Follow-up data with R and Epi

SDCC

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Chapter 1

Follow-up data in the Epi package

In the Epi-package, follow-up data is represented by adding some extra variables to a data frame. Such a data frame is called a Lexis object. The tools for handling follow-up data then use the structure of this for special plots, tabulations etc.

Follow-up data basically consists of a time of entry, a time of exit and an indication of the status at exit (normally either "alive" or "dead"). Implicitly is also assumed a status during the follow-up (usually "alive").

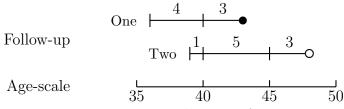


Figure 1.1: Follow-up of two persons

1.1 Timescales

A timescale is a variable that varies deterministicly within each person during follow-up, e.g.:

- Age
- Calendar time
- Time since treatment
- Time since relapse

All timescales advance at the same pace, so the time followed is the same on all timescales. Therefore, it suffices to use only the entry point on each of the time scale, for example:

- Age at entry.
- Date of entry.
- Time since treatment (at treatment this is 0).

2 1.1 Timescales Follow-up

• Time since relapse (at relapse this is 0)...

For illustration we need to load the Epi package:

```
> library(Epi)
> print( sessionInfo(), 1=F )
R version 3.4.3 (2017-11-30)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 14.04.5 LTS
Matrix products: default
BLAS: /usr/lib/openblas-base/libopenblas.so.0
LAPACK: /usr/lib/lapack/liblapack.so.3.0
attached base packages:
             datasets graphics grDevices stats
[1] utils
                                                    methods
                                                              base
other attached packages:
[1] Epi_2.25
loaded via a namespace (and not attached):
                                 MASS_7.3-49
 [1] cmprsk_2.2-7
                      zoo_1.8-0
                                                         compiler_3.4.3
 [5] Matrix_1.2-11
                      plyr_1.8.4
                                      parallel_3.4.3
                                                         survival_2.41-3
                      Rcpp_0.12.12 splines_3.4.3
 [9] etm_1.0.1
                                                         grid_3.4.3
[13] data.table_1.10.4 numDeriv_2016.8-1 lattice_0.20-35
```

In the Epi package, follow-up in a cohort is represented in a Lexis object. A Lexis object is a data frame with a bit of extra structure representing the follow-up. For the nickel data we would construct a Lexis object by:

The entry argument is a named list with the entry points on each of the timescales we want to use. It defines the names of the timescales and the entry points of the follow-up of each person. The exit argument gives the exit time on one of the timescales, so the name of the element in this list must match one of the names of the entry list. This is sufficient, because the follow-up time on all time scales is the same, in this case ageout - agein. Now take a look at the result:

```
Classes 'Lexis' and 'data.frame':
                                          679 obs. of 14 variables:
                  1934 1934 1934 1934 ...
           : num
 $ age
           : num
                  45.2 48.3 53 47.9 54.7 ...
 $ tfh
                  27.7 25.1 27.7 23.2 24.8 ...
           : num
 $ lex.dur : num
                  47.75 15 1.17 21.77 22.1 ...
 $ lex.Cst : num
                  0 0 0 0 0 0 0 0 0 0 ...
 $ lex.Xst : num
                  0 1 1 0 0 1 0 0 0 0 ...
 $ lex.id : int
                  1 2 3 4 5 6 7 8 9 10 ...
 $ id
           : num
                  3 4 6 8 9 10 15 16 17 18 ...
                  0 162 163 527 150 163 334 160 420 12 ...
 $ icd
           : num
 $ exposure: num
                  5 5 10 9 0 2 0 0.5 0 0 ...
                  1889 1886 1881 1886 1880 ...
 $ dob
           : num
           : num
                  17.5 23.2 25.2 24.7 30 ...
 $ age1st
           : num
 $ agein
                  45.2 48.3 53 47.9 54.7 ...
                  93 63.3 54.2 69.7 76.8 ...
 $ ageout
          : num
 - attr(*, "time.scales")= chr
                                "per" "age" "tfh"
 - attr(*, "time.since")= chr
                                ... ... ...
 - attr(*, "breaks")=List of 3
  ..$ per: NULL
  ..$ age: NULL
  ..$ tfh: NULL
> head( nicL )
                       tfh lex.dur lex.Cst lex.Xst lex.id id icd exposure
       per
                                                                                 dob
               age
1 1934.246 45.2273 27.7465 47.7535
                                          0
                                                  0
                                                         1
                                                            3
                                                                 0
                                                                          5 1889.019
2 1934.246 48.2684 25.0820 15.0028
                                          0
                                                         2
                                                            4 162
                                                  1
                                                                          5 1885.978
3 1934.246 52.9917 27.7465
                            1.1727
                                          0
                                                  1
                                                         3
                                                            6 163
                                                                         10 1881.255
4 1934.246 47.9067 23.1861 21.7727
                                          0
                                                  0
                                                         4
                                                            8 527
                                                                          9 1886.340
                                          0
                                                  0
                                                         5
                                                            9 150
5 1934.246 54.7465 24.7890 22.0977
                                                                          0 1879.500
6 1934.246 44.3314 23.0437 18.2099
                                                         6 10 163
                                                                          2 1889.915
   age1st
            agein ageout
1 17.4808 45.2273 92.9808
2 23.1864 48.2684 63.2712
3 25.2452 52.9917 54.1644
4 24.7206 47.9067 69.6794
5 29.9575 54.7465 76.8442
6 21.2877 44.3314 62.5413
```

The Lexis object nicL has a variable for each timescale which is the entry point on this timescale. The follow-up time is in the variable lex.dur (duration).

There is a summary function for Lexis objects that list the number of transitions and records as well as the total amount of follow-up time:

We defined the exit status to be death from lung cancer (ICD7 162,163), i.e. this variable is 1 if follow-up ended with a death from this cause. If follow-up ended alive or by death from another cause, the exit status is coded 0, i.e. as a censoring.

Note that the exit status is in the variable lex.Xst (eXit status. The variable lex.Cst is the state where the follow-up takes place (Current status), in this case 0 (alive).

It is possible to get a visualization of the follow-up along the timescales chosen by using the plot method for Lexis objects. nicL is an object of *class* Lexis, so using the function plot() on it means that R will look for the function plot.Lexis and use this function.

