

AQLSchemes

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The R package AQLSchemes has functions for retrieving individual sampling plans from the ANSI/ASQ Z1.4 Standard and the ANSI/ASQ Z1.9 Standard. The functions are interactive and query the user for the inspection level, lot size, and AQL. The **AASingle**, **AADouble**, and **AAMultiple** functions recall sampling plans from the ANSI/ASQ Z1.4 Standard, and produce a data frame with columns for the sample size (n), the acceptance number (c) and rejection numbers (r). The **AAZ19** function recalls plans from the ANSI/ASQ Z1.9 Standard and prints the sample size (n), Acceptability constant (k) and the maximum proportion nonconforming (M).

Permission has been granted by ASQ to access the ANSI/ASQ Z1.4 and Z1.9 tables through this package, but they have requested that use be limited (as much as possible) to students enrolled in the Stat 462 course in the Statistics Department at Brigham Young University. The use of the AQLSchemes package along with the Acceptance Sampling package and basic R programming and graphics functions makes it very useful for those seeking to understand the benefits of ANSI/ASQ Z1.4 and Z1.9 Sampling Schemes in relation to custom developed individual sampling plans with specified CRP and PRP.

Below are example calls and interactive dialogs for the **AASingle**, **AADouble**, and **AAMultiple** functions for creating sampling plans for normal inspection level II for a lot of 5000 items with a specified AQL of 1.5 percent.

```
## This example illustrates the use of the AASingle function to retrieve the normal
## sampling plan from the ANSI/ASQ Z1.4 Standard.
>library(AQLSchemes)
>planS<-AASingle('Normal')
What is the Inspection Level?

1:S-1
2:S-2
3:S-3
4: S-4
5:I
6:II
7:III

Selection: 6

What is the Lot Size?
1: 2-8          2: 9-15          3: 16-25          4: 26-50
5: 51-90        6: 91-150        7: 151-280        8: 281-500
9: 501-1200     10: 1201-3200    11: 3201-10,000   12: 10,001-35,000
13: 35,001-150,000 14: 150,001-500,000 15: 500,001 and over

Selection: 11

What is the AQL in percent nonconforming per 100 items?

1: 0.010   2: 0.015   3: 0.025   4: 0.040   5: 0.065   6: 0.10   7: 0.15   8: 0.25
9: 0.40    10: 0.65    11: 1.0    12: 1.5    13: 2.5    14: 4.0    15: 6.5    16: 10
```

```

17: 15      18: 25  19: 40      20: 65      21: 100      22: 150      23: 250      24: 400
25: 650      26: 1000

Selection: 12
> planS
  n c r
1 200 7 8

# This example illustrates the use of the AADouble function to retrieve the normal
# sampling plan from the ANSI/ASQ Z1.4 Standard.
>planD<-AADouble('Normal')

What is the Inspection Level?
1: S-1
2: S-2
3: S-3
4: S-4
5: I
6: II
7: III

Selection: 6

What is the Lot Size?

1: 2-8          2: 9-15          3: 16-25          4: 26-50
5: 51-90        6: 91-150         7: 151-280        8: 281-500
9: 501-1200     10: 1201-3200     11: 3201-10,000   12: 10,001-35,000
13: 35,001- 150,000  14: 150,001- 500,000  15: 500,001 and over

Selection: 11

What is the AQL in percent nonconforming per 100 items?

1: 0.010      2: 0.015      3: 0.025      4: 0.040      5: 0.065      6: 0.10      7: 0.15      8: 0.25
9: 0.40       10: 0.65      11: 1.0       12: 1.5       13: 2.5       14: 4.0       15: 6.5       16: 10
17: 15        18: 25        19: 40        20: 65        21: 100       22: 150       23: 250       24: 400
25: 650       26: 1000

Selection: 12

> planD
      n c r
first 125 3 7
second 125 8 9

# This example illustrates the use of the AAMultiple function to retrieve the normal
# sampling plan from the ANSI/ASQ Z1.4 Standard.
>planM<-AAMultiple('Normal')
What is the Inspection Level?

1: S-1
2: S-2

