How to generate new distributions in packages "distr", "distrEx"

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Abstract

In this vignette, we give short examples how to produce new distributions in packages "distr" and "distrEx". This vignette refers to package version 2.0.6.

Basically there are three ways to produce new distributions in packages "distr" and "distrEx":

- 1. automatic generation of single distribution objects by arithmetics and the like
- 2. using generating functions to produce single distribution objects
- 3. defining new distribution classes / doing it from scratch

We will give short examples of all three of them.

1 Automatic generation by arithmetics and the like

We have made available quite general arithmetical operations to our distribution objects, generating new image distribution objects automatically. As an example, try

```
> require(distr)
> N \( - \text{Norm}(mean = 2, sd = 1.3))
> P \( - \text{Pois}(lambda = 1.2))
> Z \( - 2*N + 3 + P)
> Z
```

Distribution Object of Class: AbscontDistribution

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```
> plot(Z, panel.first = grid(), lwd=2)
> p(Z)(0.4)

[1] 0.002415384
> q(Z)(0.3)

[1] 6.70507
```

 $> Zs \leftarrow r(Z)(50)$

> Zs

 [1]
 7.823008
 6.540824
 6.521669
 9.669269
 6.059693
 7.476556

 [7]
 12.730386
 7.834476
 7.536438
 15.944387
 12.999326
 6.312371

 [13]
 7.686110
 13.955380
 6.769967
 9.916557
 10.923665
 9.073019

 [19]
 9.709455
 9.265215
 4.770896
 11.671790
 4.400793
 7.803961

 [25]
 9.566977
 10.481738
 8.382077
 7.337073
 5.433704
 5.154682

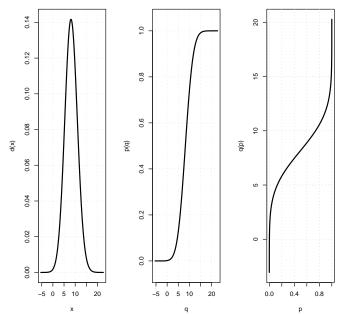
 [31]
 4.641173
 7.693903
 8.594134
 7.999718
 7.791064
 11.125091

 [37]
 10.356708
 8.807137
 8.268592
 3.099324
 10.057875
 10.243648

 [43]
 10.045342
 10.104708
 5.225103
 10.335450
 7.287580
 2.016172

 [49]
 11.729716
 8.936419

Density of AbscontDistribution
CDF of AbscontDistribution \u00fcuantile function of AbscontDistrib



Comment:

Let N an object of class "Norm" with parameters **mean=2**, **sd=1.3** and let P an object of class "Pois" with parameter **lambda=1.2**. Assigning to Z the expression 2*N+3+P, a new distribution object is generated —of class "AbscontDistribution" in our case— so that identifying N, P, Z with random variables distributed according to N, P, Z, $\mathcal{L}(Z) = \mathcal{L}(2*N+3+P)$, and writing $\mathbf{p}(Z)(0.4)$ we get $P(Z \leq 0.4)$, $\mathbf{q}(Z)(0.3)$ the 30%-quantile of Z, and with $\mathbf{r}(Z)(50)$ we generate 50 pseudo random numbers distributed according to Z, while the **plot** command generates the above figure.

There a caveats to take care about; for details refer to the (larger) vignette distr in package "distrDoc".

2 Using generating functions

If you want to generate a single distribution object (without any particular parameter) generating functions are the method of choice:

Objects of classes LatticeDistribution resp. DiscreteDistribution, AbscontDistribution, may be generated using the generating functions LatticeDistribution() resp. DiscreteDistribution() resp. AbscontDistribution(); see also the corresponding help.

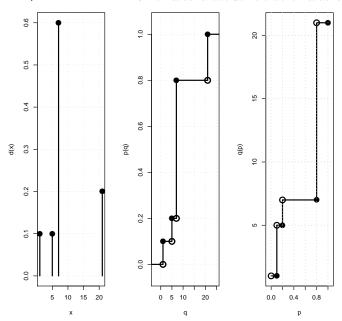
E.g., to produce a discrete distribution with support (1, 5, 7, 21) with corresponding probabilities (0.1, 0.1, 0.6, 0.2) we may write

