

When possible, I'll post the lecture notes that I work from, like the notes below, so that you'll have a list of some of the topics covered during the lecture. These may not be complete, and will be very brief, but if you find them useful then feel free to use them to supplement your own notes.

introduction

- reminder: tests on readings begin next week
- medical diagnosis problem posed at end of previous class
 - 10,000 patients
 - 100 have condition (1%)
 - test is positive for 95 (95%)
 - test is negative for 5 (5%)
 - 9900 do not have condition (99%)
 - test is positive for 495 (5%)
 - test is negative for 9505 (95%)
 - most people who have a positive test result do not have the condition
 - $P(\text{condition} \mid \text{positive}) = 95 / (95 + 495) = 0.16$
 - not a small difference; five times more likely to not have the condition

basic definitions

- random processes = a process whose outcome we cannot predict exactly
 - e.g., rolling a die; flipping a coin
 - e.g., getting sick; medical testing
 - maybe not really *random*; maybe we just don't know how to predict it
- outcomes = possible results of the random process
 - e.g., dice, numbers 1-6; coin, heads and tails
 - e.g., getting sick, yes and no; medical test, positive and negative
- sample space = set of all possible outcomes
 - e.g., dice, $\{1, 2, 3, 4, 5, 6\}$
- event = a set of outcomes
 - e.g., roll an even number, $\{2, 4, 6\}$
 - e.g., roll 4 or greater, $\{4, 5, 6\}$
 - probability of event is sum of probabilities of outcomes that make it up
 - Venn diagram: sample space and events
- notation: $P(\text{event})$; $P(\text{rolling a } 1)$, $P(1)$, $P(\text{sick})$
- probability distribution = report of all outcomes and their probabilities
 - e.g., shown in table or bar plot (or some other form, e.g., labelled sample space)
 - e.g., problem 3.11, table (gender and education)
 - outcomes are mutually exclusive; probabilities are ≥ 0 , and sum to one
 - for any event, probability is between 0 and 1

- law of large numbers: as more observations are collected, the proportion of occurrences of a particular event gets closer to the probability of that event
 - e.g., problem 3.18 (assortative mating), number of partners both having blue eyes

probability rules

- complement of an event = event does not happen
 $P(\text{not } A) = 1 - P(A)$
- general addition rule
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
 - Venn diagram
 - e.g., problem 3.8 (poverty and language), (b), (d)
- mutually exclusive (= disjoint) events = cannot both happen at the same time
 $P(A \text{ and } B) = 0$
 - Venn diagram
 - e.g., for a die, events 1 and 2
 - e.g., problem 3.8 (poverty and language), (a)
- addition rule for mutually exclusive events
 $P(A \text{ or } B) = P(A) + P(B)$
 - note how this rule follows from general addition rule
- joint probability: $P(A \text{ and } B)$ = probability that A and B are both true
- marginal probability: $P(A)$ = probability that A is true, regardless of whether B is true
- e.g., problem 3.16 (insurance and health), table
- conditional probability: $P(A | B)$ = probability that A is true, given that B is true
 $P(A | B) = P(A \text{ and } B) / P(B)$
 - Venn diagram
 - general multiplication rule: $P(A \text{ and } B) = P(B) P(A | B)$
 - also, $P(A \text{ and } B) = P(A) P(B | A)$
 - e.g., problem 3.16 (insurance and health), (d)
- independence: $P(A | B) = P(A)$
 - knowing that B occurred provides no information about whether A occurred
 - e.g., roll of two dice
- independent \neq mutually exclusive; mutually exclusive events are highly dependent
- multiplication rule for independent events
 $P(A \text{ and } B) = P(A) * P(B)$
 - note how this rule follows from general multiplication rule
 - e.g., problem 3.11 (gender and education), (c), (d)