# Follow-up data with the Lexis functions in Epi

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### Introduction

This is an introduction to the Lexis machinery in the Epi package. The machinery is intended for representation and manipulation of follow-up data (event history data) from studies where exact dates of events are known. It accommodates follow-up through multiple states and on multiple time scales.

This vignette uses an example from the Epi package to illustrate the set-up of a simple Lexis object (a data frame of follow-up intervals), as well as the subdivision of follow-up intervals needed for multistate representation and analysis of transition rates.

The first chapter is exclusively on manipulation of the follow-up representation, but it points to the subsequent chapter where analysis is based on a Lexis representation with very small follow-up intervals.

Chapter 2 uses analysis of mortality rates among Danish diabetes patients currently on insulin treatment or not to illustrate the use of the the Lexis machinery.

I owe much thanks to my colleague Lars Jorge Diaz for careful reading and many constructive suggestions.

## Chapter 1

## Representation of follow-up data in the Epi package

In the Epi-package, follow-up data is represented by adding some extra variables to a data frame. Such a data frame is called a Lexis object. The tools for handling follow-up data then use the structure of this for special plots, tabulations and modeling.

Follow-up data basically consists of a time of entry, a time of exit and an indication of the status at exit (normally either "alive" or "dead") for each person. Implicitly is also assumed a status *during* the follow-up (usually "alive").

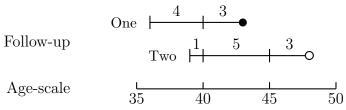


Figure 1.1: Follow-up of two persons

#### 1.1 Time scales

A time scale is a variable that varies deterministically *within* each person during follow-up, *e.g.*:

- Age
- Calendar time
- Time since start of treatment
- Time since relapse

All time scales advance at the same pace, so the time followed is the same on all time scales. Therefore, it will suffice to use only the entry point on each of the time scales, for example:

- Age at entry
- Date of entry

- Time at treatment (at treatment this is 0)
- Time at relapse (at relapse this is 0)

For illustration we need to load the Epi package:

```
> library(Epi)
> print( sessionInfo(), 1=F )
R version 3.6.0 (2019-04-26)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 14.04.6 LTS
Matrix products: default
        /usr/lib/openblas-base/libopenblas.so.0
LAPACK: /usr/lib/lapack/liblapack.so.3.0
attached base packages:
[1] utils
              datasets graphics grDevices stats
                                                        methods
                                                                  base
other attached packages:
[1] Epi_2.37
loaded via a namespace (and not attached):
 [1] Rcpp_1.0.0
                       lattice_0.20-38 zoo_1.8-4
                                                             MASS_7.3-51.1
 [5] grid_3.6.0 plyr_1.8.4 nlme_3.1-139 [9] data.table_1.12.0 Matrix_1.2-17 splines_3.6.0
                                                             etm_1.0.4
                                                             tools_3.6.0
[13] cmprsk_2.2-7 numDeriv_2016.8-1 survival_2.44-1.1 parallel_3.6.0
[17] compiler_3.6.0
                       mgcv_1.8-28
```

In the Epi package, follow-up in a cohort is represented in a Lexis object. As mentioned, a Lexis object is a data frame with some extra structure representing the follow-up. For the DMlate data — follow-up of diabetes patients in Denmark recording date of birth, date of diabetes, date of insulin use, date of first oral drug use and date of death — we can construct a Lexis object by:

```
> data( DMlate )
> head( DMlate )
              dobth
                                dodth
                                         dooad doins
       sex
                        dodm
                                                          dox
50185
       F 1940.256 1998.917
                                 NA
                                            NA
                                                  NA 2009.997
307563
       M 1939.218 2003.309
                                   NA 2007.446
                                                  NA 2009.997
                                                  NA 2009.997
294104
        F 1918.301 2004.552
                                   NA
                                            NA
       F 1965.225 2009.261
                                                  NA 2009.997
336439
                                   NA
                                            NA
        M 1932.877 2008.653
                                                  NA 2009.997
245651
                                   NA
                                            NA
        F 1927.870 2007.886 2009.923
                                                  NA 2009.923
216824
                                            NA
> dmL <- Lexis( entry = list( per=dodm,
                              age=dodm-dobth,
                              tfD=0),
+
+
                 exit = list( per=dox ),
          exit.status = factor(!is.na(dodth), labels=c("DM", "Dead") ),
                 data = DMlate )
NOTE: entry.status has been set to "DM" for all.
NOTE: Dropping 4 rows with duration of follow up < tol
> timeScales(dmL)
[1] "per" "age" "tfD"
```

(The excluded persons are persons with date of diabetes equal to date of death.)

The entry argument is a *named* list with the entry points on each of the time scales we want to use. It defines the names of the time scales and the entry points of the follow-up of each person. The exit argument gives the exit time on *one* of the time scales, so the name of the element in this list must match one of the names of the entry list. This is sufficient, because the follow-up time on all time scales is the same, in this case dox-dodm.

The exit.status is a categorical variable (a factor) that indicates the exit status — in this case whether the person (still) is in state DM or exits to Dead at the end of follow-up. In principle we should also indicate the entry.status, but the default is to assume that all persons enter in the first of the mentioned exit.states — in this case DM, because FALSE < TRUE.

Now take a look at the result:

```
> str( dmL )
Classes 'Lexis' and 'data.frame':
                                         9996 obs. of 14 variables:
          : num 1999 2003 2005 2009 2009 ...
                 58.7 64.1 86.3 44 75.8 ...
 $ age
          : num
 $ tfD
          : num
                 0 0 0 0 0 0 0 0 0 0 ...
 $ lex.dur: num
                 11.08 6.689 5.446 0.736 1.344 ...
 $ lex.Cst: Factor w/ 2 levels "DM", "Dead": 1 1 1 1 1 1 1 1 1 1 ...
  lex.Xst: Factor w/ 2 levels "DM", "Dead": 1 1 1 1 1 2 1 1 2 1 ...
 $ lex.id : int
                 1 2 3 4 5 6 7 8 9 10 ...
          : Factor w/ 2 levels "M", "F": 2 1 2 2 1 2 1 1 2 1 ...
 $ sex
                 1940 1939 1918 1965 1933 ...
 $ dobth
         : num
                 1999 2003 2005 2009 2009 ...
 $ dodm
          : num
 $ dodth
         : num
                 NA NA NA NA ...
                 NA 2007 NA NA NA ...
 $ dooad
         : num
                 NA NA NA NA NA NA NA NA NA ...
         : num
                 2010 2010 2010 2010 2010 ...
          : num
 - attr(*, "time.scales")= chr "per" "age" "tfD"
  attr(*, "time.since")= chr
  attr(*, "breaks")=List of 3
  ..$ per: NULL
  ..$ age: NULL
  ..$ tfD: NULL
> head( dmL )[,1:10]
            per
                     age tfD
                                lex.dur lex.Cst lex.Xst lex.id sex
                                                                       dobth
                                                                                 dodm
50185
      1998.917 58.66119
                           0 11.0800821
                                             DM
                                                     DM
                                                              1
                                                                  F 1940.256 1998.917
                           0 6.6885695
                                             DM
                                                      DM
                                                              2
307563 2003.309 64.09035
                                                                  M 1939.218 2003.309
                                                              3
294104 2004.552 86.25051
                           0 5.4455852
                                             DM
                                                      DM
                                                                  F 1918.301 2004.552
336439 2009.261 44.03559
                           0
                              0.7364819
                                              DM
                                                      DM
                                                              4
                                                                  F 1965.225 2009.261
245651 2008.653 75.77550
                                                              5
                           0
                              1.3442847
                                             DM
                                                      DM
                                                                  M 1932.877 2008.653
216824 2007.886 80.01643
                           0
                              2.0369610
                                                              6
                                                                  F 1927.870 2007.886
                                             DM
                                                    Dead
```

The Lexis object dmL has a variable for each time scale which is the entry point on this time scale. The follow-up time is in the variable lex.dur (duration). Note that the exit status is in the variable lex.Xst (eXit state. The variable lex.Cst is the state where the follow-up takes place (Current state), in this case DM (alive with diabetes) for all persons. This implies that *censored* observations are characterized by having lex.Cst = lex.Xst.

There is a summary function for Lexis objects that lists the number of transitions and records as well as the total amount of follow-up time; it also (optionally) prints information about the names of the variables that constitute the time scales:

```
> summary.Lexis( dmL, timeScales=TRUE )
Transitions:
    To
From DM Dead Records: Events: Risk time: Persons:
    DM 7497 2499 9996 2499 54273.27 9996
Timescales:
per age tfD
"" "" ""
```

It is possible to get a visualization of the follow-up along the time scales chosen by using the plot method for Lexis objects. dmL is an object of class Lexis, so using the function plot() on it means that R will look for the function plot.Lexis and use this function.

```
> plot( dmL )
```

The function allows quite a bit of control over the output, and a points.Lexis function allows plotting of the endpoints of follow-up:

In the above code you will note that the values of the arguments col and pch are indexed by factors, using the convention in R that the index is taken as *number of the level* of the supplied factor. Thus c("blue", "red") [dmL\$sex] is "blue" when sex is M (the first level). The results of these two plotting commands are in figure 1.2, p. 6.

#### 1.2 Splitting the follow-up time along a time scale

In next chapter we shall conduct statistical analysis of mortality rates, and a prerequisite for parametric analysis of rates is that follow-up time is subdivided in smaller intervals, where we can reasonably assume that rates are constant.

The follow-up time in a cohort can be subdivided ("split") along a time scale, for example current age. This is achieved by the splitLexis (note that it is *not* called split.Lexis). This requires that the time scale and the breakpoints on this time scale are supplied. Try:

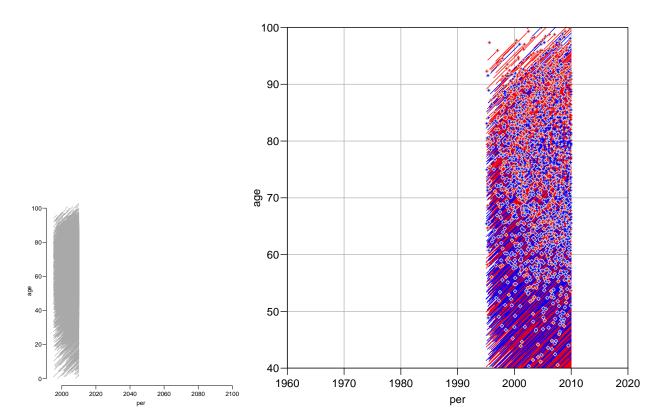


Figure 1.2: Lexis diagram of the DMlate dataset; left panel is the default version, right panel: plot with some bells and whistles. The red lines are for women, blue for men, crosses indicate deaths.

```
Transitions:
To
From DM Dead Records: Events: Risk time: Persons:
DM 18328 2499 20827 2499 54273.27 9996
```

We see that the number of persons and events and the amount of follow-up is the same in the two data sets; only the number of records differ — the extra records all have lex.Cst=DM and lex.Xst=DM.

