## Supporting Information:

Calibrating indices of avian density from non-standardized survey data: making the most of a messy situation

Péter Sólymos Steven M. Matsuoka Erin M. Bayne solymos@ualberta.ca

Subhash R. Lele Patricia Fontaine Steven G. Cumming Diana Stralberg Fiona K. A. Schmiegelow Samantha J. Song

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# Introduction

This document provides Supporting Information for the manuscript entitled "Calibrating indices of avian density from non-standardized survey data: making the most of a messy situation" by Sólymos et al. submitted to the journal *Methods in Ecology and Evolution* (ID MEE-13-02-103.R1).

The purpose of this document is to describe:

- 1. how to estimate QPAD model parameters;
- 2. how to retrieve the QPAD model parameter estimates reported in the paper;
- 3. and how to use these estimates in statistical inference and prediction.

### 1.1 Software requirements

- R for most of the calculations ((R Core Team, 2012), downloadable from http://www.r-project.org/);
- JAGS for prediction interval calculations using Markov chain Monte Carlo (MCMC) (Plummer, 2012, downloadable from http://mcmc-jags.sourceforge.net/)
- R extension packages: **detect** (Sólymos et al., 2013), **dcmle** (Sólymos, 2010) (can be installed from R console using the install.packages function).

#### 1.2 Files

- BAM\_QPAD\_coefs\_20130226.R: source-able text file with estimated model parameters (available from http://dcr.r-forge.r-project.org/qpad/BAM\_QPAD\_coefs\_20130226.R);
- BAM\_QPAD\_functions\_20130226.R: source-able text file with R functions for retrieving the information from the binary R data file (available from http://dcr.r-forge.r-project.org/qpad/BAM\_QPAD\_functions\_20130226.R).

These files can be sourced by using the load\_BAM\_QPAD() function of the **detect** R package. This function allows the user to select versions of the estimates interactively, this way updates to these files are available upon request. Using load\_BAM\_QPAD(version=1) loads the version used in this document.

# Conditional multinomial maximum likelihood estimation

The estimation procedure described in the Appendix is implemented in the cmulti function of the detect R extension package. Input data specifications are described in the help page of the function (type ?cmulti into R console).

```
library(detect)

Loading required package: Formula

Loading required package: stats4

detect 0.3-0 2013-02-26
```

## 2.1 Input data

Let us use 100 survey locations (n), and generate random values for the covariate x:

```
n <- 100
x <- rnorm(n)
X <- cbind(1, x)</pre>
```

The function simfun1 is used to simulate count data (value for tau (Effective Detection Radius; EDR) is in units of 100 m:

```
P <- CP - cbind(0, CP[, -k, drop = FALSE])
Psum <- rowSums(P, na.rm = TRUE)
PPsum <- P/Psum
Pok <- !is.na(PPsum)
N <- rpois(n, 10)
Y <- matrix(NA, ncol(PPsum), nrow(PPsum))
Ypre <- sapply(1:n, function(i) rmultinom(1, N, PPsum[i, Pok[i, ]]))
Y[t(Pok)] <- unlist(Ypre)
Y <- t(Y)
list(Y = Y, D = D)
}</pre>
```

Now let us simulate counts under the removal model using constant singing rate phi. The count matrix Y contains the number of unique individuals first observed in time intervals defined in the design matrix D which contains the enpoints of the corresponding time intervals in minutes. Note that patterns in NA values must match between the two matrices:

```
vv \leftarrow simfun1(n = n, phi = exp(-1.5))
head(vv$Y)
     [,1] [,2] [,3]
[1,]
        6
              4
                    0
[2,]
         4
               6
                   NA
[3,]
         7
              3
                   NA
[4,]
                   1
         8
              1
[5,]
                  NA
[6,]
         7
              2
                   1
head(vv$D)
     [,1] [,2] [,3]
[1,]
         3
              5
[2,]
         3
              5
                   NA
[3,]
         3
              5
                   NA
[4,]
         3
                   10
              5
[5,]
         5
             10
                   NA
[6,]
         3
              5
                   10
```

## 2.2 Removal sampling

Estimation is done using the cmulti function. The left hand side of the formula reads as count | design where count is a matrix with cell counts, design is the matrix describing the interval endpoints for the cells. The right hand side is 1 because we use a constant model, type="rem" stands for removal sampling:

When covariate x affects singing rate, the estimation is as follows:

#### 2.3 Distance sampling

Simulation and estimation for the distance sampling model with half-normal detection function is similar. We use constant tau parameter (EDR, 100 m units, useInf for unlimited distance):

```
vv \leftarrow simfun1(n = n, tau = exp(-0.2), type = "dis")
head(vv$Y)
    [,1] [,2] [,3]
[1,]
      3 4 3
           4
[2,]
       4
     3 5 2
[3,]
         9 NA
[4,]
     1
     8
         2 NA
[5,]
[6,]
           3 2
head(vv$D)
    [,1] [,2] [,3]
[1,] 0.5 1 Inf
[2,] 0.5
         1 Inf
         1 Inf
[3,] 0.5
[4,] 0.5
         1 NA
[5,] 1.0 Inf NA
[6,] 0.5
         1 Inf
m3 <- cmulti(vv$Y | vv$D ~ 1, type = "dis")</pre>
coef(m3)
log.tau_(Intercept)
          -0.2067
```

Effect of covariate x is estimated as:

#### 2.4 Convenience methods

Several methods are defined for the fitted model objects to facilitate statistical inference:

```
summary(m2)
Call:
cmulti(formula = vv$Y | vv$D ~ x, type = "rem")
Removal Sampling (homogeneous singing rate)
Conditional Maximum Likelihood estimates
Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
log.phi_(Intercept) -1.699 0.129 -13.21 < 2e-16
log.phi_x
                    -0.772
                               0.128 -6.04 1.6e-09
log.phi_(Intercept) ***
log.phi_x
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log-likelihood: -185
BIC = 379
summary(m4)
Call:
cmulti(formula = vv$Y | vv$D ~ x, type = "dis")
Distance Sampling (half-normal, circular area)
Conditional Maximum Likelihood estimates
Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
0.0315 -6.77 1.3e-11
log.tau_x
                   -0.2134
log.tau_(Intercept) ***
log.tau_x
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Log-likelihood: -176
BIC = 362
coef(m4)
log.tau_(Intercept)
                           log.tau_x
           -0.4656
                             -0.2134
vcov(m4)
```

```
log.tau_(Intercept) log.tau_x
log.tau_(Intercept) 0.0007376 -0.0001074
log.tau_x -0.0001074 0.0009932

AIC(m4)

[1] 356.4

confint(m4)

2.5 % 97.5 %
log.tau_(Intercept) -0.5188 -0.4123
log.tau_x -0.2752 -0.1516

logLik(m4)

'log Lik.' -176.2 (df=2)
```

