

Awareness Growth in Bayesian Networks

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We examine different counterexamples to Reverse Bayesianism, a popular theory that addresses the problem of awareness growth. We agree with the general skepticism toward Reverse Bayesianism, but submit that the problem of awareness growth cannot be tackled in an algorithmic manner, because subject-matter, structural assumptions need to be made explicit. Thanks to their ability to express probabilistic dependencies, we illustrate how Bayesian networks can help to model awareness growth in the Bayesian framework.

1 Introduction

Learning is modeled in the Bayesian framework by the rule of conditionalization. This rule posits that the agent's new degree of belief in a proposition H after a learning experience E should be the same as the agent's old degree of belief in H conditional on E . That is,

$$P^E(H) = P(H|E),$$

where $P()$ represents the agent's old degree of belief (before the learning experience E) and $P^E()$ represents the agent's new degree of belief (after the learning experience E).

Both E and H belong to the agent's algebra of propositions. This algebra models the agent's awareness state, the propositions taken to be live possibilities. Conditionalization never modifies the algebra and thus makes it impossible for an agent to learn something they have never thought about. Even before learning about E , the agent must already have assigned a degree of belief to any proposition conditional on E . This picture commits the agent to the specification of their 'total possible future experience' (Howson, 1976), as though learning was confined to an 'initial prison' (Lakatos, 1968).

1 But, arguably, the learning process is more complex than what conditionalization allows.
2 Not only do we learn that some propositions that we were entertaining are true or false, but
3 we may also learn new propositions that we did not entertain before. Or we may entertain
4 new propositions—without necessarily learning that they are true or false—and this change
5 in awareness may in turn change what we already believe. How should this more complex
6 learning process be modeled by Bayesianism? Call this the problem of awareness growth.

7 The algebra of propositions need not be so narrowly construed that it only contains proposi-
8 tions that are presently under consideration. The algebra may also contain propositions which,
9 though outside the agent’s present consideration, are still the object, perhaps implicitly, of
10 certain dispositions to believe.¹ But even this expanded algebra will have to be revised sooner
11 or later. The algebra of propositions could in principle contain anything that could possibly be
12 conceived, expressed, thought of. Such a rich algebra would not need to change at any point,
13 but this is an implausible model of ordinary agents with bounded resources such as ourselves.

14 Critics of Bayesianism and sympathizers alike have been discussing the problem of awareness
15 growth under different names for quite some time, at least since the eighties. This problem arises
16 in a number of different contexts, for example, new scientific theories (Chihara, 1987; Earman,
17 1992; Glymour, 1980), language changes and paradigm shifts (Williamson, 2003), and theories
18 of induction (Zabell, 1992). A proposal that has attracted considerable scholarly attention
19 in recent years is Reverse Bayesianism (Bradley, 2017; Karni & Vierø, 2015; Wenmackers
20 & Romeijn, 2016). The idea is to model awareness growth as a change in the algebra while
21 ensuring that the proportions of probabilities of the propositions shared between the old and
22 new algebra remain the same in a sense to be specified.

23 Let \mathcal{F} be the initial algebra of propositions and let \mathcal{F}^+ the algebra after the agent’s awareness
24 state has grown. Both algebras contain the contradictory and tautologous propositions \perp and
25 \top , and they are closed under connectives such as disjunction \vee , conjunction \wedge and negation \neg .
26 Denote by X and X^+ the subsets of these algebras that contain only basic propositions, namely
27 those without connectives. **Reverse Bayesianism** posits that the ratio of probabilities for any
28 basic propositions A and B in both X and X^+ —the basic propositions shared by the old and

¹Roussos (2021) notes that, for the sake of clarity, the problem of awareness growth should only address propositions which agents are *truly* unaware of (say new scientific theories), not propositions that were temporarily forgotten or set aside. This is a helpful clarification to keep in mind, although the recent literature on the topic does not make a sharp distinction between true unawareness and temporary unawareness.

1 new algebra—remain constant through the process of awareness growth:

$$\frac{P(A)}{P(B)} = \frac{P^+(A)}{P^+(B)},$$

2 where $P()$ represents the agent's degree of belief before awareness growth and $P^+()$ represents
3 the agent's degree of belief after awareness growth.

4 Reverse Bayesianism is an elegant theory that manages to cope with a seemingly intractable
5 problem. As the awareness state of an agent grows, the agent would prefer not to throw
6 away completely the epistemic work they have done previously. The agent may desire to retain
7 as much of their old degrees of beliefs as possible. Reverse Bayesianism provides a simple
8 recipe to do that. It also coheres with the conservative spirit of Bayesian conditionalization
9 which preserves the old probability distribution conditional on what is learned.

10 Unfortunately, Reverse Bayesianism does not deliver the intuitive results in all cases. There
11 is no shortage of counterexamples against it in the recent philosophical literature (Mathani,
12 2020; Steele & Stefánsson, 2021). In addition, attempts to extend traditional arguments in
13 defense of Bayesian conditionalization to the case of awareness growth seem to hold little
14 promise (Pettigrew, forthcoming). If the consensus in the literature is that Reverse Bayesianism
15 is not the right theory of awareness growth, what theory (if any) should replace it?

16 Here we offer a diagnosis of what is wrong with Reverse Bayesianism and outline an
17 alternative proposal. The problem of awareness growth—we hold—cannot be tackled in an
18 algorithmic manner because subject-matter assumptions, both probabilistic and structural, need
19 to be made explicit. So any theory of awareness growth cannot be a purely formal theory.
20 This does not mean, however, we should give up on probability theory altogether. Thanks
21 to their ability to express probabilistic dependencies, Bayesian networks can help to model
22 awareness growth in the Bayesian framework. We illustrate this claim as we examine different
23 counterexamples to Reverse Bayesianism.

24 **2 Steele and Stefánsson's Examples**

25 In this section, we rehearse two of the counterexamples to Reverse Bayesianism by Steele &
26 Stefánsson (2021). One example targets awareness expansion and the other awareness refine-
27 ment. A precise definition of expansion can be tricky to provide, but a rough characterization

1 will suffice for now. Suppose, as is customary, propositions are interpreted as sets of possible
2 worlds, where the set of all possible worlds is the possibility space. Awareness expansion
3 occurs when a new proposition is added to the algebra and its interpretation includes possible
4 worlds not in the original possibility space. So the addition of the new proposition causes the
5 possibility space to expand. By contrast, awareness refinement (roughly) occurs when the new
6 proposition added to the algebra induces a more fine-grained partition of the possibility space.

