

Likelihood ratio and evidence strength

Marcello Di Bello and Rafal Urbaniak

The fallacies we considered earlier in the book —such as the base rate fallacy, the prosecutor’s fallacy, and the defense attorney’s fallacy—show how the posterior probability of a hypothesis can be overestimated or underestimated. The posterior probability should reflect all the evidence bearing on the hypothesis, but should not be identified with the probative value or strength of the evidence. The Federal Rule of Evidence, rule 401, underscores that an item evidence has probative value if it ‘has any tendency to make a fact more or less probable than it would be without the evidence’. The emphasis here is not so much on the posterior probability of the hypothesis given the evidence, $P(H|E)$, but rather, on the extent to which an item of evidence changes—or has any tendency to change—the probability of the hypothesis. This distinction is crucial.

[add crossref](#)

Suppose the prior probability of a given hypothesis H is low, say $P(H) = .001$, but taking evidence E into account brings this probability up to .35, so $P(H|E) = .35$. This is a dramatic upward shift. Even though the posterior probability of H given E is low, E strongly favors H .¹ Conversely, suppose the prior probability of H is extremely high, say $P(H) = .999$, but taking evidence E into account brings this probability down to .75, that is, $P(H|E) = .75$. This is a dramatic downward shift. Even though the posterior probability of H given E is high, E speaks strongly against H . These examples illustrate that measuring the strength or probative value of an item of evidence solely by posterior probabilities leaves out something crucial.

The overall, global value of the evidence should be distinguished from local and incremental support (Di Bello & Verheij, 2018). If E_1, \dots, E_n is the total evidence obtained and H is the hypothesis of interest, the global value of the total evidence is simply the posterior probability of the hypothesis, $P(H|E_1, \dots, E_n)$. This notion, however, is not useful for evaluating the impact of an individual piece of evidence on the probability of the hypothesis. If a piece of evidence shifts the probability of the hypothesis say, from .00007 to .007, the posterior is still low, but the impact of the evidence is strong. Moreover, when lay witnesses and experts testify at trial, the assessment of the evidential value of their individual testimonies should precede the aggregation of their testimonies into a whole, complex body of evidence. Hence, a local and incremental notion of evidential strength or probative value is needed.

This chapter articulates a probabilistic account of probative value or evidential strength that is incremental (it tracks changes in probability) and local (it is limited to individual pieces of evidence). We prefer the expression ‘evidential strength’ because it conveys the fact that evidential value comes in degrees, as items of evidence may support hypotheses more or less strongly. Another expression we will sometimes use is ‘evidential support.’ This expression highlights that an item of evidence has value as it supports, more or less strongly, a hypothesis. We will argue that the likelihood ratio is the best measure of evidential value (or strength, support) that is incremental as well as local.

The plan is as follows. Section 1 explains why the likelihood ratio fares better than another popular measure of evidential strength, the Bayes factor. Appendix C broadens the discussion to probabilistic measures of confirmation drawing from the literature in formal epistemology, but concludes that these measures are unsuited for evidence evaluation in legal fact-finding. We then offer several illustrations of how likelihood ratios can be fruitfully deployed. Section 2 shows that they allow for a nuanced assessment of the strength of quantitative evidence, for example, DNA match evidence. To strengthen

¹Here is a more concrete example. Suppose an expert testifies that the blood found at the crime scene matches the defendant’s and it is .5 probable that a person unrelated to the crime would match by coincidence. Disregarding the match, the prior probability that the defendant is the source of the blood should be set rather low, say .01. Absent other evidence to the contrary, the defendant, as anyone else, had little to do with the crime. By running Bayes’ theorem, the posterior probability that the defendant is the source is roughly .17. We are assuming that the probability of a match if the suspect is the source is approximately 1. This might seem unimpressive, but it does not mean that the match evidence has no probative value. While the match did not make it very likely that the defendant was the source of the traces, the posterior probability is seventeen times larger than the prior.

this point, Appendix A and B show how the value of cold-hit DNA matches can be correctly assessed using likelihood ratios, a hotly debated topic among forensic scientists. Section 3 examines how likelihood ratios can help to evaluate eyewitness evidence. This should dispel the impression that likelihood ratios are only suited for explicitly quantitative evidence.

While helpful for evaluating many forms of evidence, likelihood ratios should be deployed with caution. They can be hard to interpret in practice, as they depend on the choice of two competing hypotheses. This choice is, to some extent, subjective, as we discuss in Section 4. This interpretative problem arises partly because the competing hypotheses admit of different levels of specificity: we discuss this phenomenon in Section 5, using the two-stain problem as an illustration. In Section 6 we discuss evidential relevance, a topic closely related to that of evidential value.² Likelihood ratios sometimes may categorize an item of evidence as irrelevant while intuitively the item is relevant. We explain how this problem can be addressed. The upshot is that likelihood ratios are *local* measures of evidential value. Indeed, a piece of evidence might be irrelevant with respect to a certain pair of competing hypotheses, but relevant to another. The value of an item of evidence is to be established both locally (relative to specific hypotheses) and globally (relative to the case as a whole). This suggests the need of formulating a more complex theory. We undertake this task in later chapters.

1. The likelihood ratio outperforms the Bayes factor

One measure of the strength of evidence is the likelihood of the evidence—the probability of the evidence given the hypothesis of interest, $P(E|H)$ —divided by the probability of the evidence $P(E)$. This measure is often called the *Bayes factor*:

$$\text{BF}(E, H) = \frac{P(E|H)}{P(E)}. \quad (\text{BF})$$

The Bayes factor seems a plausible measure of evidential strength as it appropriately deviates from 1, its point of neutrality. Since, by Bayes' theorem, $P(H|E) = \text{BF}(H, E) \times P(H)$, the Bayes factor is greater than one if and only if the posterior probability $P(H|E)$ is higher than the prior probability $P(H)$, $P(H) < P(H|E)$. So E *positively* supports H whenever the Bayes factor is greater than one. The greater the Bayes factor (for values above one), the greater the upward shift from prior to posterior probability, the more strongly E positively supports H . As expected, the posterior probability of H given E could still be low even if the Bayes factor is significantly above one. Conversely, again by Bayes' theorem, the probability of H given E is lower than the probability of H , $P(H) > P(H|E)$ just in case the Bayes factor is less than one. So E *negatively* supports H whenever the Bayes factor is less than one. The smaller the Bayes factor (for values below one), the greater the downward shift from prior to posterior probability, the more strongly E negatively supports H . If $P(H) = P(H|E)$, the Bayes factor equals one and the evidence has no impact on the probability of H .

So far so good. We will now identify three problems with the Bayes factor as a measure of evidential strength. The first problem is that it varies depending on the prior probability of the hypothesis. To see why, consider the denominator $P(E)$. It can be unpacked following the law of total probability:

$$P(E) = P(E|H)P(H) + P(E|\neg H)P(\neg H). \quad (1)$$

So the Bayes factor can be written in a longer form, as follows:

$$\text{BF}(E, H) = \frac{P(E|H)}{P(E|H)P(H) + P(E|\neg H)P(\neg H)}. \quad (2)$$

What should be clear from this formulation is that the Bayes factor depends on the prior probabilities $P(H)$ and $P(\neg H)$. Indeed, suppose $P(E|H) = 1$ and $P(E|\neg H) = .1$. If $P(H) = .1$, $P(E)$, the denominator, is .19, and so the Bayes factor is approximately 5.26. If, however, $P(H) = .2$, the denominator is .28 and the Bayes factor is approximately 3.57. In fact, a more general look (Figure 1) shows the prior probability can have larger impact on the Bayes factor than the likelihood $P(E|\neg H)$.

This is a strike against the Bayes factor as a measure of evidential strength in the context of legal fact-finding. For suppose an expert who is testifying in court is tasked with assessing the value of an

²An item of evidence is relevant to a hypothesis whenever it positively or negatively supports the hypothesis.

item of evidence, say a DNA or fingerprint match. This assessment should not depend on the expert's prior convictions about the plausibility of the hypothesis. Further, judges and lay jurors should be in a position to understand the expert's assessment in the same way, even if they assign different prior probabilities to the hypothesis.³

Bayes factor as a function of prior and $P(E|\sim H)$.

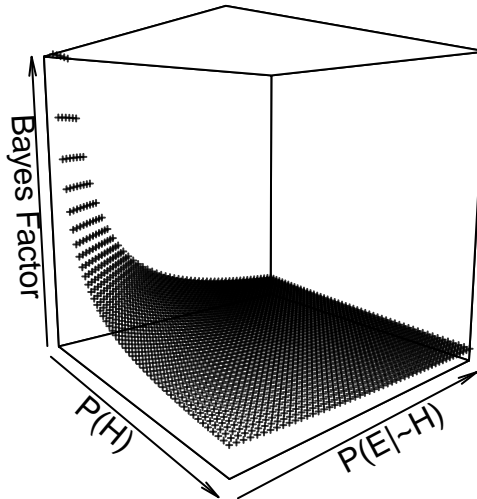


Figure 1: Impact of the prior and likelihood of E given $\neg H$ for probabilities in $(0, 0.05)$ and Bayes Factor restricted to $(0, 250)$ for visibility.

A related reason to worry about the Bayes factor is pragmatic. Whoever is tasked with assessing the strength of evidence—lay jurors, judges, or expert witnesses who are offering their expertise to judges and lay jurors—would face too great a cognitive burden. To see why, the catch-all alternative hypothesis $\neg H$ in the denominator $P(E)$ can be replaced by a more fine-grained set of alternatives, H_1, H_2, \dots, H_k , provided H and these alternatives are exclusive and cover the entire space of possibilities (that is, they form a partition). So, the denominator $P(E)$ should be unpacked, as follows:

$$P(E) = P(E|H)P(H) + \sum_{i=1}^k P(E|H_i)P(H_i). \quad (3)$$

Estimating $P(E)$ now looks quite difficult. It would require one to sift through the entire space of possibilities, as well as coming up with a sensible selection of prior probabilities for the several alternative hypotheses on hand.

A third reason to hesitate about the Bayes factor comes from an observation by Gillies (1986), also discussed by Branden Fitelson (1999). Consider the hypothesis H = “the suspect is guilty” and suppose it is a fact that E = “the suspect killed the victim.” Fact E does not establish guilt with certainty since guilt requires both *actus reus*, the killing, and *mens rea*, the intention. But clearly E provides positive support for H . Now consider a composite hypothesis $H' =$ “the suspect is guilty *and* we live in a simulation built by aliens.” Presumably, the support E provides for H' should be weaker than the support it provides for H . After all, the addition of a far-fetched hypotheses should decrease evidential support. This weakening of evidential support, however, cannot be captured by the Bayes factor since both H and H' deductively entail E . In general, suppose $H \models E$ (and so, also, $H \wedge X \models E$). Then both $P(E|H)$ and $P(E|H \wedge X)$ equal 1. But this means that $BF(H, E) = BF(H \wedge X, E) = 1/P(E)$. So the Bayes factors for the two support relations are equal. This is the problem of irrelevant conjuncts.⁴

³The requirement of prior independence is also in line with an objectivity requirement (Bickel, 2012), say, that the strength of evidence should not vary from one researcher to another.

⁴The same point can be made using irrelevant hypotheses that are not so far-fetched. For instance, suppose one hypothesis of interest is whether the victim was running in the park on a certain night, and the relevant piece of evidence is her footprints in

So suppose we are after a measure of evidential strength that (i) does not depend on priors, (ii) places no unreasonably heavy cognitive requirements, and (iii) is sensitive to the addition of irrelevant conjuncts. We will argue that a measure that satisfies these desiderata is the likelihood ratio, the ratio between $P(E|H)$ (the probability of the evidence given the hypothesis is true) and $P(E|\neg H)$ (the probability of the evidence given the hypothesis is false). Before showing that this measure satisfies the three desiderata, we address a preliminary question. Why use such a ratio in the first place?

For one thing, $P(E|H)$ by itself would not be a fine-grained enough measure. In some cases, this conditional probability may be close to one. For instance, the probability that the blood type matches if the accused is the source, $P(\text{blood match}|\text{source})$, may be close to one. Similarly, the probability that the DNA profile matches if the accused is the source, $P(\text{DNA match}|\text{source})$, may also be close to one. But obviously a DNA match is not on par with a blood type match. The DNA match should be stronger evidence than the blood type match because a specific DNA profile occurs less frequently than a specific blood type. And yet, a quantity such as $P(E|H)$, by itself, would fail to distinguish the two cases. Besides $P(E|H)$, also $P(E|\neg H)$ matters. If the accused is *not* the source, the probability of a blood type match, while relatively small, should be higher than the probability of a DNA profile match. Hence, the likelihood ratio $P(\text{DNA match}|\text{source})/P(\text{DNA match}|\neg\text{source})$ should be higher than $P(\text{blood match}|\text{source})/P(\text{blood match}|\neg\text{source})$. This would explain why a DNA match is stronger evidence than a blood type match.

Could the strength of evidence be measured by the probability $P(E|\neg H)$ alone? As before, relying solely on this quantity would leave out crucial information. Consider an example by Triggs & Buckleton (2004). In a child abuse case, the prosecutor offers evidence that a couple's child rocks and that only 3% of non-abused children rock, $P(\text{child rocks}|\neg\text{abuse}) = .3$. If it is unlikely that a child who is not abused would rock, that this child rocks might seem evidence of abuse. But this interpretation is mistaken. It could also be that 3% of abused children rock, $P(\text{child rocks}|\text{abuse}) = .3$. After all, the two conditional probabilities need not add up to 1. Similarly, learning that $P(\text{child rocks}|\text{abuse}) = .3$ does not provide enough information for evidence evaluation. One also needs information about $P(\text{child rocks}|\neg\text{abuse})$. If rocking is equally unlikely under either hypothesis, rocking cannot count as evidence of abuse. Neither of the two conditional probabilities, viewed in isolation, makes this salient.

So, both the probability of the evidence given the hypothesis and the probability of the evidence given an alternative hypothesis should be part of any good measure of evidential strength (ENFSI, 2015; Royall, 1997; Triggs & Buckleton, 2004). The Bayes factor includes both probabilities, but—as seen before—it requires too much information since it requires an assessment of the prior probability of the hypothesis. A less demanding measure that keeps track of both probabilities is the **likelihood ratio**:

$$LR(E, H, H') = \frac{P(E|H)}{P(E|H')}, \quad (LR)$$

where H' is a hypothesis that is taken to be a competing alternative to H . If the evidence is more likely given H than H' , the ratio would be above one, and if the evidence is more likely given H' than H , the ratio would be below one. As with the Bayes factor, support levels correspond to deviations from one. The greater the likelihood ratio (for values above one), the stronger the evidence in favor of H as contrasted with H' . The smaller the likelihood ratio (for values below one), the stronger the evidence in favor of the competing hypothesis H' as contrasted with H .

Because of its simplicity, the likelihood ratio is sometimes used in practice. Experts sometimes testify by offering the likelihood ratio as a measure of the strength of the evidence. An expert, for instance, may testify that the blood-staining on the jacket of the defendant is ten times more likely to be seen if the wearer of the jacket hit the victim (prosecutor's hypothesis) rather than if he did not (defense's hypothesis) (CGG Aitken, Roberts, & Jackson, 2010, p. 38). This apparent simplicity, however, can often give rise to errors in the assessment of the evidence, especially if the two hypotheses are not chosen carefully. The choice of the hypotheses that are conditioned upon is crucial. In the most straightforward case, H' is simply the negation of H . But the competing hypotheses H and H' need not be one the negation of the other, a point to which we will return later.

Let's now examine why the likelihood ratio satisfies our three desiderata. First, unlike the Bayes factor, the likelihood ratio does not depend on the prior probability of the hypothesis. It allows for a

the park. Perhaps, another hypothesis is whether she had wine at dinner later on. Clearly, whether she did is not obviously relevant to whether she was running in the park beforehand. However, one should be very hesitant to say that the evidential strength of the presence of footprints is the same relative to *she was running in the park* and *she was running in the park and had wine at dinner later on*. But this is what the Bayes factor would commit one to.

clearer separation of the impact of the priors from the impact of the evidence on the posterior probability. The relationship between likelihood ratio $P(E|H)/P(E|H')$, and prior and posterior odds is apparent in the odds version of Bayes' theorem:

$$\frac{P(H|E)}{P(H'|E)} = \frac{P(E|H)}{P(E|H')} \times \frac{P(H)}{P(H')}. \quad (4)$$

If the likelihood ratio is greater (lower) than one, the posterior odds will be greater (lower) than the prior odds of H . The likelihood ratio, then, is a measure of the upward or downward impact of the evidence on the prior odds of two hypotheses H and H' .

This feature of the likelihood ratio fits nicely with the division of labor common in legal fact-finding between experts and decision-makers, judges or lay jurors. A prominent forensic scientist recommends that 'in criminal adjudication, the values of the prior odds and the posterior odds are matters for the judge and jury, in accordance with the normal division of labor in forensic fact-finding' (Colin Aitken & Taroni, 2008, p. 194). Experts should 'not trespass on the province of the jury by commenting directly on the accused's guilt or innocence, ... and should generally confine their testimony to presenting the likelihood of their evidence under competing propositions' (CGG Aitken, Roberts, & Jackson, 2010, p. 42). If, however, experts were to report the Bayes factor, this would mean they are competent to estimate $P(E)$ directly, which is unlikely, or that they implicitly estimate it relying on their estimation of $P(H)$ in the background (if they use the law of total probability).

Second, the likelihood ratio is less cognitively burdensome than the Bayes factor. It does not require one to think about the probability of the evidence in general, $P(E)$. Think, for example, of blood match evidence. To calculate the Bayes factor, we need two things: $P(E|H)$ (which here, say, can be assumed to approximate one) and $P(E)$ (which is the probability of a blood match under any possible scenarios, whether or not the suspect is the source). Yet, direct and reliable estimation of the probability $P(E)$ is difficult. One can use the law of total probability from equation (1). This would require, besides an assessment of the conditional probabilities $P(E|H)$ and $P(E|\neg H)$, an assessment of the prior probabilities of $P(H)$ and $P(\neg H)$. Instead, the likelihood ratio would only require an assessment of the conditional probabilities. In this sense, its calculation requires less information.

Finally, unlike the Bayes factor, the likelihood ratio is not susceptible to the problem of irrelevant hypotheses. For suppose $P(E|H) = P(E|H \wedge X) = 1$, where X is an additional hypothesis that is irrelevant to H . Note that $LR(E, H) = 1/P(E|\neg H)$, while $LR(E, H \wedge X) = 1/P(E|\neg H \vee \neg X)$. Unlike with the Bayes factor, here the two denominators might differ. For example, suppose a fair coin is tossed three times. Let H = "two first tosses resulted in two heads," let E = "at least one of the two first tosses resulted in a head," and let X = "the third toss resulted in heads." Then $P(E|H) = 1$, $P(E|\neg H) = 2/3$, $LR(E, H) = \frac{1}{2/3} = 1.5$. However, $P(E|H \wedge X) = 1$, $P(E|\neg(H \wedge X)) \approx .71$, so $LR(E, H \wedge X) \approx \frac{1}{.71} = 1.4$. Thus, the support, as measured by likelihood ratio, can drop by adding a conjunct that is probabilistically irrelevant to the original hypothesis. In fact, this weakening of evidential support by adding an irrelevant conjunct holds in general given sensible assumptions.⁵

All in all, the likelihood ratio outperforms the Bayes factor on several respects. But, of course, there could be other measures of evidential strength that fare even better. Other measures worth considering come from the literature in formal epistemology on confirmation theory. The expression 'confirmation' is more commonly used in this literature instead of strength of the evidence (or value, support). An extensive discussion of these measures, however, would detract us from the main task at hand, namely the examination of the pros and cons of the likelihood ratio. We therefore relegate a discussion of confirmations measures to Appendix C. The upshot is that two questions should be distinguished. (1) To what extent should a given piece of evidence change our beliefs about a given hypothesis? Call this the question about confirmation. (2) What is the strength of a piece of evidence relative to a hypothesis? Call this the question about evidential strength. The two questions overlap to some extent. The key

⁵Branden Fitelson (2002) proved a general claim about irrelevant conjunctions. Hawthorne & Fitelson (2004) later strengthened this claim. The claim is that, if $LR(E, H, \neg H) > 1$, $P(E|X \wedge H) = P(E|H)$, and $P(X|H) \neq 1$, then $LR(E, H, \neg H) > LR(E, H \wedge X, \neg(H \wedge X))$. Crupi & Tentori (2010) raised a related problem. They point out that if $LR(E, H) \leq 1$ and X is confirmationally irrelevant conjunct to H with regard to E , then E will have the same negative or null impact on $H \wedge X$, that is $LR(E, H \wedge X) \leq LR(E, H)$. They find this counter-intuitive and argue that this can be avoided by switching to the Z confirmation measure (Crupi, Tentori, & Gonzalez, 2007). As we argue in Appendix C, the Z measure is prior-sensitive and therefore not fit for our purpose. Further, the phenomenon might not be deeply troubling either. If the likelihood ratio tracks how strongly the evidence supports a hypothesis, it should be no surprise that a more complex hypothesis—one obtained by adding an irrelevant proposition—enjoys a lower support from the same evidence.

difference is that confirmation depends on prior probabilities, and instead, in the evaluation of legal evidence in trial proceedings, the assessment of evidential strength should preferably be kept separate from the choice of prior probabilities. Confirmation measures are concerned with (1) rather than (2), and thus they are unsuitable for legal applications.