#### 2.5 Joint estimation for removal and distance sampling

The following function simulates counts for the joint estimation of singing rate and distance parameters:

```
simfun12 <- function(n = 10, phi = 0.1, c = 1, tau = 0.8, type = "rem") {
    Flat <- function(x, DIM, dur = TRUE) {</pre>
        x <- array(x, DIM)
        if (!dur) {
             x \leftarrow aperm(x, c(1, 3, 2))
        dim(x) \leftarrow c(DIM[1], DIM[2] * DIM[3])
    }
    Dparts1 <- matrix(c(5, 10, NA, 3, 5, 10, 3, 5, NA), 3, 3,
        byrow = TRUE)
    D1 <- Dparts1[sample.int(3, n, replace = TRUE), ]
    CP1 <- 1 - c * exp(-D1 * phi)
    Dparts2 \leftarrow matrix(c(0.5, 1, NA, 0.5, 1, Inf, 1, Inf, NA),
         3, 3, byrow = TRUE)
    D2 <- Dparts2[sample.int(3, n, replace = TRUE), ]</pre>
    CP2 \leftarrow 1 - \exp(-(D2/tau)^2)
    k1 \leftarrow ncol(D1)
    k2 \leftarrow ncol(D2)
    DIM \leftarrow c(n, k1, k2)
    P1 \leftarrow CP1 - cbind(0, CP1[, -k1, drop = FALSE])
    P2 \leftarrow CP2 - cbind(0, CP2[, -k2, drop = FALSE])
    Psum1 <- rowSums(P1, na.rm = TRUE)
    Psum2 <- rowSums(P2, na.rm = TRUE)
    Pflat <- Flat(P1, DIM, dur = TRUE) * Flat(P2, DIM, dur = FALSE)
    PsumFlat <- Psum1 * Psum2
    PPsumFlat <- Pflat/PsumFlat
```

```
PokFlat <- !is.na(PPsumFlat)</pre>
N <- rpois(n, 10)
Yflat <- matrix(NA, ncol(PPsumFlat), nrow(PPsumFlat))</pre>
YpreFlat <- sapply(1:n, function(i) rmultinom(1, N, PPsumFlat[i,</pre>
    PokFlat[i, ]]))
Yflat[t(PokFlat)] <- unlist(YpreFlat)</pre>
Yflat <- t(Yflat)</pre>
Y <- array(Yflat, DIM)
k1 <- dim(Y)[2]
k2 \leftarrow dim(Y)[3]
Y1 <- t(sapply(1:n, function(i) {
    count <- rowSums(Y[i, , ], na.rm = TRUE)</pre>
    nas <- rowSums(is.na(Y[i, , ]))</pre>
    count[nas == k2] <- NA</pre>
    count
}))
Y2 <- t(sapply(1:n, function(i) {
    count <- colSums(Y[i, , ], na.rm = TRUE)</pre>
    nas <- colSums(is.na(Y[i, , ]))</pre>
    count[nas == k2] <- NA</pre>
    count
}))
list(Y = Y, D1 = D1, D2 = D2, Y1 = Y1, Y2 = Y2)
```

Joint and independent estimation of constant singing rate and EDR:

```
vv <- simfun12(n = n, phi = exp(-1.5), tau = exp(-0.2))
res <- cmulti2.fit(vv$Y, vv$D1, vv$D2)
res1 <- cmulti.fit(vv$Y1, vv$D1, NULL, "rem")
res2 <- cmulti.fit(vv$Y2, vv$D2, NULL, "dis")</pre>
```

Jointly and independently estimated points estimates and standard errors are identical, the two models are orthogonal (correlation is 0):

```
round(cbind(coef.joint = res$coef, coef.indep = c(res1$coef,
   res2$coef)), 4)
    coef.joint coef.indep
[1,]
      -1.3666
                -1.3666
[2,]
       -0.2041
                  -0.2041
round(cbind(SE.joint = sqrt(diag(res$vcov)), SE.indep = c(sqrt(diag(res1$vcov)),
   sqrt(diag(res2$vcov)))), 4)
    SE.joint SE.indep
[1,] 0.0664
              0.0664
[2,] 0.0216 0.0216
ifelse(cov2cor(res$vcov) < 10^-10, 0, cov2cor(res$vcov))</pre>
    [,1] [,2]
[1,] 1 0
[2,] 0 1
```

Joint and independent estimation of covariate specific singing rate and EDR:

Jointly and independently estimated points estimates and standard errors are identical, the two models are orthogonal (correlation is 0):

```
round(cbind(coef.joint = res$coef, coef.indep = c(res1$coef,
   res2$coef)), 4)
    coef.joint coef.indep
[1,]
       -2.1104
                -2.1102
[2,]
       -1.0393
                  -1.0391
[3,] -0.5363
                -0.5363
[4,] -0.2164
                  -0.2163
round(cbind(SE.joint = sqrt(diag(res$vcov)), SE.indep = c(sqrt(diag(res1$vcov)),
    sqrt(diag(res2$vcov)))), 4)
    SE.joint SE.indep
[1,] 0.1688 0.1688
[2,] 0.1481
               0.1481
[3,] 0.0250 0.0250
[4,] 0.0270 0.0270
round(ifelse(cov2cor(res$vcov) < 10^-10, 0, cov2cor(res$vcov)),</pre>
   4)
       [,1]
             [,2] [,3] [,4]
[1,] 1.0000 0.8052
[2,] 0.8052 1.0000
                     0
                          0
[3,] 0.0000 0.0000
                     1
                          0
[4,] 0.0000 0.0000
```

# Retrieving QPAD estimates

First we need to load necessary code and data after opening R (commands can be copy-pasted from the document, # marks comments):

```
library(detect) # load detect package
# source estimates and functions
load_BAM_QPAD(version = 1)

BAM QPAD parameter estimates loaded, version 20130226
BAM QPAD access functions loaded, version 20130226
```