7 The most straightforward case of *awareness expansion* occurs when you become aware
8 of a new explanation for the evidence at your disposal which you had not considered before.
9 This can happen in many fields of inquiry: medicine, law, science, everyday affairs. Here is a
10 scenario by Steele & Stefánsson (2021):

11 FRIENDS: Suppose you happen to see your partner enter your best friend's house
12 on an evening when your partner had told you she would have to work late. At
13 that point, you become convinced that your partner and best friend are having an
14 affair, as opposed to their being warm friends or mere acquaintances. You discuss
15 your suspicion with another friend of yours, who points out that perhaps they were
16 meeting to plan a surprise party to celebrate your upcoming birthday—a possibility
17 that you had not even entertained. (Steele & Stefánsson, 2021, sec. 5, Example 2)

18 Initially, the algebra contained the hypotheses 'my partner and my best friend met to have an
19 affair' (*affair*) and 'my partner and my best friend met as friends' (*friends*). These were the
20 only explanations you considered for the fact that your partner and your best friend met one
21 night without telling you. But, when the algebra changes, a new hypothesis is added which you
22 had not considered before: your partner and your best friends met to plan a surprise party for
23 your upcoming birthday (*surprise*).

24 This change in the algebra is not innocuous. At first, the hypothesis *affair* seems more likely
25 than *friends* because the former seems a better explanation than the latter for the secretive
26 behavior of your partner and best friend.² But when the new hypothesis *surprise* is added,
27 things change: *surprise* now seems a better explanation overall and thus more likely than
28 *affair*. And since *surprise* implies *friends*, the latter must be more likely than *affair*. This
29 conclusion violates Reverse Bayesianism since the ratio of the probabilities of *friends* and

²This assumes that the prior probabilities of the two hypotheses were not strongly skewed in one direction. If you were initially nearly certain your partner could not possibly have an affair with your best friend, the fact they behaved secretly or lied to you should not affect that much the probability of the two hypotheses.

1 *affair* has changed before and after awareness expansion.

2 Steele & Stefánsson note that a quick fix is available. It is reasonable to suppose that no
3 change in the probabilities should occur so long as we confine ourselves to the old probability
4 space. With this in mind, consider the following condition, called **Awareness Rigidity**:

$$P^+(A|T^*) = P(A),$$

5 where T^* corresponds to a proposition that picks out, from the vantage point of the new
6 awareness state, the entire possibility space *before* the episode of awareness growth. Awareness
7 rigidity posits that, once a suitable proposition T^* is identified, the old probability assignments
8 remain unchanged conditional on T^* . In our running example, $\neg surprise$ is the suitable propo-
9 sition T^* : *that* there were was no surprise party in the making picks out the original possibility
10 space. Conditional on $\neg surprise$, no probability assignment should change, including the
11 probability of *affair*. This is the intended result.

12 But this is not the end of the story. Steele & Stefánsson go on to show that Awareness
13 Rigidity does not hold in other cases, what they call *awareness refinement*. As noted before,
14 these are cases in which the new proposition induces a more fine-grained partition of the
15 possibility space. Consider this scenario:

16 MOVIES: Suppose you are deciding whether to see a movie at your local cinema.
17 You know that the movie's predominant language and genre will affect your
18 viewing experience. The possible languages you consider are French and German
19 and the genres you consider are thriller and comedy. But then you realise that,
20 due to your poor French and German skills, your enjoyment of the movie will
21 also depend on the level of difficulty of the language. Since it occurs to you that
22 the owner of the cinema is quite simple-minded, you are, after this realisation,
23 much more confident that the movie will have low-level language than high-level
24 language. Moreover, since you associate low-level language with thrillers, this
25 makes you more confident than you were before that the movie on offer is a thriller
26 as opposed to a comedy. (Steele & Stefánsson, 2021, sec. 5, Example 3)

27 You initially categorized movies by just language and genre, and then you refined your catego-
28 rization by adding another variable, level of difficulty. Without considering language difficulty,

1 you assigned the same probability to the hypotheses *thriller* and *comedy*. But learning that the
2 owner was simple-minded made you think that the level of linguistic difficulty must be low and
3 the movie most likely a thriller rather than a comedy (perhaps because thrillers are simpler—
4 linguistically—than comedies). Since the probability of *thriller* goes up, this scenario violates
5 (against Reverse Bayesianism) the condition $\frac{P(\textit{thriller})}{P(\textit{comedy})} = \frac{P^+(\textit{thriller})}{P^+(\textit{Comedy})}$. For the same reason, it also
6 violates (against Awareness Rigidity) the condition $P(\textit{thriller}) = P^+(\textit{thriller}|\textit{thriller} \vee \textit{comedy})$,
7 where $\textit{thriller} \vee \textit{comedy}$ is a proposition that picks out the entire possibility space.³

8 Some might object that the probability of *thriller* goes up, not because of awareness refine-
9 ment, but because you learn that the owner is simple-minded. And if learning in the strict
10 Bayesian sense—one modeled by conditionalization—takes place, it should be no surprise that
11 some probabilities will change. We will see, however, cases of awareness refinement that do no
12 involve learning in the Bayesian sense and still violate Reverse Bayesianism and Awareness
13 Rigidity. So it is incumbent to understand under what circumstances these principles fail.

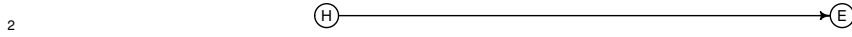
14 As will become clear, we believe that theorizing about awareness growth should be grounded
15 in the subject-matter information underlying the scenario at hand. This subject-matter takes
16 many forms. In *FRIENDS*, awareness expansion does not change the basic presupposition that
17 someone’s behavior must have a reason. In *MOVIES*, awareness refinement does not change the
18 fact that characteristics such as language, difficulty or genre may influence one’s decision to
19 select a movie for showing rather than another. Arguably, what is wrong with principles such
20 as Reverse Bayesianism or Awareness Rigidity is that they are purely formal. In contrast, we
21 need a formalism that can—at least in part—represent the relevant subject-matter information.
22 In what follows, we illustrate how Bayesian networks can serve this purpose.

23 3 Expansion with Bayesian Networks

24 A Bayesian network is a compact formalism to represent probabilistic dependencies. It consists
25 of a direct acyclic graph (DAG) accompanied by a probability distribution. The nodes in
26 the graph represent random variables that can take different values. We will use ‘nodes’ and
27 ‘variables’ interchangeably. The nodes are connected by arrows, but no loops are allowed,
28 hence the name ‘direct acyclic graph’. A simple graph structure we will use repeatedly is the

³Since *MOVIES* is a case of refinement, $\textit{thriller} \vee \textit{comedy}$ picks out the entire possibility space both before and after awareness growth.

