AquaEnv - Constants and Formulae

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1 Constants and formulae

1.1 Chemical constants used in AquaEnv

1.1.1 Elements of list PhysChemConst

absZero	-273.15	$^{\circ}\mathrm{C}$	[Dickson et al., 2007]	absolute zero
R	83.14472	(bar*cm3)/(mol*K)	Dickson et al., 2007	ideal gas constant
F	96485.3399	C/mol	[Dickson et al., 2007]	Faraday constant
е	79	= '	[Zeebe and Wolf-Gladrow, 2001]	relative dielectric constant of seawater
K_HNO2	1.584893e-3	mol/l	[Riordan et al., 2005]	approximative dissociation constant of HNO ₂ ,
				NBS pH scale, hybrid constant
K_HNO3	23.44	mol/kg-soln	[Boudreau, 1996, Soetaert et al.,	approximative dissociation constant of HNO ₃ , as-
			2007]	sumed on mol/kg-soln and free pH scale, stoichio-
				metric constant
K_H2SO4	100	mol/kg-soln	[Atkins, 1996]	approximative dissociation constant of H_2SO_4 ,
				assumed on mol/kg-soln and free pH scale, sto-
				ichiometric constant
K_HS	1.1e-12	mol/kg-soln	[Atkins, 1996]	approximative dissociation constant of HS, as-
				sumed on mol/kg-soln and free pH scale, stoichio-
				metric constant

1.1.2 Elements of list MeanMolecularMass

The list MeanMolecularMass contains mean molecular weights in g/mol. The list is taken from DOE [1994, chap. 5, p. 3] and Dickson et al. [2007, chap. 5, p. 4].

Cl	35.453
S04	(32.065+4 (15.999))
Br	79.904
F	18.998
Na	22.990
Mg	24.3050
Ca	40.078
K	39.098
Sr	87.62
В	10.811

1.1.3 Elements of list ConcRelCl

The list ConcRelCl contains relative concentrations of key chemical species in seawater with respect to chlorinity (DOE [1994, chap. 5, p. 11] and Dickson et al. [2007, chap. 5, p. 10])

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Cl	0.99889
S04	0.1400
Br	0.003473
F	0.000067
Na	0.55661
Mg	0.06626
Ca	0.02127
K	0.0206
Sr	0.00041
В	0.000232

1.2 Chlorinity Cl as a function of salinity S

Chlorinity C1 (in ‰) is calculated from salinity S using a relation given in DOE [1994, chap. 5, p. 11] and Zeebe and Wolf-Gladrow [2001, p. 100]

$$C1 = \frac{S}{1.80655} \tag{1}$$

1.3 Total concentrations of key chemical species in seawater as a function of chlorinity Cl

As described in DOE [1994, chap. 5, p. 11] and Dickson et al. [2007, chap. 5, p. 10], values in lists MeanMolecularMass and ConcRelCl are used to calculate the total concentration [X] (in mol/kg-soln) of chemical species X in seawater¹ according to the relation

$$[X] = \frac{\texttt{ConcRelCl}\$X}{\texttt{MeanMolecularMass}\$X} \, \texttt{Cl} \tag{2}$$

1.4 Ionic strength I as function of salinity S

According to DOE [1994, chapter 5, p. 13, 15], Zeebe and Wolf-Gladrow [2001, p.12], and Roy et al. [1993c, p.257], I (in mol/kg-H₂O) is calculated as

$$\mathbf{I} = \frac{19.924 \text{ S}}{1000 - 1.005 \text{ S}} \tag{3}$$

Note that the approximation $I/(mol/kg\text{-solution}) \approx 0.0199201$ S is given in Millero [1982, p. 428.]. This relationship converted into $mol/kg\text{-H}_2O$ and the last digits adjusted (from 0.0199201 to 0.019924) results in the formula used here.

1.5 Relation between water depth d and gauge pressure p

Although the relation between gauge pressure p (total pressure minus atmospheric pressure, see Feistel [2008]) and water depth d can be approximated by

$$p = 0.1 d 1.01325 \tag{4}$$

since p increases per m of water depth d by approximately $\frac{1}{10}$ of 1 atm (= 1.01325 bar Dickson et al. [2007, chap. 5, p. 3]), here, the relation given by Fofonoff and Millard [1983] as implemented in Soetaert et al. [2009] is used

$$\mathtt{d} = \frac{\left(9.72659 + \left(-2.2512\,10^{-5} + \left(2.279\,10^{-10} - 1.82\,10^{-15}\,\mathbf{p}\right)\mathbf{p}\right)\mathbf{p}\right)\mathbf{p}}{\mathtt{g} + 1.092\,10^{-6}\,\mathtt{p}}\tag{5}$$

¹Note that the solution must have seawater composition, otherwise the relation given here is void.