Print out the list of species acronyms (printing out a table linking acronyms to common and scientific names can be done by the getBAMspeciestable function):

```
getBAMspecieslist()

[1] "ALFL" "AMCR" "AMGO" "AMRE" "AMRO" "ATSP" "BAWW" "BBWA"
[9] "BCCH" "BHCO" "BHVI" "BLBW" "BLJA" "BLPW" "BOCH" "BRCR"
[17] "BTBW" "BTNW" "CAWA" "CCSP" "CEDW" "CHSP" "CMWA" "CONW"
[25] "CORA" "COYE" "CSWA" "DEJU" "EVGR" "FOSP" "GCKI" "GRAJ"
[33] "HAFL" "HETH" "HOWR" "LCSP" "LEFL" "LISP" "MAWA" "MOWA"
[41] "NAWA" "NOPA" "NOWA" "OCWA" "OSFL" "OVEN" "PAWA" "PHVI"
[49] "PISI" "PUFI" "RBGR" "RBNU" "RCKI" "REVI" "RUBL" "RWBL"
[57] "SAVS" "SOSP" "SWSP" "SWTH" "TEWA" "TRES" "VATH" "VEER"
[65] "WAVI" "WCSP" "WETA" "WEWP" "WIWA" "WIWR" "WTSP" "WWCR"
```

Print out the list of models used for estimating singing rates (sra) and effective detection radii (edr):

```
getBAMmodellist()

$sra

0

"INTERCEPT"

1

"INTERCEPT + JDAY"

2

"INTERCEPT + TSSR"
```

```
"INTERCEPT + JDAY + JDAY2"

4

"INTERCEPT + TSSR + TSSR2"

5

"INTERCEPT + JDAY + TSSR"

6

"INTERCEPT + JDAY + JDAY2 + TSSR"

7

"INTERCEPT + JDAY + TSSR + TSSR2"

8

"INTERCEPT + JDAY + JDAY2 + TSSR + TSSR2"

$edr

0

"INTERCEPT"

1

"INTERCEPT + TREE"

2

"INTERCEPT + LCC2 + LCC3 + LCC4 + LCC5"
```

Get version info:

```
getBAMversion()
[1] "2"
```

The species acronym OVEN stands for Ovenbird (Seiurus aurocapilla). This gives the estimated parameters for the Ovenbird from the models best supported by BIC:

```
summaryBAMspecies("OVEN")
BAMcorrection object for species OVEN
model.sra = 1
model.edr = 2
              Estimate Std. Error
sra_INTERCEPT 0.9005
                             0.20
               -3.8318
                             0.45
sra_JDAY
edr_INTERCEPT -0.1472
                             0.01
edr_LCC2
              -0.0777
                             0.01
edr_LCC3
               0.0147
                             0.02
                             0.01
edr_LCC4
               -0.0329
edr_LCC5
               -0.0362
                             0.02
```

The best supported model is returned by:

```
bestmodelBAMspecies("OVEN", type = "BIC")
$sra
[1] "1"
$edr
[1] "2"
```

The type argument can take values "AIC", "BIC" or "multi" (the latter returns model IDs randomly based on model weights).

It is also possible to print out other model combinations (model IDs can be looked up from the getBAMmodellist function):

```
summaryBAMspecies("OVEN", model.sra = 8, model.edr = 1)
BAMcorrection object for species OVEN
model.sra = 8
model.edr = 1
              Estimate Std. Error
sra_INTERCEPT 0.5055
                             2.85
               -1.9690
                             12.69
sra_JDAY
sra_JDAY2
               -2.0578
                            14.07
sra_TSSR
               -0.6820
                             0.63
sra_TSSR2
                             3.14
                3.1629
edr_INTERCEPT
               -0.1694
                              0.02
edr_TREE
               -0.0152
                              0.03
```

To get all possible models compared, use this:

```
selectmodelBAMspecies("OVEN")
$sra
 model logLik df nobs
                        AIC
                              BIC
                                    dAIC
                                           dBIC
                                                 weights
     0 -5178 1 6789 10357 10364 72.467 65.644 8.697e-17
     1 -5141 2 6789 10285 10299 0.000 0.000 4.736e-01
1
2
        -5178 2 6789 10359 10373 74.378 74.378 3.346e-17
3
     3 -5141 3 6789 10287 10308 2.210 9.033 1.569e-01
     4 -5177 3 6789 10361 10381 75.904 82.727 1.560e-17
5
        -5140 3 6789 10287 10307 1.819 8.642 1.907e-01
6
     6 -5140 4 6789 10289 10316 3.895 17.541 6.755e-02
7
     7 -5140 4 6789 10288 10316 3.443 17.089 8.469e-02
8
     8 -5140 5 6789 10291 10325 5.762 26.231 2.656e-02
$edr
 model logLik df nobs
                         AIC
                               BIC dAIC dBIC
                                                weights
     0 -21791 1 21742 43583 43591 57.12 25.18 3.941e-13
1
     1 -21791 2 21742 43585 43601 58.92 34.96 1.603e-13
     2 -21758 5 21742 43526 43566 0.00 0.00 1.000e+00
```

The column weights indicates model weights used by bestmodelBAMspecies with argument type="multi".

# Example data analysis

#### 4.1 The Ovenbird data set

We used a data set of Ovenbirds analyzed by Lele et al. (2011) and by Sólymos et al. (2012), available from the **detect** R extension package (data set name: oven). The Ovenbird study was conducted in Saskatchewan, Canada. Point counts were sampled according to the standards of the North American Breeding Bird Survey (3 minutes unlimited distance counts).