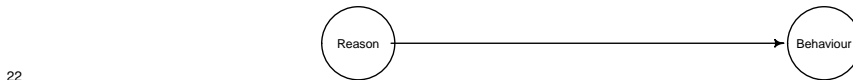
1 so-called *hypothesis-evidence idiom* (Fenton, Neil, & Lagnado, 2013):



3 where H is the hypothesis node (upstream) and E the evidence node (downstream). If an
4 arrow goes from H to E , the full probability distribution associated with the Bayesian network
5 is defined by two probability tables.⁴ One table defines the prior probabilities $P(H = h)$ for
6 all the states (or values) of H , and another table defines the conditional probabilities of the
7 form $P(E = e|H = h)$, where uppercase letters represent the variables (nodes) and lower case
8 letters represent the states (or values) of these variables. These two probability tables are
9 sufficient to specify the full probability distribution. The other probabilities—say $P(E = e)$,
10 $P(H = h|E = e)$, etc.—follow by simply applying the probability axioms. As needed, more
11 complex graphical structures can also be used.

12 Bayesian networks are relied upon in many fields, but have been rarely deployed to model
13 awareness growth (the exception is Williamson (2003)). We think instead they are a good
14 framework for this purpose. Awareness growth can be modeled as a change in the graphical
15 network—nodes and arrows are changed, added or erased—as well as a change in the probabil-
16 ity distribution from the old to the new network. In this section, we focus on cases of awareness
17 expansion and turn to refinement in the next section.

18 Recall the scenario FRIENDS from before. It can be modeled with the hypothesis-evidence
19 idiom. The graph can be made more perspicuous by labeling the downstream node ‘Behavior’
20 (the evidence or fact to be explained) and the upstream node ‘Reason’ (the explanation or
21 hypothesis about the the cause of the behavior):



23 Initially, before awareness growth, the hypothesis node *Reason* takes only two states: *friends*
24 and *affair*. These two states are meant to be exclusive and exhaustive, so *affair* functions
25 as the negation of *friends*, and vice versa. After awareness growth—specifically, awareness
26 expansion—the two states are no longer exhaustive. A third state is added: *surprise*. So, in the

⁴A major point of contention in the interpretation of Bayesian networks is is the meaning of the directed arrows. They could be interpreted causally—as though the direction of causality proceeds from the events described by the hypothesis to event described by the evidence—but they need not be; see footnote 18.

1 formalism we are using, expansion simply consists in the addition of an extra state to one of
2 the nodes of the network. The rest of the structure of the network remains intact.

3 Recall that the ratio of posterior probabilities of *friends* to *affairs* changed as a result of
4 awareness expansion. The fact that your partner and best friend met without telling you—call
5 this behavior *secretive*—initially made *affair* more likely than *friends*, but then the same fact
6 made *friends* more likely than *affair*. So, formally,

$$1 > \frac{P(\text{Reason}=\text{friends}|\text{Behavior}=\text{secretive})}{P(\text{Reason}=\text{affair}|\text{Behavior}=\text{secretive})} < \frac{P^+(\text{Reason}=\text{friends}|\text{Behavior}=\text{secretive})}{P^+(\text{Reason}=\text{affair}|\text{Behavior}=\text{secretive})} > 1.$$

7 But, despite this change in posterior probabilities, it is plausible to assume that the novel
8 state added to the upstream node *Reason* did not change the relative plausibility of the two
9 explanations initially under consideration. Even after awareness expansion, the fact that your
10 partner and best friend met without telling you—*secretive*—makes better sense in light of *affair*
11 compared to *friends*:

$$\frac{P(\text{Behavior}=\text{secretive}|\text{Reason}=\text{affair})}{P(\text{Behavior}=\text{secretive}|\text{Reason}=\text{friends})} = \frac{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{affair})}{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{friends})} > 1.$$

12 This equality holds even though the novel explanation *surprise* introduced during awareness
13 expansion makes better sense of the secretive behavior overall:⁵

$$\frac{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{surprise})}{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{affair})} > 1.$$

14 This analysis suggests a slight reformulation of conditions such as Reverse Bayesianism
15 and Awareness Rigidity. Instead of comparing posterior probabilities of hypotheses given the
16 evidence ($P(H = h|E = e)$), we can compare which explanation or hypothesis makes better
17 sense of the evidence ($P(E = e|H = h)$). So, for all values e and h of upstream node H and
18 downstream node E in the old network, consider the following constraint:

$$\frac{P(E = e|H = h)}{P(E = e|H = h)} = \frac{P^+(E = e|H = h \& X \neq x^*)}{P^+(E = e|H = h \& X \neq x^*)}, \quad (\text{C})$$

19 where x^* is the new state added and X is the node (upstream or downstream) to which the new

⁵Even though $\frac{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{surprise})}{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{affair})} > 1$ and $\frac{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{affair})}{P^+(\text{Behavior}=\text{secretive}|\text{Reason}=\text{friends})} > 1$ —so *affair* still makes better sense of the evidence than *friends* before and after awareness expansion—the posterior probability of *friends* is higher than *affair* after awareness expansion.

1 state belongs, such as $H = \textit{surprise}$ in FRIENDS.

2 In words, the constraint says that one's old assessment of the relative plausibility of two
3 competing hypotheses in light of a fixed body of evidence should remain unchanged after
4 awareness expansion. Constraint (C) is a variant of Reverse Bayesianism that only applies to
5 conditional probabilities of the form $P(E = e | H = h)$ for Bayesian networks of the form $H \rightarrow E$.
6 In addition, the constraint mimics Awareness Rigidity in that it ensures that the conditional
7 probabilities in question exclude the novel state $X = x^*$.

8 How generally does constraint (C) apply besides examples such as FRIENDS? The conjecture
9 we are putting forward is that, so long as there is no change in the structure of the network, the
10 constraint should hold. Since awareness expansion—as we have defined it—does not involve
11 any change in the structure, the constraint should hold for all cases of expansion.⁶

12 4 Mathani's Examples

13 To acquire a firmer grasp on constraint (C), we now examine a couple of examples by Mathani
14 (2020). In her reading, these examples serve as counterexamples to Reverse Bayesianism, but
15 are also meant to challenge the traditional distinction between expansion and refinement. But,
16 when modeled with Bayesian networks, they are straightforward cases of awareness expansion.