To see how records are split for each individual, it is useful to list the results for a few individuals (whom we selected with a view to the illustrative usefulness):

```
> wh.id <- c(9,27,52,484)
 subset( dmL , lex.id %in% wh.id )[,1:10]
                      age tfD
                                lex.dur lex.Cst lex.Xst lex.id sex
                                                                        dobth
            per
430048 1998.956 61.87269
                                                               9
                                                                   F 1937.083 1998.956
                               9.508556
                                                    Dead
       2000.042 52.71184
                            0
                               9.954825
                                              DM
                                                      DM
                                                              27
                                                                   M 1947.331 2000.042
                                                                   F 1936.393 1998.249
338459 1998.249 61.85626
                            0 11.748118
                                              DM
                                                      DM
                                                             52
274124 1998.260 62.37919
                            0 10.929500
                                              DM
                                                             484
                                                                   F 1935.881 1998.260
                                                    Dead
> subset( dmS1, lex.id %in% wh.id )[,1:10]
     lex.id
                                    tfD
                                         lex.dur lex.Cst lex.Xst sex
                                                                          dobth
                                                                                     dodm
                 per
                           age
14
          9 1998.956 61.87269 0.000000 3.127310
                                                       DM
                                                                DM
                                                                     F
                                                                       1937.083 1998.956
          9 2002.083 65.00000 3.127310 5.000000
15
                                                       DM
                                                                DM
                                                                     F
                                                                       1937.083 1998.956
16
          9 2007.083 70.00000 8.127310 1.381246
                                                       DM
                                                                     F
                                                                       1937.083 1998.956
                                                             Dead
54
         27 2000.042 52.71184 0.000000 2.288159
                                                       DM
                                                                DM
                                                                     Μ
                                                                       1947.331 2000.042
55
         27 2002.331 55.00000 2.288159 5.000000
                                                       DM
                                                                DM
                                                                     M 1947.331 2000.042
```

```
56
         27 2007.331 60.00000 7.288159 2.666667
                                                       DM
                                                               DM
                                                                    M 1947.331 2000.042
108
         52 1998.249 61.85626 0.000000 3.143737
                                                       DM
                                                               DM
                                                                    F
                                                                      1936.393 1998.249
109
         52 2001.393 65.00000 3.143737 5.000000
                                                       DM
                                                               DM
                                                                    F
                                                                      1936.393 1998.249
110
         52 2006.393 70.00000 8.143737 3.604381
                                                       DM
                                                               DM
                                                                    F
                                                                      1936.393 1998.249
1004
        484 1998.260 62.37919 0.000000 2.620808
                                                       DM
                                                               DM
                                                                    F 1935.881 1998.260
1005
        484 2000.881 65.00000 2.620808 5.000000
                                                               DM
                                                       DM
                                                                    F 1935.881 1998.260
1006
        484 2005.881 70.00000 7.620808 3.308693
                                                       DM
                                                             Dead
                                                                    F 1935.881 1998.260
```

The resulting object, dmS1, is again a Lexis object, and the follow-up may be split further along another time scale, for example diabetes duration, tfD. Subsequently we list the results for the chosen individuals:

```
> dmS2 <- splitLexis( dmS1, "tfD", breaks=c(0,1,2,5,10,20,30,40) )</pre>
> subset( dmS2, lex.id %in% wh.id )[,1:10]
     lex.id
                 per
                           age
                                     tfD
                                            lex.dur lex.Cst lex.Xst sex
                                                                            dobth
                                                                                       dodm
31
          9 1998.956 61.87269
                                0.000000 1.0000000
                                                         DM
                                                                 DM
                                                                       F 1937.083 1998.956
32
          9 1999.956 62.87269
                                1.000000 1.0000000
                                                                       F 1937.083 1998.956
                                                         DM
                                                                 DM
33
          9 2000.956 63.87269
                                2.000000 1.1273101
                                                         DM
                                                                 DM
                                                                       F 1937.083 1998.956
34
                                                                       F 1937.083 1998.956
          9 2002.083 65.00000
                                3.127310 1.8726899
                                                         DM
                                                                 DM
35
          9 2003.956 66.87269
                                                         DM
                                                                 DM
                                                                       F 1937.083 1998.956
                                5.000000 3.1273101
36
          9 2007.083 70.00000
                                8.127310 1.3812457
                                                         DM
                                                               Dead
                                                                       F
                                                                        1937.083 1998.956
111
         27 2000.042 52.71184
                                0.000000 1.0000000
                                                         DM
                                                                 DM
                                                                       M 1947.331 2000.042
112
         27 2001.042 53.71184
                                1.000000 1.0000000
                                                         DM
                                                                 DM
                                                                       M 1947.331 2000.042
         27 2002.042 54.71184
                                2.000000 0.2881588
                                                         DM
                                                                 DM
                                                                       M 1947.331 2000.042
113
         27 2002.331 55.00000
                                2.288159 2.7118412
                                                                 DM
                                                                       M 1947.331 2000.042
114
                                                         DM
115
         27 2005.042 57.71184
                                5.000000 2.2881588
                                                         DM
                                                                 DM
                                                                       M 1947.331 2000.042
                                                                       M 1947.331 2000.042
         27 2007.331 60.00000
116
                                7.288159 2.6666667
                                                         DM
                                                                 DM
229
         52 1998.249 61.85626
                                0.000000 1.0000000
                                                         DM
                                                                 DM
                                                                       F 1936.393 1998.249
230
         52 1999.249 62.85626
                                1.000000 1.0000000
                                                         DM
                                                                 DM
                                                                       F 1936.393 1998.249
231
         52 2000.249 63.85626
                                2.000000 1.1437372
                                                         DM
                                                                 DM
                                                                       F 1936.393 1998.249
232
         52 2001.393 65.00000
                                3.143737 1.8562628
                                                         DM
                                                                 DM
                                                                       F
                                                                        1936.393 1998.249
233
         52 2003.249 66.85626
                                                         DM
                                                                 DM
                                                                       F 1936.393 1998.249
                                5.000000 3.1437372
234
         52 2006.393 70.00000
                                8.143737 1.8562628
                                                         DM
                                                                 DM
                                                                       F 1936.393 1998.249
                                                                       F 1936.393 1998.249
235
         52 2008.249 71.85626 10.000000 1.7481177
                                                         DM
                                                                 DM
                                                                       F 1935.881 1998.260
2084
        484 1998.260 62.37919
                                0.000000 1.0000000
                                                         DM
                                                                 DM
2085
        484 1999.260 63.37919
                                1.000000 1.0000000
                                                         DM
                                                                 DM
                                                                       F 1935.881 1998.260
2086
        484 2000.260 64.37919
                                2.000000 0.6208077
                                                         DM
                                                                 DM
                                                                       F 1935.881 1998.260
2087
        484 2000.881 65.00000
                                2.620808 2.3791923
                                                         DM
                                                                 DM
                                                                       F 1935.881 1998.260
        484 2003.260 67.37919
2088
                                                         DM
                                                                 DM
                                                                        1935.881 1998.260
                                5.000000 2.6208077
                                                                       F
        484 2005.881 70.00000
                                7.620808 2.3791923
2089
                                                         DM
                                                                 DM
                                                                       F
                                                                        1935.881 1998.260
2090
        484 2008.260 72.37919 10.000000 0.9295003
                                                         DM
                                                               Dead
                                                                        1935.881 1998.260
```

A more efficient (and more intuitive) way of making this double split is to use the splitMulti function from the popEpi package:

```
> library( popEpi )
 dmM \leftarrow splitMulti(dmL, age = seq(0,100,5),
                            tfD = c(0,1,2,5,10,20,30,40),
                       drop = FALSE )
> summary( dmS2 )
Transitions:
From
        DM Dead
                  Records:
                             Events: Risk time:
                                                   Persons:
  DM 40897 2499
                     43396
                                2499
                                        54273.27
                                                       9996
> summary( dmM )
```

```
To
From DM Dead Records: Events: Risk time: Persons:
DM 40897 2499 43396 2499 54273.27 9996
```

Note we used the argument drop=FALSE which will retain follow-up also outside the window defined by the breaks. Otherwise the default for splitMulti would be to drop follow-up outside age [0,100] and tfD [0,40]. This clipping behaviour is not available in splitLexis, nevertheless this may be exactly what we want in some situations.

So we see that the two ways of splitting data yields the same amount of follow-up, but the results are not identical:

As we see, this is because the dmM object also is a data.table object; the splitMulti uses the data.table machinery which makes the splitting substantially faster — this is of particular interest if you operate on large data sets (> 100,000 records).

Thus the recommended way of splitting follow-up time is by splitMulti. But you should be aware that the result is a data.table object, which in some circumstances behaves slightly different from data.frames. See the manual for data.table.

## 1.3 Cutting follow up time at dates of intermediate events

If we have a recording of the date of a specific event as for example recovery or relapse, we may classify follow-up time as being before or after this intermediate event, but it requires that follow-up records that straddle the event be cut in two and placed in separate records, one representing follow-up before the intermediate event, and another representing follow-up after the intermediate event. This is achieved with the function cutlexis, which takes three arguments: the time point of the intermediate event, the time scale that this point refers to, and the value of the (new) state following the date. Optionally, we may also define a new time scale with the argument new.scale=.

We are interested in the time before and after inception of insulin use, which occurs at the date doins:

```
> whc <- c(names(dmL)[1:7], "dodm", "doins") # WHich Columns do we want to see?
> subset( dmL, lex.id %in% wh.id )[,whc]
            per
                     age tfD
                                lex.dur lex.Cst lex.Xst lex.id
                                                                    dodm
                                                                            doins
                               9.508556
430048 1998.956 61.87269
                           0
                                             DM
                                                   Dead
                                                             9 1998.956
                                                                               NA
22671 2000.042 52.71184
                           0 9.954825
                                             DM
                                                     DM
                                                            27 2000.042
                                                     DM
338459 1998.249 61.85626
                           0 11.748118
                                             DM
                                                            52 1998.249 2004.804
274124 1998.260 62.37919
                           0 10.929500
                                             DM
                                                   Dead
                                                            484 1998.260 2003.960
```

```
dmC <- cutLexis( data = dmL,</pre>
                     cut = dmL$doins,
+
               timescale = "per",
+
               new.state = "Ins"
+
               new.scale = "tfI",
       precursor.states = "DM" )
 whc <- c(names(dmL)[1:8], "doins") # WHich Columns do we want to see?
 subset( dmC, lex.id %in% wh.id )[,whc]
                              tfD
                                   lex.dur lex.Cst lex.Xst lex.id sex
                                                                            doins
                     age
9
      1998.956 61.87269 0.000000 9.508556
                                                                       F
                                                 DM
                                                        Dead
                                                                   9
                                                                               NΑ
27
      2000.042 52.71184 0.000000 9.954825
                                                 DM
                                                          DM
                                                                  27
                                                                       М
                                                                               NA
      1998.249 61.85626 0.000000 6.554415
                                                                  52
                                                                       F
                                                                         2004.804
                                                 DM
                                                         Ins
10048 2004.804 68.41068 6.554415 5.193703
                                                 Ins
                                                         Ins
                                                                 52
                                                                       F
                                                                         2004.804
      1998.260 62.37919 0.000000 5.700205
                                                 DM
                                                         Ins
                                                                484
                                                                       F
                                                                         2003.960
10480 2003.960 68.07940 5.700205 5.229295
                                                                       F 2003.960
                                                 Ins
                                                        Dead
                                                                484
```

(The precursor.states= argument is explained below).