First, we calculate necessary covariates for the offsets. JDAY is Julian day in the unit range (divided by 365), TSSR is calculated approximately in hours and divided by the possible max (24):

```
oven$JDAY <- oven$julian/365
oven$TSSR <- ((oven$timeday/8) - 0.75)/24
oven$xlat <- as.numeric(scale(oven$lat))  # latitude is standardized
oven$xlong <- as.numeric(scale(oven$long))  # longitude is standardized</pre>
```

Introduce variables for protocol effects, duration (3 min) and point count radius (note that this has to be in 100 metres for density per ha, use Inf for unlimited distance):

```
oven$dur <- 3
oven$dist <- Inf</pre>
```

The covariates for distance sampling are derived from the proportion of forest and proportion of deciduous forest:

```
pf <- oven$pforest
pd <- oven$pdecid
pc <- pf - pd
oven$LCC <- factor(5, levels = 1:5)  # 5=OH open habitat
oven$LCC[pf > 0.25 & pc > pd] <- "3"  # 3=SC dense conifer
oven$LCC[pf > 0.25 & pc <= pd] <- "4"  # 4=SD sparse deciduous
oven$LCC[pf > 0.6 & pc > pd] <- "1"  # 1=DC dense conifer
oven$LCC[pf > 0.6 & pc <= pd] <- "2"  # 2=DC dense deciduous
table(oven$LCC)</pre>
```

#### 4.1.1 QPAD offsets

Here is how one can calculate the offsets based on the estimates without covariate effects:

```
bc0 <- with(oven, globalBAMcorrections("OVEN", t = dur, r = dist))</pre>
summary(bc0)
      Α
                                   q
Min.
      :2.2
             Min. :0.73
                            Min.
                                  :1
             1st Qu.:0.73
1st Qu.:2.2
                            1st Qu.:1
Median :2.2
              Median:0.73
                             Median:1
Mean
      :2.2
              Mean
                     :0.73
                             Mean
              3rd Qu.:0.73
3rd Qu.:2.2
                             3rd Qu.:1
Max. :2.2 Max. :0.73
                             Max. :1
```

The offsets based on possible covariate effects can be calculated as:

```
bm <- bestmodelBAMspecies("OVEN", type = "BIC")</pre>
bc <- with(oven, localBAMcorrections("OVEN", t = dur, r = dist,
   jday = JDAY, tssr = TSSR, tree = pforest, lcc = LCC, model.sra = bm$sra,
   model.edr = bm$edr))
summary(bc)
      Α
                     p
                                    q
Min. :2.00 Min. :0.649
                              Min. :1
1st Qu.:2.00 1st Qu.:0.672
                              1st Qu.:1
Median :2.18
               Median :0.710
                              Median:1
      :2.15
Mean
               Mean
                    :0.705
                              Mean :1
3rd Qu.:2.19
               3rd Qu.:0.733
                              3rd Qu.:1
Max. :2.41
               Max. :0.766
                              Max. :1
```

### 4.2 Inference: point estimation

#### 4.2.1 Poisson GLM

Here is the general way how one can specify the offsets and estimate density, for example here using Poisson generalized linear model (GLM):

Such models are suitable for calculating point predictions for example for mapping density, but do not take into account the uncertainty associated with the singing rate and EDR estimates underlying the offsets:

#### 4.2.2 Negative Binomial GLM

The Negative Binomial model is useful in case of overdispersion due to e.g. missing covariate. The Negative Binomial GLM also uses the log link so the usual offset works. Care must be taken because the offset argument of the glm.nb function in the MASS library (Venables and Ripley, 2002) is not available. So the offset must be defined as part of the formula:

```
library(MASS)
(modNB <- glm.nb(count ~ pforest + xlong + offset(corrections2offset(bc)),</pre>
    oven))
Call: glm.nb(formula = count ~ pforest + xlong + offset(corrections2offset(bc)),
    data = oven, init.theta = 1.974529194, link = log)
Coefficients:
(Intercept) pforest xlong
-2.6161 2.7226 -0.0654
Degrees of Freedom: 890 Total (i.e. Null); 888 Residual
Null Deviance:
Residual Deviance: 673 AIC: 1490
round(cbind(Pois = coef(mod), NegBin = coef(modNB)), 3)
              Pois NegBin
(Intercept) -2.565 -2.616
            2.635 2.723
pforest
xlong
            -0.072 -0.065
```

#### 4.2.3 Poisson-Log-Normal mixed effects model

Poisson mixed effects models can be fitted for example via the **lme4** package (Bates et al., 2012) using random intercept for routes:

```
library(lme4)

Loading required package: Matrix

Loading required package: lattice
```

```
Attaching package: 'lme4'
The following object is masked from 'package:stats':
    AIC, BIC

mod4 <- glmer(count ~ pforest + xlong + (1 | route), oven, family = "poisson", offset = corrections2offset(bc))
round(cbind(Pois = coef(mod), NegBin = coef(modNB), PLn = fixef(mod4)),
    3)

Pois NegBin PLn
(Intercept) -2.565 -2.616 -2.695
pforest 2.635 2.723 2.587
xlong -0.072 -0.065 -0.093
```

#### 4.2.4 Logistic regression

For detection/non-detection situations, here is the suggested modification. Note that probability of having 1 means probability of observing non-zero (>0) counts within the sampling area. using the complementary log-log link is related to density from Poisson GLM with log link:

```
mod01 <- glm(ifelse(count > 0, 1, 0) ~ pforest + xlong, oven,
    family = binomial("cloglog"), offset = corrections2offset(bc))
round(cbind(Pois = coef(mod), NegBin = coef(modNB), PLn = fixef(mod4),
    Bin = coef(mod01)), 3)

    Pois NegBin PLn Bin
(Intercept) -2.565 -2.616 -2.695 -2.818
pforest    2.635    2.723    2.587    2.697
xlong    -0.072 -0.065 -0.093 -0.048
```

#### 4.2.5 Zero-inflated count models

It is possible to use offsets in the count distribution of zero-inflated models, such as zero-inflated Poisson or zero-inflated Negative Binomial model as implemented in the zeroinfl function of the pscl package (Zeileis et al., 2008):

```
Loading required package: colorspace
Classes and Methods for R developed in the
Political Science Computational Laboratory
Department of Political Science
Stanford University
Simon Jackman
hurdle and zeroinfl functions by Achim Zeileis
modZIP <- zeroinfl(count ~ pforest + xlong | 1, oven, dist = "poisson",</pre>
    offset = corrections2offset(bc))
modZINB <- zeroinfl(count ~ pforest + xlong | 1, oven, dist = "negbin",</pre>
    offset = corrections2offset(bc))
round(cbind(Pois = c(coef(mod), ZI = NA), NegBin = c(coef(modNB),
    NA), PLn = c(fixef(mod4), NA), Bin = c(coef(mod01), NA),
    ZIP = coef(modZIP), ZINB = coef(modZINB)), 3)
              Pois NegBin
                            PLn
                                   Bin
                                          ZIP ZINB
(Intercept) -2.565 -2.616 -2.695 -2.818 -2.273 -2.350
            2.635 2.723 2.587 2.697 2.612 2.649
xlong
            -0.072 -0.065 -0.093 -0.048 -0.049 -0.054
           NA NA NA NA -1.114 -1.368
```

