

# Package ‘CRF’

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**Version** 0.3-9

**Title** Conditional Random Fields

**Description** Implements modeling and computational tools for conditional random fields (CRF) model as well as other probabilistic undirected graphical models of discrete data with pairwise and unary potentials.

**ByteCompile** TRUE

**Depends** R (>= 2.12.0)

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Matrix (>= 1.1-2)

**License** GPL (>= 2)

**URL** <http://r-forge.r-project.org/projects/crf/>

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CRF-package

*CRF - Conditional Random Fields*

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## Description

Library of Conditional Random Fields model

## Details

CRF is R package for various computational tasks of conditional random fields as well as other probabilistic undirected graphical models of discrete data with pairwise and unary potentials. The decoding/inference/sampling tasks are implemented for general discrete undirected graphical models with pairwise potentials. The training task is less general, focusing on conditional random fields with log-linear potentials and a fixed structure. The code is written entirely in R and C++. The initial version is ported from UGM written by Mark Schmidt.

Decoding: Computing the most likely configuration

- [decode.exact](#) Exact decoding for small graphs with brute-force search

- `decode.chain` Exact decoding for chain-structured graphs with the Viterbi algorithm
- `decode.tree` Exact decoding for tree- and forest-structured graphs with max-product belief propagation
- `decode.conditional` Conditional decoding (takes another decoding method as input)
- `decode.cutset` Exact decoding for graphs with a small cutset using cutset conditioning
- `decode.junction` Exact decoding for low-treewidth graphs using junction trees
- `decode.sample` Approximate decoding using sampling (takes a sampling method as input)
- `decode.marginal` Approximate decoding using inference (takes an inference method as input)
- `decode.lbp` Approximate decoding using max-product loopy belief propagation
- `decode.trbp` Approximate decoding using max-product tree-reweighted belief propagation
- `decode.greedy` Approximate decoding with greedy algorithm
- `decode.icm` Approximate decoding with the iterated conditional modes algorithm
- `decode.block` Approximate decoding with the block iterated conditional modes algorithm
- `decode.ilp` Exact decoding with an integer linear programming formulation and approximate using LP relaxation

Inference: Computing the partition function and marginal probabilities

- `infer.exact` Exact inference for small graphs with brute-force counting
- `infer.chain` Exact inference for chain-structured graphs with the forward-backward algorithm
- `infer.tree` Exact inference for tree- and forest-structured graphs with sum-product belief propagation
- `infer.conditional` Conditional inference (takes another inference method as input)
- `infer.cutset` Exact inference for graphs with a small cutset using cutset conditioning
- `infer.junction` Exact decoding for low-treewidth graphs using junction trees
- `infer.sample` Approximate inference using sampling (takes a sampling method as input)
- `infer.lbp` Approximate inference using sum-product loopy belief propagation
- `infer.trbp` Approximate inference using sum-product tree-reweighted belief propagation

Sampling: Generating samples from the distribution

- `sample.exact` Exact sampling for small graphs with brute-force inverse cumulative distribution
- `sample.chain` Exact sampling for chain-structured graphs with the forward-filter backward-sample algorithm
- `sample.tree` Exact sampling for tree- and forest-structured graphs with sum-product belief propagation and backward-sampling
- `sample.conditional` Conditional sampling (takes another sampling method as input)
- `sample.cutset` Exact sampling for graphs with a small cutset using cutset conditioning
- `sample.junction` Exact sampling for low-treewidth graphs using junction trees
- `sample.gibbs` Approximate sampling using a single-site Gibbs sampler

Training: Given data, computing the most likely estimates of the parameters

- `train.crf` Train CRF model

- `train.mrf` Train MRF model

Tools: Tools for building and manipulating CRF data

- `make.crf` Generate CRF from the adjacent matrix
- `make.features` Make the data structure of CRF features
- `make.par` Make the data structure of CRF parameters
- `duplicate.crf` Duplicate an existing CRF
- `clamp.crf` Generate clamped CRF by fixing the states of some nodes
- `clamp.reset` Reset clamped CRF by changing the states of clamped nodes
- `sub.crf` Generate sub CRF by selecting some nodes
- `mrf.update` Update node and edge potentials of MRF model
- `crf.update` Update node and edge potentials of CRF model

### Author(s)

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### References

J. Lafferty, A. McCallum, and F. Pereira. Conditional random fields: Probabilistic models for segmenting and labeling sequence data. In *the proceedings of International Conference on Machine Learning (ICML)*, pp. 282-289, 2001.

