is due to Samuel Gross and is discussed in (**?**).

%In general, there seem to be numerous examples in which evidence is, intuitively relevant, and the evidence supports neither side's theory over the other side's theory. How is such evidence to be judged relevant from the probabilist perspective?

%

7.14 Replies to the overlapping evidence objection

%In response (inspired by the ideas put forward in the discussion by David Kaye, Bruce Hay and Roger Park), note that

It is true that if a piece of evidence E fits equally well with two competing hypotheses H and H', then $P(E|H) = P(E|H')$ and thus $LR(E, H, H')$ will equal 1. But the likelihood ratio may change depending on the selection of hypotheses. Rule 401 makes clear that relevant evidence should have 'any tendency to make the existence of *any fact that is of consequence* [emphasis ours] to the determination of the action more probable or less probable'. So the range of hypotheses to compare should be quite broad. Just because the likelihood ratio equals one for a specific selection of H and H', it does not follows that it equals one for *any* selection of H and H' which are of consequence to the determination of what happened. In *Small Town Murder*, whether the suspect was in town at all is surely of consequence for determining what happened (if he was not in town, he could not have committed the crime). The fact that he was seen driving is helpful information for establishing whether or not he was in town.

But if the range of hypotheses H and H' to compare in the likelihood ratio $LR(E, H, H')$ is quite broad, this may raise another concern. The choice of hypotheses needed to determine the relevance of an item of evidence might depend on other items of evidence, and so it might be difficult to determine relevance until one has heard all the evidence. This fact — Ronald Allen and Samuel Gross argue in (**?**) — makes the probabilistic account of relevance impractical. But, in response, David Kaye points out that deciding whether a reasonable juror would find evidence E helpful requires only looking at what hypotheses or stories the juror would reasonably consider. Since the juror will rely on several clues about which stories are reasonable, this task is computationally easier than going over all possible combinations of hypotheses (**?**).

%

7.15 Small Town Murder and bayesian networks

Legal probabilists can also offer a more principled response to *Small Town Murder* and related problems based on Bayesian networks. %rather than a reasonable choice of the competing hypotheses. %The emphasis on the logical relations between the hypotheses have been used by Fenton to address the . Let H_p be the prosecutor's hypothesis that the defendant committed the murder, and H_d the defense's hypothesis that the defendant was visiting his mother. Let E be the fact that the defendant was seen driving to the town prior to the murder. Further, suppose the prior probabilities of H_d and H_p are 50%, and the conditional probability of E on each of those hypotheses is 70% (nothing of what will be said depends on this particular choice of values). Crucially, while indeed the evidence supports both hypotheses, this example is based on a pair of hypotheses that are neither mutually exclusive nor exhaustive. A Bayesian network can be used to calculate other likelihood ratios for hypotheses that are exclusive and exhaustive.

%

Figure 1: Graphic model of Small Town Murder

Figure 2: Probability distribution of E

%

Following the calculations in (**?**), for exclusive and exhaustive hypotheses, $LR(E, H_d, \neg H_d) = 1.75$, and similarly, $LR(E, H_p, \neg H_p) = 1.75$, since $P(E|H_d) = 0.7$ and $P(E|\neg H_d) = 0.4$. The likelihood ratio of

the evidence, if it is measured against exclusive and exhaustive hypotheses, is not equal to one.[24] Such considerations should also generalize to other paradoxes of relevance.
% %For instance, in the twins problem, the LR is 1 if the hypotheses are: `the suspect committed the crime', and`the suspect's twin brother committed the crime', but is not 1 f we consider the fairly natural hypothesis that the defendant is innocent. %Similarly, %In the food tray example, Bayesian network analysis shows that the value of the evidence 'prisoner withholds tray' for the question who started the fight depends on a range of uncertain events and other pieces of evidence (such as whether indeed a parcel he was supposed to obtain was withheld; whether the prisoner inquired about this; whether and how this inquiry was answered). Considered in this context, the piece of evidence will not have a likelihood ratio of one with respect to at least some choice of sensible hypotheses. %