where p is the gauge pressure in dbar (deci-bar) and g the earth's gravity in m/s². g is calculated from the latitude lat (in degrees, -90 to 90, if not given lat=0 is assumed) as given in Fofonoff and Millard [1983] and implemented in Soetaert et al. [2009]

$$g = 9.780318 \left(1 + \left(0.0052788 + 2.3610^{-5} \sin\left(\operatorname{lat}\frac{\Pi}{180}\right)\right) \sin\left(\operatorname{lat}\frac{\Pi}{180}\right)\right)$$
 (6)

1.6 Seawater density as function of salinity S and temperature t

According to [Millero and Poisson, 1981], Millero [1982, p. 434], as reprinted in DOE [1994, chap. 5, p. 6f] the density of seawater $\rho_{SeaWater}$ (in $\frac{kg}{m^3}$; density in an object of class aquaenv) can be calculated as

$$\rho_{\text{SeaWater}} = \rho_{\text{Water}} + A S + B S^{1.5} + C S^2$$
(7)

A =
$$0.824493 - 4.0899 \cdot 10^{-3} t + 7.6438 \cdot 10^{-5} t^2 - 8.2467 \cdot 10^{-7} t^3$$
 (8)

$$+5.3875 \ 10^{-9} \ t^4$$
 (9)

$$B = -5.72466 \, 10^{-3} + 1.0227 \, 10^{-4} \, t - 1.6546 \, 10^{-6} \, t^2$$
 (10)

$$C = 4.8314 \, 10^{-4} \tag{11}$$

$$\rho_{Water} = 999.842594 + 6.793952 \, 10^{-2} \, \text{t} - 9.095290 \, 10^{-3} \, \text{t}^2$$
 (12)

$$+1.001685\ 10^{-4}\ t^3 - 1.120083\ 10^{-6}\ t^4 + 6.536332\ 10^{-9}\ t^5$$
 (13)

with t representing the temperature in °C and ρ_{Water} the density of fresh water in in kg/m³.

1.7 Gas-exchange constants, dissociation constant, and solubility products as functions of salinity S, (absolute) temperature T, and gauge pressure p

Empirical formulations for the temperature and salinity dependency of all gas exchange constants, equilibrium constants and solubility products calcuated in AquaEnv can be brought into the generic forms

$$\ln \frac{K_X}{k_o^\circ} = A + \frac{B}{T} + C \ln(T) + D T + E T^2$$
(14)

or

$$\log_{10} \frac{K_X}{k_0^{\circ}} = A' + \frac{B'}{T} + C' \log_{10}(T) + D' T + E' T^2$$
(15)

or

$$\log_{10} \frac{K_X}{k_0^{\circ}} = A'' + \frac{B''}{T} + C'' \ln(T) + D'' T + E'' T^2$$
(16)

with T being the temperature in Kelvin, S the salinity, k_0° the concentration unit of the constant, and A, B, C, D, E, and the respective variables with a prime (') being functions of salinity S. In the following we will give A, B, C, D, and E, or A', B', C', D', and E', or A'', B'', C'', D'', and E'' for each calculated constant.

1.7.1 Gas-exchange constants (Henry's constants) as functions of salinity S and temperature T

The following table shows the coefficients for gas exchange constants in AquaEnv, with fCO $_2$ being the fugacity of CO $_2$.

$\mathtt{KO_CO2}:$ solubility of \mathtt{CO}_2 in seawater					
A = 0.023517S - 167.81077	${ t CO2_sat} = f{ t CO}_2 { t KO_CO2}$				
B = 9345.17	_				
C = 23.3585	$k_0^{\circ} = \left[\frac{mol}{kg-solution\ atm}\right]$				
$D = -2.3656 \ 10^{-4} S$	_				
$E = 4.7036 \ 10^{-7} S$					
References: Weiss [1974] (original), DOE [1994, chap. 5, p.	13], Millero [1995, p. 663],				
Zeebe and Wolf-Gladrow [2001, p. 257], and Dickson et al. [2007, chap. 5, p. 12]					
${\tt KO_O2: solubility of O_2 in seawater (micromol per kg-soln and atm)}$					
A = -846.9978 - 0.037362 S	$02_{ m sat} = fO_2 { m KO_}02$				
B = 25559.07					
C = 146.4813	$k_0^{\circ} = \left[\frac{\mu mol}{kg-solution \ atm}\right]$				
D = -0.22204 + 0.00016504 S	_				
$E = -2.0564 \ 10^{-7} \ S$					
References: derived from Weiss [1970], agrees with data in Murray and Riley [1969]					

Note that the formulation for K0_02 has been derived using the formulation for a gravimetric $[O_2]_{sat}$ given in Weiss [1970, Weiss, 1970]. It has been converted from ml-O₂/kg-soln to μ mol-O₂/kg-soln using the molar volume of O₂ calculated with the virial equation using a first virial coefficient for oxygen at 273.15 Kelvin of $-22~{\rm cm}^3/{\rm mol}$ Atkins [1996], a value of 8.314472 Nm/(Kelvin mol) for the gas constant R and an ambient pressure of 101325 N/m². The expression for the Henry's constant has then been created by dividing the expression for the saturation concentration by $fO_2 = 0.20946$ atm [Williams, 2004].