Note that the process of cutting time is simplified by having all types of events referred to the calendar time scale. This is a generally applicable advice in handling follow-up data: Get all event times as dates, location of events and follow-up on other time scales can then easily be derived from this.

Note that individual 52 has had his follow-up cut at 6.55 years from diabetes diagnosis and individual 484 at 5.70 years from diabetes diagnosis. This dataset could then be split along the time scales as we did before with dmL.

We can see which of the time scales that are defined as time since entry into an intermediate state:

```
> timeSince( dmC )
  per age tfD tfI
   "" "" "Ins"
```

The names of the vector are the time scales; each element is the name of the state entry into which defines the origin of the time scale. This resulting Lexis object can then be split along one or more time scales.

The result of this can however also be achieved by cutting the split dataset dmS2 instead of dmL:

```
> dmS2C <- cutLexis( data = dmS2,
+
                       cut = dmS2$doins,
+
                 timescale = "per",
                 new.state = "Ins",
+
                 new.scale = "tfI",
         precursor.states = "DM" )
 subset( dmS2C, lex.id %in% wh.id )[,whc]
           per
                     age
                                tfD
                                      lex.dur lex.Cst lex.Xst lex.id sex
                                                                               doins
31
      1998.956 61.87269
                          0.000000 1.0000000
                                                    DM
                                                            DM
                                                                     9
                                                                         F
                                                                                  NA
                                                                     9
                                                                         F
32
      1999.956 62.87269
                          1.000000 1.0000000
                                                    DM
                                                            DM
                                                                                  NA
                                                                         F
33
      2000.956 63.87269
                          2.000000 1.1273101
                                                    DM
                                                             DM
                                                                     9
                                                                                  NA
34
      2002.083 65.00000
                          3.127310 1.8726899
                                                    DM
                                                             DM
                                                                     9
                                                                         F
                                                                                  NA
35
      2003.956 66.87269
                          5.000000 3.1273101
                                                    DM
                                                            DM
                                                                     9
                                                                         F
                                                                                  NA
      2007.083 70.00000
                                                                         F
36
                          8.127310 1.3812457
                                                    DM
                                                           Dead
                                                                     9
                                                                                  NA
      2000.042 52.71184
                                                    DM
                                                                    27
111
                          0.000000 1.0000000
                                                             DM
                                                                         Μ
                                                                                  NA
112
      2001.042 53.71184
                          1.000000 1.0000000
                                                    DM
                                                             DM
                                                                    27
                                                                         Μ
                                                                                  NA
      2002.042 54.71184
                          2.000000 0.2881588
                                                    DM
                                                            DM
                                                                    27
                                                                                  NA
113
```

```
114
      2002.331 55.00000
                          2.288159 2.7118412
                                                    DM
                                                             DM
                                                                    27
                                                                         Μ
                                                                                  NA
      2005.042 57.71184
                                                    DM
                                                             DM
                                                                    27
                                                                         М
                                                                                  NA
115
                          5.000000 2.2881588
116
      2007.331 60.00000
                          7.288159 2.6666667
                                                    DM
                                                             DM
                                                                    27
                                                                         Μ
                                                                                  NA
229
      1998.249 61.85626
                          0.000000 1.0000000
                                                    DM
                                                             DM
                                                                    52
                                                                         F
                                                                            2004.804
                                                                           2004.804
230
      1999.249 62.85626
                          1.000000 1.0000000
                                                    DM
                                                             DM
                                                                    52
                                                                         F
231
      2000.249 63.85626
                          2.000000 1.1437372
                                                    DM
                                                             DM
                                                                    52
                                                                         F 2004.804
232
      2001.393 65.00000
                          3.143737 1.8562628
                                                    DM
                                                             DM
                                                                    52
                                                                         F 2004.804
233
      2003.249 66.85626
                          5.000000 1.5544148
                                                    DM
                                                            Ins
                                                                    52
                                                                         F 2004.804
43629 2004.804 68.41068
                          6.554415 1.5893224
                                                   Ins
                                                                    52
                                                                         F 2004.804
                                                            Ins
43630 2006.393 70.00000
                          8.143737 1.8562628
                                                                    52
                                                                         F
                                                                            2004.804
                                                   Ins
                                                            Ins
43631 2008.249 71.85626 10.000000 1.7481177
                                                   Ins
                                                            Ins
                                                                    52
                                                                         F
                                                                            2004.804
2084
      1998.260 62.37919
                          0.000000 1.0000000
                                                    DM
                                                             DM
                                                                   484
                                                                         F
                                                                            2003.960
2085
      1999.260 63.37919
                          1.000000 1.0000000
                                                    DM
                                                             DM
                                                                   484
                                                                         F
                                                                            2003.960
                                                    DM
                                                                   484
2086
      2000.260 64.37919
                          2.000000 0.6208077
                                                             DM
                                                                         F
                                                                            2003.960
      2000.881 65.00000
                          2.620808 2.3791923
                                                    DM
                                                             DM
                                                                   484
                                                                         F 2003.960
2087
2088
      2003.260 67.37919
                          5.000000 0.7002053
                                                    DM
                                                            Ins
                                                                   484
                                                                         F 2003.960
45484 2003.960 68.07940
                          5.700205 1.9206023
                                                   Ins
                                                            Ins
                                                                   484
                                                                         F 2003.960
45485 2005.881 70.00000
                          7.620808 2.3791923
                                                            Ins
                                                                   484
                                                                           2003.960
                                                   Ins
45486 2008.260 72.37919 10.000000 0.9295003
                                                                   484
                                                                         F 2003.960
                                                   Ins
                                                           Dead
```

Thus it does not matter in which order we use splitLexis and cutLexis. Mathematicians would say that splitLexis and cutLexis are commutative.

Note in lex.id=484, that follow-up subsequent to the event is classified as being in state Ins, but that the final transition to state Dead is preserved. This is the point of the precursor.states= argument. It names the states (in this case DM) that will be over-written by new.state (in this case Ins), while the state Dead should not be updated even if it is after the time where the persons moves to state Ins. In other words, only state DM is a precursor to state Ins, state Dead is always subsequent to state Ins.

Note that we defined a new time scale, tfI, using the argument new.scale=tfI. This has a special status relative to the other three time scales, it is defined as time since entry into a state, namely Ins, this is noted in the time scale part of the summary of Lexis object — the information sits in the attribute time.since of the Lexis object, which can be accessed by the function timeSince() or through the summary():

```
> summary( dmS2C, timeScales=TRUE )
Transitions:
     To
From
         DM
              Ins Dead
                         Records:
                                    Events: Risk time:
                                                          Persons:
  DM
      35135 1694 2048
                            38877
                                       3742
                                               45885.49
                                                              9899
           0 5762
                   451
                             6213
                                        451
                                                8387.77
                                                              1791
  Sum 35135 7456 2499
                            45090
                                       4193
                                               54273.27
                                                              9996
Timescales:
  per
               tfD
         age
                     tfI
                "" "Ins"
```

Finally we can get a quick overview of the states and transitions by using boxes—scale.R scales transition rates to rates per 1000 PY:

```
> boxes( dmC, boxpos=TRUE, scale.R=1000, show.BE=TRUE )
```

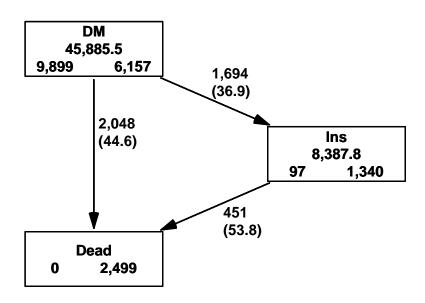


Figure 1.3: States, person years, transitions and rates in the cut dataset. The numbers in the boxes are person-years and the number of persons Beginning, resp. Ending their follow-up in each state (triggered by show.BE=TRUE). The numbers at the arrows are the number of transitions and transition rates per 1000 (triggered by scale.R=1000). ./flup-box1

## Chapter 2

## Modeling rates from Lexis objects

#### 2.1 Covariates

In the dataset dmS2C there are three types of covariates that can be used to describe mortality rates:

- 1. time-dependent covariates
- 2. time scales
- 3. fixed covariates

There is only one time-dependent covariate here, namely lex.Cst, the current state of the person's follow up; it takes the values DM and Ins according to whether the person has ever purchased insulin at a given time of follow-up.

The time-scales are obvious candidates for explanatory variables for the rates, notably age and time from diagnosis (duration of diabetes) and insulin.

#### 2.1.1 Time scales as covariates

If we want to model the effect of the time scale variables on occurrence rates, we will for each interval use either the value of the left endpoint in each interval or the middle. There is a function timeBand which returns either of these:

```
> timeBand( dmS2C, "age", "middle" )[1:10]
 [1] 57.5 57.5 62.5 62.5 62.5 67.5 67.5 62.5 67.5 67.5
> # For nice printing and column labelling we use the data.frame() function:
 data.frame( dmS2C[,c("per","age","tfD","lex.dur")],
              mid.age=timeBand( dmS2C,
                                       "age", "middle"
                                       "tfD",
                mid.t=timeBand( dmS2C,
               left.t=timeBand( dmS2C, "tfD", "left"
              right.t=timeBand( dmS2C, "tfD", "right"
               fact.t=timeBand( dmS2C, "tfD", "factor" ) )[1:15,]
                            tfD
                                   lex.dur mid.age mid.t left.t right.t
                 age
        per
  1998.917 58.66119 0.0000000 1.00000000
                                               57.5
                                                      0.5
                                                                           (0,1]
  1999.917 59.66119 1.0000000 0.33880903
                                               57.5
                                                      1.5
                                                               1
                                                                           (1,2]
  2000.256 60.00000 1.3388090 0.66119097
                                              62.5
                                                      1.5
                                                                           (1,2]
                                                               1
  2000.917 60.66119 2.0000000 3.00000000
                                                               2
                                                                      5
                                              62.5
                                                      3.5
                                                                           (2,5]
  2003.917 63.66119 5.0000000 1.33880903
                                              62.5
                                                      7.5
                                                                          (5,10]
```

```
2005.256 65.00000 6.3388090 3.66119097
                                                 67.5
                                                         7.5
                                                                  5
                                                                          10
                                                                              (5,10]
7
   2008.917 68.66119 10.0000000 1.08008214
                                                 67.5
                                                        15.0
                                                                 10
                                                                          20 (10,20]
8
   2003.309 64.09035
                       0.0000000 0.90965092
                                                 62.5
                                                         0.5
                                                                  0
                                                                           1
                                                                                (0,1]
9
   2004.218 65.00000
                       0.9096509 0.09034908
                                                 67.5
                                                         0.5
                                                                   0
                                                                           1
                                                                                (0,1]
10 2004.309 65.09035
                       1.0000000 1.00000000
                                                 67.5
                                                                   1
                                                                           2
                                                         1.5
                                                                                (1,2]
11 2005.309 66.09035
                       2.0000000 3.00000000
                                                 67.5
                                                                   2
                                                                           5
                                                         3.5
                                                                                (2,5]
12 2008.309 69.09035
                       5.0000000 0.90965092
                                                 67.5
                                                         7.5
                                                                   5
                                                                          10
                                                                               (5,10]
                                                 72.5
13 2009.218 70.00000
                       5.9096509 0.77891855
                                                         7.5
                                                                   5
                                                                          10
                                                                               (5,10]
14 2004.552 86.25051
                       0.0000000 1.00000000
                                                 87.5
                                                         0.5
                                                                   0
                                                                           1
                                                                                (0,1]
15 2005.552 87.25051
                       1.0000000 1.00000000
                                                 87.5
                                                                           2
                                                         1.5
                                                                   1
                                                                                (1,2]
```

Note that the values of these functions are characteristics of the intervals defined by breaks=, not the midpoints nor left or right endpoints of the actual follow-up intervals (which would be tfD and tfD+lex.dur, respectively).