17 The first of Mathani's examples goes like this:

18 TENANT: You are staying at Bob's flat which he shares with his landlord. You
19 know that Bob is a tenant, and that there is only one landlord, and that this landlord
20 also lives in the flat. In the morning you hear singing coming from the shower
21 room, and you try to work out from the sounds who the singer could be. At this
22 point you have two relevant propositions that you consider possible ... *landlord*
23 standing for the possibility that the landlord is the singer, and *bob* standing for
24 the possibility that Bob is the singer ... Because you know that Bob is a tenant

⁶It is crucial that the new hypothesis or explanation does not change the existing structure of the network. For consider this example. You are wondering which horse will win the race. You have done a careful study of past performances under different conditions and concluded that Red is more likely to win than Green. But you have not considered the possibility that Grey would run. If Grey does run, it will have a greater chance of winning than the others, but will also make—for some odd reason—Green a much better racer than Red. So the odds that Green will win compared to Red should now be higher. Here, the new hypothesis introduces some novel information that was not known before, say that the participation of Grey would weaken Red's performance and strengthen Green's performance. So the network should be changed in two ways: first, a new state should be added to the outcome node (Green wins, Red wins, Grey wins); and second, a new node should be added modeling the fact that Grey is participating and its participation affects Red's and Green's performance.

1 in the flat, you also have a credence in the proposition *tenant* that the singer is a
2 tenant. Your credence in *tenant* is the same as your credence in *bob*, for given
3 your state of awareness these two propositions are equivalent ... Now let's suppose
4 the possibility suddenly occurs to you that there might be another tenant living in
5 the same flat (*other*).

6 Initially, you thought the singer could either be the landlord or Bob, the tenant. Then you come
7 to the realization that a third person could be the singer, another tenant. The possibility that
8 there could be a third person in the shower—besides Bob or the landlord—is a novel explanation
9 for why you hear singing in the shower. So TENANT seems to be a standard case of expansion
10 like FRIENDS. At the same time, this scenario is a bit more complicated. The expansion in
11 awareness goes along with an interesting conceptual shift. Before awareness expansion, that
12 Bob is in the shower and that a tenant is in the shower are equivalent descriptions, but after the
13 expansion, this equivalence breaks down.

14 As Mathani shows, this scenario challenges Reverse Bayesianism. For it is natural to assign
15 $1/3$ to *landlord*, *bob* and *other* after awareness growth, and $1/2$ to *landlord* and *bob* before
16 awareness growth. That someone is singing in the shower is evidence that someone must be in
17 there, but without any more discriminating evidence, each person should be assigned the same
18 probability. Consequently, a probability of $2/3$ should be assigned to *tenant* after awareness
19 growth, but only $1/2$ before. On this picture, the proportion of *landlord* to *tenant* changes from
20 1:1 (before awareness growth) to 1:2 (after awareness growth).^{7 8}

21 But now recall that Reverse Bayesianism only applies to basic propositions, which we defined
22 earlier as propositions without connectives. So a possible fix here is to adopt the following
23 principle: if two propositions happen to be equivalent relative to some awareness state, they
24 cannot be both considered basic. In TENANT, since *bob* and *tenant* are initially equivalent
25 descriptions of the same state of affairs, they would not be considered both basic propositions.

⁷Here is more involved argument. Suppose, after you hear singing in the shower, you become sure someone is in there, but you cannot tell who. So $P(\text{landlord}) = P(\text{bob}) = 1/2$, and since *bob* and *tenant* are equivalent, also $P(\text{tenant}) = 1/2$. Now, *landlord*, *Bob* and *tenant* are all propositions that you were originally aware of, and thus Reverse Bayesianism requires that their probabilities should remain in the same proportion after your awareness grows. But note that *other* entails *tenant* and *bob* and *Other* are disjoint, so it follows that $P^+(\text{other})$ must have zero probability. If $P^+(\text{other})$ were greater than zero, the proportion of the probability of *tenant* to *landlord* (or the proportion of the probability of *bob* to *landlord*) should change.

⁸This scenario need not be a challenge for Awareness Rigidity. Much depends on the choice of the proposition T^* that picks out, from the vantage point of the new awareness state, the old possibility space prior to awareness growth. The proposition $\text{landlord} \vee \text{bob}$ does the job. For $P^+(\text{landlord}|\text{landlord} \vee \text{bob})$ and $P^+(\text{bob}|\text{landlord} \vee \text{bob})$ should both equal $1/2$, and thus $P^+(\text{other}|\text{landlord} \vee \text{bob}) = 0$, but this does not mean that $P^+(\text{other}|\text{landlord} \vee \text{tenant})$ should equal zero. This is the intended result.

1 If only *bob* is considered basic, along with *landlord*, then the proportion of the probability
 2 of *bob* and *landlord* would remain the same during awareness growth, but not the proportion
 3 of the probabilities of *tenant* and *landlord*. This yields the intuitive result. But it is odd that
 4 *landlord* would be considered basic, but not *tenant*. However, if *tenant* is considered basic
 5 along with *landlord*, Reverse Bayesianism would require that the proportion of the probability
 6 of *tenant* and *landlord* remain the same during awareness growth. This is counterintuitive.

7 So dividing propositions into basic and non-basic is riddled with difficulties. But the above
 8 discussion alerted us to the fact that a difference exists between propositions like *bob* and those
 9 like *tenant*. The latter describes a role that different people could play besides Bob. Bayesian
 10 networks can help to model the person/role distinction, as follows:



12 This subject-matter information—the distinction between people and the role they play—
 13 remain fixed throughout the process of awareness expansion. What changes is how the details
 14 are filled in. Initially, the upstream node *Person* has two possible states, representing who
 15 could be in the bathroom singing: *landlord-person* and *bob*.⁹ The downstream node *Role* has
 16 also two values, *landlord* and *tenant*. After your awareness grows, the upstream node *Person*
 17 should now have one more possible state, *other*. Crucially, note that since TENANT is a case of
 18 expansion by our definition—a state was added to a node—constraint (C) should hold. This is
 19 precisely what happens as illustrated in Table 1.¹⁰ This is good news.

20 It is now time to test the tenability of constraint (C) a bit further. So far we only considered
 21 cases of awareness expansion in which a new state was added to an *upstream* node. In
 22 FRIENDS, a state was added to the upstream node *Reason*, and in TENANT, a state was added
 23 to the upstream node *Person*. What if the new state was added to a downstream node? For
 24 consider a variation of FRIENDS. Suppose that the downstream node *Behavior* could initially
 25 take only two values, say *secretive* and *public*. You then realize the node could also take a

⁹To simplify things, the assumption here is that the evidence of singing has already ruled out the possibility that no one would be in the shower. In principle, the network should be more complex and contain another node for the evidence to be explained (the fact of singing in the shower), as follows: *Singing* \leftarrow *Person* \rightarrow *Role*.