```
> D \leftarrow DiscreteDistribution(supp = c(1,5,7,21), prob = c(0.1,0.1,0.6,0.2)) > D
```

Distribution Object of Class: DiscreteDistribution

```
> plot(D, panel.first = grid(), lwd = 2)
```

obability function of DiscreteDistr
CDF of DiscreteDistribution uantile function of DiscreteDistribution
CDF of DiscreteDi

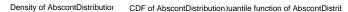


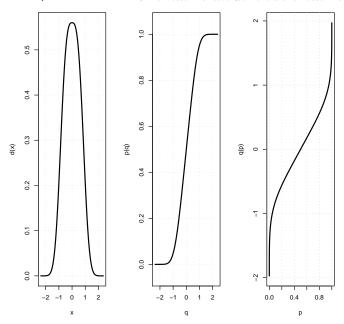
and to generate an absolutely continuos distribution with density proportional to $e^{-|x|^3}$, we write

- \rightarrow AC \leftarrow AbscontDistribution(d = function(x) exp(-abs(x)^3), withStand = TRUE)
- > AC

Distribution Object of Class: AbscontDistribution

> plot(AC, panel.first = grid(), lwd = 2)





3 Doing it from scratch

If you would like to create new parametric distributions, using already implemented r, d, p, and q functions (e.g. implementing additional distributions realized in another CRAN package), you should probably envisage introducing new distribution S4 (sub-)classes and hence better look at the implementation of some discrete and continuous parametric distribution classes in package "distr". Hint: download the .tar.gz file; extract it to some temp folder; look at subdirectories R and man

The general procedure is as follows

- 1. introduce a new subclass of class Parameter
- 2. introduce a new subclass of LatticeDistribution/DiscreteDistribution (if discrete) or of class AbscontDistribution (if continuous).
- 3. define accessor and replacement functions for the "slots" of the parameter (e.g. "size" and "prob" in the binomial case), possibly with new generics
- 4. (possibly) define a validity function
- 5. define a generating function
- 6. if existing, define particular convolution methods or similar particular methods for this new distribution class

- 7. create .Rd files for the
 - parameter class
 - distribution class
- 8. if analytic expressions are available, define particular E-, var-, skewness-, and kurtosis-methods and if so, also document¹ the corresponding methods in the distribution class .Rd file

Let's go through the steps in the example case of the Binomial implementation in packages "distr" and "distrEx":

1. in "distr", see source in R/AllClasses.R, lines 180-189

2. in "distr", see source in R/AllClasses.R, lines 829-855

```
## Class: binomial distribution
set Class ("Binom",
           prototype = prototype(
                         r = function(n) \{ rbinom(n, size = 1, prob = 0.5) \},
                         d = function(x, log = FALSE)
                                  dbinom(x, size = 1, prob = 0.5, log = log)
                         p = function(q, lower.tail = TRUE, log.p = FALSE)
                                  \mathbf{pbinom}(\mathbf{q}, \text{ size} = 1, \text{ prob} = 0.5,
                                          lower.tail = lower.tail, log.p = log.p)
                         q = function(p, lower.tail = TRUE, log.p = FALSE)
                                  \mathbf{qbinom}(\mathbf{p}, \text{ size} = 1, \text{ prob} = 0.5,
                                          lower.tail = lower.tail, log.p = log.p)
                         img = new("Naturals"),
                         param = new("BinomParameter"),
                         support = 0:1,
                         lattice = new("Lattice",
                                    pivot = 0, width = 1, Length = 2, name =
```

¹this is new, because so far, all E-, var-, skewness-, and kurtosis-methods for "basic" distributions are documented in the "distrEx" documentation to E, var, ..., but this would not be operational any longer for new derived classes, possibly defined in other, new packages

