MAP555 : Signal Processing [1]

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[1] **Warning** : This document is currently being written and should be considered unfinished and full of mistakes and typos. It should not be used yet as a pedagogical support for a course.

Contents

Chapter 1

Introduction

In this chapter we will introduce signal processing and discuss briefly the numerous fundamental problems of signal processing.

1.1 Signal processing

Signal processing is everywhere Signal processing is a field that aim at modeling signals and providing automatic processing of those signals. It has been heavily researched for several decades and signal processing methods are central part of numerous technologies in telecommunications, multi-media processing, compression and storage. In recent years, tremendous results have been obtained by using modern machine learning and artificial intelligence techniques.

Objective of this course The objective of this course is to provide an introduction to the very large field of signal processing. One fascinating aspect of signal processing is that it is at the crossroad between Physics (to generate the signals), Electronics (to measure the signals), Mathematics (to model the signals) and Computer Science (to process the signals). In this sense, Signal processing is a perfect example of a multi-disciplinary field and a lot thee existing methods are known with other names in other fields. An effort will be made to provide vocabulary coming from the signal processing community but also statistics, machine learning and computer science.

We plan on introducing in this documents both the mathematical models, the numerical algorithms used for their processing and several examples of real life applications. The implementation of the signal processing methods in Python will also be discussed with example code and existing toolboxes. Note that most of the methods are introduced very briefly, but we will always provide detailed references for a more in-depth study.

Content of the document The course begins with a short introduction of signal processing containing a few definitions and problems formulations followed by bibliographical notes. Chapter 3 provides a presentation of Fourier analysis and analog filtering with some applicative examples such as modulation and Fourier optics in astronomy. Chapter 4 introduces signal sampling and digital signal filtering that has become the de-facto standard in practical applications. It also presents the very important Fast Fourier Transform (FFT) algorithm and discuss some examples of filtering in image processing. Chapter 5 discuss the random/stochastic aspects of signals and their optimal linear filtering when modeled as as stochastic processes. The modeling of speech is taken as an example for the study of auto-regressive models. Chapter 6 briefly introduces several signal representations commonly used such as the Discrete Cosine Transform (DCT), and wavelet transforms used in JPEG encoding and image reconstruction. The short time Fourier transform will also be introduced to model non-stationary signals. Finally some recent approaches based on machine learning such as dictionary learning and deep learning signal reconstruction will be presented.

1.2 Bibliographical notes

This document was strongly inspired by a number of outstanding references books that have been published over the years. In this section we discuss a few of those strongly recommended references. Suggestions to the author are welcome to provide a curated list of "awesome" references for signal processing similar to the lists available on GitHub.

Signal processing

- Signals and Systems [Haykin and Van Veen, 2007].
- Signals and Systems [Oppenheim et al., 1997].
- Signal Analysis [Papoulis, 1977].

- Polycopiés from Stéphane Mallat and Éric Moulines [Mallat et al., 2015].
- Théorie du signal [Jutten, 2018].

Analog signal processing and Fourier Transform

- Fourier Analysis and its applications [Vretblad, 2003]
- Distributions et Transformation de Fourier [Roddier, 1985]

Digital signal processing

- https://www.numerical-tours.com/
- Discrete-time signal processing [Oppenheim and Shafer, 1999].

Random signals, stochastic processes

- Random variables and stochastic processes [Papoulis, 1965].
- [Ross et al., 1996]
- [Kay, 1993]

Signal representations

- A Wavelet tour of signal processing [Mallat, 1999].
- Wavelets and sub-band coding [Vetterli and Kovacevic, 1995].

1.3 About this document

This document contains lecture notes of MAP555 Signal Processing Course from École Polytechnique. This document is currently being written and should be considered unfinished and full of mistakes and typos. It should not be used yet as a pedagogical support for a course.

The document is available in [PDF format] and [HTML format] compiled automatically when the source is modified in the GitHub repository.

 Reader are encouraged to report typos and mistakes in the mathematical formulas and proposed correction as Pull Requests on Github.

Chapter 2

Signals and convolution

2.1 Signals and properties

2.1.1 Analog and digital signals

Analog signal We define a signal is this course as a function of time or space. For instance $x : \mathbb{R} \to \mathbb{C}$ is a complex 1D signal of time $t \in \mathbb{R}$. $x : \mathbb{R}^2 \to \mathbb{R}$ is a 2D image of space $\mathbf{p} \in \mathbb{R}^2$.

2.1.2 Properties of signals

Causality A signal $x(t)$ is causal if

$$x(t) = 0, \quad \forall x < 0$$

Example: $x(t) = \begin{cases} 0 & \text{for } t < 0 \\ \sin(t) \exp\left(-\frac{t^2}{2}\right) & \text{for } t \geq 0 \end{cases}$

Periodicity A signal $x(t)$ is periodic of period T_0 is

$$x(t - kT_0) = x(t), \forall t \in \mathbb{R}, \forall k \in \mathbb{N}$$

Example: $x(t) = \exp\left(-\frac{(t-kT_0-1)^2}{2}\right)$ for $kT_0 < t < (k+1)T_0, \quad \forall k \in \mathbb{N}$

Signal in L_p space $L_p(S)$ is the set of functions whose absolute value to the power of p has a finite integral or equivalently that

$$\|x\|_p = \int_S |x(t)|^p dt < \infty \tag{2.1}$$

- $L_1(\mathbb{R})$ is the set of absolute integrable functions
- $L_2(\mathbb{R})$ is the set of quadratically integrable functions (finite energy)
- $L_\infty(\mathbb{R})$ is the set of bounded functions

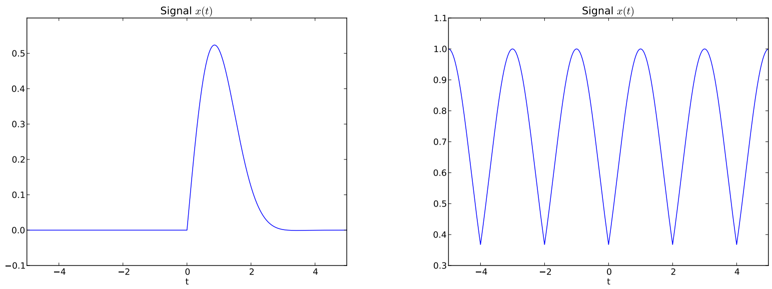

Figure 2.1: Examples of Causal signal (left) and periodic signal (right).

2.1.3 Common signals

2.2 Convolution and filtering

2.3 Fundamental signal processing problems

Chapter 3

Fourier analysis and analog filtering

3.1 Fourier transform

3.2 Frequency response and filtering

3.3 Applications of analog signal processing

Chapter 4

Digital signal processing

4.1 Sampling and Analog/Digital conversion

4.2 Digital filtering and transfer function

4.3 Finite signals and Fast Fourier Transform

4.4 Applications of DSP

Chapter 5

Random signals

5.1 Random Signals and Correlations

5.2 Frequency representation of random signals

5.3 AR modeling and linear prediction

Chapter 6

Signal representations

6.1 Short Time Fourier Transform

6.2 Common signal representations

6.3 Source separation and dictionary learning

6.4 Machine learning for signal processing

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Index