

1 Supporting Information: Instructions for replicating analyses

This document gives detailed instructions on how to carry out the tests described in the main text. In order to make first attempts at replicating the tests easier to carry out, the Supporting Information also includes copies of the data files generated by the tests described below, as well as the resulting figures.

1.1 Estimating the scale constant c

The project `render_random`, located in the ‘unity’ directory of the Supporting Information, generates the data for this test. Add this project to the Unity Hub, and open the project. In the version of the project included in the Supporting Information, many files that Unity can reconstruct have been deleted in order to reduce the size of the repository. As a result, the first time you open the project, you will have to double-click the `OutdoorsScene` object in the Assets panel in order to make it the active scene. The first time you open the project, you may also get an error message from Unity, asking whether you want to re-try opening the project. Respond that you do, and the project should open successfully on the second attempt.

When the project is open and `OutdoorsScene` is active, select the ‘Main Camera’ object in the Hierarchy view, and in the Inspector view there will be GUI elements you can use to configure the test. For this test, check the box labelled ‘Test Lambertian’, uncheck the box labelled ‘Test Tonemap’, and enter 5000 in the text box labelled ‘Samples’. Run the project, and in the Game view you will see a plane take many random orientations and colors. The random scene parameters and rendered values are recorded in a text file named `data_L1_T0.txt`, which will appear in the directory where the `render_random` project is located. The digit after the letter L indicates whether the test was run with a Lambertian material (0 = Unlit, 1 = Lambertian), and the digit after T indicates whether it was run with tonemapping (0 = no, 1 = yes). The data from the first run after opening the project often contains a few outliers, so I suggest making a first run and deleting the data file, then making a second run and using that data.

The analysis for this test is implemented in the Python script `estimate_c.py`. The module `hdrp.py` should be in the same directory as the script. Place the data file `data_L1_T0.txt` in the subdirectory `data/tonemap_off` below the script. Run the script. The script uses equations (2) and (4) in the main text, without the scale constant c , to predict the unprocessed values u_k from the random lighting and material parameters. The script uses linear regression to estimate the scale constant c , which it prints to the console. It also generates Figure 2 in the main text. All figures that can be generated using the instructions in this document are included in the ‘figures’ directory of the Supporting Information.

1.2 Testing model predictions for Lambertian material without tonemapping

This test is implemented in the script `model_test_tonemap_off.py`, and uses the same data file `data_L1_T0.txt` as the test in the previous section. Again, the module `hdrp.py` should be in the same directory as the script, and the data file `data_L1_T0.txt` should be in the subdirectory `data/tonemap_off`. The script has a boolean variable `testLambertian` that indicates whether the data to be tested comes from a run of `render_random` with a Lambertian material. Set `testLambertian` to `True`. Run the script. The

script plots post-processed values v_k predicted by equations (2) and (4) in the main text against the actual post-processed values, and also plots the prediction error as a function of the post-processed values v_k , the material color m_k , and the directional light color d_k . This is Figure 3 in the main text.

In order to make the rendering conditions similar to those in an experiment, the script does not assign a new, random exposure setting to the scene on each sample. However, the exposure can be adjusted by choosing the Global Volume object in the Hierarchy view in the render_random project, and in the Inspector view entering a value for ‘Fixed Exposure’ in the Exposure panel. The project adjusts the random lighting intensities based on this value, so that the rendered scenes are neither too dark nor oversaturated. The exposure value is saved to the data file, and the analysis scripts take it into account when predicting rendered values. Results with different exposure values are similar to those with the default value of zero, which was used for the figures in the paper.

1.3 Testing model predictions for unlit material without tonemapping

To test the model for unlit materials, run the render_random project again, as in Subsection 1.1, but this time with the box labelled ‘Test Lambertian’ unchecked. This produces a data file data_L0_T0.txt, which contains results for an unlit material. Put this file in the subdirectory data/tonemap_off. Run the same script model_test_tonemap_off.py as in Subsection 1.2, this time with the variable testLambertian set to False. The script plots post-processed values v_k predicted by equations (3) and (4) in the main text against the actual post-processed values, and also plots the prediction error as a function of the post-processed values v_k . This is Figure 4 in the main text.