```

```

3: S-3
4: S-4
5: I
6: II
7: III

Selection: 6

What is the Lot Size?

1: 2-8          2: 9-15          3: 16-25          4: 26-50
5: 51-90        6: 91-150        7: 151-280        8: 281-500
9: 501-1200     10: 1201-3200    11: 3201-10,000   12: 10,001-35,000
13: 35,001-150,000 14: 150,001- 500,000 15: 500,001 and over

Selection: 11

What is the AQL in percent nonconforming per 100 items?

1: 0.010        2: 0.015        3: 0.025        4: 0.040        5: 0.065        6: 0.10        7: 0.15        8: 0.25
9: 0.40         10: 0.65        11: 1.0         12: 1.5         13: 2.5         14: 4.0        15: 6.5        16: 10
17: 15          18: 25          19: 40          20: 65          21: 100         22: 150        23: 250        24: 400
25: 650         26: 1000

Selection: 12
> planM

      n  c  r
first  50  0  4
second 50  1  6
third  50  3  8
fourth 50  5 10
fifth  50  7 11
sixth  50 10 12
seventh 50 13 14

```

The ANSI/ASQ Z1.4 Standard is the American national standard for AQL-based Attribute Acceptance Sampling plans. They are derived from MIL-STD-105E, which is no longer supported by the U.S. Department of Defense. ANSI/ASQ Z1.4 is a scheme of sampling plans that includes normal, tightened, and reduced plans and associated switching rules for single, double, and multiple sampling. The tightened and reduced plans can be recalled using the **AASingle**, **AADouble**, **AAMultiple** functions as shown above by changing the argument 'Normal' to 'Tightened' or 'Reduced' (ie., **AASingle('Tightened')**). The switching rules provide maximum protection for consumer and supplier for a given sample size, and they must be used if the standard is to be properly applied.

The civilian standard for AQL based Variable sampling plans is ANSI/ASQ Z1.9, which is similar to the MIL-STD-414. It is also a scheme of sampling plans including normal, tightened, and reduced sampling. The normal, tightened, and reduced plans can be recalled with the **AAZ19** function. It is possible to switch back and forth between an attribute plan in ANSI/ASQ Z1.4 and a variables plan from ANSI/ASQ Z1.9 and keep essentially the same operating characteristic (for schemes with the same lot size, inspection level, and AQL).

The example below illustrates the **AAZ19** function call and interactive dialog used to recall a variable sampling plan from the ANSI/ASQ Z1.9 Standard for the same situation shown above (i.e., normal inspection level II for a lot of 5000 items with a specified AQL of 1.5 percent). The tightened and reduced plans can be recalled by substituting for the argument 'Normal', in the function call with 'Tightened' or 'Reduced'.

```

> AAZ19('Normal')

What is the Inspection Level?
1: S-3
2: S-4
3: I
4: II
5: III

Selection: 4

What is the Lot Size?

1: 2-8          2: 9-15          3: 16-25          4: 26-50
5: 51-90        6: 91-150         7: 151-280        8: 281-400
9: 401-500      10: 501-1200       11: 1201-3200     12: 3201-10,000
13: 10,001-35,000 14: 35,001-150,000 15: 150,001-500,000 16: 500,001 and over

Selection: 12

What is the AQL in percent nonconforming per 100 items?#
1: 0.10  2: 0.15  3: 0.25  4: 0.40  5: 0.65  6: 1.0  7: 1.5  8: 2.5  9: 4.0
10: 6.5  11: 10

Selection: 7
Sample size n = 75
Acceptability constant k = 1.84097
Maximum proportion non-conforming M = 0.0317