The general problem with the paradoxes of relevance is that in complex situations there is no single likelihood ratio that corresponds to a single piece of evidence. The problematic scenarios focus on a single likelihood ratio based on non-exclusive or non-exhaustive hypotheses. However, evidence can be relevant so long as it has a probabilistic impact on a sub-hypothesis involved in the case, even without having a recognizable probabilistic impact on the prosecutor's or defense's ultimate hypotheses. When this happens, it is relevant, in agreement with Rule 401 of the Federal Rules of Evidence. Bayesian networks help to see how pieces of evidence can increase or decrease the probability of different sub-hypotheses **?**.

%recommend relying on a Bayesian network to investigate in an orderly manner the way in which pieces of evidence and hypotheses interact. %in an orderly manner.

no one said the interact in an orderly manner. ;)

8 Conclusions

Where are we, how did we get here, and where can we go from here? We were looking for a probabilistically explicated condition Ψ such that the trier of fact, at least ideally, should accept any relevant claim (including G) just in case $\Psi(A,E)$.

From the discussion that transpired it should be clear that we were looking for a Ψ satisfying the following desiderata:

conjunction closure If $\Psi(A,E)$ and $\Psi(B,E)$, then $\Psi(A \wedge B,E)$.

naked statistics The account should at least make it possible for convictions based on strong, but naked statistical evidence to be unjustified.

equal treatment the condition should apply to any relevant claim whatsoever (and not just a selected claim, such as G).

Throughout the paper we focused on the first two conditions (formulated in terms of the difficulty about conjunction (DAC), and the gatecrasher paradox), going over various proposals of what Ψ should be like and evaluating how they fare. The results can be summed up in the following table:

[24] **?** offer a slightly different solution to the problem. They construct a Bayesian network with three hypotheses, also exhaustive and exclusive: in town to visit mother, in town to murder, out of town.

View	Convict iff	DAC	Gatecrasher
Threshold-based LP (TLP)	Probability of guilt given the evidence is above a certain threshold	fails	fails
Dawid's likelihood strategy	No condition given, focus on $\frac{\mathsf{P}(H\vert E)}{\mathsf{P}(H\vert\neg E)}$	- If evidence is fairly reliable, the posterior of $A \wedge B$ will be greater than the prior. - The posterior of $A \wedge B$ can still be lower than the posterior of any of A and B. - Joint likelihood, contrary do Dawid's claim, can also be lower than any of the individual likelihoods.	fails
Cheng's relative LP (RLP)	Posterior of guilt higher than the posterior of any of the defending narrations	The solution assumes equal costs of errors and independence of A and B conditional on E. It also relies on there being multiple defending scenarios individualized in terms of combinations of literals involving A and B.	Assumes that the prior odds of guilt are 1, and that the statistics is not sensitive to guilt (which is dubious). If the latter fails, tells to convict as long as the prior of guilt < 0.991.
Kaplow's decision-theoretic LP (DTLP)	The likelihood of the evidence is higher than the odds of innocence multiplied by the cost of error ratio	fails	convict if cost ratio < 110.1111

Thus, each account either simply fails to satisfy the desiderata, or succeeds on rather unrealistic assumptions. Does this mean that a probabilistic approach to legal evidence evaluation should be abandoned? No. This only means that if we are to develop a general probabilistic model of legal decision standards, we have to do better. One promising direction is to go back to Cohen's pressure against **Requirement 1** and push against it. A brief paper suggesting this direction is (Di Bello, 2019b), where the idea is that the probabilistic standard (be it a threshold or a comparative wrt. defending narrations) should be applied to the whole claim put forward by the plaintiff, and not to its elements. In such a context, DAC does not arise, but **equal treatment** is violated. Perhaps, there are independent reasons to abandon it, but the issue deserves further discussion. Another strategy might be to go in the direction of employing probabilistic methods to explicate the narration theory of legal decision standards (Urbaniak, 2018), but a discussion of how this approach relates to DAC and the gatecrasher paradox lies beyond the scope of this paper.

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