1.4 Estimating knot point coordinates u_i^* using delta functions

The data for this test is generated by the Unity project render_delta, located in the ‘unity’ directory. Load this project into the Unity Hub, open it, and double-click the OutdoorScene object to make it the active scene. Run the project. It will take around 20 minutes to finish, and the rendered scene will be simply black for much of the time. The project generates a text file data_delta.txt. The analysis is carried out by the script knots_from_delta.py. Put the file data_delta.txt in the subdirectory ‘data’ below the script, and run the script. The script generates Figure 5 in the main text, and prints estimates of the knot points to the console. These are the values in Table 2(a) in the main text.

1.5 Estimating knot point coordinates u_i^* by optimizing model predictions

This analysis requires data from six runs of render_random with six different cube files used for tonemapping. For the first run, select ‘Main Camera’ in the Hierarchy view, and then in the Inspector view, check the boxes for ‘Test Lambertian’ and ‘Test Tonemap’, enter the value 10000 for ‘Samples’, and enter the value 1 for ‘Lighting Scale’. Next, select ‘Global Volume’ in the Hierarchy view, and then in the Inspector view, in the Tonemapping panel, for the ‘Lookup Texture’ field, choose linear_max1. This selects a cube file that specifies a linear mapping from rendered values u_k in the interval $[0, 1]$ to tonemapped values in

the interval $[0, 1]$. Run the project. This will create a text file `data_L1_T1_linear_max1.txt`. Move this file to the directory `data/tonemap_on_fit`.

Repeat this procedure an additional five times, with following five filenames for the ‘Lookup Texture’ field: `square_max1`, `square_root_max1`, `linear_max58`, `square_max58`, `square_root_max58`. For these additional five runs, enter a value of 1 in the ‘Lighting Scale’ box for the filenames ending in 1, and enter a value of 58 for the filenames ending in 58. Each of these cube files specifies a mapping from rendered values u_k to tonemapped values. The tags ‘linear’, ‘square’, and ‘square_root’ in the filenames indicate the mapping. Files ending with 1 map the interval $[0, 1]$ to $[0, 1]$, and are useful in the case described in the main text where we want to map the rendered range $u_k \in [0, 1]$ to the displayable range on the monitor. Files ending with 58 map u_k in the interval $[0, 58]$ to $[0, 1]$, and are useful for estimating the knot points u_i^* , which range from almost zero to approximately 58. (Readers who are interested in how I generated these cube files can examine the Python script `make_cubes.py` in the Supporting Information.) The value entered in the ‘Lighting Scale’ box scales the lighting intensity appropriately for each cube file. Each run of the project with a different cube file will generate a data file `data_L1_T1_[name].txt`, where [name] is replaced by the name of the cube file.

With all six data files in `data/tonemap_on_fit`, run the script `knots_from_model.py`. This script makes two passes through the data. In the first pass, it uses the data from runs of `render_random` with cube files whose names end in 58, to estimate all knot points u_i^* . The analysis uses data from all three cube files whose names end in 58, in order to use a range of tonemapping data to estimate the knot points. In the second run, it uses data from runs with cube files whose names end in 1, to fine-tune the knot points in the range $[0, 1]$. The script prints its final estimates of all knot points to the console, which are the values in Table 2(b) in the main text.

