Predictor Effects Graphics Gallery

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Abstract

Predictor effect displays visualize the response surface of complex regression models by averaging and conditioning, producing a sequence of 2D line graphs, one graph or set of graphs for each predictor in the regression problem (Fox and Weisberg, 2019a,b). In this vignette, we give examples of effect plots produced by the **effects** package, and in the process systematically illustrate the optional arguments to functions in the package, which can be used to customize predictor effect plots.

1 Introduction

Predictor effect plots (Fox and Weisberg, 2019b) provide graphical summaries for fitted regression models with a linear predictor, including linear models, generalized linear models, linear and generalized linear mixed models, and many others. These graphs are an alternative to tables of fitted coefficients, which can be much harder to interpret than effect plots. Predictor effect plots are implemented in R in the effects package, documented in Fox and Weisberg (2019a). This vignette provides many examples of variations on these graphical displays that can be obtained with the effects package. Many of the details, and more complete descriptions of the data sets used as examples, are provided in the references cited at the end of the vignette.

1.1 Effects and Predictor Effect Plots

We begin with an example of a multiple linear regression, using the Prestige data set in the carData package:

The data, collected circa 1970, pertain to 102 Canadian occupations. The model lm1 is a linear model with response prestige, continuous predictors income, education, and women, and the factor predictor type, which has three levels. Before fitting the model, we reorder the levels of type as "bc" (blue-collar), "wc" (white-collar), and "prof" (professional and managerial). The predictor education represents itself in the linear model, and so it is both a predictor and a regressor, as defined in Fox and Weisberg (2019a, Sec. 4.1). The predictor income is represented by the regressor log(income). The variable women, a percentage between 0 and 100, is represented by regressors that define a polynomial of degree 2 using poly()'s default orthogonal polynomials. The variable type is a factor with three levels, so it is represented by two dummy variables defined by the default contrast-generating function in R, contr.treatment(). Finally, the formula includes an interaction between income and type. defined by multiplying each of the regressor for income by each of the regressors that represent type.

The usual numeric summary of the fit of 1m1 is a table of estimated coefficients, which we obtain via the S() function in the **car** package that is similar to, but somewhat more flexible than, the standard R summary() function:

```
education
                         2.959
                                    0.582
                                              5.09 2.0e-06
poly(women, 2)1
                        28.339
                                   10.190
                                              2.78
                                                     0.0066
poly(women, 2)2
                                              1.77
                                                     0.0800
                        12.566
                                    7.095
log(income)
                        17.514
                                    2.916
                                              6.01 4.1e-08
typewc
                         0.969
                                   39.495
                                              0.02
                                                     0.9805
typeprof
                                   30.736
                                              2.42
                                                     0.0177
                        74.276
log(income):typewc
                                    4.620
                                             -0.10
                                                     0.9199
                        -0.466
log(income):typeprof
                        -7.698
                                    3.451
                                             -2.23
                                                     0.0282
```

Residual standard deviation: 6.2 on 89 degrees of freedom

(4 observations deleted due to missingness)

Multiple R-squared: 0.879

F-statistic: 81.1 on 8 and 89 DF, p-value: <2e-16

AIC BIC 646.26 672.11

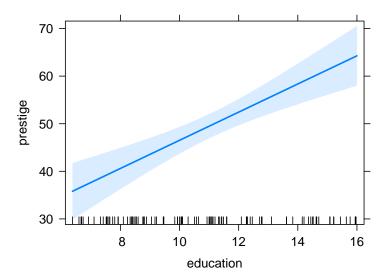
- Interpretation of the regression coefficients is straightforward only for the predictor education, where an increase of one year of education, holding other predictors fixed, corresponds to an estimated expected increase in the response of 2.959 units.
- Even ignoring the interaction, the log transformation complicates the interpretation of the effect of income.
- The predictor women is represented by two regressors, so the effect of women requires examining two coefficient estimates that are interpretable only by those knowledgeable about polynomial regression analysis. Even if raw rather than orthogonal polynomial regressors were used, via poly(women, 2, raw=TRUE) in place of poly(women, 2), interpretation of the effect of women is complicated.
- Understanding the coefficients for the main effect of type depends on the contrasts used to define the effect. The contrasts can be changed by the user, and the default contrasts in R are different from the default contrasts used by SAS or other programs, so the coefficients cannot be reliably interpreted without information not present in the regression summary.
- Finally, the interaction further complicates the interpretation of the effect of either income or type, because the interaction coefficients need to be interpreted jointly with the main effect coefficients.

Summarization of the effects of predictors using tables of coefficient estimates is often incomplete. Effects, and particularly plots of effects, can in many instances reveal the relationship of the response to the predictors more clearly. This conclusion is especially true for models with linear predictors that include interactions and multiple-coefficient terms such as regression splines and polynomials, as illustrated in this vignette.

A predictor effect plot summarizes the role of a selected *focal* predictor in a fitted regression model. The predictorEffect() function is used to compute the appropriate summary of the regression, and then the plot() function may be used to graph the resulting object, as in the following example:

```
R> library("effects")
R> e1.lm1 <- predictorEffect("education", lm1)
R> plot(e1.lm1)
```

education predictor effect plot



This graph visualizes the partial slope for education, that for each year increase in education, the fitted prestige increases by 2.959 points, when the other predictors are held fixed. The intercept of the line, which is outside the range of education on the graph, affects only the height of the line, is determined by the choices for averaging over the fixed predictors, but for any choice of averaging method, the slope of the line would be the same. The shaded area is a pointwise confidence band for the fitted values, based on standard errors computed from the covariance matrix of the fitted regression coefficients. The rug plot at the bottom of the graph shows the location of the education values.

The information that is needed to draw the plot is computed by the predictorEffect() function. The minimal arguments for predictorEffect() are the quoted name of a predictor in the model followed by the fitted model object. The essential purpose of this function is to compute fitted values from the model with education varying and all other predictors fixed at typical values (Fox and Weisberg, 2019a, Sec. 4.3). The command below displays the values of the regressors for which fitted values are computed, including a column of 1s for the intercept:

R> e1.lm1\$model.matrix

	(Intercept)	education	poly(women, 2)1	poly(women, 2)2	log(income)	typewc
1	1	6.38	0.000020998	-0.13496	8.8449	0.23469
2	1	6.58	0.000020998	-0.13496	8.8449	0.23469
3	1	6.77	0.000020998	-0.13496	8.8449	0.23469
4	1	6.97	0.000020998	-0.13496	8.8449	0.23469
5	1	7.16	0.000020998	-0.13496	8.8449	0.23469
6	1	7.36	0.000020998	-0.13496	8.8449	0.23469
7	1	7.55	0.000020998	-0.13496	8.8449	0.23469
8	1	7.75	0.000020998	-0.13496	8.8449	0.23469
9	1	7.95	0.000020998	-0.13496	8.8449	0.23469
10	1	8.14	0.000020998	-0.13496	8.8449	0.23469
11	1	8.34	0.000020998	-0.13496	8.8449	0.23469
12	1	8.53	0.000020998	-0.13496	8.8449	0.23469
13	1	8.73	0.000020998	-0.13496	8.8449	0.23469
14	1	8.92	0.000020998	-0.13496	8.8449	0.23469
15	1	9.12	0.000020998	-0.13496	8.8449	0.23469
16	1	9.32	0.000020998	-0.13496	8.8449	0.23469
17	1	9.51	0.000020998	-0.13496	8.8449	0.23469

```
18
              1
                     9.71
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
19
              1
                     9.90
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
20
                               0.000020998
                                                                   8.8449 0.23469
              1
                    10.10
                                                    -0.13496
21
              1
                    10.30
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
22
              1
                    10.50
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
23
              1
                    10.70
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
24
              1
                    10.90
                                                    -0.13496
                                                                   8.8449 0.23469
                               0.000020998
25
                                                                   8.8449 0.23469
              1
                    11.10
                               0.000020998
                                                    -0.13496
26
              1
                    11.30
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
27
                                                                   8.8449 0.23469
              1
                    11.50
                               0.000020998
                                                    -0.13496
28
              1
                    11.70
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
29
              1
                    11.90
                                                                   8.8449 0.23469
                               0.000020998
                                                    -0.13496
30
              1
                    12.10
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
31
              1
                                                                   8.8449 0.23469
                    12.30
                               0.000020998
                                                    -0.13496
                               0.000020998
32
                    12.40
                                                    -0.13496
                                                                   8.8449 0.23469
              1
33
              1
                    12.60
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
34
              1
                                                                   8.8449 0.23469
                    12.80
                               0.000020998
                                                    -0.13496
35
              1
                    13.00
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
36
                    13.20
                                                    -0.13496
                                                                   8.8449 0.23469
              1
                               0.000020998
37
              1
                    13.40
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
38
              1
                    13.60
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
39
              1
                    13.80
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
                               0.000020998
                                                    -0.13496
40
                    14.00
                                                                   8.8449 0.23469
              1
41
              1
                    14.20
                                                    -0.13496
                                                                   8.8449 0.23469
                               0.000020998
42
                                                                   8.8449 0.23469
              1
                    14.40
                               0.000020998
                                                    -0.13496
43
              1
                    14.60
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
44
              1
                    14.80
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
45
              1
                    15.00
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
46
              1
                    15.20
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
47
              1
                    15.40
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
48
              1
                    15.60
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
49
              1
                    15.80
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
50
                    16.00
                               0.000020998
                                                    -0.13496
                                                                   8.8449 0.23469
   typeprof log(income):typewc log(income):typeprof
1
    0.31633
                          2.0758
                                                 2.7979
2
    0.31633
                          2.0758
                                                 2.7979
3
    0.31633
                          2.0758
                                                 2.7979
4
    0.31633
                          2.0758
                                                 2.7979
5
    0.31633
                          2.0758
                                                 2.7979
6
    0.31633
                          2.0758
                                                 2.7979
7
    0.31633
                          2.0758
                                                 2.7979
8
    0.31633
                          2.0758
                                                 2.7979
9
    0.31633
                          2.0758
                                                 2.7979
10
    0.31633
                          2.0758
                                                 2.7979
    0.31633
11
                          2.0758
                                                 2.7979
12
    0.31633
                          2.0758
                                                 2.7979
13
    0.31633
                          2.0758
                                                 2.7979
14
    0.31633
                          2.0758
                                                 2.7979
15
    0.31633
                          2.0758
                                                 2.7979
16
    0.31633
                          2.0758
                                                 2.7979
17
    0.31633
                          2.0758
                                                 2.7979
18
   0.31633
                          2.0758
                                                 2.7979
19
    0.31633
                          2.0758
                                                 2.7979
20
    0.31633
                          2.0758
                                                 2.7979
```

