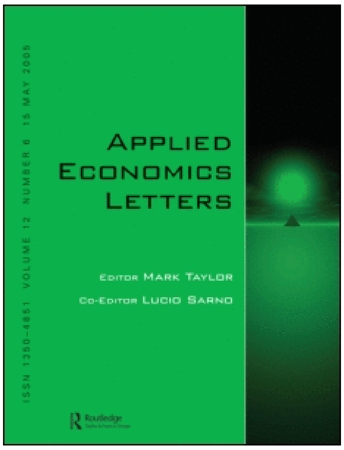

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The empirical distribution of stock returns: evidence from an emerging European market

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There is now substantial evidence that daily equity returns are not normally distributed, but instead display significant leptokurtosis and, in many cases, skewness. Considerable effort has been made in order to capture these empirical characteristics using a range of statistical distributions. However, the evidence to date is confined entirely to the returns of developed stock markets, and in particular to the USA. In this paper, the daily returns of a large emerging European stock market, Turkey, are modelled. Two very flexible families of distributions that have recently been introduced are employed: the exponential generalized beta (EGB) and the skewed generalized *t* (SGT). These distributions permit very diverse levels of skewness and kurtosis and, between them, nest most of the distribution previously considered in the literature.

I. INTRODUCTION

The distribution of equity returns plays an important role in financial economics. A particularly convenient assumption, and one that is central to much finance theory, is that equity returns are normally distributed. However, despite its prevalance in financial economics, the assumption of normality is inconsistent with the empirical evidence on the distribution of equity returns. While long-horizon returns are often found to be approximately normally distributed, there is now a large body of evidence that finds that daily equity returns display significant leptokurotisis, and in many cases, skewness. Many alternative distributions for equity returns have been proposed, some of which are based on theoretical models of investor behaviour, while others have been chosen *ad hoc*.[1] However, despite the large body of research into the distribution of equity returns, empirical evidence is confined entirely to the developed stock markets, and within these, almost exclusively to the USA.

In this paper, the distribution of daily equity returns is investigated in the largest emerging European stock market, the Istanbul Stock Exchange (ISE). At the time of writing, the ISE is the second largest emerging equity market in Europe after Greece in terms of both market capitalization and average value traded, and the twelfth largest emerging market in the world. It has also represented one of the most profitable investment opportunities to both domestic and foreign investors, consistently outperforming both emerging and developed equity markets over the last

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[1] Distributions that have been suggested include the stable Paretian (Mandelbrot, 1963; Fama, 1965; Officer, 1972; Clark, 1973; Peiro, 1994), the student *t* (Praetz, 1972; Blattberg and Bonedes, 1974; Kon, 1984; Gray and French, 1990; Peiro, 1994; Kim and Kon, 1994; Aparicio and Estrada, 1997), The Box–Tiao or power exponential (Hsu, 1982; Gray and French, 1990; Peiro, 1994; Aparicio and Estrada, 1997), the logistic (Smith, 1981; Peiro, 1994; Aparicio and Estrada, 1997), a discrete mixture of normals (Ball and Tourous, 1983; Kon , 1984; Peiro, 1994; Kim and Kon, 1994) and a Poisson mixture of normals (Press, 1967; Kim and Kon, 1994).

five years (IFC, 1999). In order to model ISE returns, one employs two very flexible families of distributions that have been recently introduced. These distributions permit very diverse levels of skewness and kurtosis and between them, nest most of the distributions previously considered in the literature. The first is the exponential generalized beta (EGB) distribution, introduced by McDonald and Xu (1995), and used to model US daily equity returns for individual stocks. The second is the skewed generalized t distribution (SGT), proposed by Theodossiou (1998) as a skewed extension of the generalised t distribution, originally introduced by McDonald and Newey (1988). Theodossiou (1998) uses the SGT to model daily returns for a range of financial assets, including aggregate equity returns for the USA, Canada and Japan.

The paper is organized as follows. Section II describes the data and presents some preliminary statistics. Section III presents the EGB and SGT distributions, and gives details of the estimation and testing methodology. The results are reported and discussed in Section IV while Section V concludes.

II. DATA

The raw data used in this paper are daily observations on the ISE composite stock price index. Data were obtained from Datastream for the five-year period 1 January 1988 to 31 December 1998 (Datastream code RI). Continuously compounded returns in domestic currency were calculated as the first difference of the natural logarithm of the series. This yields a total of 2869 observations. Table 1 reports the first four moments of the series, the minimum and maximum, skewness and excess kurtosis coefficients, and the Jarque–Bera and Kolmogorov–Smirnov statistics for normality. For ISE daily returns, normality is rejected at the 1% level using both the Jarque–Bera and Kolmogorov–Smirnov statistics. Consistent with evidence for developed markets, daily returns display a very significant leptokurtosis. There is some evidence of negative skewness, although this is almost certainly insignificant given the leptokurtosis of the data (see Peiro, 1999).