```

ANSI/ASQ Z1.9 matches the OC performance of the attribute plans in ANSI/ASQ Z1.4. The example below illustrates the use of the **OCASN** functions in the **AQLSchemes** package to create OC and ASN curves for the single double and multiple ANSI/ASQ Z1.4 attribute sampling plans (**planS**, **planD**, and **planM**) created in the previous example, and it illustrates the use of the **OCVar** function in the **Acceptance Sampling** package to get the OC curve for the ANSI/ASQ z1.9 variable sampling plan created above. Finally, the base R function **plot** is used to plot the OC curves and ASN curves for all four plans together on the same graphs in Figure 1.

```

# This example illustrates the use of the OCASN functions in the AQLSchemes package
# to create OC and ASN curves for the single double and multiple sampling plans
# (planS, planD, and planM) created in the example above
>library(AQLSchemes)
>par(mfcol=c(1,2))
>Pnc<-seq(0,.08,.001)
>SOCASN<-OCASNZ4S(planS,Pnc)
>OCS<-SOCASN$OC
>ASNS<-SOCASN$ASN
>DOCASN<-OCASNZ4D(planD,Pnc)
>OCD<-DOCASN$OC
>ASND<-DOCASN$ASN
>MOCASN<-OCASNZ4M(planM,Pnc)
>OCM<-MOCASN$OC

```

```

>ASNMC<-MOCASN$ASN
# Use Acceptance Sampling Package to get points on the OC Curves for the variable
# sampling plan
>V<-OCvar(n=75,k=1.84097,s.type="unknown",pd=Pnc)
>OCV<-V@paccept
# Plot all four OC curves on the same graph
>plot(Pnc,OCS,type='l',xlab='Proportion Nonconforming',lty=1)
>lines(Pnc, OCD, type='l', lty=2, col=2)
>lines(Pnc, OCM, type='l', lty=4, col=4)
>legend(.04, .95, c("S", "D", "M"), lty=c(1,2,4), col=c(1,2,4))

#ASNCurves
# Plot the ASN curves for the single, double attributes plan, and variable plan
# on the same graph
>plot(Pnc,ASND,type='l',lty=2,col=2,ylim=c(50,200))
>lines(Pnc,ASNMC,type='l',lty=4,col=4)
>lines(Pnc,rep(200,81),lty=1)
>lines(Pnc,rep(75,81),lty=2,col=3)
>par(mfcol=c(1,2))

```

The graphs in Figure 1 show the near OC equivalence of the ANSI/ASQ Z1.4 single double and multiple plans and the ANSI/ASQ Z1.9 variables plan. The Average Sample Number (ASN) curve for the double and multiple sampling plans (plotted on the right side of the figure above) are uniformly below the constant sample size required by the ANSI/ASQ Z1.4 single sampling plan and, except for a very low proportion nonconforming, they are uniformly above the minimal constant sample size required by the ANSI/ASQ Z1.9 variables sampling plan.

Considering the OC curves, it can be seen that they all pass through the same producer risk point ($AQL=0.013$, $1-\alpha=.975$) and the same consumer risk point ($LTPD=0.064$, $\beta = .054$). This clarifies the point made in the ANSI/ASQ z1.4 document p.6, which says “The curves for single sampling...double sampling and multiple sampling are matched as close as possible”. While the curve for the variable sampling plan has the same AQL protection for the producer, it decreases more rapidly than the attribute plans for proportions nonconforming greater than the AQL. Therefore, it provides slightly more protection to the customer for an intermediate level of lot quality.

Although the four plans have very similar OC curves, the number of samples required decreases as you move from the single \rightarrow double \rightarrow multiple \rightarrow variable sampling plans. The reduced number of samples has to be balanced against the extra administrative effort required for double and multiple sampling plans and the precision required to get numerical measurements for a variable sampling plan. However, the user can switch between the four plans at any time and keep the same producer and consumer protection.

The ANSI/ASQ Z1.4 and Z1.9 standards are recommended for in-house or U.S. domestic trade partners. The ASQ Quality Engineer and Quality Process Analyst Certifications require knowledge of ANSI/ASQ Z1.4 and Z1.9. However, the training materials and certification exams focus on the mechanics of looking up a specific plan in the tables for the published standard based on the lot size, inspection level, and AQL and in the mechanics of manually calculating specific points on the OC Curve or ASN Curve for a particular plan. These exercises do not help those preparing for the exam to understand the overall benefits of the ANSI/ASQ Scheme of sampling plans (with associated switching rules) over custom derived sampling plans that are available in commercial software products such as Minitab® or StatGraphics® or open source software, such as the Acceptance Sampling Package in R.