21	0.31633	2.0758	2.7979		
22	0.31633	2.0758	2.7979		
23	0.31633	2.0758	2.7979		
24	0.31633	2.0758	2.7979		
25	0.31633	2.0758	2.7979		
26	0.31633	2.0758	2.7979		
27	0.31633	2.0758	2.7979		
28	0.31633	2.0758	2.7979		
29	0.31633	2.0758	2.7979		
30	0.31633	2.0758	2.7979		
31	0.31633	2.0758	2.7979		
32	0.31633	2.0758	2.7979		
33	0.31633	2.0758	2.7979		
34	0.31633	2.0758	2.7979		
35	0.31633	2.0758	2.7979		
36	0.31633	2.0758	2.7979		
37	0.31633	2.0758	2.7979		
38	0.31633	2.0758	2.7979		
39	0.31633	2.0758	2.7979		
40	0.31633	2.0758	2.7979		
41	0.31633	2.0758	2.7979		
42	0.31633	2.0758	2.7979		
43	0.31633	2.0758	2.7979		
44	0.31633	2.0758	2.7979		
45	0.31633	2.0758	2.7979		
46	0.31633	2.0758	2.7979		
47	0.31633	2.0758	2.7979		
48	0.31633	2.0758	2.7979		
49	0.31633	2.0758	2.7979		
50	0.31633	2.0758	2.7979		
atti	r(,"assign")				
[1]	0 1 2 2 3 4 4 5 5				
	r(,"contrasts")				
attr(,"contrasts")\$type					
[1]	"contr.treatment"				

The focal predictor education was evaluated by default at 50 points covering the observed range of values of education. For each value of education the remaining regressors have the same fixed values for each fitted value. The fixed value for log(income) is the logarithm of the sample mean income, the fixed values for the regressors for women are computed at the mean of women in the data, and the fixed values for the regressors for type effectively take a weighted average of the fitted values at the three levels of type, with weights proportional to the number of cases in each level of the factor. Differences in the fitted values are due to education alone because all the other predictors, and their corresponding regressors, are fixed. Thus the output gives the partial effect of education with all other predictors fixed.

The computed fitted values can be viewed by printing the "eff" object returned by predictor-Effect(), by summarizing the object, or by converting it to a data frame:

```
R > e1.lm1
```

```
education predictor effect
education effect
education
```

6.38 6.58 6.77 6.97 7.16 7.36 7.55 7.75 7.95 8.14 8.34 35.805 36.397 36.959 37.550 38.113 38.704 39.267 39.858 40.450 41.012 41.604 8.53 8.73 8.92 9.12 9.32 9.51 9.71 9.9 10.1 10.3 10.5 42.166 42.758 43.320 43.912 44.504 45.066 45.658 46.220 46.811 47.403 47.995 10.7 10.9 11.1 11.3 11.5 11.7 11.9 12.1 12.3 12.4 12.6 48.587 49.178 49.770 50.362 50.954 51.545 52.137 52.729 53.321 53.617 54.208 12.8 13 13.2 13.4 13.6 13.8 14 14.2 14.4 14.6 14.8 54.800 55.392 55.984 56.575 57.167 57.759 58.351 58.942 59.534 60.126 60.718 15 15.2 15.4 15.6 15.8 16 61.309 61.901 62.493 63.085 63.676 64.268

R> summary(e1.lm1)

education effect

education

6.38 6.58 6.77 6.97 7.16 7.36 7.55 7.75 7.95 8.14 8.34 35.805 36.397 36.959 37.550 38.113 38.704 39.267 39.858 40.450 41.012 41.604 8.53 8.73 8.92 9.12 9.32 9.51 9.71 9.9 10.1 10.3 10.5 42.166 42.758 43.320 43.912 44.504 45.066 45.658 46.220 46.811 47.403 47.995 10.7 10.9 11.1 11.3 11.5 11.7 11.9 12.1 12.3 12.4 12.6 48.587 49.178 49.770 50.362 50.954 51.545 52.137 52.729 53.321 53.617 54.208 12.8 13 13.2 13.4 13.6 13.8 14 14.2 14.4 14.6 14.8 54.800 55.392 55.984 56.575 57.167 57.759 58.351 58.942 59.534 60.126 60.718 15 15.2 15.4 15.6 15.8 16 61.309 61.901 62.493 63.085 63.676 64.268

Lower 95 Percent Confidence Limits education

6.58 6.77 6.97 7.16 7.36 7.55 7.75 7.95 6.38 8.14 29.850 30.650 31.408 32.204 32.958 33.749 34.498 35.284 36.065 36.804 37.577 8.53 8.73 8.92 9.12 9.32 9.51 9.71 9.9 10.1 10.3 10.5 38.307 39.069 39.786 40.534 41.272 41.964 42.682 43.351 44.040 44.714 45.370 10.7 10.9 11.1 11.3 11.5 11.7 11.9 12.1 12.3 12.4 12.6 46.006 46.622 47.218 47.792 48.346 48.881 49.397 49.896 50.380 50.617 51.081 13 13.2 13.4 13.6 13.8 14 14.2 14.4 14.6 51.534 51.976 52.410 52.837 53.256 53.670 54.079 54.483 54.883 55.280 55.674 15 15.2 15.4 15.6 15.8 16 56.065 56.453 56.840 57.224 57.607 57.989

Upper 95 Percent Confidence Limits education

6.38 6.58 6.77 6.97 7.16 7.36 7.55 7.75 7.95 8.14 8.34 41.760 42.143 42.510 42.897 43.267 43.660 44.035 44.433 44.835 45.220 45.631 8.53 8.73 8.92 9.12 9.32 9.51 9.71 9.9 10.1 10.3 10.5 46.026 46.447 46.854 47.290 47.735 48.167 48.633 49.089 49.582 50.092 50.620 10.7 10.9 11.1 11.3 11.5 11.7 11.9 12.1 12.3 12.4 12.6 51.167 51.735 52.323 52.932 53.561 54.210 54.877 55.562 56.261 56.616 57.336 12.8 13 13.2 13.4 13.6 13.8 14 14.2 14.4 14.6 14.8 58.066 58.807 59.557 60.314 61.078 61.848 62.623 63.402 64.185 64.972 65.761 15 15.2 15.4 15.6 15.8 16 66.554 67.349 68.146 68.945 69.745 70.548

R> as.data.frame(e1.lm1)

```
education
                fit
                        se lower upper
        6.38 35.805 2.9970 29.850 41.760
1
2
        6.58 36.397 2.8923 30.650 42.143
3
        6.77 36.959 2.7937 31.408 42.510
4
             37.550 2.6909 32.204 42.897
5
        7.16 38.113 2.5943 32.958 43.267
6
        7.36 38.704 2.4939 33.749 43.660
7
        7.55 39.267 2.3998 34.498 44.035
8
        7.75 39.858 2.3024 35.284 44.433
9
        7.95 40.450 2.2068 36.065 44.835
10
        8.14 41.012 2.1179 36.804 45.220
        8.34 41.604 2.0266 37.577 45.631
11
12
        8.53 42.166 1.9423 38.307 46.026
        8.73 42.758 1.8566 39.069 46.447
13
14
        8.92 43.320 1.7785 39.786 46.854
15
        9.12 43.912 1.7000 40.534 47.290
16
        9.32 44.504 1.6262 41.272 47.735
17
        9.51 45.066 1.5608 41.964 48.167
18
        9.71 45.658 1.4977 42.682 48.633
19
        9.90 46.220 1.4440 43.351 49.089
20
       10.10 46.811 1.3946 44.040 49.582
21
       10.30 47.403 1.3534 44.714 50.092
22
       10.50 47.995 1.3213 45.370 50.620
23
       10.70 48.587 1.2988 46.006 51.167
       10.90 49.178 1.2865 46.622 51.735
24
25
       11.10 49.770 1.2846 47.218 52.323
26
       11.30 50.362 1.2932 47.792 52.932
27
       11.50 50.954 1.3122 48.346 53.561
       11.70 51.545 1.3410 48.881 54.210
28
29
       11.90 52.137 1.3790 49.397 54.877
30
       12.10 52.729 1.4256 49.896 55.562
31
       12.30 53.321 1.4798 50.380 56.261
32
       12.40 53.617 1.5096 50.617 56.616
33
       12.60 54.208 1.5739 51.081 57.336
34
       12.80 54.800 1.6439 51.534 58.066
35
       13.00 55.392 1.7189 51.976 58.807
36
       13.20 55.984 1.7983 52.410 59.557
37
       13.40 56.575 1.8816 52.837 60.314
38
       13.60 57.167 1.9683 53.256 61.078
39
       13.80 57.759 2.0578 53.670 61.848
       14.00 58.351 2.1500 54.079 62.623
40
41
       14.20 58.942 2.2444 54.483 63.402
       14.40 59.534 2.3407 54.883 64.185
42
       14.60 60.126 2.4388 55.280 64.972
43
       14.80 60.718 2.5385 55.674 65.761
44
       15.00 61.309 2.6395 56.065 66.554
45
46
       15.20 61.901 2.7417 56.453 67.349
47
       15.40 62.493 2.8450 56.840 68.146
48
       15.60 63.085 2.9493 57.224 68.945
49
       15.80 63.676 3.0544 57.607 69.745
50
       16.00 64.268 3.1604 57.989 70.548
```

The values in the column education are the values the focal predictor. The remaining columns are the fitted values, their standard errors, and lower and upper end points of 95% confidence intervals

for the fitted values. The *predictor effect plot* is simply a graph of the fitted values on the vertical axis versus the focal predictor on the horizontal axis. For a continuous focal predictor such as **education**, a line, in this case, a straight line, is drawn connecting the fitted values.

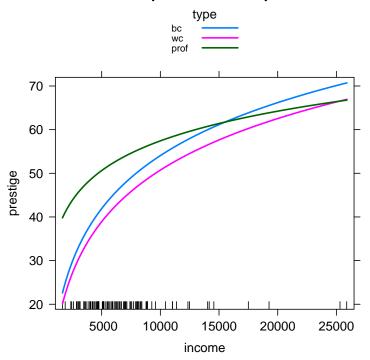
We turn next to the predictor effect plot for income. According to the regression model, the effect of income may depend on type due to the interaction between the two predictors, so simply averaging over type would be misleading. Rather, we should allow both income and type to vary, fixing the other predictors at their means or other typical values. This would require evaluating the model at $50 \times 3 = 150$ combinations of the predictors, but to save space we will only evaluate income at five values using the focal.levels argument, thus evaluating only $5 \times 3 = 15$ fitted values.