4 1.1 Timescales Follow-up

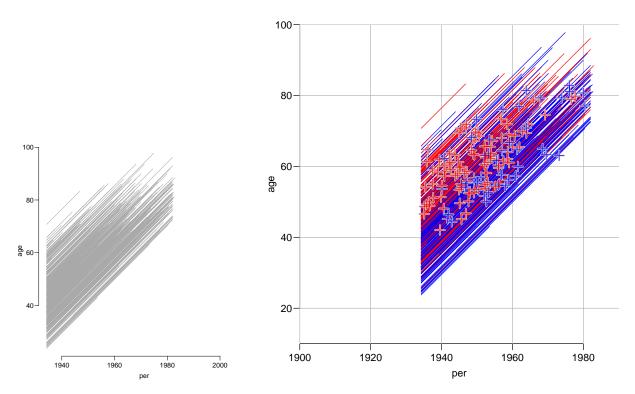


Figure 1.2: Lexis diagram of the nickel dataset; left panel the default version, right panel with bells and whistles. The red lines are for persons with exposure> 0, so it is pretty evident that the oldest ones are the exposed part of the cohort.

> plot(nicL)

The function allows quite a bit of control over the output, and a points.Lexis function allows plotting of the endpoints of follow-up:

The results of these two plotting commands are in figure ??.

Chapter 2

Subdividing follow-up for analysis

2.1 Splitting the follow-up time along a timescale

The follow-up time in a cohort can be subdivided by for example current age. This is achieved by the splitLexis (note that it is *not* called split.Lexis). This requires that the timescale and the breakpoints on this timescale are supplied. Try:

```
> nicS1 <- splitLexis( nicL, "age", breaks=seg(0,100,10) )</pre>
> summary( nicL )
Transitions:
     To
From
     0
                          Events: Risk time:
           1
              Records:
                                               Persons:
   0 542 137
                    679
                              137
                                     15348.06
                                                     679
> summary( nicS1 )
Transitions:
     Τо
                Records:
                           Events: Risk time:
                    2210
                               137
                                      15348.06
```

So we see that the number of events and the amount of follow-up is the same in the two data sets; only the number of records differ — the extra records all have lex.Cst=0 and lex.Xst=0.

To see how records are split for each individual, it is useful to list the results for a few individuals:

```
> round( subset( nicS1, id %in% 8:10 ), 2 )
                          tfh lex.dur lex.Cst lex.Xst id icd exposure
   lex.id
                    age
              per
                                                                             dob age1st
        4 1934.25 47.91 23.19
                                  2.09
                                             0
                                                      0 8 527
                                                                      9 1886.34
                                                                                  24.72
                                             0
12
        4 1936.34 50.00 25.28
                                 10.00
                                                      0
                                                        8 527
                                                                      9 1886.34
                                                                                  24.72
13
        4 1946.34 60.00 35.28
                                  9.68
                                             0
                                                        8 527
                                                                      9 1886.34
        5 1934.25 54.75 24.79
                                  5.25
                                             0
                                                      0
                                                         9 150
                                                                      0 1879.50
                                                                                  29.96
15
        5 1939.50 60.00 30.04
                                 10.00
                                             0
                                                      0
                                                        9 150
                                                                      0 1879.50
                                                                                  29.96
                                             0
                                                       9 150
16
        5 1949.50 70.00 40.04
                                  6.84
                                                                      0 1879.50
                                                                                  29.96
        6 1934.25 44.33 23.04
                                             0
                                                      0 10 163
                                                                      2 1889.91
17
                                  5.67
                                                                                  21.29
18
        6 1939.91 50.00 28.71
                                 10.00
                                             0
                                                      0 10 163
                                                                      2 1889.91
                                                                                  21.29
        6 1949.91 60.00 38.71
                                  2.54
                                                      1 10 163
                                                                      2 1889.91
                                                                                 21.29
   agein ageout
11 47.91
         69.68
12 47.91
          69.68
13 47.91
         69.68
```

```
14 54.75 76.84
15 54.75 76.84
16 54.75 76.84
17 44.33 62.54
18 44.33 62.54
19 44.33 62.54
```

The resulting object, nicS1, is again a Lexis object, and so follow-up may be split further along another timescale. Subsequently we list the results for individuals 8, 9 and 10 again:

```
> nicS2 <- splitLexis( nicS1, "tfh", breaks=c(0,1,5,10,20,30,100) )</pre>
> round( subset( nicS2, id %in% 8:10 ), 2 )
                            tfh lex.dur lex.Cst lex.Xst id icd exposure
   lex.id
               per
                     age
                                                                               dob age1st
13
        4 1934.25 47.91 23.19
                                   2.09
                                               0
                                                       0
                                                          8 527
                                                                         9 1886.34
                                                                                    24.72
        4 1936.34 50.00 25.28
                                   4.72
                                               0
                                                       0
                                                           8 527
                                                                         9 1886.34
                                                                                    24.72
14
15
        4 1941.06 54.72 30.00
                                   5.28
                                               0
                                                       0
                                                           8 527
                                                                         9 1886.34
                                                                                    24.72
                                   9.68
                                               0
                                                       0
                                                           8 527
                                                                        9 1886.34
16
        4 1946.34 60.00 35.28
                                                                                    24.72
17
        5 1934.25 54.75 24.79
                                   5.21
                                               0
                                                       0
                                                           9 150
                                                                         0 1879.50
                                                                                    29.96
18
        5 1939.46 59.96 30.00
                                   0.04
                                               0
                                                       0
                                                          9 150
                                                                         0 1879.50
                                                                                    29.96
19
        5 1939.50 60.00 30.04
                                  10.00
                                               0
                                                       0
                                                          9 150
                                                                         0 1879.50
                                                                                    29.96
20
        5 1949.50 70.00 40.04
                                   6.84
                                               0
                                                       0
                                                         9 150
                                                                         0 1879.50
                                                                                    29.96
        6 1934.25 44.33 23.04
                                               0
21
                                   5.67
                                                       0 10 163
                                                                         2 1889.91
                                                                                    21.29
22
        6 1939.91 50.00 28.71
                                   1.29
                                               0
                                                       0 10 163
                                                                         2 1889.91
                                                                                    21.29
23
        6 1941.20 51.29 30.00
                                   8.71
                                               0
                                                       0 10 163
                                                                         2 1889.91
                                                                                    21.29
        6 1949.91 60.00 38.71
                                   2.54
                                               0
                                                       1 10 163
                                                                         2 1889.91
                                                                                    21.29
   agein ageout
13 47.91
          69.68
          69.68
14 47.91
15 47.91
          69.68
16 47.91
          69.68
17 54.75
          76.84
18 54.75
          76.84
19 54.75
          76.84
20 54.75
          76.84
21 44.33
          62.54
22 44.33
          62.54
23 44.33
          62.54
24 44.33
          62.54
```

A more efficient (and more intuitive) way of making this double split is to use the splitMulti function from the popEpi package:

```
> library( popEpi )
> nicM <- splitMulti( nicL, age = seq(0,100,10),</pre>
                              tfh = c(0,1,5,10,20,30,100))
> summary( nicS2 )
Transitions:
     To
From
             1
                Records:
                           Events: Risk time:
                                                 Persons:
   0 2992 137
                    3129
                               137
                                      15348.06
                                                      679
> summary( nicM )
Transitions:
From
             1
                Records:
                           Events: Risk time:
                                                 Persons:
   0 2992 137
                    3129
                                      15348.06
                               137
                                                      679
```

So we see that the two ways of splitting data yields the same amount of follow-up, but the results are not identical:

As we see, this is because the nicM object also is a data.table object; the splitMulti uses the data.table machinery which makes the splitting substantially faster — this is of particular interest if you operate on large data sets (> 1,000,000 records).

