# BERTIN MATRICES

# AN R IMPLEMENTATION

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Contents



## 1. Bertin Plots

Among the rich material on graphical presentation of information, in (Bertin, 1977) (engl. (Bertin, 1999)) J. Bertin discusses the presentation of data matrices, with a particular view to seriation. (de Falguerolles et al., 1997) gives an appraisal of this aspect of Bertin's work. The methods illustrated in (de Falguerolles et al., 1997) have been first implemented in the Voyager system (Sawitzki, 1996). They have been partially re-implemented in R, and this paper gives an introduction to the R-implementation.

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*URL:* `http://bertin.r-forge.r-project.org/`.

The R-implementation can be downloaded as a package *bertin* from `http://bertin.r-forge.r-project.org/`. (de Falguerolles et al., 1997) is included as bertin.pdf in the documentation section of the package.

Bertin uses a small data set on hotel occupancy data to illustrate his ideas.

| | Jan | Fev | Mars | Avril | May | Juin | Juil | Aout | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ClienteleFeminine | 26 | 21 | 26 | 28 | 20 | 20 | 20 | 20 | 20 | 40 | 15 | 40 |
| Locale | 69 | 70 | 77 | 71 | 37 | 36 | 39 | 39 | 55 | 60 | 68 | 72 |
| USA | 7 | 6 | 3 | 6 | 23 | 14 | 19 | 14 | 9 | 6 | 8 | 8 |
| AmerSud | 0 | 0 | 0 | 0 | 8 | 6 | 6 | 4 | 2 | 12 | 0 | 0 |
| Europe | 20 | 15 | 14 | 15 | 23 | 27 | 22 | 30 | 27 | 19 | 19 | 17 |
| MOrientAfrique | 1 | 0 | 0 | 8 | 6 | 4 | 6 | 4 | 2 | 1 | 0 | 1 |
| Asie | 3 | 10 | 6 | 0 | 3 | 13 | 8 | 9 | 5 | 2 | 5 | 2 |
| Business | 78 | 80 | 85 | 86 | 85 | 87 | 70 | 76 | 87 | 85 | 87 | 80 |
| Touristes | 22 | 20 | 15 | 14 | 15 | 13 | 30 | 24 | 13 | 15 | 13 | 20 |
| ResDirecte | 70 | 70 | 78 | 74 | 69 | 68 | 74 | 75 | 68 | 68 | 64 | 75 |
| ResAgents | 20 | 18 | 19 | 17 | 27 | 27 | 19 | 19 | 26 | 27 | 21 | 15 |
| EquipageAeriens | 10 | 12 | 6 | 9 | 4 | 5 | 7 | 6 | 6 | 5 | 15 | 10 |
| MoinsDe20 | 2 | 2 | 4 | 2 | 2 | 1 | 1 | 2 | 2 | 4 | 2 | 5 |
| De20a55 | 25 | 27 | 37 | 35 | 25 | 25 | 27 | 28 | 24 | 30 | 24 | 30 |
| De35a55 | 48 | 49 | 42 | 48 | 54 | 55 | 53 | 51 | 55 | 46 | 55 | 43 |
| PlusDe55 | 25 | 22 | 17 | 15 | 19 | 19 | 19 | 19 | 19 | 20 | 19 | 22 |
| Prix | 163 | 167 | 166 | 174 | 152 | 155 | 145 | 170 | 157 | 174 | 165 | 156 |
| Duree | 1.65 | 1.71 | 1.65 | 1.91 | 1.9 | 2 | 1.54 | 1.6 | 1.73 | 1.82 | 1.66 | 1.44 |
| Occupation | 67 | 82 | 70 | 83 | 74 | 77 | 56 | 62 | 90 | 92 | 78 | 55 |
| Foires | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

TABLE 1. Bertin's hotel data

## 2. Bertin Matrices

To repeat from (de Falguerolles et al., 1997): In abstract terms, a Bertin matrix is a matrix of displays. Bertin matrices allow rearrangements to transform an initial matrix to a more homogeneous structure. The rearrangements are row or column permutations, and groupings of rows or columns. To fix ideas, think of a data matrix, variable by case, with real valued variables. For each variable, draw a bar chart of variable value by case. Highlight all bars representing a value above some sample threshold for that variable (Figure 1 on page 2).

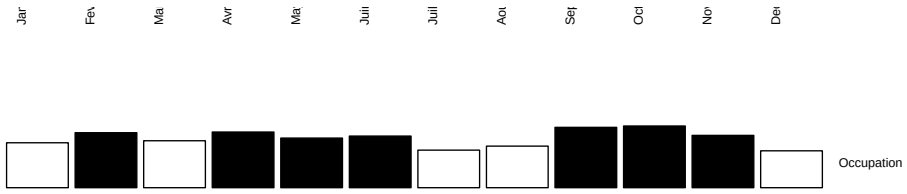



FIGURE 1. Display of one variable. Observations above average are highlighted.

Variables are collected in a matrix to display the complete data set (Figure 2 on page 3). By convention, Bertin shows variables in rows and cases in columns. To make periodic structures more visible, the data are repeated cyclically.

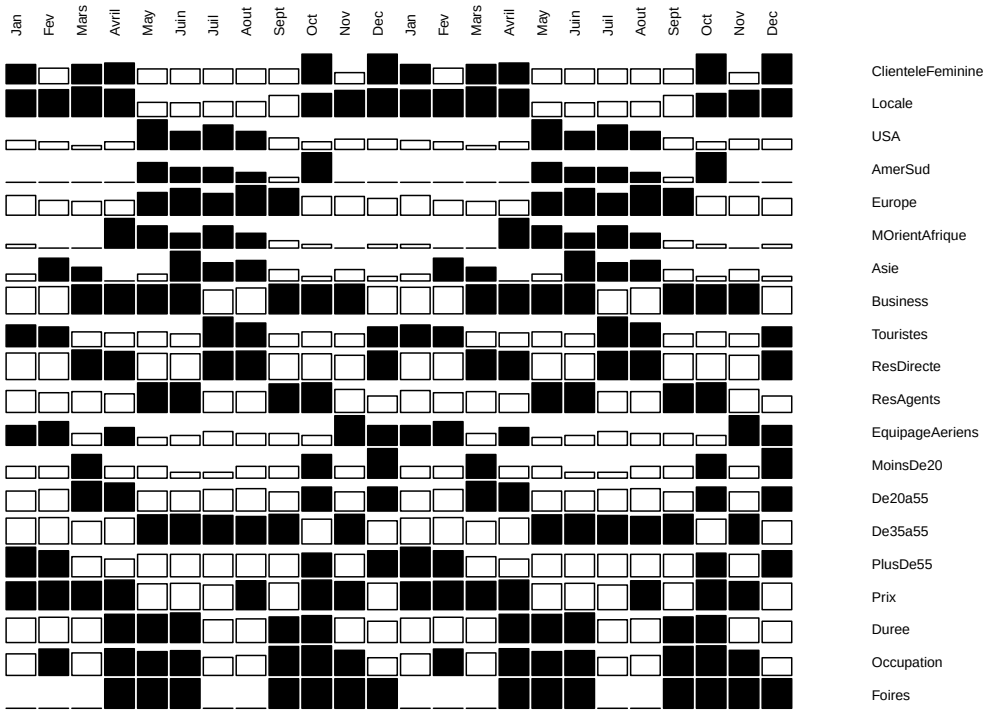



FIGURE 2. Display of a data matrix: Hotel data. Variables are shown as rows. To make periodic structures more visible, time is duplicated.

As Bertin points out, the indexing used is arbitrary. You can rearrange rows and/or columns to reveal the information of interest. If you run a hotel, of course the percentage of hotel occupation and the duration of the visits are most interesting for you. Move these variables to the top of the display, and rearrange the other variables by similarity or dissimilarity to these target variables (Figure 3 on page 4). Time points have a natural order. No rearrangement is used here.

As a second example, we use the the *USJudgeRatings* data set (Figure 4 on page 4). The data is in Table 2 (page 5).

Both cases (the judges) and variables (the qualities) allow for a rearrangement. Just sorting for row and column averages gives a more informative picture (Figure 5 on page 6). The number of contacts stands out - it has a different structure than the other variables. After all, this is not a rating variable at all, but ancillary information. There is little reason why it should go along with the rating variables. Judge G. A. Saden seems to be special. Most variables would rank him to the upper group, be his worth of retention is below average. The esteem of his integrity and demeanour go along with this. Overall, there is a very clear separation into an upper and a lower group.

We remove the variable *CONT* and re-run the analysis. Of course this changes the average scores per judge, and the arrangement changes (Figure 6 on page 6).

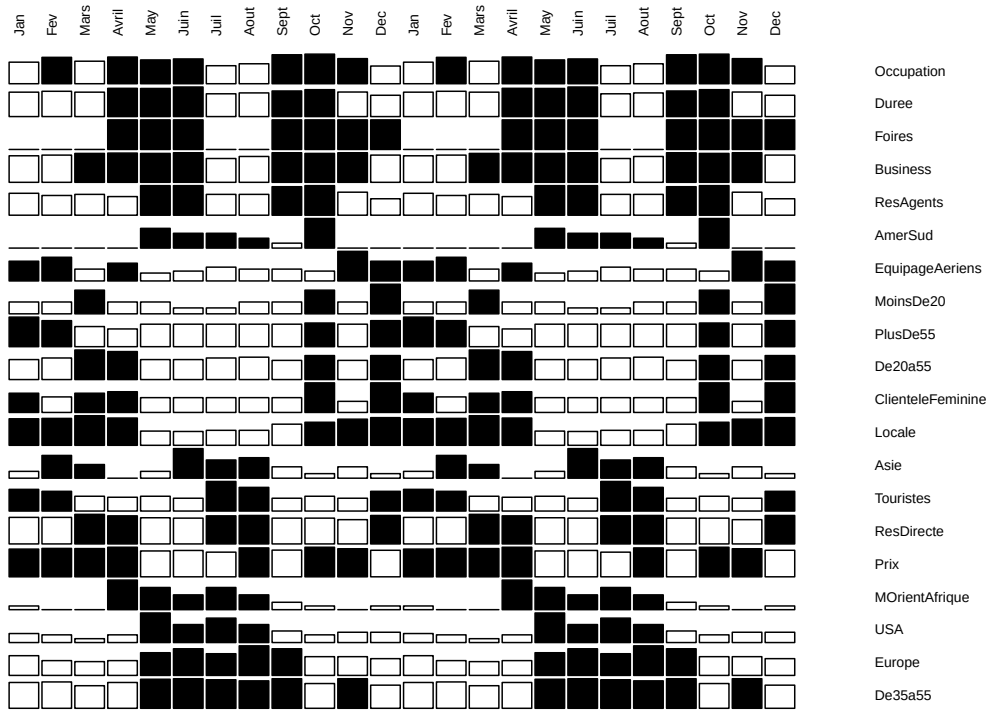



FIGURE 3. Display of a data matrix: Hotel data. Variables are rearranged by similarity to occupation and duration.

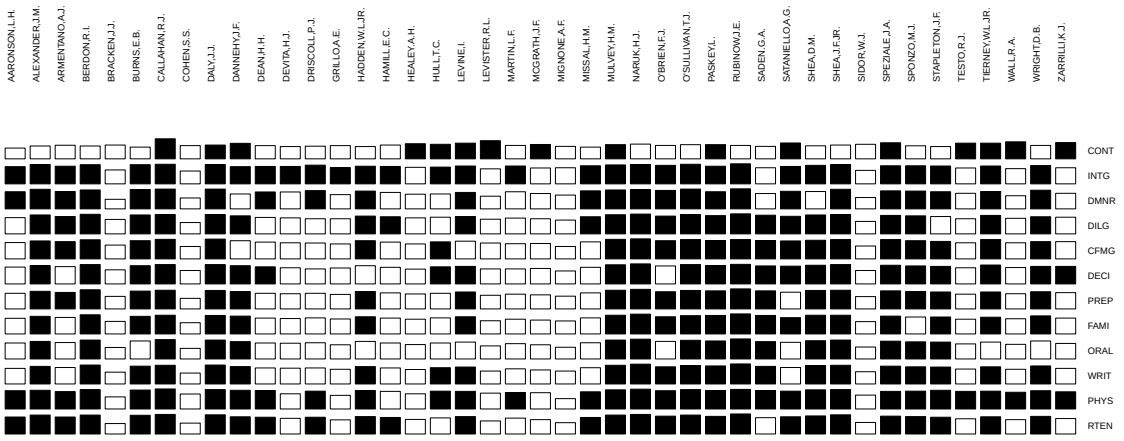



FIGURE 4. Display of a data matrix: USJudgeRatings data. Lawyers' ratings of state judges in the US Superior Court.