2. The risk of false positive and its impact

The two conditional probabilities that make up the likelihood ratio— $P(E|H)$ and $P(E|\neg H)$ —should be used in the evaluation of any form of evidence, both quantitative and non-quantitative evidence. This sections explore how DNA match evidence, the most widely used form of quantitative evidence, should be evaluated using the likelihood ratio. The argument formulated here can be generalized to any form of ‘match evidence.’ This evidence consists in a statement by an expert that the defendant matches the physical material found at the crime scene. The match can be between genetic profiles, fingerprints, blood type, shoe prints, bite marks, etc.

Suppose an expert testifies that there is a genetic or DNA match between the traces at the crime scene and a sample from the defendant. What is this statement evidence for? It is evidence that the defendant was the *source* of the traces—that the materials found at the scene originate from the defendant. The match can also be evidence that the defendant was present at the scene or committed the act, but these claims are more far-fetched, as the chain of inferences becomes weaker. We will discuss these complications later in section 5. For simplicity, let us focus on the source hypothesis.

How strong is a DNA match *qua* evidence that the defendant is the source? Following the likelihood ratio, two conditional probabilities should be compared here, $P(\text{match}|\text{source})$ and $P(\text{match}|\neg\text{source})$. When experts testify in court about a DNA match, however, they only use the **random match probability** as an indicator of evidential strength. This is the probability that a random person, unrelated to the crime, would coincidentally match the crime scene profile. The random match can be equated to the probability that someone who is not the source would match, $P(\text{match}|\neg\text{source})$. This is one of the conditional probabilities in the likelihood ratio. Should the other also be reported? In theory, it certainly should, but in practice, it may not make a difference. The random match probability is often an impressively low number, say 1 in 100 million, and alone can be a good measure of evidential strength so long as the other conditional probability, $P(\text{match}|\text{source})$, is close to one. If that is the case, that $P(\text{match}|\neg\text{source})$ is low would be enough to ensure that the likelihood ratio is significantly above one. But if

But this analysis—even assuming that $P(\text{match}|\text{source})$ equals one—leaves out other sources of error besides the random match probability. In particular, equating the probability $P(\text{match}|\neg\text{source})$ with the random match probability neglects the risk of false positive matches, and this risk is not negligible (Shaer, 2016). Note that a coincidental match is not the same thing as a false positive match. For suppose two individuals—say the perpetrator and the defendant—happen to share the same DNA profile by coincidence. If an expert states that the crime scene sample and the defendant’s sample match, this would be a coincidental match, but not a false positive match.

Unlike a coincidental match, a false positive match is often caused by a human error in a number of circumstances (see W. C. Thompson, 2013 for a more exhaustive treatment and multiple examples):

- **Cross-contamination of samples.** For instance, in Dwayne Johnson (2003) samples were accidentally swapped. In Lukis Anderson (2012), the genetic material was carried over by the paramedics. In one case, German police invested a considerable amount of time and effort searching for the so-called Phantom of Heilbronn, whose DNA profile was associated with many crimes. A bounty of 300k EUR was placed on her head. It turned out she was an innocent employee involved in the production of cotton swabs used across the country.
- **Mislabeling of samples.** For instance, in 2011 the Las Vegas Metropolitan Police Department acknowledge that samples of two men suspected of a 2001 robbery were switched, leading to the exclusion of the perpetrator and four years of incarceration of the other suspect. The mistake came to light only because the perpetrator was later on arrested for an unrelated crime.
- **Misinterpretation of test results.** Single-source sample comparison is not easily prone to misrepresentation, but evidence mixtures—often needed in sexual assault cases—are complicated to interpret. For example, Dror & Hampikian (2011) re-examined a 2002 Georgia rape trial in which two forensic scientists had concluded that the defendant could not be excluded as a contributor of the crime traces. The evidence was sent to 17 lab technicians for re-examination.

A: Bib references are missing here, is it on purpose?

One of them agreed that the defendant could not be excluded as a contributor. Twelve considered the DNA exclusionary, and four found it inconclusive. If the quantity of DNA is limited, there is uncertainty about the number of contributors and about whether any alleles are missing. Ultimately, there is an element of subjectivity in mixed DNA interpretation.

False positive DNA matches are not easy to detect for practical and theoretical reasons. Since DNA evidence carries so much weight in court, it is unusual to proceed with further DNA tests that are costly and time-consuming. It is also unusual that a defendant or their family could afford to pay for further DNA tests. Even more troubling is that errors from contamination or mislabeling of samples often cannot be detected with further DNA testing, because they will replicate the same mis-identification. Sometimes, a lab discovers its own errors and reports them, but this is rare (W. C. Thompson, 2013). Unfortunately, no serious attempt has been made to systematically quantify the relevant error rates. Anecdotal information suggest that false positive matches take place more often than coincidental matches would entail, but how often remains unclear (W. C. Thompson, 2013). Regular proficiency tests used in accredited DNA laboratories involve comparison of samples from known sources, but they are criticized for being unrealistically easy (yet, it happens that analysts fail them). Sometimes, corrective action files are made available. They usually show relatively few false positive errors.⁶ But because of the fragmentary information available, it would be premature to conclude that these errors are no cause for concern.

We should now incorporate the risk of false positive matches into a more adequate assessment of the strength of a DNA match. This can be done using the likelihood ratio, following Colin Aitken, Taroni, & Thompson (2003). We abbreviate:

- S The specimen comes from the suspect (source).
- R A match is reported (reported match).
- M There is a true match (true match).

The difference between S and M is that a true match will exist not only if the suspect is the source, but also if, even though the defendant is not the source, the profiles are in fact the same due to a random, coincidental match. Similarly, a reported match might arise not only if there is a true match, but also if a false positive error has been made. These possibilities are represented in Figure 2.

The evidence whose strength should be assessed is the *reported* match relative to the pair of hypotheses S and $\neg S$. So the likelihood ratio should have the form: $P(R|S)/P(R|\neg S)$. Taking the risk of false positive matches into account, the likelihood ratio can be written as:

$$\frac{P(R|S)}{P(R|\neg S)} = \frac{1}{P(R|M)P(M|\neg S) + P(R|\neg M)P(\neg M|\neg S)} \quad (5)$$

$$= \frac{1}{RMP + [FPP \times (1 - RMP)]} \quad (6)$$

The expression RMP stands for the random match probability $P(M|\neg S)$ and the expression FPP stands for the false positive probability $P(R|\neg M)$. This should highlight the difference between the two error probabilities. A false positive probability tracks a human error, the possibility that match may be reported (R) even without a true match ($\neg M$). The random match probability, instead, tracks a coincidence of nature—the possibility that someone who is not the source ($\neg S$) could still be—coincidentally—a true match (M). The denominator mirrors the fact that there are two ways misleading evidence can arise: there is a true match and the suspect is not the source (because of a random, coincidental match), or there is no true match, and an false positive error has been made in the identification process. The numerator simplifies to one because the probability of a false negative is assumed to be one. We will relax this assumption later one. The full derivation is given below. The reader may skip it without loss to the overall argument.

⁶For instance, the Santa Clara County district attorney's crime laboratory between 2003 and 2007 caught 14 instances of evidence cross-contamination with staff DNA, three of contamination by unknown person, and six of DNA contamination from other samples, three cases of DNA sample switch, one mistake in which the analyst reported an incorrect result, and three errors in the computation of the statistics to be reported.

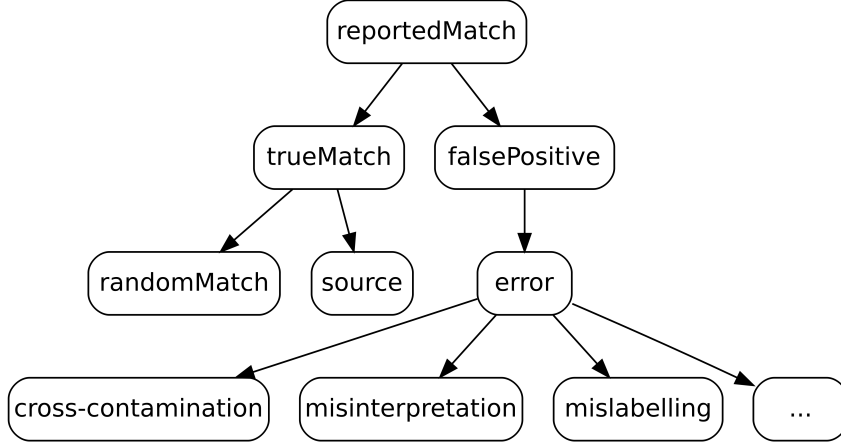


Figure 2: Dependencies between variables in the false positive problem.

By the law of total probability, the denominator $P(R|\neg S)$ can be unpacked as $P(R \wedge M|\neg S) + P(R \wedge \neg M|\neg S)$. The latter, by the chain rule, is equivalent to $P(R|M \wedge \neg S)P(M|\neg S) + P(R|\neg M \wedge \neg S)P(\neg M|\neg S)$. A similar reasoning applies to the numerator. The likelihood ratio then becomes:

$$\frac{P(R|S)}{P(R|\neg S)} = \frac{P(R|M \wedge S)P(M|S) + P(R|\neg M \wedge S)P(\neg M|S)}{P(R|M \wedge \neg S)P(M|\neg S) + P(R|\neg M \wedge \neg S)P(\neg M|\neg S)} \quad (7)$$

This expression can be simplified by assuming that a reported match (R), assuming a true match obtains (M), is independent of whether the suspect is the source (S):

$$\begin{aligned} P(R|M \wedge S) &= P(R|M \wedge \neg S) = P(R|M) \\ P(R|\neg M \wedge S) &= P(R|\neg M \wedge \neg S) = P(R|\neg M), \end{aligned} \quad (8)$$

Now, apply (8) in four places:

$$\frac{P(R|S)}{P(R|\neg S)} = \frac{P(R|M)P(M|S) + P(R|\neg M)P(\neg M|S)}{P(R|M)P(M|\neg S) + P(R|\neg M)P(\neg M|\neg S)} \quad (9)$$

In addition, for simplicity, take the probability of a false negative to be zero. We will consider the case in which it is not zero later on. In fact, the reasons for taking false positives seriously are also reasons for taking false negatives seriously, but let's deal with one problem at a time. So, let the probability of a true match if the suspect is a source be one:

$$P(M|S) = 1 \quad \text{so also} \quad P(\neg M|S) = 0, \quad (10)$$

and let the probability that a true match is reported also be one:

$$P(R|M) = 1. \quad (11)$$

Finally, using (10) and (11) in the numerator yields the desired formula:

$$\frac{P(R|S)}{P(R|\neg S)} = \frac{1}{P(R|M)P(M|\neg S) + P(R|\neg M)P(\neg M|\neg S)} \quad (12)$$

Figure 3 shows the impact of the probability of a false positive match (between 0 and 0.05 probability) assuming random match probabilities in the order of 10^{-9} (often reported in the case of two single-source samples over ten or more loci) and 10^{-3} (sometimes obtained by means of less discriminating tests when the comparison involves a mixed sample). Even a small increase in the false positive probability can lower the likelihood ratio dramatically.

Impact of false positive probability on likelihood ratio

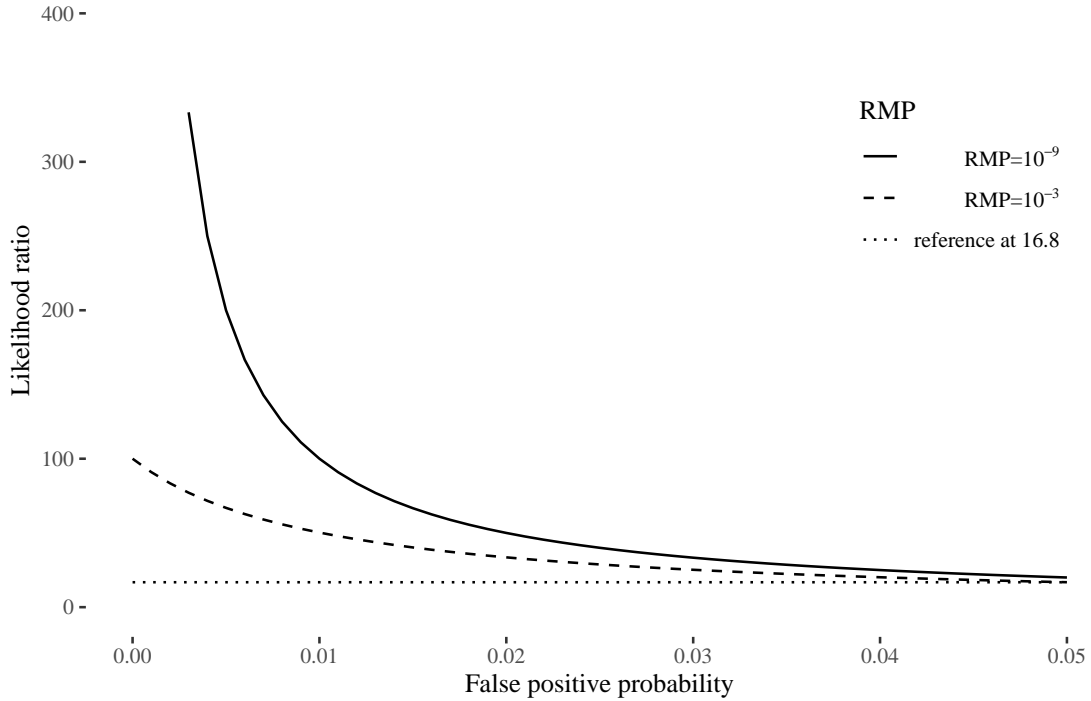


Figure 3: Impact of the false positive probability on the likelihood ratio for two values of RMP. The horizontal reference line is at 16.8, the likelihood reached at RMP= 10^{-3} for FPP=0.05. At the same value of FPP, the LR for RMP 10^{-9} is 20.

We have not yet considered the possibility of false negative matches. The formula for the likelihood ratio should include this possibility. This can be done quite easily.

The rest of this section needs rewriting

$$\frac{P(R|S)}{P(R|\neg S)} = \frac{P(R|M)P(M|S) + P(R|\neg M)P(\neg M|S)}{P(R|M)P(M|\neg S) + P(R|\neg M)P(\neg M|\neg S)} \quad (13)$$

$$= \frac{1 - FPP}{(1 - FPP)RMP + [FPP \times (1 - RMP)]} \quad (14)$$

Buckleton, Bright, & Taylor (2018) derive a more general formula for the likelihood ratio. First, they make the conceptual distinction between the probability that an error occurs (E) and the probability that a match is reported if it does. Mind your head: here E stands for error, not for evidence! In terms of our notation, we have:

$$e = P(E) = P(E|S) = P(E|\neg S)$$

That is, we denote the probability of error as e , and we assume it doesn't depend on whether the prosecution hypothesis is true (whether the suspect is the source).

Separately, the formula includes the probability of a reported match if an error occurs, also assumed to be independent of whether the prosecution hypothesis is true:

$$k = P(R|E, S) = P(R|E, \neg S)$$

Further, it is assumed that the probability of false negatives is zero ($P(R|S, \neg E) = 1$) and the probability of reported match if no error occurs and the defense hypothesis is true is RMP ($P(R|\neg E, \neg S) = RMP$).

Now the derivation:

$$\begin{aligned}
 LR &= \frac{P(R|S)}{P(R|\neg S)} \\
 &= \frac{P(R|\neg E, S)P(\neg E|S) + P(R|E, S)P(E|S)}{P(R|\neg E, \neg S)P(\neg E|\neg S) + P(R|E, \neg S)P(E|\neg S)} \\
 &= \frac{1(1-e) + ke}{RMP(1-e) + ke} = \frac{1-e+ke}{RMP-e \times RMP + ke} \\
 &= \frac{1-(1-k)e}{RMP(1-e) + ke}
 \end{aligned}$$

Let's take it slow. First, the likelihood ratio is just the ratio of probabilities of a reported match (1) if the suspect is the source and (2) if the suspect is not the source.

Now, think of $P(R|S)$. This can be split into two possible scenarios: an error has not been made, or an error has been made. Accordingly, the numerator in the second line uses the law of total probability to split $P(R|S)$ into these two options.

Similarly, $P(R|\neg S)$ can be split into two cases: the suspect is not the source, but we are dealing with a random match, or the suspect is not the source, and an error has been made. An application of the law of total probability in the denominator mirrors this. The rest of the argument is just rewriting in terms of abbreviations, and algebraic manipulation.

Note now that if you think of an error as something that guarantees a mistaken identification, k becomes 1 and e becomes the false positive rate. On this assumption straightforward algebraic manipulation gives:

$$\begin{aligned}
 \frac{1-(1-k)e}{RMP(1-e) + ke} &= \frac{1-e+e}{RMP(1-e) + e} \\
 &= \frac{1}{RMP-e \times RMP + e} = \frac{1}{1+e(1-RMP)}
 \end{aligned}$$

which is the same as the formula obtained by Colin Aitken, Taroni, & Thompson (2003) if we take e to be FPP, as we should on the assumption that $k = 1$.

An analogous reasoning can be used to study the impact of false negative probability on the value of exculpatory DNA evidence. Consider the probability of no match being reported if an error has been made, analogous to k above:

$$l = P(\neg R|E, S) = P(\neg R|E, \neg S) = P(\neg R|E)$$

Now, the likelihood ratio calculations, assuming $l = 1$, go as follows:

$$\begin{aligned}
 LR(\neg R, S, \neg S) &= \frac{P(\neg R|S)}{P(\neg R|\neg S)} \\
 &= \frac{P(\neg R|\neg E, S)P(\neg E|S) + P(\neg R|E, S)P(E|S)}{P(\neg R|\neg E, \neg S)P(\neg E|\neg S) + P(\neg R|E, \neg S)P(E|\neg S)} \\
 &= \frac{0(1-e) + le}{(1-RMP)(1-e) + le} \\
 &= \frac{le}{1-RMP-e+eRMP+le} = \frac{le}{1-RMP+e(l+RMP-1)} \\
 &= \frac{e}{1-RMP+eRMP} = \frac{e}{1+(e-1)RMP}
 \end{aligned}$$

If the error rate is 0, then the numerator is 0 and so is the LR, as it should. In such a case, the evidence is completely exculpatory, the posterior probability that the suspect is the source will be also 0. If the error rate is not 0, the numerator simply is the probability of error, and the numerator takes values between $1 - RMP$ and 1, depending on the value of e . Quite crucially, 1 in the denominator is decreased by

$(1 - e)RMP$, which with usually very low RMP in the case of DNA evidence is a very small change as compared to one, so the denominator stays very close to 1 even if e is very high, and the LR effectively simply is $\approx e/1 = e$. The lines in Figure ??, strictly speaking, do not overlap, but the difference between them (with RMP being fairly low) is negligible.

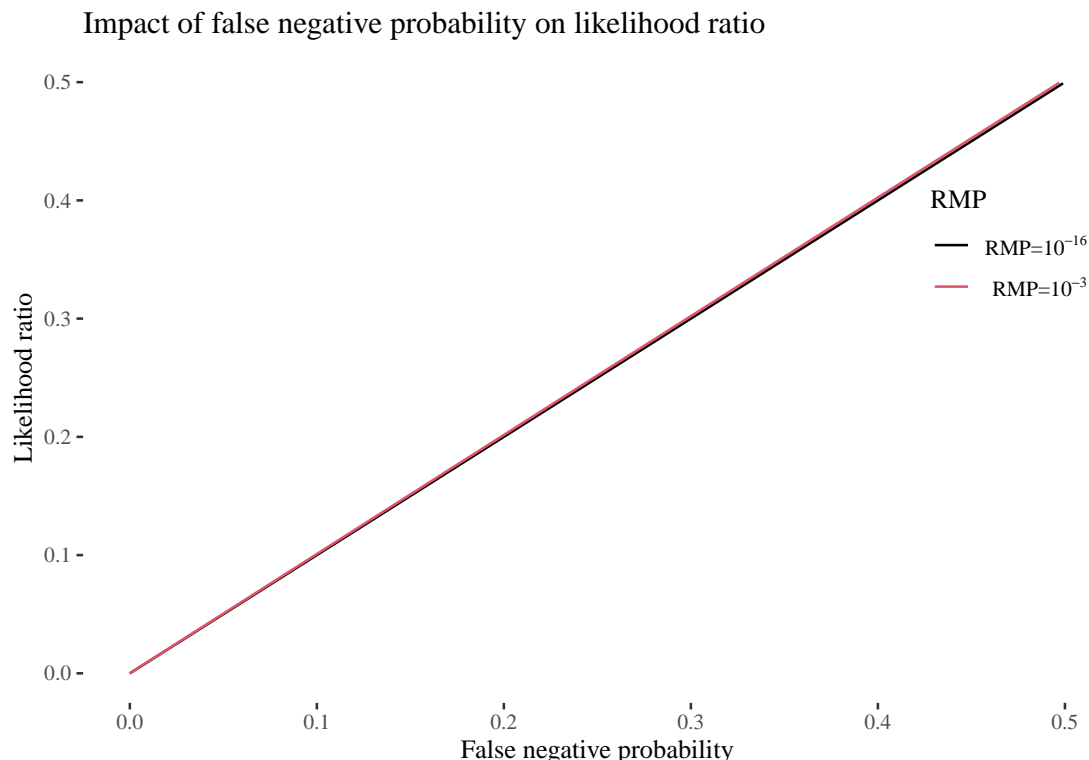


Figure 4: Impact of the false negative probability on the likelihood ratio of exculpatory evidence for two values of RMP .