The R package AQLSchemes automates the process of retrieving particular sampling plans, and the programming features in R and availability of the complimentary AcceptanceSampling package make it easy to compute and graph OC, and ASN, curves for comparing more than one plan on the same graph, as shown in the example above.

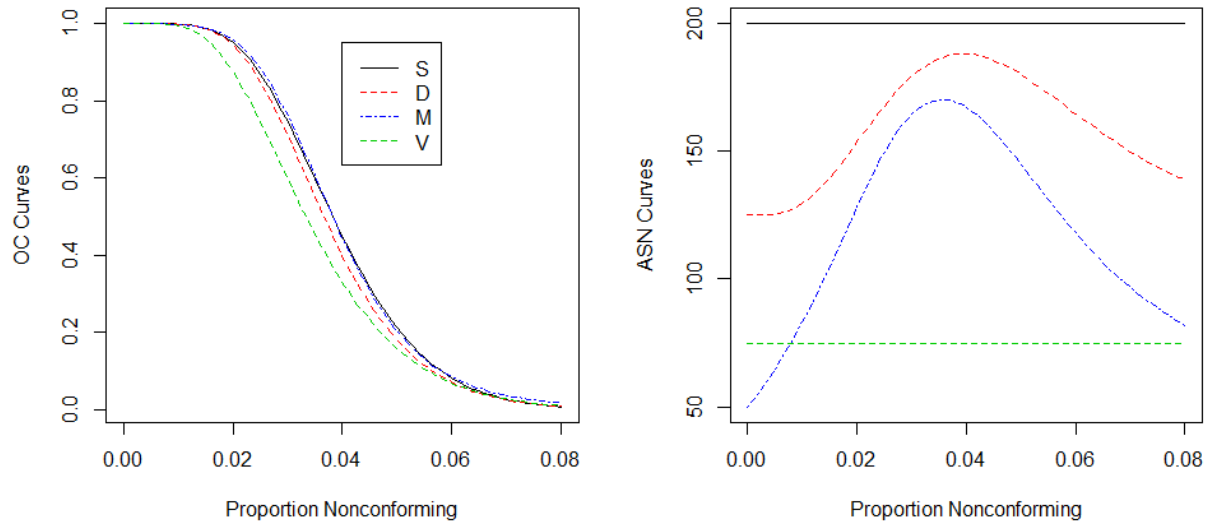


Figure 1: Operating Characteristic and ASN Curves

As another example, the R code below produces the OC curves and constant ASN curves for the ANSI/ASQ Z1.4 Single Sampling plans for Normal, Tightened, and Reduced sampling for inspection level II, Lot Size = 1750, and AQL=1.5 %. The R code then compares these curves in Figure 2. The plans (`planSN` for normal inspection, `planST` for tightened inspection, and `planSR` for reduced inspection) were obtained earlier using the `AASingle` function that was illustrated earlier to obtain `planS`).

```
# Create the OC and ASN Curves
>Pnc<-seq(0,.08,.001)
>SOCASNN<-OCASNZ4S(planSN,Pnc)
>OCSN<-SOCASNN$OC
>ASNSN<-SOCASNN$ASN
>SOCASNT<-OCASNZ4S(planST,Pnc)
>OCST<-SOCASNT$OC
>ASNST<-SOCASNT$ASN
>SOCASNR<-OCASNZ4S(planSR,Pnc)
>OCSR<-SOCASNR$OC
>ASNSR<-SOCASNR$ASN
# plot the OC and ASN curves
>par(mfcol=c(1,2))
>plot(Pnc,OCSN,type='l',xlab='Proportion Nonconforming',lty=1,ylab="OC Curves")
>lines(Pnc,OCST,type='l',lty=2,col=2)
>lines(Pnc,OCSR,type='l',lty=4,col=4)
>legend(.04,.95,c("Normal","Tightened","Reduced"),lty=c(1,2,4),col=c(1,2,4))
#ASNCurves
# Plot the ASN curves for the single, double attributes plan, and variable plan
# on the same graph
>plot(Pnc,ASNSN,type='l',lty=1,ylim=c(40,160),xlab='Proportion Nonconforming',
>ylab="ASN Curves")
>lines(Pnc,ASNST,type='l',lty=2,col=2)
>lines(Pnc,ASNSR,type='l',lty=4,col=4)
>par(mfcol=c(1,2))
```