¹⁰More generally, these conditional probabilities do not change during awareness growth:

$$P(\text{Role} = \text{landlord} | \text{Person} = \text{landlord}) = P^+(\text{Role} = \text{landlord} | \text{Person} = \text{landlord})$$

$$P(\text{Role} = \text{landlord} | \text{Person} = \text{bob}) = P^+(\text{Role} = \text{landlord} | \text{Person} = \text{bob})$$

$P(\text{Role} \text{Person})$		Person	
Role	tenant	landlord-person	bob
	landlord	0	1
	TOTAL	1	0
		1	1

$P^+(\text{Role} \text{Person})$		Person		
Role	tenant	landlord-person	bob	other
	landlord	0	1	1
		1	0	0
	TOTAL	1	1	1

$P(\text{Person})$		Person	
	landlord-person	bob	
	1/2	1/2	

$P^+(\text{Person})$		Person	
	landlord-person	bob	other
	1/3	1/3	1/3

Table 1: The table displays a plausible probability distribution for the TENANT scenarios. Constraint (C) is met.

third value, say *ambiguous*. Initially, *secretive* and *public* are considered exhaustive, but that is no longer true after the addition of *ambiguous*. So the old conditional probabilities will change, specifically, $P(E = e|H = h) \neq P^+(E = e|H = h)$, where H is the upstream node and E downstream. However, if we exclude the the novel state, there should be no change, so $P(E = e|H = h \ \& \ E \neq e^*) = P^+(E = e|H = h \ \& \ E \neq e^*)$, where e^* is the novel state added to the downstream node E . So constraint (C) should again be satisfied.

The same analysis applies to a more complex example by Mathani:

COIN: You know that I am holding a fair ten pence UK coin which I am about to toss. You have a credence of 0.5 that it will land *heads*, and a credence of 0.5 that it will land *tails*. You think that the tails side always shows an engraving of a lion. So you also have a credence of 0.5 that it will land with the lion engraving face-up (*lion*): relative to your state of awareness *tails* and *lion* are equivalent.... Now let's suppose that you somehow become aware that occasionally ten pence coins have an engraving of Stonehenge on the tails side (*stonehenge*).

The propositions *tails* and *lion* are equivalent prior to awareness growth. Suppose you initially gave *tails* and *lion* the same credence. If they are basic propositions, Reverse Bayesianism would require that their relative proportions should stay the same after awareness grow. The

1 same is true of *heads* and *tails*. But since *lion* and *stonehenge* are incompatible and the latter
 2 entails *tails*, you should have $P^+(stonehenge) = 0$, an undesirable conclusion.

3 Mathani observes that this scenario blurs the distinction between expansion and refinement.
 4 For one thing, COIN seems a case of refinement. The space of possibilities is held fixed—the
 5 coin could come up heads or tails—but the options for tails are further refined, for tails could
 6 be *lion* or *stonehenge*. On the other hand, a new possibility has been added after awareness
 7 growth, namely *stonehenge*, which had not been considered before. This would indicate that
 8 COIN is a case of refinement. This ambiguity makes it difficult to settle whether the scenario is
 9 a challenge for Awareness Rigidity.¹¹

10 These difficulties disappear if the scenario is modeled using Bayesian networks. The
 11 definition of awareness expansion we have been working with is simple: whenever a new state
 12 is added to one of the nodes in the network, awareness expansion takes place. The novel state
 13 can be added to any node in the network. Each node, with its range of states, characterizes
 14 an exhaustive partition of the possibility space. Whenever a new state is added to a node, the
 15 partition associated with the node expands. (Refinement, as we will see in the next section,
 16 consists in the addition of a new node, not in the addition of a new state to an existing node.)

17 By the definition of expansion just given, COIN counts as a case of expansion, but it is
 18 structurally different from the more straightforward cases such as FRIENDS and TENANT.
 19 Bayesian networks can help to model the difference precisely. The scenario can be modeled by
 20 this familiar graph structure:



22 The upstream node *outcome* has two states, *tails* and *heads*. These two states remain the same
 23 throughout. What changes are the states associated with the *image* node downstream. Before
 24 awareness growth, the node *image* has two states: *lions* and *heads-image*.¹² You assume that
 25 *Image = lions* is true if and only if *Outcome = tails* is true. Then, you come to the realization
 26 that the imagines for tails could include a lion or a stonehenge engraving. So, after awareness

¹¹If it is a case of refinement, $heads \vee tails$ would pick out the entire possibility space even before awareness growth. If so, by Awareness Rigidity, $P^+(tails|heads \vee tails)$ and $P^+(lion|heads \vee tails)$ should both equal $1/2$ since these were their probabilities before awareness growth. But these assignments would force $P^+(stonehenge|heads \vee tails)$ to zero. To avoid this odd result for awareness Rigidity, one might argue that $heads \vee tails$ picks out a possibility space larger than the old one, because it also includes the possibility of *stonehenge*. So which is it?

¹²The heads side must have some image, not specified in the scenario.

1 growth, the node *Image* contains three states: *lion*, *stonehenge* and *heads-image*.

2 To some extent, COIN has the same structure as TENANT—they are modeled by the same
3 networks structure—but there is also an important asymmetry that is apparent by comparing
4 their Bayesian networks. In the network for COIN, the states of the upstream node remain fixed
5 while a new state is added to the downstream node. In the network for TENANT, the opposite
6 happens: the states of the downstream node remain fixed, while a new state is added to the
7 upstream node. Specifically, after awareness expansion, no new state is added to upstream
8 node *Outcome*, but an additional state, *other*, is added to the downstream node *Person*.

9 The states of the upstream node model what we tend to view—somewhat loosely speaking—
10 as more causally fundamental compared to the states of downstream nodes. In TENANT, who is
11 the person singing in the shower is more fundamental, while the proposition that describe their
12 roles are derivatives, for multiple people could play the same role. In *Coin*, that an outcome
13 (heads or tails) could be instantiated by different kinds of engravings is considered a derivative
14 fact. What seems causally fundamental is the type of outcome, not the engraving. Bayesian
15 networks offer a language to model these differences that are crucial to model episodes of
16 awareness expansion.

