Solving Differential Equations in R (book) - DAE examples

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Abstract

This vignette contains the R-examples of chapter 6 from the book:

Soetaert, K., Cash, J.R. and Mazzia, F. (2012). Solving Differential Equations in R. that will be published by Springer.

Chapter 6. Solving Differential Algebraic Equations in R.

Here the code is given without documentation. Of course, much more information about each problem can be found in the book.

Keywords: differential algebraic equations, initial value problems, examples, R.

1. A simple DAE of Index 2

```
resdae <- function (t, y, dy, p) {
  r1 <- dy[1] - y[2]
  r2 <- y[1] - cos(t)
  list(c(r1, r2))
}
library(deTestSet)
yini  <- c(y1 = cos(0), y2 = -sin(0))
dyini <- c(-sin(0), -cos(0))
times <- seq(from = 0, to = 10, by = 0.1)
index <- c(1, 1, 0)
out1   <- mebdfi(times = times, res = resdae, y = yini,
                 atol = 1e-10, rtol = 1e-10, dy = dyini,
                 parms = NULL, nind = index)
max (abs(out1[,"y1"] - cos(times)), abs(out1[,"y2"] + sin(times)))
```

```
[1] 2.349123e-09
```

```
fundae <- function (t, y, p) {
  f1 <- y[2]
  f2 <- y[1] - cos(t)
  list(c(f1, f2))
```

```
}
M <- matrix(nrow = 2, ncol = 2, data = c(1, 0, 0, 0))
out2 <- radau(times = times, fun = fundae, y = yini,
              atol = 1e-10, rtol = 1e-10, mass = M,
              parms = NULL, nind = index)
max (abs(out2[,"y1"] - cos(times)), abs(out2[,"y2"] + sin(times)))
```

```
[1] 5.476366e-07
```

2. A Nonlinear Implicit DAE of index 1

```
implicit <- function(t, y, dy, parms) {
   list(t*y^2*dy^3 - y^3*dy^2 + t*(t^2+1)*dy - t^2*y)
}
yini  <- sqrt(3/2)
times <- seq(from = 1, to = 10, by = 0.1)
library(rootSolve)
rootfun <- function (dy, y, t)
  t*y^2*dy^3 - y^3*dy^2 + t*(t^2+1)*dy - t^2*y
dyini <- multiroot(f = rootfun, start = 0, y = yini,
                  t = times[1] )$root
dyini
```

```
[1] 0.8164966
```

```
out   <- mebdfi(times = times, res = implicit, y = yini,
               dy = dyini, parms = NULL)
out2  <- daspk (times = times, res = implicit, y = yini,
               dy = dyini, parms = NULL)
max(abs(out [,2]- sqrt(times^2+0.5)))
```

```
[1] 3.017694e-06
```

```
max(abs(out2[,2]- sqrt(times^2+0.5)))
```

```
[1] 5.689474e-05
```

```
implicit2 <- function (t, y, p) {
   f1 <- y[2]
   f2 <- t*y[1]^2*y[2]^3-y[1]^3*y[2]^2+t*(t^2+1)*y[2]-t^2*y[1]
   list(c(f1, f2))
 }
M <- matrix(nrow = 2, ncol = 2, data = c(1, 0, 0, 0))
yini_li      <- c(yini,dyini)
out3 <- radau(times = times, fun = implicit2, y = yini_li,
              mass = M, parms = NULL)
out4 <- gamd (times = times, fun = implicit2, y = yini_li,
              mass = M, parms = NULL)
max(abs(out3[,2]- sqrt(times^2+0.5)))
```

```
[1] 3.41116e-08
```

```
max(abs(out4[,2]- sqrt(times^2+0.5)))
```

```
[1] 1.348e-06
```