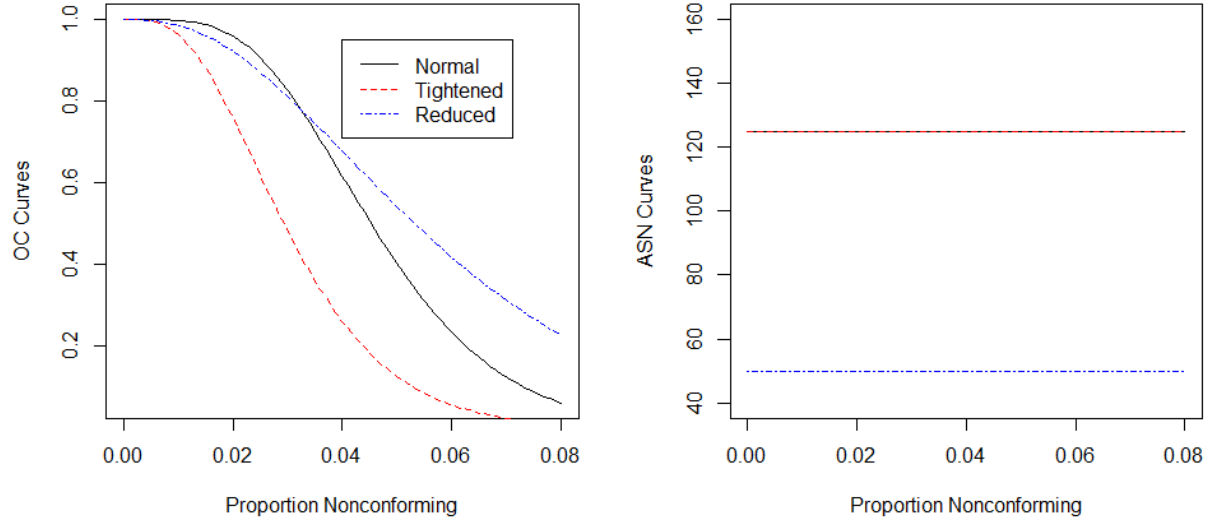


Figure 2: Operating Characteristic and ASN Curves

The OC curves on the left in Figure 2 show that the normal sampling plan offers more protection for the producer since the OC=probability of accepting a lot is higher for good quality lots with a low proportion nonconforming. On the other hand, the tightened sampling plan offers more protection for the customer since the OC=probability of accepting is lower for poor quality lots with a higher proportion nonconforming.

Stephens and Larson(1967) investigated the properties of MIL-STD-105E, which is also relevant to ANSI/ASQ Standard Z1.4. When ignoring the reduced plan (the use of which requires authority approval) and considering only the normal-tightend system, the sampling scheme can be viewed as a two-state Markov chain with the two states being normal inspection and tightened inspection. They showed the OC or probability of accepting by this scheme is given by the following equation:

$$Pr(accept) = \frac{aP_N + bP_T}{a + b} \quad (1)$$

where P_N is the probability of accepting under normal inspection, P_T is the probability of accepting under tightened inspection, and

$$a = \frac{2 - P_N^4}{(1 - P_N)(1 - P_N^4)} \quad (2)$$

$$b = \frac{1 - P_T^5}{(1 - P_T)P_T^5}.$$

In addition, $a/(a + b)$ is the steady state probability of being in the normal sampling state, and $b/(a + b)$ is

the steady state probability of being in the tightened sampling state. Therefore, the average sample number (ASN) for the normal-tightened scheme is given by the equation

$$ASN = \frac{an_N + bn_T}{a + b}. \quad (3)$$

To illustrate the benefit of using the ANSI/ASQ Standard Z1.4 sampling scheme for attribute sampling, consider the case of using a single sampling scheme for a continuing stream of lots with lot sizes between 151 and 280. If the required AQL is 1.0 (or 1%), then the normal inspection plan is $n = 50$, with $c = 1$, and the tightened inspection plan is $n = 80$, with $c = 1$.