These functions are intended for modeling time scale variables as factors (categorical variables) in which case the coding must be independent of the censoring and mortality pattern — it should only depend on the chosen grouping of the time scale. Modeling time scales as *quantitative* should not be based on these codings but directly on the values of the time-scale variables, notably the left endpoints of the intervals.

#### 2.1.2 Differences between time scales

Apparently, the only fixed variable is sex, but formally the dates of birth (dobth), diagnosis (dodm) and first insulin purchase (doins) are fixed covariates too. They can be constructed as origins of time scales referred to the calendar time scale. Likewise, and possibly of greater interest, we can consider these origins on the age scale, calculated as the difference between age and another time scale.

These would then be age at birth (hardly relevant), age at diabetes diagnosis and age at insulin treatment.

#### 2.1.3 Keeping the relation between time scales

The midpoint (as well as the right interval endpoint) should be used with caution if the variable age at diagnosis dodm-dobth is modeled too; the age at diabetes is logically equal to the difference between current age (age) and time since diabetes diagnosis (tfD):

```
> summary( (dmS2$age-dmS2$tfD) - (dmS2$dodm-dmS2$dobth) )
Min. 1st Qu. Median Mean 3rd Qu. Max.
0 0 0 0 0 0
```

This calculation refers to the *start* of each interval — which are in the time scale variables in the Lexis object. But when using the middle of the intervals, this relationship is not preserved:

If all three variables are to be included in a model, we must make sure that the *substantial* relationship between the variables be maintained. One way is to recompute age at diabetes

diagnosis from the two midpoint variables, but more straightforward would be to use the left endpoint of the intervals, that is the time scale variables in the Lexis object.

If we dissolve the relationship between the variables age, tfD and age at diagnosis by grouping we may obtain identifiability of the three separate effects, but it will be at the price of an arbitrary allocation of a linear trend between them.

For the sake of clarity, consider current age, a, duration of disease, d and age at diagnosis e, where

current age = age at diagnosis + disease duration, i.e.  $a = e + d \Leftrightarrow e + d - a = 0$ 

If we model the effect of the quantitative variables a, e and d on the log-rates by three functions f, g and h:  $\log(\lambda) = f(a) + g(d) + h(e)$  then for any  $\kappa$ :

$$\log(\lambda) = f(a) + g(d) + h(e) + \kappa(e + d - a)$$

$$= (f(a) - \kappa a) + (g(d) + \kappa d) + (h(e) + \kappa e)$$

$$= \tilde{f}(a) + \tilde{g}(d) + \tilde{h}(e)$$

In practical modeling this will turn op as a singular model matrix with one parameter aliased, corresponding to some arbitrarily chosen value of  $\kappa$  (depending on software conventions for singular models). This is well known from age-period-cohort models.

Thus we see that we can move any slope around between the three terms, so if we achieve identifiability by using grouping of one of the variables we will in reality have settled for a particular value of  $\kappa$ , without known why we chose just that. The solution is to resort to predictions which are independent of the particular parametrization or choose a particular parametrization with explicit constraints.

#### 2.2 Modeling of rates

As mentioned, the purpose of subdividing follow-up data in smaller intervals is to be able to model effects of time scale variables as parametric functions. When we split along a time scale we can get intervals that are as small as we want; if they are sufficiently small, an assumption of constant rates in each interval becomes reasonable.

In a model that assumes a constant occurrence rate in each of the intervals the likelihood contribution from each interval is the same as the likelihood contribution from a Poisson variate D, say, with mean  $\lambda \ell$  where  $\lambda$  is the rate and  $\ell$  is the interval length, and where the value of the variate D is 1 or 0 according to whether an event has occurred or not. Moreover, the likelihood contributions from all follow-up intervals from a single person are conditionally independent (conditional on having survived till the start of the interval in question). This implies that the total contribution to the likelihood from a single person is a product of terms, and hence the same as the likelihood of a number of independent Poisson terms, one from each interval.

Note that variables are neither Poisson distributed (e.g. they can only ever assume values 0 or 1) nor independent — it is only the likelihood for the follow-up data that happens to be the same as the likelihood from independent Poisson variates. Different models can have the same likelihood, a model cannot be inferred from the likelihood.

Parametric modeling of the rates is obtained by using the *values* of the time scales for each interval as *quantitative* explanatory variables, using for example splines. And of

course also the values of the fixed covariates and the time-dependent variables for each interval. Thus the model will be one where the rate is assumed constant in each (small) interval, but where a parametric form of the *size* of the rate in each interval is imposed by the model, using the time scale as a quantitative covariate.

#### 2.2.1 Interval length

In the first chapter we illustrated cutting and splitting by listing the results for a few individuals across a number of intervals. For illustrational purposes we used 5-year age bands to avoid excessive listings, but since the doubling time for mortality on the age scale is only slightly larger than 5 years, the assumption about constant rates in each interval would be pretty far fetched if we were to use 5 year intervals.

Thus, for modeling purposes we split the follow-up in 3 month intervals. When we use intervals of 3 month's length it is superfluous to split along multiple time scales — the precise location of tightly spaced splits will be irrelevant from any practical point of view. splitLexis and splitMulti will allocate the actual split times for all of the time scale variables, so these can be used directly in modeling.

So we split the cut dataset in 3-month intervals along the age scale:

```
> dmCs \leftarrow splitMulti(dmC, age = seq(0,110,1/4))
> summary( dmCs, t=T )
Transitions:
     Tο
From
          DM
               Ins Dead
                         Records:
                                     Events: Risk time:
                                                          Persons:
  DM
     189669
              1694 2048
                             193411
                                        3742
                                                45885.49
                                                              9899
           0 34886
                    451
                             35337
                                         451
                                                 8387.77
                                                               1791
  Sum 189669 36580 2499
                            228748
                                        4193
                                                54273.27
                                                              9996
Timescales:
                    tfI
              tfD
               "" "Ins"
```

We see that we now have 228,748 records and 9996 persons, so about 23 records per person. The total risk time is 54,275 years, a bit less than 3 months per record as expected.

#### 2.2.2 Practicalities for splines

In this study we want to look at how mortality depend on age (age) and time since start of insulin use (tfI). If we want to use splines in the description we must allocate knots for anchoring the splines at each of the time scales, either by some *ad hoc* method or by using some sort of penalized splines. The latter will not be treated here; it belongs in the realm of the mgcv package.

Here we shall use the former approach and allocate 5 knots on each of the time-scales. We allocate knots so that we have the events evenly distributed between the knots. Since the insulin state starts at 0 for all individuals we include 0 as the first knot, such that any set of natural splines with these knots will have the value 0 at 0 on the time scale.

In the Epi package there is a convenience wrapper, Ns, for the natural spline generator ns, that takes the smallest and the largest of a set of supplied knots to be the boundary knots, so the explicit definition of the boundary knots becomes superfluous.

Note that it is a feature of the Ns (via the features of ns) that any generated spline function is 0 at the leftmost knot.

#### 2.2.3 Poisson model

Number of Fisher Scoring iterations: 8

A model that describes mortality rates as only a function of age would then be:

```
> ma <- glm( (lex.Xst=="Dead") ~ Ns(age,knots=a.kn),
              family = poisson,
+
              offset = log(lex.dur),
                data = dmCs )
> summary( ma )
Call:
glm(formula = (lex.Xst == "Dead") ~ Ns(age, knots = a.kn), family = poisson,
    data = dmCs, offset = log(lex.dur))
Deviance Residuals:
   Min
         1Q
                 Median
                                3Q
                                        Max
-0.5883 -0.1688 -0.1144 -0.0766
                                     4.5958
Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)
                                           -99.16
                                                     <2e-16
                      -3.82830
                                0.03861
Ns(age, knots = a.kn)1 1.36254
                                   0.08723
                                             15.62
                                                     <2e-16
                       1.49446
                                             21.83
Ns(age, knots = a.kn)2
                                   0.06845
                                                     <2e-16
Ns(age, knots = a.kn)3 2.63557
                                   0.07058
                                             37.34
                                                     <2e-16
                                   0.05769
                                             33.66
                                                     <2e-16
Ns(age, knots = a.kn)4 1.94173
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 27719 on 228747
                                    degrees of freedom
Residual deviance: 25423 on 228743 degrees of freedom
AIC: 30431
```

The offset,  $\log(\text{lex.dur})$  comes from the fact that the likelihood for the follow-up data during  $\ell$  time is the same as that for independent Poisson variates with mean  $\lambda \ell$ , and that the default link function for the Poisson family is the log, so that we are using a linear model for the log-mean,  $\log(\lambda) + \log(\ell)$ . But when we want a model for the log-rate  $(\log(\lambda))$ , the term  $\log(\ell)$  must still be included as a covariate, but with regression coefficient fixed to 1; a so-called *offset*. This is however a technicality; it just exploits that the likelihood of a particular Poisson model and that of the rates model is the same.

In the Epi package is a glm family, poisreg that has a more intuitive interface, where the response is a 2-column matrix of events and person-time, respectively. This is in concert with the fact that the outcome variable in follow-up studies is bivariate: (event, risk time).

```
> Ma <- glm(cbind(lex.Xst=="Dead",lex.dur) ~ Ns(age,knots=a.kn),
            family = poisreg, data = dmCs )
> summary( Ma )
Call:
glm(formula = cbind(lex.Xst == "Dead", lex.dur) ~ Ns(age, knots = a.kn),
   family = poisreg, data = dmCs)
Deviance Residuals:
   Min
         1Q
                Median
                              3Q
                                      Max
-0.5883 -0.1688 -0.1144 -0.0766
                                   4.5958
Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)
                     -3.82830 0.03861 -99.15
Ns(age, knots = a.kn)1 1.36254 0.08723 15.62
                                                  <2e-16
Ns(age, knots = a.kn)2 1.49446 0.06845
                                           21.83
                                                  <2e-16
Ns(age, knots = a.kn)3 2.63557
                              0.07058
                                           37.34
                                                  <2e-16
Ns(age, knots = a.kn)4 1.94173
                                0.05769
                                          33.66
                                                  <2e-16
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 27719 on 228747 degrees of freedom
Residual deviance: 25423 on 228743 degrees of freedom
AIC: 30431
Number of Fisher Scoring iterations: 7
```

Exploiting the multistate structure in the Lexis object there is a multistate convenience wrapper for glm with the poisreg family, that just requires specification of the transitions in terms of from and to. Although it is called glm. Lexis it is not an S3 method for Lexis objects:

```
> Xa <- glm.Lexis( dmCs, from="DM", to="Dead",
                   formula = ~ Ns(age,knots=a.kn) )
stats::glm Poisson analysis of Lexis object dmCs with log link:
Rates for the transition: DM->Dead
```

The result is a glm object but with an extra attribute, Lexis:

```
> attr( Xa, "Lexis" )
$data
[1] "dmCs"
$trans
[1] "DM->Dead"
$formula
~Ns(age, knots = a.kn)
<environment: 0xbe99468>
$scale
[1] 1
```

There are similar wrappers for gam and coxph models, gam.Lexis and coxph.Lexis, but these will not be elaborated in detail.