At this early point, let us put Bertin's work in place. Visualising information is but one aspect. In statistics, as we see it today, visualisation may be one part of an analysis. The outcome will be a decision leading to an action. Then there is a loss (or gain) depending on the action taken on the one hand, and the "true" state of

| | CONT | INTG | DMNR | DILG | CFMG | DECI | PREP | FAMI | ORAL | WRIT | PHYS | RTEN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AARONSON,L.H. | 5.70 | 7.90 | 7.70 | 7.30 | 7.10 | 7.40 | 7.10 | 7.10 | 7.10 | 7.00 | 8.30 | 7.80 |
| ALEXANDER,J.M. | 6.80 | 8.90 | 8.80 | 8.50 | 7.80 | 8.10 | 8.00 | 8.00 | 7.80 | 7.90 | 8.50 | 8.70 |
| ARMENTANO,A.J. | 7.20 | 8.10 | 7.80 | 7.80 | 7.50 | 7.60 | 7.50 | 7.50 | 7.30 | 7.40 | 7.90 | 7.80 |
| BERDON,R.I. | 6.80 | 8.80 | 8.50 | 8.80 | 8.30 | 8.50 | 8.70 | 8.70 | 8.40 | 8.50 | 8.80 | 8.70 |
| BRACKEN,J.J. | 7.30 | 6.40 | 4.30 | 6.50 | 6.00 | 6.20 | 5.70 | 5.70 | 5.10 | 5.30 | 5.50 | 4.80 |
| BURNS,E.B. | 6.20 | 8.80 | 8.70 | 8.50 | 7.90 | 8.00 | 8.10 | 8.00 | 8.00 | 8.00 | 8.60 | 8.60 |
| CALLAHAN,R.J. | 10.60 | 9.00 | 8.90 | 8.70 | 8.50 | 8.50 | 8.50 | 8.50 | 8.60 | 8.40 | 9.10 | 9.00 |
| COHEN,S.S. | 7.00 | 5.90 | 4.90 | 5.10 | 5.40 | 5.90 | 4.80 | 5.10 | 4.70 | 4.90 | 6.80 | 5.00 |
| DALY,J.J. | 7.30 | 8.90 | 8.90 | 8.70 | 8.60 | 8.50 | 8.40 | 8.40 | 8.40 | 8.50 | 8.80 | 8.80 |
| DANNEHY,J.F. | 8.20 | 7.90 | 6.70 | 8.10 | 7.90 | 8.00 | 7.90 | 8.10 | 7.70 | 7.80 | 8.50 | 7.90 |
| DEAN,H.H. | 7.00 | 8.00 | 7.60 | 7.40 | 7.30 | 7.50 | 7.10 | 7.20 | 7.10 | 7.20 | 8.40 | 7.70 |
| DEVITA,H.J. | 6.50 | 8.00 | 7.60 | 7.20 | 7.00 | 7.10 | 6.90 | 7.00 | 7.00 | 7.10 | 6.90 | 7.20 |
| DRISCOLL,P.J. | 6.70 | 8.60 | 8.20 | 6.80 | 6.90 | 6.60 | 7.10 | 7.30 | 7.20 | 7.20 | 8.10 | 7.70 |
| GRILLO,A.E. | 7.00 | 7.50 | 6.40 | 6.80 | 6.50 | 7.00 | 6.60 | 6.80 | 6.30 | 6.60 | 6.20 | 6.50 |
| HADDEN,W.L.JR. | 6.50 | 8.10 | 8.00 | 8.00 | 7.90 | 8.00 | 7.90 | 7.80 | 7.80 | 7.80 | 8.40 | 8.00 |
| HAMILL,E.C. | 7.30 | 8.00 | 7.40 | 7.70 | 7.30 | 7.30 | 7.30 | 7.20 | 7.10 | 7.20 | 8.00 | 7.60 |
| HEALEY.A.H. | 8.00 | 7.60 | 6.60 | 7.20 | 6.50 | 6.50 | 6.80 | 6.70 | 6.40 | 6.50 | 6.90 | 6.70 |
| HULL,T.C. | 7.70 | 7.70 | 6.70 | 7.50 | 7.40 | 7.50 | 7.10 | 7.30 | 7.10 | 7.30 | 8.10 | 7.40 |
| LEVINE,I. | 8.30 | 8.20 | 7.40 | 7.80 | 7.70 | 7.70 | 7.70 | 7.80 | 7.50 | 7.60 | 8.00 | 8.00 |
| LEVISTER,R.L. | 9.60 | 6.90 | 5.70 | 6.60 | 6.90 | 6.60 | 6.20 | 6.00 | 5.80 | 5.80 | 7.20 | 6.00 |
| MARTIN,L.F. | 7.10 | 8.20 | 7.70 | 7.10 | 6.60 | 6.60 | 6.70 | 6.70 | 6.80 | 6.80 | 7.50 | 7.30 |
| MCGRATH,J.F. | 7.60 | 7.30 | 6.90 | 6.80 | 6.70 | 6.80 | 6.40 | 6.30 | 6.30 | 6.30 | 7.40 | 6.60 |
| MIGNONE,A.F. | 6.60 | 7.40 | 6.20 | 6.20 | 5.40 | 5.70 | 5.80 | 5.90 | 5.20 | 5.80 | 4.70 | 5.20 |
| MISSAL,H.M. | 6.20 | 8.30 | 8.10 | 7.70 | 7.40 | 7.30 | 7.30 | 7.30 | 7.20 | 7.30 | 7.80 | 7.60 |
| MULVEY,H.M. | 7.50 | 8.70 | 8.50 | 8.60 | 8.50 | 8.40 | 8.50 | 8.50 | 8.40 | 8.40 | 8.70 | 8.70 |
| NARUK,H.J. | 7.80 | 8.90 | 8.70 | 8.90 | 8.70 | 8.80 | 8.90 | 9.00 | 8.80 | 8.90 | 9.00 | 9.00 |
| O'BRIEN,F.J. | 7.10 | 8.50 | 8.30 | 8.00 | 7.90 | 7.90 | 7.80 | 7.80 | 7.80 | 7.70 | 8.30 | 8.20 |
| O'SULLIVAN,T.J. | 7.50 | 9.00 | 8.90 | 8.70 | 8.40 | 8.50 | 8.40 | 8.30 | 8.30 | 8.30 | 8.80 | 8.70 |
| PASKEY,L. | 7.50 | 8.10 | 7.70 | 8.20 | 8.00 | 8.10 | 8.20 | 8.40 | 8.00 | 8.10 | 8.40 | 8.10 |
| RUBINOW,J.E. | 7.10 | 9.20 | 9.00 | 9.00 | 8.40 | 8.60 | 9.10 | 9.10 | 8.90 | 9.00 | 8.90 | 9.20 |
| SADEN.G.A. | 6.60 | 7.40 | 6.90 | 8.40 | 8.00 | 7.90 | 8.20 | 8.40 | 7.70 | 7.90 | 8.40 | 7.50 |
| SATANIELLO,A.G. | 8.40 | 8.00 | 7.90 | 7.90 | 7.80 | 7.80 | 7.60 | 7.40 | 7.40 | 7.40 | 8.10 | 7.90 |
| SHEA,D.M. | 6.90 | 8.50 | 7.80 | 8.50 | 8.10 | 8.20 | 8.40 | 8.50 | 8.10 | 8.30 | 8.70 | 8.30 |
| SHEA,J.F.JR. | 7.30 | 8.90 | 8.80 | 8.70 | 8.40 | 8.50 | 8.50 | 8.50 | 8.40 | 8.40 | 8.80 | 8.80 |
| SIDOR,W.J. | 7.70 | 6.20 | 5.10 | 5.60 | 5.60 | 5.90 | 5.60 | 5.60 | 5.30 | 5.50 | 6.30 | 5.30 |
| SPEZIALE,J.A. | 8.50 | 8.30 | 8.10 | 8.30 | 8.40 | 8.20 | 8.20 | 8.10 | 7.90 | 8.00 | 8.00 | 8.20 |
| SPONZO,M.J. | 6.90 | 8.30 | 8.00 | 8.10 | 7.90 | 7.90 | 7.90 | 7.70 | 7.60 | 7.70 | 8.10 | 8.00 |
| STAPLETON,J.F. | 6.50 | 8.20 | 7.70 | 7.80 | 7.60 | 7.70 | 7.70 | 7.70 | 7.50 | 7.60 | 8.50 | 7.70 |
| TESTO,R.J. | 8.30 | 7.30 | 7.00 | 6.80 | 7.00 | 7.10 | 6.70 | 6.70 | 6.70 | 6.70 | 8.00 | 7.00 |
| TIERNEY,W.L.JR. | 8.30 | 8.20 | 7.80 | 8.30 | 8.40 | 8.30 | 7.70 | 7.60 | 7.50 | 7.70 | 8.10 | 7.90 |
| WALL,R.A. | 9.00 | 7.00 | 5.90 | 7.00 | 7.00 | 7.20 | 6.90 | 6.90 | 6.50 | 6.60 | 7.60 | 6.60 |
| WRIGHT,D.B. | 7.10 | 8.40 | 8.40 | 7.70 | 7.50 | 7.70 | 7.80 | 8.20 | 8.00 | 8.10 | 8.30 | 8.10 |
| ZARRILLI,K.J. | 8.60 | 7.40 | 7.00 | 7.50 | 7.50 | 7.70 | 7.40 | 7.20 | 6.90 | 7.00 | 7.80 | 7.10 |

TABLE 2. US judge ratings

the world on the other. This is the common decision theoretical setting. Statistics has formulated a few standard problems, and given suggestions how to handle these. In our Hotel example, the problem can be seen as a prediction problem: find a prediction model to predict occupation and duration, based on the other variables. More specifically this is a control problem, and the statistical contribution is to find a regression model for occupation and duration, based on the other variables. The visualisation can be seen as one way to hint at a regression model. There are very few classical problems. Regression is one of them, and prediction is closely related. Classification and clustering is another, closely related pair of problems, and their relation to Bertin matrices should be obvious. The USJudgeRatings can be looked at as a classification problem.

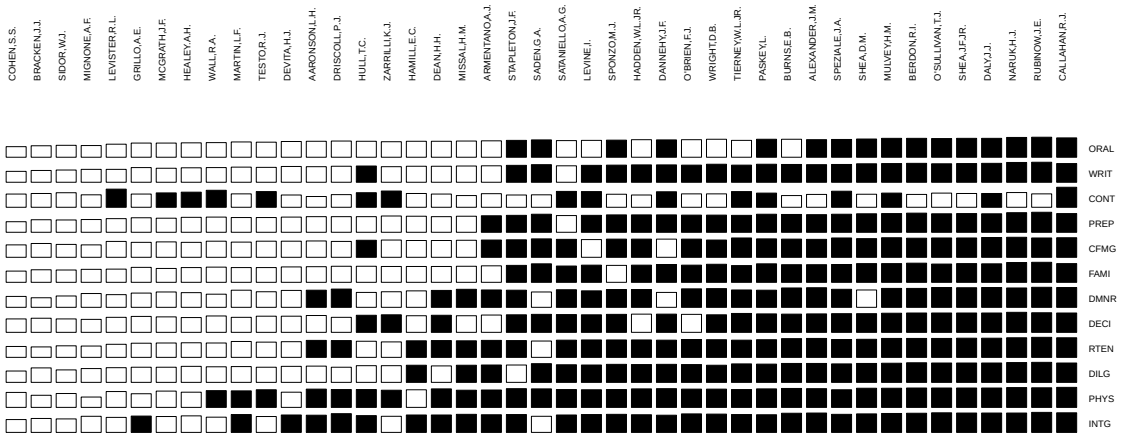



FIGURE 5. Display of a data matrix: USJudgeRatings data. Lawyers' ratings of state judges in the US Superior Court.

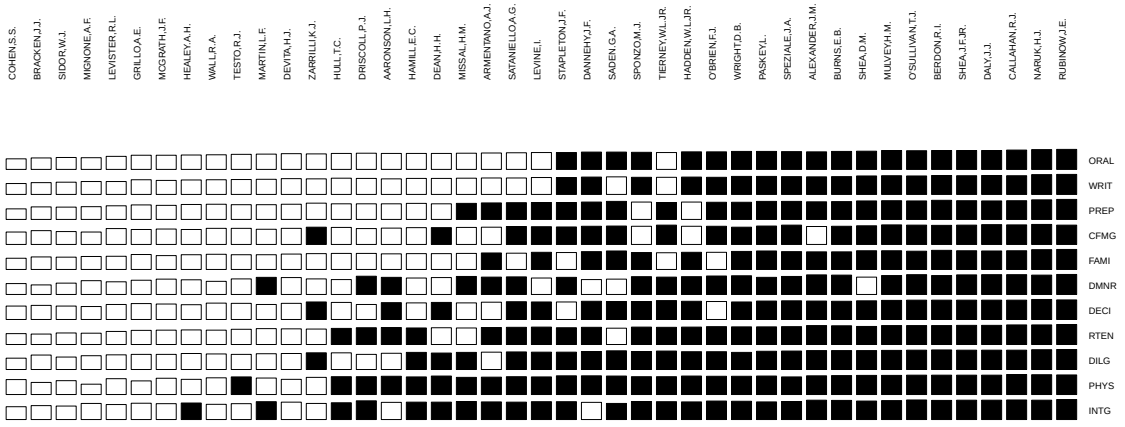



FIGURE 6. Display of a data matrix: USJudgeRatings data. Lawyers' ratings of state judges in the US Superior Court. Variable *CONT* removed.

## 3. Work flow

Bertin matrices usually are part of a work flow.