Thus the recommended way of splitting follow-up time is by splitMulti. But you should be aware that the result is a data.table object, which in some circumstances behaves slightly different from data.frames. See the manual for data.table.

2.1.1 Time scales as covariates

If we want to model the effect of these timescale variables on occurrence rates, we will for each interval use either the value of the left endpoint in each interval or the middle. There is a function timeBand which returns either of these:

```
> timeBand( nicM, "age", "middle" )[1:20]
 [1] 45 45 55 65 75 85 95 45 55 55 65 55 45 55 65 55 65 75
 # For nice printing and column labelling use the data.frame() function:
  data.frame( nicS2[,c("lex.id","per","age","tfh","lex.dur")],
                                         "age", "middle"
               mid.age=timeBand( nicS2,
                 mid.t=timeBand( nicS2,
                                         "tfh",
                                                 "middle"
                                         "tfh",
                left.t=timeBand( nicS2,
                                         "tfh",
               right.t=timeBand( nicS2,
                                                "right"
                fact.t=timeBand( nicS2,
                                         "tfh", "factor" ) )[1:20,]
   lex.id
                                 tfh lex.dur mid.age mid.t left.t right.t
                                                                               fact.t
                per
                        age
        1 1934.246 45.2273 27.7465
                                                                              (20,30]
1
                                      2.2535
                                                   45
                                                         25
                                                                 20
                                                                         30
2
        1 1936.500 47.4808 30.0000
                                      2.5192
                                                   45
                                                         65
                                                                 30
                                                                         100 (30,100]
3
                                                   55
                                                         65
                                                                 30
                                                                         100 (30,100]
        1 1939.019 50.0000 32.5192 10.0000
4
        1 1949.019 60.0000 42.5192 10.0000
                                                   65
                                                         65
                                                                 30
                                                                         100 (30,100]
5
        1 1959.019 70.0000 52.5192 10.0000
                                                   75
                                                         65
                                                                 30
                                                                         100 (30,100]
6
        1 1969.019 80.0000 62.5192 10.0000
                                                                        100 (30,100]
                                                   85
                                                         65
                                                                 30
7
        1 1979.019 90.0000 72.5192
                                                   95
                                                                         100 (30,100]
                                      2.9808
                                                         65
                                                                 30
8
        2 1934.246 48.2684 25.0820
                                      1.7316
                                                   45
                                                         25
                                                                 20
                                                                         30
                                                                              (20,30]
9
        2 1935.978 50.0000 26.8136
                                      3.1864
                                                   55
                                                         25
                                                                 20
                                                                         30
                                                                              (20,30]
10
        2 1939.164 53.1864 30.0000
                                                   55
                                                         65
                                                                 30
                                                                         100 (30,100]
                                      6.8136
11
        2 1945.978 60.0000 36.8136
                                      3.2712
                                                   65
                                                         65
                                                                 30
                                                                         100 (30,100]
12
        3 1934.246 52.9917 27.7465
                                      1.1727
                                                   55
                                                         25
                                                                 20
                                                                         30
                                                                              (20,30]
                                                                              (20,30]
        4 1934.246 47.9067 23.1861
                                      2.0933
                                                   45
                                                         25
                                                                 20
                                                                         30
13
        4 1936.340 50.0000 25.2794
14
                                      4.7206
                                                   55
                                                         25
                                                                 20
                                                                         30
                                                                              (20,30]
15
        4 1941.060 54.7206 30.0000
                                      5.2794
                                                   55
                                                         65
                                                                 30
                                                                         100 (30,100]
                                                                         100 (30,100]
16
        4 1946.340 60.0000 35.2794
                                      9.6794
                                                   65
                                                         65
                                                                 30
17
        5 1934.246 54.7465 24.7890
                                                   55
                                                         25
                                                                 20
                                                                              (20,30]
                                      5.2110
                                                                         30
        5 1939.457 59.9575 30.0000
                                                   55
                                                         65
                                                                 30
                                                                         100 (30,100]
18
                                      0.0425
                                                                         100 (30,100]
19
        5 1939.500 60.0000 30.0425 10.0000
                                                   65
                                                         65
                                                                 30
20
        5 1949.500 70.0000 40.0425
                                                   75
                                                                 30
                                                                         100 (30,100]
```

Note that these are characteristics of the intervals defined by breaks=, not the midpoints nor left or right endpoints of the actual follow-up intervals (which would be tfh and tfh+lex.dur, respectively).

These functions are intended for modeling timescale variables as factors (categorical variables) in which case the coding must be independent of the censoring and mortality pattern — it should only depend on the chosen grouping of the timescale. Modeling timescales as *quantitative* should not be based on these codings but directly on the values of the time-scale variables.

2.1.2 Differences between time scales

The midpoint (as well as the left and right interval endpoint) should be used with caution if the variable age1st is modeled too; the age at hire is logically equal to the difference between current age (age) and time since hire (thf):

```
> summary( (nicS2$age-nicS2$tfh) - nicS2$age1st )
        Min. 1st Qu. Median Mean 3rd Qu. Max.
-7.105e-15 0.000e+00 0.000e+00 2.214e-17 0.000e+00 7.105e-15
```

This calculation refer to the *start* of each interval — the time scale variables in the Lexis object. But when using the middle of the intervals, this relationship is not preserved:

If all three variable are to be included in a model, you must make sure that the *substantial* relationship between the variables be maintained. One way is to recompute age at first hire from the two midpoint variables, but more straightforward would be to use the left endpoint of the intervals, that is the time scale variables in the Lexis object. The latter approach however requires that the follow-up is split in fairly small chunks.

2.2 Cutting follow up time at a specific date

If we have a recording of the date of a specific event as for example recovery or relapse, we may classify follow-up time as being before or after this intermediate event, but it requires that follow-up records that straddle the event be cut into two record. This is achieved with the function cutlexis, which takes three arguments: the time point, the timescale, and the value of the (new) state following the date.