Interestingly, there is a sense in which the situation is not symmetric when we compare FPP to FNP. With FPP, the LR is 50 when $e = 0.01$ for $RMP = 10^{-3}$, while for the same e and RMP it is 0.01 for FNP, an hundredfold decrease. This illustrates that the exculpatory value of DNA evidence is higher than its incriminating value, even if the error rates and random match probabilities are the same.

Some conceptual symmetry can be regained though. Suppose RMP is really low as compared to FPP and let's ignore it in our approximation. Then, the likelihood ratio of the incriminating evidence becomes $1/FPP$ and the likelihood ratio of exculpatory evidence becomes $FNP/1$. However, the change rate of these differ. While $d/dx(1/x) = d/dx(x^{-1}) = -1/x^2$, $d/dx(x/1) = 1$, and the derivatives look quite different (Figure 5)

The key lesson from this section is as follows. First, likelihood ratios are useful in the evaluation and comparison of the impact of positive and negative error rates; and it makes clear that this impact is not the same, contrary to what one may intuitively think. Second, the likelihood ratio analysis reveals that even a seemingly small error rate in a sense trumps random match probability: if you think there are good reasons to worry about random matches, the analysis shows that there are much better reasons to worry about error rates. While in fact error rates have not been properly studied, the above application of likelihood ratio to investigate the potential impact of hypothetical error rates is still useful in guiding further research, as it clearly suggests that error rates may have serious impact on the value of various types of evidence, and so require attention.

3. Eyewitness identification and likelihood ratio

So far we paid attention to DNA evidence, a form of quantitative evidence widely used in trial proceedings. But the question arises whether evidence that is not explicitly quantitative, for example, eyewitness

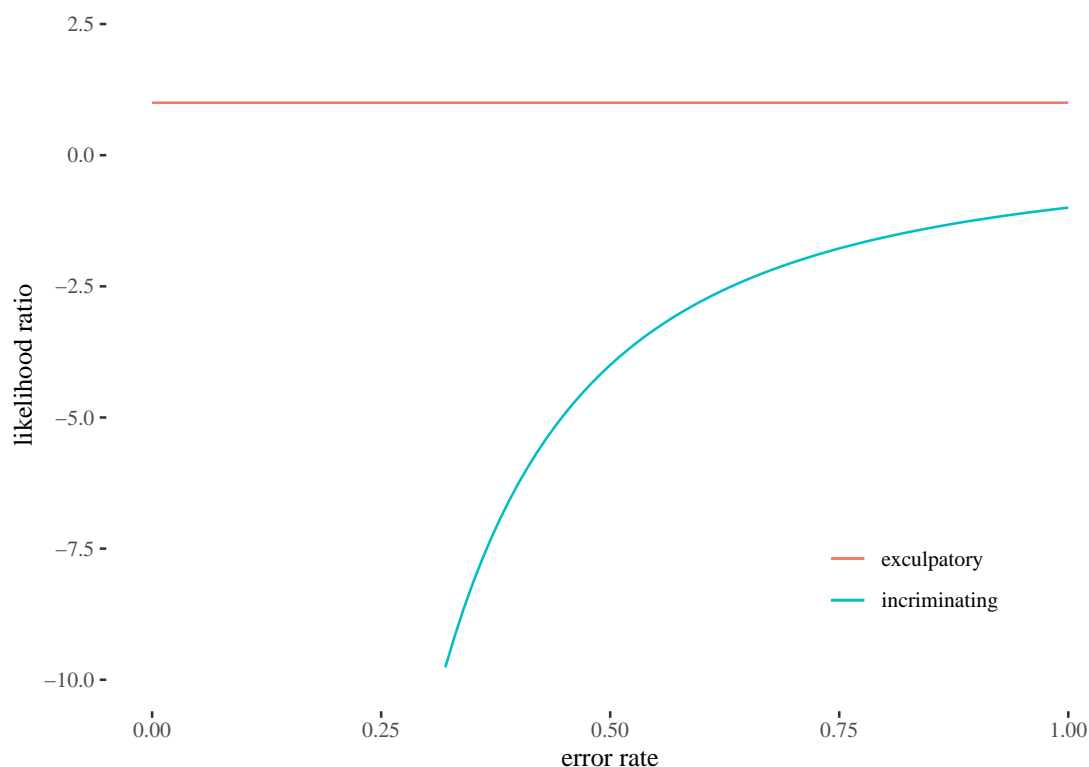


Figure 5: Derivatives of incriminatory and exculpatory likelihood ratios, range restricted to (-10,2).

testimony, can also be evaluated using likelihood ratio. We show that the answer is affirmative. In fact, there is no sharp divide between quantitative and non-quantitative evidence. The statement that the crime scene DNA matches the defendant's DNA is qualitative, but its value is best assessed by means of numerical information, such as error rates and the random match probability. A witness testifying 'I saw him!' carries a lot of weight in a court of law, but again, its value is best assessed by means of numerical information, in particular, data about the risks of false identifications. We should avoid exaggerating or downplaying these risks. Quantitative information can be used to this effect. We will show how this information can be combined in a principled manner into the likelihood ratio.

There is plenty of quantitative information about the risks of false eyewitness identifications. Let's begin with statistics from the criminal justice system. The rate of false convictions in death penalty cases in the United States is estimated at about 4% (Gross, O'Brien, Hu, & Kennedy, 2014). How much can be attributed to false eyewitness identifications is hard to say exactly, but presumably quite a bit. In fact, a study of 340 exoneration in the years 1989-2003 shows that around 90% of false convictions in rape cases resulted from a false eyewitness identification. This percentage comes close to 100% in rape convictions in which the victim and the defendant were of different races. In murder cases, 43% of false convictions resulted from a false identification by one or multiple eyewitnesses.

Field studies offer an equally discouraging picture. These studies show that, in line-up identifications, eyewitnesses select filler individuals at a rate of 20-24% (Klobuchar, Steblay, & Caligiuri, 2006). That is, around 20% of the time, an innocent person in police line-up is incorrectly identified as the perpetrator. Similarly, a field study in Greater London (Wright & McDaid, 1996) and another study in Sacramento, California (Behrman & Davey, 2001) suggest false identification rate of around 20%. Even in experimental settings—where witnesses are less emotionally taxed—eyewitnesses identify a filler from a line-up in approximately 20% of the cases (S. G. Thompson, 2007).

In light of the empirical results that we have discussed and similar studies, the justice systems has grown suspicious of eyewitness evidence over the last twenty years. But blanket statements that eyewitnesses are unreliable—even when backed up by well-researched statistics—are not very helpful. The risks of mistaken identification should be assessed in the individual circumstances. It may be that, on average, eyewitness identifications are unreliable, but *this* particular identification can be perfectly

REFERENCE
MISSING

reliable. Judges and jurors should make determinations about the reliability of a specific eyewitness identification, not about eyewitness reliability in general. Cross-examination is often thought to be the tool for this individualized assessment of the risk of error.⁷ Unfortunately, empirical studies suggest that cross-examination might be quite ineffective at detecting false identifications. In a series of experiments, subjects were asked to cross-examine eyewitnesses to determine whether witnesses made accurate or mistaken identifications. Subjects showed little or no ability to make such discrimination (Wells & Olson, 2003, p. 285). In another experiment, a representative sample of 48 witnesses was cross-examined. Subjects ($n = 96$) viewing the cross-examinations showed little ability to distinguish accurate from false identifications (Lindsay, Wells, & Rumpel, 1981).

reference missing

The ineffectiveness of cross-examination might stem from its reliance on intuitive, folk assessment of the risks of a false identification, not on well-researched quantitative data. The research on eyewitness testimony can improve this state of affairs since a great deal of it aims to identify the conditions under which an identification goes awry. These conditions are usually divided into system variables and estimator variables (Behrman & Davey, 2001; Wells & Olson, 2003). The former refer to how the identification took place in a regimented setting, say whether it was a line-up or a showup;⁸ whether the line-up was simultaneous or sequential; whether the witness identified a suspect prior to the line-up; etc. Estimator variables, instead, refer to the environmental conditions. These variables include distance, lighting conditions, time, the presence of a weapon, etc.

An additional factor to consider is the confidence of the witness in the identification. It is a point of contention whether or not confidence is positively correlated with an accurate identification. A meta-analysis by Wixted & Wells (2017) offers a nuanced but overall optimistic picture. Their analysis focuses on identifications under *pristine conditions*. They require, for example, a double-blind line-up containing one suspect and at least five fillers with no resemblance to the suspect. The witness is cautioned that the offender might not be in the line-up and there is no expectation that they identify someone. Needless to say, very few police departments run their lineups in pristine conditions. But, as it turns out, if the identification occurs under pristine conditions, the high confidence of the witness in the identification is strongly predictive of an accurate identification. Most interestingly, witnesses with high confidence under pristine conditions should be around 90% accurate, a rather encouraging figure.⁹

So, a more individualized assessment of the risks of false eyewitness identification can be made by taking into account estimator and system variables, as well as other relevant factors such as eyewitness confidence. Cross-examination serves a similar purpose. Questions about time of day, distance, stress level, etc. elicit information about estimator variable as well as system variables. Empirical research on eyewitness testimony can supply numerical information about the significance of these variables. It can help to answer questions such as, how much does poor visibility or distance increase the risk of a false identification? Ideally, properly formatted data about all the factors we discussed could be used to develop a multivariate model. This model would allow an assessment of the risks of a false identification under a variety of relatively well-specified circumstances elicited during cross-examination. We are far from reaching this state of maturity, but even with the current research, an expert who is aware of the literature we have cited and of the specific circumstances of the case could assess the risks of a false identification quantitatively. An untrained jury member unaided by numerical data—even after cross-examination—is unlikely to arrive at a better assessment of the risks of error involved.

So consider two scenarios in which we numerically estimate the reliability of the eyewitness evidence in line with the research we just discussed. In one, such an expert recognizes the identification conditions as pristine and testifies: the probability of the testimony if the suspect in fact is the perpetrator ($P(E|H)$) is $.9\% \pm .05$, and the false positive probability ($P(E|\neg H)$) is $.1 \pm .03$. In scenario two, the conditions were not pristine, and the expert testifies that $P(E|H) = .8 \pm .05$ and $P(E|\neg H) = .25 \pm .05$. We get two different *ranges* of likelihood ratios, lr_1 and lr_2 . In the first case, the minimum and the maximum are as follows:

$$\min(lr_1) = .95/.07 \approx 13.57 \quad \max(lr_1) = .85/.13 \approx 6.53$$

⁷The evidence law scholars Henry Wigmore in his monumental treatise on the law of evidence famously asserted that ‘cross-examination is greatest legal engine ever invented for the discovery of truth.’ This assertion has been subject to little empirical testing.

⁸A showup refers to the observation of a single suspect by a witness in the field, typically at the crime scene. A lineup refers to the presentation of the suspect and several foils, either live or via photographs.

⁹The extent to which initial high confidence under pristine conditions is indicative of accuracy depends on the base rate of target-present lineups. In lab studies the base rate is about 50%, but in real-life circumstances, the best estimate is about 35%.

so the expert can, say, testify that the likelihood ratio is in the range of 6.5-13.5. In the second scenario, the minimum and the maximum are as follows:

$$\min(lr_1) = .85/.2 = 4.25 \quad \max(lr_1) = .75/.3 \approx 2.5$$

so the expert can testify that the likelihood ratio is in the range of 2.5-4.25.

Now, suppose further evidence is put forward to the effect that the suspect blood type matches the crime scene sample. Say the probability of a match if the suspect is the source is simply 1, while the probability of a random match is .05. The likelihood of this evidence alone is $1/.05 = 20$. Assuming independence, the joint likelihood for total evidence is obtained by multiplying separate likelihood ratios. Now, without any commitment to the priors, the impact of likelihood ratio ranges on any prior probability can be quantified and visualized, as in Figure 6.

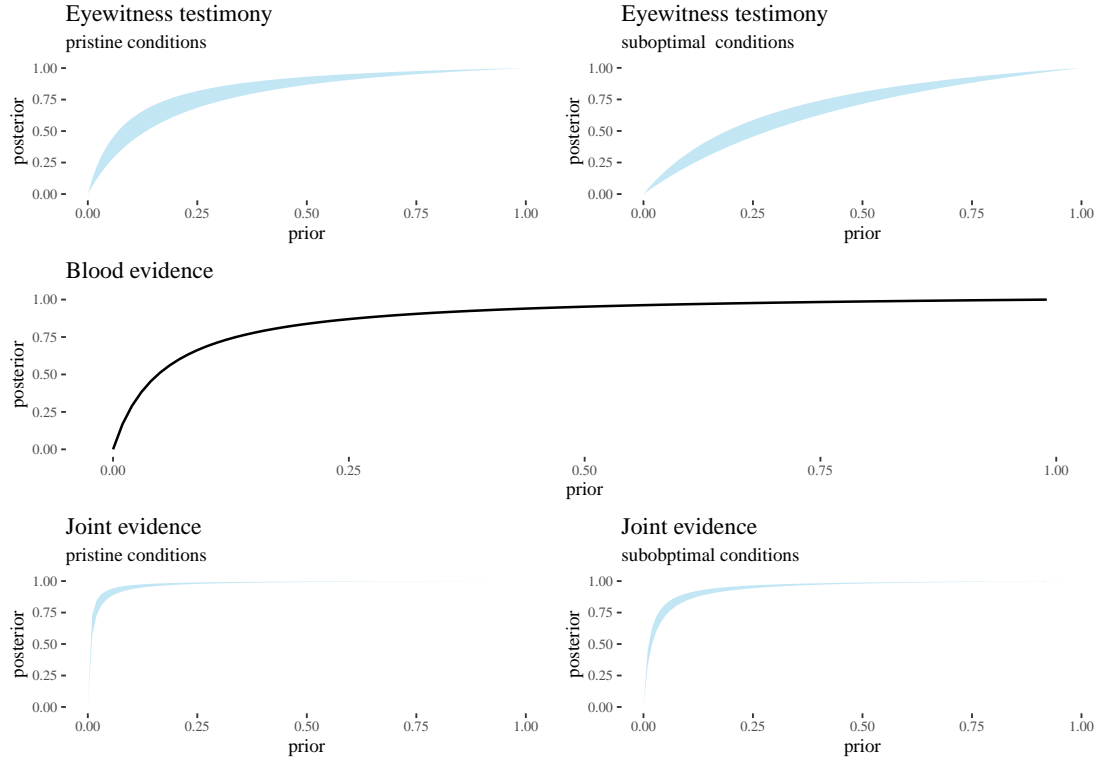


Figure 6: Impact of converging items of evidence on the posteriors.

What about conflicting evidence? Suppose this time the conditions were pristine, the expert's evaluation of the eyewitness evidence in pristine conditions is as before, but the witness identified someone else than the suspect (evidence E), while DNA evidence (evidence D) supports the prosecution hypothesis H with $LR = 300$ (this is, say, because we take the false positive probability seriously). Then, the range of plausible likelihood ratios for eyewitness evidence alone is:

$$\left[\frac{.07}{.95}, \frac{.13}{.85} \right] \approx [.073, .15]$$

In contrast, if the eyewitness is a friend of the suspect, so that you estimate $P(E|H)$ to be .6, while the conditions were sub-optimal, so that $P(E|\neg H) = .8 \pm .05$, the likelihood ratio range is .7 – .8 and the impact of such eyewitness evidence is quite different.

The key observation here is that whether exculpatory eyewitness testimony can in fact be sufficient to acquit in light of incriminating DNA evidence depends on the particulars: both on the quality of the eyewitness testimony and on the realistic assessment of the likelihood ratio for the DNA evidence (incorporating the probability of false positives), and on what priors other pieces of evidence led us to

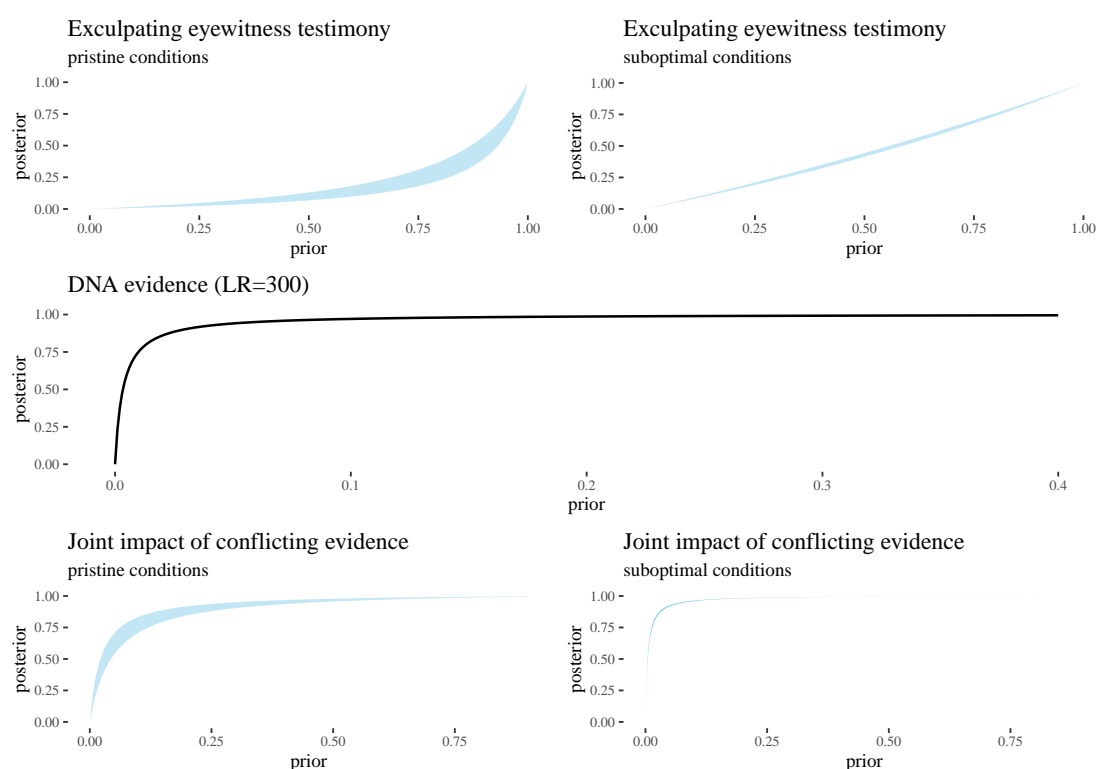


Figure 7: Impact of conflicting items of evidence on the posteriors.

so far prior to the evaluation of such items of evidence. The devil is in the detail, and it is not always the case that one should trump the other.

The reader might be disappointed: simply saying “it depends” does not seem to help much. However, keep in mind that there are well known items in the literature, on one hand, in psychology, illustrating that human jurors tend to value eyewitness evidence over statistical evidence (Niedermeier, Kerr, & Messeé, 1999; Wells, 1992; Wells & Olson, 2003), and, on the other hand, in epistemology, attempting to explain this intuition in a philosophically principled manner (Bolinger, 2020; Smith, 2017; Thomson, 1986). In line with a more balanced view (Redmayne, 2008), our approach suggests that while the psychological effect exists, there are multiple cases in which the preference for eyewitness evidence is mistaken, and relegates the identification and evaluation of factors that contribute to its proper evaluation to actual empirical research. If the situation is complicated, the best that can be done is take its complexity seriously, and quantitative methods are much more fit for this task than juror’s intuitions or general philosophical statements.

Here is an example of real-life complexities that the considerations in this section can clarify to some extent. In *The People of the State of New York v. Bansheen Rush* (Supreme Court of New York, 1995), a rape victim identified the defendant from photographic a few weeks after the event, identified the defendant in a corporeal lineup two weeks later, but at trial identified a spectator as her assailant, so the previous identifying evidence has been precluded.¹⁰ The defendant had been seen in the vicinity of the crime location three days before the crime. A DNA expert provided a positive match and testified that the random match probability is 1 in 500 million. The court found the DNA evidence alone to legally sufficient to support the guilty verdict.