The from= and to= can even be omitted, in which case all possible transitions *into* any of the absorbing states is modeled:

```
> xa <- glm.Lexis( dmCs, formula = ~ Ns(age,knots=a.kn) )
stats::glm Poisson analysis of Lexis object dmCs with log link:
Rates for transitions: DM->Dead, Ins->Dead
We can check if the four models fitted are the same:
> c( deviance(ma), deviance(Ma), deviance(Xa), deviance(xa) )
[1] 25422.92 25422.92 20902.31 25422.92
```

Oops! the model Xa is apparently not the same as the other three? This is because the explicit specification from="DM", to="Dead", omits modeling contributions from the Ins  $\rightarrow$  Dead transition — the output actually said so. The other three models all use both transitions — and assume that the two transition rates are the same, *i.e.* that start of insulin has no effect on mortality. We shall relax this assumption later.

The parameters from the model do not have any direct interpretation *per se*, but we can compute the estimated mortality rates for a range of ages using ci.pred with a suitably defined prediction data frame.

Note that if we use the poisson family of models, we must specify all covariates in the model, including the variable in the offset, lex.dur (remember that this was a covariate with coefficient fixed at 1). We set the latter to 1000, because we want the mortality rates per 1000 person-years. Using the poisreg family, the prediction will ignore any value of lex.dur specified in the prediction data frame, the returned rates will be per unit in which lex.dur is recorded.

```
> nd <- data.frame( age=40:85, lex.dur=1000 )</pre>
> pr.0 <- ci.pred( ma, newdata = nd )</pre>
                                              # mortality per 100 PY
> pr.a <- ci.pred( Ma, newdata = nd )*1000 # mortality per 100 PY</pre>
> summary(pr.0/pr.a)
                   2.5%
                               97.5%
    Estimate
Min.
       :1
             Min.
                   :1
                          Min. :1
1st Qu.:1
             1st Qu.:1
                          1st Qu.:1
Median :1
             Median :1
                          Median:1
Mean
      :1
             Mean
                   :1
                          Mean
             3rd Qu.:1
3rd Qu.:1
                          3rd Qu.:1
Max.
        :1
             Max.
                     :1
                          Max.
> matshade( nd$age, pr.a, plot=TRUE,
            type="1", lty=1,
log="y", xlab="Age (years)",
+
            ylab="DM mortality per 1000 PY")
```

#### 2.3 Time dependent covariate

A Poisson model approach to mortality by insulin status, would be to assume that the rate-ratio between patients on insulin and not on insulin is a fixed quantity, independent of time since start of insulin, independent of age. This is commonly termed a proportional hazards assumption, because the rates (hazards) in the two groups are proportional along the age (baseline time) scale.

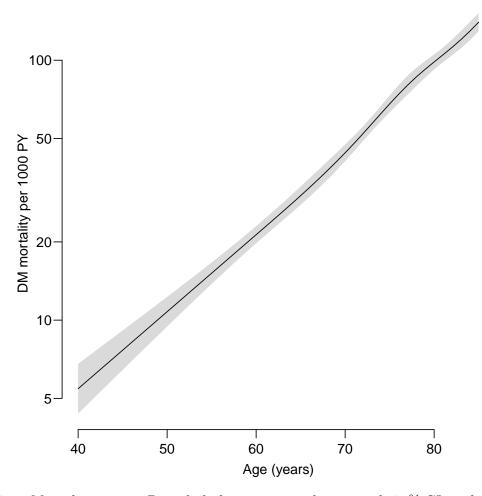


Figure 2.1: Mortality among Danish diabetes patients by age with 95% CI as shaded area. We see that the rates increase linearly on the log-scale, that is exponentially by age. ./flup-pr-a

```
> pm <- glm( cbind(lex.Xst=="Dead",lex.dur)</pre>
                                               ~ Ns(age,knots=a.kn)
                                               + lex.Cst + sex,
              family=poisreg, data = dmCs )
> round( ci.exp( pm ), 3 )
                        exp(Est.)
                                     2.5%
                                           97.5%
(Intercept)
                                           0.024
                            0.022
                                    0.021
Ns(age, knots = a.kn)1
                            4.248
                                    3.581
                                           5.040
Ns(age, knots = a.kn)2
                            5.008
                                    4.376
                                           5.731
Ns(age, knots = a.kn)3
                            16.832 14.624 19.373
Ns(age, knots = a.kn)4
                            7.994
                                           8.968
                                    7.126
lex.CstIns
                                           2.200
                            1.985
                                    1.791
sexF
                            0.668
                                    0.617
                                           0.724
```

So we see that persons on insulin have about twice the mortality of persons not on insulin and that women have 2/3 the mortality of men.

#### 2.3.1 Time since insulin start

If we want to test whether the excess mortality depends on the time since start if insulin treatment, we can add a spline terms in tfI. But since tfI is a time scale defined as time

since entry into a new state (Ins), the variable tfI will be missing for those in the DM state, so before modeling we must set the NAs to 0, which we do with tsNA20 (acronym for timescale NAs to zero):

As noted before we could do this simpler with glm.Lexis, even without the from= and to= arguments, because we are modeling all transitions *into* the absorbing state (Dead):

The coding of the effect of tfI is so that the value is 0 at 0, so the meaning of the estimate of the effect of lex.Cst is the RR between persons with and without insulin, immediately after start of insulin:

We see that the effect of sex is pretty much the same as before, but the effect of lex.Cst is much larger, it now refers to a different quantity, namely the RR at tfI=0. If we want to see the effect of time since insulin, it is best viewed jointly with the effect of age:

```
> ndI <- data.frame( expand.grid( tfI=c(NA,seq(0,15,0.1)),</pre>
                                   ai = seq(40,80,10)),
+
                     sex="M",
                     lex.Cst="Ins" )
> ndI <- transform( ndI, age=ai+tfI )</pre>
> head( ndI )
  tfI ai sex lex.Cst age
 NA 40
         M
                 Ins
                       NA
1
         M
2 0.0 40
                 Ins 40.0
3 0.1 40
         M
                 Ins 40.1
4 0.2 40
         M
                 Ins 40.2
5 0.3 40
         Μ
                 Ins 40.3
6 0.4 40
                 Ins 40.4
         M
> ndA <- data.frame( age= seq(40,100,0.1), tfI=0, lex.Cst="DM", sex="M" )
> pri <- ci.pred( Pm, ndI ) * 1000
> pra <- ci.pred( Pm, ndA ) * 1000
> matshade( ndI$age, pri, plot=TRUE, las=1,
            xlab="Age (years)", ylab="DM mortality per 1000 PY",
            log="y", lty=1, col="blue" )
> matshade( ndA$age, pra )
```

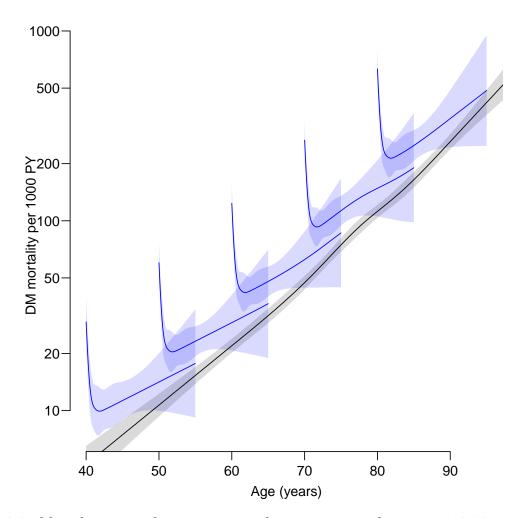


Figure 2.2: Mortality rates of persons on insulin, starting insulin at ages 40,50,...,80 (blue), compared with persons not on insulin (black curve). Shaded areas are 95% CI../flup-ins-time

In figure 2.2, p. 21, we see that mortality is high just after insulin start, but falls by almost a factor 3 during the first year. Also we see that there is a tendency that mortality in a given age is smallest for those with the longest duration of insulin use.

#### 2.4 The Cox model

Note that in the Cox-model the age is used as response variable, slightly counter-intuitive. Hence the age part of the linear predictors is not in that model:

There is also a multistate wrapper for Cox models, requiring a l.h.s. side for the formula= argument:

```
> Cm <- coxph.Lexis( tsNA20(dmCs),
+ form= age ~ Ns(tfI,knots=i.kn) + lex.Cst + sex )</pre>
```

```
model survival::coxph analysis of Lexis object tsNA20(dmCs):
Rates for transitions DM->Dead, Ins->Dead
> cbind( ci.exp( cm ), ci.exp( Cm ) )
                                    2.5%
                     exp(Est.)
                                            97.5% exp(Est.)
                                                                 2.5%
                                                                         97.5%
Ns(tfI, knots = i.kn)1 0.2984062 0.19417148 0.4585960 0.2984062 0.19417148 0.4585960
Ns(tfI, knots = i.kn)2 0.3871151 0.29011380 0.5165495 0.3871151 0.29011380 0.5165495
Ns(tfI, knots = i.kn)3 0.1239128 0.06287008 0.2442238 0.1239128 0.06287008 0.2442238
Ns(tfI, knots = i.kn)4 0.4405121 0.34839015 0.5569932 0.4405121 0.34839015 0.5569932
                     5.6700284 4.45011220 7.2243623 5.6700284 4.45011220 7.2243623
lex.CstIns
lex.CstDead
                     sexF
                     0.6753202 0.62316569 0.7318397 0.6753202 0.62316569 0.7318397
```

We can compare the estimates from the Cox model with those from the Poisson model we must add NAs because the Cox-model does not give the parameters for the baseline time scale (age), but also remove one of the parameters, because coxph parametrizes factors (in this case lex.Cst) by all defined levels and not only by the levels present in the dataset at hand (note the line of 1.0000000s in the print above):

```
> round( cbind( ci.exp( Pm ),
         rbind(matrix(NA,5,3),
                ci.exp(cm)[-6,])), 3)
                                    2.5% 97.5% exp(Est.)
                       exp(Est.)
                                                            2.5% 97.5%
(Intercept)
                           0.022
                                   0.021
                                          0.024
                                                       NA
                                                              NA
Ns(age, knots = a.kn)1
                           4.208
                                   3.546
                                          4.993
                                                       NA
                                                              NA
                                                                    NΑ
Ns(age, knots = a.kn)2
                           5.012
                                  4.380
                                          5.736
                                                       NA
                                                              NA
                                                                    NA
Ns(age, knots = a.kn)3
                           16.560 14.386 19.063
                                                       NA
                                                              NA
                                                                    NA
Ns(age, knots = a.kn)4
                           7.921
                                   7.061
                                          8.885
                                                       NA
                                                              NA
                                                    0.298 0.194 0.459
Ns(tfI, knots = i.kn)1
                           0.298
                                   0.194
                                          0.458
Ns(tfI, knots = i.kn)2
                           0.385
                                   0.289
                                          0.514
                                                    0.387 0.290 0.517
Ns(tfI, knots = i.kn)3
                           0.125
                                   0.064
                                          0.246
                                                    0.124 0.063 0.244
Ns(tfI, knots = i.kn)4
                           0.438
                                   0.346
                                          0.553
                                                    0.441 0.348 0.557
lex.CstIns
                           5.632
                                   4.430
                                          7.160
                                                    5.670 4.450 7.224
sexF
                            0.674
                                   0.622
                                          0.730
                                                    0.675 0.623 0.732
```

Thus we see that the Poisson and Cox gives pretty much the same results. You may argue that Cox requires a smaller dataset, because there is no need to subdivide data in small intervals before insulin use. But certainly the time after insulin inception need to be if the effect of this time should be modeled.