Now we define the age for the nickel workers where the cumulative exposure exceeds 50 exposure years:

```
> subset( nicL, id %in% 8:10 )
                       tfh lex.dur lex.Cst lex.Xst lex.id id icd exposure
                                                                                 dob
4 1934.246 47.9067 23.1861 21.7727
                                                 0
                                                         4 8 527
                                                                         9 1886.340
                                         0
                                                         5 9 150
5 1934.246 54.7465 24.7890 22.0977
                                         0
                                                  0
                                                                         0 1879.500
                                                         6 10 163
6 1934.246 44.3314 23.0437 18.2099
                                                  1
                                                                         2 1889.915
   age1st
            agein ageout
4 24.7206 47.9067 69.6794
5 29.9575 54.7465 76.8442
6 21.2877 44.3314 62.5413
```

```
> agehi <- nicL$age1st + 50 / nicL$exposure</pre>
> nicC <- cutLexis( data = nicL,
                   cut = agehi,
              timescale = "age",
+
+
              new.state = 2,
       precursor.states = 0 )
> subset( nicC, id %in% 8:10 )
                       tfh lex.dur lex.Cst lex.Xst lex.id id icd exposure
                age
683 1934.246 47.9067 23.1861 21.7727
                                                               9 1886.340
                                   2 2 4 8 527
   1934.246 54.7465 24.7890 22.0977
                                       0
                                              0
                                                      5 9 150
                                                                    0 1879.500
                                      0 2
                                                    6 10 163
   1934.246 44.3314 23.0437 1.9563
                                                                    2 1889.915
685 1936.203 46.2877 25.0000 16.2536
                                       2
                                              1
                                                      6 10 163
                                                                     2 1889.915
    age1st
            agein ageout
683 24.7206 47.9067 69.6794
   29.9575 54.7465 76.8442
   21.2877 44.3314 62.5413
685 21.2877 44.3314 62.5413
```

(The precursor.states= argument is explained below). Note that individual 6 has had his follow-up split at 25 years since hire where 50 exposure-years were attained. This could also have been achieved in the split dataset nicS2 instead of nicL, try:

```
> subset( nicS2, id %in% 8:10 )
   lex.id
                                      tfh lex.dur lex.Cst lex.Xst id icd exposure
                                                                                                    dob
                  per
                            age
         4 1934.246 47.9067 23.1861 2.0933 0
                                                                                           9 1886.340
13
                                                                       0 8 527
          4 1936.340 50.0000 25.2794
                                            4.7206
                                                                       0 8 527
                                                                                           9 1886.340
                                                                      0 8 527
          4 1941.060 54.7206 30.0000
                                            5.2794
                                                                                          9 1886.340

      0
      0
      8
      527

      0
      0
      9
      150

      0
      0
      9
      150

      0
      0
      9
      150

      0
      0
      10
      163

      0
      0
      10
      163

      0
      0
      10
      163

      0
      1
      10
      163

16
          4 1946.340 60.0000 35.2794
                                            9.6794
                                                            0
                                                                      0 8 527
                                                                                          9 1886.340
17
          5 1934.246 54.7465 24.7890 5.2110
                                                                                          0 1879.500
         5 1939.457 59.9575 30.0000 0.0425
18
                                                                                          0 1879.500
19
         5 1939.500 60.0000 30.0425 10.0000
                                                                                          0 1879.500
20
         5 1949.500 70.0000 40.0425 6.8442
                                                                                          0 1879.500
21
         6 1934.246 44.3314 23.0437 5.6686
                                                                                          2 1889.915
22
         6 1939.915 50.0000 28.7123 1.2877
                                                                                          2 1889.915
23
          6 1941.203 51.2877 30.0000 8.7123
                                                                                         2 1889.915
          6 1949.915 60.0000 38.7123 2.5413
                                                                                          2 1889.915
     age1st
                agein ageout
13 24.7206 47.9067 69.6794
14 24.7206 47.9067 69.6794
15 24.7206 47.9067 69.6794
16 24.7206 47.9067 69.6794
17 29.9575 54.7465 76.8442
18 29.9575 54.7465 76.8442
19 29.9575 54.7465 76.8442
20 29.9575 54.7465 76.8442
21 21.2877 44.3314 62.5413
22 21.2877 44.3314 62.5413
23 21.2877 44.3314 62.5413
24 21.2877 44.3314 62.5413
> agehi <- nicS2$age1st + 50 / nicS2$exposure
> nicS2C <- cutLexis( data = nicS2,
                            cut = agehi,
                     timescale = "age",
+
                     new.state = 2,
            precursor.states = 0 )
> subset( nicS2C, id %in% 8:10 )
```

The same results would have emerged if we had used the nicM dataset (the data.table object). Mathematicians would say that splitLexis and cutLexis are commutative.

Note that follow-up subsequent to the event is classified as being in state 2, but that the final transition to state 1 (death from lung cancer) is preserved. This is the point of the precursor.states= argument. It names the states (in this case 0, "Alive") that will be over-written by new.state (in this case state 2, "High exposure"), while state 1 ("Dead") should not be updated even if it is after the time where the persons moves to state 2. In other words, only state 0 is a precursor to state 2, state 1 is always subsequent to state 2. Even if you at a high exposure level, death is pretty final.

If the intermediate event is to be used as a time-dependent variable in a Cox-model, then lex.Cst should be used as the time-dependent variable, and lex.Xst==1 as the event.

Chapter 3

Modeling rates

3.1 Background

The purpose of subdividing follow-up data is to be able to model the effects of the time scale variables as parametric functions.

In a model that assumes a constant occurrence rate in each of the intervals the likelihood contribution from each interval is the same as the likelihood contribution from a Poisson variate D, say, with mean $\lambda\ell$ where λ is the rate and ℓ is the interval length, and where the value of the variate D is 1 or 0 according to whether an event has occurred or not. Moreover, the likelihood contributions from all follow-up intervals from a single person are conditionally independent (conditional on having survived till the start of the interval in question). This implies that the total contribution to the likelihood from a single person is a product of terms, and hence the same as the likelihood of a number of independent Poisson terms, one from each interval.

Parametric modeling of the rates is obtained by using the *value* of the timescale for each interval as quantitative explanatory variables, using for example splines. Thus the model will be one where the rate is assumed constant in each interval, but where a parametric form of the *size* of the rate in each interval is imposed by the model, using the timescale as a covariate.

3.2 Practicalities

In the nickel worker study we might want to look at the effects of age and time since hire. If we want to use splines we must allocate knots for anchoring the splines at each of the time scales, either by some *ad hoc* method or by using some sort of penalized splines. The letter will not be treated here.