In light of our discussion so far, this deserves a few remarks. First, the circumstances of the first two are not described in sufficient detail to decide how the existing research applies. To correctly deploy the results, information would be needed on how much the two first identification conditions departed from pristine conditions and in what respects. Moreover, it is known that the quality of testimony decreases with time, and is even less reliable in a stressful situation such as the trial. So, overall, the three

¹⁰Interestingly, Articles 60.25 and 60.30 of the *Rules of Evidence and Related Matters* referred to at this point by the court do not say anything about mixed eyewitness evidence of this sort: they only define eyewitness evidence as requiring recognition after the event in a proper procedure (not at trial).

R: try to come up with more references in this passage, think about this.

testimonies jointly still constituted positive identifying evidence, albeit weakened by the mistake (made in the most stressful conditions and with much more time between the crime and the identification), but without further details, it is hard to judge the extent or estimate the joint reliability quantitatively. Second, while the random match probability has been fairly low, some doubts remain as no estimates of false positive probability are available, and so it is difficult to estimate its impact on the final likelihood ratio of the DNA evidence.

Again, this might be disappointing, if the reader expected a precise numerical evaluation. However, this is still far from useless: it suggests that the eyewitness evidence preclusion was over-cautions, that more attention should have been paid to the detailed circumstances of the first two identifications, and that more reliable research is needed on false positive probabilities in DNA identifications for such cases to facilitate the handling of such unusual cases. If these elements had been in place, one could, in principle, be more specific about the value of the joint evidence.

The discussion in this section points towards an approach that we will be developing in other chapters. The evaluation of eyewitness testimony turns out to be quite complicated and depends on various factors. This multivariate nature of the problem makes Bayesian networks a plausible tool for the representation of the factors at play and for the assessment of how they interact. Moreover, eyewitness evidence is often subject to cross-examination. While in general it is meant to elicit further information that would change the evaluation of the evidence, there are a few different ways the situation might develop, and several argumentative moves that can be made in such a context. On one hand, the additional evidence obtained can **rebutt** the eyewitness evidence, if it supports a hypothesis incompatible with the original one, as might happen when the eyewitness makes a statement contradicting something they have said previously. On the other, the new evidence might **udercut** the eyewitness testimony if it leads to the re-evaluation of the witness's reliability without leading to a new hypothesis, as might happen when the eyewitness provides new evidence about the lightning conditions. Such moves, again, can be suitably modeled in terms of Bayesian networks (Di Bello, 2021). Here we just mention this connection to Bayesian networks, and it will be explored more extensively later on.

crossref

4. Hypothesis choice

The point so far is that the likelihood ratio is a fruitful conceptual framework for assessing the strength of the evidence. However, its use is not devoid of challenges, and we move to a discussion of how they arise.

Now, we turn to a difficulty with the use of likelihood ratio in the context of legal fact-finding: likelihood ratios is sensitive to the hypothesis choice. We discuss this problem in general in this section, (i) showing how an ad hoc hypothesis can always lead to a useless likelihood ratio, (ii) discussing a real case in which the choice of hypothesis was fairly confusing and had impact on evidence evaluation, (iii) indicating that the problem doesn't arise if the hypotheses are exclusive and exhaustive, and (iv) arguing that this theoretical resolution may be unhelpful in practice, as there seem often to be good reasons why experts do not use such pairs of hypotheses. Further, in Section 5 we further illustrate how large shifts in the likelihood ratio can take place even if the hypotheses chosen seem reasonable, and argue that another clear source of variability of likelihood ratio is the choice of hypothesis levels. The two sections taken together paint a difficulty with using likelihood ratio alone as a measure of evidential strength and suggest that a more elaborate framework, employing likelihood ratios, but not restricted to their reporting, would be more useful.

One major difficulty, however, is the choice of the hypotheses H and H' that should be compared. Generally speaking, the hypotheses should in some sense compete with one another—say, in a criminal trial, H is the hypothesis put forward by the prosecution and H' is the hypothesis put forward by the defense. Presumably, the two hypotheses should be something that the two parties disagree about. But this minimal constraint offers too little guidance and leaves open the possibility for manipulations and misinterpretations of the evidence. What follows outlines some of the main arguments in the literature on this topic.

Consider a stylized DNA evidence case. Suppose the prosecutor puts forward the hypothesis that the suspect left the traces found at the crime scene. This hypothesis is well supported by laboratory analyses showing that the defendant genetically matches the traces.

The defense, however, responds by putting forward the following *ad hoc* hypothesis: 'The crime stain was left by some unknown person who happened to have the same genotype as the suspect.' Since the

probability of the DNA match given either hypothesis is 1, the likelihood ratio equals 1 (Evetts, Jackson, & Lambert, 2000). The problem generalizes. For any item of evidence and any given prosecutor's hypothesis H , there is an *ad hoc* competing hypothesis H^* such that $P(E|H)/P(E|H^*) = 1$.

Hypothesis H^* is simply a just-so hypothesis, one that is selected only because it explains the evidence just as well as hypothesis H does (Mayo, 2018). If no further constraints are placed on the choice of the competing hypotheses—it would seem—no evidence could ever incriminate a defendant. This is unsettling.

One reply might be that this phenomenon need not be so damning in practice. Judges and jurors, after all, will often recognize *ad hoc* hypotheses for what they are—artificial theories that should not be taken seriously. Perhaps, the reasonable expectations of the participants in a trial will suffice to constrain the choice of hypotheses in just the right way.

One issue with this reply is that it is not principled. Fair enough, maybe in practice fact-finders will avoid *ad hoc* hypotheses. But why should they avoid them, other than because otherwise the resulting likelihood ratio will be useless? Moreover, as we will soon see, hypothesis choice has impact on likelihood ratio even if no *ad hoc* hypothesis seems to be used. Moreover, real cases tend to be quite complex, and it is not always obvious whether a certain choice of competing hypotheses, none of which are obviously *ad hoc*, is legitimate or not. In what follows we will illustrate some of these issues, and argue that due to this sensitivity, likelihood ratio alone is not informative enough, if not supplied by careful considerations of hypothesis choice.

Here is an example that illustrates how even when the competing hypotheses are not obviously *ad hoc*, the absence of a clear rationale for their choice may create confusions in the assessment of the evidence. In *R. v. Barry George* (2007 EWCA Crim 2722). Barry George was accused of murdering TV celebrity Jill Dando. A key piece of evidence at play was:

- E A single particle of firearm residue (FDR) was found one year later in George's coat pocket and it matched the residue from the crime scene. This was the key incriminating evidence against him.

The defense argued that, since it was only one particle, there must have been contamination. The experts for the prosecution, however, testified that it was not unusual that a single particle would be found on the person who fired the gun. George was convicted, and his first appeal was unsuccessful.

After the first appeal, Dr. Evett from the Forensic Science Service worried that the evidence had not been properly assessed at trial. The jurors were presented with the conditional probability $P(\text{residue}|H_d)$ of finding the firearm residue in George's coat given the defense hypothesis H_d that George *did not* fire the gun. This probability was estimated to be quite low, indicating that the evidence spoke against the defense's hypothesis. But the jurors were not presented with the conditional probability $P(\text{residue}|H_p)$ of finding the same evidence given the prosecutor's hypothesis H_p that George *did* fire the gun that shot Dando. An expert witness, Mr. Keeley, was asked to provide both conditional probabilities and estimated them to be $1/100$, which indicated that the firearm residue had no probative value. After new guidelines for reporting low level FDR in 2006, the FSS re-assessed the evidence and concluded that it was irrelevant. George appealed again in 2007, and relying on Keeley's estimates, won the appeal.

At first, this case seems a good illustration of how likelihood ratios help to correctly assess the value of the evidence presented at trial. But this reading of the case would be overly optimistic. In fact, a close study of the trial transcript shows that Keeley's choice of hypotheses was not systematic and the likelihood ratio based on them was therefore really hard to interpret (Fenton, Berger, Lagnado, Neil, & Hsu, 2014). For instance, Mr Keeley is reported to have said:

- It was necessary to balance the likelihood that the particle came from a gun fired by the appellant and the likelihood that it came from some other source. Both were unlikely but both were possible.

Keeley compared the hypothesis that the particle found in George's pocket came from a gun fired by George himself, and the alternative hypothesis that the particle came from another source. In line with the quotation, Keeley said that the prior probabilities of both hypotheses should be low. But this is mathematically impossible if they were exhaustive and exclusive.

On another occasion, Keeley took the prosecutor's hypothesis to be 'The particle found in George's pocket came from the gun that killed Dando' and the defense hypothesis to be 'The particle on George's pocket was inserted by contamination.' The problem is that the evidence is a logical consequence of

either of them, so the conditional probability of the evidence given each of these hypothesis is one. Crucially, they are therefore useless for the evaluation of the weight of evidence, because in such case the likelihood ratio will always be one for trivial reasons. The most charitable reading of the trial transcript suggests that the expert had in mind the hypotheses ‘George was the man who shot Dando’ and ‘The integrity of George’s coat was corrupted.’ But these hypotheses are neither exhaustive nor exclusive, and Keeley gave no clear criterion for why these hypotheses should be compared in the likelihood ratio (see Fenton, Berger, Lagnado, Neil, & Hsu, 2014 for further details).

The confusion in the Barry George case is attributable to the absence of clear rules for choosing the hypotheses in the likelihood ratio. One such rule could be: pick competing hypotheses that are exclusive (they cannot be both true) and exhaustive (they cannot be both false). In this way, the parties would not be able to pick *ad hoc* hypotheses and skew the assessment of the evidence in their own favor.

Besides blocking partisan interpretations of the evidence, there are other principled reasons to follow the exclusive-and-exhaustive rule, specifically, the fact that when the hypotheses are not exclusive or exhaustive, the likelihood ratio might deliver counter-intuitive results and cause confusion in the assessment of the strength of the evidence. If two competing hypotheses, H_p and H_d are not mutually exclusive, it is possible that they both make the evidence equally likely (the likelihood ratio is one), and yet the posterior probabilities of the hypotheses given the evidence are higher than their prior probabilities.

For instance, let H_p stand for ‘The defendant is guilty’ and H_d for ‘The defendant was not at the crime scene’. Both hypotheses might be true. Let E stand for ‘Ten minutes before the crime took place the defendant—seen at a different location—was overheard on the phone saying *go ahead and kill him*.’ It is conceivable that the likelihood ratio should equal one in this context, yet the posterior probabilities of each hypothesis, given E , should be higher than the prior probability. So, intuitively, the evidence should positively support each hypothesis, contrary to what the likelihood ratio would suggest.

Further, when the two competing hypotheses are not exhaustive, the likelihood ratio may once again clash with our intuitions. The likelihood ratio might then equal one even though the evidence lowers their posterior probability. For example, suppose Fred and Bill attempted to rob a man. The victim resisted, was struck on the head and died. Say H_p stand for ‘Fred struck the fatal blow’ and H_d stand for ‘Bill struck the fatal blow.’ The hypotheses are not exhaustive. A missing hypothesis is ‘The man did not die from the blow.’ Suppose E is the information that the victim had a heart attack six months earlier. The likelihood ratio $P(E|H_p)/P(E|H_d)$ equals one since $P(E|H_p) = P(E|H_d)$. Yet E reduces the probability of both H_p and H_d . So, in this case, the evidence should negatively support each hypothesis, contrary to what the likelihood ratio suggests.

Here is a challenge that we need to face when using likelihood ratios as measures of evidential strength: conceptually, it seems ideal for the interpretability of the result that a hypothesis be compared with its negation. However, practically, this might not be viable, as often there are multiple important different ways a hypothesis can turn out to be false, and the assessment of the probability given simply its negation is quite challenging.

In other words, whether the exhaustive-and-exclusive rule would be a good guiding principle, however, is not clear-cut. Requiring that the hypotheses be always exclusive and exhaustive hypotheses is not without complications either. For consider an expert who decides to formulate the defense hypothesis by negating the prosecution hypothesis, say, ‘the defendant did not hit the victim in the head.’ This choice of defense hypothesis can be unhelpful in assessing the evidence, because the required probabilities are hard to estimate. For instance, what is the probability that the suspect would carry such and such blood stain if he did not hit the victim in the head? This depends on whether he was present at the scene, what he was doing at the time and many other circumstances.

(Reader warning: this passage will discuss hypothesis choice in a rape case.) Similarly, in a rape case, it is hard to estimate the probability of the matching evidence if the suspect did not have the intercourse with the victim. Instead, what is considered is the hypothesis that someone else, unrelated to the suspect, had intercourse with the victim. As (Evetts, Jackson, & Lambert, 2000) point out, in many real life rape cases the choice of a particular hypothesis to be used by the expert in the evaluation of the strength of the evidence (of, say, the lack of semen in a rape case), will depend on contextual factors. Sometimes it will be ‘intercourse did not take place,’ sometimes it will be ‘the intercourse took place, but the complainant used a vagina douche,’ or sometimes ‘another sexual act took place.’ More often than not, the hypotheses chosen will not be mutually exclusive.

Moreover, comparing exclusive and exhaustive hypotheses can also be unhelpful for jurors or judges

R: added this passage in light of Sophie’s comments

making a decision at trial. In a paternity case, for example, the expert should not compare the hypotheses ‘The accused is the father of the child’ and its negation, but rather, ‘The accused is the father of the child’ and ‘The father of the child is a man unrelated to the putative father’ (Biedermann, Hicks, Taroni, Champod, & Aitken, 2014). The choice of the latter pair of competing hypotheses is preferable. Even though the relatives of the accused are potential fathers, considering such a far-fetched possibility would make the assessment of the evidence more difficult than needed. At the same time, if the defense hypothesis is too specific, *ad hoc* and entails the evidence, it won’t be of much use. For example, take ‘The crime stain was left by some unknown person who happened to have the same genotype as the suspect.’ The probability of a DNA match given this hypothesis would be 1. But usually the probability of the DNA match given the prosecution’s hypothesis, say ‘The crime stain was left by the suspect,’ is also 1. This would result in a rather uninformative likelihood ratio of 1. Another feature of such specific explanations is that it’s hard to reasonably estimate their prior probability, and so hard to use them in arguments between opposing sides. (Evvett, Jackson, & Lambert, 2000).

So, it seems, the choice of competing hypotheses lies between two extremes. Exclusive and exhaustive hypotheses guard against *ad hoc* hypotheses, arbitrary comparisons and ensure a more objective assessment of the evidence. Unfortunately, exhaustive and exclusive hypothesis cover the entire space of possibilities, and sifting through this space may be cognitively unfeasible. So, in this respect, comparing more circumscribed hypotheses is preferable. The danger of doing so, however, is slipping into arbitrariness as likelihood ratios heavily depend on the hypotheses that are compared. The more latitude in the choice of the hypotheses, the more variable the likelihood ratio as a measure of evidential value. This is a particularly troubling phenomenon, as competing hypotheses can concern any factual dispute, from minute details such as whether the cloth used to suffocate the victim was red or blue, to ultimate questions such as whether the defendant stabbed the victim.

To add another complication, the likelihood ratio varies across hypotheses formulated at different levels of granularity: offense, activity and source level hypotheses. It is even possible that, at the source level, the likelihood ratio favors one side, say the prosecution, but at the offence level, the likelihood ratio favors the other side, say the defense, even though the hypotheses at the two levels are quite similar. Further, a likelihood ratio that equals 1 when source level hypotheses are compared may tip in favor of one side or the other when offence level hypotheses are compared (Fenton, Berger, Lagnado, Neil, & Hsu, 2014). This variability makes the likelihood ratio a seemingly arbitrary—and easily manipulable—measure of evidential value.¹¹

The likelihood ratio can be misleading, but this risk is mitigated when its assessment is accompanied by a careful discussion of a number of issues, such as: which hypotheses are being compared; how they are formulated; their level of granularity (that is, source, activity and offense level); why the hypotheses are (or are not) exclusive and exhaustive, and why other hypotheses are ruled out as unworthy of consideration.

This is our first warning sign when it comes to relying on likelihood ratios. In real-life applications, multiple hypotheses and pairs thereof are at play, and their choice does matter. For this reason, merely reporting likelihood ratios might be misleading,¹² and such a presentation of evidence is best accompanied by a careful discussion of which hypotheses were considered, which were chosen and why, and how they all are related to the evidence. Later on we will argue that a clear treatment of such issues is immensely assisted by the use of Bayesian networks in evidence presentation and evaluation. This will be a recurring point in this book.

5. Levels of Hypotheses and the two-stain problem

Since we want to put likelihood ratios in their proper place when it comes to evidence evaluation and reporting, we would like to obtain a deeper understanding of the factors that might lead to their variation

¹¹ Note that in our terminology the offence hypothesis is still factual: it represents the facts that according to the law should be established in order to a certain legal decision to be applicable. Note also that the supposed fact-law distinction is quite tricky, with some (Allen & Pardo, 2003) arguing that much of the effort to properly delineate matters as questions of law or fact is doomed, as there is no essential difference. On this approach, there are no significant epistemological or analytical differences between the concepts; there are only pragmatic differences, which are reflected in the three dichotomies of the conventional meaning of the terms, the judge-jury relationship, and the general-specific spectrum, and legal questions are part of the more general category of factual questions.

¹² Similarly, BF would be sensitive to the choice of the prosecution hypothesis, but since BF faces other problems we already discussed, we will not get into this issue.

You keep mentioning this dispute between Taroni research group and Fenton research group about LR, but I never got the impression of deep disagreement. Maybe I missed something, do you have any specific papers in mind?

R added footnote here, check.

depending on the hypothesis choice. let's take a closer look at one systematic dimension of hypothesis choice sensitivity: likelihood ratios change with the levels of hypothesis under consideration. In this section we take a closer look at this issue.

To some approximation, hypotheses can be divided into three **levels: offence, activity, and source** level hypotheses. At the offence level, the issue is one of guilt or innocence, as in the statement 'Smith intentionally attacked the victim with a knife.' At the activity level, hypotheses do not include information about intent but simply describe what happened and what those involved did or did not do. An example of activity level hypothesis is 'Smith bled at the scene.' Finally, source level hypotheses describe the source of the traces, such as 'Smith left the stains at the crime scene,' without specifying how the traces got there. Overlooking differences in hypothesis level can lead to serious confusions. To illustrate, consider a case in which a DNA match is the primary incriminating evidence. In testifying about the DNA match at trial, experts will often assess the probability that a random person, unrelated to the crime, would coincidentally match the crime stain profile¹³ The random match probability is often an impressively low number, say 1 in 100 million or lower, at least excluding the possibility that relatives or identical twins would coincidentally match (Donnelly, 1995). This should count as a strong evidence against the suspect. But how exactly? RMP—the probability that a random person from the population matches the crime stain profile—is taken to be the probability that the suspect is a match if in fact he is innocent and is usually estimated as the frequency of a given profile in the relevant population. As we already know from the chapter in which we discussed probabilistic fallacies, RMP is not the posterior probability of innocence, since $P(\text{match}|\text{innocence})$ should not be confused with $P(\text{innocence}|\text{match})$. To confuse the two would be to commit the prosecutor's fallacy. Further, it is tempting to equate the random match probability to $P(\text{match}|\text{innocence})$ and together with the prior $P(\text{innocence})$ use Bayes' theorem to calculate the posterior probability of innocence $P(\text{innocence}|\text{match})$. But this also might be a mistake. Equating the random match probability with $P(\text{match}|\text{innocence})$ overlooks the difference between offense, activity and source level hypothesis. It is hasty to assume that, in one way or another, a DNA match can speak directly to the question of guilt or innocence. Even if the suspect actually left the genetic material at the scene—source level proposition—the match does not establish guilt. Even if the defendant did visit the scene and came into contact with the victim, it does not follow that he committed the crime he was accused of. It is true, that in many circumstances the random match probability and the posterior probability of innocence given a match would both be very low, but such issues need to be considered and took into consideration in DNA evidence evaluation.