17 So what about constraint (C)? Plausible probability distributions for the Bayesian networks
18 associated with the scenario COIN is displayed in Table 2. It is easy to check that constraint
19 (C) is once again satisfied. So, the constraint has outperformed both Reverse Bayesianism and
20 Awareness Rigidity. But our objective here is not to replace one formal constraint with another.
21 As noted in the introduction, we think that awareness growth cannot be modeled in a purely
22 formal matter. The success of constraint (C) relies on the right network structure. How the
23 networks should be built is based on our subject-matter knowledge—for example, that people’s
24 behavior must have a reason; that multiple peoples can play different roles; or that heads and
25 tails can be associated with different specific engravings. And sometimes awareness growth
26 may even require the very structure of the network to change. This is our next topic.

27 **5 Refinement with Bayesian Networks**

28 We turn now from cases of awareness expansion to cases of awareness refinement. The previous
29 section illustrated why, when no change in structure in the Bayesian network occurs, constraint

$P(\text{Image} \text{Outcome})$		<i>Outcome</i>	
<i>Image</i>	<i>lion</i>	<i>heads</i>	<i>tails</i>
	<i>heads-image</i>	0	1
		1	0
	TOTAL	1	1
$P^+(\text{Image} \text{Outcome})$		<i>Outcome</i>	
<i>Image</i>	<i>lion</i>	<i>heads</i>	<i>tails</i>
	<i>stonehenge</i>	0	1/2
	<i>heads-image</i>	0	1/2
		1	0
	TOTAL	1	1
$P(\text{Outcome}) = P^+(\text{Outcome})$		<i>Outcome</i>	
	<i>heads</i>	<i>tails</i>	
	1/2	1/2	

Table 2: Table displays a plausible probability distribution for the COIN scenario. Constraint (C) is met.

(C) holds. But, of course, awareness expansion may sometimes require to change the structure of the network. What will happen to the constraint then? It will likely fail, at least sometimes. The challenge is to develop a method to determine when it holds and when it fails. This method will afford us a firmer foundation for a general theory of awareness growth.

In the framework of Bayesian networks, expansion consists in adding states to existing nodes in the network. Refinement, instead, can be modeled by adding nodes to the network without adding any new state to existing nodes. Intuitively, refinement takes place when an epistemic agent acquires a more-fined grained picture of the situation, say instead of thinking that the political spectrum is divided into liberals and conservatives, the political spectrum can be further divided into traditional-liberal, new-liberal, traditional-conservative and new-conservative. The political spectrum is still divided into liberal and conservative—no expansion occurred—but the two categories have been further refined.¹³

Although there is no shortage of counterexamples to Reverse Bayesianism when it comes to awareness refinement, we will use our own. Recall that MOVIES—the refinement-based counterexample to Reverse Bayesianism by Steele & Stefánsson—suffered from a possible objection. The example contained awareness refinement paired with a standard case of Bayesian learning by conditionalization. Some might argue that conditionalization, not awareness refinement, is responsible for the change in probabilities. To alleviate this worry, we will work with our own example which can be more clearly interpreted as mere awareness refinement.

¹³This example is from Pettigrew (forthcoming).

1 Our own example will also allow us to underscore the role of subject-matter assumptions in
2 theorizing about awareness growth. So consider this scenario:

3 LIGHTING: You have evidence that favors a certain hypothesis, say a witness saw
4 the defendant around the crime scene. You give some weight to this evidence.
5 In your assessment, that the defendant was seen around the crime scene (your
6 evidence) raises the probability that the defendant was actually there (your hypoth-
7 esis). But now you ask, what if it was dark when the witness saw the defendant?
8 In light of your realization that it could have been dark, you wonder whether (and
9 if so how) you should change the probability that you assigned to the hypothesis
10 that the defendant was around the crime scene.

11 As your awareness grows, you do not learn anything specific about the lighting conditions,
12 neither that they were bad nor that they were good. You simply wonder what they were, a
13 variable you had previously not considered. So no Bayesian conditioning takes place in the strict
14 sense, although broadly speaking some new information has been introduced.¹⁴ Something has
15 changed in your epistemic state—you have a more fine-grained assessment of what could have
16 happened—but it is not clear what you should do in this scenario. Since the lighting conditions
17 could have been bad but could also have been good, perhaps you should just stay put until you
18 learn something more specific.

19 In what follows, we illustrate how Bayesian networks helps to model what is going on in
20 LIGHTING and conclude that you should probably revise downward your confidence in the
21 hypothesis that the defendant was around the crime scene. The starting point of our analysis is
22 the usual hypothesis-evidence idiom, repeated below for convenience:



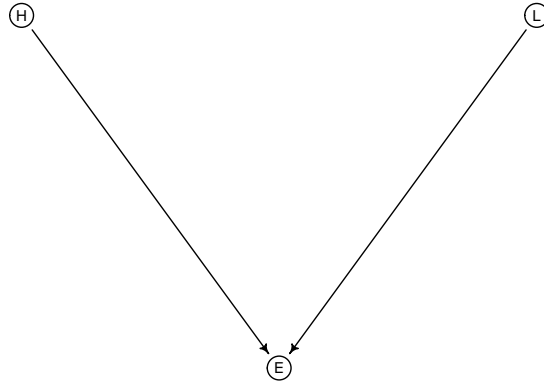
24 Since you trust the evidence, you think that the evidence is more likely under the hypothesis
25 that the defendant was present at the crime scene than under the alternative hypothesis:

$$P(E=seen|H=present) > P(E=seen|H=absent)$$

¹⁴The process of awareness growth in LIGHTING adds only one extra variable, lighting conditions, while MOVIES adds two extra variables, language difficulty and whether the owner is simple-minded or not. Further, MOVIES contains a clear-cut case of Bayesian updating, that the owner *is* simple-minded. This is not so in LIGHTING. Strictly speaking, you are learning that it is *possible* that the lighting conditions were bad. However, you are not conditioning on the proposition ‘the lighting conditions were bad’ or ‘the lighting conditions were good’. So you are not learning about the lighting conditions in the sense of Bayesian updating.

1 The inequality is a qualitative ordering of how plausible the evidence is in light of competing
 2 hypotheses. No matter the numbers, by the probability calculus, it follows that the evidence
 3 raises the probability of the hypothesis $H=present$.

4 Now, as you wonder about the lighting conditions, the graph should be amended:



5
 6 where the node L can have two values, $L=good$ and $L=bad$. Commonsense as well as psycho-
 7 logical findings suggest that when the visibility deteriorates, people's ability to identify faces
 8 worsen. So a plausible way to modify your assessment of the evidence is as follows:

$$P^+(E=seen|H=present \wedge L=good) > P^+(E=seen|H=absent \wedge L=good)$$

9

$$P^+(E=seen|H=present \wedge L=bad) = P^+(E=seen|H=absent \wedge L=bad)$$

10 In words, if the lighting conditions were good, you still trust the evidence like you did before
 11 (first line), but if the lighting conditions were bad, you regard the evidence as no better than
 12 chance (second line). These probabilistic constraints are plausible, but should ultimately be
 13 grounded on verifiable empirical regularities.