Mark Schmidt. UGM: Matlab code for undirected graphical models. <http://www.di.ens.fr/~mschmidt/Software/UGM.html>

### Examples

```
library(CRF)
data(Small)
decode.exact(Small$crf)
infer.exact(Small$crf)
sample.exact(Small$crf, 100)
```

---

Chain

*Chain CRF example*

---

### Description

This data set gives a chain CRF example

### Usage

```
data(Chain)
```

**Format**

A list containing two elements:

- crf The CRF
- answer A list of 4 elements:
  - decode The most likely configuration
  - node.bel The node belief
  - edge.bel The edge belief
  - logZ The logarithmic value of CRF normalization factor Z

---

clamp.crf	<i>Make clamped CRF</i>
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---

**Description**

Generate clamped CRF by fixing the states of some nodes

**Usage**

```
clamp.crf(crf, clamped)
```

**Arguments**

crf	The CRF generated by <a href="#">make.crf</a>
clamped	The vector of fixed states of nodes

**Details**

The function will generate a clamped CRF from a given CRF by fixing the states of some nodes. The vector clamped contains the desired state for each node while zero means the state is not fixed. The node and edge potentials are updated to the conditional potentials based on the clamped vector.

**Value**

The function will return a new CRF with additional components:

original	The original CRF.
clamped	The vector of fixed states of nodes.
node.id	The vector of the original node ids for nodes in the new CRF.
node.map	The vector of the new node ids for nodes in the original CRF.
edge.id	The vector of the original edge ids for edges in the new CRF.
edge.map	The vector of the new edge ids for edges in the original CRF.

**See Also**

[make.crf](#), [sub.crf](#), [clamp.reset](#)

## Examples

```
library(CRF)
data(Small)
crf <- clamp.crf(Small$crf, c(0, 0, 1, 1))
```

---

clamp.reset	<i>Reset clamped CRF</i>
-------------	--------------------------

---

## Description

Reset clamped CRF by changing the states of clamped nodes

## Usage

```
clamp.reset(crf, clamped)
```

## Arguments

crf	The clamped CRF generated by <a href="#">clamp.crf</a>
clamped	The vector of fixed states of nodes

## Details

The function will reset a clamped CRF by changing the states of fixed nodes. The vector `clamped` contains the desired state for each node while zero means the state is not fixed. The node and edge potentials are updated to the conditional potentials based on the clamped vector.

## Value

The function will return the same clamped CRF.

## See Also

[make.crf](#), [clamp.crf](#)

## Examples

```
library(CRF)
data(Small)
crf <- clamp.crf(Small$crf, c(0, 0, 1, 1))
clamp.reset(crf, c(0,0,2,2))
```

---

Clique	<i>Clique CRF example</i>
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---

**Description**

This data set gives a clique CRF example

**Usage**

```
data(Clique)
```

**Format**

A list containing two elements:

- crf The CRF
- answer A list of 4 elements:
  - decode The most likely configuration
  - node.bel The node belief
  - edge.bel The edge belief
  - logZ The logarithmic value of CRF normalization factor Z

---

crf.nll	<i>Calculate CRF negative log likelihood</i>
---------	--

---

**Description**

Calculate the negative log likelihood of CRF model

**Usage**

```
crf.nll(par, crf, instances, node.fea = NULL, edge.fea = NULL,
        node.ext = NULL, edge.ext = NULL, infer.method = infer.chain, ...)
```

**Arguments**

par	The parameter vector of CRF
crf	The CRF
instances	The training data matrix of CRF model
node.fea	The list of node features
edge.fea	The list of edge features
node.ext	The list of extended information of node features
edge.ext	The list of extended information of edge features
infer.method	The inference method used to compute the likelihood
...	Other parameters need by the inference method

## Details

This function calculates the negative log likelihood of CRF model as well as the gradient. This function is intended to be called by optimization algorithm in training process.

In the training data matrix `instances`, each row is an instance and each column corresponds a node in CRF. The variables `node.fea`, `edge.fea`, `node.ext`, `edge.ext` are lists of length equal to the number of instances, and their elements are defined as in [crf.update](#) respectively.

## Value

This function will return the value of CRF negative log-likelihood.

## See Also

[crf.update](#), [train.crf](#)

---

<code>crf.update</code>	<i>Update CRF potentials</i>
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---

## Description

Update node and edge potentials of CRF model

## Usage

```
crf.update(crf, node.fea = NULL, edge.fea = NULL, node.ext = NULL,
           edge.ext = NULL)
```

## Arguments

<code>crf</code>	The CRF
<code>node.fea</code>	The node features matrix with dimension (n.nf, n.nodes)
<code>edge.fea</code>	The edge features matrix with dimension (n.ef, n.edges)
<code>node.ext</code>	The extended information of node features
<code>edge.ext</code>	The extended information of edge features

## Details

This function updates `node.pot` and `edge.pot` of CRF model by using the current values of parameters and features.

There are two ways to model the relationship between parameters and features. The first one exploits the special structure of features to reduce the memory usage. However it may not suitable for all circumstances. The other one is more straightforward by explicitly specifying the coefficients of each parameter to calculate the potentials, and may use much more memory. Two approaches can be used together.