#### 4.2.6 Classification and regression trees (CART)

This is how a CART model can be specified using the offset approach using the **rpart** package (Therneau et al., 2012). Note that the specification has the linear predictor for the Poisson rate on the response scale ( $\lambda = f(x)$  when method = "poisson", see package vignette), which needs to be standardized by the correction, and not via offsets (Fig. 4.1):

```
library(rpart)
oven$C <- corrections(bc)</pre>
(cart <- rpart((count/C) ~ pforest + xlong, data = oven, method = "poisson"))</pre>
n = 891
node), split, n, deviance, yval
      * denotes terminal node
 1) root 891 836.30 0.35030
   2) pforest< 0.3616 489 197.90 0.08832
     4) pforest< 0.1168 283 43.00 0.02592 *
     5) pforest>=0.1168 206 121.80 0.17730 *
   3) pforest>=0.3616 402 406.50 0.66850
     6) pforest< 0.93 301 278.80 0.54370
      12) xlong< -0.579 72 51.94 0.29850 *
      13) xlong>=-0.579 229 214.60 0.62040
        26) xlong>=0.007684 161 132.40 0.46700 *
        27) xlong< 0.007684 68 63.07 0.96430
          54) pforest< 0.6831 38 37.75 0.64020 *
          55) pforest>=0.6831 30 16.18 1.31400 *
     7) pforest>=0.93 101 102.20 1.02500 *
```

Alternatively, one can use the "anova" with a log transformed response variable. Note that predicted values need to be back-transformed (Fig. 4.1):

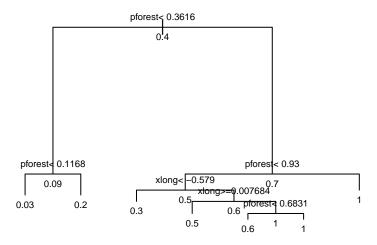
```
(cart2 <- rpart(log(count + 0.5) ~ pforest + xlong + offset(corrections2offset(bc)),</pre>
    data = oven, method = "anova"))
n = 891
node), split, n, deviance, yval
      * denotes terminal node
 1) root 891 431.50000 -0.68130
   2) pforest< 0.4036 521 89.56000 -0.97700
    4) pforest< 0.2014 373 30.71000 -1.04500 *
    5) pforest>=0.2014 148 52.78000 -0.80550 *
   3) pforest>=0.4036 370 232.30000 -0.26490
     6) pforest< 0.8809 212 119.80000 -0.43000
      12) xlong>=1.673 22 6.92200 -0.87140 *
      13) xlong< 1.673 190 108.10000 -0.37890
        26) xlong< -0.579 50 20.29000 -0.68070 *
        27) xlong>=-0.579 140 81.62000 -0.27110 *
     7) pforest>=0.8809 158 98.94000 -0.04337
      14) xlong>=1.693 23 9.82600 -0.61080
        28) pforest>=0.9159 11
                                 0.08456 -1.06600 *
        29) pforest< 0.9159 12
                                 5.36900 -0.19330 *
      15) xlong< 1.693 135 80.45000 0.05330
        30) xlong< -1.193 55 33.41000 -0.13270
          60) xlong>=-1.402 23 13.86000 -0.51300 *
          61) xlong< -1.402 32 13.83000 0.14070 *
        31) xlong>=-1.193 80 43.83000 0.18120 *
opar \leftarrow par(mfrow = c(2, 1), xpd = NA)
plot(cart)
text(cart)
title(main = "poisson")
plot(cart2)
text(cart2)
title(main = "anova")
par(opar)
```

The **rpart** based tree model has issues with handling zero observations, therefore its use is not highly recommended for sparse counts.

#### 4.2.7 Boosted regression trees

This code snipped demonstrates how the offsets can be specified using the **gbm** R package (Ridgeway, 2013) (Fig. 4.2). Of course one might want to use more covariates in such cases, and more advanced settings for determining learning rate e.g. through the **dismo** R package (Hijmans et al., 2013). Note the use of Poisson distribution, where the gradient function is calculated on the log scale ( $(\lambda = e^{f(x)})$ , see package vignette for specifications), therefore the use of the additive offset is justified:

## poisson



#### anova

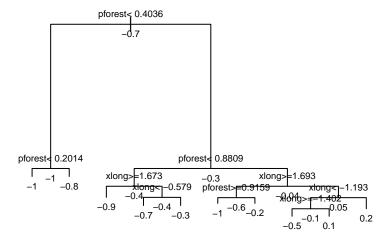


Figure 4.1: Regression tree analysis of the Ovenbird data set using QPAD offsets.

#### 4.2.8 Regularization approaches

This example shows the regularized Poisson GLM using the elastic net penalty using the glmnet R package (Friedman et al., 2010) (Fig. 4.3):

```
library(glmnet)
Loaded glmnet 1.9-3
enet <- glmnet(model.matrix(mod), mod$y, family = "poisson",</pre>
   offset = corrections2offset(bc))
coef(enet)
4 x 55 sparse Matrix of class "dgCMatrix"
  [[ suppressing 55 column names 's0', 's1', 's2' ... ]]
(Intercept) -1.063 -1.1573 -1.246 -1.3310 -1.411 -1.4876
(Intercept) .
                 0.2278   0.430   0.6116   0.776   0.9256
pforest
xlong
(Intercept) -1.560 -1.629 -1.694 -1.755 -1.813 -1.868
(Intercept) . .
pforest 1.063 1.188 1.304 1.410 1.509 1.600
xlong
(Intercept) -1.920 -1.968 -2.013 -2.056 -2.096 -2.133
(Intercept) . . . .
pforest 1.684 1.761 1.833 1.900 1.961 2.018
xlong
(Intercept) -2.168 -2.200 -2.230 -2.258 -2.284 -2.308
```