```
gettext(
                                      "lattice of a Binomial distribution"
            contains = "LatticeDistribution"
3. in "distr", see source in R/BinomialDistribution.R, lines 8-15, and 43-53
  ## Access Methods
  ## Replace Methods
  setReplaceMethod("size", "BinomParameter",
                     function(object, value){ object@size ← value; object})
  setReplaceMethod("prob", "BinomParameter",
                     function(object, value){ object@prob ← value; object})
  ## wrapped access methods
  setMethod("prob", "Binom", function(object) prob(param(object)))
setMethod("size", "Binom", function(object) size(param(object)))
  ## wrapped replace methods
  setMethod ("prob ←", "Binom",
             function(object, value) new("Binom", prob = value,
                                            size = size(object))
  setMethod("size ←", "Binom",
             function (object, value) new ("Binom", prob = prob (object),
                                            size = value))
  and R/AllGenerics, lines 143-146
  if(!isGeneric("size"))
     setGeneric("size", function(object) standardGeneric("size"))
  if(!isGeneric("prob"))
     setGeneric("prob", function(object) standardGeneric("prob"))
4. in "distr", see source in R/BinomialDistribution.R, lines 18-32
  setValidity("BinomParameter", function(object){
    if(length(prob(object)) != 1)
      stop ("prob_has_to_be_a_numeric_of_length_1")
    if(prob(object) < 0)
      stop ("prob_has_to_be_in_[0,1]")
    if(prob(object) > 1)
      stop ("prob_has_to_be_in_[0,1]")
    if(length(size(object)) != 1)
      stop("size_has_to_be_a_numeric_of_length_1")
    if(size(object) < 1)</pre>
      stop("size_has_to_be_a_natural_greater_than_0")
    if(!identical(floor(size(object)), size(object)))
```

```
stop("size_has_to_be_a_natural_greater_than_0")
    else return (TRUE)
  })
5. in "distr", see source in R/BinomialDistribution.R, line 41
  Binom ← function(size = 1, prob = 0.5) new("Binom", size = size, prob = prob)
6. in "distr", see source in R/BinomialDistribution.R, lines 54-68
  ## Convolution for two binomial distributions Bin(n1,p1) and Bin(n2,p2)
  ## Distinguish cases
  ## p1 == p2 und p1 != p2
  setMethod("+", c("Binom", "Binom"),
             function (e1, e2)
               newsize \leftarrow size(e1) + size(e2)
                if (isTRUE(all.equal(prob(e1),prob(e2))))
                   return(new("Binom", prob = prob(e1), size = newsize,
                               .withArith = TRUE)
               return(as(e1, "LatticeDistribution") + e2)
             })
7. in "distr", see sources in
    • man/BinomParameter-class.Rd
       \name{BinomParameter-class}
       \docType{ class}
       \alias {BinomParameter-class}
       \alias { initialize , BinomParameter-method }
       \title{Class "BinomParameter"}
       description { The parameter of a binomial distribution, used by Binom-class}
       \section{Objects from the Class}{
       Objects can be created by calls of the form
             Usually an object of this class is not needed on its own, it is generated
       automatically when an object of the class Binom
       is instantiated.
       \section{Slots}{
        \describe{
           the probability of a binomial distribution }
           \item{\code{size}:}{Object of class \code{"numeric"}:
                 the size of a binomial distribution }
          \widetilde{\operatorname{code}}_{\operatorname{name}}: {\operatorname{Object} \ of \ \operatorname{class} \ \operatorname{code}_{\operatorname{name}}: }
                 a name / comment for the parameters }
      }
```

```
\section{Extends}{
  Class \code{"Parameter"}, directly.
  \section { Methods } {
     \describe{
       \item{initialize}{\code{signature(.Object = "BinomParameter")}:
               initialize method }
       \item{prob}{\code{signature(object = "BinomParameter")}: returns the slot
               \code{prob} of the parameter of the distribution }
       \operatorname{item} \{\operatorname{prob} \leftarrow \} \{\operatorname{code} \{\operatorname{signature} (\operatorname{object} = \operatorname{"BinomParameter"})\} : \operatorname{modifies} \operatorname{the slot}
               \code{prob} of the parameter of the distribution }
       \item{size}{\code{signature(object = "BinomParameter")}: returns the slot
       \label{local_code} $$ \code{size} of the parameter of the distribution } $$ \code{size} \leftarrow {\code{signature(object = "BinomParameter")}}: modifies the slot $$ \code{signature(object = "BinomParameter")}$$ :
               \code{size} of the parameter of the distribution}
  }
  \author{
    Thomas Stabla \email{statho3@web.de},\cr
     Florian Camphausen \email{fcampi@gmx.de},\cr
     Peter Ruckdeschel \email{Peter.Ruckdeschel@itwm.fraunhofer.de},\cr
     Matthias Kohl \email{Matthias.Kohl@stamats.de}
     }
  \seealso{
  \code{\link{Binom-class}}
  \code{\link{Parameter-class}}
  \examples{
      W ← new("BinomParameter", prob=0.5, size=1)
      size (W) # size of this distribution is 1. size (W) \leftarrow 2 # size of this distribution is now 2.
  \keyword{ distribution }
  \setminus \mathbf{concept} \{ \mathbf{parameter} \}
  \concept{Binomial distribution}
  \concept{S4 parameter class}
• man/Binom-class.Rd
  \name{Binom-class}
  \docType{ class}
  \alias {Binom-class}
  \alias {Binom}
  \alias { initialize , Binom-method }
  \title{Class "Binom" }
  \description{The binomial distribution with \code{size} \eqn{= n}, by default
     \{eqn\{=1\}, and \}
     \langle code\{prob\} \rangle = p\}, by default \langle eqn\{=0.5\} \rangle, has density
     p(x) = choose(n,x) p^x (1-p)^(n-x)
     for \{x = 0, \{x \in \mathbb{N}\}.
    C. f. \setminus code\{ \setminus link [stats:Binomial] \{rbinom\} \}
```