1.7.2 Stoichiometric acid base dissociation constants as functions of salinity S and temperature T

The following table gives the coefficients of stoichiometric acid base dissociation constants in AquaEnv. Note that if some of the coefficients A to E are not listed, they are to be considered zero. Note also that given references somtimes contain the formulae in different units or on different pH scales. The formulae provided in this table yield the dissociation constants on different pH scales and concentration units. In AquaEnv, constants that are not already on the free pH scale and in mol/kg-soln are converted to the free pH scale and mol/kg-soln.

$\text{K.HSO4}: \text{HSO}_4^- \rightleftharpoons \text{H}^+ + \text{SO}_4^{2-} \text{ ("dickson")}$			free pH scale
$A = 324.57 \sqrt{\left(\frac{I}{m^{\circ}}\right)} - 771.54 \frac{I}{m^{\circ}} + 141.328$	K_HSO4	=	$\frac{[\mathrm{H}^{+}]_{F} [\mathrm{SO}_{4}^{2-}]}{[\mathrm{HSO}_{4}^{-}]}$
$B = 35474 \frac{I}{m^{\circ}} + 1776 \left(\frac{I}{m^{\circ}}\right)^{2} - 13856 \sqrt{\left(\frac{I}{m^{\circ}}\right)} - 2698 \left(\frac{I}{m^{\circ}}\right)^{\frac{3}{2}} - 4276.1$	k°	=	$\frac{mol}{kg\mathrm{-H}_2\mathrm{O}}$
$C = 114.723 \frac{I}{m^{\circ}} - 47.986 \sqrt{\left(\frac{I}{m^{\circ}}\right)} - 23.093$	m°	=	$\frac{mol}{kg-\mathrm{H_2O}}$
References: DOE [1994, c. 5, p. 13], Zeebe and Wolf-Gladrow [2001,	p. 260], L)icksoı	n [1990b] (original)
$\text{K.HSO4}: \text{HSO}_4^- \rightleftharpoons \text{H}^+ + \text{SO}_4^{2-} \text{ ("khoo")}$			free pH scale
$A = 6.3451 + 0.5208 \sqrt{\left(\frac{1}{m^{\circ}}\right)}$	K_HSO4	=	$\frac{[\mathrm{H}^{+}]_{F} [\mathrm{SO}_{4}^{2-}]}{[\mathrm{HSO}_{4}^{-}]}$
B = -647.59	k°	=	$\frac{mol}{kg-H_2O}$
D = -0.019085	m°	=	$\frac{mol}{kg-\mathrm{H}_2\mathrm{O}}$
References: Khoo et al. [1977] (original), Roy et al. [1993b], Millero [1995], Lev	vis an	d Wallace [1998]

