Solutions to problems on probability

Diez et al. give solutions to odd numbered problems at the back of the book. Here are solutions to the assigned even numbered problems.

3.2 (a) The outcome on the fourth spin is independent of the outcomes on the first three spins. If all slots are equally likely, then the probability of landing in a red slot on the fourth spin is 18/38 = 0.47.

(b) If the ball lands in a red slot 300 times in a row, then it seems unlikely that all slots are equally likely. I estimate that the probability of landing in a red slot on the 301st spin is around 1.0.

(c) It's hard to say which answer I'm more confident in. In a typical roulette wheel, all slots are equally likely, so I'm quite confident in my answer to (a). But if the ball lands in a red slot on 300 spins in a row, then I'm also quite sure that the probability of landing in a red slot must be near 1.0, so I'm also confident in my answer to (b). If the roulette wheels in parts (a) and (b) are the same wheel, then I would have more confidence in my answer to (b).

3.4. If the rolls of the die are independent, then the probability of rolling two sixes twice in a row is:

P(four sixes) = P(6) x P(6) x P(6) x P(6) = (1/6)^4 = 0.0008

The probability of rolling two threes twice in a row is exactly the same:

P(four threes) = P(3) x P(3) x P(3) x P(3) = (1/6)^4 = 0.0008

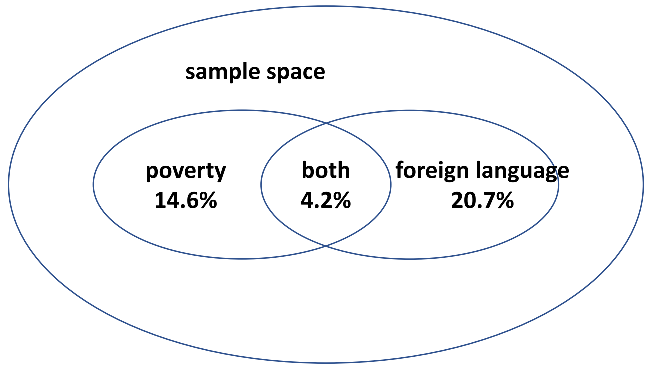
3.6. (a) P(sum of one) = 0

(b) P(sum of 5) = P(1 and 4) + P(2 and 3) + P(3 and 2) + P(4 and 1)
= P(1) P(4) + P(2) P(3) + P(3) P(2) + P(4) P(1)
= (1/6)(1/6) + (1/6)(1/6) + (1/6)(1/6) + (1/6)(1/6)
= 4/36 = 0.11

(c) P(sum of 12) = P(6 and 6) = P(6) P(6) = 1/36

3.8. (a) There are some people who are below the poverty line and who speak a foreign language at home, so no, these events are not disjoint.

(b) Venn diagram:



(c) P(below poverty line and no foreign language) = 14.6% - 4.2% = 10.4%

(d) P(below poverty line or foreign language) = 14.6% + 20.7% - 4.2% = 31.1%

(e) P(above poverty line and no foreign language) = 100% - 31.1% = 68.9%

(f) P(below poverty line) = 14.6%

P(below poverty line | foreign language)
= P(below poverty line and foreign language) / P(foreign language)
= 4.2% / 20.7% = 20.3%

These two probabilities are different, so being below the poverty line and speaking a foreign language at home are not independent events.

3.10. (a) P(first correct question is 5)
= P(incorrect) P(incorrect) P(incorrect) P(incorrect) P(correct)
= (3/4)(3/4)(3/4)(3/4)(1/4) = 0.079

(b) P(all correct) = P(correct) ^{5} = (1/4) ^{5} = 0.00098

(c) P(at least one correct) = 1 - P(all incorrect)
= 1 - P(wrong) ^{5} = 1 - (3/4) ^{5} = 0.76

3.14. P(jelly | peanut butter) = P(jelly and peanut butter) / P(peanut butter)
= 0.78 / 0.89 = 0.88

3.16. (a) Some people are in excellent health and have health coverage, so these events are not mutually exclusive.

(b) P(excellent health) = 0.2329

(c) P(excellent health | health coverage)
= P(excellent health and health coverage) / P(health coverage)
= 0.2099 / 0.8738 = 0.2402

(c) P(excellent health | no health coverage)
= P(excellent health and no health coverage) / P(no health coverage)
= 0.0230 / 0.1262 = 0.1823

(d) Having excellent health and health coverage are not independent. Having health coverage changes the probability of having excellent health.

3.18. (a) P(male or female has blue eyes) = (78 + 23 + 13 + 19 + 11) / 204 = 0.71

(b) P(female blue | male blue) = P(female blue and male blue) / P(male blue)
= 78 / 114 = 0.68

(c) P(female blue | male brown) = P(female blue and male brown) / P(male brown)
= 19 / 54 = 0.35

P(female blue | male green) = P(female blue and male green) / P(male green)
= 11 / 36 = 0.31

(d) It does not seem like the eye colours of male and female partners are independent. If a male has blue eyes, it is much more likely that the female partner also has blue eyes. But maybe these differences are just due to random chance! To make such conclusions with confidence we will need the methods of inferential statistics that we will cover in the upcoming lectures.