# 1 Supporting Information: Instructions for replicating analyses

- <sup>2</sup> This document gives detailed instructions on how to carry out the tests described in the main text. In order to make first attempts
- at replicating the tests easier to carry out, the Supporting Information also includes copies of the data files generated by the tests
- described below, as well as the resulting figures.

### $_{5}$ 1.1 Estimating the scale constant c

- 6 The project render\_random, located in the 'unity' directory of the Supporting Information, generates the data for this test. Add
- this project to the Unity Hub, and open the project. In the version of the project included in the Supporting Information, many
- 8 files that Unity can reconstruct have been deleted in order to reduce the size of the repository. As a result, the first time you open
- 9 the project, you will have to double-click the OutdoorsScene object in the Assets panel in order to make it the active scene. The
- first time you open the project, you may also get an error message from Unity, asking whether you want to re-try opening the
- project. Respond that you do, and the project should open successfully on the second attempt.
- When the project is open and OutdoorsScene is active, select the 'Main Camera' object in the Hierarchy view, and in the Inspector view there will be GUI elements you can use to configure the test. For this test, check the box labelled 'Test Lambertian', uncheck the box labelled 'Test Tonemap', and enter 5000 in the text box labelled 'Samples'. Run the project, and in the Game view you will see a plane take many random orientations and colors. The random scene parameters and rendered values are recorded in a text file named data\_L1\_T0.txt, which will appear in the directory where the render\_random project is located. The digit after the letter L indicates whether the test was run with a Lamberitan material (0 = Unlit, 1 = Lambertian), and the digit after T indicates whether it was run with tonemapping (0 = no, 1 = yes). The data from the first run after opening the project often contains a few outliers, so I suggest making a first run and deleting the data file, then making a second run and using that data.
- The analysis for this test is implemented in the Python script estimate\_c.py. The module hdrp.py should be in the same directory as the script. Place the data file data\_L1\_T0.txt in the subdirectory data/tonemap\_off below the script. Run the script. The script uses equations (2) and (4) in the main text, without the scale constant c, to predict the unprocessed values  $u_k$  from the random lighting and material parameters. The script uses linear regression to estimate the scale constant c, which it prints to the console. It also generates Figure 1 in the main text. All figures that can be generated using the instructions in this document are included in the 'figures' directory of the Supporting Information.

### 27 1.2 Testing model predictions for Lambertian material without tonemapping

This test is implemented in the script model\_test\_tonemap\_off.py, and uses the same data file data\_L1\_T0.txt as the test in the previous section. Again, the module hdrp.py should be in the same directory as the script, and the data file data\_L1\_T0.txt should be in the subdirectory data/tonemap\_off. The script has a boolean variable testLambertian that indicates whether the data to be tested comes from a run of render\_random with a Lambertian material. Set testLambertian to True. Run the script. The

script plots post-processed values  $v_k$  predicted by equations (2) and (4) in the main text against the actual post-processed values, and also plots the prediction error as a function of the post-processed values  $v_k$ , the material color  $m_k$ , and the directional light color  $d_k$ . This is Figure 2 in the main text.

In order to make the rendering conditions similar to those in an experiment, the script does not assign a new, random exposure setting to the scene on each sample. However, the exposure can be adjusted by choosing the Global Volume object in the Hierarchy view in the render\_random project, and in the Inspector view entering a value for 'Fixed Exposure' in the Exposure panel. The project adjusts the random lighting intensities based on this value, so that the rendered scenes are neither too dark nor oversaturated. The exposure value is saved to the data file, and the analysis scripts take it into account when predicting rendered values. Results with different exposure values are similar to those with the default value of zero, which was used for the figures in the paper.

### 1.3 Testing model predictions for unlit material without tonemapping

To test the model for unlit materials, run the render\_random project again, as in Subsection 1.1, but this time with the box labelled 'Test Lambertian' unchecked. This produces a data file data\_L0\_T0.txt, which contains results for an unlit material. Put this file in the subdirectory data/tonemap\_off. Run the same script model\_test\_tonemap\_off.py as in Subsection 1.2, this time with the variable testLambertian set to False. The script plots post-processed values  $v_k$  predicted by equations (3) and (4) in the main text against the actual post-processed values, and also plots the prediction error as a function of the post-processed values  $v_k$ . This is Figure 3 in the main text.

# 49 1.4 Estimating knot point coordinates $u_i^*$ using delta functions

The data for this test is generated by the Unity project render\_delta, located in the 'unity' directory. Load this project into
the Unity Hub, open it, and double-click the OutdoorScene object to make it the active scene. Run the project. It will take
around 20 minutes to finish, and the rendered scene will be simply black for much of the time. The project generates a text file
data\_delta.txt. The analysis is carried out by the script knots\_from\_delta.py. Put the file data\_delta.txt in the subdirectory 'data'
below the script, and run the script. The script generates Figure 4 in the main text, and prints estimates of the knot points to the
console. These are the values in Table 2(a) in the main text.