The drawback of the Cox-modeling is that it is not possible to show the absolute rates as we did in the graph above.

#### 2.5Marginal effect of time since insulin

When we plot the marginal effect of tfI from the two models we get pretty much the same; here we plot the RR relative to tfI=2 years. Note that we are deriving the RR as the ratio of two sets of predictions, from the data frames nd and nr — for further details consult the help page for ci.lin, specifically the use of a list as the ctr.mat argument:

```
> nd <- data.frame( tfI=seq(0,15,,151), lex.Cst="Ins", sex="M" )</pre>
                                           , lex.Cst="Ins", sex="M" )
> nr <- data.frame( tfI=</pre>
                               2
> ppr <- ci.exp( pm, list(nd,nr), xvars="age" )</pre>
> cpr <- ci.exp( cm, list(nd,nr) )</pre>
```

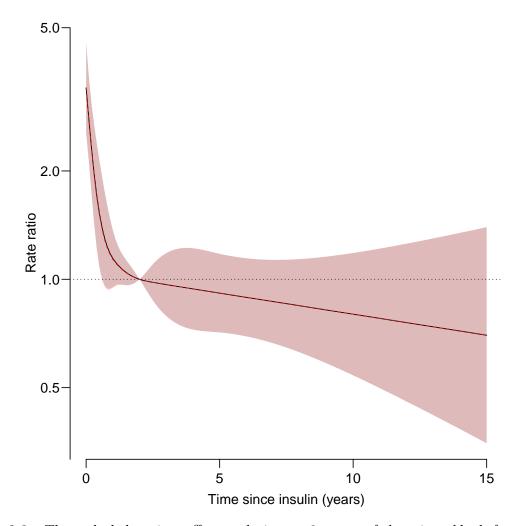


Figure 2.3: The naked duration effects relative to 2 years of duration, black from Poisson model, red from Cox model. The two sets of estimates are identical, and so are the standard errors, so the two shaded areas overlap almost perfectly.

./flup-leff

In figure 2.3, p. 23, we see that the duration effect is exactly the same from the two modeling approaches.

We will also want the RR relative to the non-insulin users — recall these are coded 0 on the tfI variable:

```
+ ylab="Rate ratio relative to non-Insulin",
+ lty=c(1,2), log="y", plot=T)
```

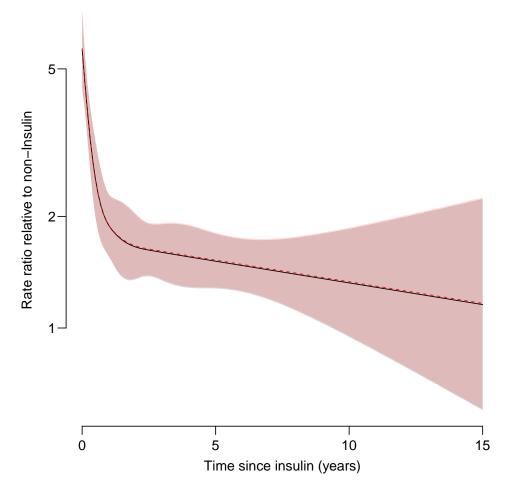


Figure 2.4: Insulin duration effect (state Ins) relative to no insulin (state DM), black from Poisson model, red from Cox model. The shape is the same as the previous figure, but the RR is now relative to non-insulin, instead of relative to insulin users at 2 years duration. The two sets of estimates are identical, and so are the standard errors, so the two shaded areas overlap almost perfectly.

./flup-IeffR

In figure 2.4, p. 24, we see the effect of increasing duration of insulin use for a fixed age which is a bit artificial, so we would like to see the *joint* effects of age and insulin duration. What we cannot see is how the duration affects mortality relative to current age (at the age attained at the same time as the attained tfI).

Another way of interpreting this curve is as the rate ratio relative to a person not on insulin, so we see that the RR (or hazard ratio, HR as some call it) is over 5 at the start of insulin (the lex.Cst estimate), and decreases to about 1.5 in the long term.

Both figure 2.3 and 2.4 indicate a declining RR by insulin duration, but only from figure 2.2 it is visible that mortality actually is *increasing* by age after some 2 years after insulin start. This point would not be available if we had only fitted a Cox model where we did not have access to the baseline hazard as a function of age.

#### 2.6 Age×duration interaction

The model we fitted assumes that the RR is the same regardless of the age at start of insulin — the hazards are multiplicative. Sometimes this is termed the proportional hazards assumption: For *any* fixed age the HR is the same as a function of time since insulin, and vice versa.

A more correct term would be "main effects model" — there is no interaction between age (the baseline time scale) and other covariates. So there is really no need for the term "proportional hazards"; well defined and precise statistical terms for it has existed for aeons.

#### 2.6.1 Age at insulin start

In order to check the consistency of the multiplicativity assumption across the spectrum of age at insulin inception, we can fit an interaction model. One approach to this would be using a non-linear effect of age at insulin use (for convenience we use the same knots as for age) — note that the prediction data frames are the same as we used above, because we do not compute age at insulin use as a separate variable, but rather enter it as the difference between current age (age) and insulin duration (tfI).

At first glance we might think of doing:

But this will fit a model with a rate-ratio between persons with and without insulin that depends only on age at insulin start for the time *after* insulin start, the RR at tfI=0 will be the same at any age, which really is not the type of interaction we wanted.

We want the age-tfI term to be specific for the insulin exposed so we may use one of two other approaches, that are conceptually alike but mathematically different:

```
> Im <- glm.Lexis( tsNA20(dmCs),</pre>
                   formula = ~ Ns(age
                                         ,knots=a.kn)
+
                              + Ns( tfI,knots=i.kn)
                              + Ns((age-tfI)*(lex.Cst=="Ins"),knots=a.kn)
+
                              + lex.Cst + sex )
stats::glm Poisson analysis of Lexis object tsNA20(dmCs) with log link:
Rates for transitions: DM->Dead, Ins->Dead
> im <- glm.Lexis( tsNA20(dmCs),</pre>
                   formula = ~ Ns(age
                                          ,knots=a.kn)
+
                             + Ns( tfI,knots=i.kn)
+
                              + lex.Cst:Ns(age-tfI,knots=a.kn)
                              + lex.Cst + sex )
stats::glm Poisson analysis of Lexis object tsNA20(dmCs) with log link:
Rates for transitions: DM->Dead, Ins->Dead
```

The first model (Im) has a common age-effect (Ns(age,...) for persons with and without diabetes and a RR depending on insulin duration tfI and age at insulin (age-tfI). Since

the linear effect of these two terms are in the model as well, a linear trend in the RR by current age (age) is accommodated as well.

The second model allows age-effects that differ non-linearly between person with and without insulin, because the interaction term lex.Cst:Ns(age-tfI... for persons not on insulin is merely an age term (since tfI is coded 0 for all follow-up not on insulin).

We can compare the models fitted:

```
> anova( imx, Im, im, test='Chisq')
Analysis of Deviance Table
Model 1: cbind(trt(Lx$lex.Cst, Lx$lex.Xst) %in% trnam, Lx$lex.dur) ~ Ns(age,
    knots = a.kn) + Ns(tfI, knots = i.kn) + Ns(age - tfI, knots = a.kn) +
    lex.Cst + sex
Model 2: cbind(trt(Lx$lex.Cst, Lx$lex.Xst) %in% trnam, Lx$lex.dur) ~ Ns(age,
    knots = a.kn) + Ns(tfI, knots = i.kn) + Ns((age - tfI) *
    (lex.Cst == "Ins"), knots = a.kn) + lex.Cst + sex
Model 3: cbind(trt(Lx$lex.Cst, Lx$lex.Xst) %in% trnam, Lx$lex.dur) ~ Ns(age,
    knots = a.kn) + Ns(tfI, knots = i.kn) + lex.Cst:Ns(age -
    tfI, knots = a.kn) + lex.Cst + sex
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1
     228734
                 25096
     228733
                 25087
                            8.9631 0.002755
3
     228730
                 25082
                       3
                            4.6804 0.196749
```

so we see that the models indeed are different, and moreover that the last model does not provide substantial further improvement, by allowing non-linear RR along the age-scale.

We can illustrate the different estimated rates from the three models in figure 2.5, p. 27:

We can also plot the RRs only from these models (figure 2.6, p. 28); for this we need the reference frames, and the machinery from ci.exp allowing a list of two data frames:

```
> ndR <- transform( ndI, tfI=0, lex.Cst="DM" )</pre>
> cbind( head(ndI), head(ndR) )
  tfI ai sex lex.Cst age tfI ai sex lex.Cst
1 NA 40
           Μ
                 Ins
                       NA
                             0 40
                                    Μ
                                           DM
                                                NA
2 0.0 40
           Μ
                 Ins 40.0
                             0 40
                                    Μ
                                           DM 40.0
3 0.1 40
           Μ
                 Ins 40.1
                             0 40
                                    Μ
                                           DM 40.1
4 0.2 40
          Μ
                 Ins 40.2
                             0 40
                                    M
                                           DM 40.2
5 0.3 40
          Μ
                 Ins 40.3
                             0 40
                                    Μ
                                           DM 40.3
6 0.4 40
         M
                 Ins 40.4
                             0 40
                                           DM 40.4
                                    Μ
```

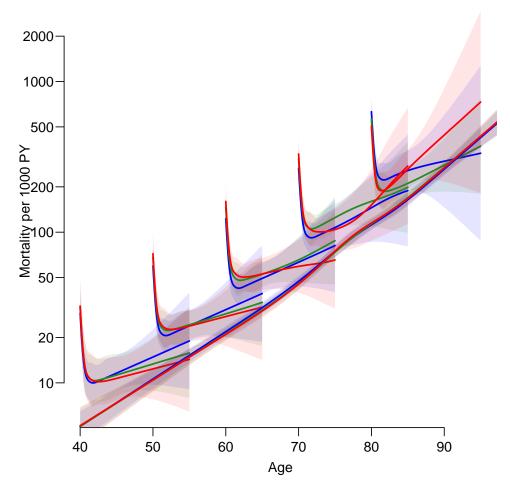


Figure 2.5: Age at insulin as interaction between age and duration. Blue curves are from the naive interaction model  $\mathtt{imx}$  with identical RR at  $\mathtt{tfI} = 0$  at any age; green curves are from the interaction model with age at insulin, from the model  $\mathtt{Im}$  with only linear differences by age, and red lines from the full interaction model  $\mathtt{im}$ .