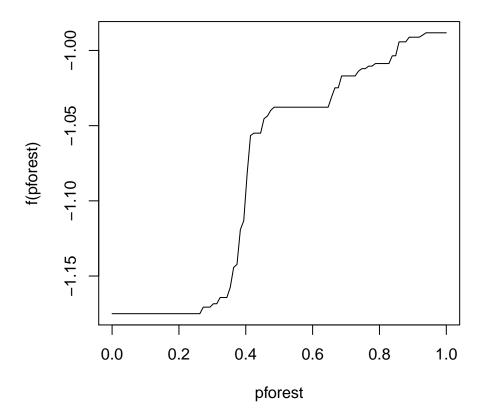


Figure 4.2: Influence plot for the proportion of forest based on the boosted regression tree analysis of the Ovenbird data set using QPAD offsets.

```
pforest 2.070 2.119 2.163 2.204 2.242 2.277
xlong
           . . . . . .
(Intercept) -2.329090 -2.34873 -2.36684 -2.38351 -2.39885
(Intercept) . . . .
pforest 2.308253 2.33641 2.36219 2.38579 2.40738
           -0.004871 -0.01096 -0.01647 -0.02147 -0.02601
xlong
(Intercept) -2.41296 -2.42592 -2.43782 -2.44867 -2.45869
(Intercept) .
pforest
          2.42714 2.44521 2.46174 2.47676 2.49057
           -0.03013 -0.03387 -0.03726 -0.04035 -0.04315
xlong
(Intercept) -2.4679 -2.47627 -2.48397 -2.49102 -2.49747
(Intercept) . . . . . .
pforest 2.5032 2.51473 2.52527 2.53489 2.54368
xlong
           -0.0457 -0.04801 -0.05012 -0.05204 -0.05378
(Intercept) -2.50336 -2.50876 -2.51362 -2.51812 -2.5222
(Intercept) . . . . .
pforest 2.55170 2.55902 2.56561 2.57171 2.5773 xlong -0.05536 -0.05681 -0.05812 -0.05932 -0.0604
(Intercept) -2.52601 -2.5294 -2.53258 -2.53545 -2.53806
(Intercept) .
pforest
          2.58237 2.5870 2.59124 2.59510 2.59862
xlong
           -0.06139 -0.0623 -0.06312 -0.06386 -0.06454
(Intercept) -2.54037 -2.54255 -2.54454 -2.54636 -2.54802
(Intercept) .
pforest 2.60173 2.60466 2.60734 2.60978 2.61200
           -0.06516 -0.06573 -0.06624 -0.06671 -0.06714
xlong
(Intercept) -2.54953
(Intercept) .
pforest 2.61403 xlong -0.06753
plot(enet)
```

## 4.3 Inference: parameter uncertainty

#### 4.3.1 Non-parametric bootstrap

Here is a simple implementation of the non-parametric bootstrap for Poisson GLM using fixed offset. The hbootindex function creates indices for the bootstrap iterations by taking into account the grouped nature of the data set. It resamples routes first, then resamples stops within the resampled routes.

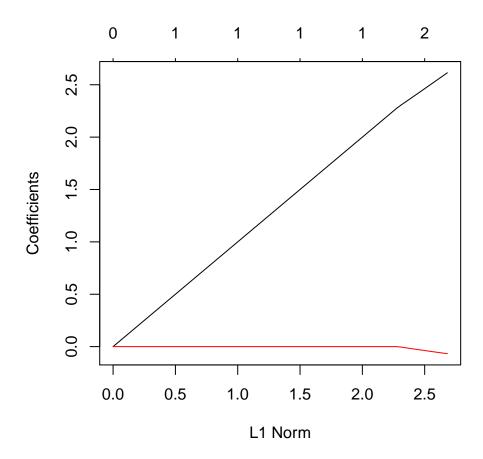


Figure 4.3: Coefficent profile from regularized Poisson GLM analysis of the Ovenbird data set based on QPAD offsets.

See how standard errors compare between Wald-type and bootstrap approach:

To incorporate uncertainty w.r.t. singing rate and distance sampling parameter estimates, one can use nonparametric bootstrap to generate offsets. These offsets will represent the uncertainty in singing rate and distance sampling parameter estimates, so the error can be propagated through the GLM.

For this, we can use the boot argument in the functions globalBAMcorrections and localBAMcorrections making the call B times:

```
modB1 <- t(sapply(1:length(Bi), function(i) {
   bc <- with(oven, localBAMcorrections("OVEN", t = dur, r = dist,
        jday = JDAY, tssr = TSSR, tree = pforest, lcc = LCC,
        model.sra = 5, model.edr = 1, boot = TRUE))
   coef(glm(count ~ pforest + xlong, oven[Bi[[i]], ], family = "poisson",
        offset = corrections2offset(bc)[Bi[[i]]]))
}))</pre>
```

We can compare standard errors again:

The error in offsets can be propagated through other related count models similarly.

It is also possible to take into account model selection uncertainty in the QPAD estimates by randomly choosing among the possible singing rate and distance models based on model weights (with and without the parametric bootstrap procedure in the offsets switched by the boot argument):

```
modB2 <- t(sapply(1:length(Bi), function(i) {</pre>
    mm <- bestmodelBAMspecies("OVEN", type = "multi")</pre>
    bc <- with(oven, localBAMcorrections("OVEN", t = dur, r = dist,
        jday = JDAY, tssr = TSSR, tree = pforest, lcc = LCC,
        model.sra = mm$sra, model.edr = mm$edr, boot = FALSE))
    coef(glm(count ~ pforest + xlong, oven[Bi[[i]], ], family = "poisson",
        offset = corrections2offset(bc)[Bi[[i]]]))
}))
modB3 <- t(sapply(1:length(Bi), function(i) {</pre>
    mm <- bestmodelBAMspecies("OVEN", type = "multi")</pre>
    bc <- with(oven, localBAMcorrections("OVEN", t = dur, r = dist,
        jday = JDAY, tssr = TSSR, tree = pforest, lcc = LCC,
        model.sra = mm$sra, model.edr = mm$edr, boot = TRUE))
    coef(glm(count ~ pforest + xlong, oven[Bi[[i]], ], family = "poisson",
        offset = corrections2offset(bc)[Bi[[i]]]))
}))
round(cbind(wald_fixed = sqrt(diag(vcov(mod))), boot_fixed = sqrt(diag(cov(modB))),
    boot_boot = sqrt(diag(cov(modB1))), boot_multi = sqrt(diag(cov(modB2))),
    boot_bmulti = sqrt(diag(cov(modB2)))), 3)
            wald_fixed boot_fixed boot_boot boot_multi
(Intercept)
                 0.117
                            0.254 0.257
                                                  0.254
pforest
                 0.149
                            0.313
                                       0.315
                                                  0.313
                            0.084
xlong
                 0.040
                                       0.084
                                                  0.084
            boot_bmulti
(Intercept)
                  0.254
pforest
                  0.313
xlong
                  0.084
```

It is clear that the uncertainty in this case is driven by parameter uncertainty, and not model uncertainty w.r.t. the offsets used.