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K_HF: HF \rightleftharpoons H<sup>+</sup> + F<sup>-</sup> ("dickson"
                                                                                                                free pH scale
                                                                                                                [H^{+}]_{F}[F^{-}]
                 \frac{I}{I} - 12.641
                                                                                                                  \frac{[\text{HF}]}{[\text{HF}]}
                                                                                            K\_HF
                                                                                                               m^{\circ} = \underline{\frac{moi}{kg - \mathrm{H_2O}}}
                                                                                            k^{\circ}
B = 1590.2
References: Dickson and Riley [1979a, p. 91] (original), Dickson and Millero [1987, p. 1783],
                Roy et al. [1993b, p. 257], DOE [1994, c. 5, p. 15], Millero [1995, p. 664],
                Zeebe and Wolf-Gladrow [2001, p. 260]
K_HF: \overline{HF} \rightleftharpoons \overline{H^+ + F^-} ("perez")
                                                                                                                total pH scale
                                                                                                                \frac{[\mathrm{H}^+]_F [\mathrm{F}^-]}{[\mathrm{HF}]}
A = -9.68 + 0.111 \sqrt{S}
                                                                                            K_HF
                                                                                            k^{\circ}
                                                                                                                     mol
B = 874
                                                                                                                kg-solution
References: Perez and Fraga [1987, p. 91] (original), Dickson et al. [2007, chap. 5, p. 14]
K_CO2: CO_2(aq) + H_2O \ (\rightleftharpoons H_2CO_3) \ \rightleftharpoons H^+ + HCO_3^- \ ("roy"; high salinities: S > 5)
                                                                                                                total pH scale
                                                                                                                [\mathrm{H^+}] [\mathrm{HCO_3^-}]
A = 2.83655 - 0.20760841 \sqrt{s} + 0.08468345 s - 0.00654208 s^{\frac{3}{2}}
                                                                                            K_C02
                                                                                                                \frac{\text{[CO}_2(\text{aq})]}{mol}
\frac{mol}{kg-\text{H}_2\text{O}}
                                                                                            k^{\circ}
B = -2307.1266 - 4.0484 \sqrt{S}
C = -1.5529413
References: Roy et al. [1993b, p. 254] (original), DOE [1994, c. 5, p.14], Millero [1995, p. 664],
                Zeebe and Wolf-Gladrow [2001, p. 255]
K_CO2: CO_2(aq) + H_2O \ (\rightleftharpoons H_2CO_3) \ \rightleftharpoons H^+ + HCO_3^- ("roy"; low salinities: S \le 5)
                                                                                                                total pH scale
A = 290.9097 - 228.39774 \sqrt{S} + 54.20871 S - 3.969101 S^{\frac{3}{2}} - 0.00258768 S^{2}
                                                                                                                \frac{\mathrm{[H^+]\ [HCO_3^-]}}{\mathrm{[CO_2(aq)]}} \\ \frac{mol}{kg-\mathrm{H_2O}}
B = -14554.21 + 9714.36839 \sqrt{s} - 2310.48919 s + 170.22169 s^{\frac{3}{2}}
                                                                                            K_C02
                                                                                            k^{\circ}
C = -45.0575 + 34.485796 \sqrt{S} - 8.19515 S + 0.60367 S^{\frac{3}{2}}
References: Roy et al. [1993b, p. 256] (original, based on a temperature dependency restated in
               Millero [1979], originally given in Harned and Davis [1943]. Note that there is a typesetting
                error in Roy et al. [1993b]: The third value for B is 2310.48919, not 310.48919)
                Millero [1995, p. 664] (the typesetting error is corrected here. also, here it is mentioned that
                this formula should be used for S \leq 5. Note that both functions do not always intersect at
                S=5. The true intersection is a function of t, is calculated in AquaEnv, and is used to
                decide which formula to use.)
K_CO2: CO_2(aq) + H_2O \ (\rightleftharpoons H_2CO_3) \rightleftharpoons H^+ + HCO_3^-
                                                                                                                total pH scale
                                                                                                                [\mathrm{H^+}] [\mathrm{HCO_3^-}]
A" = 61.2172 + 0.011555 S - 0.0001152 S^{2}
                                                                                            K_C02
                                                                                                                 [CO_2(aq)]
                                                                                            k^{\circ}
B'' = -3633.86
                                                                                                                    \overline{mol}
                                                                                                                \frac{1}{kg-solution}
C'' = -9.67770
References: Lueker et al. [2000] (original), Dickson et al. [2007, chap. 5, p.13-14]
K_CO2: CO_2(aq) + H_2O \ (\rightleftharpoons H_2CO_3) \ \rightleftharpoons H^+ + HCO_3^- \ ("millero")
                                                                                                                SW pH scale