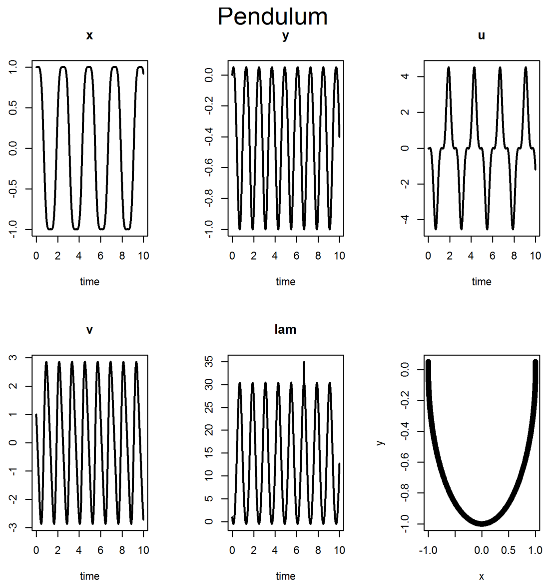

Figure 1: Solution of the pendulum problem. See book for explanation.

3. The Pendulum Problem

```
library(deTestSet)
pendulum <- function (t, y, dy, parms) {
 list(c(-dy[1] + y[3]          ,
        -dy[2] + y[4]          ,
        -dy[3] -y[5]*y[1]      ,
        -dy[4] -y[5]*y[2] - 9.8,
         y[1]^2 + y[2]^2 -1
     ))
}
yini  <- c(x = 1,  y = 0, u = 0,  v = 1   , lam = 1)
dyini <- c(dx = 0,dy = 1,du = -1,dv = -9.8,dlam = 3*9.8)
times <- seq(from = 0, to = 10, by = 0.01)
index3 <- c(2, 2, 1)
out3 <- mebdfi (y = yini, dy = dyini, res = pendulum,
              parms = NULL, times = times,
              nind = index3)

plot(out3, lwd = 2)
plot(out3[, 2:3])
mtext(side = 3, outer = TRUE, line = -1.5,
     "Pendulum", cex = 1.5)
```

4. The Car Axis problem

```
caraxis <- function(t, y, dy, parms) {
 with(as.list(y), {
   f <- rep(0, 10)
   yb <- r * sin(w * t)
   xb <- sqrt(L^2  - yb^2)
   Ll <- sqrt(xl^2 + yl^2)
   Lr <- sqrt((xr - xb)^2 + (yr - yb)^2)
   f[1:4] <- y[5:8]
   f[5] <- 1/k*((L0-Ll)*xl/Ll + lam1*xb + 2*lam2*(xl-xr))
   f[6] <- 1/k*((L0-Ll)*yl/Ll + lam1*yb + 2*lam2*(yl-yr)) -g
   f[7] <- 1/k*((L0-Lr)*(xr - xb)/Lr    - 2*lam2*(xl-xr))
   f[8] <- 1/k*((L0-Lr)*(yr - yb)/Lr    - 2*lam2*(yl-yr)) -g
   f[9] <- xb * xl + yb * yl
   f[10]<- (xl - xr)^2 + (yl - yr)^2 - L^2

   delt       <- dy - f
   delt[9:10] <- -f[9:10]

   list(delt)
 })
}
eps <- 0.01; M <- 10; k <- M * eps * eps/2
L <- 1; L0 <- 0.5; r <- 0.1; w <- 10; g <- 9.8
yini <- c(xl = 0,     yl = L0, xr = L,     yr = L0,
         ul = -L0/L, vl = 0,  ur = -L0/L, vr = 0,
         lam1 = 0, lam2 = 0)
library(rootSolve)
rootfun <- function (dyi, y, t)
  unlist(caraxis(t, y, dy = c(dyi, 0, 0),
        parms = NULL)) [1:8]
dyini <- multiroot(f = rootfun, start = rep(0,8),
                  y = yini, t = 0)$root
(dyini <- c(dyini,0,0))
```

```
 [1] -0.500000  0.000000 -0.500000  0.000000  0.000000 -9.799999  0.000000 -9.799999
 [9]  0.000000  0.000000
```

```
caraxis(t = 0, yini, dyini, NULL)
```

```
[[1]]
 [1] 2.512380e-09 0.000000e+00 2.512380e-09 0.000000e+00 0.000000e+00 8.108556e-07
 [7] 0.000000e+00 8.108556e-07 0.000000e+00 0.000000e+00
```

```
index <- c(4, 4, 2)
times <- seq(from = 0, to = 3, by = 0.01)
```