#### 4.3.2 Bayesian and frequentist approach to hierarchical modeling

In the next example we consider a Poisson-Log-Normal generalized linear mixed model with a random intercept for routes, which also incorporates detectability related uncertainty.

This model uses a global maximization technique called data cloning (Lele et al., 2007, 2010), which takes advantage of Bayesian MCMC techniques for maximum likelihood estimation. The software implementation is described in Sólymos (2010). Note that using a single clone is identical to the Bayesian hierarchical modeling.

```
library(dcmle) # load dcmle package

Loading required package: dclone
dclone 2.0-0 2013-07-10

Loading required package: rjags
Linked to JAGS 3.3.0

Loaded modules: basemod, bugs

Loading required package: R2WinBUGS

Loading required package: boot

Attaching package: 'boot'
```

```
The following object is masked from 'package:survival':
The following object is masked from 'package:lattice':
       melanoma
dcmle 0.2-4 2013-06-19
Attaching package: 'dcmle'
The following objects are masked from 'package:coda':
        chanames, crosscorr.plot, gelman.diag,
gelman.plot, geweke.diag, heidel.diag,
raftery.diag, varnames
load.module("glm") # load glm module for JAGS
module qlm loaded
model <- function() {</pre>
    for (i in 1:n) {
        Y[i] ~ dpois(lam[i])
        log(lam[i]) <- inprod(X[i, ], beta) + E[gr[i]] + log(A[i] *</pre>
            p[i])
        p[i] <- 1 - exp(-3 * phi[i])
        A[i] <- 3.141593 * tau[i]^2
        log(phi[i]) <- inprod(Z1[i, ], theta01)</pre>
        log(tau[i]) <- inprod(Z2[i, ], theta02)</pre>
    for (j in 1:m) {
        E[j] ~ dnorm(0, 1/exp(log.sigma)^2)
    for (k in 1:np) {
        beta[k] ~ dnorm(pr[k], 1)
    log.sigma ~ dnorm(-2, 0.01)
    theta01 ~ dmnorm(theta1, Sigma1)
    theta02 ~ dmnorm(theta2, Sigma2)
dat <- list(Y = oven$count, X = model.matrix(~pforest + xlong,</pre>
    oven), np = 3, n = nrow(oven), m = length(unique(oven$route)),
    gr = dciid(as.integer(as.factor(oven$route))), pr = coef(mod),
    theta1 = coefBAMspecies("OVEN", bm$sra, bm$edr)$sra, Z1 = model.matrix(~JDAY,
        oven), Sigma1 = solve(vcovBAMspecies("OVEN", bm$sra,
        bm$edr)$sra), theta2 = coefBAMspecies("OVEN", bm$sra,
        bm$edr)$edr, Z2 = model.matrix(~LCC, oven), Sigma2 = solve(vcovBAMspecies("OVEN",
        bm$sra, bm$edr)$edr))
dcf <- makeDcFit(model = model, data = dat, params = c("beta",</pre>
    "log.sigma"), multiply = c("n", "m"), unchanged = c("np",
    "pr", "theta1", "theta2", "Sigma1", "Sigma2"))
cl <- makePSOCKcluster(3) # parallel computing for speed up</pre>
K \leftarrow c(1, 2) # sequence for the number of clones to use
parLoadModule(cl, "glm") # load glm module for JAGS on workers
[[1]]
NULL
```

```
[[2]]
NULL
[[3]]
NULL
dcm <- dcmle(dcf, n.clones = K, n.iter = 1000, cl = cl, partype = "parchains")</pre>
Fitting model with 1 clone
Parallel computation in progress
Loading required package: snow
Attaching package: 'snow'
The following objects are masked from 'package:parallel':
       clusterApply, clusterApplyLB, clusterCall,
clusterEvalQ, clusterExport, clusterMap,
clusterSplit, makeCluster, parApply, parCapply,
parLapply, parRapply, parSapply, splitIndices,
stopCluster
Fitting model with 2 clones
Parallel computation in progress
Warning: chains convergence problem, see R.hat values
stopCluster(cl) # close cluster
dcm
Call:
dcmle(x = dcf, n.clones = K, cl = cl, n.iter = 1000, partype = "parchains")
Coefficients:
  beta[1]
          beta[2]
                      beta[3] log.sigma
 -2.66274 2.59990 -0.08568 -3.55219
```

The model reveals results similar to the GLM example above, Proportion of forest cover around points had a significant positive effect on density, while longitude had a negative non-significant effect. The estimate of the route level random effect was 0.56 (SE 0.1).