                                                                                                                [{\rm H}^{+}] [{\rm HCO}_{3}^{-}]
A'' = 126.34048 - 0.0331 S + 0.0000533 S^2 - 13.4191 \sqrt{S}
                                                                                            K_C02
                                                                                                                 [CO_2(aq)]
                                                                                            k^{\circ}
B'' = -6320.813 + 6.103 S + 530.123 \sqrt{S}
                                                                                                                \frac{n}{kg-solution}
C'' = -19.568224 + 2.06950 \sqrt{S}
References: Millero et al. [2006] (original)
K_HCO3: HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} ("roy"; high salinities: S > 5)
                                                                                                                total pH scale
                                                                                                                [H^+][CO_3^{2-}]
A = -9.226508 - 0.106901773 \sqrt{S} + 0.1130822 S - 0.00846934 S^{\frac{3}{2}}
                                                                                            K_HCO3
                                                                                                                  [HCO_3^-]
                                                                                                                   mol
                                                                                            k^{\circ}
B = -3351.6106 - 23.9722 \sqrt{S}
                                                                                                                \overline{kg\mathrm{-H}_2\mathrm{O}}
C = -0.2005743
References: Roy et al. [1993b, p. 254] (original), DOE [1994, c. 25, p.15], Millero [1995, p. 664],
                Zeebe and Wolf-Gladrow [2001, p. 255]
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K_HCO3: HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} ("roy"; low salinities: S \le 5)
                                                                                                       total pH scale
A = 207.6548 - 167.69908 \sqrt{S} + 39.75854 S - 2.892532 S^{\frac{3}{2}} - 0.00613142 S^{2}
                                                                                                        [H^+][CO_3^{2-}]
B = -11843.79 + 6551.35253 \sqrt{S} - 1566.13883 S + 116.270079 S^{\frac{3}{2}}
                                                                                     K_HCO3
                                                                                                          [HCO_3^-]
C = -33.6485 + 25.928788 \sqrt{S} - 6.171951 S + 0.45788501 S^{\frac{3}{2}}
                                                                                     k^{\circ}
                                                                                                       \frac{mol}{kg-H_2O}
References: Roy et al. [1993b, p. 256] (original, based on a temperature dependence restated in
               Millero [1979], originally given in Harned and Scholes [1941]), Millero [1995, p. 664]
               (here it is mentioned that this formula should be used for S \leq 5. Note that both functions
              do not always intersect at S=5. The true intersection is a function of t, is calculated in
              AquaEnv, and is used to decide which formula to use.)
K_HCO3: HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} ("lueker"
                                                                                                       total pH scale
                                                                                                        [H^{+}][CO_{3}^{2-}]
A" = -25.9290 + 0.01781 S - 0.0001122 S^{2}
                                                                                     K_HCO3
                                                                                                         [HCO<sub>3</sub>]
B'' = -471.78
                                                                                     k^{\circ}
                                                                                                        \overline{kg-solution}
C" = 3.16967
References: Lucker et al. [2000] (original), Dickson et al. [2007, chap. 5, p.14]
                                                                                                       SW pH scale
K_HCO3: HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} ("millero")
                                                                                                        [H^{+}][CO_{3}^{2-}]
A'' = 90.18333 - 0.1248 S + 0.0003687 S^2 - 21.0894 \sqrt{S}
                                                                                     K_HCO3
                                                                                                          [\mathrm{HCO}_3^-]
B'' = -5143.692 + 20.051 S + 772.483 \sqrt{S}
                                                                                     k^{\circ}
                                                                                                           mol
                                                                                                        \frac{kq-solution}{kq-solution}
C'' = -14.613358 + 3.3336 \sqrt{S}
References: Millero et al. [2006] (original)
K_W: H_2O \rightleftharpoons H^+ + OH^-
                                                                                                       total pH scale
A = 148.9652 - 5.977 \sqrt{S} - 0.01615 S
                                                                                     K_W
                                                                                                        [H^{+}][OH^{-}]
                                                                                                            mol
