Sparse shifts can provide identifiability