R> e2.lm1 <- predictorEffect("income", lm1, focal.levels=5)
R> as.data.frame(e2.lm1)

```
income type
                  fit
                          se
                             lower upper
1
     2000
            bc 25.863 3.3037 19.299 32.428
2
     8000
            bc 50.142 2.3737 45.426 54.859
3
   10000
            bc 54.050 2.7996 48.487 59.613
4
   20000
           bc 66.190 4.4814 57.285 75.094
5
   30000
            bc 73.291 5.5708 62.222 84.360
6
            wc 23.290 4.5674 14.214 32.365
     2000
7
     8000
            wc 46.922 2.3106 42.331 51.513
8
   10000
            wc 50.726 3.0575 44.651 56.802
9
   20000
            wc 62.543 5.7716 51.075 74.011
10
   30000
            wc 69.455 7.4432 54.665 84.244
11
    2000 prof 41.630 4.4812 32.726 50.534
    8000 prof 55.237 2.3316 50.605 59.870
12
13
   10000 prof 57.428 2.4552 52.549 62.306
   20000 prof 64.231 3.6170 57.045 71.418
14
15 30000 prof 68.211 4.5680 59.135 77.288
```

To draw the predictor effects plot we recalucluate the fitted values using the default focal.values=50 to get more accurate curves:

income predictor effect plot

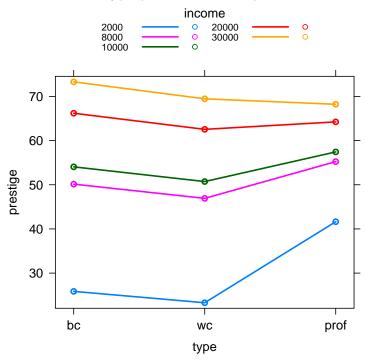


The focal predictor income is displayed on the horizontal axis. There is a separate line shown for the fitted values at each level of type. The lines are curved rather than straight because income appears in the model in log-scale but is displayed in the predictor effect plot in arithmetic (i.e., dollar) scale. The lines in the graph are not parallel because of the interaction between log(income) and type. For type = "prof", the fitted values of prestige are relatively high for lower values of income, and are relatively less affected by increasing values of income. If the number of levels for evaluating income were five rather than 50 the lines on the plot would be somewhat less accurate.

The predictor effect plot for type uses the same fitted values as the plot for income, but we now get five lines, one for each of the values of income selected by the predictorEffect() function:

R> plot(predictorEffect("type", lm1), lines=list(multiline=TRUE))

type predictor effect plot



Here we use both the predictorEffect() and plot() functions in the same command. Because the horizontal axis is now a factor, the fitted values are displayed explicitly as points, and the lines that join the points are merely a visual aid representing *profiles* of fitted values. Fitted prestige increases with income for all levels of type, but, as we found before, when type = "prof", fitted prestige is relatively high for lower income.

These initial examples use only default arguments for predictorEffect() and plot(), apart from the multiline argument to plot() to put all the fitted lines in the same graph. We explain how to customize predictor effect plots in subsequent sections of this vignette.

1.2 General Outline for Constructing Predictor Effect Plots

Using the **effects** package to draw plots usually entails the following steps:

- Fit a regression model with a linear predictor. The package supports models created by lm(), glm(), lmer() and glmer() in the lme4 package, lme() in the nlme package, and many other regression-modeling functions (see ?Effect).
- 2. The regression model created in the first step is then used as input to either predictorEffect(), to get the effects for one predictor, or predictorEffects, to get effects for one or more predictors. These functions do the averaging needed to get fitted values that will ultimately be plotted. There are many arguments for customizing the computation of the effects. The two predictor effect functions call the more basic Effect() function, and almost all of the material in this vignette applies to Effect() as well.
- 3. Use the generic plot() function to draw a graph or graphs based on the object created in Step 2.

1.3 How predictorEffect() Chooses Conditioning Predictors

Suppose that you select one focal predictor for which you want to draw a predictor effect plot. The predictorEffect() function divides the predictors in a model formula into three groups:

- 1. The focal predictor.
- 2. The *conditioning group*, consisting of all predictors with at least one interaction in common with the focal predictor.
- 3. The *fixed group*, consisting of all other predictors, that is, those with no interactions in common with the focal predictor.

The predictors in the fixed group are all evaluated at typical values, usually their means, effectively averaging out the influence of these predictors on the fitted value. Fitted values are computed for all combinations of levels of the predictors in the conditioning group, with each continuous predictor in the conditioning group replaced by a few discrete values spanning the range of the predictor, for example, replacing years of education by a discrete variable with the values 8, 12, and 16 years.

Suppose that we fit a model with R formula

$$y \sim x1 + x2 + x3 + x4 + x2:x3 + x2:x4$$
 (1)

or, equivalently,

$$y \sim x1 + x2*x3 + x2*x4$$

There are four predictor effect plots for this model, one for each predictor selected in turn as the focal predictor:

Focal	Conditioning	Fixed
Predictor	Group	Group
x1	none	x2, x3, x4
x2	x3, x4	x1
x3	x2	x1, x4
x4	x2	x1 x3

The predictor x1 does not interact with any of the other predictors, so its conditioning set is empty and all the remaining predictors are averaged over; x2 interacts with both x3 and x4; x3 interacts only with x2; and x4 interacts with x2.

1.4 The Effect() Function

Prior to late 2018, the primary function in **effects** for computing and displaying effects was the Effect() function. Whereas the predictorEffect() function automatically determines the conditioning group and the fixed group of predictors, the Effect() function puts that burden on the user. Each call to predictorEffect() is equivalent to a specific call to the Effect() function as follows. Suppose that m is the fitted model produced by the formula in (1); then

```
predictorEffect("x1", m) is equivalent to Effect("x1", m);
predictorEffect("x2", m) is equivalent to Effect(c("x2", "x3", "x4"), m);
predictorEffect("x3", m) is equivalent to Effect(c("x3", "x2"), m); and
predictorEffect("x4", m) is equivalent to Effect(c("x4", "x2"), m).
```

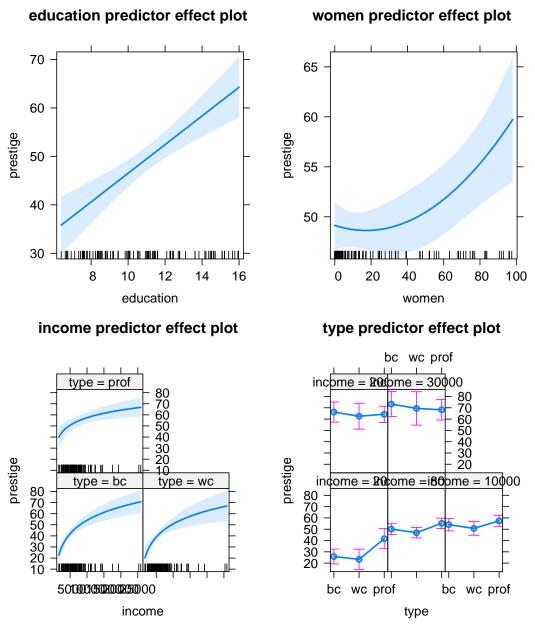
The predictorEffect() function determines the correct call to Effect() based on the choice of focal predictor and on the structure of main effects and interactions in the linear predictor for the model. It then uses the Effect() function to do the computing. As a result, most of the arguments to predictorEffect() are documented in help("Effect") rather than in help("predictorEffect").

¹The **effects** package also includes the older allEffects() function, which computes effects for each high-order term in a model with a linear predictor. As we explain in Fox and Weisberg (2019b), we prefer predictor effects to high-order term effects, and so, although its use is similar to predictorEffects(), we won't describe allEffects() in this vignette. There is also an older effect() function (with a lowercase "e"), which is a less flexible version of Effect(), and which calls Effect() to perform computations; effect() is retained only for backwards comparability.

1.5 The predictorEffects() Function

This function, whose name ends with the plural "plots", computes the values needed for one or more predictor effect plots, and by default for *all* of the predictors in the model. For example, the following command produces all of the predictor effect plots for the model we fit to the Prestige data:

R> eall.lm1 <- predictorEffects(lm1)
R> plot(eall.lm1)



The predictor effect plots for this model are displayed in an array of graphs. The plots for income and type have a separate panel for each level of the conditioning variable because the default argument lines=list(multiline=FALSE) was used. Confidence bounds are shown by default when multiline=FALSE.

The resulting object eall.lm1 is a list with four elements, where eall.lm1[[1]] is the summary for the first predictor effect plot, eall.lm1[[2]] for the second plot, and so on. The following

equivalent commands draw the same array of predictor effect plots:

```
R> plot(eal1.lm1)
R> plot(predictorEffects(lm1))
R> plot(predictorEffects(lm1, ~ income + education + women + type))

If you want only the predictor effect plots for type and education, in that order, you could enter
R> plot(predictorEffects(lm1, ~ type + education))

Similarly, the commands
R> plot(predictorEffects(lm1, ~ women))
R> plot(predictorEffects(lm1)[[3]])
R> plot(predictorEffect("women",lm1))
```

all produce the same graph, the predictor effect plot for women.

Predictor effect plots in an array can be a useful shortcut for drawing many graphs quickly, but can lead to problems with the displayed graphs. For example, the horizontal axis labels for the plot for <code>income</code> are overprinted, and the labels at the top of the plots for <code>type</code> with conditioning variable <code>income</code> are larger than the available space. These problems can often be fixed using optional arguments described later in this vignette.

2 Optional Arguments for the predictorEffect() and Effect() Functions

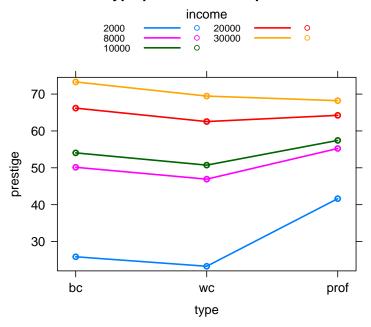
This section comprises a catalog of the arguments available to modify the behavior of the predictorEffect() and Effect() functions. The information provided by help("Effect") is somewhat more comprehensive, if terser, explaining for example exceptions applying to "svyglm" objects or for plotting residuals.

2.1 focal.levels and xlevels: Options for the Focal Predictor and Predictors in the Conditioning Group

Numeric predictors in the conditioning group need to be discretized to draw a predictor effect plot. For example the predictor effect plot for type in model lm1 consists of a separate line, or a separate panel, for each discrete value of income:

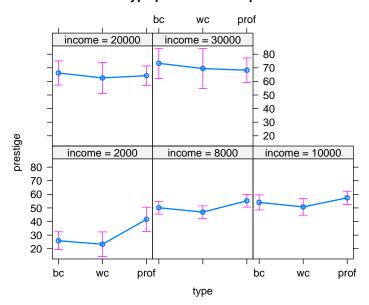
```
R> e3.lm1 <- predictorEffect("type", lm1)
R> plot(e3.lm1, lines=list(multiline=TRUE))
```

type predictor effect plot



R> plot(e3.lm1, lines=list(multiline=FALSE)) # the default

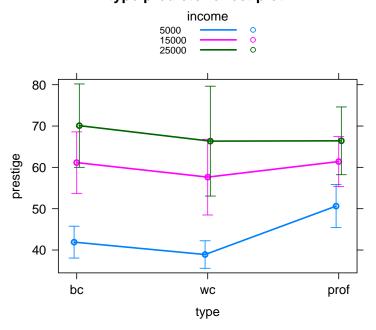
type predictor effect plot



The numeric conditioning predictor income is evaluated by default at five equally spaced values, and then rounded to "nice" numbers.