The first way uses the objects `node.par` and `edge.par` to define the structure of features and provides the feature information in variables `node.fea` and `edge.fea`. The second way directly provides the feature information in variables `node.ext` and `edge.ext` without any prior assumption on feature structure. `node.ext` is a list and each element has the same structure as `node.pot`. `edge.ext` is a list and each element has the same structure as `edge.pot`.



In detail, the node potential is updated as follows:

$$node.pot[n, i] = \sum_f par[node.par[n, i, f]] * node.fea[f, n] + \sum_k par[k] * node.ext[[k]][n, i]$$

and the edge potential is updated as follows:

$$edge.pot[[e]][i, j] = \sum_f par[edge.par[[e]][i, j, f]] * edge.fea[f, e] + \sum_k par[k] * edge.ext[[k]][[e]][i, j]$$

### Value

This function will directly modify the CRF and return the same CRF.

### See Also

[crf.nll](#), [train.crf](#)

---

decode.block	<i>Decoding method using block iterated conditional modes algorithm</i>
--------------	---

---

### Description

Computing the most likely configuration for CRF

### Usage

```
decode.block(crf, blocks, decode.method = decode.tree, restart = 0,
  start = apply(crf$node.pot, 1, which.max), ...)
```

### Arguments

crf	The CRF
blocks	A list of vectors, each vector containing the nodes in a block
decode.method	The decoding method to solve the clamped CRF
restart	Non-negative integer to control how many restart iterations are repeated
start	An initial configuration, a good start will significantly reduce the seraching time
...	The parameters for decode.method

### Details

Approximate decoding with the block iterated conditional modes algorithm

### Value

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

### Examples

```
library(CRF)
data(Small)
d <- decode.block(Small$crf, list(c(1,3), c(2,4)))
```

---

decode.chain	<i>Decoding method for chain-structured graphs</i>
--------------	--

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.chain(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact decoding for chain-structured graphs with the Viterbi algorithm.

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.chain(Small$crf)
```

---

decode.conditional	<i>Conditional decoding method</i>
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---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.conditional(crf, clamped, decode.method, ...)
```

**Arguments**

crf	The CRF
clamped	The vector of fixed values for clamped nodes, 0 for unfixed nodes
decode.method	The decoding method to solve clamped CRF
...	The parameters for <code>decode.method</code>

**Details**

Conditional decoding (takes another decoding method as input)

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.conditional(Small$crf, c(0,1,0,0), decode.exact)
```

---

 decode.cutset

*Decoding method for graphs with a small cutset*


---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.cutset(crf, cutset, engine = "default", start = apply(crf$node.pot,
  1, which.max))
```

**Arguments**

<code>crf</code>	The CRF
<code>cutset</code>	A vector of nodes in the cutset
<code>engine</code>	The underlying engine for cutset decoding, possible values are "default", "none", "exact", "chain", and "tree".
<code>start</code>	An initial configuration, a good start will significantly reduce the searching time

**Details**

Exact decoding for graphs with a small cutset using cutset conditioning

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.cutset(Small$crf, c(2))
```

---

decode.exact	<i>Decoding method for small graphs</i>
--------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.exact(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact decoding for small graphs with brute-force search

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.exact(Small$crf)
```

---

decode.greedy	<i>Decoding method using greedy algorithm</i>
---------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.greedy(crf, restart = 0, start = apply(crf$node.pot, 1, which.max))
```

**Arguments**

crf	The CRF
restart	Non-negative integer to control how many restart iterations are repeated
start	An initial configuration, a good start will significantly reduce the searching time

**Details**

Approximate decoding with greedy algorithm

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.greedy(Small$crf)
```

---

<code>decode.icm</code>	<i>Decoding method using iterated conditional modes algorithm</i>
-------------------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.icm(crf, restart = 0, start = apply(crf$node.pot, 1, which.max))
```

**Arguments**

<code>crf</code>	The CRF
<code>restart</code>	Non-negative integer to control how many restart iterations are repeated
<code>start</code>	An initial configuration, a good start will significantly reduce the searching time

**Details**

Approximate decoding with the iterated conditional modes algorithm

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.icm(Small$crf)
```

---

decode.ilp	<i>Decoding method using integer linear programming</i>
------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.ilp(crf, lp.rounding = FALSE)
```

**Arguments**

crf	The CRF
lp.rounding	Boolean variable to indicate whether LP rounding is need.

**Details**

Exact decoding with an integer linear programming formulation and approximate using LP relaxation

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.ilp(Small$crf)
```

---

decode.junction	<i>Decoding method for low-treewidth graphs</i>
-----------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.junction(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact decoding for low-treewidth graphs using junction trees

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.junction(Small$crf)
```

---

decode.lbp	<i>Decoding method using loopy belief propagation</i>
------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.lbp(crf, max.iter = 10000, cutoff = 1e-04, verbose = 0)
```