Here we shall use the former approach and allocate 5 knots on each of the two time-scales. We allocate knots so that we have the event evenly distributed between the knots:

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In the Epi package there is a convenience wrapper for the natural spline generator ns, Ns, that takes the smallest and the largest of a set of supplied knots to be the boundary knots.

3.3 Models for rates

3.3.1 One time scale

A model that only models lung cancer mortality rates as a function of age would then be:

```
> ma <- glm( (lex.Xst==1) ~ Ns(age,knots=a.kn),
             family = poisson,
             offset = log(lex.dur),
              data = nicM )
> summary( ma )
Call:
glm(formula = (lex.Xst == 1) ~ Ns(age, knots = a.kn), family = poisson,
    data = nicM, offset = log(lex.dur))
Deviance Residuals:
         1Q Median
   Min
                               3Q
                                       Max
-0.5074 -0.3896 -0.2143 -0.1203
                                     3.7904
Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)
                        -4.6591
                                    0.1324 -35.199
                                                  < 2e-16
Ns(age, knots = a.kn)1
                        0.1671
                                    0.2970
                                            0.563
                                                   0.57371
Ns(age, knots = a.kn)2
                       -0.1315
                                    0.3727
                                            -0.353
                                                   0.72411
Ns(age, knots = a.kn)3
                        0.7827
                                    0.2885
                                            2.713
                                                   0.00667
                                    0.2780
Ns(age, knots = a.kn)4 -0.3717
                                           -1.337
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 1024.38
                           on 3128
                                    degrees of freedom
Residual deviance: 979.16
                           on 3124
                                    degrees of freedom
AIC: 1263.2
```

The offset, $\log(\text{lex.dur})$ comes from the fact that the likelihood for the follow-up data is the same as that for independent Poisson variates with mean $\lambda \ell$, and that the default link function for the Poisson family is the log, so that we are using a linear model for the log-mean, that is $\log(\lambda) + \log(\ell)$. But when we want a model for the log-rate $(\log(\lambda))$, the term $\log(\ell)$ must be included as a covariate with regression coefficient fixed to 1; a so-called offset.

The parameters from the model do not have any direct interpretation *per se*, but we can compute the estimated lung cancer incidence rates for a range of ages using ci.pred with a suitably defined prediction data frame. Note that we must specify all covariates in the model, also the variable in the offset, lex.dur. We set the latter to 1000, because we want the lung cancer mortality rates per 1000 PY. By default ci.pred yields a prediction on the response-scale, that is the rate-scale:

```
> nd <- data.frame( age=40:85, lex.dur=1000 )
> pr.a <- ci.pred( ma, newdata = nd )</pre>
```

Number of Fisher Scoring iterations: 7

Modeling rates 3.3 Models for rates 13

```
> matplot( nd$age, pr.a,
+ type="l", lty=1, col=1, lwd=c(3,1,1),
+ log="y", xlab="Age (years)",
+ ylab="Lunng cancer mortality per 1000 PY")
```

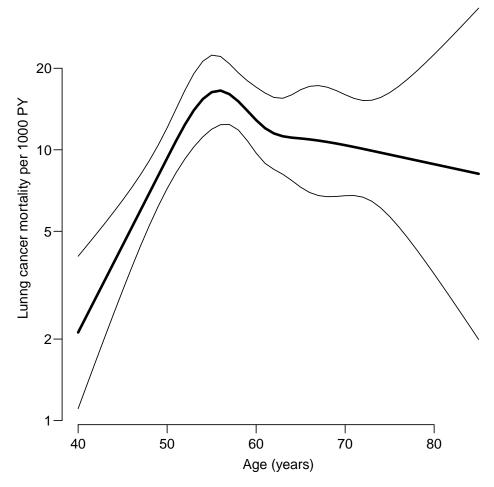


Figure 3.1: Lung cancer mortality among Nickel smelter workers by age. We see that the rates increase till about 55 years, and from then on is approximately flat. .../flup-pr-a

3.3.2 More time scales

There may however also be an effect of time since hire too, so we can add this term to the model:

```
-0.6308 -0.3730 -0.2170 -0.1180
                                      3.8903
Coefficients:
                       Estimate Std. Error z value Pr(>|z|)
(Intercept)
                       -4.65125
                                   0.14844 -31.335
                                                      <2e-16
Ns(age, knots = a.kn)1 0.19029
                                    0.32601
                                              0.584
                                                      0.5594
Ns(age, knots = a.kn)2
                        0.04239
                                    0.40857
                                              0.104
                                                      0.9174
Ns(age, knots = a.kn)3
                        0.87848
                                    0.37395
                                              2.349
                                                      0.0188
Ns(age, knots = a.kn)4
                        0.08124
                                    0.37567
                                              0.216
                                                      0.8288
Ns(tfh, knots = t.kn)1
                        0.05961
                                    0.45702
                                              0.130
                                                      0.8962
Ns(tfh, knots = t.kn)2 - 0.30254
                                    0.39214
                                             -0.771
                                                      0.4404
                                    0.37493
Ns(tfh, knots = t.kn)3 - 0.08144
                                             -0.217
                                                      0.8281
Ns(tfh, knots = t.kn)4 - 0.63400
                                    0.34055
                                             -1.862
                                                      0.0626
(Dispersion parameter for poisson family taken to be 1)
                                     degrees of freedom
    Null deviance: 1024.4
                           on 3128
Residual deviance:
                    970.7
                           on 3120
                                     degrees of freedom
AIC: 1262.7
Number of Fisher Scoring iterations: 7
```

This model has two time scales, age and time since hire, so it makes little sense to report the effect of age for a *fixed* value of time since hire — the time since hire increases by age. Instead we can show the mortality rates for persons hired at different ages, and report the

In order to get a feeling for the values that can be use we look at age1st

```
> summary( nickel$age1st )
  Min. 1st Qu. Median Mean 3rd Qu. Max.
  10.78  21.80  26.16  26.74  30.63  52.19
```

joint effect of increasing age and time since hire.

Thus we shall show mortality rates in ages 20–90 for persons hired in ages 15, 25, 35 and 45:

```
> nd <- data.frame( expand.grid( age=c(20:90,NA), age1st=seq(15,45,10) ) )</pre>
> nd <- transform( nd, tfh = ifelse( age > age1st, age-age1st, NA ),
                    lex.dur = 1000)
> # makes no sense to have age < age1st
> nd <- transform( nd, age = ifelse( age > age1st, age, NA ) )
> head( nd )
  age age1st tfh lex.dur
1
  20
          15
               5
                     1000
2
  21
          15
               6
                     1000
3
  22
          15
               7
                     1000
4
  23
          15
               8
                     1000
5
  24
          15
               9
                     1000
6
   25
          15
                     1000
              10
```

With this in place we can plot the estimated rates as before, only now we will get 4 separate lines. The purpose of inserting an NA on the age-scale in the expand.grid is that the different instances of age1st be separated by NAs, and hence will not be connected when we plot the curves. The downside of this trick is that lines cannot be plotted with different colors or symbols.