In a first step, we transfer the input data to allow for common, or comparable scales. In the Hotel example, Bertin rescales by the maximum value of each variable. The dichotomous variable Faires is encoded as 0/1. Our implementation default is to

rescale for *(0, max)* for positive variables, *(min, 0)* for negative variables, *(min, max)* for general variables. Out preferred, or recommended rescaling however is to use ranks. We use the term *score* for the rescaled variables. Orientation of the data set is critical convention in this step. Usually, rescaling should be by variable, not by case. Depending on the orientation, this can lead for example to ranks by row or by column. We allow global scaling as an additional option for those situations where all data are already on a common scale. Following Bertin, our implementation default is to expect variables in rows, but we provide the means to switch to the R convention with variables in columns. The raw data may come in data frames, or lists, or views on a data base, and the original convention should be preserved. The scores however are a matrix, or an array (which we consider a stacked list of matrices in our context.) We prefer to keep these in Bertin conventions, that is variables are in rows.

In a second step, the scores are translated to visualisation attributes. Colour is handled in two steps. The scores are translated to a colour index, which is used together with a colour palette to determine the display colour for a data element. This allows rapid experiments with various colour palettes, as long as the length of the palettes are compatible. We strongly recommend to always look at the inverted colour palette together with a chosen one to mitigate the effects of colour perception. Simple image displays limit the visualisation attributes to colour. *rect* for example allows rectangle geometry, colour, and border width. Shading and line types should be considered as an alternative for print media.

Visualisation attributes may reflect different aspects. So for example in the classical Bertin display, height of a rectangle is used to reflect the value of a data element, colour is used to show an indicator whether the value is above or below variable mean.

A third step controls the actual placement of the graphical elements. With a matrix layout, it is specified by possible permutations of rows and of columns. This may be related to information used in the first two steps, but should be considered an independent step. A vector or row orders and of column orders is the critical information from this step. Various seriation methods apply. This is where Bertin's ideas about "internal mobility" as a characteristics of modern graphics come to action. The typical situation is to select scores and display attributes, and then search for optimal or good seriations. The arrangement often leads to hard optimisation problems. Placing this step later allows to use information from score transformation and attributes, which may allow more efficient algorithms. In the end, we may be better with a good solution which helps to solve the practical problem, instead of an optimal solution to a theoretical one. These may differ considerably.

The final step is to merge these informations and render a display.

## 4. Scores

In principle, scores can be generated using an appropriate score function and `apply` or any of its variants. As examples, and for convenience, we provide a small collection of score functions.

As an illustration, each is applied by row to the Hotel data set, and the result is shown using a default Bertin plot. A second plot shows a poor man's regression: assuming that the hotel occupancy, variable 19, is the parameter of interest. Sort the scores by correlation to the score of occupancy.

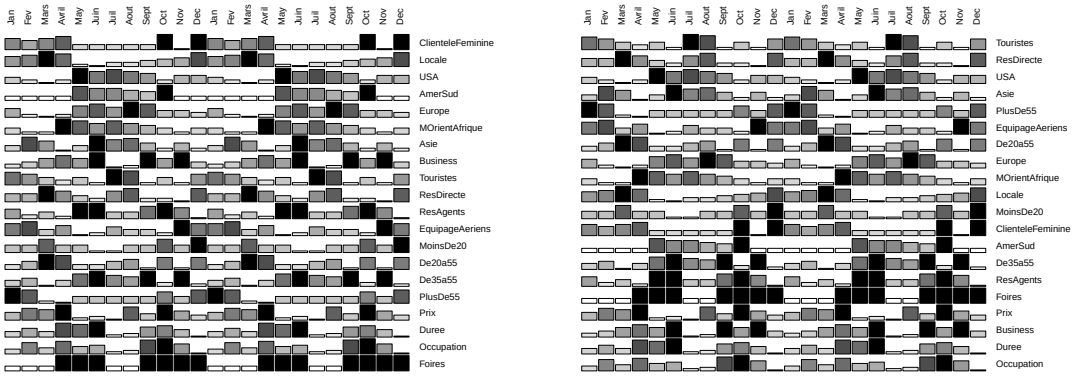
`var.orientation = "byrow"` is the default, it can be omitted. If needed in other data sets, add `var.orientation = "bycolumn"` or `var.orientation = "global"`. This allows to follow our design decision to keep the original data following the original conventions. The results here are matrices. They can be transposed at convenience.

4.1. **Ranks.**

$$x \mapsto rank(x)$$

*Input*

```
oldpar <-par(mfcol=c(1,2))
hotelrank <- bertinrank(Hotel2)
plot.bertin(hotelrank)
hotelrankorder <- bertin:::ordercor(hotelrank, 19)
plot.bertin(hotelrank, roworder=hotelrankorder)
par(oldpar)
```

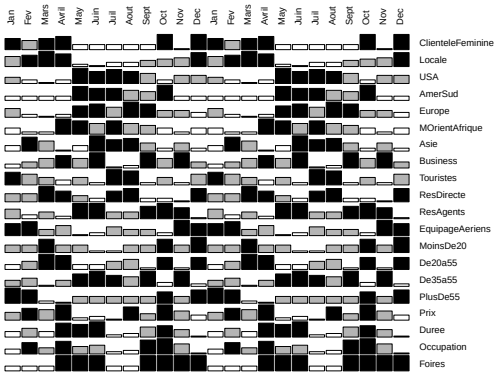


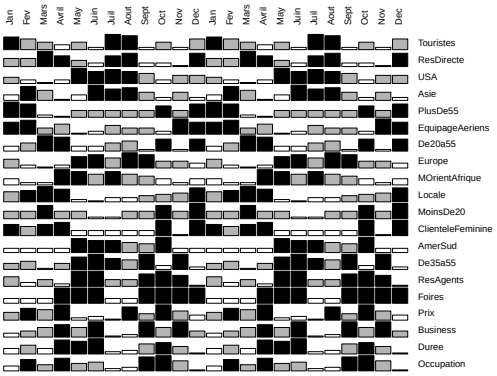

Rank scores have a sound basis. The bring us back to the range of rank statistics. However, since they scale any rank difference by unit steps, they convey order, but not quantitative differences. Preferably they are combined with colour palettes which do not suggest a quantitative scale. The default colour palette uses 256 steps of grey and suggests a quantitative order, whereas the ranks by variable provide at most 12 steps in this example.

It is preferable to use a palette with reduced resolution. For 12 rank values, a scale with 3 or 4 steps should do.

*Input*

```
oldpar <-par(mfcol=c(1,2))
hotelrank <- bertinrank(Hotel2)
plot.bertin(hotelrank,
        palette = gray((2:0 / 2)^0.5))
hotelrankorder <- bertin:::ordercor(hotelrank, 19)
plot.bertin(hotelrank, roworder=hotelrankorder,
        palette = gray((2:0 / 2)^0.5))
par(oldpar)
```


Jan
Fev
Mars
Avril
May
Juin
Juil
Aout
Sept
Oct
Nov
Dec
ClienteleFeminine
Locale
USA
AmerSud
Europe
MOrientAfrique
Asie
Business
Touristes
ResDirecte
ResAgents
EquipageAeriens
MoinsDe20
De20a55
De35a55
PlusDe55
Prix
Duree
Occupation
Foires

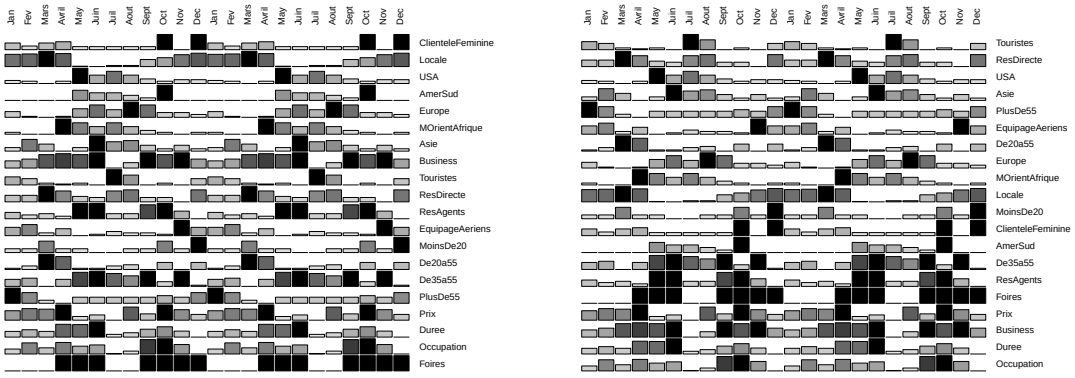

Jan
Fev
Mars
Avril
May
Juin
Juil
Aout
Sept
Oct
Nov
Dec
Touristes
ResDirecte
USA
Asie
PlusDe55
EquipageAeriens
De20a55
Europe
MOrientAfrique
Locale
MoinsDe20
ClienteleFeminine
AmerSud
De35a55
ResAgents
Foires
Prix
Business
Duree
Occupation

4.2. **z Scores.**

$$x \mapsto \frac{x - mean(x)}{sd(x)}$$

*Input*

```
oldpar <-par(mfcol=c(1,2))
hotelzscore <- bertinzscore(Hotel2)
plot.bertin(hotelzscore)
hotelzscoreorder <- bertin:::ordercor(hotelzscore, 19)
plot.bertin(hotelzscore, roworder=hotelzscoreorder)
par(oldpar)
```




Bertin uses to highlight "above average" observations. If the data is not degenerate, this corresponds to *bertinzscore > 0*.

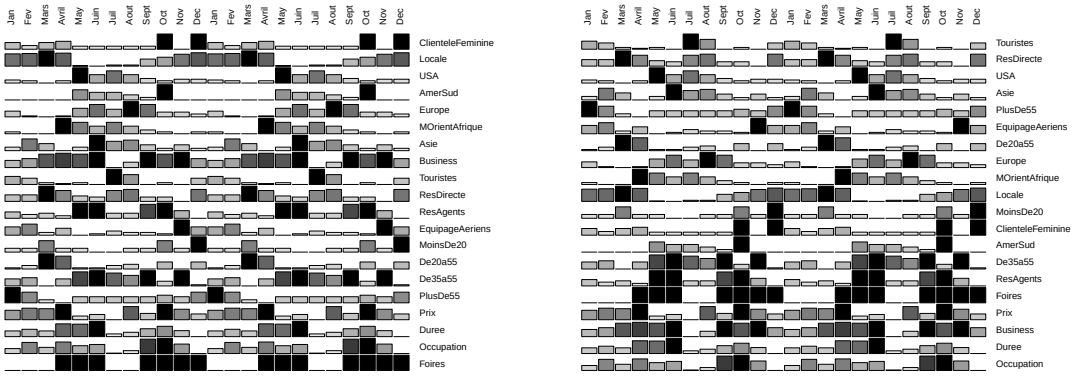
Since z Scores center the variables around the mean, they require positive and negative values for display. This leads in effect to a reduction of the display space to a half, which should be compensated by a more expressive choice of colour coding.

4.3. **Range Scores.**

$$x \mapsto \frac{x - min(x)}{maxd(x) - min(x)}$$

Input

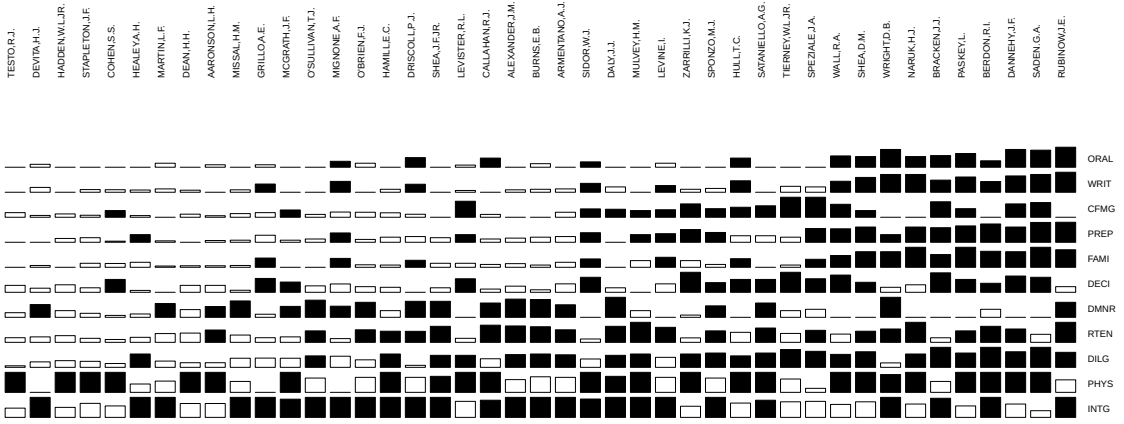
```
oldpar <-par(mfcol=c(1,2))
hotelrangescore <- bertinrangescore(Hotel2)
plot.bertin(hotelrangescore)
hotelrangescoreorder <- bertin:::ordercor(hotelrangescore, 19)
plot.bertin(hotelrangescore, roworder=hotelrangescoreorder)
par(oldpar)
```




This score just rescales to $[0, 1]$. On contrast to ranks, it preserves quantitative proportions.