# 56 1.5 Estimating knot point coordinates $u_i^*$ by optimizing model predictions

This analysis requires data from six runs of render\_random with six different cube files used for tonemapping. For the first run, select 'Main Camera' in the Hierarchy view, and then in the Inspector view, check the boxes for 'Test Lambertian' and 'Test Tonemap', enter the value 10000 for 'Samples', and enter the value 1 for 'Lighting Scale'. Next, select 'Global Volume' in the Hierarchy view, and then in the Inspector view, in the Tonemapping panel, for the 'Lookup Texture' field, choose linear\_max1. This selects a cube file that specifies a linear mapping from rendered values  $u_k$  in the interval [0, 1] to tonemapped values in

the interval [0, 1]. Run the project. This will create a text file data\_L1\_T1\_linear\_max1.txt. Move this file to the directory data/tonemap\_on\_fit.

Repeat this procedure an additional five times, with following five filenames for the 'Lookup Texture' field: square\_max1, square\_root\_max1, linear\_max58, square\_max58, square\_root\_max58. For these additional five runs, enter a value of 1 in the 'Lighting Scale' box for the filenames ending in 1, and enter a value of 58 for the filenames ending in 58. Each of these cube files specifies a mapping from rendered values  $u_k$  to tonemapped values. The tags 'linear', 'square', and 'square\_root' in the filenames indicate the mapping. Files ending with 1 map the interval [0, 1] to [0, 1], and are useful in the case described in the main text where we want to map the rendered range  $u_k \in [0, 1]$  to the displayable range on the monitor. Files ending with 58 map  $u_k$  in the interval [0, 58] to [0, 1], and are useful for estimating the knot points  $u_i^*$ , which range from almost zero to approximately 58. (Readers who are interested in how I generated these cube files can examine the Python script make\_cubes.py in the Supporting Information.) The value entered in the 'Lighting Scale' box scales the lighting intensity appropriately for each cube file. Each run of the project with a different cube file will generate a data file data\_L1\_T1\_[name].txt, where [name] is replaced by the name of the cube file.

With all six data files in data/tonemap\_on\_fit, run the script knots\_from\_model.py. This script makes two passes through the data. In the first pass, it uses the data from runs of render\_random with cube files whose names end in 58, to estimate all knot points  $u_i^*$ . The analysis uses data from all three cube files whose names end in 58, in order to use a range of tonemapping data to estimate the knot points. In the second run, it uses data from runs with cube files whose names end in 1, to fine-tune the knot points in the range [0, 1]. The script prints its final estimates of all knot points to the console, which are the values in Table 2(b) in the main text.

### 1.6 Testing model predictions for Lambertian material with tonemapping

This test requires new samples from render\_random, of the same kind generated in the previous section. We could re-use the data from the previous section, but that would mean using the same data for estimating the knot points  $u_i^*$  and for testing model predictions based on those knot points, which could result in overfitting. Instead, re-run the procedure described in the first two paragraphs of the previous section. However, this time you will only need to generate data files using the cube files ending in 1, since we will just test the rendering model for unprocessed values  $u_k$  in the range [0, 1]. Put the resulting three data files in data/tonemap\_on\_test. Run model\_test\_tonemap\_on.py with the variable testLambertian set to True. This produces Figure 6 in the main text.

### 1.7 Testing model predictions for unlit material with tonemapping

This test requires data like that used in the previous section, but from an unlit material. Re-run the procedure described in the first two paragraphs of Section 1.5, for the three cube files ending in 1, but with the 'Test Lambertian' box unchecked. Put the resulting three data files in data/tonemap\_on\_test, and run model\_test\_tonemap\_on.py, with the variable testLambertian set to False. This produces Figure 7 in the main text.

#### 4 1.8 Testing achromatic gamma correction

This test uses the project caldemo, which is a simple orientation discrimination experiment, located in the 'unity' folder. Load this project into the Unity Hub, start it, and double-click the OutdoorScene object to make it the active scene. In the Hierarchy view, select the Main Camera object, and in the Inspector view, uncheck the box labelled 'Chromatic Characterization'. In the Hierarchy view, select the Global Volume object, and in the Inspector view, in the Tonemapping panel, check the box labelled 'Mode', and from the drop-down list select 'External'. In the same panel, check the box labelled 'Lookup Texture', and for the selection box to the right, select the cube file named 'linearize\_achromatic'.

Run the project. You will see a scene with a pill-shaped object that is approximately vertical, but tilted slightly clockwise or counterclockwise. Press F if it appears to be tilted counterclockwise, and J if it appears to be tilted clockwise. After your response, a new trial will begin, and the object will appear at a new orientation. (If keypresses have no effect, try clicking the Game view, i.e., the rendered scene, as this view must be active in order for the project to register keypresses.) This stimulus-response sequence will continue indefinitely. The project does not record any experimental data, and this task is simply a placeholder to illustrate how characterization routines can be integrated into an experiment.

The project includes a cube file called linearize\_achromatic. At any point in the experiment, you can press 1 to turn on tonemapping with this file, and press 2 to turn off tonemapping. Try this a few times, and you will see the appearance of the rendered scene change.