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```
> pr.at <- ci.pred( mat, newdata = nd )
> matplot( nd$age, pr.at,
+ type="l", lty=1, col=1, lwd=c(3,1,1),
+ log="y", xlab="Age (years)",
+ ylab="Lunng cancer mortality per 1000 PY")
```

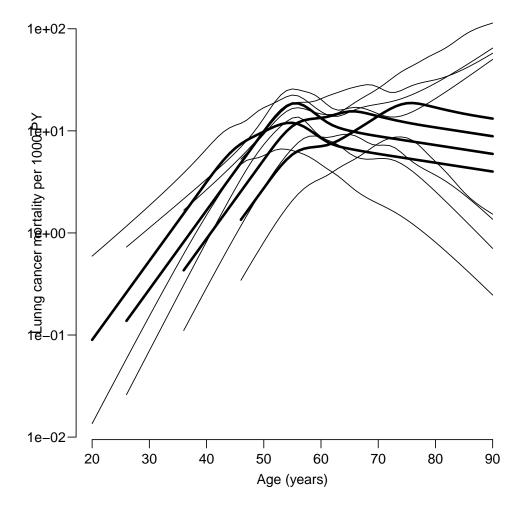


Figure 3.2: Lung cancer mortality among Nickel smelter workers by age and age at hire 15, 25,35 and 45. Each line (except the first) starts at the age of hire; we see that the later in life you are hired, the smaller the initial risk, but the higher the eventual risk of lung cancer death.

./flup-pr-at

We can check whether the effect of time since hire is actually improving the model:

We see a pretty strong indication that this is the case.

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3.3.3 Difference between time scales

However it might be the case that it really is the age at first hire that is the main determinant (recall that age - thf = age1st), so we could fit a model with this variable instead — a model with only 1 timescale, namely age.

```
> ( f.kn <- with( subset( nicM, lex.Xst==1 ), quantile( age1st, (1:5-0.5)/5 ) ) )</pre>
              30%
                       50%
                                70%
                                          90%
20.25860 22.55422 26.00000 28.36578 33.96910
> maf <- update( ma, . ~ . + Ns(age1st,knots=f.kn) )</pre>
> summary( maf )
Call:
glm(formula = (lex.Xst == 1) ~ Ns(age, knots = a.kn) + Ns(age1st,
    knots = f.kn), family = poisson, data = nicM, offset = log(lex.dur))
Deviance Residuals:
    Min
              1Q
                   Median
                                3Q
                                        Max
-0.5696 -0.3671 -0.2257
                          -0.1197
                                     3.7777
Coefficients:
                          Estimate Std. Error z value Pr(>|z|)
                                      0.17564 -26.340 < 2e-16
(Intercept)
                          -4.62646
Ns(age, knots = a.kn)1
                           0.21589
                                      0.29742
                                                0.726 0.46792
Ns(age, knots = a.kn)2
                          -0.06427
                                      0.37653
                                               -0.171 0.86446
Ns(age, knots = a.kn)3
                           0.79456
                                      0.29345
                                                2.708
                                                       0.00678
Ns(age, knots = a.kn)4
                          -0.31305
                                      0.27976
                                               -1.119
                                                        0.26314
Ns(age1st, knots = f.kn)1 -0.15145
                                      0.38279
                                                -0.396
                                                        0.69237
Ns(age1st, knots = f.kn)2
                          0.04607
                                      0.27980
                                                0.165
                                                        0.86923
Ns(age1st, knots = f.kn)3 0.26374
                                      0.26156
                                                1.008
                                                       0.31331
Ns(age1st, knots = f.kn)4 -0.22878
                                      0.23117
                                               -0.990 0.32234
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 1024.4
                           on 3128
                                    degrees of freedom
Residual deviance: 973.2
                          on 3120
                                    degrees of freedom
AIC: 1265.2
Number of Fisher Scoring iterations: 7
> anova( maf, ma, mat, test="Chisq" )
Analysis of Deviance Table
Model 1: (lex.Xst == 1) ~ Ns(age, knots = a.kn) + Ns(age1st, knots = f.kn)
Model 2: (lex.Xst == 1) ~ Ns(age, knots = a.kn)
Model 3: (lex.Xst == 1) ~ Ns(age, knots = a.kn) + Ns(tfh, knots = t.kn)
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1
       3120
                973.20
2
       3124
                979.16 - 4
                           -5.9624
                                    0.20198
       3120
                970.70 4
                            8.4626
                                    0.07603
```

We see that there is much less indication that the age at first hire has an effect.

For the sake of completeness we can draw the predicted values from the maf model on top of the ones from the mat model:

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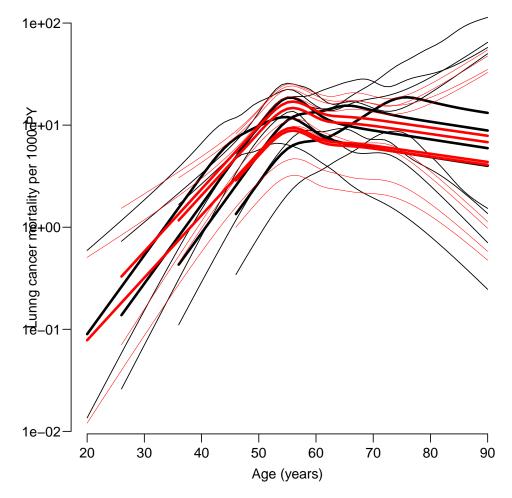


Figure 3.3: Lung cancer mortality among Nickel smelter workers by age and age at hire 15, 25,35 and 45. Each line (except the first) starts at the age of hire; we see that the later in life you are hired, the smaller the initial risk, but the higher the eventual risk of lung cancer death. The red lines are from the model maf where the lines are constrained to be parallel, and which gives a worse fit to data.

./flup-pr-at-af

$3.3.4 \quad \text{The complete picture} --\text{exercise}$

We could fit the remaining models where one or more of the three variables are included, and compare all of them:

```
> maft <- update( mat, . ~ . + Ns(age1st,knots=f.kn) )
> summary( maft )
```

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```
Call:
glm(formula = (lex.Xst == 1) ~ Ns(age, knots = a.kn) + Ns(tfh,
    knots = t.kn) + Ns(age1st, knots = f.kn), family = poisson,
    data = nicM, offset = log(lex.dur))
Deviance Residuals:
                   Median
    Min
              1Q
                                 3Q
                                         Max
-0.5899 -0.3579 -0.2224 -0.1185
                                      3.8687
Coefficients: (1 not defined because of singularities)
                           Estimate Std. Error z value Pr(>|z|)
(Intercept)
                                       0.16481 -28.612
                           -4.71537
                                                          <2e-16
Ns(age, knots = a.kn)1
                           0.01671
                                       0.35152
                                                0.048
                                                          0.9621
Ns(age, knots = a.kn)2
                           -0.11682
                                       0.44638
                                                -0.262
                                                          0.7935
Ns(age, knots = a.kn)3
                           0.47689
                                       0.50638
                                                0.942
                                                          0.3463
Ns(age, knots = a.kn)4
                          -0.18241
                                       0.47318 - 0.385
                                                          0.6999
Ns(tfh, knots = t.kn)1
                           0.35272
                                       0.51329
                                                0.687
                                                          0.4920
Ns(tfh, knots = t.kn)2
                          -0.11034
                                       0.43043
                                                -0.256
                                                          0.7977
Ns(tfh, knots = t.kn)3
                           0.26874
                                                 0.547
                                                          0.5844
                                       0.49133
Ns(tfh, knots = t.kn)4
                           -0.30302
                                       0.43585
                                                -0.695
                                                          0.4869
Ns(age1st, knots = f.kn)1 - 0.10650
                                       0.37476
                                                -0.284
                                                          0.7763
                                                0.860
Ns(age1st, knots = f.kn)2 0.17245
                                       0.20063
                                                          0.3900
                                       0.24239
                                                1.954
                                                          0.0507
Ns(age1st, knots = f.kn)3 0.47357
Ns(age1st, knots = f.kn)4
                                            NA
                                                    NA
                                                              NA
                                 NΑ
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 1024.38 on 3128 degrees of freedom
Residual deviance: 966.31 on 3117 degrees of freedom
AIC: 1264.3
Number of Fisher Scoring iterations: 7
> mft <- update( maft, . ~ . - Ns(age,knots=a.kn) )</pre>
> mf <- update( maf , . ~ . - Ns(age,knots=a.kn) )
> mt <- update( mat , . ~ . - Ns(age,knots=a.kn) )</pre>
> allp <- anova( maft, mat, ma, maf, mf, mft, mt, mat,
                 maf, maft, mft,
                 test="Chisq" )
> mall <- as.matrix( allp )</pre>
> cbind( mod = c("maft", "mat", "ma", "maf", "mf", "mft", "mt", "mat", "maf", "maft", "mft"),
         round( allp[,1:5], 3 ) )
    mod Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1 maft
            3117
                   966.306 NA NA
2
             3120
   \mathtt{mat}
                     970.697 -3
                                  -4.391
                                              0.222
3
             3124
                     979.160 -4
                                  -8.463
                                             0.076
    ma
4
   maf
            3120
                     973.197 4
                                   5.962
                                             0.202
5
                    1011.593 -4
    {\tt mf}
             3124
                                  -38.396
                                             0.000
                     971.120 4
6
   mft
             3120
                                   40.473
                                             0.000
7
             3124
                      985.734 -4
                                  -14.614
                                             0.006
    mt
8
             3120
                      970.697 4
    mat
                                   15.037
                                              0.005
9
             3120
                      973.197 0
                                   -2.500
                                                 NA
    \mathtt{maf}
10 maft
             3117
                     966.306 3
                                   6.892
                                             0.075
                     971.120 -3
11 mft
             3120
                                   -4.814
                                             0.186
```

1. Explain why there are NAs among the parameters in the model maf.

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2. Draw a graph (a "DAG") with the models as nodes and the tests as vertices, put the p-values on the vertices and use the result to argue that the model with age an time since hire is actually the most sensible description in this case.

Chapter 4

Competing risks — multiple types of events

If we want to consider death from lung cancer and death from other causes as separate events we can code these as for example 1 and 2.

```
> data( nickel )
> nicL <- Lexis( entry = list( per = agein+dob,
                             age = agein,
                             tfh = agein-age1st ),
                exit = list( age = ageout ),
          exit.status = (icd > 0) + (icd %in% c(162,163)),
                 data = nickel )
> summary( nicL )
Transitions:
    To
From 0 1 2 Records: Events: Risk time: Persons:
  0 47 495 137
                    679
                             632
                                 15348.06
> subset( nicL, id %in% 8:10 )
                     tfh lex.dur lex.Cst lex.Xst lex.id id icd exposure
             age
4 1934.246 47.9067 23.1861 21.7727
                                 0 1 4 8 527 9 1886.340
                                                 5 9 150
6 10 163
                                            1
2
                                  0
0
                                                                  0 1879.500
5 1934.246 54.7465 24.7890 22.0977
6 1934.246 44.3314 23.0437 18.2099
  age1st
          agein ageout
4 24.7206 47.9067 69.6794
5 29.9575 54.7465 76.8442
6 21.2877 44.3314 62.5413
```

In order to have a more readable output we can label the states, we can enter the names of these in the states parameter, try for example:

```
> str( nicL )
```

```
Classes 'Lexis' and 'data.frame':
                                         679 obs. of 14 variables:
          : num 1934 1934 1934 1934 ...
                  45.2 48.3 53 47.9 54.7 ...
 $ age
           : num
                  27.7 25.1 27.7 23.2 24.8 ...
 $ tfh
           : num
 $ lex.dur : num 47.75 15 1.17 21.77 22.1 ...
 $ lex.Cst : Factor w/ 3 levels "Alive", "D.oth", ...: 1 1 1 1 1 1 1 1 1 1 ...
 $ lex.Xst : Factor w/ 3 levels "Alive", "D.oth",..: 1 3 3 2 2 3 2 2 2 2 ...
          : int
                  1 2 3 4 5 6 7 8 9 10 ...
 $ id
           : num
                  3 4 6 8 9 10 15 16 17 18 ...
 $ icd
                  0 162 163 527 150 163 334 160 420 12 ...
           : num
                  5 5 10 9 0 2 0 0.5 0 0 ...
 $ exposure: num
          : num
                  1889 1886 1881 1886 1880 ...
                  17.5 23.2 25.2 24.7 30 ...
 $ age1st : num
 $ agein
           : num
                  45.2 48.3 53 47.9 54.7 ...
 $ ageout : num
                  93 63.3 54.2 69.7 76.8 ...
 - attr(*, "time.scales")= chr
                               "per" "age" "tfh"
 - attr(*, "time.since")= chr
 - attr(*, "breaks")=List of 3
  ..$ per: NULL
  ..$ age: NULL
  ..$ tfh: NULL
```

Note that the Lexis function automatically assumes that all persons enter in the first level (given in the states= argument), corresponding to the numerical values given in exit.status.