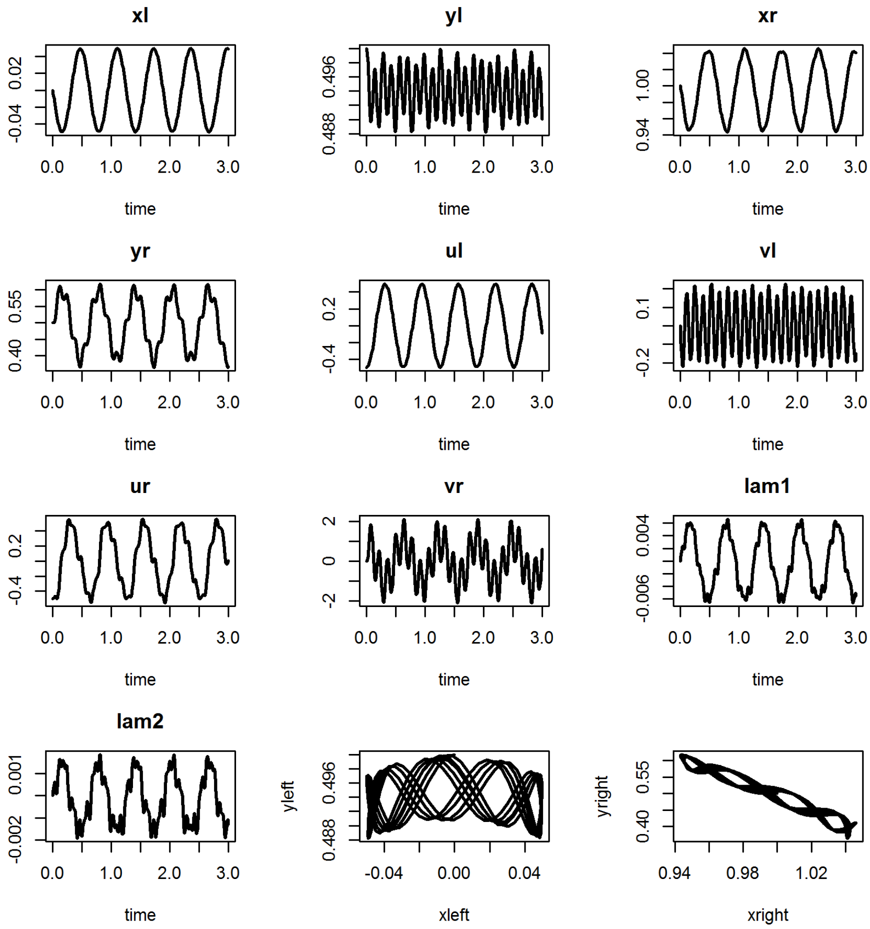

Figure 2: The car axis problem. See book for explanation.

```
out <- mebdfi(y = yini, dy = dyini, times = times,
              res = caraxis, parms = parameter, nind = index)

par(mar = c(4, 4, 3, 2))
plot(out, lwd = 2, mfrow = c(4,3))
plot(out[,c("xl", "yl")], xlab = "xleft", ylab = "yleft",
    type = "l", lwd = 2)
plot(out[,c("xr", "yr")], xlab = "xright", ylab = "yright",
    type = "l", lwd = 2)
```