Table 1. *Preliminary statistics*

Mean	0.0023
Standard deviation	0.0276
Minimum	−0.1558
Maximum	0.1570
Skewness	−0.1597
Excess kurtosis	2.8673
Jarque–Bera test	994.6460***
Kolmogorov–Smirnov test	0.0647***
No of observations	2869

Notes: The table reports statistics for daily ISE returns for the period 1.1.88 to 31.12.98. *** significance at the 1% level.

III. METHODOLOGY

The exponential generalized beta distribution

The exponential generalized beta (EGB) distribution, introduced by McDonald and Xu (1995), is a flexible five-parameter distribution that allows very diverse levels of skewness and kurtosis, and nests more than thirty well-known distributions. The probability density function is defined by

$$EGB(z; \delta, \sigma, c, p, q) = \frac{e^{p(z-\delta)/\sigma}(1-(1-c)e^{(z-\delta)/\sigma})^{q-1}}{|\sigma|B(p,q)(1+ce^{(z-\delta)/\sigma})^{p+q}} \quad (1)$$

for

$$-\infty < \frac{z-\delta}{\sigma} < \ln\left(\frac{1}{1-c}\right)$$

with $0 \leq c \leq 1$, $\sigma > 0$, $p > 0$ and $q > 0$. $B(.)$ is the beta function.

The parameter σ determines the scale of EGB distribution, while δ is a location parameter. The parameters p and q control the shape and skewness of the EGB density. By restricting the parameters of the EGB distribution, many other well-known distributions are obtained, including the exponential generalized beta of the first type (EGB1) or generalized exponential distribution ($c = 0$); the exponential generalized beta of the second type (EGB2) or type IV generalized logistic distribution ($c = 1$); the exponential generalized gamma (EGG) or generalized Gompertz ($q \to \infty$); the exponential power ($c = 0, q = 1$); the exponential ($c = 0, q = 1, \delta \to \infty$); the generaized Gumbel or type III generalized logistic ($c = 1$, $p = q$); the exponential Burr type 12 (EBR12) or Burr type 2 (BR2, $c = 1$, $p = 1$); the exponential Weibull (EW) or extreme value type I ($c = 1$, $p = 1$, $q \to \infty$); and the exponential Fisk or logistic ($c = 1$, $p = q = 1$). For a more detailed discussion of the distributions that are nested within the EGB, see McDonald and Xu (1995).

The skewed generalized t distribution

The skewed generalized t (SGT) distribution, introduced by Theodossiou (1998), is a skewed extension of the generalized t distribution, originally proposed by McDonald and Newey (1988). Like the EGB, the SGT is a flexible distribution that allows for very diverse levels of skewness and kurtosis, and nests many other well-known distributions. The probability density function of the SGT distribution is given by[2]

[2] The probability density function of Theodossiou (1998) has been modified in order to include a non-zero location parameter.

$$f(x|m,k,n,\lambda,\sigma^2) = \begin{cases} f_1 & \text{for} \quad x < m \\ f_2 & \text{for} \quad x \geq m \end{cases} \tag{2}$$

where

$$f_1 = C(1 + (k/(n-2))\theta^{-k}(1-\lambda)^{-k}|(x-m)/\sigma|^k)^{-(n+1)/k} \tag{3}$$

$$f_2 = C(1 + (k/(n-2))\theta^{-k}(1+\lambda)^{-k}|(x-m)/\sigma|^k)^{-(n+1)/k} \tag{4}$$

and C and θ are given by

$$C = \frac{1}{2\sigma} kB\left(\frac{1}{k}, \frac{n}{k}\right)^{-3/2} B\left(\frac{3}{k}, \frac{n-2}{k}\right)^{1/2} S \tag{5}$$

$$\theta = \frac{1}{S}\left(\frac{k}{n-2}\right)^{1/k} B\left(\frac{1}{k}, \frac{n}{k}\right) B\left(\frac{3}{k}, \frac{n-2}{k}\right)^{-1/2} \tag{6}$$

with S given by

$$S = \left[1 + 3\lambda^2 - 4\lambda^2 B\left(\frac{2}{k}, \frac{n-1}{k}\right)^2 B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1} \times B\left(\frac{3}{k}, \frac{n-2}{k}\right)^{-1}\right]^{1/2} \tag{7}$$

and $-1 < \lambda < 1$, $n > 0$, $\sigma > 0$ and $k > 2$.