The R code below computes the OC curves and the ASN curves for the scheme, the normal sampling plan, and the tightened plan, then plots them on the same graph in Figure 3.

```
> library(AQLSchemes)
> planSN<-AASingle('Normal')
What is the Inspection Level?

1: S-1
2:   S-2
3:   S-3
4: S-4
5: I
6: II
7: III

Selection: 6

What is the Lot Size?

1: 2-8      2: 9-15      3: 16-25
4: 26-50    5: 51-90      6: 91-150
7: 151-280  8: 281-500    9: 501-1200
10: 1201-3200 11: 3201-10,000 12: 10,001-35,000
13: 35,001-150,000 14: 150,001-500,000 15: 500,001 and over

Selection: 7

What is the AQL in percent nonconforming per 100 items?

1: 0.010    2: 0.015    3: 0.025    4: 0.040    5: 0.065    6: 0.10    7: 0.15
8: 0.25     9: 0.40    10: 0.65    11: 1.0     12: 1.5     13: 2.5     14: 4.0
15: 6.5     16: 10     17: 15     18: 25     19: 40     20: 65     21: 100
22: 150     23: 250    24: 400     25: 650     26: 1000

Selection: 11
> planSN
  n c r
1 50 1 2
> planST<-AASingle('Tightened')

What is the Inspection Level?

1: S-1
```


2: S-2
 3: S-3
 4: S-4
 5: I
 6: II
 7: III

Selection: 6

What is the Lot Size?

1: 2-8	2: 9-15	3: 16-25
4: 26-50	5: 51-90	6: 91-150
7: 151-280	8: 281-500	9: 501-1200
10: 1201-3200	11: 3201-10,000	12: 10,001-35,000
13: 35,001-150,000	14: 150,001-500,000	15: 500,001 and over

Selection: 7

What is the AQL in percent nonconforming per 100 items?

1: 0.010	2: 0.015	3: 0.025	4: 0.040	5: 0.065	6: 0.10	7: 0.15
8: 0.25	9: 0.40	10: 0.65	11: 1.0	12: 1.5	13: 2.5	14: 4.0
15: 6.5	16: 10	17: 15	18: 25	19: 40	20: 65	21: 100
22: 150	23: 250	24: 400	25: 650	26: 1000		

Selection: 11

```
> planST
  n c r
1 80 1 2
> Pnc<-seq(0,.08,.001)
> OCASN<-OCASN4S(planSN,Pnc)
> PN<-OCASN$OC
> OCAST<-OCASN4S(planST,Pnc)
> PT<-OCAST$OC
> a<-(2-PN^4)/((1-PT)*(1-PN^4))
> b<-(1-PT^5)/((1-PT)*PT^5)
> PS<-(a*PN+b*PT)/(a+b)
> ASNS<-(a*50+b*80)/(a+b)
> ## Plot all four OC curves on the same graph
> par(mfcol=c(1,2))
> plot(Pnc,PS,type='l',xlab='Proportion Nonconforming',lty=1,ylab="OC Curves")>
lines(Pnc,PN,type='l',lty=2,col=2)
> lines(Pnc,PT,type='l',lty=4,col=4)
> legend(.04,.95,c("Scheme","Normal","Tightened"),lty=c(1,2,4),col=c(1,2,4))
# ASNCurves
# Plot the ASN curves for the single, double attributes plan, and variable
# plan on the same graph
> plot(Pnc,ASNS,type='l',lty=1,ylim=c(40,100),xlab='Proportion Nonconforming',
  ylab='ASN Curves')
> lines(Pnc,rep(50,81),type='l',lty=2,col=2)
> lines(Pnc,rep(80,81),lty=4,col=4)
```