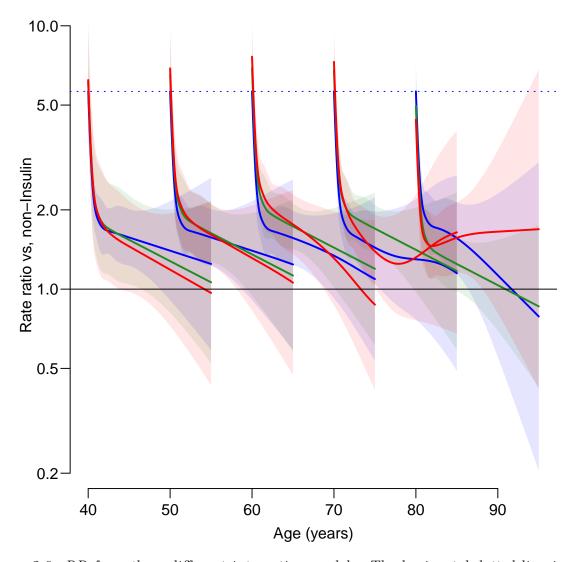


Figure 2.6: RR from three different interaction models. The horizontal dotted line is at the estimated effect of lex.Cst, to illustrate that the first model (blue) constrains this initial HR to be constant across age. The green curves are the extended interaction model, and the red the full one.

#### 2.6.2 General interaction

As a final illustration we may want to explore a different kind of interaction, not defined from the duration — here we simplify the interaction by not using the second-last knot in the interaction terms — figure 2.7, p. 30. Note again that the prediction code is the same:

```
> gm <- glm.Lexis( tsNA20(dmCs),
                      formula = ~ Ns(age,knots=a.kn)
+
                                  + Ns(tfI,knots=i.kn)
+
                                  + lex.Cst:Ns(age,knots=a.kn):Ns(tfI,knots=i.kn)
                                  + lex.Cst + sex )
stats::glm Poisson analysis of Lexis object tsNA20(dmCs) with log link:
Rates for transitions: DM->Dead, Ins->Dead
> pgi <- ci.pred( gm, ndI )</pre>
> pga <- ci.pred( gm, ndA )</pre>
> par(mar=c(3,3,1,1), mgp=c(3,1,0)/1.6, las=1, bty="n")
> matshade( ndI$age, cbind( pgi, pii )*1000, plot=T,
+ lty=c("solid","21"), lend="butt", lwd=2, log="y",
+ xlab="Age (years)", ylab="Mortality rates per 1000 PY",
              alpha=c(0.2,0.1), col=c("black","red"))
> matshade( ndA$age, cbind( pga, pia )*1000,
              lty=c("solid","21"), lend="butt", lwd=2,
              alpha=c(0.2,0.1), col=c("black","red"))
```

This is in figure 2.7, p. 30.

#### 2.6.3 Evaluating interactions

Here we see that the interaction effect is such that in the older ages the length of insulin use has an increasing effect on mortality.

Even though there is no statistically significant interaction between age and time since start of insulin, it would be illustrative to show the RR as a function of age at insulin and age at follow-up:

This is in figure 2.8, p. 31.

The advantage of the parametric modeling (be that with age at insulin or general spline interaction) is that it is straight-forward to *test* whether we have an interaction.

#### 2.7 Separate models

In the above we insisted on making a joint model for the DM $\rightarrow$ Dead and the Ins $\rightarrow$ Dead transitions, but with the complications demonstrated it would actually have been more sensible to model the two transitions separately:

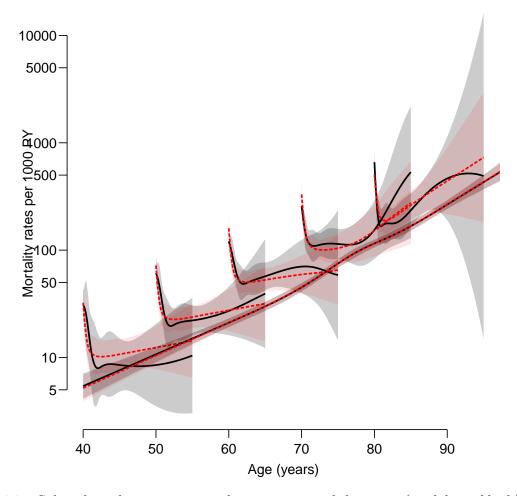


Figure 2.7: Spline-by-spline interaction between age and duration (model gm, black), and the interaction using a non-linear effect of age at entry (model im, red), corresponding to the red curves in figure 2.5.

./flup-splint

```
> dmd <- glm.Lexis( dmCs,
                      from="DM", to="Dead",
formula = ~ Ns(age,knots=a.kn)
                                 + sex)
stats::glm Poisson analysis of Lexis object dmCs with log link:
Rates for the transition: DM->Dead
  ind <- glm.Lexis( dmCs,</pre>
                      from="Ins", to="Dead",
                      formula = ~ Ns(age,knots=a.kn)
                                 + Ns(tfI,knots=i.kn)
                                 + Ns(age-tfI,knots=a.kn)
                                 + sex )
stats::glm Poisson analysis of Lexis object dmCs with log link:
Rates for the transition: Ins->Dead
> ini <- ci.pred( ind, ndI )</pre>
> dmi <- ci.pred( dmd, ndI )</pre>
> dma <- ci.pred( dmd, ndA )</pre>
```

The estimated mortality rates are shown in figure ??, p. ??, using:

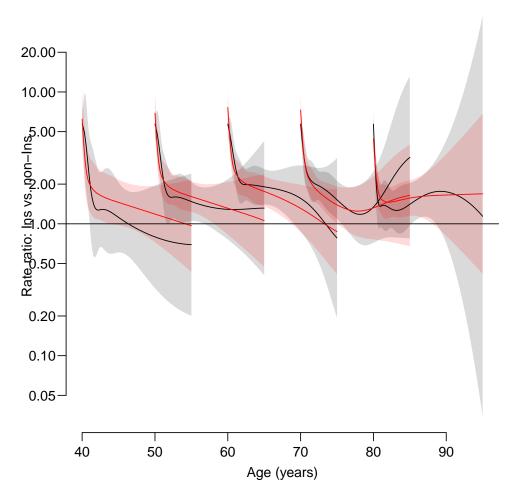


Figure 2.8: The effect of duration of insulin use at different ages of follow-up (and age at insulin start). Estimates are from the model with an interaction term using a non-linear effect of age at insulin start (model im, red) and using a general spline interactions (model gm, black). It appears that the general interaction over-models a bit. .../flup-RR-int

The estimated RRs are computed using that the estimates from the two models are uncorrelated, and hence qualify for ci.ratio (this and the previous graph appear in figure 2.9, p. 32)

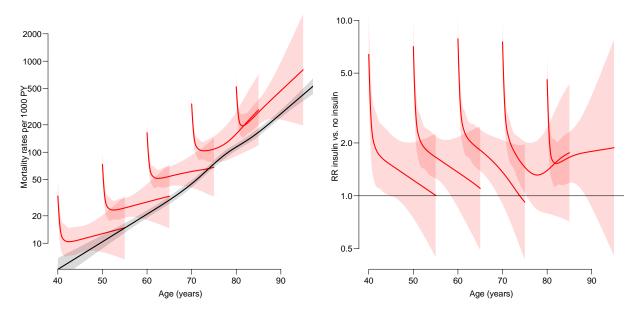


Figure 2.9: Left panel: Mortality rates from separate models for the two mortality transitions; the DM $\rightarrow$ Dead transition modeled by age alone; Ins $\rightarrow$ Dead transition modeled with spline effects of current age, time since insulin and and age at insulin. Right panel: Mortality HR of insulin vs. no insulin.

## Chapter 3

#### More states

#### 3.1 Subdividing states

It may be of interest to subdivide the states following the intermediate event according to whether the event has occurred or not. This will enable us to address the question of the fraction of the patients that ever go on insulin.

This is done by the argument split.states=TRUE.

```
> dmCs <- cutLexis( data = dmS2,
                       cut = dmS2$doins,
                 timescale = "per",
+
                new.state = "Ins"
                new.scale = "tfI"
         precursor.states = "DM"
             split.states = TRUE )
> summary( dmCs )
Transitions:
     То
            Ins Dead Dead(Ins)
From
         DM
                                  Records:
                                             Events: Risk time:
                                                                  Persons:
  DM 35135 1694 2048
                              0
                                     38877
                                                3742
                                                       45885.49
                                                                      9899
          0 5762
                                                 451
                                                        8387.77
                    0
                             451
                                      6213
                                                                      1791
  Sum 35135 7456 2048
                             451
                                      45090
                                                4193
                                                       54273.27
                                                                      9996
```

We can illustrate the numbers and the transitions (figure 3.1, p. 34)

Note that it is only the mortality rates that we have been modeling, namely the transitions DM $\rightarrow$ Dead and Ins $\rightarrow$ Dead(Ins). If we were to model the cumulative risk of using insulin we would also need a model for the DM $\rightarrow$ Ins transition. Subsequent to that we would then compute the probability of being in each state conditional on suitable starting conditions. With models where transition rates depend on several time scales this is not a trivial task. This is treated in more detail in the vignette on simLexis.

#### 3.2 Multiple intermediate events

We may be interested in starting either insulin or OAD (oral anti-diabetic drugs), thus giving rise to more states and more time scales. This can be accomplished by the

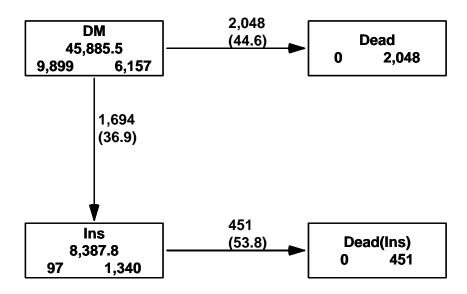


Figure 3.1: Transitions between 4 states: the numbers in the boxes are person-years (middle), and below the number of persons who start, respectively end their follow-up in each of the states. ./flup-box4

mcutLexis function, that generalizes cutLexis:

> dmM <- mcutLexis( dmL,</pre>

```
timescale = "per",
                       wh = c("doins", "dooad"),
              new.states = c("Ins","OAD"),
              new.scales = c("tfI", "tfO"),
        precursor.states = "DM",
             ties.resolve = TRUE )
       9996 records with tied events times resolved.
 Results only reproducible if the seed for the random number generator is set.
> summary( dmM, t=T )
Transitions:
     To
From
            DM Dead OAD
                           Ins OAD-Ins Ins-OAD Records:
                                                            Events: Risk time:
                                                                                  Persons:
          2830 1056 2957
                           689
  DM
                                      0
                                              0
                                                      7532
                                                                4702
                                                                       22920.21
                                                                                      7532
  OAD
             0
                992 3327
                             0
                                   1005
                                              0
                                                      5324
                                                                1997
                                                                       22965.30
                                                                                      5324
                                            172
             0
                 152
                        0
                           462
                                     0
                                                       786
                                                                 324
                                                                        3883.14
                                                                                       786
  Ins
                 266
                        0
                             0
                                    739
                                                                        3770.53
  OAD-Ins
             0
                                              0
                                                      1005
                                                                 266
                                                                                      1005
  Ins-OAD
             0
                  33
                        0
                             0
                                      0
                                             139
                                                       172
                                                                  33
                                                                         734.09
                                                                                       172
          2830 2499 6284 1151
                                            311
                                                     14819
                                                                7322
                                                                       54273.27
                                                                                      9996
                                   1744
Timescales:
               tfD
                     tfI
                           tf0
               "" "Ins" "OAD"
```