```
\section{Objects from the Class}{
Objects can be created by calls of the form \code{Binom(prob, size)}.
This object is a binomial distribution.
\section{Slots}{
    \describe{
         \item{\code{img}:}{Object of class \code{"Naturals"}: The space of the
          image of this distribution has got dimension 1 and the
          name "Natural_Space". }
         \item{\code{param}:}{Object of class \code{"BinomParameter"}: the parameter
                      of this distribution (\code{prob}, \code{size}), declared at its
                      instantiation }
         \item{\code{r}:}{Object of class \code{"function"}: generates random
                      numbers (calls function \code{rbinom}) }
         \item{\code{d}:}{Object of class \code{"function"}: density function (calls
                      function \code{dbinom}) }
         \item{\code{p}:}{Object of class \code{"function"}: cumulative function
                       (calls function \code{pbinom}) }
         \widetilde{q}:{\ode{q}:}{\ode{q}:} Object of class \operatorname{code{"function"}: inverse of the }
                        cumulative function (calls function \code{qbinom}).
        The quantile is defined as the smallest value x such that F(x) >= p, where
                          F is the cumulative function. }
         \label{local_code} $$ \left( \operatorname{code} \left( \operatorname{support} \right) : \right) \in \operatorname{Class'} \left( \operatorname{code} \left( \operatorname{"numeric"} \right) : a \right) = \left( \operatorname{sorted} \right) . $$
                           vector containing the support of the discrete density function}
   }
\section { Extends } {
Class \code{"DiscreteDistribution"}, directly.\cr
Class \code{"UnivariateDistribution"}, by class \code{"DiscreteDistribution"}.\cr
Class \code{"Distribution"}, by class \code{"DiscreteDistribution"}.
\section { Methods } {
    \backslash describe\{
         \begin{array}{lll} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & &
                              distributions with equal probabilities the exact convolution
                             formula is implemented thereby improving the general numerical
                             accuracy.}
         \item{initialize}{\cdot\code{signature(.Object = "Binom")}: initialize method }
         \item{prob}{\code{signature(object = "Binom")}: returns the slot \code{prob}
                             of the parameter of the distribution }
         \item{prob \( \) \} \\ \code \{ \signature (object = "Binom") \\}: modifies the slot
        \label{lem:code:prob} $$ \operatorname{code}\{\operatorname{prob}\}$ of the parameter of the distribution } $$ \operatorname{size}\{\operatorname{code}\{\operatorname{signature}(\operatorname{object} = \operatorname{"Binom"})\}: returns the slot $$ \operatorname{code}\{\operatorname{size}\}$ 
                             of the parameter of the distribution }
         \item{size \( \) \} \\ \code \{ \signature \( \) object = "Binom" \) \}: modifies the slot
                             \code{size} of the parameter of the distribution }
}
\author{
    Thomas Stabla \email{statho3@web.de},\cr
    Florian Camphausen \email{fcampi@gmx.de},\cr
    Peter Ruckdeschel \email{Peter.Ruckdeschel@itwm.fraunhofer.de},\cr
    Matthias Kohl \email{Matthias.Kohl@stamats.de}
```