Few forms of evidence can speak directly to offense level hypotheses. Circumstantial evidence that is more amenable to a probabilistic quantification, such as DNA matches and other trace evidence, does not. Eyewitness testimony may speak more directly to offense level hypotheses, but it is also less easily amenable to a probabilistic quantification. This makes it difficult to assign probabilities to offense level hypotheses. Experts are usually not supposed to comment directly on offense level hypotheses, but they often comment on activity level and source level hypotheses. In moving from source to activity level, however, additional sources of uncertainty come into play. The assessment of activity level hypotheses depends on additional variables other than those on which the assessment of source level hypotheses depends. For example, the probability of finding such and such quantity of matching glass if the suspect smashed the window depends on how the window was smashed, when it was smashed, and what the suspect did after the action. Another problem arises due to recent improvements in DNA profiling technology. Since today investigators are able to obtain profiles from minimal amounts of genetic material, transfer probabilities (that is, probabilities that the material has been transferred accidentally, rather due to the source's participation in a crime under consideration) become more difficult to assess, as more opportunities of transfer arise. If small traces such as dust speckles can be used as evidence, the possibility that the traces were brought to the scene accidentally becomes more likely. For this reason, moving beyond source level hypotheses requires a close collaboration between scientists, investigators and attorneys (see Cook, Evett, Jackson, & Jones, 1998 for a discussion). The choice and formulation of the hypotheses are up for revision as new evidence is obtained or facts about what happened are accepted (Evett, Jackson, & Lambert, 2000). A case study that further illustrates both advantages and limitations of the likelihood ratio as a measure of evidential strength is the two-stain problem, originally formulated by Evett (1987). The key limitation is due to the combination of two circumstances: first, that likelihood ratios vary depending on the choice of hypotheses being compared; second, that it is

R: clarified transfer probabilities

¹³For a survey of developments and complications of this model, see (Foreman et al., 2003).

not always clear which hypotheses should be compared. To illustrate what is at stake, what follows begins with Evett's original version of the two-stain problem (which does not pose any challenge to the likelihood ratio) and then turns to a more complex version (which suggests that likelihood ratios, in and of themselves, are insufficiently informative). Suppose two stains from two different sources were left at the crime scene, and the suspect's blood matches one of them. More precisely, the two items of evidence are as follows:

- E_1 The blood stains at the crime scene are of types γ_1 and γ_2 of estimated frequencies q_1 and q_2 respectively.
- E_2 The suspect's blood type is γ_1 .

Let the first hypothesis be that the suspect was one of the two men who committed the crime and the second hypothesis the negation of the first.

- H_p The suspect was one of the two men who committed the crime.
- H_d The suspect was not one of the two men who committed the crime.

Evett (1987) shows that the likelihood ratio of the match relative to these two hypotheses is $1/2q_1$ where q_1 is the estimated frequency of the characteristics of the first stain. Surprisingly, the likelihood ratio does not depend on the frequency associated with the second stain. To understand Evett's argument, consider first the likelihood ratio:

$$\frac{P(E_1 \wedge E_2 | H_p)}{P(E_1 \wedge E_2 | H_d)} = \frac{P(E_1 | E_2 \wedge H_p)}{P(E_1 | E_2 \wedge H_d)} \times \frac{P(E_2 | H_p)}{P(E_2 | H_d)}.$$

Notice that the suspect's blood type as reported in E_2 is independent of whether or not he participated in the crime, that is, $P(E_2 | H_p) = P(E_2 | H_d)$. So the likelihood reduces to:

$$\frac{P(E_1 \wedge E_2 | H_p)}{P(E_1 \wedge E_2 | H_d)} = \frac{P(E_1 | E_2 \wedge H_p)}{P(E_1 | E_2 \wedge H_d)}.$$

The numerator $P(E_1 | E_2 \wedge H_p)$ is the probability that one of the stains is γ_1 and the other γ_2 given that the suspect is guilty and has profile γ_1 . The probability that one of the stains is γ_1 is simply 1, and assuming blood type does not affect someone's propensity to commit a crime, the probability that the second stain is γ_2 equals its relative frequency in the population, q_2 . So the numerator is $1 \times q_2 = q_2$. % Next, consider the denominator $P(E_1 | E_2 \wedge H_d)$. If H_d is true, the fact that the suspect has profile γ_1 is irrelevant for the crime scene profiles. The crime was committed by two randomly selected men with profiles γ_1 and γ_2 , who can be seen as two random samples from the general population as far as their blood profiles are concerned. There are two ways of picking two men with such profiles (γ_1, γ_2 and γ_2, γ_1), each having probability $q_1 q_2$. So the denominator equals $2q_1 q_2$. By putting numerator and denominator together, we have:

$$\frac{q_2}{2q_1 q_2} = \frac{1}{2q_1}.$$

which completes the argument. In general, if there are n bloodstains of different phenotypes, the likelihood ratio is $1/nq_1$, or in other words, the likelihood ratio depends on the number of stains but not on the frequency of the other characteristics.

Consider now a more complex two-stain scenario. Suppose a crime was committed by two people, who left two stains at the crime scene: one on a pillow and another on a sheet. John Smith, who was arrested for a different reason, genetically matches the DNA on the pillow, but not the one on the sheet. What likelihood ratio should we assign to the DNA match in question? Meester & Sjerps (2004a) argue that there are three plausible pairs of hypotheses associated with numerically different likelihood ratios (see their paper for the derivations). The three options are listed below, where R is the random match probability of Smith's genetic profile and δ the prior probability that Smith was one of the crime scene donors.

| H_p | H_d | LR |
|--|--|---------------------------|
| Smith was one of the crime scene donors. | Smith was not one of the crime scene donors. | $R/2$ |
| Smith was the pillow stain donor. | Smith was not one of the crime scene donors. | R |
| Smith was the pillow stain donor. | Smith was not the pillow stain donor. | $R(2-\delta)/2(1-\delta)$ |

Two facts are worth noting here. First, even though the likelihood ratios associated with the hypotheses in the table above are numerically different, the hypotheses are in fact equivalent conditional on the evidence. After all, Smith was one of the crime scene donors just in case he was the pillow stain donor, because he is excluded as the stain sheet donor. Smith was not one of the crime scene donors just in case he was not the pillow stain donor, because he is excluded as the sheet stain donor. Second, the example illustrates that sometimes the likelihood ratio is sensitive to the prior probability (after all, δ occurs in the third likelihood ratio in the table).

In addition, even though the likelihood ratios are numerically different, their posterior probabilities given the evidence are the same.

To see why, note that the prior odds of the three H_p 's in the table should be written in terms of δ . Following Meester & Sjerps (2004a), the prior odds of the first hypothesis in the table are $\delta/1-\delta$. The prior odds of the second hypothesis are $(\delta/2)/(1-\delta)$. The prior odds of the third hypothesis are $(\delta/2)/(1-(\delta/2))$. In each case, the posterior odds — the result of multiplying the prior odds by the likelihood ratio — are the same: $R \times \delta/2(1-\delta)$. So despite differences in the likelihood ratio, the posterior odds of equivalent hypotheses are the same so long as the priors are appropriately related (this point holds generally).

Dawid (2004) cautions that the equivalence of hypotheses, conditional on the evidence, does not imply that they can all be presented in court. He argues that the only natural hypothesis for the two-stain problem is that Smith is guilty as charged. Meester & Sjerps (2004b) reply that focusing on the guilt hypothesis is beyond the competence of expert witnesses who should rather select pairs of hypotheses on which they are competent to comment. Some such pairs of hypotheses, however, will not be exclusive and exhaustive. When this happens, as seen earlier, the selection of hypotheses is prone to arbitrariness. To avoid this problem, Meester & Sjerps (2004a) recommend that the likelihood ratio should be accompanied by a tabular account of how a choice of prior odds (or prior probabilities) will impact the posterior odds, for a sensible range of priors. In this way, the impact of the likelihood ratio is made clear, no matter the hypotheses chosen. This strategy concedes that likelihood ratios, in and of themselves, are insufficiently informative, and that they should be combined with other information, such as a range of priors, to allow for an adequate assessment of the evidence.¹⁴

The sensitivity of the likelihood ratio to the choice of hypotheses is not confined to the two-stain problem or alike scenarios. Recall our discussion of DNA matches in cold-hit cases. When the suspect is identified through a database search of different profiles, Taroni, Biedermann, Bozza, Garbolino, & Aitken (2014) and David J. Balding & Donnelly (1996) have argued that the likelihood ratio of the match—which usually equals $1/\gamma$ where γ is the random match probability—should be adjusted by the database search ratio. This proposal tacitly assumes that the hypothesis of interest is something like ‘the defendant is the true source of the crime traces.’ This assumption is eminently plausible but not uncontroversial.

The National Research Council (NRC II) recommended in 1996 that the likelihood ratio of the match $1/\gamma$ be divided by the size of the database. In defending this proposal, Stockmarr (1999) argues the likelihood ratio of the match in cold-hit cases should be divided by the size of the database. Stockmarr believes we should evaluate the likelihood ratio using hypotheses that can be formulated prior to the database search, such as ‘The true source of the crime traces is among the suspects in the database,’ while others insist on using ‘The defendant is the true source of the crime traces.’ Now, interestingly, while these approaches lead to different LR evaluations, they are equivalent conditional on the evidence: given the same evidence, they read to the same posterior. This is another example of how likelihood ratios on their own might be insufficiently informative to allow for an adequate assessment of the evidence. We will discuss these issues in more depth later on. First, we will look at an argument against likelihood ratio being useful as a measure of evidential relevance, as dealing with it is conceptually more straightforward.

This section emphasizes the warning message we left you with at the end of the previous section: even if likelihood ratios are legitimate tool of evidence evaluation, they should be used with care, and

¹⁴The reference class problem is lurking in the background. David J. Balding (2004) argues that, in order to calculate the probability of a match given the evidence, the class of possible culprits should be identified, and different choices of such a class might lead to different likelihood ratios. On the problem of priors see. On the reference class problem, see Chapter XXX.

accompanied with an explicit discussion how they fit into a larger picture of various items of evidence and hypotheses at play. Again, we will argue that a prominent way to do so is to employ Bayesian networks in evidence presentation and evaluation.

6. Relevance and the small-town murder scenario

Another objection to likelihood ratios is that they might suggest evidence is irrelevant even if reported with respect to a pair of exclusive and exhaustive hypotheses. The objection indeed sometimes applies when the key hypotheses are the main guilt hypothesis and its negation. But we will argue that this is not an objection to the use of likelihood ratios, but rather against reporting them in isolation. When justice is done to the complexities at play, appropriate likelihood ratios still evaluate such evidence as relevant. Again, the point is that there is no escape from using a more elaborate evidence presentation tool than just reporting likelihood ratios in isolation.

The U.S. Federal Rules of Evidence define relevant evidence as one that has ‘any tendency to make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without the evidence’ (rule 401). This definition is formulated in a probabilistic language. Legal probabilists interpret it by relying on the likelihood ratio, a standard probabilistic measure of evidential relevance [Lempert (1977); Lyon1996relevance; aitken2004statistics; aitken2010fundamentals; sullivan2016LikelihoodStoryTheory]. The likelihood ratio is the probability of observing the evidence given that the prosecutor’s or plaintiff’s hypothesis is true, divided by the probability of observing the same evidence given that the defense’s hypothesis is true. Let E be the evidence, H the prosecutor’s or plaintiff’s hypothesis, and H' the defense’s hypothesis. Recall that the likelihood ratio, $LR(E, H, H')$, is defined as follows:

$$LR(E, H, H') = \frac{P(E|H)}{P(E|H')}$$

On this interpretation, relevance depends on the choice of the competing hypotheses. H_p and H_d are used as examples, but other competing hypotheses H and H' could also be used. When there are no ambiguities, $LR(E, H_p, H_d)$ will be shortened into the less cumbersome $LR(E)$. On the approach under consideration, a piece of evidence is relevant—in relation to a pair of hypotheses H and H' —provided the likelihood ratio $LR(E, H, H')$ is different from one and irrelevant otherwise. For example, the bloody knife found in the suspect’s home is relevant evidence in favor of the prosecutor’s hypothesis because we think it is far more likely to find such evidence if the suspect committed the crime (prosecutor’s hypothesis) than if he did not (defense’s hypothesis) (Finkelstein, 2009). In general, for values greater than one, $LR(E, H, H') > 1$, the evidence supports the prosecutor’s or plaintiff’s hypothesis H , and for values below one, $LR(E, H, H') < 1$, the evidence supports the defense’s hypothesis H' . If the evidence is equally likely under either hypothesis, $LR(E, H, H') = 1$, the evidence is considered irrelevant.

This account of relevance has been challenged by cases in which the evidence is intuitively relevant and yet its likelihood ratio, arguably, equals one. Here is one of them (The difficulty has been formulated by Ronald Allen, see the multi-authored discussion in Park et al., 2010):

Small Town Murder. A person accused of murder in a small town was seen driving to the small town at a time prior to the murder. The prosecution’s theory is that he was driving there to commit the murder. The defense theory is an alibi: he was driving to the town because his mother lives there to visit her. The probability of this evidence if he is guilty equals that if he is innocent, and thus the likelihood ratio is 1 . . . , and under what is suggested as the “Bayesian” analysis, it is therefore irrelevant. Yet, every judge in every trial courtroom of the country would admit it [as relevant evidence] . . . and I think everyone on this list would say it is relevant. And so we have a puzzle.

Seeming counterexamples of this sort abound, here are a few of them:

- Suppose a prisoner and two guards had an altercation because the prisoner refused to return a food tray. The prisoner had not received a package sent to him by his family and kept the tray in protest. According to the defense, the prisoner was attacked by the guards, but according to the prosecution, he attacked the guards. The information about the package sent to the prisoner and the withholding of the tray fails to favor either version of the facts, yet it is relevant evidence (Pardo, 2013).

- In response to an eyewitness testimony the defendant claims that his identical twin is the culprit. The testimony is unable to favor any of the two options and yet is considered relevant.
- Suppose the evidence at issue is that a fight occurred and the only dispute is over who started it.
- Or suppose the defendant was stopped because of speeding three minutes after an aborted bank robbery and 1/2 a mile away from the site. The prosecution says this is evidence of guilt: it shows the defendant was escaping. The defense responds that this is evidence of innocence: no bank robber would speed and attract attention.
- Or, in a murder case, the defendant is the victim's son. Is that relevant to show he's guilty? Is it relevant to show he's innocent? The answer seems to be yes, to both questions (this example is due to Samuel Gross and is discussed in (Park et al., 2010)).

In general, there seem to be numerous examples in which evidence is, intuitively relevant, and the evidence supports neither side's theory over the other side's theory. How is such evidence to be judged relevant from the probabilist perspective?

In response (inspired by the ideas put forward in the discussion by David Kaye, Bruce Hay and Roger Park), note that it is true that if a piece of evidence E fits equally well with two competing hypotheses H and H' , then $P(E|H) = P(E|H')$ and thus $LR(E, H, H')$ will equal 1. But the likelihood ratio may change depending on the selection of hypotheses. Rule 401 makes clear that relevant evidence should have 'any tendency to make the existence of *any fact that is of consequence* [emphasis ours] to the determination of the action more probable or less probable.' So the range of hypotheses to compare should be quite broad. Just because the likelihood ratio equals one for a specific selection of H and H' , it does not follow that it equals one for *any* selection of H and H' which are of consequence to the determination of what happened. In *Small Town Murder*, whether the suspect was in town at all is surely of consequence for determining what happened (if he was not in town, he could not have committed the crime). The fact that he was seen driving is helpful information for establishing whether or not he was in town.

But if the range of hypotheses H and H' to compare in the likelihood ratio $LR(E, H, H')$ is quite broad, this may raise another concern. The choice of hypotheses needed to determine the relevance of an item of evidence might depend on other items of evidence, and so it might be difficult to determine relevance until one has heard all the evidence. This fact—Ronald Allen and Samuel Gross argue in (Park et al., 2010)—makes the probabilistic account of relevance impractical. But, in response, David Kaye points out that deciding whether a reasonable juror would find evidence E helpful requires only looking at what hypotheses or stories the juror would reasonably consider. Since the juror will rely on several clues about which stories are reasonable, this task is computationally easier than going over all possible combinations of hypotheses (Park et al., 2010).

Legal probabilists can also offer a more principled response to *Small Town Murder* and related problems based on Bayesian networks. Let H_p be the prosecutor's hypothesis that the defendant committed the murder, and H_d the defense's hypothesis that the defendant was visiting his mother. Let E be the fact that the defendant was seen driving to the town prior to the murder. For the sake of the example, suppose the prior probability of H_d is .5, the prior of H_p is .02, the conditional probability of E if H_d alone is true is .7, the conditional probability of E given H_p alone is true is .6 (say, the suspect is more likely to move fast and less likely to be seen if he's guilty), and the probability of E if both hypotheses are true is .8 (it is a bit higher if he visited two spots and therefore drove around a bit more), and the conditional probability of E is .05 if both are false (say, because he'd have no business being in the town if they are both false). The following Bayesian network can be used to calculate the relevant likelihood ratios.

Now, $P(E|H_d) = 0.604$, $P(E|\neg H_d) = 0.063$, so $LR(E, H_d, \neg H_d) \approx 9.587$, and $P(E|H_p) = 0.74$, $P(E|\neg H_p) = 0.27$, and so $LR(E, H_p, \neg H_p) \approx 3$. Crucially, this example is based on a pair of hypotheses that are neither mutually exclusive nor exhaustive. As it might in such a case happen, both likelihood ratios are above one, and so the evidence is not neutral, despite the apparent intuitions relied on in the counterexample. Moreover, while the evidence supports each of the hypotheses, the difference in likelihood ratios indicates that this is so to different extents.¹⁵

Now, perhaps, one could insist that one at least can think of a case in which the conditional probabilities: $P(E|H_d)$ and $P(E|H_p)$ on one hand, and $P(E|\neg H_d)$ and $P(E|\neg H_p)$, are exactly the same, in which

¹⁵de Zoete, Fenton, Noguchi, & Lagnado (2019) offer a slightly different solution to the problem. They construct a Bayesian network with three hypotheses, also exhaustive and exclusive: in town to visit mother, in town to murder, out of town. The gist of the resolution holds, though.

R: added this passage in light of Sophie's comments.

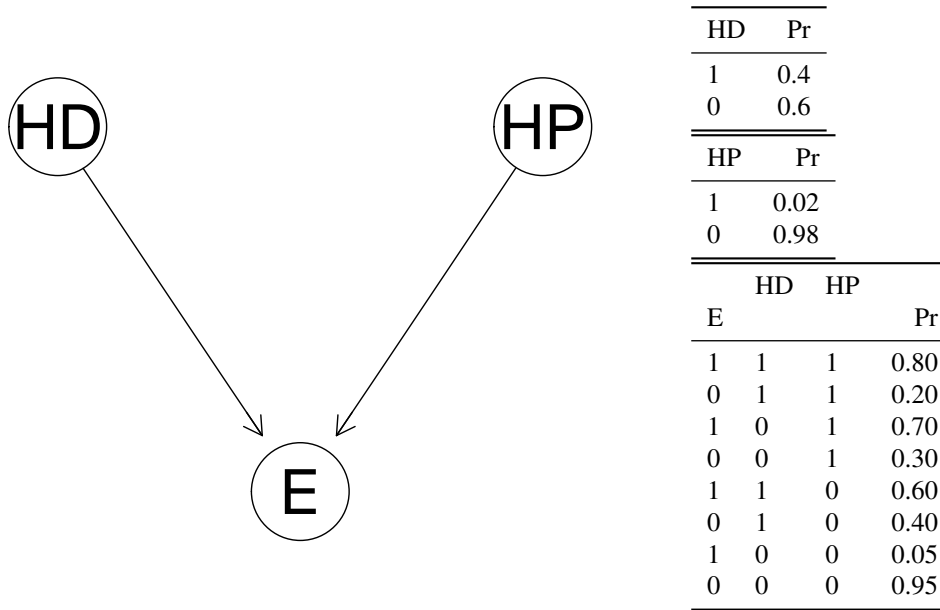


Figure 8: Bayesian network and CPTs for a small town murder scenario.

case the likelihood ratio calculations would fail to shift the probability balance between the hypothesis. First, such cases would be quite likely very artificial and unlikely to occur in reality. Second, even in such cases, we need to recall that Rule 401 talks about the existence of any fact that is of consequence to the determination of the action, and so it is enough that there is *some* relevant hypothesis whose determination becomes easier when the evidence is considered. In the artificial variant of the small town murder scenario, even if H_d and H_p have the same LR, there is another pair of relevant hypotheses that does not: *the suspect was in town* and its negation.

Analogous considerations should also generalize to other paradoxes of relevance. For instance, in the twins problem, the LR might be 1 if the hypotheses are: ‘the suspect committed the crime,’ and ‘the suspect’s twin brother committed the crime,’ but is not 1 if we consider the fairly natural hypothesis that the defendant is innocent.

Similarly, in the food tray example, Bayesian network analysis shows that the value of the evidence ‘prisoner withholds tray’ for the question who started the fight depends on a range of uncertain events and other pieces of evidence (such as whether indeed a parcel he was supposed to obtain was withheld; whether the prisoner inquired about this; whether and how this inquiry was answered). Considered in this context, the piece of evidence will not have a likelihood ratio of one with respect to at least some choice of sensible hypotheses.

The general problem with the paradoxes of relevance is that in complex situations there is no single likelihood ratio that corresponds to a single piece of evidence. The problematic scenarios focus on a single likelihood ratio based on non-exclusive or non-exhaustive hypotheses. However, evidence can be relevant so long as it has a probabilistic impact on a sub-hypothesis involved in the case, even without having a recognizable probabilistic impact on the prosecutor’s or defense’s ultimate hypotheses. When this happens, it is relevant, in agreement with Rule 401 of the Federal Rules of Evidence. Bayesian networks help to see how pieces of evidence can increase or decrease the probability of different sub-hypotheses (de Zoete, Fenton, Noguchi, & Lagnado, 2019). Thus, while likelihood ratios are useful, they need to be properly presented in the context.

