

Solution to Kelley and Donnelly, problem 14.2

Q: What is the correlation between hours of study (X) and exam score (Y) in this table?

| z_x | X | Y | z_y | $z_x z_y$ |
|---------|---|----|---------|-----------|
| -0.4767 | 3 | 86 | 0.0000 | 0.0000 |
| 1.4302 | 5 | 95 | 1.4021 | 2.0053 |
| 0.4767 | 4 | 92 | 0.9348 | 0.4456 |
| 0.4767 | 4 | 83 | -0.4674 | -0.2228 |
| -1.4302 | 2 | 78 | -1.2464 | 1.7826 |
| -0.4767 | 3 | 82 | -0.6232 | 0.2971 |

Answer

We'll use the formula for the sample Pearson correlation:

$$r = 1/(n-1) \sum_i z_{xi} z_{yi}$$

To be safe we'll keep four decimal places during the calculation.

1. Find the mean and standard deviation of X and Y.

$$\begin{aligned} \bar{X} &= 3.5000 & s_X &= 1.0488 \\ \bar{Y} &= 86.0000 & s_Y &= 6.4187 \end{aligned}$$

2. Calculate the z scores of each X and Y score. (Shown in red, above.)
3. Multiply the pairs of z scores from X and Y. (Shown in blue, above.)
4. Add up the products of the pairs of z scores (shown in blue) and divide by n-1, where here the sample size is n = 6.

$$r = 4.3078 / (6-1) = 0.8616$$

The Pearson correlation between hours of study and exam score is 0.8616.