In this example using three values of 5000, 15000, 25000 for the conditioning predictor income produces a simpler graph:

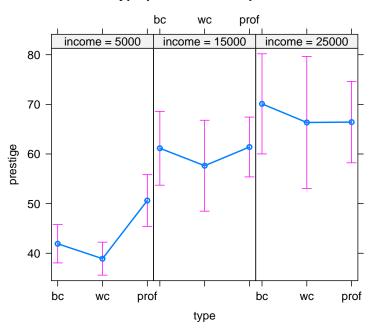
type predictor effect plot



R> plot(e3.lm1,

- + lines=list(multiline=FALSE), # the default
- + lattice=list(layout=c(3, 1)))

type predictor effect plot



The argument xlevels is a list of sub-arguments that control how numeric predictors are discretized when used in the conditioning group. For example, xlevels=list(x1=c(2, 4, 7), x2=6) would use the values 2, 4, and 7 for the levels of the predictor x1, use 6 equally spaced values for the predictor x2, and use the default of 5 values for any other numeric predictors. Numeric predictors in the *fixed* group are not affected by the xlevels argument. We use the layout sub-argument of the lattice argument to arrange the panels of the second graph in 3 columns and 1 row (see Section 3.4.2).

See help("plot.eff") for information on the quantiles argument, which provides an alternative method of setting xlevels when partial residuals are displayed, as discussed in Section 4.

The number of points at which a numeric focal predictor is evaluated is controlled by the focal.levels argument. The default of focal.levels=50 is recommended for drawing graphs, but if the goal is to produce a table of fitted values a smaller value such as focal.values=5 produces more compact output. With the predictorEffects function the focal.levels can be set separately for each focal predictor; see help("predictorEffects").

2.2 fixed: Options for Predictors in the Fixed Group

Predictors in the fixed group are replaced by "typical" values of the predictors. Fitted values are then computed using these typical values for the fixed group, varying the values of predictors in the conditioning group and of the focal predictor. The user can control how the fixed values are determined by specifying the fixed.predictors argument. This argument takes a list of subarguments that allow for controlling each predictor in the fixed group individually, with different rules for factors and numeric predictors.

2.2.1 Factor Predictors

For a fixed factor, imagine computing the fitted values evaluating the factor at each of its levels. The fitted value that is used is the weighed average of these within-level fitted values, with weights proportional to the number of observations at each level of the factor. This is the default approach, and is an appropriate notion of "typical" if the data at hand are viewed as a random sample from a population, and so the sample fraction at each level estimates the population fraction.

A second approach is to average the level-specific fitted values with equal weights at each level. This may be appropriate, for example, in designed experiments in which the levels of a factor are assigned by an investigator. The latter method is invoked by setting fixed.predictors=list(given.values="equal").

You can construct other weighting schemes for averaging over the levels of a factor, as described on the help page for the Effect() function.

2.2.2 Numeric Predictors

For numeric predictors in the fixed group, the default method of selecting a typical value is to apply the mean() function to the sample values of each predictor. The specification fixed.predictors=list(typical=median) would instead use the median() function; in general, typical can be any function that takes a numeric vector as its argument and returns a single number.

Other sub-arguments to fixed.predictors apply to the use of offsets, and to the survey package; see the help page for the Effect() function.

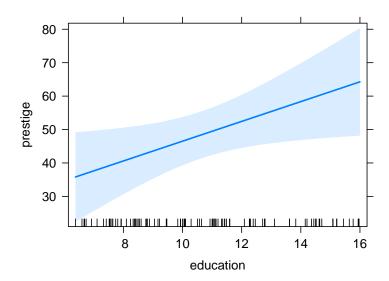
2.3 se: Standard Errors and Confidence Intervals

Standard errors and confidence intervals for fitted values are computed by default. The default corresponds to setting the argument se=list(compute=TRUE, type="pointwise", level=.95). Setting se=FALSE omits standard errors, type="scheffe" uses wider Scheffé intervals that adjust for simultaneous inference, and level=.8, for example, produces 80% intervals.

Standard errors are based by default on the "usual" sample covariance matrix of the estimated regression coefficients. You can replace the default coefficient covariance matrix with some other estimate, such as one obtained from the bootstrap or a sandwich coefficient covariance matrix estimator, by setting the vcov. argument either to a function that returns a coefficient covariance matrix, such as hccm() in the car package for linear models, or to a matrix of the correct size; for example:

+ vcov.=hccm)
R> plot(e4.lm1)

education predictor effect plot



This plot displays 99% Scheffé intervals based on a robust coefficient covariance matrix computed by the sandwich method; see help("hccm").

2.4 residuals: Computing Residuals

The argument residuals=TRUE computes and saves residuals, providing the basis for adding partial residuals to subsequent effect plots, a topic that we discuss in Section 4.

3 Arguments for Plotting Predictor Effects

The arguments described in Section 2 are for the predictorEffect() function or the Effect() function. Those arguments modify the computations that are performed, such as methods for averaging and fixing predictors, and for computing standard errors. Arguments to the plot() methods for the predictor effect and effect objects produced by the predictorEffect() and Effect() functions are described in this section, and these change the appearance of an effect plot or modify the quantities that are plotted. These optional arguments are described in more detail in help("plot.eff").

In late 2018, we reorganized the plot() method for effect objects by combining arguments into five major groups of related sub-arguments, with the goal of simplifying the specification of effect plots. For example, the lines argument group is a list of sub-arguments for determining line type, color, and width, whether or not multiple lines should be drawn on the same graph, and whether plotted lines should be smoothed. The defaults for these sub-arguments are the choices we generally find the most useful, but they will not be the best choices in all circumstances. The cost of reorganizing the arguments in this manner is the necessity of specifying arguments as lists, some of whose elements are themselves lists, requiring the user to make sure that parentheses specifying the possibly nested lists properly balanced.

In addition to the five argument groups that we describe below, the plot() method for effect objects accepts the arguments main for the main title of the graph and id for identifying points in effect plots that include residuals as discussed in Section 4.

Finally, the plot() method for effect objects retains a number of "legacy" arguments shown in help("plot.eff"). These arguments have been kept so existing scripts using the effects package would not break, but they are all duplicated as sub-arguments of the five argument groups. The

legacy arguments work but they may not be supported forever, so we encourage you to use the newer argument groups and sub-arguments.

3.1 axes Group: Specify Axis Characteristics

The axes argument group has two major sub-arguments, x for the horizontal axis, y for the vertical axis, and two minor sub-arguments, the grid argument, which adds a background grid to the plot, and the alternating argument, for changing the placement of axis-tick labels in multi-panel plots.

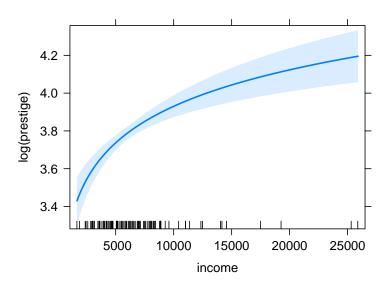
3.1.1 x: Horizontal Axis Specification

We introduce another linear model fit to the Prestige data set to serve as an example:

The default predictor effect plot for income is

R> plot(predictorEffects(lm2, ~ income))

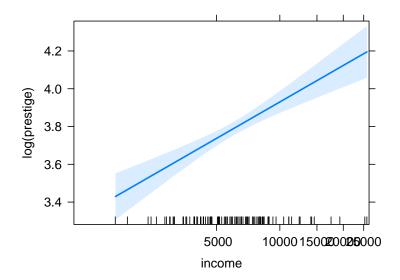
income predictor effect plot



The plot is curved because the predictor income is represented by its logarithm in the model formula, but the default predictor effect plot uses the predictor income, not the regressor log(income), on the horizontal axis. The x sub-argument can be used transform the horizontal axis, for example to replace income by log(income):

```
R> plot(predictorEffects(lm2, ~ income),
+ axes=list(
+ x=list(income=list(transform=list(trans=log, inverse=exp)))
+ ))
```

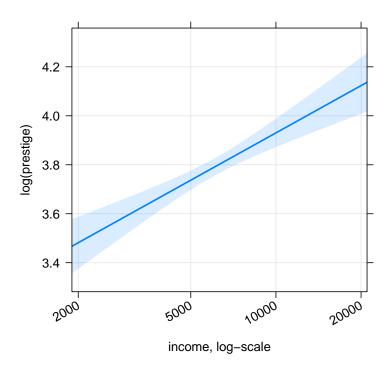
income predictor effect plot



The transformation changes the scale on the horizontal axis to log-scale, but leaves the tick labels in arithmetic scale, and the graph is now a straight line because of the change to log-scale. This plot has several obviously undesirable features with regard to the range of the horizontal axis and over-printing of tick marks. We show next that additional arguments to plot() can correct these defects.

A more elaborate version of the graph illustrates all the sub-arguments to x in lines:

Transformed Plot



We use the top-level argument main="Transformed Plot" to set the title of the plot. The axes argument is a list with two sub-arguments, grid to turn on the background grid, and x to modify the horizontal axis.

The x sub-argument is itself a list with three elements: The sub-arguments rotate and rug set the rotation angle for the tick labels and suppress the rug plot, respectively. The additional sub-argument is a list called income, the name of the focal predictor. If you were drawing many predictor effect plots you would supply one list named for each of the focal predictors. All of the sub-arguments for income are displayed in the example code above. The sub-argument transform=list(trans=log, inverse=exp) specifies how to transform the x-axis. The ticks and lim sub-arguments set the tick marks and range for the horizontal axis.

This is admittedly a *complex command*, but it allows you to fine-tune the graph to look the way you want. In specifying nested argument lists, you may encounter problems getting the parentheses in the right places. Be careful, indent your code to clarify the structure of the command, and be patient!

3.1.2 y: Vertical Axis Specification for Linear Models

The model lm2 has a transformed response log(prestige), and "untransforming" the response to arithmetic scale may be desirable. This can be accomplished with the y sub-argument, which has two sub-arguments named transform and type that together control the scale and labeling of the vertical axis.

There are three options for drawing the predictor effect plot for a numeric response like log(prestige):

```
axes=list(y=list(transform=list(trans=log, inverse=exp),
                               lab="prestige", type="rescale")))
   # Replace log(presige) by prestige:
   plot(predictorEffects(lm2, ~ education),
           main="Prestige in Arithmethic Scale",
           axes=list(y=list(transform=exp, lab="prestige")))
+
                                                                         Prestige in Arithmethic Scale
           Default log(prestige)
                                        log(prestige), Arithmetic Ticks
                                      70
      4.2
                                                                      70
                                      60
                                                                      60
      4.0
                                      50
      3.8
                                                                      50
```

40

3.6

10 12

education

14

The first plot is the default, with a log-response. In the second plot, the transform sub-argument specifies the transformation of the response and its inverse, and the sub-argument type="rescale" changes the tick marks on the vertical axis to arithmetic scale. In the third version, with transform=exp, lab="prestige", the vertical axis now is in arithmetic scale, not log scale, although that may not be completely obvious in the example because $\log(x)$ is nearly linear over the range of approximately 35 to 65 for fitted prestige in this graph: Look closely to see that the axis ticks marks in the second graph are unequally spaced, while those in the third graph are equally spaced and the plotted line in the latter is slightly curved. The help page ?plot.eff provides a somewhat more detailed explanation of these options.