**Arguments**

<code>crf</code>	The CRF
<code>max.iter</code>	The maximum allowed iterations of termination criteria
<code>cutoff</code>	The convergence cutoff of termination criteria
<code>verbose</code>	Non-negative integer to control the tracing information in algorithm

**Details**

Approximate decoding using max-product loopy belief propagation

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.lbp(Small$crf)
```

---

decode.marginal	<i>Decoding method using inference</i>
-----------------	--

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.marginal(crf, infer.method, ...)
```

**Arguments**

crf	The CRF
infer.method	The inference method
...	The parameters for infer.method

**Details**

Approximate decoding using inference (takes an inference method as input)

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.marginal(Small$crf, infer.exact)
```

---

decode.sample	<i>Decoding method using sampling</i>
---------------	---------------------------------------

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.sample(crf, sample.method, ...)
```

**Arguments**

crf	The CRF
sample.method	The sampling method
...	The parameters for sample.method



**Details**

Approximate decoding using sampling (takes a sampling method as input)

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.sample(Small$crf, sample.exact, 10000)
```

---

decode.trbp	<i>Decoding method using tree-reweighted belief propagation</i>
-------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.trbp(crf, max.iter = 10000, cutoff = 1e-04, verbose = 0)
```

**Arguments**

<code>crf</code>	The CRF
<code>max.iter</code>	The maximum allowed iterations of termination criteria
<code>cutoff</code>	The convergence cutoff of termination criteria
<code>verbose</code>	Non-negative integer to control the tracing informtion in algorithm

**Details**

Approximate decoding using max-product tree-reweighted belief propagation

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.trbp(Small$crf)
```

---

decode.tree	<i>Decoding method for tree- and forest-structured graphs</i>
-------------	---

---

**Description**

Computing the most likely configuration for CRF

**Usage**

```
decode.tree(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact decoding for tree- and forest-structured graphs with max-product belief propagation

**Value**

This function will return the most likely configuration, which is a vector of length `crf$n.nodes`.

**Examples**

```
library(CRF)
data(Small)
d <- decode.tree(Small$crf)
```

---

duplicate.crf	<i>Duplicate CRF</i>
---------------	----------------------

---

**Description**

Duplicate an existing CRF

**Usage**

```
duplicate.crf(crf)
```

**Arguments**

crf	The existing CRF
-----	------------------

**Details**

This function will duplicate an existing CRF. Since CRF is implemented as an environment, normal assignment will only copy the pointer instead of the real data. This function will generate a new CRF and really copy all data.

**Value**

The function will return a new CRF with copied data

**See Also**

[make.crf](#)

---

get.logPotential	<i>Calculate the log-potential of CRF</i>
------------------	---

---

**Description**

Calculate the logarithmic potential of a CRF with given configuration

**Usage**

```
get.logPotential(crf, configuration)
```

**Arguments**

crf	The CRF
configuration	The vector of states of nodes

**Details**

The function will calculate the logarithmic potential of a CRF with given configuration, i.e., the assigned states of nodes in the CRF.

**Value**

The function will return the log-potential of CRF with given configuration

**See Also**

[get.potential](#)

---

get.potential	<i>Calculate the potential of CRF</i>
---------------	---------------------------------------

---

**Description**

Calculate the potential of a CRF with given configuration

**Usage**

```
get.potential(crf, configuration)
```

**Arguments**

crf	The CRF
configuration	The vector of states of nodes

**Details**

The function will calculate the potential of a CRF with given configuration, i.e., the assigned states of nodes in the CRF.

**Value**

The function will return the potential of CRF with given configuration

**See Also**

[get.logPotential](#)

---

infer.chain	<i>Inference method for chain-structured graphs</i>
-------------	---

---

**Description**

Computing the partition function and marginal probabilities

**Usage**

```
infer.chain(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact inference for chain-structured graphs with the forward-backward algorithm

**Value**

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor <code>Z</code> .

**Examples**

```
library(CRF)
data(Small)
i <- infer.chain(Small$crf)
```

---

infer.conditional	<i>Conditional inference method</i>
-------------------	-------------------------------------

---

## Description

Computing the partition function and marginal probabilities

## Usage

```
infer.conditional(crf, clamped, infer.method, ...)
```

## Arguments

crf	The CRF
clamped	The vector of fixed values for clamped nodes, 0 for unfixed nodes
infer.method	The inference method to solve the clamped CRF
...	The parameters for infer.method

## Details

Conditional inference (takes another inference method as input)

## Value

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor <code>Z</code> .

## Examples

```
library(CRF)
data(Small)
i <- infer.conditional(Small$crf, c(0,1,0,0), infer.exact)
```

---

infer.cutset	<i>Inference method for graphs with a small cutset</i>
--------------	--

---

## Description

Computing the partition function and marginal probabilities

## Usage

```
infer.cutset(crf, cutset, engine = "default")
```

## Arguments

crf	The CRF
cutset	A vector of nodes in the cutset
engine	The underlying engine for cutset decoding, possible values are "default", "none", "exact", "chain", and "tree".

## Details

Exact inference for graphs with a small cutset using cutset conditioning

## Value

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor $Z$ .

## Examples

```
library(CRF)
data(Small)
i <- infer.cutset(Small$crf, c(2))
```

---

infer.exact	<i>Inference method for small graphs</i>
-------------	--

---

**Description**

Computing the partition function and marginal probabilities

**Usage**

```
infer.exact(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact inference for small graphs with brute-force counting

**Value**

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor $Z$ .