Input

```
USJudgeRatings1rngt <- t(bertinrangescore(USJudgeRatings1))
colmeans <- apply(USJudgeRatings1rngt, 2, mean)
colorder<- order(colmeans)
rowmeans <-  apply(USJudgeRatings1rngt, 1, mean)
roworder <- order(rowmeans)
col=ifelse(USJudgeRatings1rngt[,]>rowmeans, "black","white")
bertinrect(USJudgeRatings1rngt[roworder, colorder],col=(col)[roworder, colorder], sepw
```




## 5. Permutation, Seriation, Arrangement

As Bertin has pointed out,

> *Ce point est fondamental. C'est la mobilité interne de l'image qui charactérise la graphique moderne.* [Bertin 1977, p. 5]

Once we have solved the problem, the problem can often be formulated as an optimisation problem. But while we are searching for a solution, experimenting is necessary. In our implementation, we separate two branches of experiment. Finding an adequate display is one branch. This amounts to building up a collection of proven models. A specific data set at hand can contribute by hinting at specific needs and simplify model choice. Building a collection of models and model choice is repeated not so often. Stability of implementation has priority over speed. We will provide a small number of basic model implementations.

The classic Bertin display shown above is one of the examples to represent certain models. Following the ideas, but deviating in the details, is to use a simple grey scale image for representation. This may be not the most informative variant. But it is most economic in the use of display space (Figure 7 on page 14).

For a chosen display, we have to compare different arrangements (seriations, for example). If we allow for interactive work, speed of display has priority. We try to cache the information that is invariant of the permutation.

As a final aspect, display space is limited. The number of variables and cases that can be displayed simultaneously is limited by the pixel size of the display. We can increase it by one or two magnitudes by using e series of detail displays. Any display calibration however should be constant for this series We try to allow for this global calibration.

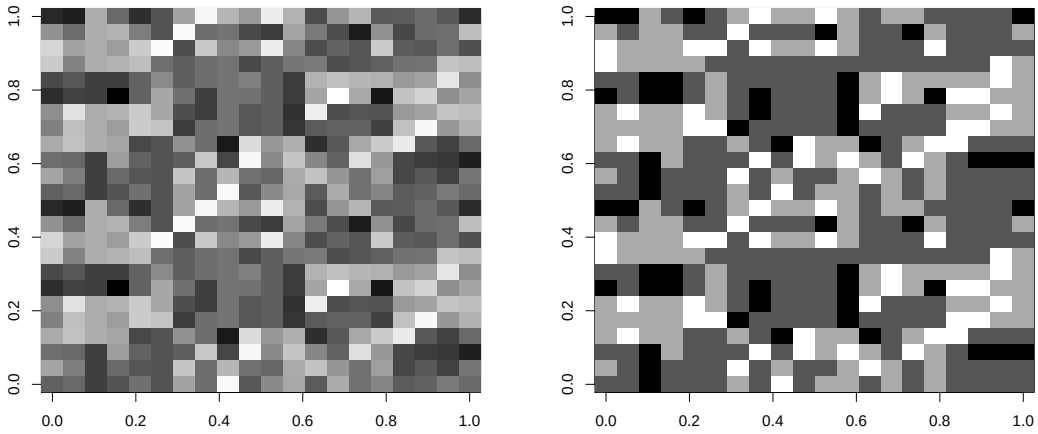



FIGURE 7. Display of a data matrix as grey scale image. Variables are rearranged by similarity to occupation and duration.

The restriction to a matrix structure is arbitrary and can be omitted. Bertin has been working as a cartographer, and his main work applies to geographical data. What we call the Bertin matrices has been introduced in the very beginning of his book and are but a starting point.

## 6. Colour, perception and pitfalls

Colour may need some experiments to find an informative choice. To allow easy experiments, we use a two step procedure. Based on the original data or the score, we derive a colour index. From an abstract point of view, this is just another score with values in $1, \ldots nrColours$. Colours are provided as a colour palette, and the index is used to select the colour to apply. See `help(palette)` for information on colour palettes.

Some colour palettes that may be useful for Bertin displays are provided.

There is an abundant literature on chosing colour palettes for the visual display of quantitative information. Please read.

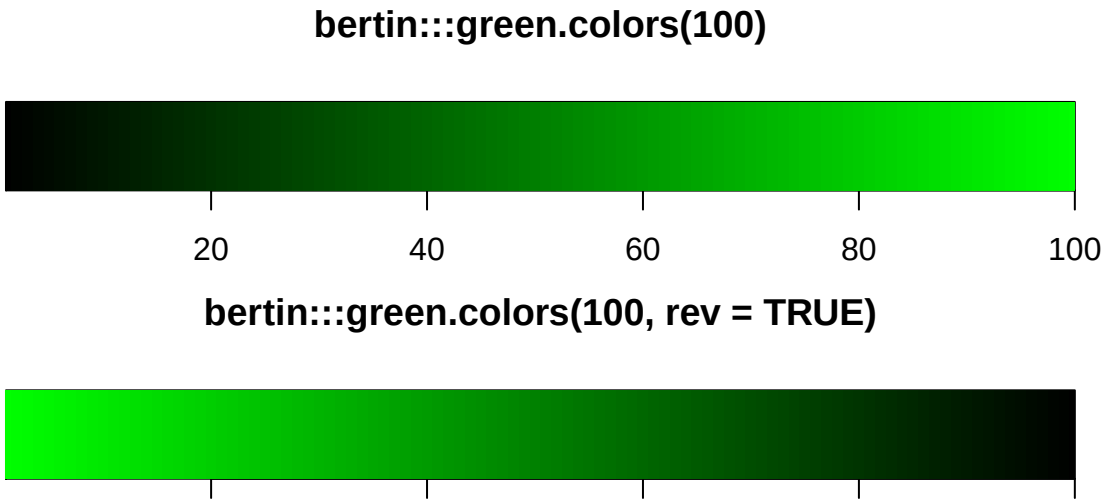
Be aware that not all people perceive colour the same way. In perticular, if you are using red and green in you coulour palette, about 6% of the male popultion will have difficulties.

Do trivial tests. If you have a smat colour scale: give a sample to some frinds and ask tehm tio tell which shoes higher and which shows lower values.

Colour displays are a means to convey some information. Check that your choices serve this purpose. In particular: if it is of importance to recognize high or low values, chose a colour scale thet does not focus on differences in neutral values.

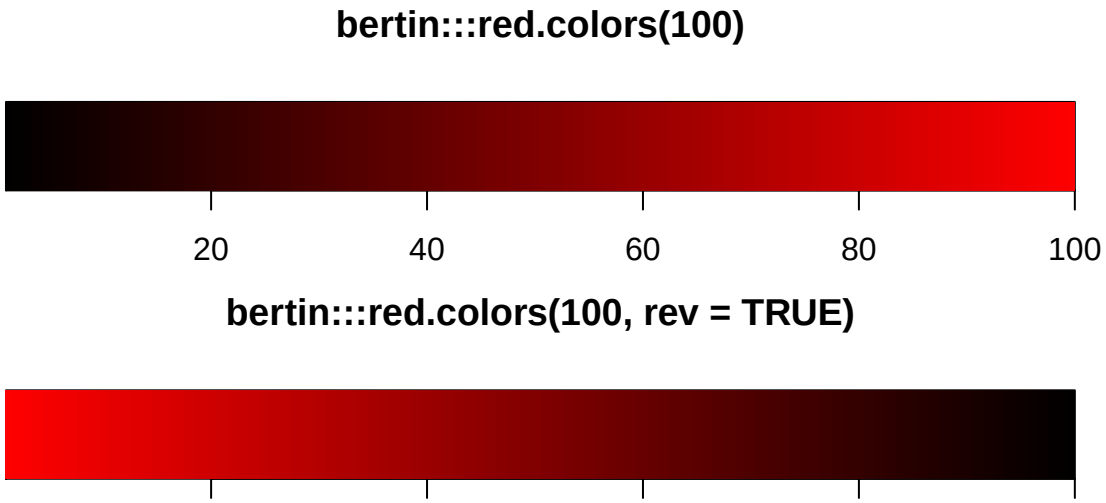
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::green.colors(100))
colramp(bertin:::green.colors(100, rev=TRUE))
par(oldpar)
```



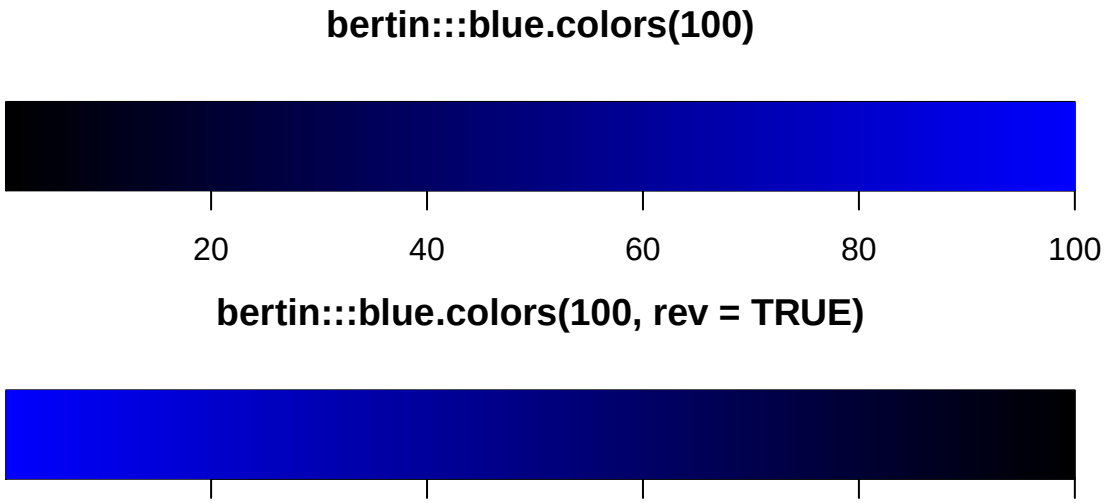
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::red.colors(100), horizontal=TRUE)
colramp(bertin:::red.colors(100, rev=TRUE),horizontal=TRUE)
par(oldpar)
```



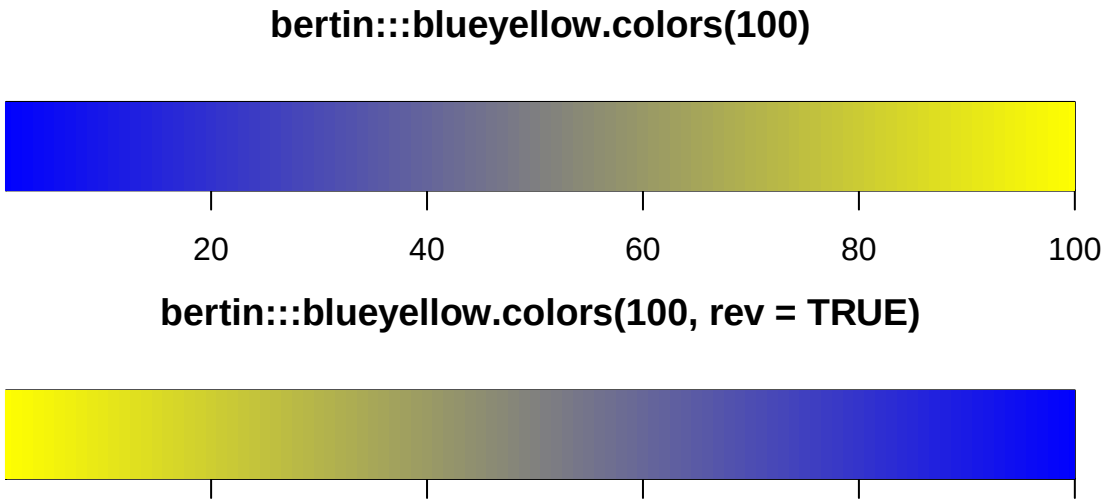
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::blue.colors(100), horizontal=TRUE)
colramp(bertin:::blue.colors(100, rev=TRUE),horizontal=TRUE)
par(oldpar)
```



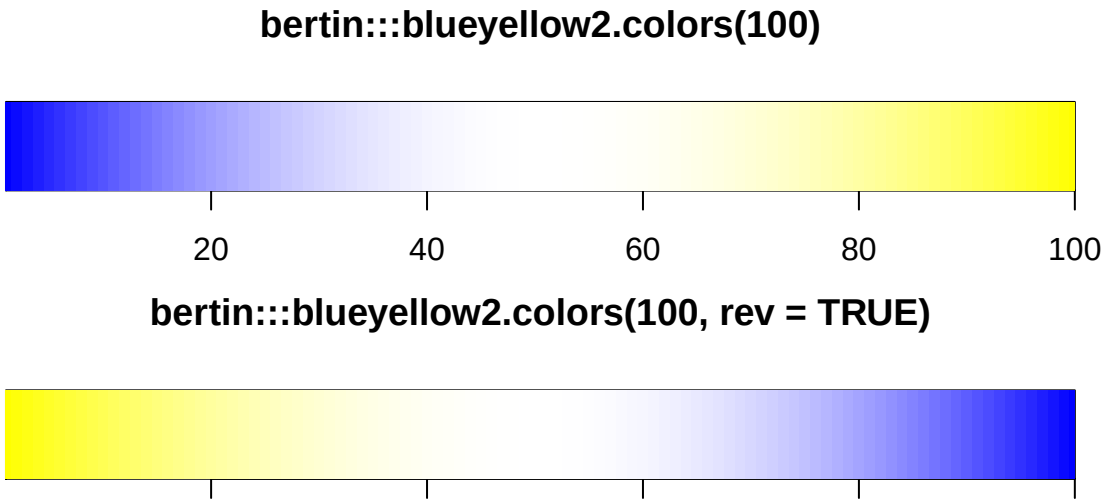
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::blueyellow.colors(100),horizontal=TRUE)
colramp(bertin:::blueyellow.colors(100, rev=TRUE),horizontal=TRUE)
par(oldpar)
```