8 10 12 14

education

40 30

> 10 12

education

14

As a second example we will reconstruct Figure 7.10 in Fox and Weisberg (2019a, Sec. 7.2). In that section, we fit a linear mixed-effects model to data from the Blackmore data frame in the carData package. Blackmore includes longitudinal data on amount of exercise for girls hospitalized for eating disorders and for similar control subjects who were not hospitalized. We transformed the response variable in the model, hours of exercise, using a transformation in a modified Box-Cox power family that allows zero or negative responses, explained briefly by Fox and Weisberg (2019a, Sec. 3.4) and more thoroughly by Hawkins and Weisberg (2017). The fitted model is

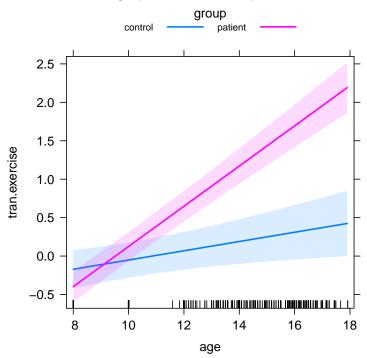
```
R> library("lme4") # for lmer()
Loading required package: Matrix
R> Blackmore$tran.exercise <- bcnPower(Blackmore$exercise,
                                          lambda=0.25, gamma=0.1)
  mm1 <- lmer(tran.exercise ~ I(age - 8)*group +
R>
                   (I(age - 8) | subject), data=Blackmore)
```

This model, with numeric predictor age and factor predictor group, is a linear mixed model with random intercepts and slopes for age that vary by subject. The response variable is a transformation of exercise similar to the fourth root with adjustment for zero values; see help("bcnPower").

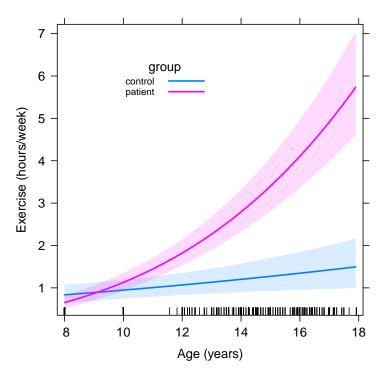
The predictor effect plot for the fixed effect of age is

```
R> e1.mm1 <- predictorEffect("age", mm1)</pre>
R> plot(e1.mm1, lines=list(multiline=TRUE), confint=list(style="auto"))
```

age predictor effect plot



The plot clearly shows the difference between the "control" and "patient" groups, with the fitted response for the latter having a larger slope. The graph is hard to decode, however, because the vertical axis is approximately in the scale of the fourth-root of hours of exercise, so untransforming the response may produce a more informative plot. Because the bcnPower() transformation is complex, the car package includes the function bcnPowerInverse() to reverse the transformation:



The response scale is now in hours per week, and we see that hours of exercise increase more quickly on average in the patient group for older subjects. We use additional arguments in this plot to match Fox and Weisberg (2019a, Fig. 7.10), including moving the key inside of the graph (see Section 3.4.1), changing the axis labels, and removing the main title to the plot.²

3.1.3 y: Vertical Axis Specification for Generalized Linear Models

Transforming the vertical axis for generalized linear models also uses the y sub-argument to the axes argument. You typically do not need to specify the transform argument because plot() obtains the right functions from the regression model's family component. The type sub-argument has the same three possible values as for linear models, but their interpretation is somewhat different:

- 1. Effect plots in type="link" or linear predictor scale, in which the horizontal axis of each plot is a predictor and the vertical axis is in the scale of the linear predictor. For logistic regression, for example, the vertical axis is in log-odds (logit) scale. For Poisson regression with the log-link, the vertical axis is in log-mean (log-count) scale.
- 2. Predictor effect plots in type="response" or mean scale are obtained by "untransforming" the y axis using the inverse of the link function. For the log-link, this corresponds to transforming the y axis and plotting $\exp(y)$. For logistic regression, $y = \log[p/(1-p)]$ and, solving for p, $p = \exp(y)/[1 + \exp(y)] = 1/[1 + \exp(-y)]$, so the plot in mean scale uses $1/[1 + \exp(-y)]$ on the vertical axis.
- 3. We also provide a third option, type="rescale", which plots in linear predictor (e.g., logit) scale, but labels the tick marks on the vertical axis in mean (e.g., probability) scale. This third option, which retains the linear structure of the model but labels the vertical axis on the usually more familiar mean scale, is the default.

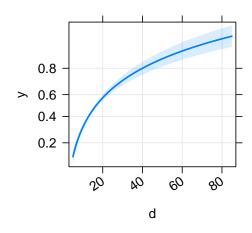
We use the Blowdown data from the alr4 package to provide examples. These data concern the probability of blowdown y, a tree being uprooted as the result of a major straight-line wind storm

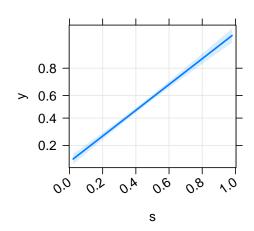
²The code shown for this graph in Fox and Weisberg (2019a) uses "legacy" arguments, and is therefore somewhat different from the code given here. Both commands produce the same plot, however.

in the Boundary Waters Canoe Area Wilderness in 1999, modeled as a function of the diameter d of the tree, the local severity **s** of the storm, and the species **spp** of the tree. We fit a main-effects model and then display all three predictor effect plots:

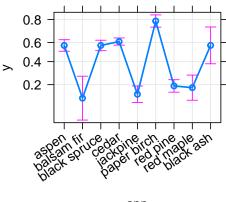
d predictor effect plot

s predictor effect plot





spp predictor effect plot



spp

The rug=FALSE sub-argument to x suppresses the rug plot that appears by default at the bottom of graphs for numeric predictors, and the grid sub-argument to axes adds background grids. The rotate sub-argument prints the horizontal tick labels at an angle to avoid overprinting.

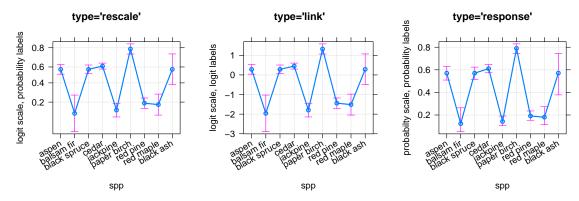
Interpretation of GLM predictor effect plots in link scale is similar to predictor effect plots for linear models, and all the modifications previously described can be used for these plots. Because the default is type="rescale", the vertical axis is in linear predictor scale, which is the log-odds for logit for this logistic regression example, but the vertical axis labels are in probability scale, so the tick-marks are not equally spaced.

The next three graphs illustrate the possible values of the argument type:

```
+ lab="logit scale, probability labels"),
+ x=list(rotate=30),
+ grid=TRUE))

R> plot(e1.gm1, main="type='link'",
+ axes=list(y=list(type="link",
+ lab="logit scale, logit labels"),
+ x=list(rotate=30),
+ grid=TRUE))

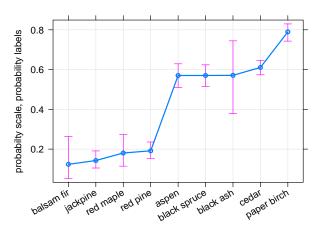
R> plot(e1.gm1, main="type='response'",
+ axes=list(y=list(type="response", grid=TRUE,
+ lab="probability scale, probability labels"),
+ x=list(rotate=30),
+ grid=TRUE))
```



The first two graphs show the same plot, but in the first the tick-marks on the vertical axis are unequally spaced and are in probability scale, while in the second the tick-marks are equally spaced and are in log-odds scale. In the third graph, the vertical axis as been transformed to probability scale, and the corresponding tick-marks are now equally spaced.

The plot for species would be easier to understand if the levels of the factor were ordered according to the estimated log-odds of blowdown. First, we need to recover the fitted values in link scale, which are log-odds of blowdown for a logistic model. The fitted log-odds are stored in as.data.frame(e1.gm1)\$fit using the e1.gm1 object previously computed:

type='response', ordered



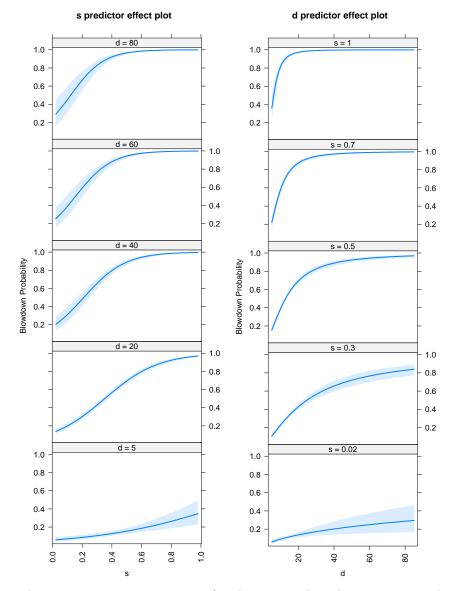
The separation of species into two groups of lower and higher probability species is reasonably clear after ordering, with paper birch more susceptible to blowdown than the other species and possibly in a group by itself.

3.2 lines Group: Specifying Plotted Lines

The lines argument group allows the user to specify the color, type, thickness, and smoothness of lines. This can be useful, for example, if the colors used by **effects** by default are for some reason unacceptable, such as for publications in which only black or gray-scale lines are permitted. The most common use of this argument group is to allow more than one line to be plotted on the same graph or panel via the multiline sub-argument.

3.2.1 multiline and z.var: Multiple Lines in a Plot

Default predictor effect plots with conditioning predictors generate a separate plot for each level of the conditioning variable, or each combination of levels if there is more than one conditioning variable. For an example, we add the log(d):s interaction to the model gm1, and generate the predictor effect plots for s and for d:

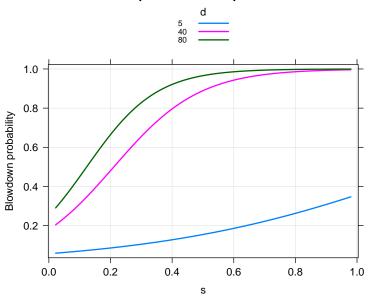


Setting the sub-argument type="response" for the y axis plots the response on the probability scale. Setting layout=c(1, 5) arranges each predictor effect plot in 1 column of 5 rows. See the description of the lattice argument in Section 3.4.

