

Test 1 - answers

*Aids permitted: a simple calculator with arithmetic and square root
(no scientific calculators, no statistical functions)*

Time permitted: 90 minutes

A. Multiple choice questions

Answer all questions in this section. Each question is worth two marks.

1. What is the difference between a parameter and a statistic?
(a) a parameter describes a population; a statistic describes a sample
(b) a parameter describes a sample; a statistic describes a population
(c) we can never measure a parameter
(d) we can never measure a statistic
(e) a statistic is more descriptive than a parameter
2. Which of the following is not a type of variable in statistics?
(a) nominally scaled
(b) ordinal
(c) interval
(d) fractional
(e) ratio
3. Using students in two existing classrooms as subjects in two different conditions of an experiment (one class for each condition) is an example of
(a) a correlational research design
(b) a randomized control trial
(c) an experimental research design
(d) an uncontrolled research design
(e) a quasi-experimental research design
4. The probability of any two events occurring together is $P(A \text{ and } B) =$
(a) $P(A) + P(B)$
(b) $P(A) \times P(B)$
(c) $P(A) \times P(A|B)$
(d) $P(A) \times P(B|A)$
(e) $P(A|B) + P(B|A)$
5. Which of the following pairs of events are both mutually exclusive and independent?
(a) an even roll of a die and an odd roll of a die
(b) an even roll of a die and a roll of a die that is less than 4
(c) an even roll of a die and a roll of a die that gives number 6
(d) all of the above
(e) none of the above

6. You roll two fair, six-sided dice that have the numbers 1 to 6 on different sides. If the two rolls are independent, what is the probability of the first die rolling a 1 and the second die **not** rolling a 1?

- (a) 0.027
- (b) 0.139
- (c) 0.500
- (d) 0.552
- (e) 0.694

7. These are the 31st to 40th scores (i.e., score number 31, score number 32, etc.) in a sorted list of 78 scores. What is the median of the full list of 78 scores?

...	31st	32nd	33rd	34th	35th	36th	37th	38th	39th	40th	...
	4.3	4.9	5.0	5.5	5.9	6.5	7.0	7.2	7.3	7.5	

- (a) 4.3
- (b) 5.25
- (c) 6.5
- (d) 7.25
- (e) 7.4

8. In a positively skewed distribution, the _____ is _____ the _____.

- (a) median, lower than, mean
- (b) mean, lower than, median
- (c) median, equal to, mean
- (d) none of the above

9. What is the sample variance of the following four numbers? 0.1, 1.1, 2.3, 3.4

- (a) 0.52
- (b) 0.86
- (c) 1.24
- (d) 1.43
- (e) 2.06

10. A distribution that is symmetrical but has a higher peak than the normal distribution is called

- (a) leptokurtic
- (b) platykurtic
- (c) asymptotic
- (d) supranormal
- (e) skewed

11. What proportion of the standard normal distribution (mean zero, standard deviation one) has a value less than -1.5 or greater than 1.5?

- (a) 0.0045
- (b) 0.0668
- (c) 0.1336
- (d) 0.7500
- (e) 0.9332

12. In a population that has mean 2 and standard deviation 1, what raw score is the 40th percentile?

- (a) 0.40
- (b) 1.50
- (c) 1.75
- (d) 2.00
- (e) 2.25

Answers

1 a 2 d 3 e 4 d 5 e 6 b 7 e 8 a 9 e 10 a 11 c 12 c

B. Written answer questions

Answer any two of the following three questions. If you answer all three, we will only mark the first two. Each question is worth ten marks. Show your work in a way we can understand.

1. You measure the length and colour of a sample of woolly bear caterpillars. Here is a table showing the number of each type of caterpillar in the sample.

	$\leq 5 \text{ cm}$	$> 5 \text{ cm}$
black	10	40
brown	20	80

(a) What is the marginal probability of a caterpillar being black? (2.5 marks)

The sample size is $10 + 20 + 40 + 80 = 150$.