#### 4.4 Prediction

Point predictions (unconditional on the observations) and associated prediction intervals can be calculated by the same MCMC technique:

```
pmodel <- function() {</pre>
    for (i in 1:n) {
        Y[i] ~ dpois(lam[i])
        log(lam[i]) <- inprod(X[i, ], tmp[1:np]) + E[i]</pre>
        E[i] ~ dnorm(0, 1/exp(tmp[np + 1])^2)
    tmp[1:(np + 1)] ~ dmnorm(cf, Sig)
}
pf <- 10:0/10
ND <- expand.grid(pforest = pf, xlong = c(-1.8, 0, 1.8))
Xnew <- model.matrix(~pforest + xlong, ND)</pre>
## grouping is random (so this is at region and not route
## level)
dat1 <- list(X = Xnew, np = 3, n = nrow(Xnew), cf = coef(dcm),</pre>
    Sig = solve(vcov(dcm)))
dcf2p1 <- makeDcFit(model = pmodel, data = dat1, params = c("lam"))</pre>
pm1 <- dcmle(dcf2p1, n.clones = 1)</pre>
Compiling model graph
   Resolving undeclared variables
   Allocating nodes
   Graph Size: 330
Initializing model
pm1 <- as.matrix(pm1)</pre>
ND$mean <- colMeans(pm1)</pre>
ND$cl1 <- apply(pm1, 2, quantile, probs = 0.025)
ND$cl2 <- apply(pm1, 2, quantile, probs = 0.975)
## drawing the figure
op \leftarrow par(las = 1)
plot(mean ~ pforest, ND, col = as.integer(as.factor(ND$xlong)),
    ylim = c(0, max(ND$c12)), type = "n", xlab = "Proportion of forest",
    ylab = "Density (males / ha)", axes = FALSE)
polygon(c(pf, rev(pf)), c(ND$cl1[ND$xlong > 0], rev(ND$cl2[ND$xlong <</pre>
    0])), border = NA, col = "grey")
polygon(c(pf, rev(pf)), c(ND$mean[ND$xlong > 0], rev(ND$mean[ND$xlong <
    0])), border = NA, col = "black")
lines(pf, ND$mean[ND$xlong == 0], col = "white", lwd = 2)
box(bty = "l")
axis(1, tck = 0.02)
axis(2, tck = 0.02)
par(op)
```

Predicted density in areas with 100% forest cover was 1.05 males / ha ranging from 0.9 to 1.25 depending on longitude. This prediction almost equals the density estimate of 0.99 (95% confidence limits, 0.85-1.12) by Bayne (2000) based on territory mapping in the same region (Fig. 4.4).

Point predictions conditional on the observations and associated prediction intervals given the observations can be calculated in a similar manner (Fig. 4.5).

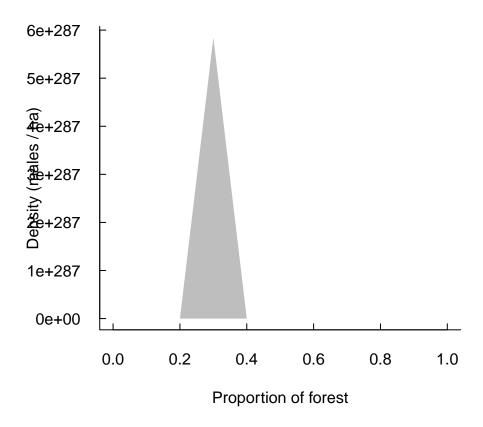


Figure 4.4: Predicted density of Ovenbird as a function of forest cover, based on a generalized linear mixed model (GLMM) fitted to a data set from Saskatchewan, Canada. Grey shade represents 95% prediction intervals incorporating uncertainties related to the random effect and the detectability corrections. Black shade indicates range of variation in mean predictions due to longitude, white line represents the prediction at the centroid of the study area.

```
dat2 \leftarrow list(Y = dat\$Y, X = dat\$X, np = 3, n = nrow(dat\$X), cf = coef(dcm),
    Sig = solve(vcov(dcm)))
dcf2p2 <- makeDcFit(model = pmodel, data = dat2, params = c("lam"))</pre>
pm2 <- dcmle(dcf2p2, n.clones = 1, n.chains = 1)</pre>
Compiling model graph
   Resolving undeclared variables
   Allocating nodes
   Graph Size: 8052
Initializing model
pm2 <- as.matrix(pm2)</pre>
pp <- data.frame(mean = colMeans(pm2), cl1 = apply(pm1, 2, quantile,</pre>
   probs = 0.025), cl2 = apply(pm1, 2, quantile, probs = 0.975))
summary(pp)
                                          c12
     mean
                      cl1
 Min. :0.089 Min. : 0.00e+00 Min. :8.83e+169
 1st Qu.:0.126
                1st Qu.: 0.00e+00 1st Qu.:1.94e+199
 Median: 0.230 Median: 0.00e+00 Median: 1.73e+210
 Mean :0.482 Mean :1.28e-149 Mean :1.77e+286
 3rd Qu.:0.777 3rd Qu.: 0.00e+00
                                     3rd Qu.:3.99e+229
 Max. :1.807
               Max. :4.21e-148
                                     Max. :5.84e+287
boxplot(pp$mean ~ dat$Y, xlab = "Observed counts", ylab = "Predicted mean density")
```

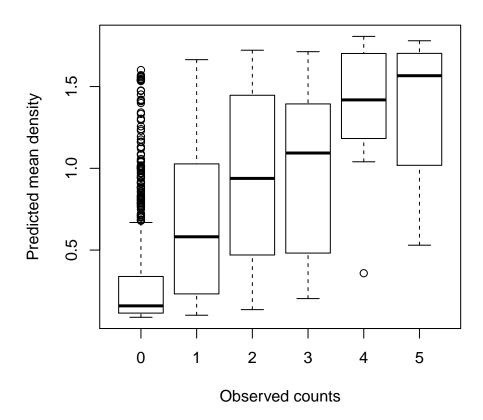


Figure 4.5: The relationship between observed counts and predicted mean density conditional on the observations based on the Ovenbird data set (males / ha).

# Calculating offsets based on independent estimates

Using the estimates derived from our extensive BAM data can help in data deficient situations, but the QPAD approach for using offsets allows the use of custom derived estimates. The only requirement is that estimates need to be consistent with the QPAD approach.

One can create a table with corrections using independently derived point estimates for singing rates (phi) and distance parameter (tau):

Similarly, one can supply a vector of values for phi and tau:

Custom values can be derived from fitted models based on the conditional likelihood estimating procedure:

```
1 3.142 0.8861 0.3903
2 3.142 0.9892 0.5108
3 3.142 0.6783 0.2915
4 3.142 0.8722 0.3816
5 3.142 0.9586 0.4528
6 3.142 0.7224 0.3090
```

# Further examples

Further examples can be found at the BAM website (http://www.borealbirds.ca) under the Results tab.

These include spatial maps of expected mean abundance for Bird Conservation Regions (BCRs) within provinces and territories of Canada, and relative densities within various land cover classes per spatial units. Information is available for 70 bird species.

For example, the Ovenbird results (density estimates, habitat associations, and more) are available at the BAM website.

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