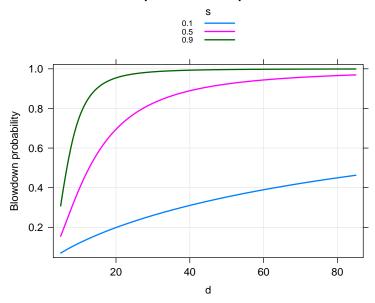
The predictor effect plot for s conditions on the level of d, and displays the plot of the fitted values for y versus s in a separate panel for each value of d. Similarly, the predictor effect plot for d displays a separate panel for each conditioning level of s. Confidence bands are displayed by default around each fitted line. These two graphs are based on the same fitted values, with the values of the interacting predictors s and d varying, and fixing the factor predictor spp to its distribution in the data, as described in Section 2.2.1. Concentrating on the graph at the right for the focal predictor d, when s is very small the probability of blowdown is estimated to be in the range of about .05 to .3 for any value of d, but for larger values of s, the probability of blowdown increases rapidly with d. Similar comments can be made concerning the predictor effect plot for s.

Setting multiline=TRUE superimposes the lines for all the conditioning values in a single graph. In the example below, we reduce the number of levels of the conditioning variable for each predictor effect plot to three each to produce simpler graphs, although this is not required. The xlevels argument changes the number of levels for the conditioning predictors, but does not affect the number of levels for the focal predictor. This latter quantity could be changed with the focal.levels argument, but the default value of 50 evaluations is apparopriate for graphing effects.

s predictor effect plot



d predictor effect plot

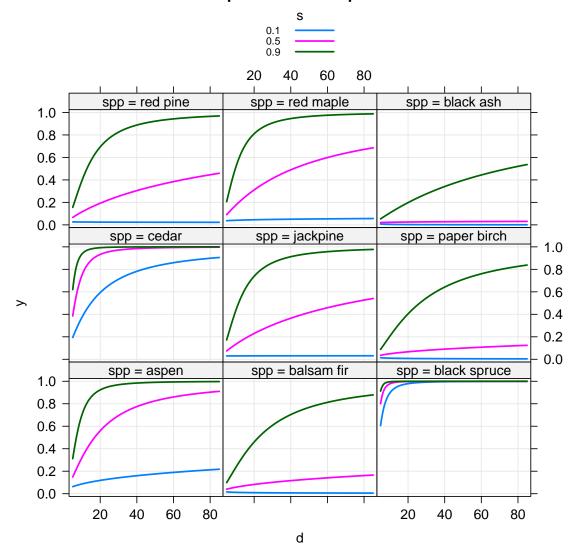


In each graph, we kept, more or less, the lowest, middle, and highest values of the conditional predictor for the interaction. We also added a grid to each graph. Multiline plots by default omit

confidence bands or intervals, but these can be included using the **confint** argument discussed in Section 3.3. By default, different values of the conditioning predictor are distinguished by color, and a key is provided. The placement and appearance of the key are controlled by the **key.args** sub-argument in the **lattice** group discussed in Section 3.4.1.

When the conditioning group includes two or more predictors, and certainly when it includes three or more predictors, multiline plots are almost always helpful because otherwise the resulting array of panels becomes too complicated. Suppose that we add the spp:log(d) interaction to the model. The predictor effect plot for d now includes both s and spp in the conditioning set because d interacts with both of these predictors:

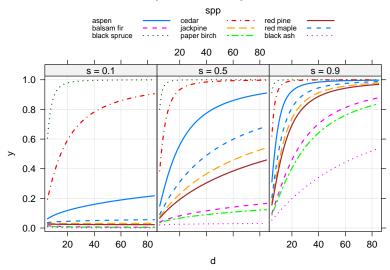
d predictor effect plot



This plot now displays the lines for all conditioning values of s within the panel for each level of the conditioning factor spp. Compare this graph to the much more confusing plot in which different

lines are drawn for the nine levels of the conditioning factor spp, obtained by using the z.var sub-argument in the lines group:

d predictor effect plot



The z.var sub-argument for lines selects the predictor that determines the lines within a panel and the remaining predictors, here just s, distinguish the panels. The default choice of z.var is usually, but not always, appropriate. We also use the lattice argument to display the array of panels in 3 columns and 1 row, and differentiate the lines by line type and color using arguments discussed next.

3.2.2 col, lty, lwd, spline: Line Color, Type, Width, Smoothness

Different lines in the same plot are differentiated by default using color. This can be modified by the sub-arguments lty, lwd and col to set line types, widths, and colors, respectively. For example, in the last graph shown you can get all black lines of different line types using lines=list(multiline=TRUE, col="black", lty=1:9), or using a gray scale, lines=list(multiline=TRUE, col=gray((1:9)/10)).

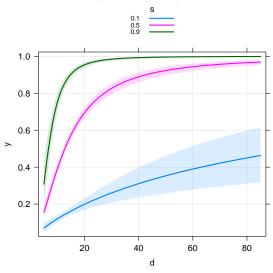
The plot() method for effect objects by default uses smoothing splines to interpolate between plotted points. Smoothing can be turned off with splines=FALSE in the lines argument, but we rarely expect this to be a good idea. The number of values at which the focal predictor is evaluated is set with the focal.levels argument, and it defaults to 50. In any case more than three evaluations should be used for a reasonable spline approximation.

3.3 confint: Specifying Confidence Interval Inclusion and Style

The confint argument group controls the inclusion and appearance of confidence intervals and regions. This argument has three sub-arguments. The style sub-argument is either "bars", for confidence bars typically around the estimated adjusted mean for a factor level, "bands" for shaded confidence bands typically for numeric focal predictors, "auto" to let the program automatically choose between "bars" and "bands", "lines" to draw only the edges of confidence bands with no shading, or "none" to suppress confidence intervals. The default is "auto" when multiline=FALSE

and "none" when multiline=TRUE. Setting confint="auto" produces bars for factors and bands for numeric predictors. For example:

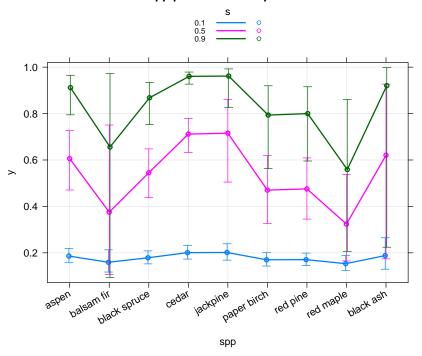
d predictor effect plot



In this example the confidence bands are well separated, so including them in a multiline graph isn't problematic; in other cases, overlapping confidence bands produce an artistic but uninterpretable mess.

With a factor focal predictor, we get:

spp predictor effect plot



The error bars for the various levels of s are slightly staggered to reduce over-plotting.

Two additional arguments, col and alpha, control respectively the color of confidence bars and regions and the transparency of confidence regions. Users are unlikely to need these options. Finally, the type of confidence interval shown, either pointwise or Scheffé corrected for multiple comparisons, is controlled by the se argument to the predictorEffect() or Effect() function (see Section 2.3).

3.4 lattice: Specifying Standard lattice Package Arguments

The plot() methods defined in the effects package use functions in the lattice package (Sarkar, 2008), such as xyplot(), to draw effect plots, which often comprise rectangular arrays of panels. In particular, the plot() method for the "eff" objects returned by the Effect() function are "trellis" objects, which can be manipulated in the normal manner. "Printing" a returned effect-plot object displays the plot in the current R graphics device.

The lattice group of arguments to the plot() method for effect objects may be used to specify various standard arguments for lattice graphics functions such as xyplot(). In particular, you can control the number of rows and columns when panels are displayed in an array, modify the key (legend) for the graph, and specify the contents of the "strip" displayed in the shaded region of text above each panel in a lattice array. In addition, the array sub-argument, for advanced users, controls the layout of multiple predictor effect plots produced by the predictorEffects() function.

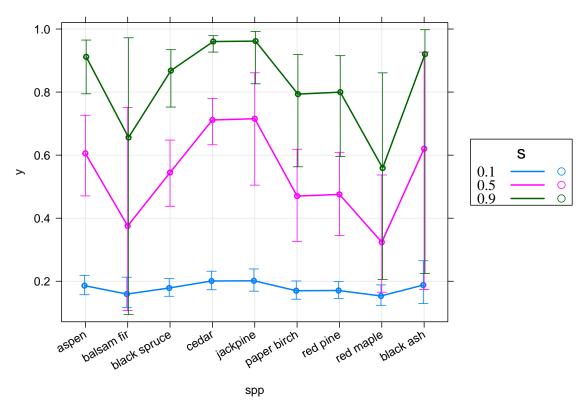
3.4.1 key.args: Modifying the Key

A user can modify the placement and appearance of the key with the key.args sub-argument, which is itself a list. For example:

```
R> plot(predictorEffects(gm5, ~ spp, xlevels=list(s=c(0.1, 0.5, 0.9))),
+ rug=FALSE,
+ axes=list(grid=TRUE,
+ y=list(type="response"),
+ x=list(rotate=30)),
+ lines=list(multiline=TRUE),
```

```
+ confint=list(style="auto"),
+ lattice=list(key.args=list(space="right",
+ columns=1,
+ border=TRUE,
+ fontfamily="serif",
+ cex=1.25,
+ cex.title=1.5)))
```

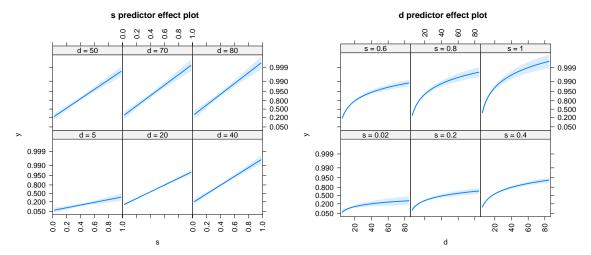
spp predictor effect plot



The sub-argument <code>space="right"</code> moves the key to the right of the graph, overriding the default <code>space="top"</code>. Alternatively the key can be placed inside the graph using the <code>x</code>, <code>y</code>, and <code>corner</code> sub-arguments, as illustrated in the graph on page 24. The choices for <code>fontfamily</code> are "sans" and "serif", and affect only the key; the rest of the plot uses "sans". The sub-arguments <code>cex</code> and <code>cex.title</code> control the relative sizes of the key entries and the key title, respectively. Finally, any argument documented in <code>help("xyplot")</code> in the <code>key</code> section can be set with this argument. If you use the default <code>space="top"</code> for placement of the key, you may wish to adjust the number of columns in the key, particularly if the level names are long.

3.4.2 layout

The layout sub-argument to the lattice argument allows a user to customize the layout of multiple panels in an effect plot; for example:



Here, the layout sub-argument specifies an array of 3 columns and 2 rows for each of the predictor effect plots.