7. Conclusions

We have introduced likelihood ratio as a measure of evidential strength, and argued that it is more fit for this purpose in legal contexts than the Bayes Factor (and, in an appendix, than other probabilistic confirmation measure).

Then, we moved to the utility of this notion in theorizing about important factors at play in legal

evidence evaluation: false positive probability of DNA evidence, which should be taken more seriously, and in the combining of identifying eyewitness evidence with scientific evidence (and, in appendices on the value of cold-hit DNA matches). Such considerations, while perhaps fail to provide us with precise quantitative estimations, tell us what we do not, but should know: false positive probabilities, and more relevant details of the circumstances of an eyewitness identification. Had such information been available, at least rough quantitative estimates of the weight of joint evidence seem in principle viable, and they seem to be better than mere intuitive judgment of untrained jury members (also, proper considerations of cold-hit matches results in a *prima facie* counter-intuitive result: such matches have higher evidential weight than matches not obtained through a database search, and doubts about their actionability have sources elsewhere than in the mere fact that the matches were obtained through a database search).

R: revised this in light of Sophie's comments.

While, conceptually, likelihood ratio seems to be the right tool for the job of being a local measure of evidential strength, there are some risks to using likelihood ratios in legal evidence evaluation. Likelihood ratios are sensitive of this measure to hypothesis choice. In real-life complex cases multiple hypothesis are available, and proper evidence evaluation needs to go beyond a single likelihood ratio statement. The context of an evidence evaluation and such choices should be made clear and transparent, and proper incorporation of the likelihood ratios in the judging of a whole case requires a more elaborate framework and representation of such complexities. A useful tool fit for this task, we will argue, is Bayesian Network representation.

A. The cold-hit confusion

We have been quite negative in the last two sections. What we are after, though, is a balanced perspective. We are not saying that likelihood ratios should be disposed of: rather that they should be reported for individual pieces of evidence and used to evaluate the impact of combined evidence, but only when proper attention is paid to the structure of the problem. To re-emphasize the utility of likelihood ratios when care is paid to both the hypothesis formulation, and to what the evidence in fact is, we illustrate it by showing how it can be used to resolve a lengthy debate in legal evidence studies: the value of a cold-hit DNA match. First, we spend some time explaining the problem in Appendix A, and then show how LR can be deployed to resolve it in Appendix B.

First, we explain what a cold hit is and give an example of case based on a cold hit, the murder of Diana Sylvester. In that case, instead of random match probability, the defense focused on another measure, database match probability (DMP). We explain what it is and how it was recommended to the second National Research Council of DNA Technology in Forensic Science report. Then we explain how NRC tried to motivate their recommendation, and why these recommendation are not convincing. This sets up the stage for the next appendix: if DMP is not the right tool to evaluate cold-hit evidence, what is?

DNA evidence is one of the most widely used forms of quantitative evidence currently in use. It may be used to corroborate other evidence in a case, or as the primary incriminating evidence. For example, suppose different investigative leads point to an individual, Mark Smith, as the perpetrator. The investigators also find several traces at the crime scene left by the perpetrator. Laboratory analyses show that the genetic profile associated with the traces matches Smith. In this scenario, the DNA match corroborates the other evidence against Smith. In contrast, suppose the police has no other investigative lead except the traces left at the crime scene. Hoping to find the perpetrator, the police run the genetic profile associated with the traces through a database of profiles and find a match, a so-called **cold-hit**. Cold-hit DNA matches have been the focus of intense discussion in recent years. Since in cold-hit cases there is little or no other evidence, cold-hit matches are often the primary item of evidence against the defendant. Some believe that this circumstance weakens the case. Others disagree. This debate illustrates how probability theory—in particular, the likelihood ratio—can help to assess the strength of evidence at trial. What follows examines some of the main arguments.

For concreteness, consider the California rape and murder case of Diana Sylvester. In 2008, many years after the crime, John Puckett was identified as a unique 9-loci match through a database search of 338,000 profiles. He was the only individual in the database who matched the traces collected from the victim Diana Sylvester in 1972. According to an expert witness, the particular pattern of alleles present in the material was (conservatively) expected to occur randomly among Caucasian men with a frequency of 1 in 1.1 million. This is the **random match probability (RMP)**. The random match

probability—often interpreted as the probability that someone who is not the source would coincidentally match, $P(\text{match}|\neg\text{source})$ —is a common measure of the strength of a DNA match. The lower the RMP, the more strongly incriminating the match. The rationale here is that a low random match probability suggests that it is unlikely that two people would share the same DNA profile. In line with what we already discussed, strictly speaking, a match is a strong evidence that the defendant is the source only if the probability that the person who left the traces (the ‘source’) would match is significantly greater than RMP. In practice, when it comes to DNA evidence, it is often assumed that $P(\text{match}|\text{source})$ is very high.

Although clearly 1 in 1.1 million should not be confused with the probability of Puckett’s innocence, the small figure indicates it is very unlikely that a random person unrelated to the crime would match. The match is therefore strong evidence of Puckett’s guilt. Assuming that the probability of a match if Puckett indeed was the source was (practically) 1, the likelihood ratio is simply 1.1×10^6 .

During the pretrial hearing, however, Bicka Barlow, the DNA expert for the defense, pointed out that this was a cold-hit case. No evidence tied Puckett to the crime other than the cold-hit match, Puckett’s previous rape convictions, and the fact that he was in the area at the time of the murder. In order to correctly assess the probative value of the cold-hit match, Barlow argued, the random match probability should be multiplied by the size of the database. The result of such a multiplication is called the **database match probability (DMP)**. In Puckett’s case, the multiplication of $1/1.1 \times 10^6$ by 338,000 resulted in a database match probability of approximately .3.

which is a less impressive number than the original RMP (the likelihood ratio for the DMP is approximately 3.25). According to this calculation, it was no longer very unlikely that an unrelated person from the database would match, and so the cold-hit DNA match was no longer strong evidence of guilt. At least, this was Barlow’s argument.

Barlow followed a 1996 report by the National Research Council called NRC II (National Research Council, 1996), preceded by an earlier report on DNA evidence called NRC I (National Research Council, 1992). NRC II recommended precisely what Barlow did: that in cold-hit cases RMP should be multiplied by the database size, yielding DMP. The underlying idea was that the larger the size of the dataset, the higher the database match probability, and the lower the strength of the match. This correction was meant to guard against the heightened risk of mistaken matches for the innocent people in the database. To see, however, if this was sound advice, we need to look under the hood.

The NRC formed the Committee on DNA Technology in Forensic Science, which issued its first report in 1992. In that report they advised against using cold hit results as evidence, and insisted that only the frequencies related to loci not used in the original identification should be presented at trial, that is, that the evidence used to identify the suspect should not be used as evidence against the suspect.

This recommendation has been criticized by many because it underestimates the value of cold-hit matches. The problem was, given a certain amount of evidence the expert, prior to suspect identification, had to make a somewhat subjective decision of how to divide the evidence into two items: one to be used only in the suspect identification, and one to be used only in the trial itself as evidence against the suspect. This overly limited the utility of the evidence and introduced an unnecessary element of subjectivity.¹⁶

NRC II withdrew the earlier recommendation. However, the contrast between low RMP and the frequency of DNA matches in actual database searches was indeed stark. For instance, the Arizona Department of Public Safety searched for matching profiles in a database comprising 65,000 individuals. The search found 122 pairs of people whose DNA partially matched at 9 out of 13 loci; 20 pairs people who matched at 10 loci; and one pair of people who matches at 12 loci. So it is not that unlikely to find two people in a database who share the same genetic profiles (examples of fairly high counts of DNA matches in database searches was actually used by Barlow in the Diana Sylvester case). In light of this contrast, NRC II recommended the use of DMP rather than RMP. NRC II recommended also that in cold-hit cases the likelihood ratio R associated with the DNA match should be divided by $d + 1$. Their first recommendation was about a correction of the random match probability, and this second recommendation is about the likelihood ratio.

One argument by NRC employed an analogy involving coin tosses. If you toss several different coins at once and all show heads on the first attempt, this seems strong evidence that the coins are biased. If, however, you repeat this experiment sufficiently many times, it is almost certain that at some point all

¹⁶It also opened the gate for multiple testing with various evidence division points, and multiple testing leads to its own statistical problems. But let’s put this issue aside.

coins will land heads. This outcome should not count as evidence that the coins are biased. According to NRC II, repeating the coin toss experiment multiple times is analogous to trying to find a match by searching through a database of profiles. As the size of the database increases, so does the number of attempts at finding a match, and it is more likely that someone in the database who had nothing to do with the crime would match.

Another argument provided by NRC II compared a database trawl to multiple hypothesis testing, and multiple hypothesis testing should be avoided if possible in light of classical statistical methods.

Third, NRC II was concerned with the fact that in cold-hit cases the identification of a particular defendant occurs after testing several individuals. This concern has to do with the data-dependency of one's hypothesis: seemingly, the hypothesis 'at least one person in a given database matches the DNA profile in question' changes its content with the choice of the database.

We will start with the coin analogy. It is in fact unclear how the analogy with coin tossing translates to cold-hit cases. Searching a larger database no doubt increases the probability of finding a match at some point, but is the increase as fast as the Arizona Department of Public Safety examples and the coin analogy suggest? Quite crucially, following (Donnelly & Friedman, 1999) we need to pay attention to what hypotheses are tested, what probabilistic methods the context recommends, and what exactly the evidence we obtained is. For instance, one hypothesis of interest is what we will call a *general match hypothesis*:

(General match hypothesis) At least one of the profiles in the database of size n matches the crime sample.

The general match hypothesis is what NRC II seems to have been concerned with. If for each data point $RMP = \gamma$ were held constant, and if random matches with different data points $match_1, match_2, \dots, match_d$ excluded each other, the probability of there being at least one random match would be the same as the probability of their disjunction and could be calculated by the additivity axiom:

$$\begin{aligned} P(\text{at least one match}) &= P(\text{match}_1 \vee \text{match}_2 \vee \dots \vee \text{match}_d) \\ &= \sum_i^d P(\text{match}_i) = \gamma \times d \end{aligned}$$

This calculation would result in the outcome recommended by NRC II, if the value of the evidence were to be a function of the probability of (General match hypothesis).

The first question is, whether a directly additive calculation should be applied to database matches. Notice that in applications DMP does not really behave like probability. Take a simple example. Suppose a given profile frequency is .1 and you search for this particular profile in a database of size 10. Does the probability of a match equal $.1 \times 10 = 1$? The answer is clearly negative. Multiplication by database size would make sense if we thought of it as addition of individual match probabilities, provided matches exclude each other and so are not independent. Here is a coin analogy. Suppose I toss a die, and my database contains $n = \text{three different numbers: } 1, 2 \text{ and } 3$. Then, for each element of the database, the probability p of each particular match is $1/6$, and the probability of *at least one* match is $1/6 + 1/6 + 1/6 = 1/6 \times 3 = n \times p = 1/2$. We could use addition in such a situation because each match excludes the other matches, a condition that is not satisfied in the database scenario.

Another reason why DMP is problematic can be seen by taking a limiting case. Suppose everyone in the world is recorded in the database. In this case, a unique cold-hit match would be extremely strong evidence of guilt, since everybody except for one matching individual would be excluded as a suspect. But if RMP were to be multiplied by the size of the database, the probative value of the match as measured by DMP should be extremely low. This is highly counter-intuitive.

Even without a world database, the NRC II proposal remains problematic, since it sets up a way for the defendant to arbitrarily weaken the weight of cold-hit DNA matches. It is enough to make more tests against more profiles in more databases. Even if all the additional profiles are excluded (intuitively, pointing even more clearly to the defendant as the perpetrator), the NRC II recommendation would require to devalue the cold-hit match even further. This, again, is highly counter-intuitive.

Perhaps a somewhat more sensible answer is obtained by assuming the independence of nomatch for the members of the database and deploying a solution similar to the one used in the birthday problem. Here, the idea would be—assuming matches for different data points are independent and have constant

RMP — to calculate:

$$\begin{aligned} P(\text{match}) &= 1 - P(\text{nomatch}) \\ &= 1 - (1 - \gamma)^d \end{aligned}$$

where γ is RMP, and d is the database size. This would be in line with using the binomial distribution to calculate the probability of no match:

$$\begin{aligned} \text{dbinom}(0, d, \gamma) &= \binom{n}{0} \gamma^0 (1 - \gamma)^{d-0} \\ &= 1 \times 1 \times (1 - \gamma)^d \end{aligned}$$

Now, assuming indeed that γ is constant and that matches between data points are independent, the dependence of the probability of at least one match on the database size can be pictured as in Figure ??.

If we use the RMP and database size used in the Puckett case, the calculated probability of at least one match is 0.2645501. Not exactly the DMP postulated by the defendant, but pretty close. The question is, should this number be the probability used to evaluate the evidential impact of the cold hit?

One problem is, whether the independence assumption is satisfied in the database search problem is unclear. After all, if you are informed about the match frequencies in the database, and they teach you that since two arbitrary database points quite likely do not match, if the sample matches one of them, it is less likely to match the other one. And the independence assumption is not benign. We will illustrate it with a somewhat distant, but a very striking example, coming from (Barnett, 2020). Suppose you consider whether your effort of casting a vote in the upcoming election is worth it in a context where there are 500k other voters. One of the probabilities you might be interested in is the probability that your vote would make a difference. If we apply the binomial model to the problem, the probability that a candidate will receive exactly k votes if n people vote is supposed to be $\binom{n}{k} p^k (1 - p)^{(n-k)}$, where votes of the population members are supposed to be independent and estimated to have the same probability p of being for the candidate. For instance, if 500k people vote and $p = 0.5$, the probability that the candidate will receive exactly 250k votes is 0.0011284, which is around $1/886$ and much higher than $1/n$. This fairly high chance made some claim that the chance that your voice is decisive if the chances are equal is fairly high in such circumstances. However, note that if $p = .505$, the probability that the candidate will receive exactly 250k votes is $1.5651281 \times 10^{-14}$, which is less than one in a trillion. This lead some (Brennan, 2012; *Democracy and decision*, 1993) to claim that outside of the very specific circumstances, decision-theoretic arguments for the rationality of voting are hopeless. Barnett, however, points out that such a sensitivity to success probability simply makes the binomial model inappropriate for the voting context, observing that its calculations also disagree with empirical estimates which are not too far from $1/n$ (Gelman, Katz, & Tuerlinckx, 2002; Gelman, King, & Boscardin, 1998; Mulligan & Hunter, 2003). This sensitivity arises, because within the binomial model the more trials (voters) there are, the more tightly the results will tend to cluster around the probability of success. To observe how unrealistic that is, keep $p = .505$ and ask yourself how probable it is that the voting result will be between 50.4% and 50.6%. Sure, this outcome might be quite likely, but the binomial certainly overestimates it at 0.8427212. Another unrealistic estimate obtained by the binomial model is the estimate of the probability of an upset (that the leading candidate will lose). With $p = .505$ this is $\text{pbinom}(249999, 500000, .505)$, which turns out to be extremely and unrealistically low: $7.5994495 \times 10^{-13}$.

Coming back to our original problem, the binomial estimate of the probability of a match is also quite sensitive to RMP, as illustrated in Figure ??. The bottom line is that if we have reasons to think the independence assumption is not satisfied, the binomial model is not appropriate. So, it seems, it is not appropriate for the database match problem either.

The binomial model, however, is useful, in its simplicity, for illustrating an important distinction whose conflation underlies one of the involved arguments. You might have been surprised learning that while the expert testified that RMP on 9 loci for Puckett was 1 in 1.1 million, the Arizona Department of Public Safety found 122 9-loci matches among 65,000 individuals. After all, $122/65000$ is 0.0018769, which is much higher than the reported RMP.

Crucially, notice that there is a difference between having a sample and looking for a match in a database of size n and taking a database of size n and checking all pairs that occur within it for a match. In the former case, you are making n comparisons. In the latter case, the number of comparisons is $\binom{n}{2}$, which is much higher. If $n = 65000$, there are 2.1124675×10^9 pairs to compare, so while the binomial

M: Pretty interesting, but in the voting model, it seems that feedback mechanisms between voters affect behaviour, but not so in the genetic case. Can you spell out the analogy more clearly?

Not sure what you mean, we should talk about this.

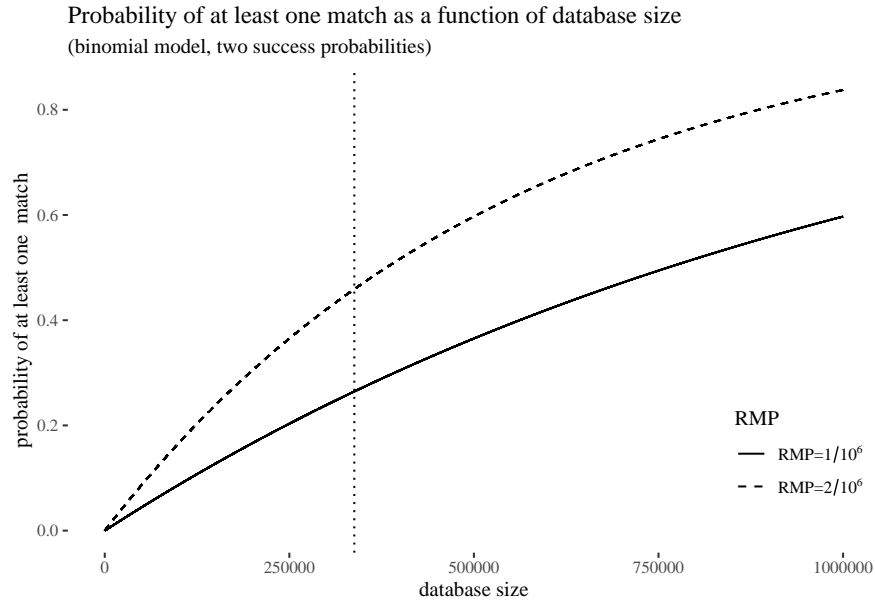


Figure 9: Binomial model of the database search problem. The probability of at least one match depending on the database size, assuming independence and constant RMP used in the Puckett case as compared with the binomial estimate for $p=2/1.1e6$ (dashed line). The actual database size marked with a vertical line."

estimate of the probability of at least one match for n comparisons (the former case) is 0.057379, it is approximately 1 for $\binom{n}{2}$ comparisons. For the impact it has on the Arizona Department of Public Safety statistics, consider the binomial estimate of the probability of at least 122 matches among all pairs as a function of the database size, even for relatively low database size range (up to 50000), as illustrated in Figure 10.

From this perspective, it is no surprise there were so many matching pairs among all the pairs from the database. Unfortunately, this frequency does not estimate the probability of at least one match in the set-up we are actually interested in. After all, in a cold-hit scenario we do have a sample outside of the database and make n comparisons, instead of testing all possible pairs from the database for a match.

Before we move on, it is worthwhile to think about how the example at hand actually constitutes evidence against a certain probabilistic model that one might initially want to deploy in this context. Note how the Arizona statistics constitute some empirical evidence against the adequacy of the binomial model. While the binomial estimate probability of at least 122 matches with an external sample for $n = 6500$ is pretty much 0, let us look at the most likely number of matches if we test $\binom{6500}{2}$ pairs, as estimated by the binomial model. We illustrate it with an 89% highest density interval in Figure 11. So, if the expert's estimate and the binomial model are both adequate, we indeed should be surprised by the presence of 122 matches. But this is because this number is surprisingly low: instead we should expect a much higher number, around 2000 of them! Of course, it is unlikely that in fact all possible pairs have been compared, and it is hard to evaluate this evidence against the binomial model unless we know the exact number of comparisons made.

Now that we used the imperfect binomial model to clear up at least one confusion, let us put it aside, and focus on an even deeper problem with using the probability of (General match hypothesis) instead of RMP in evidence evaluation. Probabilistic epistemology recommends that once we obtain new evidence, our new degrees of belief should be the probabilities obtained by conditionalizing on this evidence. Crucially, we should update on the total evidence we obtained rather than only on a part of it. Here the question is, does (General match hypothesis) exhaust what we have learned from our database match?