There are $10 + 40 = 50$ black caterpillars.

$$P(\text{black}) = 50 / 150 = 0.33$$

(b) What is the joint probability of a caterpillar being brown and longer than 5 cm? (2.5 marks)

There are 80 caterpillars that are brown and longer than 5 cm.

$$P(\text{brown AND longer than 5 cm}) = 80 / 150 = 0.53$$

(c) What is the conditional probability that a caterpillar is longer than 5 cm, given that it is brown? (2.5 marks)

There are $20 + 80 = 100$ brown caterpillars, and 80 of these are longer than 5 cm.

$$P(\text{longer than 5 cm} \mid \text{brown}) = 80 / 100 = 0.80$$

(d) Is the length of a caterpillar (i.e., whether it is $\leq 5 \text{ cm}$ or $> 5 \text{ cm}$) independent of its colour? (2.5 marks)

$$P(\text{longer than 5 cm} \mid \text{black}) = 40 / 50 = 0.80$$

$$P(\text{longer than 5 cm} \mid \text{brown}) = 80 / 100 = 0.80$$

$$P(\text{longer than 5 cm}) = (40 + 80) / 150 = 0.80$$

Yes, the length of a caterpillar is independent of its colour.

2. We measure the masses of five woolly bear caterpillars that we find near the shore of Lake Ontario. They are 12 g, 15 g, 11 g, 9 g, and 10 g.

(a) Find the 60th percentile of the sample using the index method. (four marks)

$$n = 5$$

$$i = (60/100) \times n = 3$$

The 60th percentile is the average of the 3rd and 4th ranked samples, which is $(11 + 12) / 2 = 11.5$.

(b) Assume that the sample is taken from a normally distributed population. Calculate the mean and standard deviation of the sample, and then use Appendix A to estimate the 60th percentile of the population. (six marks)

$$M = 57 / 5 = 11.4$$

$$\sigma = \text{square_root}[(1/4)(671 - (1/5) \times 57^2)] = 2.3$$

Appendix A shows that the 60th percentile has a z score of approximately 0.25. This corresponds to a raw score of $x = 11.4 + 0.25 \times 2.3 = 12.0$.

3. Suppose the life span of a woolly bear caterpillar has a population mean of 5.5 years and a population standard deviation of 2.5 years.

(a) What is the probability of a caterpillar living longer than 7.5 years? (four marks)

$$\text{A raw score of 7.5 has a z score of } (x - \mu) / \sigma = (7.5 - 5.5) / 2.5 = 0.80.$$

Appendix A shows that the proportion of the standard normal distribution to the left of 0.80 is 0.7881, so the area to the right is $1 - 0.7881 = 0.2119$.

The probability of a caterpillar living longer than 7.5 years is 0.2119.

(b) What is the probability of two randomly chosen caterpillars living longer than 7.5 years? (two marks)

We define two events as follows.

A = caterpillar 1 lives longer than 7.5 years

B = caterpillar 2 lives longer than 7.5 years

If the events are independent, then

$$P(A \text{ and } B) = P(A) \times P(B) = 0.2119 \times 0.2119 = 0.0449$$

The probability of two caterpillars living longer than 7.5 years is 0.0449.

(c) What is the probability that a woolly bear caterpillar lives between 3.5 and 7.5 years?
(four marks)

The raw score 3.5 has z score $(x - \mu) / \sigma = (3.5 - 5.5) / 2.5 = -0.80$.
Appendix A shows that the proportion of the standard normal distribution to the left of 0.80 is 0.7881, so the area to the left of -0.80 is $1 - 0.7881 = 0.2119$.

The raw score 7.5 has z score $(x - \mu) / \sigma = (7.5 - 5.5) / 2.5 = 0.80$.
Appendix A shows that the proportion of the standard normal distribution to the left of 0.80 is 0.7881

The probability that a woolly bear caterpillar lives between 3.5 and 7.5 years is $0.7881 - 0.2119 = 0.5762$.