**Examples**

```
library(CRF)
data(Small)
i <- infer.exact(Small$crf)
```

---

infer.junction	<i>Inference method for low-treewidth graphs</i>
----------------	--

---

**Description**

Computing the partition function and marginal probabilities

**Usage**

```
infer.junction(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

Exact decoding for low-treewidth graphs using junction trees

**Value**

This function will return a list with components:

<code>node.bel</code>	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
<code>edge.bel</code>	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
<code>logZ</code>	The logarithmic value of CRF normalization factor <code>Z</code> .

**Examples**

```
library(CRF)
data(Small)
i <- infer.junction(Small$crf)
```

---

`infer.lbp`

*Inference method using loopy belief propagation*

---

**Description**

Computing the partition function and marginal probabilities

**Usage**

```
infer.lbp(crf, max.iter = 10000, cutoff = 1e-04, verbose = 0)
```

**Arguments**

<code>crf</code>	The CRF
<code>max.iter</code>	The maximum allowed iterations of termination criteria
<code>cutoff</code>	The convergence cutoff of termination criteria
<code>verbose</code>	Non-negative integer to control the tracing information in algorithm

**Details**

Approximate inference using sum-product loopy belief propagation

**Value**

This function will return a list with components:

<code>node.bel</code>	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
<code>edge.bel</code>	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
<code>logZ</code>	The logarithmic value of CRF normalization factor <code>Z</code> .



**Examples**

```
library(CRF)
data(Small)
i <- infer.lbp(Small$crf)
```

infer.sample

*Inference method using sampling***Description**

Computing the partition function and marginal probabilities

**Usage**

```
infer.sample(crf, sample.method, ...)
```

**Arguments**

crf	The CRF
sample.method	The sampling method
...	The parameters for sample.method

**Details**

Approximate inference using sampling (takes a sampling method as input)

**Value**

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor <code>Z</code> .

**Examples**

```
library(CRF)
data(Small)
i <- infer.sample(Small$crf, sample.exact, 10000)
```

---

infer.trbp	<i>Inference method using tree-reweighted belief propagation</i>
------------	--

---

## Description

Computing the partition function and marginal probabilities

## Usage

```
infer.trbp(crf, max.iter = 10000, cutoff = 1e-04, verbose = 0)
```

## Arguments

crf	The CRF
max.iter	The maximum allowed iterations of termination criteria
cutoff	The convergence cutoff of termination criteria
verbose	Non-negative integer to control the tracing information in algorithm

## Details

Approximate inference using sum-product tree-reweighted belief propagation

## Value

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor <code>Z</code> .

## Examples

```
library(CRF)
data(Small)
i <- infer.trbp(Small$crf)
```

infer.tree

*Inference method for tree- and forest-structured graphs***Description**

Computing the partition function and marginal probabilities

**Usage**

```
infer.tree(crf)
```

**Arguments**

crf                      The CRF

**Details**

Exact inference for tree- and forest-structured graphs with sum-product belief propagation

**Value**

This function will return a list with components:

node.bel	Node belief. It is a matrix with <code>crf\$n.nodes</code> rows and <code>crf\$max.state</code> columns.
edge.bel	Edge belief. It is a list of matrices. The size of list is <code>crf\$n.edges</code> and the matrix <code>i</code> has <code>crf\$n.states[crf\$edges[i,1]]</code> rows and <code>crf\$n.states[crf\$edges[i,2]]</code> columns.
logZ	The logarithmic value of CRF normalization factor $Z$ .

**Examples**

```
library(CRF)
data(Small)
i <- infer.tree(Small$crf)
```

Loop

*Loop CRF example***Description**

This data set gives a loop CRF example

**Usage**

```
data(Loop)
```

**Format**

A list containing two elements:

- `crf` The CRF
- `answer` A list of 4 elements:
  - `decode` The most likely configuration
  - `node.bel` The node belief
  - `edge.bel` The edge belief
  - `logZ` The logarithmic value of CRF normalization factor  $Z$

---

`make.crf`
*Make CRF*


---

**Description**

Generate CRF from the adjacent matrix

**Usage**