3.4.3 array: Multiple Predictor Effect Plots

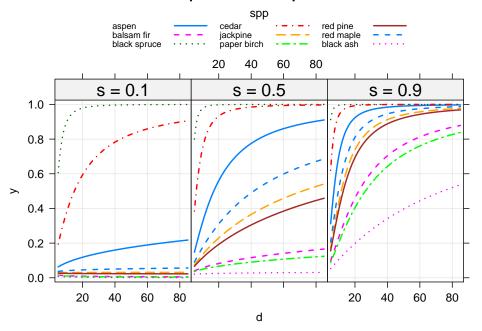
If you create several predictor effect objects with the predictorEffects() function, the plot() method for the resulting object uses the lattice array argument to divide the plotting region into sub-plots so that the predictor effect plots, each potentially with several panels, are drawn without overlapping. The user can also supply this argument directly to create a custom meta-array of predictor effect plots; see help("plot.eff") for details.

3.4.4 strip

Lattice graphics with more than one panel typically provide a title at the top of each panel in an area called the *strip*. The default strip title contains the name of the conditioning predictor and the value to which it is set in the panel; if there are more than one conditioning predictor, then all of their names and corresponding values are shown. For example:

```
R> plot(predictorEffects(gm4, ~ d, xlevels=list(s=c(0.1, 0.5, 0.9)),
          axes=list(grid=TRUE,
                    x=list(rug=FALSE),
+
                    y=list(type="response")),
          lines=list(multiline=TRUE, z.var="spp", lty=1:9),
          lattice=list(layout=c(3, 1),
+
                        strip=list(factor.names=TRUE,
                                   values=TRUE,
                                   cex=1.5)))
Error: <text>:10:0: unexpected end of input
8:
                                 values=TRUE,
                                 cex=1.5)))
9:
```

d predictor effect plot

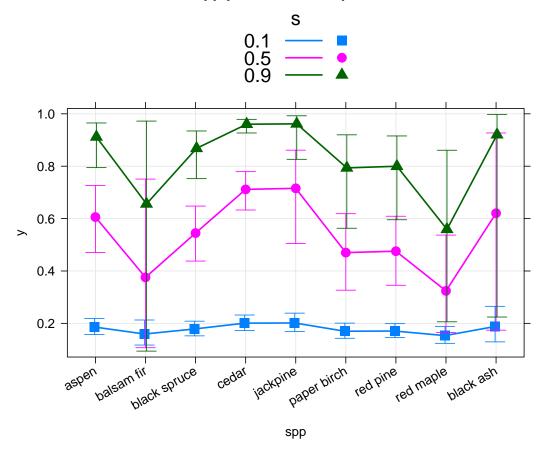


Setting factor.names=FALSE (the default is TRUE) displays only the value, and not the name, of the conditioning predictor in each strip; usually, this is desirable only if the name is too long to fit, in which case you may prefer to rename the predictor. Setting values=FALSE replaces the conditioning value with a line in the strip that represents the value: The line is at the left of the strip for the smallest conditioning value, at the right for the largest value, and in a proportional intermediate position in between the two extremes. The most generally useful sub-argument is cex, which allows you to reduce or expand the relative size of the text in the strip, in this case increasing the size to 150% of standard size.

3.5 symbols: Plotting symbols

Symbols are used to represent adjusted means when the focal predictor is a factor. You can control the symbols used and their relative size:

spp predictor effect plot

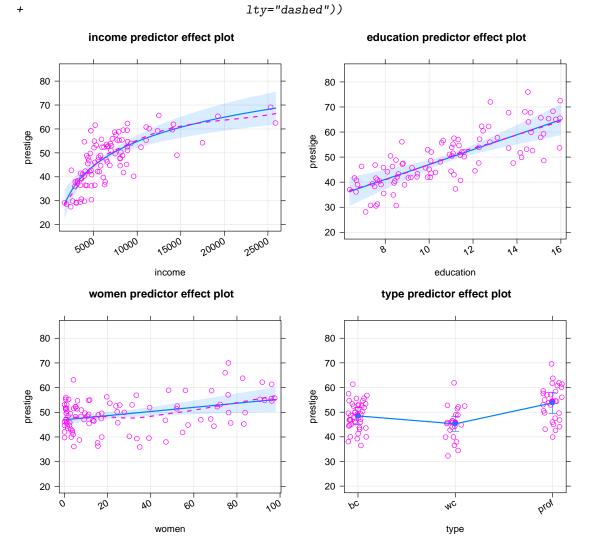


We use the pch sub-argument to set the symbol number for plotted symbols; you can enter the commands plot(1:25, pch=1:25) and lines(1:25, lty=2, type="h") to see the 25 plotting symbols in R. The sub-argument pch can also be a character vector, such as letters[1:10]. In this example, we set cex=1.5 to increase the symbol size by the factor 1.5. Because only one value is given, it is recycled and used for all of the symbols. We need to change the size of the symbols in the key separately, as we do here via the key.args sub-argument to the lattice argument (see Section 3.4.1).

4 Displaying Residuals in Predictor Effect Plots

Fox and Weisberg (2019b) introduce methodology for adding partial residuals to a predictor effect or effect plot. This can be desirable to display variation in data around a fitted partial regression surface or to diagnose possible lack of fit, as the resulting plots are similar to traditional component-plus-residual plots (Fox and Weisberg, 2019a, Sec. 8.4).

The predictor effect plot for a numeric focal predictor that does not interact with other predictors is equivalent to a standard component-plus-residual plot; for example:



The partial residuals to be plotted are computed using the residuals argument to the predictorEffect(), predictorEffects(), or Effect() function. For the numeric predictors income, education, and women, the plotted points are each equal to a point on the fitted blue line, representing the partial fit, plus the corresponding residual. For income, the fitted partial-regression line in curved because of the log transformation of the predictor, but the partial-regression function is a straight line for the other two numeric predictors.

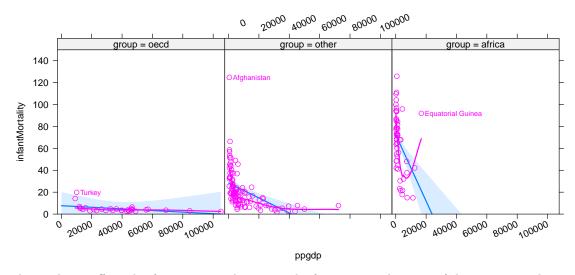
The dashed line produced by lty="dashed" in the same magenta color as the plotted points on the graph, is a loess nonparametric-regression smooth of the points. The sub-argument smooth=TRUE is the default if residuals are present in the effect object to be plotted. The sub-argument span=0.75 adjust the span of the loess smoother from the default of 2/3—an unnecessary adjustment here specified simply to illustrate how to set the span. If the model adequately represents the data, then the dashed magenta line should approximately match the solid blue partial-regression line, which represents the fitted model.

For the factor type, the points have been jittered horizontally to separate them visually, because the only possible horizontal coordinates are at the three distinct factor levels. Smooths are not fit to factors and instead the conditional means of the partial residuals are plotted as magenta dots; in the current model, the magenta dots and the blue dots representing the fitted adjusted means of the response at the levels of name necessarily match.

The plot() method for effect objects has a partial.residuals argument, with several subarguments that control how partial residuals are displayed. In the command above, we used the sub-argument smooth=TRUE to add the smoother, which is the default when residuals are included in the effect object, and lty="dashed" to change the line type for the smooth from the default solid line to a dashed line. All the smooth sub-arguments are described in help("plot.eff").

For a second example, we turn to a linear model with an interaction, modelling infantMortality rate as a function of ppgdp, per person GDP, and country group in the UN data set in the carData package:

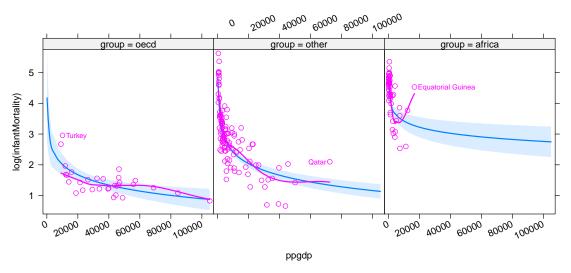
ppgdp predictor effect plot



The predictor effect plot for ppgdp conditions on the factor group because of the interaction between these two predictors. Several problems are apparent in this plot: The id argument is used to identify the most unusual point in each panel, as described in detail in help("plot.eff"). Turkey has higher than predicted infant mortality for the "oecd" group; Afghanistan, in the "other" group, has infant mortality much higher than predicted; and Equatorial Guinea is clearly unusual for the "africa" group. In addition, the smooths through the points do not match the fitted lines in the "other" and "africa" groups. We use the command options(scipen=10) to suppress annoying scientific notation in the tick-mark labels, and instead rotate these labels so that they fit without over-plotting.

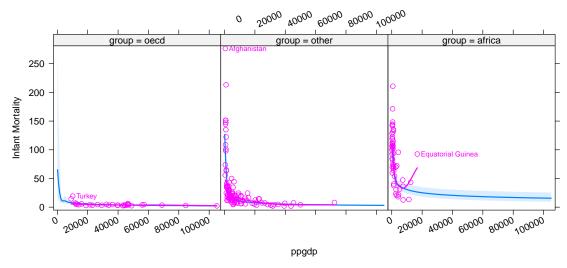
Log-transforming both the predictor ppgdp and the response infantMortality produces a better fit to the data:

ppgdp predictor effect plot



Equatorial Guinea is still anomalous, however. Rescaling the vertical axis to arithmetic scale produces a slightly different, but possibly useful, picture:

ppgdp predictor effect plot



Partial residuals can be added to effect plots for linear or generalized linear models in the default link scale, and to effect plots for linear or generalized linear mixed models.

4.1 Using the Effect() Function With Partial Residuals

In most instances, predictor effect plots produced by predictorEffect() or predictorEffects() visualize a fitted model in the most natural manner, but sometimes in looking for lack of fit, we

want to plot against arbitrary combinations of predictors. The more general Effect() function is capable of doing that.

Recall, for example, the additive model 1m2 fit to the Prestige data:

R> S(1m2)

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.6073	0.3666	1.66	0.10100
log(income)	0.2787	0.0458	6.09	0.000000026
education	0.0656	0.0162	4.05	0.00011
typewc	0.0325	0.0634	0.51	0.61014
typeprof	0.1255	0.0965	1.30	0.19662

Residual standard deviation: 0.177 on 93 degrees of freedom (4 observations deleted due to missingness)

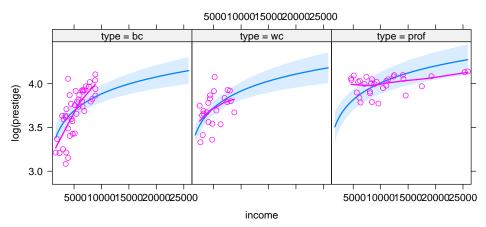
Multiple R-squared: 0.792

F-statistic: 88.5 on 4 and 93 DF, $\,$ p-value: <2e-16

AIC BIC -54.36 -38.85

Plotting partial residuals for the predictors income and type simultaneously reveals an unmodeled income × type interaction:

income*type effect plot



5 Polytomous Categorical Responses

The effects package produces special graphs for ordered and unordered polytomous categorical response variables. In an ordinal regression, the response is an ordered categorical variable with three or more levels. For example, in a study of women's labor force participation that we introduce below, the response is not working outside the home, working part time, or working full time. The proportional-odds model Fox and Weisberg (2019a, Sec. 6.9) estimates the probability of a response in each of these three categories given a linear combination of regressors defined by a set of predictors, assuming a logit link function.