```
\seealso{
\code\{\link\{BinomParameter-class\}\}\
\code{\link{DiscreteDistribution-class}}
\code{\link{Naturals-class}}
\code{\link[stats:Binomial]{rbinom}}
\examples{
     B ← Binom(prob=0.5, size=1) # B is a binomial distribution with prob=0.5 and size=1.
     r\left( \mathrm{B}
ight) (1) # # one random number generated from this distribution, e.g. 1
     d(B)(1) # Density of this distribution is 0.5 for x=1.
    p(B)(0.4) # Probability that x<0.4 is 0.5.
    \begin{array}{l} (B)\,(.1) \ \# \ \text{$x$=0$ is the smallest value $x$ such that $p(B)(x)$>=0.1.} \\ \text{size}\,(B) \ \# \ \text{size} \ \text{of this distribution is 1.} \\ \text{size}\,(B) \ \leftarrow \ 2 \ \# \ \text{size} \ \text{of this distribution is now 2.} \end{array}
    C \leftarrow \text{Binom}(\text{prob} = 0.5, \text{ size} = 1) # C is a binomial distribution with prob=0.5 and size=1. D \leftarrow \text{Binom}(\text{prob} = 0.6, \text{ size} = 1) # D is a binomial distribution with prob=0.6 and size=1.
    E\leftarrow B+C # E is a binomial distribution with prob=0.5 and size=3. F\leftarrow B+D # F is an object of class LatticeDistribution. G\leftarrow B+as(D,"DiscreteDistribution") ## DiscreteDistribution
\keyword{ distribution }
\concept{discrete distribution}
\concept{lattice distribution}
\concept{Binomial family}
\concept{Binomial distribution}
\concept{S4 distribution class}
\concept{generating function}
```

• you could have: man/Binom.Rd for the generating function; in the Binomial case, documentation is in Binom-class.Rd; but in case of the Gumbel distribution, in package "distrEx", there is such an extra .Rd file

8. in "distrEx", see sources in

• Expectation.R, lines 309-314

• Functionals.R, lines 159-166

```
 \begin{array}{lll} setMethod("var", signature(x = "Binom"), \\ & & function(x, \dots) \{ \\ & fun \leftarrow NULL; cond \leftarrow NULL \\ & & if((hasArg(fun))||(hasArg(cond))) \\ & & return(var(as(x, "DiscreteDistribution"), \dots)) \\ & & else \\ & & return(size(x)*prob(x)*(1-prob(x))) \\ & \}) \end{array}
```

• skewness.R, lines 60-67

```
setMethod("skewness", signature(x = "Binom"),
      function (x, \dots)
      fun \leftarrow NULL; cond \leftarrow NULL
      if ((hasArg(fun))||(hasArg(cond)))
          return(skewness(as(x,"DiscreteDistribution"),...))
      else
           return((1-2*prob(x))/sqrt(size(x)*prob(x)*(1-prob(x))))
• kurtosis.R, lines 70-78
  setMethod("kurtosis", signature(x = "Binom"),
      function (x, \dots)
      fun \leftarrow NULL; cond \leftarrow NULL
      if ((hasArg(fun))||(hasArg(cond)))
          return(kurtosis(as(x,"DiscreteDistribution"),...))
      else
           p \leftarrow \operatorname{prob}(x)
           return ((1-6*p*(1-p))/(size(x)*p*(1-p)))
```

The procedure will be similar for any new class of distributions.

Comment In the classes in package "distr" (historically the "oldest" in the development of this project), we still use initialize methods; this is no longer needed, if you provide generating functions; for this "more recent" approach, confer the realization of class Gumbel in package "distrex".

4 Help needed / collaboration welcome

You are — as announced on http://distr.r-forge.r-project.org — very welcome to collaborate in this project! See in particular http://distr.r-forge.r-project.org/HOWTO-collaborate.txt With this you should be able to start working.

References

- [1] Kohl M., Ruckdeschel P. and Stabla T. General Purpose Convolution Algorithm for Distributions in S4-Classes by means of FFT. unpublished manual
- [2] Ruckdeschel P., Kohl M., Stabla T., and Camphausen F. S4 Classes for Distributions. *R-News*, **6**(2): 10–13. http://CRAN.R-project.org/doc/Rnews/Rnews_2006-2.pdf