```
make.crf(adj.matrix = NULL, n.states = 2, n.nodes = 2)
```

**Arguments**

<code>adj.matrix</code>	The adjacent matrix of CRF network.
<code>n.states</code>	The state numbers of nodes.
<code>n.nodes</code>	The number of nodes, which is only used to generate linear chain CRF when <code>adj.matrix</code> is NULL.

**Details**

The function will generate an empty CRF from a given adjacent matrix. If the length of `n.states` is less than `n.nodes`, it will be used repeatedly. All node and edge potentials are initialized as 1.

Since the CRF data are often very huge, CRF is implemented as an environment. The assignment of environments will only copy the addresses instead of real data, therefore the variables using normal assignment will refer to the exactly same CRF. For complete duplication of the data, please use [duplicate.crf](#).

**Value**

The function will return a new CRF, which is an environment with components:

<code>n.nodes</code>	The number of nodes.
<code>n.edges</code>	The number of edges.
<code>n.states</code>	The number of states for each node. It is a vector of length <code>n.nodes</code> .
<code>max.state</code>	The maximum number of states. It is equal to <code>max(n.states)</code> .
<code>edges</code>	The node pair of each edge. It is a matrix with 2 columns and <code>n.edges</code> rows. Each row denotes one edge. The node with smaller id is put in the first column.
<code>n.adj</code>	The number of adjacent nodes for each node. It is a vector of length <code>n.nodes</code> .

<code>adj.nodes</code>	The list of adjacent nodes for each node. It is a list of length <code>n.nodes</code> and the <code>i</code> -th element is a vector of length <code>n.adj[i]</code> .
<code>adj.edges</code>	The list of adjacent edges for each node. It is similar to <code>adj.nodes</code> while contains the edge ids instead of node ids.
<code>node.pot</code>	The node potentials. It is a matrix with dimension <code>(n.nodes, max.state)</code> . Each row <code>node.pot[i,]</code> denotes the node potentials of the <code>i</code> -th node.
<code>edge.pot</code>	The edge potentials. It is a list of <code>n.edges</code> matrixes. Each matrix <code>edge.pot[[i]]</code> , with dimension <code>(n.states[edges[i,1]], n.states[edges[i,2]])</code> , denotes the edge potentials of the <code>i</code> -th edge.

**See Also**

[duplicate.crf](#), [clamp.crf](#), [sub.crf](#)

**Examples**

```
library(CRF)

nNodes <- 4
nStates <- 2

adj <- matrix(0, nrow=nNodes, ncol=nNodes)
for (i in 1:(nNodes-1))
{
  adj[i,i+1] <- 1
  adj[i+1,i] <- 1
}

crf <- make.crf(adj, nStates)

crf$node.pot[1,] <- c(1, 3)
crf$node.pot[2,] <- c(9, 1)
crf$node.pot[3,] <- c(1, 3)
crf$node.pot[4,] <- c(9, 1)

for (i in 1:crf$n.edges)
{
  crf$edge.pot[[i]][1,] <- c(2, 1)
  crf$edge.pot[[i]][2,] <- c(1, 2)
}
```

---

make.features

---

*Make CRF features*


---

**Description**

Make the data structure of CRF features

**Usage**

```
make.features(crf, n.nf = 1, n.ef = 1)
```

**Arguments**

crf	The CRF
n.nf	The number of node features
n.ef	The number of edge features

**Details**

This function makes the data structure of features need for modeling and training CRF.

The parameters `n.nf` and `n.ef` specify the number of node and edge features, respectively.

The objects `node.par` and `edge.par` define the corresponding parameters used with each feature. `node.par` is a 3-dimensional arrays, and element `node.par[n, i, f]` is the index of parameter associated with the corresponding node potential `node.pot[n, i]` and node feature `f`. `edge.par` is a list of 3-dimensional arrays, and element `edge.par[[e]][i, j, f]` is the index of parameter associated with the corresponding edge potential `edge.pot[[e]][i, j]` and edge feature `f`. The value 0 is used to indicate the corresponding node or edge potential does not depend on that feature.

For detail of calculation of node and edge potentials from features and parameters, please see [crf.update](#).

**Value**

This function will directly modify the CRF and return the same CRF.

**See Also**

[crf.update](#), [make.par](#), [make.crf](#)

---

make.par

*Make CRF parameters*


---

**Description**

Make the data structure of CRF parameters

**Usage**

```
make.par(crf, n.par = 1)
```

**Arguments**

crf	The CRF
n.par	The number of parameters

**Details**

This function makes the data structure of parameters need for modeling and training CRF. The parameters are stored in `par`, which is a numeric vector of length `n.par`.

**Value**

This function will directly modify the CRF and return the same CRF.

**See Also**

[crf.update](#), [make.features](#), [make.crf](#)

---

mrf.nll

*Calculate MRF negative log-likelihood*

---

**Description**

Calculate the negative log-likelihood of MRF model

**Usage**