1.6 Testing model predictions for Lambertian material with tonemapping

This test requires new samples from `render_random`, of the same kind generated in the previous section. We could re-use the data from the previous section, but that would mean using the same data for estimating the knot points u_i^* and for testing model predictions based on those knot points, which could result in overfitting. Instead, re-run the procedure described in the first two paragraphs of the previous section. However, this time you will only need to generate data files using the cube files ending in 1, since we will just test the rendering model for unprocessed values u_k in the range $[0, 1]$. Put the resulting three data files in `data/tonemap_on_test`. In the Python script `model_test_tonemap_on.py`, set the variable `testLambertian` to `True`. In the same script, uncomment the appropriate line to set the variable `u_knot` to the knot points estimated either from delta functions or by optimization. Alternatively, set `u_knot` to the knot points estimated from your own run of these tests. Run `model_test_tonemap_on.py`. With knot points estimated from delta functions, this produces Figure 6 in the main text, and with knot points from optimization, it produces Figure 7.

1.7 Testing model predictions for unlit material with tonemapping

This test requires data like that used in the previous section, but from an unlit material. Re-run the procedure described in the first two paragraphs of Section 1.5, for the three cube files ending in 1, but with the ‘Test Lambertian’ box unchecked. Put the resulting three data files in `data/tonemap_on_test`, and run `model_test_tonemap_on.py`, with the variable `testLambertian` set to `False`, and the variable `u_knot` set to the knot points from optimization. This produces Figure 8 in the main text.

1.8 Testing achromatic gamma correction

This test uses the project `caldemo`, which is a simple orientation discrimination experiment, located in the ‘unity’ folder. Load this project into the Unity Hub, start it, and double-click the `OutdoorScene` object to make it the active scene. In the Hierarchy view, select the Main Camera object, and in the Inspector view, uncheck the box labeled ‘Chromatic Characterization’. In the Hierarchy view, select the Global Volume object, and in the Inspector view, in the Tonemapping panel, check the box labelled ‘Mode’, and from the drop-down list select ‘External’. In the same panel, check the box labelled ‘Lookup Texture’, and for the selection box to the right, select the cube file named ‘linearize_achromatic’.

Run the project. You will see a scene with a pill-shaped object that is approximately vertical, but tilted slightly clockwise or counterclockwise. Press F if it appears to be tilted counterclockwise, and J if it appears to be tilted clockwise. After your response, a new trial will begin, and the object will appear at a new orientation. (If keypresses have no effect, try clicking the Game view, i.e., the rendered scene, as this view must be active in order for the project to register keypresses.) This stimulus-response sequence will continue indefinitely. The project does not record any experimental data, and this task is simply a placeholder to illustrate how characterization routines can be integrated into an experiment.

The project includes a cube file called `linearize_achromatic`. At any point in the experiment, you can press 1 to turn on tonemapping with this file, and press 2 to turn off tonemapping. Try this a few times, and you will see the appearance of the rendered scene change.

Press 2 to turn off tonemapping. Press 3 to enter characterization mode. The rendered scene will go black. Each time you press the spacebar, the scene will become a uniform grey field, of a slightly lighter shade of grey. After eleven presses of the spacebar, the experimental scene will return. This characterization mode displays a uniform field with post-processed values v_k from 0.0 to 1.0 in steps of 0.1, and gives you the opportunity to use a photometer to measure the luminance generated by each value.

You can now make luminance characterization measurements. Press 2 to turn off tonemapping if you haven’t done that already, and press 3 to enter characterization mode. Measure and record the luminance of the black field ($v_k = 0.0$). Press the spacebar, and then measure the luminance of the next lighter field ($v_k = 0.1$). Continue this for each value of v_k up to 1.0. To help keep track of what stimulus is being shown, in the message bar at the bottom of the Unity window, and in the Console window, after each time you press the spacebar, a message like this appears: ‘next characterization stimulus shown: 0.10, 0.10, 0.10’.

Record the characterization measurements in a text file. Make the first line ‘m,k,lum’ to label the columns. Make each

subsequent line have the format '0.0,0.213', where the value before the comma is the displayed v_k , and the value after the comma is the recorded luminance. Name the file `data_achromatic.T0.txt`, and put it in the directory `data/characterize`. In the Supporting Information, this directory contains an example of such a file, to show how its contents should be formatted. (I have used the name 'm_k' for the first column, because in the project, the values in this column are assigned as the material color coordinates m_k of the unlit material that fills the scene in characterization mode. However, as shown in the main text, with tonemapping turned off this means that the resulting post-processed values are also $v_k = m_k$.)