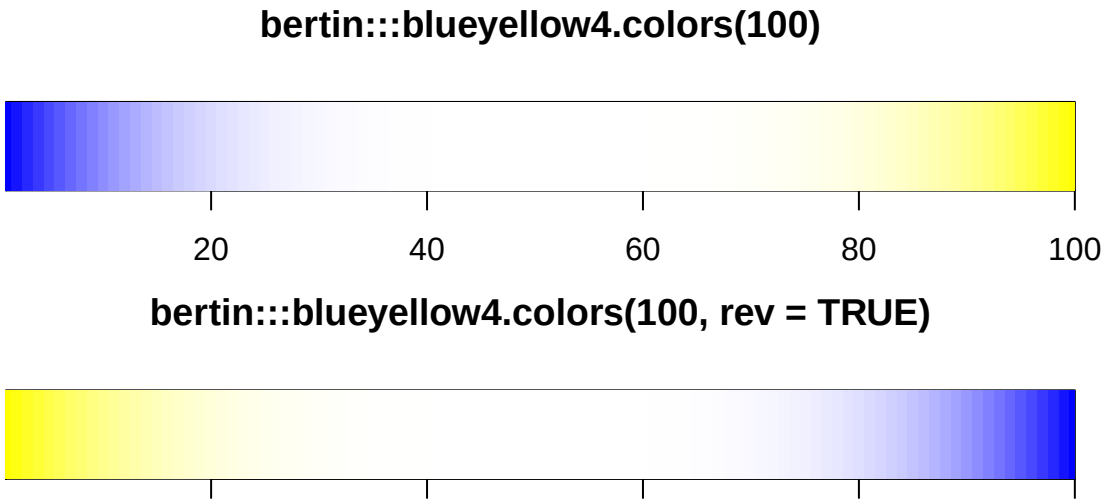
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::blueyellow2.colors(100),horizontal=TRUE)
colramp(bertin:::blueyellow2.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```



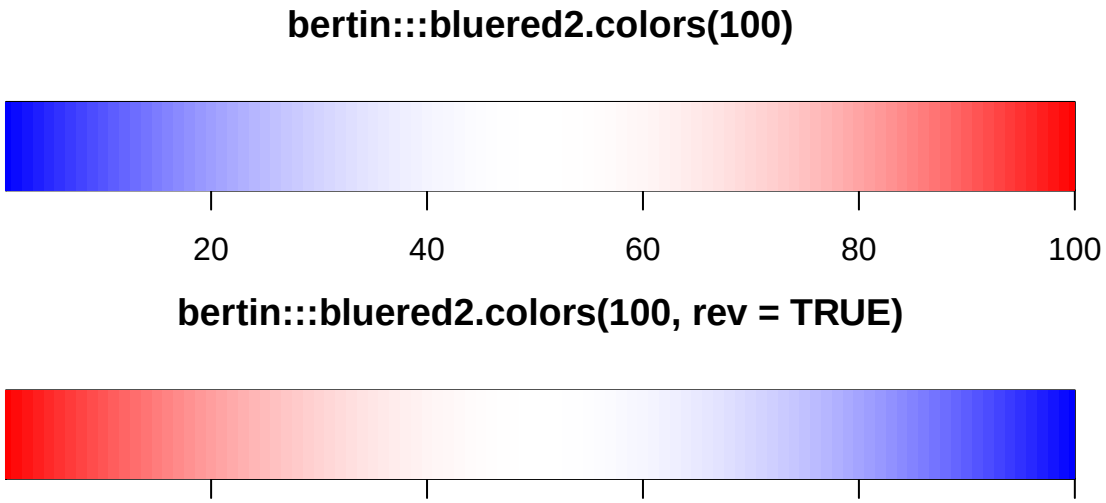
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::blueyellow4.colors(100),horizontal=TRUE)
colramp(bertin:::blueyellow4.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```



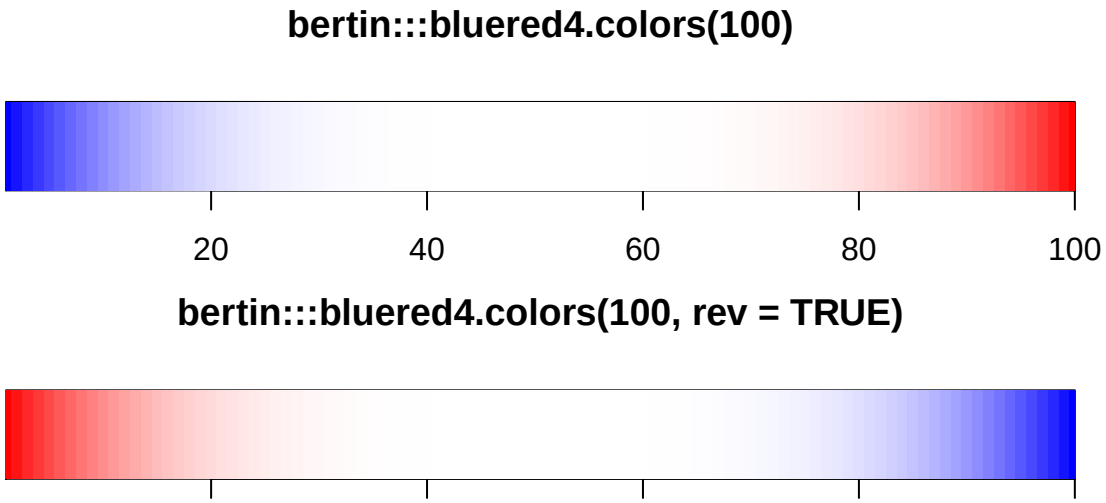
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::bluered2.colors(100),horizontal=TRUE)
colramp(bertin:::bluered2.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```



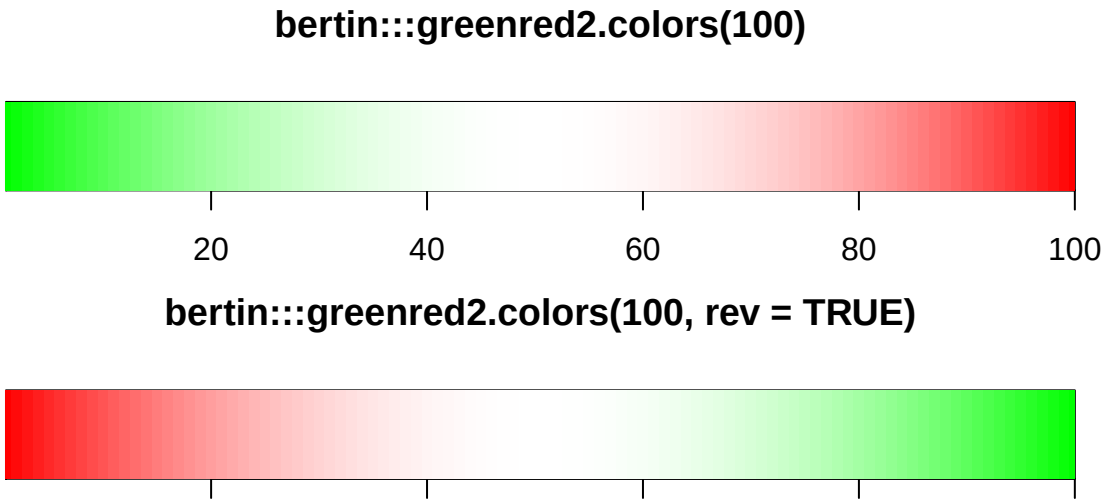
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::bluered4.colors(100),horizontal=TRUE)
colramp(bertin:::bluered4.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```



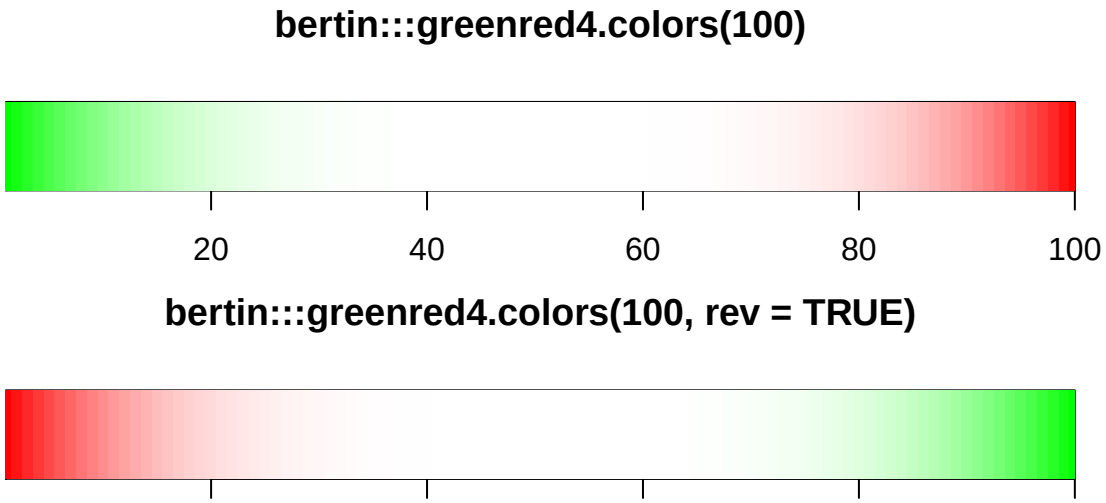
Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::greenred2.colors(100),horizontal=TRUE)
colramp(bertin:::greenred2.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```

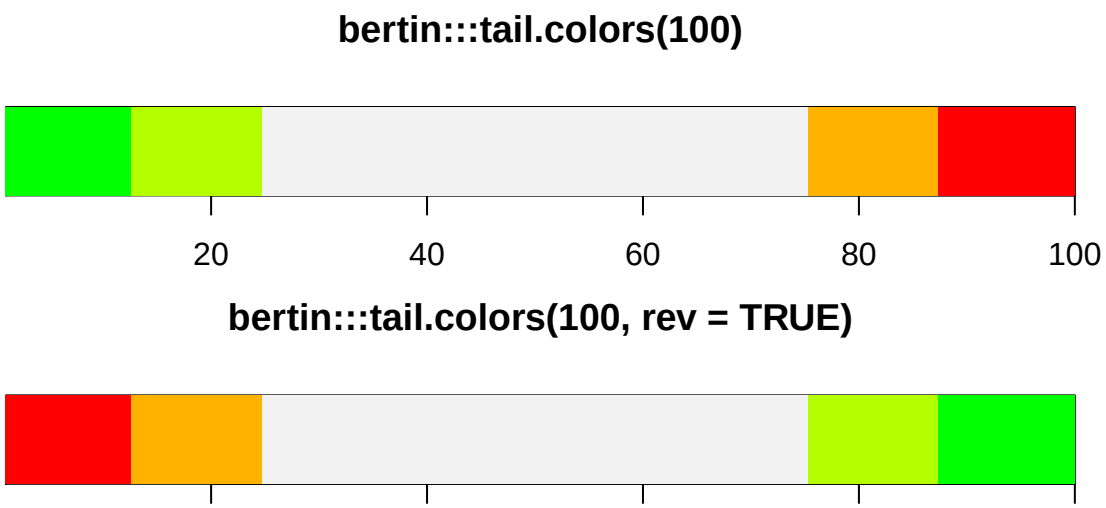


Input

```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::greenred4.colors(100),horizontal=TRUE)
colramp(bertin:::greenred4.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```



Input
```
oldpar <- par(mfrow=c(2,1), mar=c(1, 1, 4, 1) + 0.1)
colramp(bertin:::tail.colors(100),horizontal=TRUE)
colramp(bertin:::tail.colors(100,rev=TRUE),horizontal=TRUE)
par(oldpar)
```




Perception is an active process, and any visual presentation may be swayed by the intricacies of perception. Colour perception is particularly complex. When working with colour (and this includes black and white), we strongly suggest to have a look at the image with inverted colours as well.

Here is a sample implementation. On the R level, provide a plotting function

Input
```
sampleimagem <- function(z,
        col = grey((1:256)/256), xlab, ylab, main,
        colinvert=FALSE){
        if (colinvert) col <- col[length(col):1]
        # x1, x2. y1, y2
        oldpar <- par(fig=c(0, 1, 0.2, 1),
                mar=c(2.5,1.5,0.5,0.5), new=FALSE)
        imagem(z, col=col)

        par(yaxt="n", fig=c(0, 1, 0, 0.2),
                mar=c(3.5,12.0,0.5,12.0), new=TRUE)
#        colramp(col=col, horizontal=TRUE)
        zrange <- range(z, finite=TRUE)
        image(z=t(matrix(seq(zrange[1],zrange[2],length.out=length(col)),
                1, length(col))),
                zlim=zrange,main="", ylab="", xlab="", col=col)
        par(oldpar)
}
```

and run it with *colinvert=FALSE* and *colinvert=TRUE* . If your are using *Sweave*, use two separate chunks, and place the figure output side by side in TEX.

*Input*