To get us started thinking about this question, consider a coin analogy which Donnelly & Friedman (1999, p. 950) found more adequate than the one proposed by the NRC. Imagine a biased coin whose physical appearance is indistinguishable from the fair coins in a piggy bank. The biased coin is the perpetrator and the piggy bank is the database containing innocent people. After the biased coin is thrown into the bank with the other coins, someone picks out a handful of coins at random and flips each

External vs. internal comparison probabilities of at least 122 matches
(binomial model)

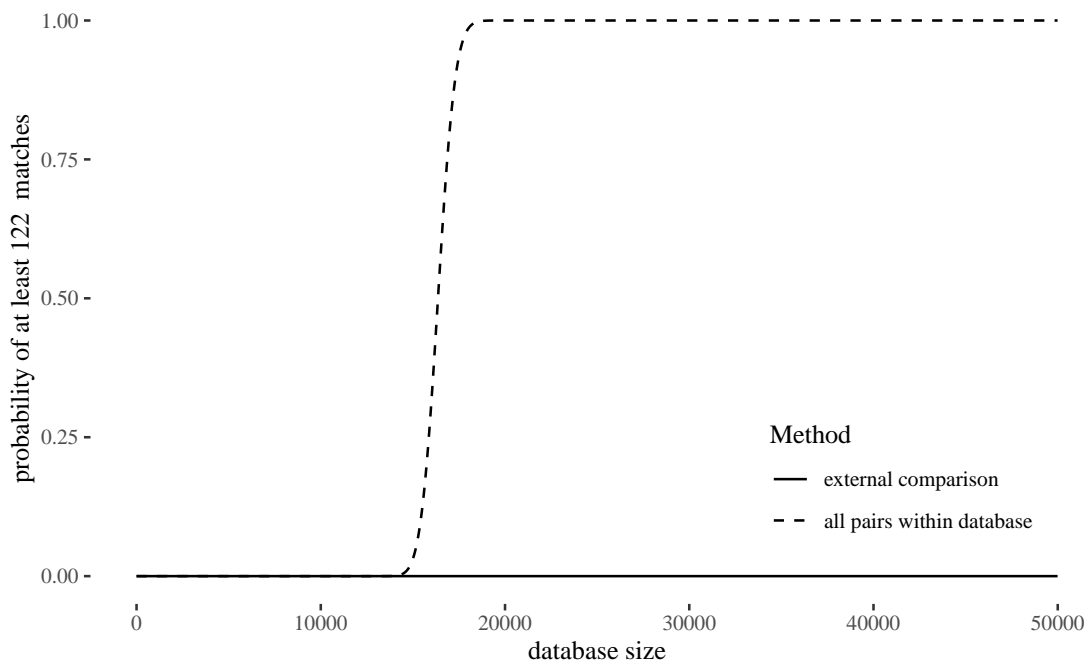


Figure 10: Binomial model of the database search problem. The probability of at least 122 matches depending on the database size for n comparisons with an external sample, and for all possible pairs among n datapoints (dashed), assuming $RMP=1/1.1e6$.

of them twenty times. Each coin lands heads approximately ten times—except for one coin, which lands heads on all twenty flips. The fact that other coins seem unbiased makes the claim that this one is biased better supported.

Coming back to DNA matches, think about the following scenario: first, you identified the suspect by some means other than a database trawl. Then, it turned out his DNA profile matches the crime scene stain. Fine, here it seems uncontroversial that this constitutes strong further incriminating evidence. Now, imagine a further database search in a database not containing this suspect finds no matches. Would you think that this information supports the guilt hypothesis? If your answer is yes, then you do have the intuition that the lack of matches with other people (whose profiles, in this particular case, happen to be in a database) strengthens the evidence.

The key lesson here (and in the complete-world database scenario we already discussed) is that we not only learned that there was a match in the database of size n , but also that in $n - 1$ cases there was no match, and this information also has evidential value. In line with this, contrary to NRC II, Donnelly & Friedman (1999) argues that if potential suspects in the database are excluded as sources, this should increase, not decrease, the probability that the defendant who matches the crime traces is the source. A cold-hit match, then, is stronger and not weaker evidence of guilty than ordinary DNA matches.

Before moving on to a better model that captures how this could be, let us look at another argument put forward by NRC, an analogy to multiple hypothesis testing. NRC claimed that there is an analogy between searching for a match in a database and multiple hypothesis testing, which is a dubious research practice. In classical hypothesis testing, if the probability of type I error in a single test of a hypothesis is 5%, the probability of at least one type I error will increase by testing the same hypothesis multiple times. In analogy—the argument goes—we need to correct for the increased risk of type I error, and just as the Bonferroni correction requires that the p -value threshold be divided by the number of tests, NRC II requires that the estimated probability of a random match should be multiplied by the number of comparisons.

This analogy with multiple testing, however, is misplaced. As David J. Balding (2002) points out, multiple testing consists in testing the *same* hypothesis multiple times against new evidence. In cold-hit

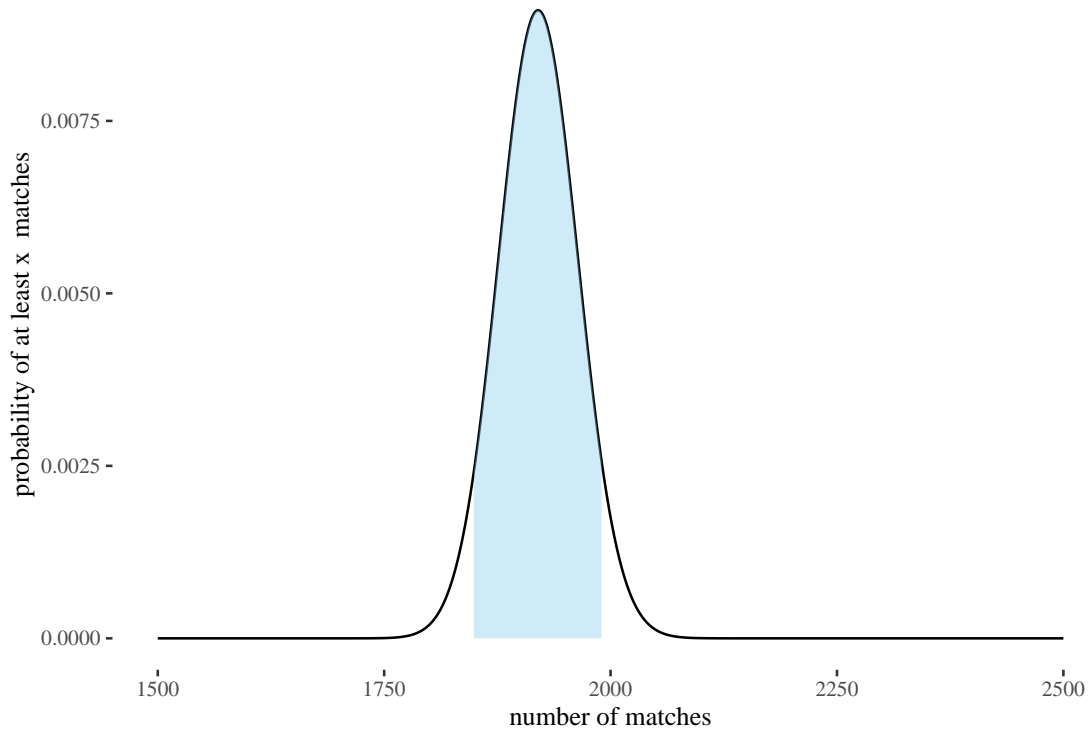


Figure 11: Binomial probability density of n matches in pairwise comparison within a database of size 65000, assuming $p = 1/1.1e6$, with 89% highest density interval = (1849, 1990) shaded in blue.

cases, no such multiple testing is involved. Rather, multiple hypotheses—each concerning a different individual in the database—are tested only once and then excluded if the test is negative. From this perspective, for each $1 < i < n$, the following hypothesis is tested:

(Particular match hypothesis) Profile i in the database matches the crime sample.

and the hypothesis that the defendant is the source was one of the many hypotheses subject to testing. The cold-hit match supports that hypothesis and rules out multiple other hypotheses.

B. Likelihood ratio and cold-hit DNA matches

To find a better path towards the resolution of the database search problem, let us look at another recommendation of NRC II, that in cold-hit cases the likelihood ratio R associated with the DNA match should be divided by $n + 1$, where n is the database size. This approach has been defended by (Stockmarr, 1999), who points out that since the suspect is identified after the databases search, the hypothesis is formulated *ad hoc*. Without the correction, then, the likelihood ratio would be data-dependent. For instance, he insists that hypotheses such as *JS was one of the crime scene donors* are evidence-dependent in the case of database search, “since we had no way of knowing prior to the search that Smith would be the person that matched” (p. 672). Instead, Stockmarr claims, we should evaluate LR using hypotheses that can be formulated prior to the search, such as *the true perpetrator is among the suspects identified from the database*. And indeed, the likelihood of this hypothesis is as NRC II suggests, k/np , where k is the number of matching profiles, n the database size, and p the random match probability (see Stockmarr, 1999 for a derivation).

Dawid, in a discussion with Stockmarr (Dawid & Stockmarr, 2001) points out that Stockmarr’s hypotheses, while not depending on the result of the search, depend on the data themselves (because they change with the database size). More importantly, he also indicates that Stockmarr’s hypotheses are composite and the assessment of LR therefore requires additional assumptions about the priors. Once these are used with Stockmarr’s own LR, the posterior is the same as the one obtained using the

methods proposed by the critics of NCR II. This is a particular case of a general phenomenon that we will discuss later on: different hypotheses might result in different LR, but be equivalent conditional on the evidence, and so result in the same posterior probabilities. This general phenomenon indicates how LR on its own might be insufficiently informative.¹⁷

Putting Stockmarr's defense and its problems aside, the NRC II recommendation is questionable on more principled grounds. Suppose R is not too high, say because the identified profile is common since the crime scene DNA is degraded and only a few markers could be used. Then, $n + 1$ can be greater than R , so $R/(n + 1) < 1$. The match would then be exculpatory, a very counter-intuitive result. Moreover, if the defendant on trial is the source, the probability that he would match is practically 1. If he is not, the probability that he would still match equals the random match probability. Neither of these probabilities change because other suspects have been tested in the database search. In fact, if potential suspects are excluded as potential sources, this should increase, not decrease, the probability that the defendant who matches the crime traces is the source.

A more principled way to assess cold-hit matches based on the likelihood ratio, exists. The proposal draws from the literature on the so-called **island problem**, studied by (Dawid, 1994; Dawid & Mortera, 1996; Eggleston, 1978). Let the prosecutor's hypothesis H_p be 'The suspect is the source of the crime traces' and the defense's hypothesis H_d be 'The suspect is not the source of the crime traces.' Let E be the DNA match between the crime stain and the suspect (included in the database) and D the information that none of the $n - 1$ profiles in the database matches the crime stain. The likelihood ratio associated with E and D should be (David J. Balding & Donnelly, 1996; Taroni, Biedermann, Bozza, Garbolino, & Aitken, 2014):

$$V = \frac{P(E, D|H_p)}{P(E, D|H_d)}.$$

Since $P(A \wedge B) = P(A|B)P(B)$, for any statement A and B , this ratio can be rewritten as:

$$V = \frac{P(E|H_p, D)}{P(E|H_d, D)} \times \frac{P(D|H_p)}{P(D|H_d)}. \quad (15)$$

The first ratio in (15) is roughly $1/\gamma$, where γ is the random match probability. The second ratio—call it the **database search ratio**—requires some more work. Consider first the denominator $P(D|H_d)$. If the suspect is not the source (H_d), someone else is, either someone who is in the database or someone not in the database. Let S stand for The source is someone in the database. By the law of total probability,

$$P(D|H_d) = P(D|S, H_d)P(S|H_d) + P(D|\neg S, H_d)P(\neg S|H_d). \quad (16)$$

If the source is someone in the database (S) and the suspect is not the source (H_d), it is very unlikely that no one in the database would match (D), so $P(D|S, H_d) \approx 0$. The equality in (16) therefore simplifies to:

$$P(D|H_d) = P(D|\neg S, H_d)P(\neg S|H_d),$$

The database search ratio would therefore be:

$$\frac{P(D|H_p)}{P(D|H_d)} = \frac{P(D|H_p)}{P(D|\neg S, H_d)P(\neg S|H_d)}.$$

Note that $P(D|H_p) = P(D|\neg S, H_d)$ because whether the suspect is the source (H_p) or not (H_d) does not affect whether there is a match in a database that does not contain the source ($\neg S$). Let the probability that no person in the database other than the suspect would match (D), assuming the suspect was in fact the source, be ψ_{n-1} . Notice that $P(D|\neg S, H_d)$ is the probability that no one other than the suspect matches in the database that does not contain the real source, if the suspect is not the source. So this conditional probability can also be estimated as ψ_{n-1} .¹⁸ Let $P(S|H_d) = \varphi$. The database search ratio then would reduce to

¹⁷See however, Stockmarr's own reply (*ibidem*).

¹⁸If the prior probability that the perpetrator is in the database was high, the calculations would need to be different. But normally, this prior is not too high.

$$\frac{P(D|H_p)}{P(D|H_d)} = \frac{1}{1 - \varphi}.$$

As the database gets larger, φ increases and the database search ratio also increases. This ratio equals one only if no one in the database could be the source, that is, $\varphi = 0$.

Since the likelihood ratio V of the cold-hit match results by multiplying the likelihood ratio of the DNA match and the database search ratio, V will always be greater than the mere likelihood ratio of the match (except for the unrealistic case in which $\varphi = 0$). Thus, a cold-hit DNA match should count as stronger evidence than a DNA match of a previously identified suspect.

Dawid & Mortera (1996) study different database search strategies and consider the possibility that information about the match is itself uncertain, but the general point remains. Under reasonable assumptions, ignoring the database search would give a conservative assessment of the evidential strength of the cold-hit match. Donnelly & Friedman (1999), with slightly different assumptions, derived the formula $R \times [1 + md/N]$, where $R = 1/\gamma$, d is the database size, N the number of people in population not in database, and m is an optional multiplier reflecting how much more likely persons in the database are thought to be the source when compared to the rest of the population. The expression cannot be less than γ . If no other profile has been tested, $d = 0$ and LR is simply the regular DNA match LR. If N is zero, that is, everyone in population is in the database, the result is infinitely large.

This proposal is able to accommodate different apparently competing intuitions. First, consider the intuition that as the size of the database grows, it is more likely that someone in the database would match. This intuition is captured by the fact that φ increases proportionally to the size of the database even though this increase does not imply that the evidential value of the cold-hit match should decrease. Second, there is intuitive resistance to basing a conviction on a cold-hit match, although this resistance is less strong in case of an ordinary match (more on this later in Section ??). This preference for convictions based on an ordinary DNA match seems in tension with the claim that a cold-hit match is stronger evidence of guilt than an ordinary match. There is a way to make sense of this, though. The key is to keep in mind that the evidential strength—measured by the likelihood ratio—should not be confused with the posterior probability of guilt given the evidence. Even if a cold-hit match is stronger evidence of guilty, this fact does not imply that the posterior probability of the defendant's guilt should be higher.

If the cold-hit match is the only evidence of guilt, the posterior probability of guilt may well be lower compared to cases in which other evidence, such as investigative leads, supplements the DNA match. This lower posterior probability would justify the intuitive resistance towards convictions in cold-hit cases, despite the fact that a cold-hit match alone is stronger evidence than a DNA match obtained otherwise and taken on its own. Moreover, it is possible that the intuitive assessment of cold-hit evidence takes to some extent the impact of false positive probability into account.

Fix crossref later

C. Confirmation measures

At this point a philosophically minded reader might recall that there is an important notion in the vicinity—that of confirmation—and that there is a vast philosophical literature on probabilistic explication of that notion. Natural questions arise: how is the notion of confirmation related to the notion of evidence strength, and why almost none of the probabilistic explications of confirmation have not been deployed in legal probabilism?

The key question behind the enterprise we are going to take a look at is: when does a piece of evidence confirm a theory, and how are these requirements to be explicated probabilistically to agree with both successful scientific practice and sensible philosophical principles? The hope is that having answered these questions would facilitate both rational reconstructions of various developments in the history of science, and a critical evaluation of various ongoing scientific investigations.

The underlying probabilistic idea is that the level of confirmation of a theory (T) by a piece of evidence (E) is a function of an agent's degrees of belief. The first stab might be, let's simply identify the confirmation level with $P(T|E)$. This, however, is way too quick. Multiple factors come into the assessment of this conditional probability, and two agents can agree on the extent to which E confirms T without agreeing on the posterior probability of T (identified with $P(T|E)$), because the agents might disagree about the prior probability of T and this might have an impact on the posterior.

Still, some requirements on confirmation measures can be formulated in terms of probabilities. One

M: We need a proper conclusion here. Now it feels as though LR are a good thing, at least for cold-hits. But is this just an exception? Do LR have a very narrow applicability, say only for DNA evidence? So where do we stand exactly? And what about BF and Bayesian networks and priors? Need general morals here.

I think I now reiterated our point too many times throughout the chapter, we'll discuss this.

usual assumption (Sprenger & Hartmann, 2019) is that the level of confirmation is to be a continuous function of $P(T)$ and $P(E|T)$ which is non-decreasing in the first argument and non-increasing in the second argument. That is, increasing the prior, should not lower the confirmation level, and increasing the likelihood should not increase the confirmation level. Let's call this condition the *prior-posterior dependence*.¹⁹

One consequence of the prior-posterior-dependence—called *Final Probability Incrementality* is that confirmation of T by E , $c(T, E)$ should track the posterior order-wise, that is $c(T, E) > c(T, E')$ just in case $P(T|E) > P(T|E')$.

Another requirement is that there should be a neutral point n such that E confirms (disconfirms) T just in case $c(T, E) > n$ ($c(T, E) < n$) and is neutral exactly at n . This is called the *qualitative-quantitative bridge*.

Yet another requirement is *local equivalence*. Theories that are logically equivalent given the evidence should receive equal confirmation from this evidence. Interestingly, all confirmation measures which satisfy prior-posterior dependence, qualitative-quantitative bridge, and local equivalence are strictly increasing functions of $P(H|E)$. Such measures are said to explicate confirmation as *firmness of belief*. Moreover, all functions satisfying these three conditions are ordinally equivalent.²⁰

However, another notion of confirmation seems often at play. For instance, even if the posterior T is low, one might still think that a given experiment speaks strongly in favor of T . And relatedly, E can lower the posterior of T while still leading the posterior to be sufficiently high for the firmness confirmation measure to be above the neutrality threshold. Another feature of confirmation as firmness is that if, in this sense, T confirms H , then for any H' that is excluded by H , T disconfirms H' . But now think of the small town murder scenario we already discussed: the fact that the suspect was seen in town seems to support both the prosecution hypothesis that he committed the murder, and the defense hypothesis, that he was in town to visit his mother. Confirmation as firmness cannot capture such intuitions, as relevance cannot be captured as a function of the posterior alone.

For such reasons, following the second edition of (Carnap, 1962), it is customary to distinguish another notion in the vicinity: confirmation as increase in firmness of belief. If we replace local equivalence with tautological equivalence $c(T, \top) = c(T', \top)$, where \top is a logical tautology—the idea being that hypotheses are equally supported by empty evidence—we end up with another class of confirmation measures, those meant to capture *probabilistic relevance*. On this approach, E confirms (disconfirms) T just in case $P(H|E) > P(H)$ ($P(H|E) < P(H)$).

Here is a list of key confirmation measures available on the market (Sprenger & Hartmann, 2019), normalized so that they all have neutral points at 0:

$$D(T, E) = P(T|E) - P(T) \quad (\text{Difference})$$

$$Lr(T, E) = \log \left(\frac{P(T|E)}{P(T)} \right) \quad (\text{Log-ratio})$$

$$LL(T, E) = \log \left(\frac{P(E|T)}{P(E|\neg T)} \right) \quad (\text{Log-likelihood})$$

$$K(T, E) = \frac{P(E|T) - P(E|\neg T)}{P(E|T) + P(E|\neg T)} \quad (\text{Kemeny-Oppenheim})$$

$$Z(T, E) = \begin{cases} \frac{P(T|E) - P(T)}{1 - P(T)} & \text{if } P(T|E) \geq P(T) \\ \frac{P(T|E) - P(T)}{P(T)} & \text{if } P(T|E) < P(T) \end{cases} \quad (\text{Generalized entailment})$$

$$S(T, E) = P(T|E) - P(T|\neg E) \quad (\text{Christensen-Joyce})$$

$$C(T, E) = P(E)(P(T|E) - P(T)) \quad (\text{Carnap})$$

$$R(T|E) = 1 - \frac{P(\neg T|E)}{P(\neg T)} \quad (\text{Rips})$$

(Log-likelihood), our good old likelihood ratio, and (Kemeny–Oppenheim) are ordinally equivalent

¹⁹Some formulations (Crupi, 2015) are a bit more general and include background knowledge K . In that setting, the corresponding requirement is called *Formality* and takes the confirmation to be a function of $P(H \wedge E|K)$, $P(H|K)$ and $P(E|K)$. For the sake of simplicity, we will suppress the reference to K , unless required by the context.

²⁰Measure c is ordinally equivalent to measure c' just in case always $c(E, T) \geq c(E', T')$ iff $c'(E, T) \geq c'(E', T')$.