```
mrf.nll(par, crf, instances, infer.method = infer.chain, ...)
```

**Arguments**

<code>par</code>	The parameter vector of CRF
<code>crf</code>	The CRF
<code>instances</code>	The training data matrix of MRF model
<code>infer.method</code>	The inference method used to compute the likelihood
<code>...</code>	Other parameters need by the inference method

**Details**

This function calculates the negative log-likelihood of MRF model as well as the gradient. This function is intended to be called by optimization algorithm in training process. Before calling this function, the MRF sufficient statistics must be calculated and stored in object `par.stat` of CRF.

In the training data matrix `instances`, each row is an instance and each column corresponds a node in CRF.

**Value**

This function will return the value of MRF negative log-likelihood.

**See Also**

[mrf.stat](#), [mrf.update](#), [train.mrf](#)

---

mrf.stat	<i>Calculate MRF sufficient statistics</i>
----------	--

---

**Description**

Calculate the sufficient statistics of MRF model

**Usage**

```
mrf.stat(crf, instances)
```

**Arguments**

crf	The CRF
instances	The training data matrix of MRF model

**Details**

This function calculates the sufficient statistics of MRF model. This function must be called before the first calling to [mrf.nll](#). In the training data matrix `instances`, each row is an instance and each column corresponds to a node in CRF.

**Value**

This function will return the value of MRF sufficient statistics.

**See Also**

[mrf.nll](#), [train.mrf](#)

---

mrf.update	<i>Update MRF potentials</i>
------------	------------------------------

---

**Description**

Update node and edge potentials of MRF model

**Usage**

```
mrf.update(crf)
```

**Arguments**

crf	The CRF
-----	---------

**Details**

The function updates `node.pot` and `edge.pot` of MRF model.



**Value**

This function will directly modify the CRF and return the same CRF.

**See Also**

[mrf.nll](#), [train.mrf](#)

---

Rain	<i>Rain data</i>
------	------------------

---

**Description**

This data set gives an example of rain data used to train CRF and MRF models

**Usage**

```
data(Rain)
```

**Format**

A list containing two elements:

- rain A matrix of 28 columns containing raining data (1: rain, 2: sunny). Each row is an instance of 28 days for one month.
- months A vector containing the months of each instance.

**References**

Mark Schmidt. UGM: Matlab code for undirected graphical models. <http://www.di.ens.fr/~mschmidt/Software/UGM.html>

---

sample.chain	<i>Sampling method for chain-structured graphs</i>
--------------	--

---

**Description**

Generating samples from the distribution

**Usage**

```
sample.chain(crf, size)
```

**Arguments**

crf	The CRF
size	The sample size

**Details**

Exact sampling for chain-structured graphs with the forward-filter backward-sample algorithm

**Value**

This function will return a matrix with size rows and `crf$n.nodes` columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.chain(Small$crf, 100)
```

---

sample.conditional	<i>Conditional sampling method</i>
--------------------	------------------------------------

---

**Description**

Generating samples from the distribution

**Usage**

```
sample.conditional(crf, size, clamped, sample.method, ...)
```

**Arguments**

<code>crf</code>	The CRF
<code>size</code>	The sample size
<code>clamped</code>	The vector of fixed values for clamped nodes, 0 for unfixed nodes
<code>sample.method</code>	The sampling method to solve the clamped CRF
<code>...</code>	The parameters for <code>sample.method</code>

**Details**

Conditional sampling (takes another sampling method as input)

**Value**

This function will return a matrix with size rows and `crf$n.nodes` columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.conditional(Small$crf, 100, c(0,1,0,0), sample.exact)
```

---

sample.cutset	<i>Sampling method for graphs with a small cutset</i>
---------------	---

---

**Description**

Generating samples from the distribution

**Usage**

```
sample.cutset(crf, size, cutset, engine = "default")
```

**Arguments**

crf	The CRF
size	The sample size
cutset	A vector of nodes in the cutset
engine	The underlying engine for cutset sampling, possible values are "default", "none", "exact", "chain", and "tree".

**Details**

Exact sampling for graphs with a small cutset using cutset conditioning

**Value**

This function will return a matrix with size rows and crf\$n.nodes columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.cutset(Small$crf, 100, c(2))
```

---

sample.exact	<i>Sampling method for small graphs</i>
--------------	---

---

**Description**

Generating samples from the distribution

**Usage**

```
sample.exact(crf, size)
```

**Arguments**

crf	The CRF
size	The sample size

**Details**

Exact sampling for small graphs with brute-force inverse cumulative distribution

**Value**

This function will return a matrix with size rows and `crf$n.nodes` columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.exact(Small$crf, 100)
```

---

sample.gibbs

*Sampling method using single-site Gibbs sampler*


---

**Description**

Generating samples from the distribution

**Usage**

```
sample.gibbs(crf, size, burn.in = 1000, start = apply(crf$node.pot, 1,
  which.max))
```

**Arguments**

crf	The CRF
size	The sample size
burn.in	The number of samples at the beginning that will be discarded
start	An initial configuration

**Details**

Approximate sampling using a single-site Gibbs sampler

**Value**

This function will return a matrix with size rows and `crf$n.nodes` columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.gibbs(Small$crf, 100)
```