We see that we now have two time scales defined as entry since into states.

```
> wh <- c(subset(dmM,lex.Cst=="Ins-OAD")$lex.id[1:2],
          subset(dmM,lex.Cst=="OAD-Ins")$lex.id[1:2])
> options( width=110 )
```

```
> print( subset( dmM, lex.id %in% wh )[,c('lex.id',names(dmM[1:8]),c("doins","dooad"))],
         digits=6, row.names=FALSE )
                       tf0
                                                         lex.dur lex.Cst lex.Xst
 lex.id
              tfI
                               per
                                        age
                                                  tfD
     18
                        NA 1996.75 61.7221 0.0000000 1.1690623
                                                                      DM
                                                                              OAD 2005.99 1997.
               NA 0.00000 1997.92 62.8912 1.1690623 8.0793977
                                                                      OAD OAD-Ins 2005.99 1997.
     18 0.0000000 8.07940 2005.99 70.9706 9.2484600 4.0027379 DAD-Ins DAD-Ins 2005.99 1997.
     25
               NΑ
                        NA 2003.69 60.3422 0.0000000 1.8809035
                                                                      DM
                                                                              OAD 2008.64 2005.
     25
                NA 0.00000 2005.57 62.2231 1.8809035 3.0691307
                                                                      OAD OAD-Ins 2008.64 2005.
     25 0.0000000 3.06913 2008.64 65.2923 4.9500342 1.3579740 DAD-Ins DAD-Ins 2008.64 2005
                        NA 2009.25 53.2183 0.0000000 0.0316523
     20
                NΑ
                                                                      DM
                                                                              Ins 2009.28 2009
     20 0.0000000
                        NA 2009.28 53.2500 0.0316523 0.0105728
                                                                      Ins Ins-OAD 2009.28 2009
     20 0.0105728 0.00000 2009.29 53.2606 0.0422251 0.7079461 Ins-OAD Ins-OAD 2009.28 2009
                        NA 2008.37 63.9316 0.0000000 0.0930869
                                                                              Ins 2008.46 2008
                                                                      DM
                        NA 2008.46 64.0246 0.0930869 0.2135524
                                                                      Ins Ins-OAD 2008.46 2008.
     38 0.2135524 0.00000 2008.67 64.2382 0.3066393 1.3251198 Ins-OAD
                                                                             Dead 2008.46 2008.
> summary( dmM, t=T )
Transitions:
     To
From
            DM Dead
                     OAD
                           Ins OAD-Ins Ins-OAD
                                                 Records:
                                                            Events: Risk time:
                                                                                 Persons:
  DM
          2830 1056 2957
                           689
                                      0
                                              0
                                                      7532
                                                               4702
                                                                       22920.21
                                                                                     7532
  OAD
                992 3327
                             0
                                   1005
                                              0
                                                      5324
                                                               1997
                                                                       22965.30
                                                                                     5324
             0
                                                       786
                                                                324
             0
                 152
                        0
                           462
                                      0
                                            172
                                                                       3883.14
                                                                                      786
  Ins
                                    739
                 266
                        0
                             0
                                              0
                                                      1005
                                                                266
                                                                        3770.53
  OAD-Ins
             0
                                                                                     1005
  Ins-OAD
             0
                  33
                        0
                             0
                                      0
                                            139
                                                       172
                                                                 33
                                                                        734.09
                                                                                      172
          2830 2499 6284 1151
  Sum
                                   1744
                                            311
                                                     14819
                                                               7322
                                                                       54273.27
                                                                                     9996
Timescales:
  per
              tfD
                     tfI
                           tf0
        age
         11-11
               "" "Ins" "OAD"
```

We can also illustrate the transitions to the different states, as in figure 3.2:

```
> boxes( dmM, boxpos=list(x=c(15,80,40,40,85,85),
+ y=c(50,50,90,10,90,10)),
+ scale.R=1000, show.BE=TRUE)
```

We may not be interested in whether persons were prescribed insulin before OAD or vice versa, in which case we would combine the levels with both insulin and OAD to one, regardless of order (figure 3.3):

```
> summary( dmMr <- Relevel( dmM, list('OAD+Ins'=5:6), first=FALSE) )</pre>
                old
                        new
      type
   lex.Cst
                 DM
1
                         DM
2
  lex.Cst
               Dead
3
  lex.Cst
                OAD
                        OAD
4
   lex.Cst
                Ins
5
   lex.Cst OAD-Ins OAD+Ins
6
  lex.Cst Ins-OAD OAD+Ins
7
  lex.Xst
                 DM
                         DM
8
  lex.Xst
               Dead
                       Dead
9
  lex.Xst
                OAD
                        OAD
10 lex.Xst
                Ins
11 lex.Xst OAD-Ins OAD+Ins
12 lex.Xst Ins-OAD OAD+Ins
```

Transitions:

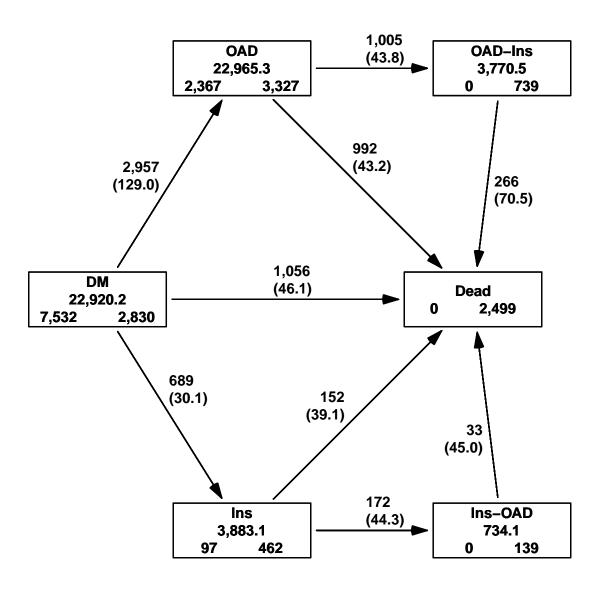


Figure 3.2: Boxes for the dmM object. The numbers in the boxes are person-years (middle), and below the number of persons who start, respectively end their follow-up in each of the states.

./flup-mbox

```
То
                           Ins OAD+Ins
                                         Records:
From
            DM Dead OAD
                                                    Events: Risk time:
                                                                         Persons:
  DM
          2830 1056 2957
                            689
                                      0
                                                       4702
                                                               22920.21
                                                                              7532
                                              7532
  OAD
              0
                992 3327
                             0
                                   1005
                                              5324
                                                       1997
                                                               22965.30
                                                                              5324
                           462
                                                        324
  Ins
              0
                 152
                        0
                                    172
                                               786
                                                                3883.14
                                                                               786
  OAD+Ins
              0
                 299
                        0
                             0
                                    878
                                              1177
                                                        299
                                                                4504.62
                                                                              1177
  Sum
          2830 2499 6284 1151
                                   2055
                                             14819
                                                       7322
                                                               54273.27
                                                                              9996
> boxes(dmMr, boxpos=list(x=c(15,50,15,85,85),
                            y=c(85,50,15,85,15)),
+
                scale.R=1000, show.BE=TRUE )
```

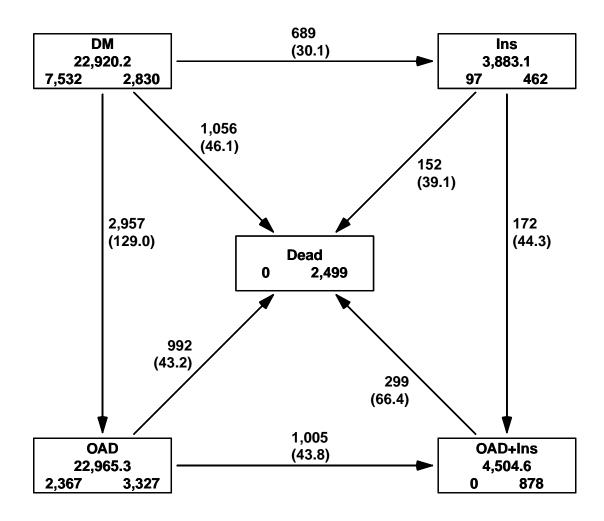


Figure 3.3: Boxes for the dmMr object with collapsed states. The numbers in the boxes are person-years (middle), and below the number of persons who start, respectively end their follow-up in each of the states.

./flup-mboxr

Diagrams as those in figures 3.2 and 3.3 gives an overview of the possible transitions, which states it might be relevant to collapse, and which transitions to model and how.

## Chapter 4

#### Lexis functions

The Lexis machinery has evolved over time since it was first introduced in a workable version in Epi\_1.0.5 in August 2008.

Over the years there have been additions of tools for handling multistate data. Here is a list of the current functions relating to Lexis objects with a very brief description; it does not replace the documentation. Unless otherwise stated, functions named something.Lexis (with a ".") are S3 methods for Lexis objects, so you can skip the ".Lexis" in daily use.

#### Define

Lexis defines a Lexis object

#### Cut and split

cutLexis cut follow-up at intermediate event
mcutLexis cut follow-up at several intermediate events
countLexis cut follow-up at intermediate event count the no. events so far
splitLexis split follow up along a time scale
splitMulti split follow up along a time scale — from the popEpi package, faster
 and has simpler syntax than splitLexis
addCov.Lexis add clinical measurements at a given date to a Lexis object

#### Boxes and plots

boxes.Lexis draw a diagram of states and transitions plot.Lexis draw a standard Lexis diagram points.Lexis add points to a Lexis diagram lines.Lexis add lines to a Lexis diagram PY.ann.Lexis annotate life lines in a Lexis diagram

#### Summarize and query

summary.Lexis overview of transitions, risk time etc.
levels.Lexis what are the states in the Lexis object
nid.Lexis number of persons in the Lexis object — how many unique values of
lex.id are present
entry entry time
exit exit time

Lexis functions 39

status status at entry or exit
timeBand factor of time bands
timeScales what time scales are in the Lexis object
timeSince what time scales are defined as time since a given state
breaks what breaks are currently defined
absorbing what are the absorbing states
transient what are the transient states
preceding, before which states precede this
succeeding, after which states can follow this
tmat.Lexis transition matrix for the Lexis object

#### Manipulate

subset.Lexis, [ subset of a Lexis object merge.Lexis merges a Lexis objects with a data.frame cbind.Lexis bind a data.frame to a Lexis object rbind.Lexis put two Lexis objects head-to-foot transform.Lexis transform and add variables tsNA20 turn NAs to 0s for time scales Relevel.Lexis, factorize.Lexis reorder and combine states bootLexis bootstrap sample of persons (lex.id) in the Lexis object

#### Simulate

simLexis simulate a Lexis object from specified transition rate models nState, pState count state occupancy from a simulated Lexis object plot.pState, lines.pState plot state occupancy from a pState object

#### Stack

stack.Lexis make a stacked object for simultaneous analysis of transitions—
returns a stacked.Lexis object
subset.stacked.Lexis subsets of a stacked.Lexis object
transform.stacked.Lexis transform a stacked.Lexis object

#### Interface to other packages

msdata.Lexis interface to mstate package etm.Lexis interface to etm package crr.Lexis interface to cmprsk package

#### Statistical models — these are not S3 methods