```
hotelrk <- bertinrank(Hotel)
sampleimagem(hotelrk)
```

See Figure 8 on page 27 left.

*Input*

```
sampleimagem(hotelrk, colinvert=TRUE)
```

See Figure 8 on page 27 right.

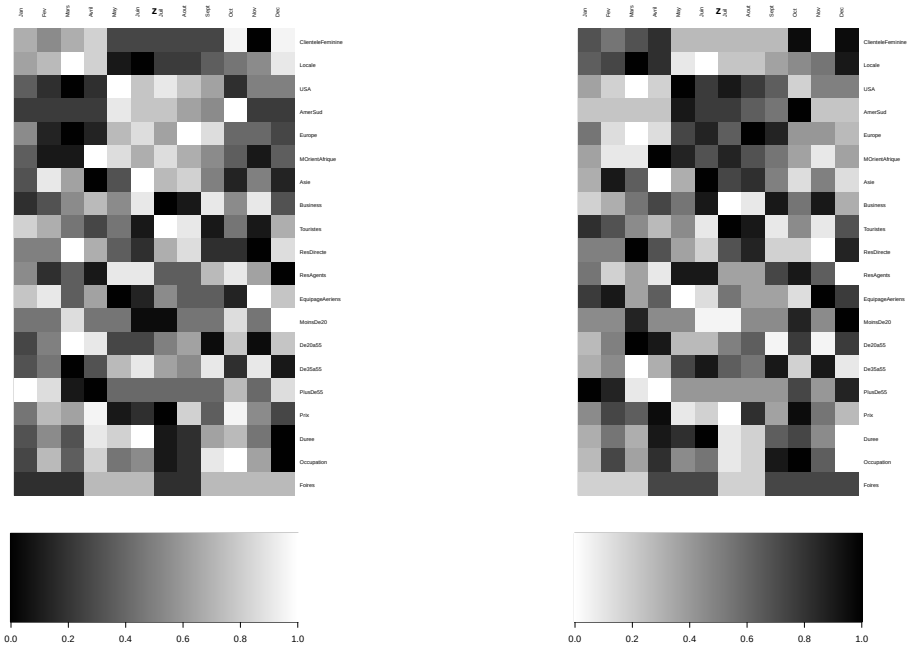



FIGURE 8. Test matrix: same information, but colour table inverted on the right.

## 7. Coordinate System and Conventions

We provide prototypes for the display of Bertin matrices. To simplify the implementation of extensions, we chose a coordinate system that allots unit square to each matrix cell. If we want to add separator lines to structure our data, we have to allocate additional space *sepwd*, measured in user space.

*image* requires increasing scales for both axes. To align with matrix conventions, we have to transform the row numbers.

We return the relevant graphical parameters as an invisible result in the basic functions *imagem()* and *bertinrect*. The *usr* parameters are set to have $(1, 1)$ as the

bottom left corner, and `x[i, j]` has a unit square figure area with bottom left coordinates at $nrow(x) - i + 1, j$, with drawing coordinates following the usual plot conventions, that is the $y$ axis is pointing upwards.

## 8. Test matrices

To test the implementation, a series of matrices is provided. Each matrix is shown in four displays: as an image using default setting of the low level function `imagem`, as an image using the default setting of `image.bertin`, as a rectangle display using the low level function `bertinrect` and as a display using the default setting of `plot.bertin`.

### 8.1. **Random Matrices.**

Input

```
nrow <- 5
ncol <- 3
BMunif <- matrix(runif(nrow*ncol), nrow, ncol)
colnames(BMunif) <- colnames(BMunif, do.NULL=FALSE)
rownames(BMunif) <- rownames(BMunif, do.NULL=FALSE)
BMnorm <- matrix(rnorm(nrow*ncol), nrow, ncol)
colnames(BMnorm) <- colnames(BMnorm, do.NULL=FALSE)
rownames(BMnorm) <- rownames(BMnorm, do.NULL=FALSE)
```

8.2. **Pure Vanilla.** The most simple case: all variables are on a common scale, and the sequence is given (no seriation possible) or irrelevant (no seriation necessary).

If we want to build test matrices, there are two free parameters to be set, for example

Input

```
BMExplRows=8
BMExplCols=6
```

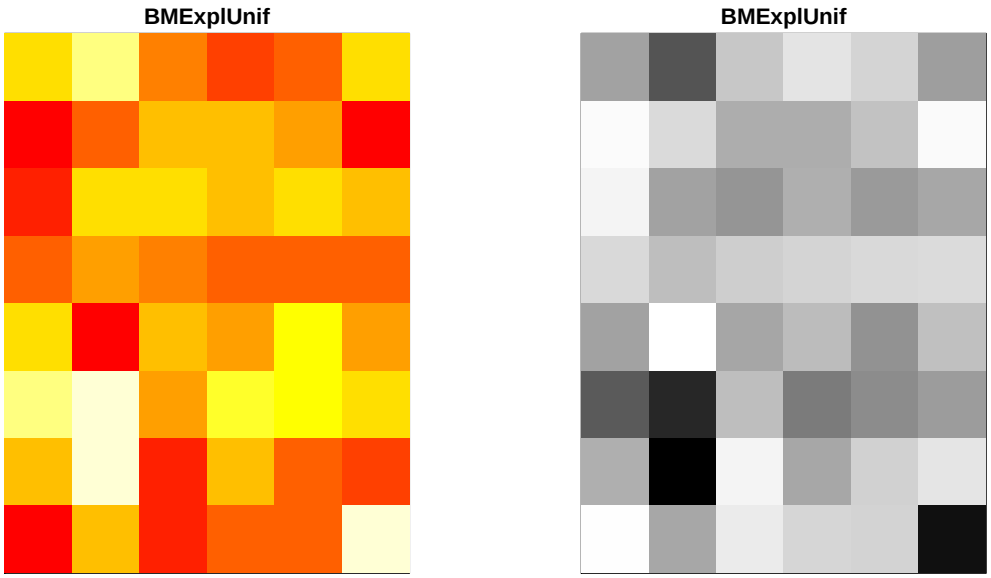
Typical cases are :

Input

```
BMExplUnif  <- matrix( runif(BMExplRows*BMExplCols),
        nrow= BMExplRows, ncol= BMExplCols)
BMExplNorm  <- matrix( rnorm(BMExplRows*BMExplCols),
        nrow= BMExplRows, ncol= BMExplCols)
```

Input

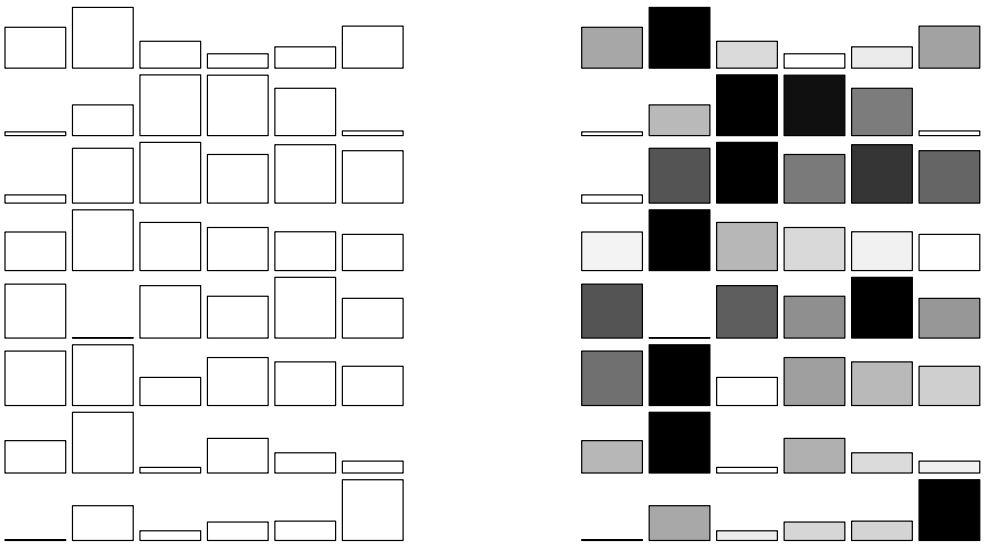
```
oldpar <- par(mfrow=c(1,2))
imagem(BMExplUnif)
image.bertin(BMExplUnif,useRaster=FALSE)
par(oldpar)
```




Random uniform left: *imagem()* right: *image.bertin()* using default settings

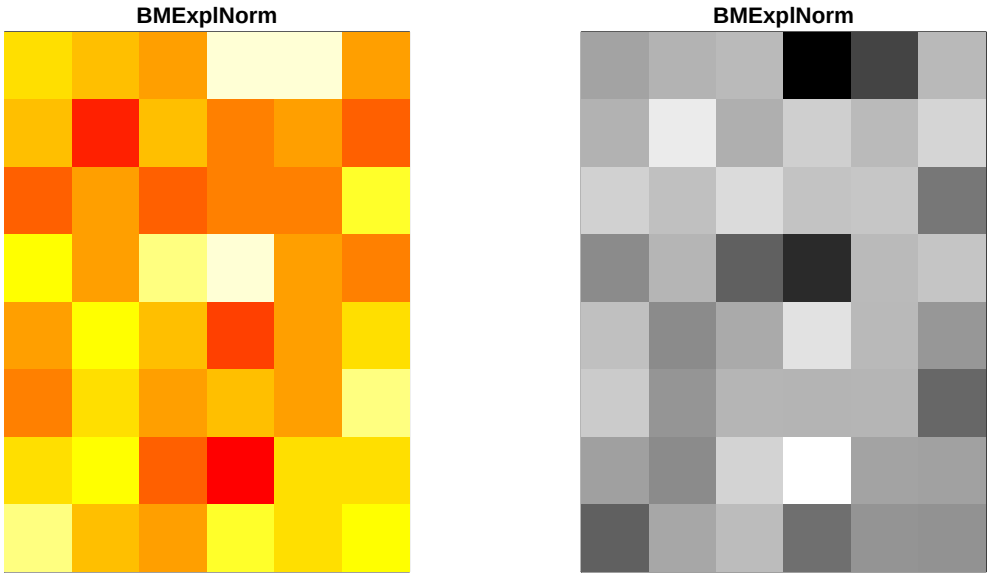
*Input*

```
oldpar <- par(mfrow=c(1,2))
bertinrect(BMExplUnif)
plot.bertin(BMExplUnif)
par(oldpar)
```



*Input*

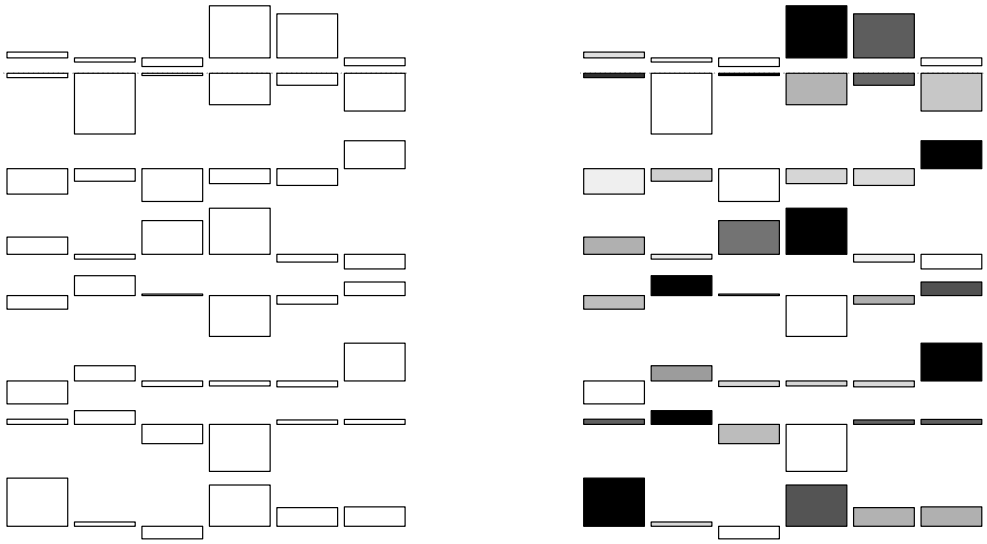
```
oldpar <- par(mfrow=c(1,2))
imagem(BMExplNorm)
image.bertin(BMExplNorm)
par(oldpar)
```




Random normal left: *imagem()* right: *image.bertin()* using default settings

*Input*

```
oldpar <- par(mfrow=c(1,2))
bertinrect(BMExplNorm)
plot.bertin(BMExplNorm)
par(oldpar)
```



Input

8.3. **Vanilla.** The next round of test cases are numeric, but not on a common scale. We provide some test vectors which we can use to construct various test matrices.

Input