The parameter m determines the location of the random variable x, while σ is a scale parameter. The parameters k and n control the height and tails of the density, and consequently its kurtosis, while the parameter λ determines its skewness, with a symmetric distribution obtaining when $\lambda = 0$. The parameter n has the degrees of freedom interpretation in the case that $\lambda = 0$ and $k = 2$. As with the EGB, by restricting the parameters of the SGT, many other well-known distributions are obtained, including the generalized t (McDonald and Newey, 1988); the skewed t (Hansen, 1994, $k = 2$); the student t ($k = 2$, $\lambda = 0$); the normal ($k = 2$, $\lambda = 0$, $n \to \infty$); the Cauchy ($k = 2$, $\lambda = 0$, $n = 1$); the power exponential or Box–Tiao ($\lambda = 0$, $n \to \infty$); the Laplace or double exponential ($k = 1$, $\lambda = 0$, $n \to \infty$); and the uniform ($k \to \infty$, $\lambda = 0$, $n \to \infty$).

Estimation of the EGB and SGT distributions

The parameters of the EGB and SGT distributions are estimated by maximum likelihood (ML), using the BFGS algorithm with a convergence criterion of 0.0001 applied to the log likelihood function value. Robust standard errors are reported that allow for mis-specification of the underlying distribution (see White, 1982). The sample mean and standard deviation were used as starting values for the location and spread parameters for both distributions. Starting values for the shape parameters were chosen on the basis of empirical results reported elsewhere (Bookstaber and McDonald, 1987; McDonald and Xu, 1995; McDonald, 1996; Theodossiou, 1998). A logistic transformation is used to restrict the range of the parameter c in the EGB distribution, and the parameter λ in the SGT distribution. The likelihood function is then maximized with respect to the transformed parameter, and the inverse of this transformation used to recover the ML estimate of the original parameter by the invariance property of ML.

The validity of the restrictions that are implied by each of the nested distributions is tested using a likelihood ratio statistic, $LR = -2(\ln L_0 - \ln L_1)$, where $\ln L_0$ is the maximum log likelihood of the restricted distribution and lnL_1 is the maximum log likelihood of the unrestricted distribution. Under the null hypothesis that the restrictions are valid, the likelihood ratio statistic has an asymptotic chi-squared distribution with k degrees of freedom, where k is the numer of restrictions imposed. In order to compare the relative performance of non-nested distributions, both within each of the EGB and SGT families and across the two, we report the Akaike Information Criterion (AIC). The AIC is based on the maximum log-likelihood function value, but is adjusted for the loss of degrees of freedom that results from the estimation of additional parameters.

IV. RESULTS

Table 2 reports the results. The first section of each panel reports the ML parameter estimates, together with robust standard errors in parentheses. The second section reports the mean, variance, skewness and excess kurtosis of the estimated distribution.[3] The third section reports the maximum log-likelihood function value, the LR statistic to test the restricted distribution against the parent distribution, and the AIC. The results for the normal distribution are reported in the last column of the second panel of Table 2.

From Table 2, it is clear that both the EGB and SGT distributions provide a very substantial improvement over the normal distribution, with the SGT yielding a marginally better fit than the EGB. All of the distributions nested by the EGB are rejected at the 1% level by the LR statistic except the EGB2, which is rejected at the 10% level. By the AIC, which makes some allowance for the additional par-

[3] The skewness and excess kurtosis coefficients are computed as $m_3/m_2^{3/2}$ and $m_4/m_2^2 - 3$, respectively, where m_2, m_3 and m_4 are the second, third and fourth central moments of the estimated distribution. The first four moments of each distribution of EGB family were computed using the formulae given in McDonald and Xu (1995, p. 150–51) and McDonaold (1996, p. 437). The first four moments of each distribution of the SGT family were computed using the formulae given in Theodossiou (1998, p. 1651).

Table 2. *Estimated Parameters and Moments of the EGB and SGT Distribution for ISE Daily Returns*