Press 2 to turn off tonemapping. Press 3 to enter characterization mode. The rendered scene will go black. Each time you press the spacebar, the scene will become a uniform grey field, of a slightly lighter shade of grey. After eleven presses of the spacebar, the experimental scene will return. This characterization mode displays a uniform field with post-processed values  $v_k$  from 0.0 to 1.0 in steps of 0.1, and gives you the opportunity to use a photometer to measure the luminance generated by each value.

You can now make luminance characterization measurements. Press 2 to turn off tonemapping if you haven't done that already, and press 3 to enter characterization mode. Measure and record the luminance of the black field ( $v_k = 0.0$ ). Press the spacebar, and then measure the luminance of the next lighter field ( $v_k = 0.1$ ). Continue this for each value of  $v_k$  up to 1.0. To help keep track of what stimulus is being shown, in the message bar at the bottom of the Unity window, and in the Console window, after each time you press the spacebar, a message like this appears: 'next characterization stimulus shown: 0.10, 0.10, 0.10'.

Record the characterization measurements in a text file. Make the first line 'm\_k,lum' to label the columns. Make each subsequent line have the format '0.0,0.213', where the value before the comma is the displayed  $v_k$ , and the value after the comma is the recorded luminance. Name the file data\_achromatic\_T0.txt, and put it in the directory data/characterize. In the Supporting Information, this directory contains an example of such a file, to show how its contents should be formatted. (I have used the name 'm\_k' for the first column, because in the project, the values in this column are assigned as the material color coordinates  $m_k$  of the unlit material that fills the scene in characterization mode. However, as shown in the main text, with tonemapping turned off this means that the resulting post-processed values are also  $v_k = m_k$ .)

After you have created and saved the data file, run char\_achromatic\_1.py. This script will load your characterization data, and use it to generate a cube file that linearizes the mapping from  $v_k$  to displayed luminance. The cube file will be saved in the subdirectory 'cube', with the name 'linearize\_achromatic.cube'. In the project caldemo, delete the existing asset with that name, and load this new cube file into the project.

Run the project caldemo again. Press 1 to turn tonemapping on. The experiment should now be luminance-calibrated, i.e., the luminance at each pixel should be proportional to the rendered value  $u_k$ . To test this, press 3 to enter characterization mode again, this time with tonemapping on. As before, measure the luminance of the black display, and then press the spacebar to continue through values of  $v_k$  from 0.0 to 1.0. Measure and record the luminance for each value. Using the same file format as before, save this data in a text file data/characterize/data\_achromatic\_T1.txt. Note that the filename should now include the string T1 to indicate that the data was recorded with tonemapping on.

Run the script char\_achromatic\_2.py. This script will load the new test data, and plot luminance against rendered values  $u_k$ .

The mapping should be linear. The figures from this section, showing luminance with and without tonemapping, are merged into Figure 8(a) in the main text.

## 1.9 Testing chromatic gamma correction

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This test is similar to the one in the previous section, except that here we record and model XYZ coordinates instead of luminance.

In the project caldemo, in the Hierarchy view, select the Main Camera object, and in the Inspector view, check the box labelled 'Chromatic Characterization'. In the Hierarchy view, select the Global Volume object, and in the Inspector view, in the Tonemapping panel, check the box labelled 'Mode', and from the drop-down list select 'External'. In the same panel, check the box labelled 'Lookup Texture', and for the selection box to the right, select the cube file named 'linearize\_chromatic'. (Note that checking or unchecking the Chromatic Characterization box does not automatically switch the Lookup Texture value between linearize\_achromatic and linearize\_chromatic, and this must be done manually.)

Run the project. Press 2 to disable tonemapping. Press 3 to enter characterization mode. The screen will go black, and each time you press the spacebar, a new set of chromatic post-processed color coordinates  $\mathbf{v}$  will be assigned to the unlit material that fills the scene. Each coordinate  $v_k$  will step independently from 0.0 to 1.0 in steps of 0.1. A message is printed to the message bar and Console view to indicate what stimulus is being shown. Using a colorimeter or spectrophotometer, record the XYZ coordinates for each stimulus.

Record the measurements in a text file. Make the first line 'm\_r,m\_g,m\_b,x,y,z' to label the columns. Each subsequent line should record six comma-separated values: the three stimulus color coordinates  $v_r, v_g, v_b$ , and the three XYZ colorimetric coordinates. Save the file as data/characterize/data\_chromatic\_T0.txt. The Supporting Information contains an example of such a file.

Run the script char\_chromatic\_1.py. This script will load your characterization data, and use it to generate a cube file, saved as 'cube/linearize\_chromatic.cube'. In the caldemo project, replace the existing asset with that name by the new cube file.

Run the project again. Press 1 to turn tonemapping on. The experiment should now be color-calibrated. To test this,

press 3 to enter characterization mode again, this time with tonemapping on. Measure the XYZ coordinates of the black display, and then press the spacebar to continue through additional values of **v**. Measure and record the XYZ coordinates for each value. Using the same file format as for the first set of color characterization measurements, save this data in a text file data/characterize/data\_chromatic\_T1.txt.

Run the script char\_chromatic\_2.py. This script will load the new test data, and plot the primary activations against rendered values  $u_k$ . The mapping should be linear. The figures from this section are merged into Figure 8(b) in the main text.