

TEST FOR TREND WITH A MULTINOMIAL OUTCOME

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1. INTRODUCTION

- Consider a study in which a multinomial outcome with K possible unordered values is measured in subjects belonging to one of G ordered groups. The size of each group, n_i , is defined by the study design, and will be treated as fixed. Let $\mathbf{p}_i = (p_{i1}, \dots, p_{iK})^\top$ denote the probabilities of the multinomial outcomes in the i th group. The hypothesis of interest is to evaluate the homogeneity of these probabilities across the groups with a targeted alternative of a trend in at least one of the categories. Formally, we consider testing $H_0 = \bigcap_{j=1}^K H_{0j}$ versus $H_1 = \bigcup_{j=1}^K H_{1j}$, where

$$\begin{aligned} H_{0j} : p_{1j} &= \dots = p_{Gj} \\ H_{1j} : p_{1j} &\leq \dots \leq p_{Gj} \text{ or } p_{1j} \geq \dots \geq p_{Gj} \text{ with at least one inequality} \end{aligned} \tag{1}$$

The test is based on the following result:

Theorem 1. *Let $\mathcal{J} \subset \{1, \dots, K\}$, then under $H_{0\mathcal{J}} = \bigcap_{j \in \mathcal{J}} H_{0j}$ as $N \rightarrow \infty$*

$$W_{\mathcal{J}} = \sum_{j \in \mathcal{J}} (1 - p_{\cdot j}) T_j^2 + \left(\sum_{j \in \mathcal{J}} p_{\cdot j} \right) T_{\mathcal{J}}^2 \xrightarrow{d} \chi_d^2, \tag{2}$$

where $d = \min(|\mathcal{J}|, K - 1)$, $T_{\mathcal{J}} = [\sum_{i=1}^G \sum_{j \in \mathcal{J}} n_{ij}(c_i - \bar{c})] / \sqrt{p_{\cdot \mathcal{J}}(1 - p_{\cdot \mathcal{J}})s^2}$ denotes the Cochran-Armitage trend test statistic for testing for marginal trend in $p_{i\mathcal{J}} = \sum_{j \in \mathcal{J}} p_{ij}$, $i = 1, \dots, G$.

2. IMPLEMENTING THE OVERALL TEST

The main `multiCA.test` function is a generic, with methods for a matrix and formula input.

```
"../R/aaa-generics.R" ?≡
```

```
#'Multinomial Cochran-Armitage trend test
#
#'#The \code{multiCA.test} performs a multinomial generalization of the
#'# Cochran-Armitage trend test.
#
#
#'#@export
#'#@param x a two-dimensional matrix or a formula
#'#@param \dots other arguments
#'#@return an object of class "htest" with the results of the test
#'#@author Aniko Szabo
#'#@references Szabo, A. (2016) Test for trend with a multinomial outcome.
#'#@keywords nonparametric
#'#@examples
#
#'#data(stroke)
#'### using formula interface
#'#multiCA.test(Type ~ Year, weight=Count, data=stroke)
#
#'### using matrix interface
#'#strk.mat <- xtabs(Count ~ Type + Year, data=stroke)
#'#multiCA.test(strk.mat)
#
#'#@name multiCA.test

multiCA.test <- function(x,...) UseMethod("multiCA.test")
```

◇
Uses: multiCA.test ?.

The default method uses a two-dimensional contingency matrix with the outcomes as rows and ordered groups as columns.

```
"../R/multiCA.R" ?≡
```

```
#'#rdname multiCA.test
#'#method multiCA.test default
#'#@export
#'#@param scores numeric vector of the same length as the number of ordered groups. Defaults to linearly i
#'#@param outcomes integer or character vector defining the set of outcomes (by row index or row name) ove

multiCA.test.default <- function(x, scores=1:ncol(object), outcomes=1:nrow(object),
...){
  if (!is.matrix(x)) stop("x should be a two-dimensional matrix")
  if (length(scores) != ncol(x)) stop("The length of the score vector should equal the number of columns

  K <- nrow(x)
  full <- length(outcomes) == K #full test

  nidot <- apply(x, 2, sum)
  n <- sum(nidot)

  cbar <- sum(nidot * scores)/n

  s2 <- sum(nidot * (scores - cbar)^2)
  pdot <- prop.table(rowSums(x))[outcomes]
  nonz <- (pdot > 0)
```

```

if (!any(nonz)) return(1)

X <- x[outcomes, ,drop=FALSE] %*% (scores - cbar)

if (full || sum(pdot) >= 1){
  Tt <- ( sum(X[nonz]^2 / pdot[nonz])) / s2
} else {
  Tt <- (sum(X)^2 / (1-sum(pdot)) + sum(X[nonz]^2 / pdot[nonz])) / s2
}

df <- length(outcomes) - full
p.value <- pchisq(Tt, df=df, lower=FALSE)

res <- list(statistic = Tt, parameter = df, p.value = p.value,
            method="Multinomial Cochran-Armitage trend test",
            dname <- deparse(substitute(x)))
class(res) <- "htest"
return(res)
}

```

Defines: multiCA.test ?.

3. FILES

"../R/aaa-generics.R" Defined by ?.
 "../R/multiCA.R" Defined by ?.

4. MACROS

None.

5. IDENTIFIERS

multiCA.test: ?, ?.