Panel A	EGB	EGB2	EGB1	EGG	BR2	EW Gompertz	Gen. Gumbel	Efisk logistics
δ	0.0003 (0.0033)	0.0011 (0.0006)	0.2676 (0.0281)	−1.3173 (0.2117)	0.0002 (0.0006)	0.0137 (0.0006)	0.0006 (0.0006)	0.0023 (0.0005)
σ	0.0002 (0.0009)	0.0004 (0.0011)	0.2509 (0.0245)	0.2842 (0.0328)	0.0139 (0.0002)	0.0317 (0.0003)	0.0002 (0.0007)	0.0144 (0.0002)
p	0.0117 (0.0455)	0.0244 (0.0561)	56.432 (11.881)	103.45 (21.788)	[1.0]	[1]	[q]	[1.0]
q	0.0108 (0.0469)	0.0223 (0.0581)	111.17 (23.683)	[∞]	0.9012 (0.0138)	[∞]	0.0113(0.0337)	[1.0]
c	1.0000 (0.1383)	[1.0]	[0.0]	–	[1.0]	–	[1.0]	[1.0]
Mean	0.0019	0.0022	0.0017	−0.0002	0.0026	−0.0046	0.0006	0.0023
Variance	7.7407×10^{-4}	7.4080×10^{-4}	1.4381×10^{-3}	7.8458×10^{-4}	6.8559×10^{-4}	1.6524×10^{-3}	7.6661×10^{-4}	6.8420×10^{-4}
Skewness	0.1693	0.1907	−0.0733	−0.09856	0.1168	-1.1395	0.0000	0.0000
Kurtosis	3.0179	3.0191	0.0195	0.0194	1.2743	2.4000	3.0000	1.2000
ln L	6437.79	6433.18	6323.38	6225.44	6366.61	5791.30	6432.00	6365.21
LR	-	9.20*	228.81***	424.69***	142.35***	1292.97***	11.57***	145.16***
AIC	6432.79	6429.18	6319.38	6222.44	6363.61	5789.30	6429.00	6363.21
Panel B	**SGT**	**Generalised t**	**Skewed t**	**P. exponential**	**Student t**	**Laplace**	**Cauchy**	**Normal**
k	0.9289 (0.0548)	0.9366 (0.0442)	[2.0]	0.9317 (0.0415)	[2.0]	[1.0]	[2.0]	[2.0]
n	363.08 (46.965)	433.31 (42.389)	3.3894 (0.1934)	[∞]	3.4162 (0.2698)	[∞]	[1.0]	[∞]
σ	0.0282 (0.0004)	0.0281 (0.0005)	0.0297 (0.0008)	0.0282 (0.0007)	0.0296 (0.0009)	0.0276 (0.0006)	0.0127 (0.0003)	0.0273 (0.0006)
m	0.0024 (0.0004)	0.0004 (0.0008)	0.0027 (0.0004)	0.0004 (0.0001)	0.0022 (0.0005)	0.0007 (0.0004)	0.0014 (0.0004)	0.0024 (0.0007)
λ	0.0561 (0.0104)	[0.0]	0.0402 (0.0173)	[0.0]	[0.0]	[0.0]	[0.0]	[0.0]
Mean	2.3792×10^{-3}	4.1369×10^{-4}	2.7409×10^{-3}	4.0303×10^{-4}	2.1925×10^{-3}	5.6352×10^{-4}	NA	2.3206×10^{-3}
Variance	7.9272×10^{-4}	7.9102×10^{-4}	8.8338×10^{-4}	7.9270×10^{-4}	8.7664×10^{-4}	7.6180×10^{-4}	NA	7.6292×10^{-4}
Skewness	0.2643	0.0000	0.6070	0.0000	0.0000	0.0000	NA	0.0000
Kurtosis	3.8170	3.6703	NA	3.6579	NA	3.0000	NA	0.0000
ln L	6441.86	6433.88	6399.40	6434.04	6397.78	6432.53	6171.59	6224.06
LR	–	15.99***	84.96***	15.67***	88.20***	18.69***	540.56***	435.63***
AIC	6436.86	6429.88	6395.40	6431.04	6396.78	6430.53	6169.59	6222.06

Notes: Sample mean = 0.0023, variance = 7.6300×10^{-4}, skewness = −0.1597, kurtosis = 2.8673. Parameters reported in [ ] are the restricted values implied by that distribution. Robust standard errors of the parameter estimates are reported in ( ). Mean, variance, skewness and kurtosis are reported for each distribution and are calculated using the estimated parameters. *LR* is the likelihood ratio test of the restrictions on the EGB and SGT implied by each distribution. ***, **, and * denote significance at the 1%, 5% and 10% level, respectively.

ameters involved in its estimation, the EGB is again the preferred distribution. All of the distributions nested by the SGT are rejected at the 1% level by the LR statistic. By the AIC, the preferred distribution from the SGT family is the SGT itself. On the basis of the AIC, the SGT is to be preferred to the EGB. Interestingly, many of the distributions previously considered in the literature, including the logistic, student t, power exponential and Laplace distributions, are very strongly rejected.

V. SUMMARY AND CONCLUSIONS

This paper provides evidence on the return characteristics of a large emerging European stock market, using two very flexible families of distributions that have recently been introduced, the EGB and SGT. Consistent with evidence from developed equity markets, it is found that daily equity returns are highly non-normal. Moreover, one finds that we can reject the restrictions on both the SGT and EGB distributions that are implied by many distributions that have previously been considered in the literature, including the student-t, logistic, power-exponential and Laplace. The preferred distribution overall is the SGT, which provides a marginally better fit to the data than the EGB. These results should be of interest to both researchers and practitioners in emerging markets, and could be used as input to a number of finance applications, such as the calculation of value-at-risk and the pricing equity options.

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