---

sample.junction	<i>Sampling method for low-treewidth graphs</i>
-----------------	---

---

**Description**

Generating samples from the distribution

**Usage**

```
sample.junction(crf, size)
```

**Arguments**

crf	The CRF
size	The sample size

**Details**

Exact sampling for low-treewidth graphs using junction trees

**Value**

This function will return a matrix with size rows and `crf$n.nodes` columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.junction(Small$crf, 100)
```

---

`sample.tree`*Sampling method for tree- and forest-structured graphs*

---

**Description**

Generating samples from the distribution

**Usage**

```
sample.tree(crf, size)
```

**Arguments**

<code>crf</code>	The CRF
<code>size</code>	The sample size

**Details**

Exact sampling for tree- and forest-structured graphs with sum-product belief propagation and backward-sampling

**Value**

This function will return a matrix with `size` rows and `crf$n.nodes` columns, in which each row is a sampled configuration.

**Examples**

```
library(CRF)
data(Small)
s <- sample.tree(Small$crf, 100)
```

---

`Small`*Small CRF example*

---

**Description**

This data set gives a small CRF example

**Usage**

```
data(Small)
```

**Format**

A list containing two elements:

- crf The CRF
- answer A list of 4 elements:
  - decode The most likely configuration
  - node.bel The node belief
  - edge.bel The edge belief
  - logZ The logarithmic value of CRF normalization factor Z

---

sub.crf	<i>Make sub CRF</i>
---------	---------------------

---

**Description**

Generate sub CRF by selecting some nodes

**Usage**

```
sub.crf(crf, subset)
```

**Arguments**

crf	The CRF generated by <a href="#">make.crf</a>
subset	The vector of selected node ids

**Details**

The function will generate a new CRF from a given CRF by selecting some nodes. The vector subset contains the node ids selected to generate the new CRF. Unlike [clamp.crf](#), the potentials of remaining nodes and edges are untouched.

**Value**

The function will return a new CRF with additional components:

original	The original CRF data.
node.id	The vector of the original node ids for nodes in the new CRF.
node.map	The vector of the new node ids for nodes in the original CRF.
edge.id	The vector of the original edge ids for edges in the new CRF.
edge.map	The vector of the new edge ids for edges in the original CRF.

**See Also**

[make.crf](#), [clamp.crf](#)

## Examples

```
library(CRF)
data(Small)
crf <- sub.crf(Small$crf, c(2, 3))
```

---

train.crf

*Train CRF model*


---

## Description

Train the CRF model to estimate the parameters

## Usage

```
train.crf(crf, instances, node.fea = NULL, edge.fea = NULL,
  node.ext = NULL, edge.ext = NULL, nll = crf.nll, trace = 0)
```

## Arguments

crf	The CRF
instances	The training data matrix of CRF model
node.fea	The list of node features
edge.fea	The list of edge features
node.ext	The list of extended information of node features
edge.ext	The list of extended information of edge features
nll	The function to calculate negative log likelihood
trace	Non-negative integer to control the tracing information of the optimization process

## Details

This function train the CRF model.

In the training data matrix `instances`, each row is an instance and each column corresponds a node in CRF. The variables `node.fea`, `edge.fea`, `node.ext`, `edge.ext` are lists of length equal to the number of instances, and their elements are defined as in [crf.update](#) respectively.

## Value

This function will directly modify the CRF and return the same CRF.

## See Also

[crf.update](#), [crf.nll](#), [make.crf](#)



---

`train.mrf`*Train MRF model*

---

**Description**

Train the MRF model to estimate the parameters

**Usage**

```
train.mrf(crf, instances, nll = mrf.nll, trace = 0)
```

**Arguments**

<code>crf</code>	The CRF
<code>instances</code>	The training data matrix of CRF model
<code>nll</code>	The function to calculate negative log likelihood
<code>trace</code>	Non-negative integer to control the tracing information of the optimization process

**Details**

This function trains the Markov Random Fields (MRF) model, which is a simple variant of CRF model.

In the training data matrix `instances`, each row is an instance and each column corresponds a node in CRF.

**Value**

This function will directly modify the CRF and return the same CRF.

**See Also**

[mrf.update](#), [mrf.stat](#), [mrf.nll](#), [make.crf](#)

---

`Tree`*Tree CRF example*

---

**Description**

This data set gives a tree CRF example

**Usage**

```
data(Tree)
```

**Format**

A list containing two elements:

- `crf` The CRF
- `answer` A list of 4 elements:
  - `decode` The most likely configuration
  - `node.bel` The node belief
  - `edge.bel` The edge belief
  - `logZ` The logarithmic value of CRF normalization factor  $Z$

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