```
# Test vectors, used to build a matrix
Bzero <- rep(0, BMExplCols)
Bone <- rep(1, BMExplCols)
Bmone <- rep(-1, BMExplCols)
Binc <- (1:BMExplCols)/BMExplCols
Bdec <- (BMExplCols:1)/BMExplCols
Bstep <- c(Bmone[1:floor(BMExplCols/2)],
        Bone[(1+floor(BMExplCols/2)):BMExplCols])
Bhat <- Bone
Bhat[(floor(BMExplCols/3)+1):(BMExplCols-floor(BMExplCols/3)) ] <- 0.5
Bnazero <- rep(c(NA,0),length.out= BMExplCols)
Bnanzero <- rep(c(NaN,0),length.out= BMExplCols)
Binf <- rep(c(Inf,0,-Inf),length.out= BMExplCols)
```

8.3.1. *Basic test matrices.*

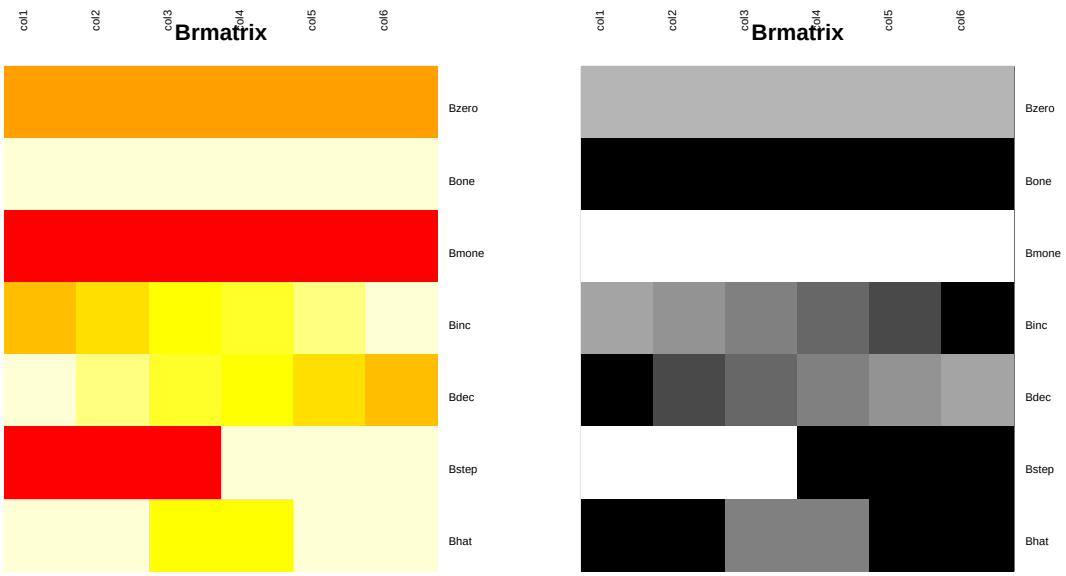
Input

```
Brmatrix <- rbind(Bzero, Bone, Bmone, Binc, Bdec, Bstep, Bhat)
colnames(Brmatrix) <- colnames(Brmatrix,FALSE)
```

| | col1 | col2 | col3 | col4 | col5 | col6 |
|---|---|---|---|---|---|---|
| Bzero | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Bone | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Bmone | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 |
| Binc | 0.17 | 0.33 | 0.50 | 0.67 | 0.83 | 1.00 |
| Bdec | 1.00 | 0.83 | 0.67 | 0.50 | 0.33 | 0.17 |
| Bstep | -1.00 | -1.00 | -1.00 | 1.00 | 1.00 | 1.00 |
| Bhat | 1.00 | 1.00 | 0.50 | 0.50 | 1.00 | 1.00 |

*Input*

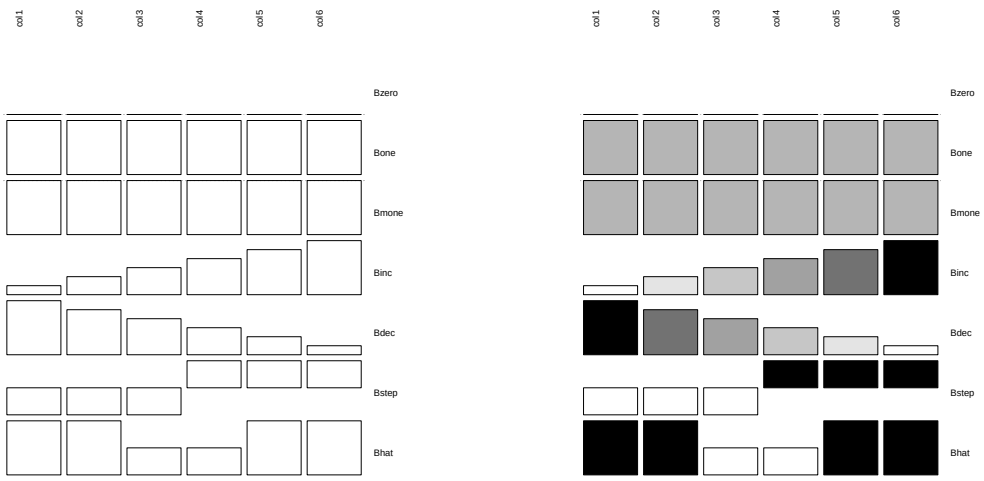
```
oldpar <- par(mfrow=c(1,2))
imagem(Brmatrix)
image.bertin(Brmatrix,useRaster=FALSE)
par(oldpar)
```




Test matrix by row. Left: *imagem()* right: *image.bertin()* using default settings

*Input*

```
oldpar <- par(mfrow=c(1,2))
bertinrect(Brmatrix)
plot.bertin(Brmatrix)
par(oldpar)
```




Test matrix by row. Left: ***bertinrect()*** right: ***plot.bertin()*** using default settings

R may use internal housekeeping to keep matrix columns homogeneous. Check! Use row matrix and column matrix for tests.

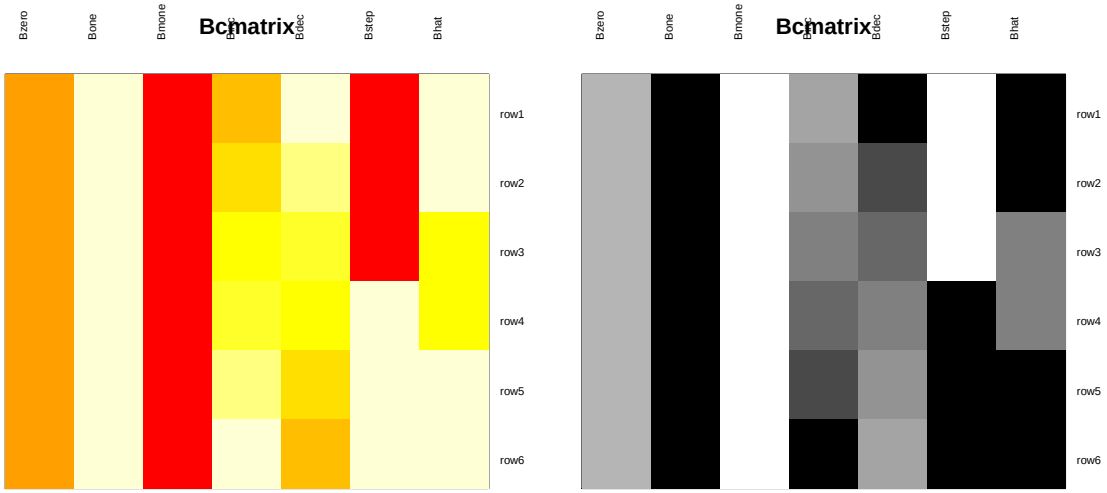
*Input*

```
Bcmatrix <- cbind(Bzero, Bone, Bmone, Binc, Bdec, Bstep, Bhat)
rownames(Bcmatrix) <- rownames(Bcmatrix,FALSE)
```

| | Bzero | Bone | Bmone | Binc | Bdec | Bstep | Bhat |
|---|---|---|---|---|---|---|---|
| row1 | 0.00 | 1.00 | -1.00 | 0.17 | 1.00 | -1.00 | 1.00 |
| row2 | 0.00 | 1.00 | -1.00 | 0.33 | 0.83 | -1.00 | 1.00 |
| row3 | 0.00 | 1.00 | -1.00 | 0.50 | 0.67 | -1.00 | 0.50 |
| row4 | 0.00 | 1.00 | -1.00 | 0.67 | 0.50 | 1.00 | 0.50 |
| row5 | 0.00 | 1.00 | -1.00 | 0.83 | 0.33 | 1.00 | 1.00 |
| row6 | 0.00 | 1.00 | -1.00 | 1.00 | 0.17 | 1.00 | 1.00 |

*Input*

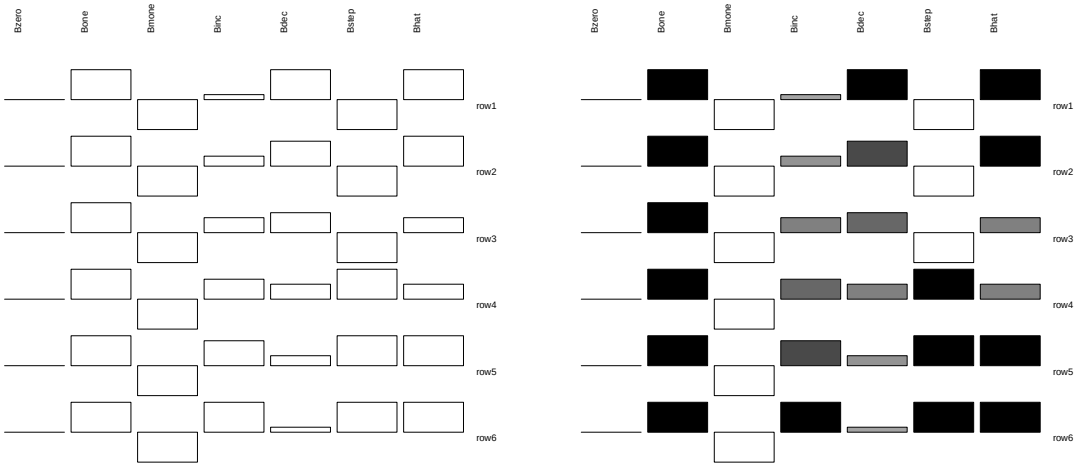
```
oldpar <- par(mfrow=c(1,2))
imagem(Bcmatrix)
image.bertin(Bcmatrix,useRaster=FALSE)
par(oldpar)
```




Test matrix by column. Left: *imagem()* right: *image.bertin()* using default settings

*Input*

```
oldpar <- par(mfrow=c(1,2))
bertinrect(Bcmatrix)
plot.bertin(Bcmatrix)
par(oldpar)
```




Test matrix by column. Left: *bertinrect()* right: *plot.bertin()* using default settings

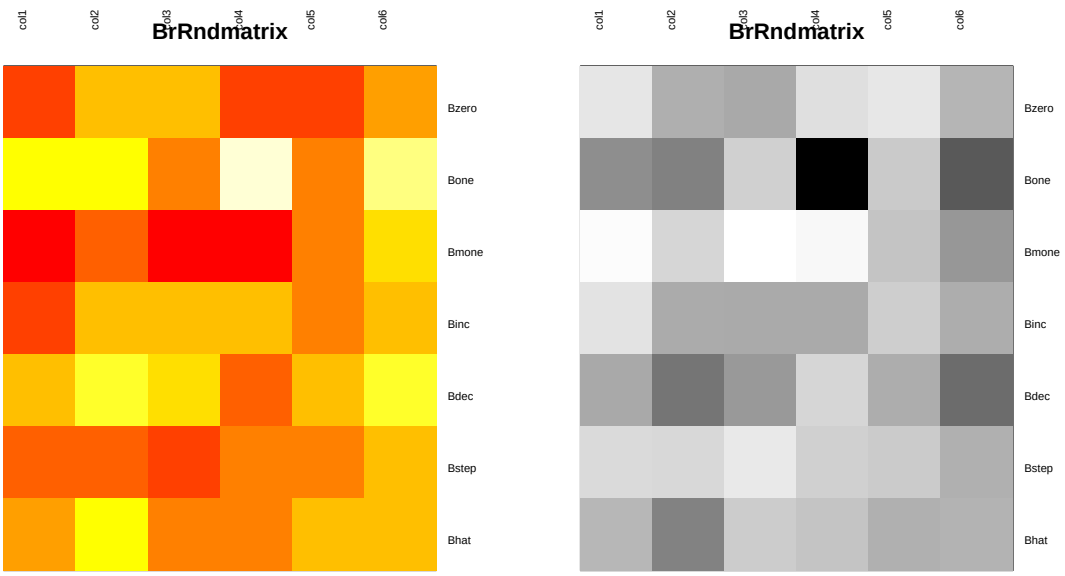
Basic test matrices with normal random error

Input