14 Despite the change in awareness, you have not learned anything in the strict sense. Your
 15 new stock of evidence does not contain neither the information that the lighting conditions
 16 were bad nor that they were good. But the Bayesian network structure that represents your
 17 epistemic state is now more fine-grained. The network contains the new variable L which it
 18 did not contain prior to the episode of awareness growth. In addition—and this is the crucial
 19 point—the new variable bears certain *structural relationships* with the variables H and E . The
 20 graphical network represents a direct probabilistic dependency between the lighting conditions
 21 L and the witness sensory experience E , but does not allow for any direct dependency between
 22 the lighting conditions and the fact that the defendant was (or was not) at the crime scene.

1 There is no direct arrow between the nodes L and H . This structure of dependencies captures
 2 our causal intuitions about the scenario: the lighting conditions do affect what the witness
 3 could see, but do not directly affect what the defendant might have done.

4 Without Bayesian networks, episodes of awareness growth could only be modeled by the
 5 addition of new propositions that were not previously in the algebra. But this approach would
 6 fail to capture crucial information. When awareness growth takes place against the background
 7 of an intuitive causal structure of the world—as in the case of LIGHTING—this structure should
 8 also be modeled. Bayesian networks offer a formal framework that can do precisely that.

9 This model of causal structure can now guide us to decide whether the restricted version of
 10 Reverse Bayesianism, what we called constraint (C), holds in this scenario. Specifically, we
 11 need to assess whether the following holds:

$$\frac{P(E = \text{seen} | H = \text{present})}{P(E = \text{seen} | H = \text{absent})} = \frac{P^+(E = \text{seen} | H = \text{present})}{P^+(E = \text{seen} | H = \text{absent})}.$$

12 The question here is whether you should assess the evidence at your disposal—that the witness
 13 saw the defendant at the crime scene—any differently than before.¹⁵ As noted earlier, without
 14 a clear model of the scenario, it might seem that you should simply stay put. After all, besides
 15 the sensory experience of the witness, you have gained no novel information about the lighting
 16 conditions. Should you thus conclude that the evidence has the same value before and after the
 17 realization that lighting could have been bad?

18 The evidence would have the same value if the likelihood ratios associated with it relative to
 19 the competing hypotheses were the same before and after awareness growth. But, in changing
 20 the probability function from $P()$ to $P^+()$, it would be quite a coincidence if this were true. In
 21 our example, many possible probability assignments violate this equality. If before awareness
 22 growth you thought the evidence favored the hypothesis $H=\text{present}$ to some extent, after the
 23 growth in awareness, the evidence is likely to appear less strong. To see this is tedious, so we
 24 relegate the details to a footnote.¹⁶ If this is correct, constraint (C) has been violated. Reverse
 25 Bayesianism is also violated since the ratio of the probabilities of $H=\text{present}$ to $E=\text{seen}$, before

¹⁵Note that since no new state was added to an existing node, the condition $X \neq x^*$ in constraint (C) (where x^* is the new state added to an existing node X) is redundant here.

¹⁶By the law of total probability, the right hand side of the equality in (C) should be expanded, as follows:

$$\frac{P^+(E = e | H = h)}{P^+(E = e | H = h')} = \frac{P^+(E = \text{seen} \wedge L = \text{good} | H = \text{present}) + P^+(E = \text{seen} \wedge L = \text{bad} | H = \text{present})}{P^+(E = \text{seen} \wedge L = \text{good} | H = \text{absent}) + P^+(E = \text{seen} \wedge L = \text{bad} | H = \text{absent})}.$$

1 and after awareness growth, has changed:

$$\frac{P^{E=seen}(H=present)}{P^{E=seen}(E=seen)} \neq \frac{P^{+,E=seen}(H=present)}{P^{+,E=seen}(E=seen)},$$

2 where $P^{E=seen}()$ and $P^{+,E=seen}()$ represent the agent's degrees of belief, before and after aware-
 3 ness growth, updated by the evidence $E=seen$.¹⁷

4 The general lesson to be learned here has to do with the importance of formalizing subject-
 5 matter assumptions and the role of Bayesian networks in modeling awareness growth. Modeling
 6 the causal assumptions in LIGHTING allowed us to see that constraint (C)—as well as Reverse
 7 Bayesianism more generally—should fail here. But, as we noted already, constraint (C) might
 8 hold in other, structurally different cases of refinement. For consider this scenario:

9 VERACITY: A witness saw that the defendant was around the crime scene and

For concreteness, let's use some numbers:

$$P(E=seen|H=present) = P^+(E=seen|H=present \wedge L=good) = .8$$

$$P(E=seen|H=absent) = P^+(E=seen|H=absent \wedge L=good) = .4$$

$$P^+(E=seen|H=present \wedge L=bad) = P^+(E=seen|H=absent \wedge L=bad) = .5.$$

$$P^+(L=bad) = P^+(L=good) = .5.$$

So the ratio $\frac{P(E=seen|H=present)}{P(E=seen|H=absent)}$ equals 2. After the growth in awareness, the ratio $\frac{P^+(E=seen|H=present)}{P^+(E=seen|H=absent)}$ will drop to $\frac{.65}{.45} \approx 1.44$. The calculations here rely on the dependency structure encoded in the Bayesian network (see starred step below).

$$\begin{aligned} P^+(E=seen|H=present) &= P^+(E=seen \wedge L=good|H=present) + P^+(E=seen \wedge L=bad|H=present) \\ &= P^+(E=seen|H=present \wedge L=good) \times P^+(L=good|H=present) \\ &\quad + P^+(E=seen|H=present \wedge L=bad) \times P^+(L=bad|H=present) \\ &= * P^+(E=seen|H=present \wedge L=good) \times P^+(L=good) \\ &\quad + P^+(E=seen|H=present \wedge L=bad) \times P^+(L=bad) \\ &= .8 \times .5 + .5 \times .5 = .65 \end{aligned}$$

$$\begin{aligned} P^+(E=seen|H=absent) &= P^+(E=seen \wedge L=good|H=absent) + P^+(E=seen \wedge L=bad|H=absent) \\ &= P^+(E=seen|H=absent \wedge L=good) \times P^+(L=good|H=absent) \\ &\quad + P^+(E=seen|H=absent \wedge L=bad) \times P^+(L=bad|H=absent) \\ &= * P^+(E=seen|H=absent \wedge L=good) \times P^+(L=good) \\ &\quad + P^+(E=seen|H=absent \wedge L=bad) \times P^+(L=bad) \\ &= .4 \times .5 + .5 \times .5 = .45 \end{aligned}$$

This argument can be repeated with many other numerical assignments.

