The problem

Here is a difficult problem for those of you who like puzzles. This is completely optional, and I will not test you on it.

Three prisoners, A, B, and C, with apparently equally good records have applied for parole. The parole board has decided to release two of the three, and the prisoners know this but not which two. A warder friend of prisoner A knows who are to be released. Prisoner A realizes that it would be unethical to ask the warder if he, A, is to be released, but thinks of asking for the name of one prisoner other than himself who is to be released. He thinks that before he asks, his chances of release are 2/3. He thinks that if the warder says "B will be released", his own chances have now gone down to 1/2, because either A and B or B and C are to be released. And so A decides not to reduce his chances by asking. However, A is mistaken in his calculations. Explain.

Note: We're not looking for a completely verbal explanation. Instead, use the rules of probability to find the probability that A will be released, if he does and if he does not ask the warder. After you find the answer, compare your reasoning to prisoner A's reasoning, and describe where he went wrong.

from: Mosteller, F. (1965). *Fifty challenging problems in probability with solutions.* Dover Publications.

Solution

Let's write down all possible outcomes in a format like ABB, where the first two letters (AB) mean that prisoners A and B will be released, and the third letter (B) means that the warder says prisoner B will be released. If all three prisoners have the same chance of being released, and if the warder will never say that A will be released, then we have the following four outcomes and probabilities:

$$P(\text{ ABB }) = 1/3$$

$$P(\text{ ACC }) = 1/3$$

$$P(\text{ BCB }) = 1/6$$

$$P(\text{ BCC }) = 1/6$$

The event that A is released consists of outcomes ABB and ACC, which together have a probability of 2/3. So before he asks the warder, A has probability 2/3 of being released.

The event that the warder says B will be released consists of outcomes ABB and BCB. So...

$$P(\text{A released } | \text{warder says B released}) = \frac{P(\text{A released and warder says B released})}{P(\text{warder says B released})} \tag{1}$$
$$= \frac{P(ABB)}{P(ABB, BCB)} \tag{2}$$
$$= (1/3)/(1/2) \tag{3}$$
$$= 2/3 \tag{4}$$

Thus if the warder says B will be released, A still has a 2/3 probability of being released. A similar argument shows that A has probability 2/3 of being released if the warder says that C will be released. So no matter what the warder says, A's probability of being released is unchanged.

Where did A go wrong in his calculation? He seems to have calculated this:

$$P(\text{A released } | \text{B released}) = \frac{P(\text{A released and B released})}{P(\text{B released})} \tag{5}$$
$$= \frac{P(ABB, ACC)}{P(ABB, BCB, BCC)} \tag{6}$$
$$= (1/3)/(2/3) \tag{7}$$
$$= 1/2 \tag{8}$$

That is, he treated the events “B is released” and “the warder says B is released” as the same, which they are not.