                                                                                     k^{\circ}
                                                                                                         \frac{\text{mol}}{\text{kg-solution}}
B = -13847.26 + 118.67 \sqrt{S}
C = -23.6521 + 1.0495 \sqrt{S}
References: Millero [1995, p.670] (original), DOE [1994, c. 5, p. 18] (update 1997 cites Millero [1995]),
               Zeebe and Wolf-Gladrow [2001, p. 258], Dickson et al. [2007, chap. 5, p.16]
K_BOH3: B(OH)_3 \rightleftharpoons H^+ + B(OH)_4^-
                                                                                                       total pH scale
                                                                                                        [H^{+}][B(OH)_{4}^{-}]
A = 148.0248 + 137.1942 \sqrt{s} + 1.62142s
                                                                                     K\_BOH3 =
                                                                                                          [B(OH)_3]
B = -8966.90 - 2890.53 \sqrt{S} - 77.942S + 1.728 S^{\frac{3}{2}} - 0.0996 S^{2}
                                                                                                            mol
                                                                                     k^{\circ}
                                                                                                        \overline{kg-solution}
C = -24.4344 - 25.085 \sqrt{S} - 0.2474 S
D = 0.053105 \sqrt{s}
References: Dickson [1990a, p. 763] (or.), DOE [1994, c. 5, p. 14], Millero [1995, p. 669],
               Zeebe and Wolf-Gladrow [2001, p. 262], agrees with data in Roy et al. [1993a]
K_NH4: NH_4^+ \rightleftharpoons H^+ + NH_3
                                                                                                       SW pH scale
                                                                                                        [H<sup>+</sup>] [NH<sub>3</sub>]
A = -0.25444 + 0.46532 \sqrt{S} - 0.01992 S
                                                                                     K_NH4
                                                                                                       \frac{mol}{kg-solution}
B = -6285.33 - 123.7184 \sqrt{S} + 3.17556 S
                                                                                     k^{\circ}
References: Millero [1995, p. 671], Millero et al. [1995] (original),
              corrections of Millero [1995] in Lewis and Wallace [1998] give pH scale
K_H2S: H_2S \rightleftharpoons H^+ + HS^-
                                                                                                       total pH scale
                                                                                                        [H<sup>+</sup>] [HS<sup>-</sup>]
A = 225.838 + 0.3449 \sqrt{S} - 0.0274 S
                                                                                     K_H2S
                                                                                                          B = -13275.3
                                                                                     k^{\circ}
                                                                                                        \overline{kg-solution}
C = -34.6435
References: Millero [1995, p. 671], Millero et al. [1988] (original),
              corrections of Millero [1995] in Lewis and Wallace [1998] give pH scale
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K_H3P04: $H_3PO_4 \rightleftharpoons H^+ + H_2PO_4^-$		total pH scale			
$A = 115.525 + 0.69171 \sqrt{S} - 0.01844 S$	K_H3P04 =	$\frac{[\mathrm{H^+}] [\mathrm{H_2PO_4^-}]}{[\mathrm{H_3PO_4}]}$			
$B = -4576.752 - 106.736 \sqrt{S} - 0.65643 S$	$k^{\circ} =$	$\frac{mol}{kg-solution}$			
C = -18.453		kg-solution			
References: DOE [1994, chap. 5, p 16], Millero [1995, p.670], (original	1)				
Dickson et al. [2007, chap. 5, p.15] agrees with data in D	ickson and R	iley [1979b]			
$K_{H2PO4}: H_2PO_4^- \rightleftharpoons H^+ + HPO_4^{2-}$		total pH scale			
$A = 172.0883 + 1.3566 \sqrt{s} - 0.05778 s$	K_H2P04 =	$\frac{[H^+] [HPO_4^{2-}]}{[H_0PO^-]}$			
$B = -8814.715 - 160.340 \sqrt{S} + 0.37335 S$	$k^{\circ} =$	$rac{[\mathrm{H_2PO}_4^-]}{mol} \ rac{mol}{kg-solution}$			
C = -27.927		κy -solution			
References: DOE [1994, chap. 5, p 16], Millero [1995, p.670] (original	1),				
Dickson et al. [2007, chap. 5, p.15], agrees with data in I	Dickson and I	Riley [1979b]			
$\texttt{K.HP04}: \texttt{HPO}_4^{2-} \rightleftharpoons \texttt{H}^+ + \texttt{PO}_4^{3-}$		total pH scale			
$A = -18.141 + 2.81197 \sqrt{S} - 0.09984 S$	K_HP04 =	$\frac{[\mathrm{H^+}] [\mathrm{PO}_4^{3-}]}{[\mathrm{HPO}_4^{2-}]}$			
$B = -3070.75 + 17.27039 \sqrt{s} - 44.99486 s$	$k^{\circ} =$	$\frac{mol}{kg-solution}$			
References: DOE [1994, chap. 5, p 17], Millero [1995, p.670] (original),					
Dickson et al. [2007, chap. 5, p.15], agrees with data in I	Dickson and I	Riley [1979b]			
K_SiOH4: $Si(OH)_4 \rightleftharpoons H^+ + SiO(OH)_3^-$		total pH scale			
$A = 117.385 + 3.5913 \sqrt{\frac{I}{m^{\circ}}} - 1.5998 \frac{I}{m^{\circ}} + 0.07871 \left(\frac{I}{m^{\circ}}\right)^{2}$	K_SiOH4 =	$\frac{[\mathrm{H}^+] [\mathrm{SiO}(\mathrm{OH})_3^-]}{[\mathrm{Si}(\mathrm{OH})_4]}$			
$B = -8904.2 - 458.79 \sqrt{\frac{1}{m^{\circ}}} + 188.74 \frac{1}{m^{\circ}} - 12.1652 \left(\frac{1}{m^{\circ}}\right)^{2}$	k° =	$rac{mol}{kg- ext{H}_2 ext{O}}$			
C = -19.334	$m^{\circ} =$	$rac{mo ilde{l}}{kg- ext{H}_2 ext{O}}$			
References: Millero et al. [1988] (original), DOE [1994, chapter 5, p 1	[7], Millero [1	995, p.671]			
K_SiOOH3: $SiO(OH)_3^- \rightleftharpoons H^+ + SiO_2(OH)_2^{2-}$		total pH scale			
A = 8.96	K_SiOOH3=	$\frac{[H^{+}] [SiO_{2}(OH)_{2}^{2-}]}{[SiO(OH)_{3}^{-}]}$			
B = -4465.18	k° =	$\frac{mol}{kg-\mathrm{H}_2\mathrm{O}}$			
D = 0.021952		y 1120			
References: Wischmeyer et al. [2003] (original; including corrections)	by co-author	D. Wolf-Gladrow)			