¹⁷The scenario also violates Awareness Rigidity which requires that $P^+(A|T^*) = P(A)$, where T^* corresponds to a proposition that picks out, from the vantage point of the new awareness state, the entire possibility space before the episode of awareness growth. In LIGHTING, however, T^* does not change, so Awareness Rigidity would require that $P^+(A) = P(A)$, and instead in the scenario, we have

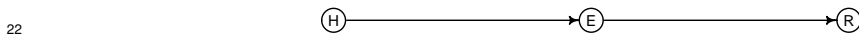
$$P^+(H=present|E=seen) \neq P(H=present|E=seen).$$

1 you initially took this to be evidence that the defendant was actually there. But
 2 then you worry that the witness might be lying or misremembering what happened.
 3 Perhaps, the witness was never there, made things up or mixed things up. Should
 4 you reassess the evidence at your disposal? If so, how?

5 It might seem that this scenario is no different from LIGHTING. The realization that lighting
 6 could be bad should make you less confident in the truthfulness of the sensory evidence. And
 7 the same conclusion should presumably follow from the realization that the witness could be
 8 lying. But, upon closer scrutiny, things are not that simple. To run the two scenarios together
 9 would be a mistake.

10 The evidence at your disposal in LIGHTING is the sensory evidence—the experience of
 11 seeing—and the possibility of bad lighting does affect the quality of your visual experience.
 12 So, if lighting was indeed bad, this would warrant lowering your confidence in the truthfulness
 13 of the visual experience. But the possibility of lying in VERACITY does not affect the quality
 14 of the visual experience; it only affects the quality of the *reporting* of that experience. So, if
 15 the witness did lie, this would not warrant lowering your confidence in the truthfulness of the
 16 visual experience, only in the truthfulness of the reporting. The distinction between the visual
 17 experience and its reporting is crucial here. Bayesian networks help to model this distinction
 18 precisely, and then see why LIGHTING and VERACITY are structurally different.

19 The graphical network should initially look like the initial DAG for LIGHTING, consisting
 20 of the hypothesis node H upstream and the evidence node E downstream. As your awareness
 21 grows, the graphical network should be updated by adding another node R further downstream:



23 As before, the hypothesis node H bears on the whereabouts of the defendant and has two values,
 24 $H=present$ and $H=absent$. Note the difference between E and R . The evidence node E bears
 25 on the visual experience had by the witness. The reporting node R , instead, bears on what the
 26 witness reports to have seen. The chain of transmission from ‘visual experience’ to ‘reporting’
 27 may fail for various reasons, such as lying or misremembering.

28 In VERACITY, the conditional probabilities, $P(E = e|H = h)$ should be the same as $P^+(E =$
 29 $e|H = h)$ for any values e and h of the variables H and E that are shared before and after
 30 awareness growth. In comparing the old and new Bayesian network, this equality falls out from

1 their structure, as the connection between H and E remains unchanged. Thus, constraint (C) is
2 perfectly fine in scenarios such as VERACITY.

3 This does not mean that the assessment of the probability of the hypothesis $H=present$ should
4 undergo no change. If you worry that the witness could have lied, this should presumably
5 make you less confident about $H=present$. To accommodate this intuition, VERACITY can
6 be interpreted as a scenario in which an episode of awareness refinement takes place together
7 with a form of retraction. At first, after the learning episode, you update your belief based on
8 the *visual experience* of the witness. But after the growth in awareness, you realize that your
9 learning is in fact limited to what the witness *reported* to have seen. The previous learning
10 episode is retracted and replaced by a more careful statement of what you learned: instead
11 of conditioning on $E=seen$, you should condition on what the witness reported to have seen,
12 $R=seen-reported$. This retraction will affect the probability of the hypothesis $H=present$.

13 Where does this leave us? Refinement cases that might at first appear similar can be
14 structurally different in important ways, and this difference can be appreciated by looking at
15 the Bayesian networks used to model them. In modeling VERACITY, the new node is added
16 downstream, while in modeling LIGHTING, it is added upstream. This difference affects how
17 probability assignments should be revised. Since the conditional probabilities associated with
18 the upstream nodes are unaffected, constraint (C) is satisfied in VERACITY. By contrast, since
19 the conditional probabilities associated with the downstream node will often have to change,
20 the constraint fails in LIGHTING.

21 This further corroborates our working hypothesis: subject-matter assumptions—in the case
22 of LIGHTING and VERACITY, causal assumptions—about how we conceptualize a specific
23 scenario are the guiding principles about how we update the probability function through
24 awareness growth, not formal principles like Reverse Bayesianism, Awareness Rigidity or even
25 constraint (C).

26 **6 Towards a general theory**

27 We conclude with some programmatic remarks. We think that the awareness of agents grows
28 while holding fixed certain material structural assumptions, based on commonsense, semantic

1 stipulations or causal dependency.¹⁸ To model awareness growth, we need a formalism that
 2 can express these material structural assumptions. This can be done using Bayesian networks,
 3 and we offered some illustrations of this strategy. These material assumptions also guide
 4 us in formulating the adequate conservative constraints, and these will inevitably vary on
 5 a case-by-case basis. The literature on awareness growth from a Bayesian perspective is
 6 primarily concerned with a formal, almost algorithmic solution to the problem. Insofar as
 7 Reverse Bayesianism is an expression of this formalistic aspiration, we agree with Steele and
 8 Stefánsson that we are better off looking elsewhere.

9 Awareness growth can occur in different ways. The key question is to what extent probability
 10 assignments that were made prior to the episode of awareness growth can be retained. There
 11 seems to be no clear rule that can decide that. We propose the following procedure. Construct a
 12 Bayesian network prior to awareness growth and compare it with the new Bayesian network
 13 after awareness growth. If the new arrows and nodes are all downstream, the old probabilities
 14 table should not be changed. The paradigmatic cases of this are scenarios VERACITY and
 15 COIN. If, instead, the new arrows and nodes are upstream, the old probabilities tables
 16 should be changed. The paradigmatic examples are LIGHTING and TENANT.

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¹⁸ Arrows in Bayesian networks are often taken to represent causal relationships, but other interpretations exist. Schaffer (2016) discusses an interpretation in which arrows represent grounding relations rather than causality.

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