```
BrRndmatrix <- Brmatrix+rnorm(nrow(Brmatrix)*ncol(Brmatrix))
```

Input

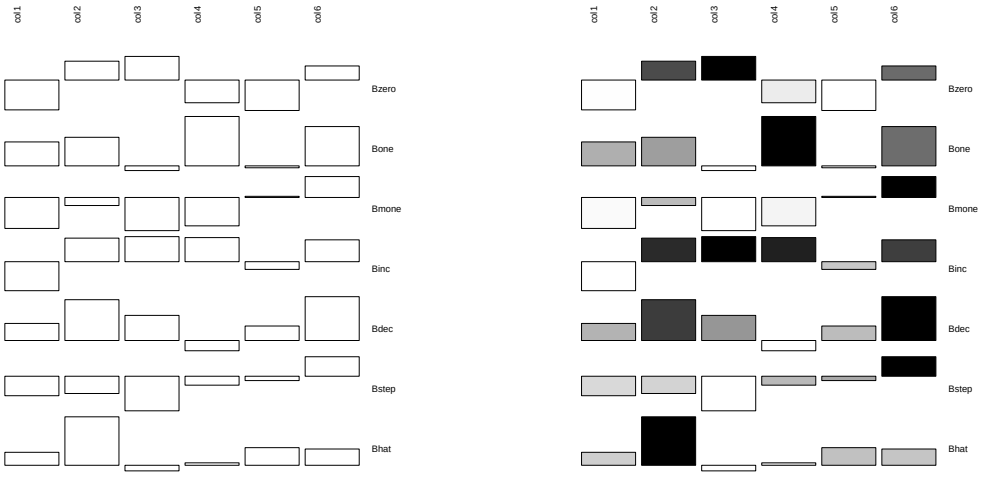
```
oldpar <- par(mfrow=c(1,2))
imagem(BrRndmatrix)
image.bertin(BrRndmatrix,useRaster=FALSE)
par(oldpar)
```




Test matrix by row with normal random error. Left: *imagem()* right: *image.bertin()* using default settings

Input

```
oldpar <- par(mfrow=c(1,2))
bertinrect(BrRndmatrix)
plot.bertin(BrRndmatrix)
par(oldpar)
```




Test matrix by row with normal random error. Left: *bertinrect()* right: *plot.bertin()* using default settings

## 8.4. **Test matrices with IEEE specials.**

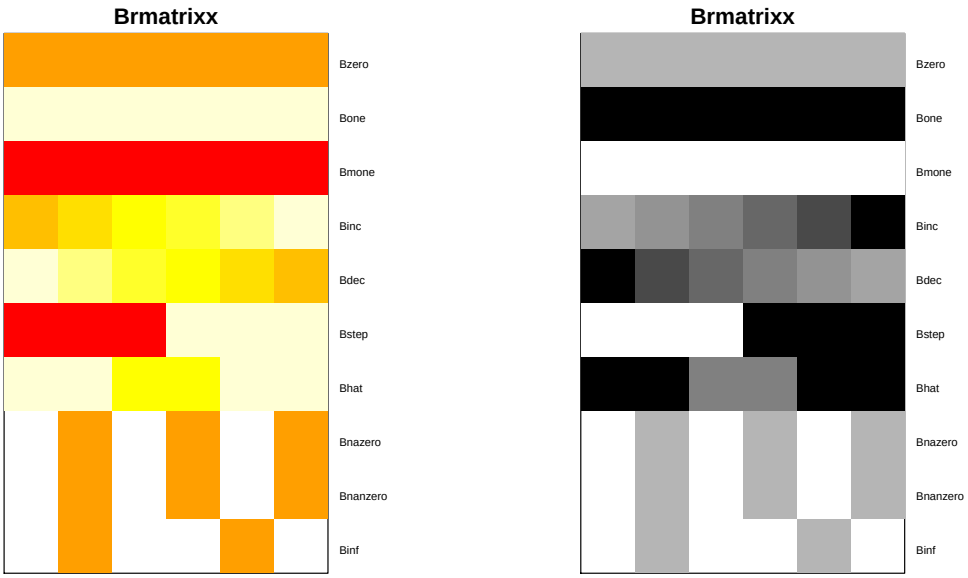
*Input*

```
Brmatrixx <- rbind(Bzero, Bone, Bmone, Binc, Bdec, Bstep, Bhat,
        Bnazero, Bnanzero, Binf)
```

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Bzero | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Bone | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Bmone | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 |
| Binc | 0.17 | 0.33 | 0.50 | 0.67 | 0.83 | 1.00 |
| Bdec | 1.00 | 0.83 | 0.67 | 0.50 | 0.33 | 0.17 |
| Bstep | -1.00 | -1.00 | -1.00 | 1.00 | 1.00 | 1.00 |
| Bhat | 1.00 | 1.00 | 0.50 | 0.50 | 1.00 | 1.00 |
| Bnazero | | 0.00 | | 0.00 | | 0.00 |
| Bnanzero | | 0.00 | | 0.00 | | 0.00 |
| Binf | Inf | 0.00 | -Inf | Inf | 0.00 | -Inf |

*Input*

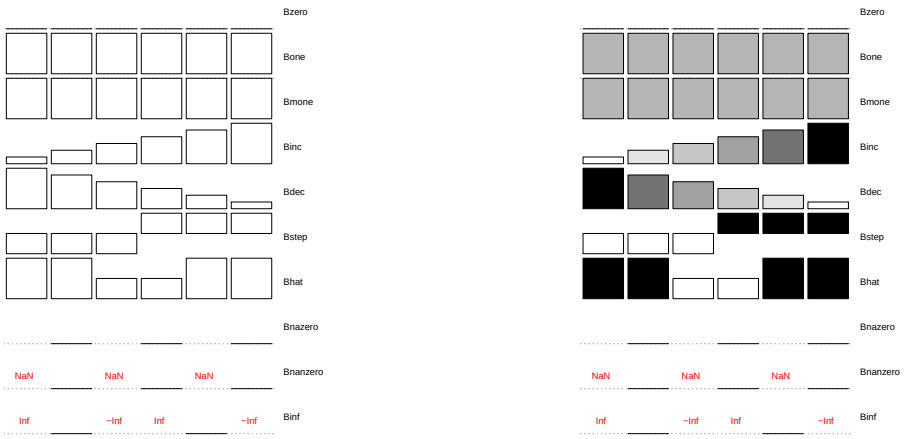
```
oldpar <- par(mfrow=c(1,2))
imagem(Brmatrixx)
image.bertin(Brmatrixx,useRaster=FALSE)
par(oldpar)
```




Test matrix with IEEE specials by row. Left: *imagem()* right: *image.bertin()* using default settings

*Input*

```
oldpar <- par(mfrow=c(1,2))
bertinrect(Brmatrixx)
plot.bertin(Brmatrixx)
par(oldpar)
```



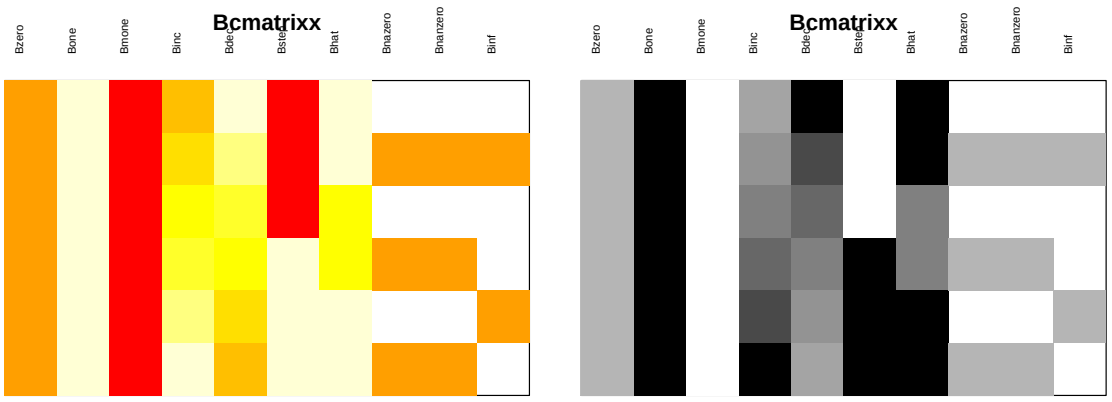

Test matrix with IEEE specials by row. Left: *bertinrect()* right: *plot.bertin()* using default settings

*Input*

```
Bcmatrixx <- cbind(Bzero, Bone, Bmone, Binc, Bdec, Bstep, Bhat,
        Bnazero, Bnanzero,Binf)
```

Input

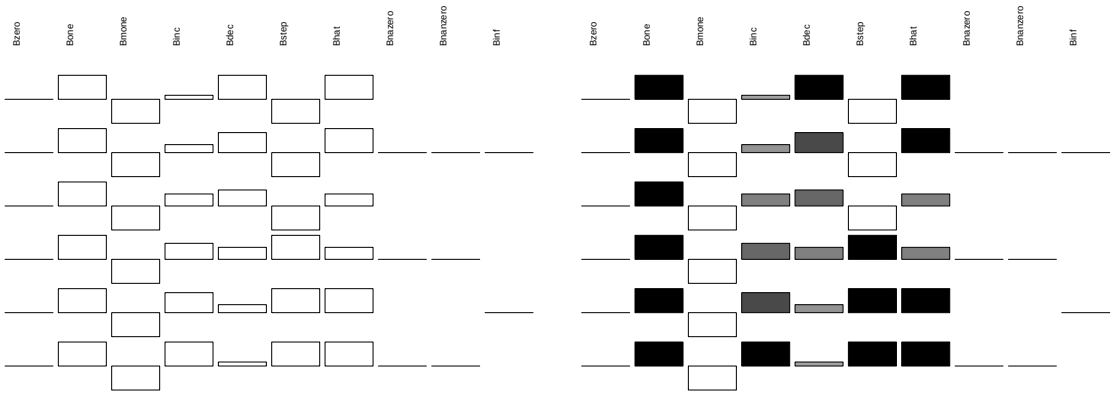
```
oldpar <- par(mfrow=c(1,2))
imagem(Bcmatrixx)
image.bertin(Bcmatrixx,useRaster=FALSE)
par(oldpar)
```




Test matrix with IEEE specials by column. Left: *imagem()* right: *image.bertin()* using default settings

Input

```
oldpar <- par(mfrow=c(1,2))
bertinrect(Bcmatrixx)
plot.bertin(Bcmatrixx)
par(oldpar)
```



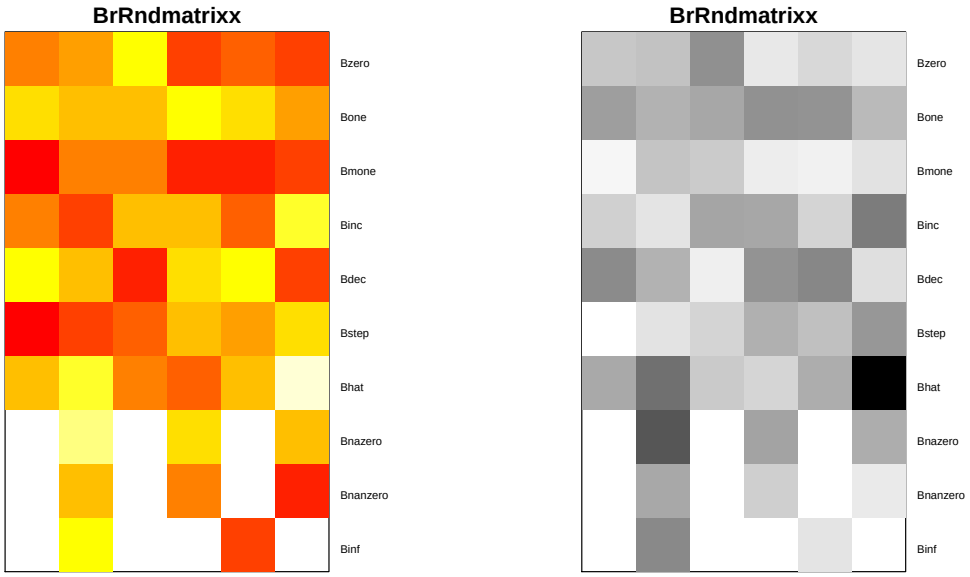

Test matrix with IEEE specials by column. Left: *bertinrect()* right: *plot.bertin()* using default settings

Input

```
BrRndmatrixx <- Brmatrixx+rnorm(nrow(Brmatrixx)*ncol(Brmatrixx))
```

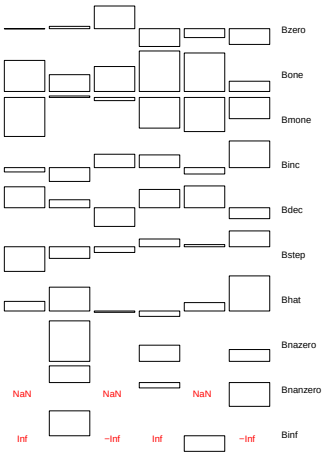
Input

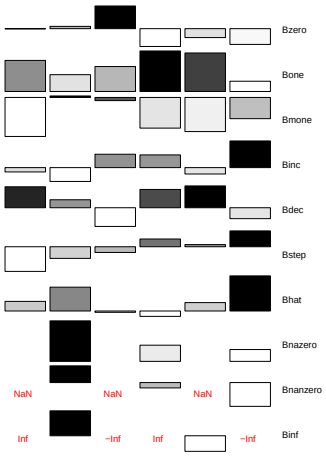
```
oldpar <- par(mfrow=c(1,2))
imagem(BrRndmatrixx)
image.bertin(BrRndmatrixx,useRaster=FALSE)
par(oldpar)
```



*Input*

```
oldpar <- par(mfrow=c(1,2))
bertinrect(BrRndmatrixx)
plot.bertin(BrRndmatrixx)
par(oldpar)
```



## References

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`$Author: gsawitzki $`
textwidth: 6.00612in linewidth:6.00612in

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