(and no other pair on the list is). The former makes calculations of joint support additive, and the former has the nice feature of ranging from -1 to 1 and having 0 as a neutral point, so if one accepts the use of likelihood ratio, these two also can be used on some occasions when these additional features are justified. Further grouping and assessment of the confirmation measures for a given purpose is facilitated by the following facts:

- One might require that E always confirms the disjunction of excluding hypotheses more than one of them just in case it also confirms the other one (*disjunction of alternative hypotheses*). This can happen only if the confirmation measure is a strictly increasing function of the difference measure. Whether this is an intuitive requirement in our context is unclear.
- One might require that confirmation should track likelihood— $c(T, E) > c(T, E')$ (*Law of likelihood*)—just in case $P(E|T) > P(E'|T)$. This can happen only if the measure is a strictly increasing function of the Bayes factor. At least in legal applications, the law of likelihood is suspicious, as our example with rocking child abuse victims discussed on page 1 indicates.
- You might wish that confirmation be *contrapositive* ($c(T, E) = c(\neg E, \neg T)$) and *commutative* ($c(H, E) = c(E, H)$). The only measures that satisfy both are relative distance measures, that is, they are strictly increasing functions of the generalized entailment measure.
- One might require that if E and E' are conditionally independent given T and $\neg T$, then $c(T, E)$ should be identical with $c(T, E|E')$ (the confirmation obtained when E' is added to the background knowledge). This condition is called *modularity*. This condition holds only if a confirmation measure is a strictly increasing function of the likelihood ratio.

Moreover, if you require strict additivity: $c(H, E \wedge E') = c(H, E) + c(H, E'|E)$, the only measure that satisfies the disjunction of alternative hypotheses is the difference measure, the only measure that satisfies the law of likelihood is the log-ratio measure, and the only one that satisfies modularity is the log-likelihood measure.

Some unity can be brought into the picture (Crupi, Tentori, & Gonzalez, 2007) by normalizing by what happens with a measure where logical consequence or exclusion is involved. For instance, if $E|T$, $D(E, T) = P(\neg T)$ and if $E|\neg T$, $D(E, T) = -P(T)$. So the normalized version has the form:

$$D_n(E, T) = \begin{cases} D(E, T)/P(\neg T) & \text{if } P(T|E) \geq P(T) \\ D(E, T)/P(H) & \text{otherwise.} \end{cases}$$

Interestingly, analogous normalization of measures other than (Generalized entailment) leads to the same single new Bayesian measure of confirmation: (Generalized entailment). Another reason one might have to like this measure is as follows. Take any $k > 0$ and say $v(E, T) = k$ iff $E \models T$, $v(E, T) = -k$ iff $E \models \neg T$ and $v(E, T) = 0$ otherwise. The *logical closure requirement* is that if $v(E, T) > v(E', T')$, then $c(E, T) > c(E', T')$. It turns out that all measures ordinally equivalent to the listed measures other than (Log-likelihood), likelihood ratio, (Generalized entailment) fail to satisfy this condition and Z , likelihood ratio, (Kemeny-Oppenheim) and (Kemeny-Oppenheim) succeed at satisfying it.

Now, what reasons do we have to not use some of the measures we introduced? First, some insights are obtained by considering the abstract requirements. Crucially, (1) final probability incrementality with prior-posterior dependence exclude (Carnap) and (Christensen-Joyce), (2) (Carnap) and (Log-ratio) have the unintuitive consequence that $C(T, E) = C(E, T)$ (call this *symmetry*), (3) (Difference), (Generalized entailment), (Log-ratio), (Carnap) and (Rips) depend on the prior of T , and (4) logical closure requirement excludes many of the measures.

Moreover, there is an issue with Z (Branded Fitelson, 2021). Say E and E' are confirmationally independent regarding H just in case both $c(T, E|E') = c(T, E)$ and $c(T, E'|E) = c(T, E')$. Say E and E' are conflicting evidence regarding T iff $P(T|E) > P(T)$ while $P(T|E') < P(T)$. Branded Fitelson (2021) has proven that any measure ordinally equivalent with Z , however, excludes the fairly intuitive possibility of the existence of confirmationally independent and yet conflicting evidence (he also gives a clear example of such a case).

Last but not least, for legal applications it seems that dependence on the prior probability is undesirable. We propose that at least two conceptual takes on confirmation is available. On one hand, say a scientific community pretty much agrees on the status of a given theory prior to an experiment. Then, after the experiment, it is a legitimate question what impact the experiment has on the status of that theory, and perhaps it makes sense that the prior status of that theory plays a role. On the other hand, in legal context, we would like (1) the expert's assessment not to depend on the expert's prior convictions about the hypothesis, and (2) the expert's statement to mean the same for various agents involved in the

| Measure | Reason not to use |
|--------------------------------|---|
| (Difference) | dependence on priors, logical closure failure |
| (Log-ratio) and (Bayes factor) | satisfies law of likelihood, symmetry, dependence on priors, failure to satisfy logical closure |
| (Generalized entailment) | dependence on priors, independent conflicting evidence |
| (Christensen-Joyce) | excluded by final probability incrementality with prior-posterior dependence |
| (Carnap) | excluded by final probability incrementality with prior-posterior dependence, symmetry, logical closure failure |
| (Rips) | dependence on priors, failure of logical closure |
| (Christensen-Joyce) | excluded by final probability incrementality with prior-posterior dependence |
| (Kemeny-Oppenheim) | none of the above, but unnecessarily complex |
| (Log likelihood) | none of the above, but logarithms are hard for humans |
| (Likelihood ratio) | none of the above |

Table 1: Reasons not to use various confirmation measures in legal fact-finding applications.

fact-finding process, even if they assign different priors to the hypothesis. For this reason, we propose that dependence on priors in legal evidence evaluation is an undesirable feature of a confirmation measure.

Thus, the general picture obtained, pictured in Table 1, seems to suggest that the likelihood ratio is a decent choice for our applications. This, of course, also applies to (Log likelihood) and (Kemeny-Oppenheim), which are ordinally equivalent to likelihood ratio, but the reasons to not use them in a legal context are that (Kemeny-Oppenheim) is conceptually more complex than likelihood ratio, and that thinking in terms of logarithms is not very natural for human agents.

Finally, here is a somewhat abstract, yet important approach to confirmation measures due to (Heckerman, 1988), which constitutes a fairly strong motivation to use likelihood ratio. Suppose you have background information e , and consider a hypothesis H when you obtain a piece of evidence E . A belief update, $U(H, E, e)$, together with your prior stance of H , should determine your posterior belief in H . If we denote conditional belief in H given E , without assuming it's probabilistic yet, as $H|E$, this means that there should be an f such that:

$$H|Ee = f(U(H, E, e), H|e)$$

where, on this approach, $H|Ee$, $U(H, E, e)$, and $H|e$, are all real numbers, and f is required to be continuous in both arguments and monotonically increasing when the other is held constant. Call this the **update requirement**. Bayesian updating is a particular case: the selection of H and e , with a joint distribution in the background, determines a function from $P(H|E)$ to $P(H|Ee)$.

Further, assume the **consistency property**: if the arguments are logically equivalent, then belief update yields the same value:

$$[H_1 \Leftrightarrow H_2, E_1 \Leftrightarrow E_2] \Rightarrow U(H_1, E_1, e_1) = U(H_2, E_2, e_2)$$

The next assumption has it that when you update on two items of evidence, the order of updating shouldn't matter and the result should be the same as updating on the joint evidence. The **combination property** requires that there is a function g that is continuous in both arguments and monotonically increasing in each argument when the other is held constant such that:

$$U(H, E_1 E_2, e) = g(U(H, E_1, e), U(H, E_2, E_1 e))$$

A general result in group theory by Aczel is that any continuous monotonic function of two arguments that satisfies the associativity relation must be additive in some transformed space. In this particular case, the result entails that any update that satisfies the update condition, and the consistency and combination properties is the arithmetic difference of a posterior and prior belief up to an arbitrary monotonic transformation, that is, that there are monotonic functions h and i such that:

$$h(U(H, E, e)) = i(H|Ee) - i(H|e)$$

In the probabilistic context, where $H|E$ is $P(H|E)$, the likelihood ratio satisfies the update requirement and has the combination and consistency properties. Accordingly, the transformation that makes it additive is the logarithmic function.

R: so here's my account of the Heckerman's result, take a look

Now, the *independence correspondence property* has it that if E_1 and E_2 are conditionally independent given H and given $\neg H$, then $U(H, E_2, E_1 e) = U(H, E_2, e)$. The key result is that any probabilistic update satisfying the independence correspondence must be a monotonic transformation of the likelihood ratio. This means that there is a list of fairly intuitive general conditions on what an update function should behave like which entails that it is just a variant of the likelihood ratio.

References

- Aitken, CGG, Roberts, P., & Jackson, G. (2010). Fundamentals of probability and statistical evidence in criminal proceedings (Practitioner Guide No. 1), Guidance for judges, lawyers, forensic scientists and expert witnesses. *Royal Statistical Society's Working Group on Statistics and the Law*.
- Aitken, Colin, & Taroni, F. (2008). Fundamentals of statistical evidence - a primer for legal professionals. *The International Journal of Evidence and Proof*, 12(3), 181–207. SAGE Publications.
- Aitken, Colin, Taroni, F., & Thompson, W. (2003). How the probability of a false positive affects the value of DNA evidence. *Journal of Forensic Science*, 48(1), 1–8. ASTM International.
- Allen, R. J., & Pardo, M. S. (2003). The myth of the law-fact distinction. *Northwestern University Law Review*, 97(4), 1769–1808.
- Balding, David J. (2002). The DNA Database Search Controversy. *Biometrics*, 58(1), 241–244.
- Balding, David J. (2004). Comment on: Why the effect of prior odds should accompany the likelihood ratio when reporting DNA evidence. *Law, Probability and Risk*, 3(1), 63–64. Oxford Univ Press.
- Balding, David J., & Donnelly, P. (1996). Evaluating DNA Profile Evidence When the Suspect Is Identified Through a Database Search. *Journal of Forensic Sciences*, 41(4), 13961J.
- Barnett, Z. (2020). Why you should vote to change the outcome. *Philosophy and Public Affairs*, 48(4), 422–446.
- Behrman, B. W., & Davey, S. L. (2001). Eyewitness identification in actual criminal cases: An archival analysis. *Law and Human Behavior*, 25(5), 475–491.
- Bickel, D. R. (2012). The strength of statistical evidence for composite hypotheses: Inference to the best explanation. *Statistica Sinica*, 1147–1198. JSTOR.
- Biedermann, A., Hicks, T., Taroni, F., Champod, C., & Aitken, C. (2014). On the use of the likelihood ratio for forensic evaluation: Response to Fenton et al. *Science & Justice*, 54(4), 316–318.
- Bolinger, R. J. (2020). Explaining the justificatory asymmetry between statistical and individualized evidence. *The social epistemology of legal trials*. Routledge.
- Brennan, J. (2012). *The ethics of voting*. Princeton University Press.
- Buckleton, J. S., Bright, J.-A., & Taylor, D. (2018). *Forensic DNA evidence interpretation*. CRC press.
- Carnap, R. (1962). Logical foundations of probability. Citeaser.
- Cook, R., Evett, I., Jackson, G., & Jones, P. (1998). A hierarchy of propositions: Deciding which level to address in casework. *Science & Justice*, 38(4), 231–239.
- Crupi, V. (2015). Confirmation. In E. N. Zalta (Ed.), *Stanford encyclopedia of philosophy*.
- Crupi, V., & Tentori, K. (2010). Irrelevant conjunction: Statement and solution of a new paradox, 77(1), 1–13. University of Chicago Press. Retrieved from <https://doi.org/10.1086/650205>
- Crupi, V., Tentori, K., & Gonzalez, M. (2007). On Bayesian Measures of Evidential Support: Theoretical and Empirical Issues. *Philosophy of Science*, 74(2), 229–252.
- Dawid, A. P. (1994). The island problem: Coherent use of identification evidence. In P. R. Freeman & A. F. M. Smith (Eds.), *Aspects of uncertainty: A tribute to D. V. Lindley* (pp. 159–170). John Wiley & Sons, New York.
- Dawid, A. P. (2004). Which likelihood ratio? (Comment on ‘Why the effect of prior odds should accompany the likelihood ratio when reporting DNA evidence,’ by Ronald Meester and Marjan Sjerps). *Law, probability and risk*, 3(1), 65–71. Oxford Univ Press.
- Dawid, A. P., & Mortera, J. (1996). Coherent Analysis of Forensic Identification Evidence. *Journal of the Royal Statistical Society. Series B (Methodological)*, 58(2), 425–443.
- Dawid, A. P., & Stockmarr, A. (2001). Comment on Stockmarr’s “Likelihood Ratios for Evaluating DNA Evidence When the Suspect Is Found through a Database Search.” *Biometrics*, 57(3), 976–980.
- de Zoete, J. C., Fenton, N., Noguchi, T., & Lagnado, D. (2019). Resolving the so-called “probabilistic paradoxes in legal reasoning” with Bayesian networks. *Science & Justice*, 59(4), 367–379.

- Democracy and decision: The pure theory of electoral preference*. (1993). Cambridge University Press.
- Di Bello, M. (2021). A probabilistic analysis of cross-examination using bayesian networks. Wiley. Retrieved from <https://doi.org/10.1111/phis.12209>
- Di Bello, M., & Verheij, B. (2018). Evidential reasoning. *Handbook of legal reasoning and argumentation* (pp. 447–493). Springer.
- Donnelly, P. (1995). Nonindependence of matches at different loci in DNA profiles: Quantifying the effect of close relatives on the match probability. *Heredity*, 75(1), 26–34.
- Donnelly, P., & Friedman, R. D. (1999). DNA Database Searches and the Legal Consumption of Scientific Evidence. *Michigan Law Review*, 97(4), 931.
- Dror, I. E., & Hampikian, G. (2011). Subjectivity and bias in forensic DNA mixture interpretation. *Science & Justice*, 51(4), 204–208. Elsevier BV. Retrieved from <https://doi.org/10.1016/j.scijus.2011.08.004>
- Eggleston, R. (1978). *Evidence, proof and probability* (Vol. 2). Weidenfeld; Nicolson London.
- ENFSI. (2015). *Guidelines for evaluative reporting in forensic sciences*.
- Evet, I. W. (1987). On meaningful questions: A two-trace transfer problem. *Journal of the Forensic Science Society*, 27(6), 375–381. Elsevier BV. Retrieved from [https://doi.org/10.1016/s0015-7368\(87\)72785-6](https://doi.org/10.1016/s0015-7368(87)72785-6)
- Evet, I. W., Jackson, G., & Lambert, J. A. (2000). More on the hierarchy of propositions: Exploring the distinction between explanations and propositions. *Science & Justice*, 40(1), 3–10.
- Fenton, N., Berger, D., Lagnado, D., Neil, M., & Hsu, A. (2014). When ‘neutral’ evidence still has probative value (with implications from the Barry George Case). *Science & Justice*, 54(4), 274–287.
- Finkelstein, M. (2009). *Basic concepts of probability and statistics in the law*. Springer.
- Fitelson, Branden. (1999). The plurality of bayesian measures of confirmation and the problem of measure sensitivity. *Philosophy of Science*, 66, S362–S378. University of Chicago Press. Retrieved from <https://doi.org/10.1086/392738>
- Fitelson, Branden. (2002). Putting the irrelevance back into the problem of irrelevant conjunction. *Philosophy of Science*, 69(4), 611–622. University of Chicago Press.
- Fitelson, Branden. (2021). *A problem for confirmation measure Z*. [online manuscript].
- Foreman, L., Champod, C., Evett, I. W., Lambert, J., Pope, S., & others. (2003). Interpreting DNA evidence: A review. *International Statistical Review*, 71(3), 473–495. International Statistical Institute.
- Gelman, A., Katz, J. N., & Tuerlinckx, F. (2002). The mathematics and statistics of voting power. *Statistical Science*, 420–435. JSTOR.
- Gelman, A., King, G., & Boscardin, W. J. (1998). Estimating the probability of events that have never occurred: When is your vote decisive? *Journal of the American Statistical Association*, 93(441), 1–9. Taylor & Francis.
- Gillies, D. (1986). In defense of the popper-miller argument. *Philosophy of Science*, 53(1), 110–113. University of Chicago Press. Retrieved from <https://doi.org/10.1086/289295>
- Gross, S. R., O’Brien, B., Hu, C., & Kennedy, E. H. (2014). Rate of false conviction of criminal defendants who are sentenced to death. *Proceedings of the National Academy of Sciences*, 111(20), 7230–7235.
- Hawthorne, J., & Fitelson, B. (2004). Discussion: Re-solving irrelevant conjunction with probabilistic independence, 71(4), 505–514. University of Chicago Press. Retrieved from <https://doi.org/10.1086/423626>
- Heckerman, D. (1988). An axiomatic framework for belief updates. *Machine Intelligence and Pattern Recognition*, 5, 11–22.
- Klobuchar, A., Steblay, N. K. M., & Caligiuri, H. L. (2006). Improving eyewitness identifications: Hennepin county’s blind sequential lineup pilot project. *Cardozo Pub. L. Pol’y & Ethics J.*, 4, 381–413. HeinOnline.
- Lempert, R. O. (1977). Modeling relevance. *Michigan Law Review*, 75, 1021–1057. JSTOR.
- Lindsay, R. C. L., Wells, G. L., & Rumpel, C. M. (1981). Can people detect eyewitness-identification accuracy within and across situations? *Journal of Applied Psychology*, 66(1), 79–89.
- Mayo, D. (2018). *Statistical inference as severe testing*. Cambridge University Press.
- Meester, R., & Sjerps, M. (2004a). Why the effect of prior odds should accompany the likelihood ratio when reporting DNA evidence. *Law, Probability and Risk*, 3(1), 51–62.

- Meester, R., & Sjerps, M. (2004b). Response to Dawid, Balding, Triggs and Buckleton. *Law, Probability and Risk*, 3(1), 83–86.
- Mulligan, C. B., & Hunter, C. G. (2003). The empirical frequency of a pivotal vote. *Public Choice*, 116(1), 31–54. Springer.
- National Research Council. (1992). *DNA technology in forensic science* [NRC I]. Committee on DNA technology in Forensic Science, National Research Council.
- National Research Council. (1996). *The evaluation of forensic DNA evidence* [NRC II]. Committee on DNA technology in Forensic Science, National Research Council.
- Niedermeier, K. E., Kerr, N. L., & Messeé, L. A. (1999). Jurors' use of naked statistical evidence: Exploring bases and implications of the Wells effect. *Journal of Personality and Social Psychology*, 76(4), 533–542.
- Pardo, M. S. (2013). The Nature and Purpose of Evidence Theory. *Vanderbilt Law Review*, 66, 547–613.
- Park, R. C., Tillers, P., Moss, F. C., Risinger, D. M., Kaye, D. H., Allen, R. J., Gross, S. R., et al. (2010). Bayes Wars Redivivus – An Exchange. *International Commentary on Evidence*, 8(1).
- Redmayne, M. (2008). Exploring the proof paradoxes. *Legal Theory*, 14(4), 281–309. Cambridge University Press.
- Royall, R. M. (1997). *Statistical evidence: A likelihood paradigm*. Chapman; Hall/CRC.
- Shaer, M. (2016). The false promise of DNA testing. *The Atlantic*. Retrieved from <https://www.theatlantic.com/magazine/archive/2016/06/a-reasonable-doubt/480747/>
- Smith, M. (2017). When does evidence suffice for conviction? *Mind*.
- Sprenger, J., & Hartmann, S. (2019). *Bayesian philosophy of science*. Oxford University Press.
- Stockmarr, A. (1999). Likelihood Ratios for Evaluating DNA Evidence When the Suspect is Found Through a Database Search. *Biometrics*, 55(3), 671–677.
- Supreme Court of New York, K. C. (1995). *The people of the state of new york, plaintiff, v. Basheen rush, defendant*.
- Taroni, F., Biedermann, A., Bozza, S., Garbolino, P., & Aitken, C. (2014). *Bayesian networks for probabilistic inference and decision analysis in forensic science* (2nd ed.). John Wiley & Sons.
- Thompson, S. G. (2007). Beyond a reasonable doubt-reconsidering uncorroborated eyewitness identification testimony. *UC Davis L. Rev.*, 41, 1487–1545. HeinOnline.
- Thompson, W. C. (2013). Forensic DNA evidence: The myth of infallibility. In S. Krinsky & J. Gruber (Eds.), *Genetic explanations: Sense and nonsense* (pp. 227–347). Harvard University Press.
- Thomson, J. J. (1986). Liability and individualized evidence. *Law and Contemporary Problems*, 49(3), 199–219. JSTOR.
- Triggs, C. M., & Buckleton, J. S. (2004). Comment on: Why the effect of prior odds should accompany the likelihood ratio when reporting DNA evidence. *Law, Probability and Risk*, 3, 73–82.
- Wells, G. L. (1992). Naked statistical evidence of liability: Is subjective probability enough? *Journal of Personality and Social Psychology*, 62(5), 739–752. American Psychological Association.
- Wells, G. L., & Olson, E. A. (2003). Eyewitness Testimony. *Annual Review of Psychology*, 54(1), 277–295.
- Wixted, J. T., & Wells, G. L. (2017). The Relationship Between Eyewitness Confidence and Identification Accuracy: A New Synthesis. *Psychological Science in the Public Interest*, 18(1), 10–65.
- Wright, D. B., & McDaid, A. T. (1996). Comparing system and estimator variables using data from real line-ups. *Applied Cognitive Psychology*, 10, 75–84.