When we cut at a date as in this case, the date where cumulative exposure exceeds 50 exposure-years, we get the follow-up *after* the date classified as being in the new state if the exit (lex.Xst) was to a state we defined as one of the precursor.states:

```
> nicL$agehi <- nicL$age1st + 50 / nicL$exposure
> nicC <- cutLexis( data = nicL,
                     cut = nicL$agehi,
               timescale = "age",
               new.state = "HiExp"
        precursor.states = "Alive" )
> subset( nicC, id %in% 8:10 )
                          tfh lex.dur lex.Cst lex.Xst lex.id id icd exposure
         per
                 age
683 1934.246 47.9067 23.1861 21.7727
                                        HiExp
                                                 D.oth
                                                            4 8 527
                                                                             9 1886.340
    1934.246 54.7465 24.7890 22.0977
                                        Alive
                                                 D.oth
                                                            5 9 150
                                                                             0 1879.500
    1934.246 44.3314 23.0437
                                                            6 10 163
                                                                            2 1889.915
                              1.9563
                                        Alive
                                                 HiExp
685 1936.203 46.2877 25.0000 16.2536
                                        HiExp D.lung
                                                            6 10 163
                                                                             2 1889.915
     age1st
              agein
                     ageout
683 24.7206 47.9067 69.6794 30.27616
    29.9575 54.7465 76.8442
    21.2877 44.3314 62.5413 46.28770
685 21.2877 44.3314 62.5413 46.28770
> summary( nicC, scale=1000 )
Transitions:
     To
        Alive HiExp D.oth D.lung
From
                                   Records:
                                             Events: Risk time:
                 83
                      279
                                        466
                                                  427
                                                           10.77
                                                                       466
  Alive
           39
                               65
                                                                       296
  HiExp
            0
                  8
                       216
                               72
                                        296
                                                  288
                                                            4.58
  Sum
           39
                 91
                      495
                              137
                                        762
                                                 715
                                                           15.35
                                                                       679
```

Note that the persons-years is the same, but that the number of events has changed. This is because events are now defined as any transition, including the transitions to HiExp.

Also note that (so far) it is necessary to specify the variable with the cut points in full, using only cut=agehi would give an error.

4.1 Subdividing states

679

HiExp

time.scale time.since

per

age

tfh

tfe

Sum

1

2

3

4

Timescales:

It may be of interest to subdivide the states following the intermediate event according to whether the event has occurred or not. That is done by the argument split.states=TRUE.

Moreover, it will also often be of interest to introduce a new timescale indicating the time since intermediate event. This can be done by the argument new.scale=TRUE, alternatively new.scale="tfe", as illustrated here:

```
> nicC <- cutLexis( data = nicL,
+
                      cut = nicL$agehi,
+
               timescale = "age",
               new.state = "HiExp"
               new.scale = "tfe",
            split.states = TRUE,
        precursor.states = "Alive" )
 subset( nicC, id %in% 8:10 )
                          tfh
                                   tfe lex.dur lex.Cst
                                                               lex.Xst lex.id id icd
                  age
         per
683 1934.246 47.9067 23.1861 17.63054 21.7727
                                                         D.oth(HiExp)
                                                                            4 8 527
                                                  HiExp
    1934.246 54.7465 24.7890
                                    NA 22.0977
                                                                            5 9 150
                                                  Alive
                                                                 D.oth
    1934.246 44.3314 23.0437
                                        1.9563
                                                  Alive
                                                                 HiExp
                                                                            6 10 163
685 1936.203 46.2877 25.0000
                               0.00000 16.2536
                                                  HiExp D.lung(HiExp)
                                                                            6 10 163
    exposure
                  dob age1st
                                 agein
                                         ageout
                                                   agehi
683
           9 1886.340 24.7206 47.9067 69.6794 30.27616
5
           0 1879.500 29.9575 54.7465 76.8442
6
           2 1889.915 21.2877 44.3314 62.5413 46.28770
           2 1889.915 21.2877 44.3314 62.5413 46.28770
> summary( nicC, scale=1000, timeScales=TRUE )
Transitions:
     To
From
        Alive HiExp D.oth D.lung D.lung(HiExp) D.oth(HiExp)
                                                               Records:
                                                                          Events: Risk time:
  Alive
           39
                 83
                       279
                               65
                                                                                        10.77
                                               0
                                                            0
                                                                     466
                                                                              427
                                              72
            0
                  8
                                0
                                                           216
                                                                     296
                                                                              288
  HiExp
                         0
                                                                                         4.58
           39
                 91
                       279
                               65
                                              72
                                                           216
                                                                     762
                                                                              715
                                                                                        15.35
Transitions:
     То
From
         Persons:
              466
  Alive
              296
  HiExp
```

Note that the timeScales=TRUE to summary lists the timescales available in the object, and also indicates which of them that are defined as time since entry to a particular state. This facility is not used here, but it is needed when simulating follow-up data — see the vignette on simLexis.

With 6 different states it is quite difficult to get an overview of the transitions between states from the summary(). Therefore there is function that gives a graphical display of the states showing the transitions between the states:

```
> boxes( nicC, boxpos = list(x=c(10,10,80,80,80,80),
+ y=c(75,25,87,63,13,37)),
+ scale.Y = 1000,
+ show.BE = TRUE )
```

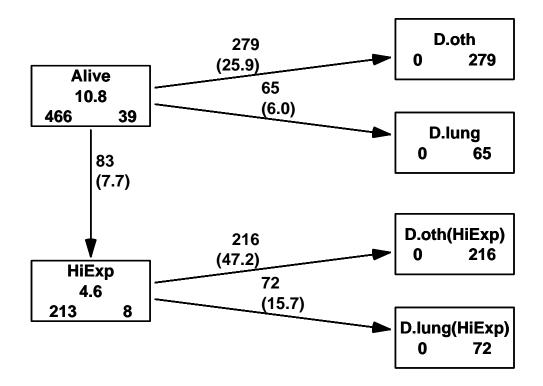


Figure 4.1: Transitions between states; the number in the middle of each box is the personyears (in 1000s — since scale.Y=1000), the numbers at the bottom of the boxes are the number that start, respectively end their follow-up in each state. The numbers on the arrows are the number of transitions and crude transition rates (the latter in events per 1000 PY). The function boxes.Lexis has a zillion arguments to fine-tune the appearance of the display in terms of colors etc. ./flup-nic-box