5. The Transistor Amplifier

```
library(deSolve)
Transistor <- function(t, u, du, pars) {
  delt <- vector(length = 8)
   uin  <- 0.1 * sin(200 * pi * t)
   g23  <- beta * (exp( (u[2] - u[3]) / uf) - 1)
   g56  <- beta * (exp( (u[5] - u[6]) / uf) - 1)

   delt[1] <- (u[1] - uin)/R0
   delt[2] <- u[2]/R1 + (u[2]-ub)/R2 + (1-alpha) * g23
   delt[3] <- u[3]/R3 - g23
   delt[4] <- (u[4] - ub) / R4 + alpha * g23
   delt[5] <- u[5]/R5 + (u[5]-ub)/R6 + (1-alpha) * g56
   delt[6] <- u[6]/R7 - g56
   delt[7] <- (u[7] - ub) / R8 + alpha * g56
   delt[8] <- u[8]/R9
   list(delt)
}
ub <- 6; uf <- 0.026; alpha <- 0.99; beta <- 1e-6; R0 <- 1000
R1 <- R2 <- R3 <- R4 <- R5 <- R6 <- R7 <- R8 <- R9 <- 9000
C1 <- 1e-6; C2 <- 2e-6; C3 <- 3e-6; C4 <- 4e-6; C5 <- 5e-6
mass <- matrix(nrow = 8, ncol = 8, byrow = TRUE, data = c(
      -C1,C1, 0,  0,  0,  0,  0,  0,
      C1,-C1, 0,  0,  0,  0,  0,  0,
      0,  0,-C2,  0,  0,  0,  0,  0,
      0,  0,  0,-C3, C3,  0,  0,  0,
      0,  0,  0, C3,-C3,  0,  0,  0,
      0,  0,  0,  0,  0,-C4,  0,  0,
      0,  0,  0,  0,  0,  0,-C5, C5,
      0,  0,  0,  0,  0,  0, C5,-C5
))
yini <- c(0, ub/(R2/R1+1), ub/(R2/R1+1),
         ub, ub/(R6/R5+1), ub/(R6/R5+1), ub, 0)
names(yini) <- paste("u", 1:8, sep = "")
ind   <- c(8, 0, 0)
times <- seq(from = 0, to = 0.2, by = 0.001)
out <- radau(func = Transistor, y = yini, parms = NULL,
            times = times, mass = mass, nind = ind)

plot(out, lwd = 2, which = c("u1", "u5", "u8"),
    mfrow = c(1, 3))
```

Affiliation:

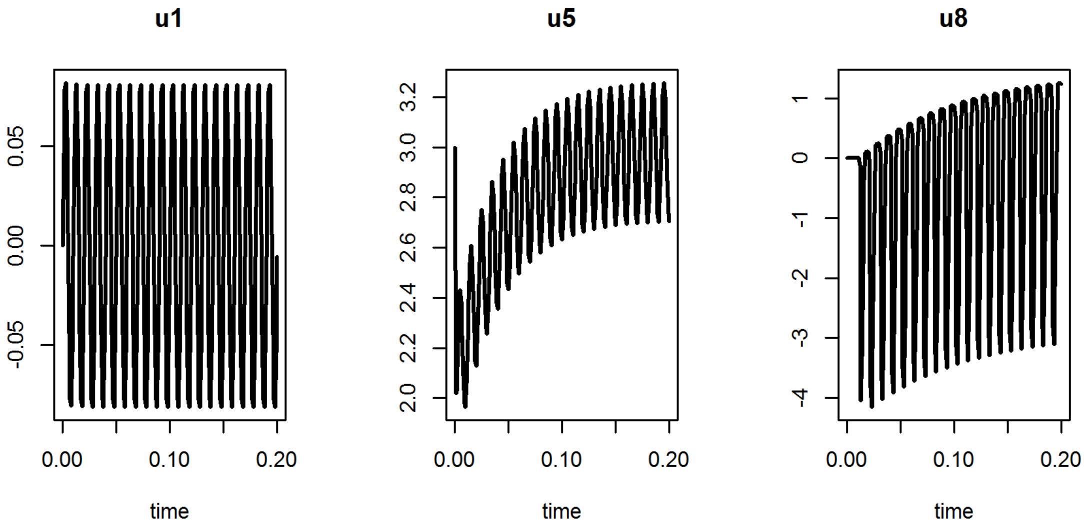

Figure 3: The transistor amplifier. See book for more information.

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