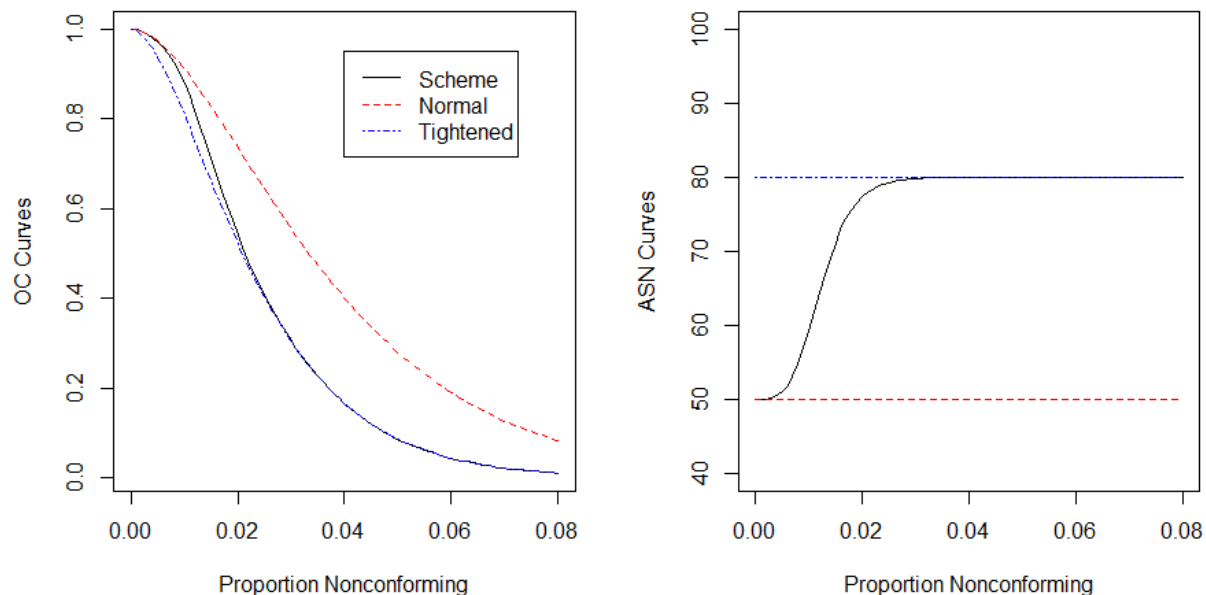


Figure 3: Operating Characteristic and ASN Curves

It can be seen from the OC curves in Figure 3 that the scheme offers higher protection for the producer (like the normal plan) by having a high OC=probability of acceptance for good quality lots with a low proportion of nonconforming items. At the same time, the OC curve for the scheme drops very rapidly and offers higher protection for the customer by having a low OC=probability of acceptance for low quality lots with a high proportion of nonconforming items. The producer risk point for the scheme is $PRP=(.007, .94555)$ and the customer risk point is $CRC=(.047, .1051135)$

The ASN curve on the right side of Figure 3 shows that the average sample number for the scheme is equal or close the the sample size for the normal sampling plan ($n=50$) when the lot quality is good, but increases to the sample size for the tightened plan when the lot quality degrades (i.e., proportion nonconforming increases).

As final example of the benefit of the ANSI/ASQ scheme in comparison to a custom sampling plan, consider comparing the OC curves of the ANSI/ASQ sampling scheme to the OC curve of a custom single sampling plan that passes through the same producer risk point PRP and customer risk point CRP. The R code segment shown below illustrates the use of the **find.plan** function in the Acceptance Sampling package to find a custom single sampling plan whose OC curve passes through the same PRP and CRP as the OC curve for the scheme shown in Figure 3. Next, the OC curve for the scheme (PS), and the OC curve for custom plan (OCCS) are plotted on the same graph in Figure 4. OCCS was found with the R function **pbinom**, in the code above, and PS was found in the previous R code example,

It can be seen in Figure 4 that the OC curves for the scheme and the custom single sampling plan are very similar and pass through the same PRP and CRP. The sample size for the scheme is either $n=50$, or $n=80$ depending on whether the switching rules call for normal or tightened sampling. On the other hand, the sample size for the custom plan is $n=111$, which is a 38.75% to 122% increase over that required by the scheme.