After you have created and saved the data file, run `char_achromatic_1.py`. This script will load your characterization data, and use it to generate a cube file that linearizes the mapping from v_k to displayed luminance. The cube file will be saved in the subdirectory 'cube', with the name 'linearize_achromatic.cube'. In the project `caldemo`, delete the existing asset with that name, and load this new cube file into the project.

Run the project `caldemo` again. Press 1 to turn tonemapping on. The experiment should now be luminance-calibrated, i.e., the luminance at each pixel should be proportional to the rendered value u_k . To test this, press 3 to enter characterization mode again, this time with tonemapping on. As before, measure the luminance of the black display, and then press the spacebar to continue through values of v_k from 0.0 to 1.0. Measure and record the luminance for each value. Using the same file format as before, save this data in a text file `data/characterize/data_achromatic.T1.txt`. Note that the filename should now include the string T1 to indicate that the data was recorded with tonemapping on.

Run the script `char_achromatic_2.py`. This script will load the new test data, and plot luminance against rendered values u_k . The mapping should be linear. The figures from this section, showing luminance with and without tonemapping, are merged into Figure 9(a) in the main text.

1.9 Testing chromatic gamma correction

This test is similar to the one in the previous section, except that here we record and model XYZ coordinates instead of luminance.

In the project `caldemo`, in the Hierarchy view, select the Main Camera object, and in the Inspector view, check the box labelled 'Chromatic Characterization'. In the Hierarchy view, select the Global Volume object, and in the Inspector view, in the Tonemapping panel, check the box labelled 'Mode', and from the drop-down list select 'External'. In the same panel, check the box labelled 'Lookup Texture', and for the selection box to the right, select the cube file named 'linearize_chromatic'. (Note that checking or unchecking the Chromatic Characterization box does not automatically switch the Lookup Texture value between `linearize_achromatic` and `linearize_chromatic`, and this must be done manually.)

Run the project. Press 2 to disable tonemapping. Press 3 to enter characterization mode. The screen will go black, and each time you press the spacebar, a new set of chromatic post-processed color coordinates \mathbf{v} will be assigned to the unlit material that fills the scene. Each coordinate v_k will step independently from 0.0 to 1.0 in steps of 0.1. A message is printed to the message bar and Console view to indicate what stimulus is being shown. Using a colorimeter or spectrophotometer, record the XYZ coordinates for each stimulus.

Record the measurements in a text file. Make the first line 'm_r,m_g,m_b,x,y,z' to label the columns. Each subsequent line

158 should record six comma-separated values: the three stimulus color coordinates v_r, v_g, v_b , and the three XYZ colorimetric
159 coordinates. Save the file as data/characterize/data_chromatic_T0.txt. The Supporting Information contains an example of such
160 a file.

161 Run the script char_chromatic_1.py. This script will load your characterization data, and use it to generate a cube file, saved
162 as 'cube/linearize_chromatic.cube'. In the caldemo project, replace the existing asset with that name by the new cube file.

163 Run the project again. Press 1 to turn tonemapping on. The experiment should now be color-calibrated. To test this,
164 press 3 to enter characterization mode again, this time with tonemapping on. Measure the XYZ coordinates of the black
165 display, and then press the spacebar to continue through additional values of v . Measure and record the XYZ coordinates for
166 each value. Using the same file format as for the first set of color characterization measurements, save this data in a text file
167 data/characterize/data_chromatic_T1.txt.

168 Run the script char_chromatic_2.py. This script will load the new test data, and plot the primary activations against rendered
169 values u_k . The mapping should be linear. The figures from this section are merged into Figure 9(b) in the main text.