We illustrate the proportional-odds model with the Womenlf data set in the carData package, for young married Canadian women's labor-force participation, using the polr() function in the MASS package to fit the model:

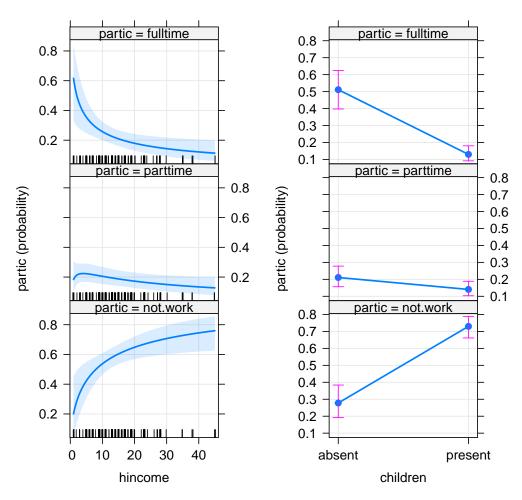
```
R> library("MASS") # for polr()
R> Womenlf$partic <- factor(Womenlf$partic,</pre>
         levels=c("not.work", "parttime", "fulltime")) # order response levels
R> or1 <- polr(partic ~ log(hincome) + children, data=Womenlf)</pre>
R> S(or1)
Re-fitting to get Hessian
Call: polr(formula = partic ~ log(hincome) + children, data = Womenlf)
Coefficients:
                Estimate Std. Error z value
                                                   Pr(>|z|)
                                                     0.0042
log(hincome)
                  -0.666
                               0.233
                                       -2.86
childrenpresent
                  -1.948
                               0.287
                                       -6.80 0.0000000001
 Intercepts (Thresholds):
                  Estimate Std. Error z value Pr(>|z|)
not.work|parttime
                    -2.747
                                 0.654
                                          -4.20 0.000027
parttime|fulltime
                    -1.837
                                 0.640
                                         -2.87
                                                  0.0041
Residual Deviance: 441.12
logLik
             df
                    AIC
                             BIC
-220.56
              4
                 449.12 463.40
```

The response variable partic initially has its levels in alphabetical order, which does not correspond to their natural ordering. We therefore start by reordering the levels to increase from "not.work", to "parttime" work, to "fulltime" work. The predictors are the numeric variable hincome (husband's income), which enters the model in log-scale, and the factor children, presence of children in the household.

The model summary is relatively complex, and is explained in Fox and Weisberg (2019a, Sec. 6.9). Predictor effect plots greatly simplify interpretation of the fitted model:

hincome predictor effect plot

children predictor effect plot

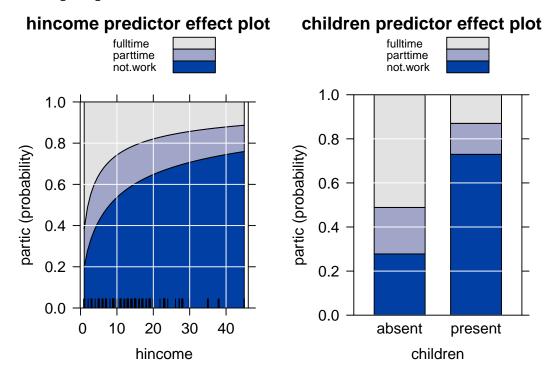


Unlike predictor effect plots for generalized linear models, the default scaling for the vertical axis is the probability scale, equivalent to axes=list(y=list(type="response")) for a GLM, and the alternative is axes=list(y=list(type="logit")), which is analogous to type="link" for a GLM.³ Confidence bands are present by default, unless turned off with the argument confint=list(style="none"). Numeric focal predictors are by default evaluated at 50 points. The plot for hincome suggests high probability of full-time work if husband's income is low, with the probability of full-time work sharply decreasing to about \$15,000 and then nearly leveling off at about .1 to .2. The probability of not working rapidly increases with husband's income, while the probability of working part time is fairly flat. A similar pattern is apparent for children present in the home, with full-time work much less prevalent and not working much more prevalent when children are present than when they are absent.

Stacked area plots are sometimes more useful for examining polytomous response models; for example:

Re-fitting to get Hessian

³The logits plotted, however, correspond to the individual-level probabilities and are not the ordered logits in the definition of the proportional-odds model.



For each fixed value on the horizontal axis, the vertical axis "stacks" the probabilities in the three response categories. For example, with children absent from the household and hincome set to its mean, nearly 30% of women did not work outside the home, about 20% worked part time, and the remaining approximate 50% worked full time.

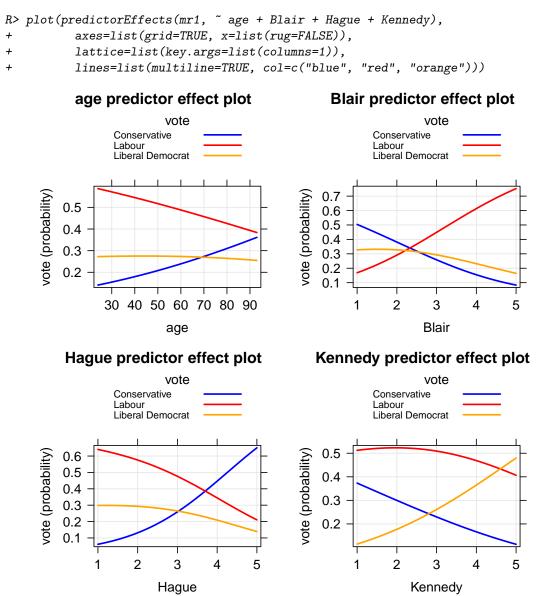
Some ordinal-response models produced by the functions clm(), clm2(), and clmm() in the **ordinal** package can be used with the **effects** package. To work with model objects produced by these functions, you must also load the **MASS** package.

The effects package can also draw similar graphs for the more general multinomial logit model, in which the polytomous categorical response has unordered levels (see Fox and Weisberg, 2019a, Sec. 6.7). The details of the model, its parameters, and its assumptions are different from those of the proportional-odds model and other ordered-response models, but predictor effect plots for these models are similar.

As an example, we use the BEPS data set in the **carData** package, consisting of about 1,500 observations from the 1997-2001 British Election Panel Study. The response variable, vote, is party choice, one of "Liberal Democrat", "Labour", or "Conservative". There are numerous predictors of vote in the data set, and we fit the model

There are nine predictors, seven of which are scales with values between 0 and 5 concerning respondents' attitudes; these predictors enter the model as main effects. The remaining two predictors are scales between 0 and 3 for political.knowledge and between 1 and 11 for Europe (attitude toward European integration of the UK in the European Union, with high values representing "Euroscepticism", a negative attitude toward Europe); these predictors enter the model with a two-factor interaction.

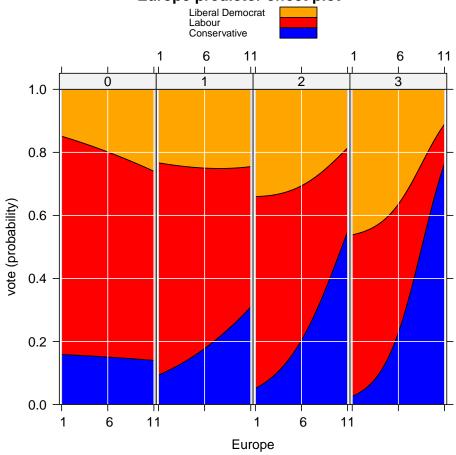
Drawing all nine predictor effect plots simultaneously is not a good idea because the plots won't fit reasonably in a single display. We therefore draw only a few of the plots at a time:

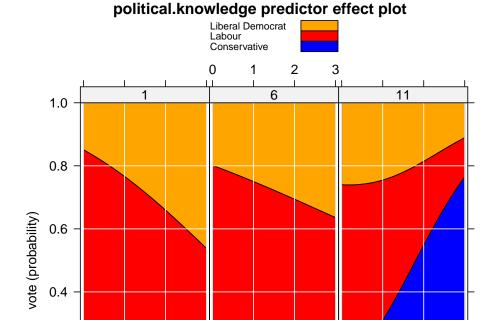


We use optional arguments to get a multiline plot, with a grid and no rug plot, and to modify the key. The color specification for the lines represents the traditional colors of the three parties. Interpreting these plots is challenging: For example, the probability of voting Labour decreases with age, increases with attitude toward the Labour leader Blair, strongly decreases with attitude toward the Conservative leader Hague, and is relatively unaffected by attitude toward the Liberal Democrat leader Kennedy. In general, a positive attitude toward a party leader increases the probability of voting for that leader's party, as one would expect. Of course, the causal direction of these relationships is unclear.

We next turn to the interaction between Europe and political.knowledge, this time drawing a stacked area display.

Europe predictor effect plot





We draw the graphs separately so that we can use 50 values for the focal predictor in each plot, producing smoother curves separating the stacked areas. The lines argument is used to specify the colors for the stacked areas representing the parties. Both effect plots are of nearly the same fitted values, in the first graph with Europe varying and conditioning on political.knowledge, and in the second with political.knowledge varying and conditioning on Europe. Setting strip=list(factor.names=FALSE) suppresses the names of the conditioning predictor in each effect plot; these names are too long for the strips at the tops of the panels. From the first graph, preference for the Conservative Party increases with Europe for respondents with high political knowledge, but not for those with low political knowledge. More generally, voters with high political knowledge are more likely to align their votes with the positions of the parties, Eurosceptic for the Convervatives, pro-Europe for Labour and the Liberal Democrats, than are voters with low political knowledge.

political.knowledge

1

2

3

6 The Lattice Theme for the effects Package

The effects package uses the xyplot() and barchart() functions in the standard lattice package (Sarkar, 2008) to draw effect plots. The lattice package has many options for customizing the appearance of graphs that are collected into a lattice theme. We have created a custom theme for use with the effects package that automatically supersedes the default Lattice theme when the effects package is loaded, unless the lattice package has been previously loaded. You can invoke the effects package theme directly by the command

R> effectsTheme()

0.2

0.0

0

1

2

3

⁴Not exactly the same because in each plot the focal predictor takes on 50 values and the conditioning predictor 3 values.

You can also customize the **effects** package theme; see help("effectsTheme"). Finally, because plot() methods in the **effects** package return lattice objects, these objects can be edited and manipulated in the normal manner, for example by functions in the latticeExtra package (Sarkar and Andrews, 2016).

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