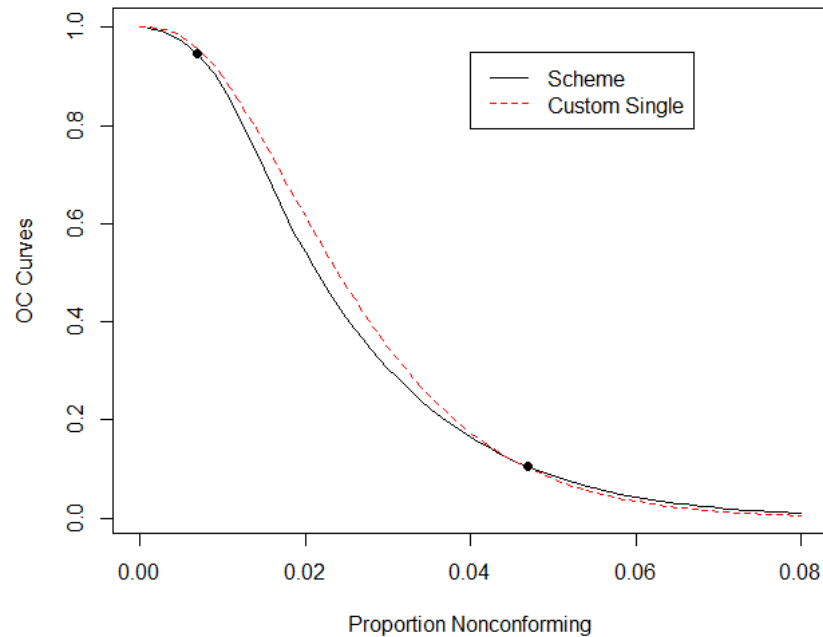


Figure 4: Illustrates Common PRP and CRP

```
>library(AcceptanceSampling)
>find.plan(PRP=c(.007,.94554689),CRP=c(.047,.10511354))
  $n
[1] 111

  $c
[1] 2

  $r
[1] 3

>OCCS<-pbinom(2,size=111,prob=Pnc,lower.tail=TRUE)
>Rp<-c(.007,.047)
>Pa<-c(.945547,.1051135)
# Plot both OC curves on the same graph
>par(mfcol=c(1,1))
>plot(Pnc,PS,type='l',xlab='Proportion Nonconforming',lty=1,ylab="OC Curves")
>lines(Pnc,OCCS,type='l',lty=2,col=2)
>lines(Rp,Pa,type='p',col=1,bg=1, pch = 21:21.)
> legend(.04,.95,c("Scheme","Custom Single"),lty=c(1,2),col=c(1,2))
```

The ANSI/ASQ Z1.4 switching rules also apply to double and multiple sampling. Therefore, the double and multiple sampling schemes will also have a lower average sample size (ASN) than a comparable custom double or multiple sampling plan whose OC curve passes through the same PRP and CRP.

The Variables sampling plans in ANSI/ASQ Z1.9 also includes switching rules that are somewhat more complicated than the rules for ANSI/ASQ Z1.4. These switching rules must be followed in order to properly apply the standard. Following the rules will again be economically effective in reducing the sample size and increasing the protection over what could be achieved with custom designed single plans alone (Schilling and Neubauer(2017)).

References

- Stephens K. and Larson K. (1967) An evaluation of the MIL-STD-105D system of sampling plans, *Industrial Quality Control* 23(7).
- Schilling E. G. and Neubauer D.V. (2017) *Acceptance Sampling and Quality Control, 3rd Edition.*, CRC Press, Boca Raton, Fl, 33487-2742, p.313.