1.8 Stoichiometric solubility products as functions of salinity S and temperature T

The following table shows the coefficients for the stoichiometric solubility products for calcite and aragonite in $\mathsf{AquaEnv}$.

Ksp_calcite: solubility product of calcite					
$A' = -171.9065 - 0.77712 \sqrt{S} - 0.07711 S + 0.0041249 S^{1.5}$	$Ksp_cal \ = \ [\mathrm{CO}_3^{2-}] \ [\mathrm{Ca}^{2+}]$				
$B' = 2839.319 + 178.34 \sqrt{s}$	_				
C' = 71.595	$k_0^{\circ} = \left[\left(\frac{mol}{kg-solution} \right)^2 \right]$				
$D' = -0.077993 + 0.0028426 \sqrt{\mathtt{S}}$					
References: Mucci [1983] (original), Boudreau [1996, p. 160],					
(note that the second value for A' is -0.77712 not -0.7712 as cited in Boudreau [1996])					

Ksp_aragonite: solubility product of aragonite				
$A' = -171.945 - 0.068393 \sqrt{S} - 0.10018 S + 0.0059415 S^{1.5}$	Ksp_ara =	$[CO_3^{2-}][Ca^{2+}]$		
$B' = 2903.293 + 88.135 \sqrt{\mathtt{S}}$				
C' = 71.595	$k_0^{\circ} =$	$\left\lceil \left(\frac{mol}{kg-solution}\right)^2\right\rceil$		
$D' = -0.077993 + 0.0017276 \sqrt{S}$		L J		
References: Mucci [1983] (original), Boudreau [1996, p. 160],				
(note that the second value for D' is 0.0017276 not 0.001727 as cited in Boudreau [1996])				

1.9 Pressure correction of dissociation constants and solubility products

Pressure has an effect on the stoichiometric acid-base dissociation constants and the stoichiometric solubility products given in the previous sections. As described in Millero [1995, p. 675] using corrections and assumptions from Lewis and Wallace [1998, p. A-7] the effect of pressure can be accounted for by the equation

$$K_{corr} = K \left(-\frac{a_0 + a_1 t + a_2 t^2}{R T} p + \frac{b_0 + b_1 t + b_2 t^2}{1000 R T} 0.5 p^2 \right)$$
 (17)

Where $K_{\rm corr}$ is the pressure corrected constant and K is the uncorrected constant, both on matching units, e.g., mol/kg-soln, T is the absolute temperature in Kelvin, t is the temperature in °C, R is the ideal gas constant in (bar cm³)/(mol Kelvin), and p is the gauge pressure (total pressure minus one atm, see Feistel [2008] for a definition) in bar. The a and b coefficients (according to Millero [1995] which is partly a restatement of Millero [1979], corrected by Lewis and Wallace [1998]) for constants in AquaEnv (stored in the data frame DeltaPcoeffs) are given in the following table².

Note that, while not stated in Millero [1995], it can be inferred from Lewis and Wallace [1998] and the code given by van Heuven et al. [2009], that the pressure correction is valid for K_HF and H_H2SO4 on the free scale and for all other dissociation constants on the seawater pH scale. To be consistent with Lewis and Wallace [1998] and van Heuven et al. [2009], in AquaEnv all dissociation constants obtained from the original formulae are first converted to the free or seawater scale respectively using scale conversion factors with K_HF and H_H2SO4 being not pressure corrected. Then the pressure correction is applied. Subsequently, the dissociation constants are converted to the desired pH scale with scale conversion factors with K_HF and H_H2SO4 being pressure corrected.

	a_0	a_1	a_2	b_0	b_1	b_2
K_HSO4	-18.03	0.0466	$0.3160 \ 10^{-3}$	- 4.53	0.0900	0
K_HF	-9.78	-0.0090	$-0.9420 \ 10^{-3}$	- 3.91	0.0540	0
K_C02	-25.50	0.1271	$0.0000 \ 10^{-3}$	- 3.08	0.0877	0
K_HCO3	-15.82	-0.0219	$0.0000 \ 10^{-3}$	1.13	-0.1475	0
K_W	-25.60	0.2324	$-3.6246 \ 10^{-3}$	- 5.13	0.0794	0
K_BOH3	-29.48	0.1622	$2.6080 \ 10^{-3}$	- 2.84	0.0000	0
K_NH4	-26.43	0.0889	$-0.9050 \ 10^{-3}$	- 5.03	0.0814	0
K_H2S	-14.80	0.0020	$-0.4000 \ 10^{-3}$	2.89	0.0540	0
K_H3P04	-14.51	0.1211	$-0.3210 \ 10^{-3}$	- 2.67	0.0427	0
K_H2P04	-23.12	0.1758	$-2.6470 \ 10^{-3}$	- 5.15	0.0900	0
K_HPO4	-26.57	0.2020	$-3.0420 \ 10^{-3}$	- 4.08	0.0714	0
K_SiOH4	-29.48	0.1622	$2.6080 \ 10^{-3}$	- 2.84	0.0000	0
K_SiOOH3	-29.48	0.1622	$2.6080 \ 10^{-3}$	- 2.84	0.0000	0
Ksp_calcite	-48.76	0.5304	$0.0000 \ 10^{-3}$	-11.76	0.3692	0
$Ksp_aragonite$	-45.96	0.5304	$0.0000 \ 10^{-3}$	-11.76	0.3692	0

 $^{^{2}}$ Note that in Lewis and Wallace [1998] it is stated that the a values for $H_{2}O$ and $H_{2}S$ are freshwater values! And that the coefficients for the silicate species are assumed to be the same as the ones for the borate species.

1.10 Conversion factors

The following list gives a basic list of concentration and pH scale conversion factors used in AquaEnv. All other conversion factors, e.g., to be used in the function convert, are calculated from the factors given here. Note that the factors given below are multiplicative factors that can be used to convert e.g. dissociation constants or proton concentration values. To convert pH values, one needs to use the negative decadal logarithm of the factors below as an additive term. molal2molin signifies conversion from mol/kg-H₂O to mol/kg-soln, free2tot signifies conversion from the free to the total pH scale, free2sws signifies conversion from the free to the seawater pH scale (for a general treatment of the free, total and seawater pH scale see Dickson [1984] and Zeebe and Wolf-Gladrow [2001]), and free2nbs signifies conversion from the free to the NBS pH scale [Durst, 1975].

molal2molin	(1 - 0.001005 S)	Roy et al. [1993b, p. 257], DOE [1994, chap. 5, p. 15]
free2tot	$(1+\frac{S_T}{KHSO4})$	
	N_1100 1	Dickson [1984, p. 2302], DOE [1994, chap. 5, p. 16], Zeebe and Wolf-Gladrow [2001, p. 57, p. 261]
free2sws	$(1 + \frac{S_T}{K_HSO4} + \frac{F_T}{K_HF})$, , , , , , , , , , , , , , , , , , , ,
free2nbs	$\gamma_{ m H^+}$ / molal2molin	Dickson [1984, p. 2303], Zeebe and Wolf-Gladrow [2001]
	, ,	Dickson [1984], Lewis and Wallace [1998], Zeebe and Wolf-
		Gladrow [2001]

In the above table S is salinity, $S_T = [SO_4^{2-}] + [HSO_4^-] \approx [SO_4^{2-}]$, $F_T = [HF] + [F^-] \approx [F^-]$, both in mol/kg-soln, and γ_{H^+} is the activity coefficient for the proton. The dissociation constants K_HSO4 and K_HF are on the free pH scale and in mol/kg-soln as well. Note that, as given in Dickson [1984, p. 2303] and Dickson and Riley [1979a, p. 91f] all concentrations appearing in the definition for the total and the seawater pH scale are molal, i.e. mol/kg-H₂O, concentrations. But in Roy et al. [1993b, p. 257] and in DOE [1994, chap.. 4, SOP 6, p. 1] it is stated, that concentrations for the seawater and total pH scale are molin, i.e. mol/kg-soln. To be consistent with DOE [1994] and Dickson et al. [2007] mol/kg-soln is chosen here for the free, total and seawater scale. To be consistent with Lewis and Wallace [1998], the NBS scale is based on the proton concentration on mol/kg-H₂O.

1.11 Activity coefficient for the proton

In AquaEnv a complex ion-interaction model like, e.g., Millero and Pierrot [1998] is not implemented. According to Zeebe and Wolf-Gladrow [2001] the activity coefficient for the proton $\gamma_{\rm H^+}$ can be approximated by the Davies equation as long as the ionic strength of the solution in question remains below 0.5 mol/kg-H₂O. This means for solutions with a salinity of less than 24.48. Since NBS scale pH values are mostly not used for open ocean applications but mainly in brackish and fresh waters, the Davies equation has been assumed to be a sufficient approximation for $\gamma_{\rm H^+}$. Important to note, however, is that the conversion from and to the NBS pH scale in AquaEnv for salinities above 24.48 is only a rough approximation!. The Davies equation is used as given in Zeebe and Wolf-Gladrow [2001]

$$\gamma_{\rm H^+} = 10^{-\left(1.82\ 10^6\ (\epsilon\ {\rm T})^{-\frac{3}{2}}\right)\left(\frac{\sqrt{\rm (I)}}{1+\sqrt{\rm (I)}} - 0.2\ {\rm I}\right)}$$
(18)

where ϵ is the relative dielectric constant of seawater (PhysChemConst\$e in AquaEnv), T is the temperature in Kelvin, and I is the ionic strength in mol/kg-H₂O. Note that the squared charge of the ion before the brackets with the ionic strength terms which is present in the generic form of the Davies equation has been omitted here since for the proton, this factor is 1.

1.12 The revelle factor

In Zeebe and Wolf-Gladrow [2001, p.73] the revelle factor is given as

$$\text{revelle} = \frac{d[\text{CO}_2]}{[\text{CO}_2]} / \frac{d[\sum \text{CO}_2]}{[\sum \text{CO}_2]} \bigg|_{[\text{TA}] = \text{const.}}$$
(19)

in AquaEnv revelle is calculated numerically.

1.13 Partial derivatives of total alkalinity

The values for dTAdKdKdS, dTAdKdKdT, dTAdKdKdd, dTAdKdKdSumH2SO4, and dTAdKdKdSumHF are calculated numerically as described in ?.

The values for dTAdH, dTAdSumCO2, dTAdSumBOH3, dTAdSumH2SO4, and dTAdSumHF are calculated analytically as given in Hofmann et al. [2008].

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