

Structural Breaks in Inflation Dynamics within the European Monetary Union

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Abstract

The aim of this paper is to investigate how a country's decision to join one or all of the steps leading towards the common European currency changed its inflation rate dynamics. Our attempt is to model the inflation rates via a Generalized Logistic Distribution (GL) and then test for breaks in its parameters.

Keywords: inflation rate, structural break, EMU, Generalized Logistic Distribution, euro.

1. Introduction

The [European Central Bank \(2010\)](#) defines price stability “as a year-on-year increase in the Harmonized Index of Consumer Prices (HICP) for the euro area of below 2%”. [Emerson, Gros, Italianer, Pisani-Ferry, and Reichenbach \(1992\)](#) emphasize the fact that a high inflation rate is also more variable and uncertain and thus causes more relative price variability, leading to a less efficient price mechanism. The impact of lower inflation rates seems to be rather beneficial, as [Crespo-Cuaresma and Silgoner \(2004\)](#) point out, who suggest that an inflation rate around 1.6% has favorable effects on economic growth. For now, the question of interest centers around the way in which a country's decision to join the Economic and Monetary Union (EMU) changed its inflation rate dynamics.

Throughout the literature, there is still a considerable degree of uncertainty as to what extent the introduction of the euro, or monetary unions in general, affect the inflation rate. We provide a brief summary of the most important papers in this area, the data used and the conclusion reached:

Authors	Applied method	Data (region)	Conclusions
Busetti, Forni, Harvey, and Venditti (2007)	Unit root tests on inflation differentials	monthly CPI ¹ (EMU countries)	Inflation rates diverge after 1998.
Caporale and Kon-tonikas (2009)	GARCH framework	monthly seasonally adjusted CPI (12 EMU countries)	No consistent pattern can be found for the degree of persistence of inflation.

¹Consumer price index

Crespo-Cuaresma, Balázs, and Silgoner (2007)	T2 test for significance in changes in price dispersion	Price level data on over 160 products and services (27 European cities)	Price convergence took place at beginning of 1990s. Not much evidence that the euro further narrowed price differentials.
Holtemöller (2007)	Dynamic stochastic general equilibrium model; theoretical analysis	no real data (modeled similar to new EU Member States)	Joining a monetary union decreases the variability of the inflation rate.
Morana (2000)	Markov switching ARFIMA ² model	monthly HICP ³ and M3 (aggregated euro Area inflation rate)	Finds 3 regimes (roughly 1980—1984, 1984—1993 and 1993—2000) governing core inflation rate.
Palomba, Sarno, and Zazzaro (2009)	Autoregressive metric in ARIMA ⁴ context to measure similarity	monthly seasonally unadjusted CPI (EU 25 Member States)	The degree of similarity of short-run inflation dynamics across euro area countries is still weak.
Slačik (2008)	ARDL ⁵ model combined with Bayesian Model Averaging techniques	Various variables (EU 12)	Inflationary jumps from joining a monetary union, in the case of the Czech Republic, may not be completely avoided, but are rather unlikely.

The clear message here is that there is no clear message up to now. Most studies fail to find evidence of common behaviour of the inflation time series of the Member States of the EMU. Our paper aims to help to fill this gap.

²Auto regressive fractionally integrated moving average

³Harmonized index of consumer prices

⁴Auto regressive integrated moving average

⁵Autoregressive distributed lags

2. Data

The data is obtained from the [Organisation for Economic Cooperation and Development \(2010\)](#). Inflation is measured as the monthly change in the HICP rates from Jan. 1990—Mar. 2010. Countries included are Austria, Belgium, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, the Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, and the United Kingdom (Latvia and Lithuania as well as Bulgaria and Romania are excluded due to data scarcity).

The countries can be divided into three different groups. First the euro countries (Austria, Belgium, Estonia (although Estonia has not yet introduced the euro, but in 2011), France, Finland, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Slovakia, Slovenia and Spain). Cypria, Malta, Andorra, Monaco, San Marino and the Vatican are left out due their minor importance. Second, the EU members not participating in the ERM II (Exchange Rate Mechanism): the Czech Republic, Hungary, Poland, the United Kingdom and Sweden. Denmark stands on its own as a member of the EU and the ERM II, but not yet a member of the EMU.

It should be pointed out however, that these monthly series do still contain all the seasonality.⁶

3. Model

Our main question of interest is whether or not a country's decision to join one or all of the steps leading towards a common European currency changed its inflation rate dynamics. The economic reason why this should be so is rather clear cut for more than half of the countries in our sample. We would expect changes in the dynamics of all Eastern European countries due to their transition process away from previously centrally planned economies towards free market economies. All of these countries experienced some problems with very high inflation rates at the beginning and they all were successful in eliminating excess inflation afterwards. We might also assume, that the former constituent republics of Yugoslavia follow a rather similar pattern. In the case of the Southern European countries, with a history of high inflation and less pronounced fiscal discipline, we would also expect inflation dynamics to change towards a regime of lower inflation. The case for the Central European countries of course is different. There is no reason why the European Central Bank (ECB) should actually follow the rather stable monetary policies of the former German central bank in a one-to-one fashion.

Our approach follows one simple idea: we want to track the evolution of mean, variance and skewness of the inflation series over time. Thereby we hope to be able to find and estimate possible breakpoints in the data. These breakpoints – where any breakpoint would suggest a change in dynamics – are then examined for each country individually. We focus on countries rather than the whole euro-area due to the still large degree of heterogeneity in the sample.

⁶We would like to mention that in the case of Belgium and Luxembourg, the seasonality patterns can be traced back to changes in HICP definition and legal requirements. In addition to Belgium and Luxembourg, there are a number of other states, where the sales periods are regulated by law. France has an individual time span regulated by each department. This should pose no problem, since there is no global state level solution. In Greece there are also two set sales-periods in winter and summer. In Italy the sales periods are fixed each year by the Chambers of Commerce. Portugal has two set sales periods as well, but for two month (January and February as well as Mid-July to Mid-September. The source for this information is [Konsument Europa \(2010\)](#).

If a number of countries would have the same break dates, this would possibly indicate some common factor responsible for changes in the dynamics. We hope to be able to trace out similarities and differences in the countries and thereby improve our understanding of what the monetary union actually meant for the inflation rate dynamics.

3.1. Generalized Logistic Distribution

The first problem to solve was the selection of an appropriate distribution reflecting the properties of the data in use in as good a manner as possible. Most of the time series exhibit asymmetric properties (with sometimes rather strong skewness) and – compared to the normal distribution – a fatter tail. This ruled out all the symmetric distributions. In the field of the asymmetric distributions there was of course still no supply shortage, but we have chosen the Generalized logistic distribution (GL) due to its relative low degree of complexity, in comparison – for example – with the skewed-t-distribution. With its three parameters, we are able to trace mean, variance and skewness of the inflation rates. For our inflation series $y_t = 100 \cdot \log(HICP_t/HICP_{t-1})$ we therefore assume a GL distribution as defined in [Johnson, Kotz, and Balakrishnan \(1995\)](#):

$$f(y|\theta, \sigma, \delta) = \frac{\frac{\delta}{\sigma} \cdot \exp^{-\frac{y-\theta}{\sigma}}}{(1 + \exp^{-\frac{y-\theta}{\sigma}})^{(\delta+1)}}$$

with location (θ), scale (σ) and shape (δ). For $\delta=1$ the distribution simplifies to the logistic distribution, for $\delta<1$ it is skewed to the left and for $\delta>1$ it is skewed to the right.⁷

In econometrics, the logistic distribution is often used in income distributions and growth models. This is due to its fatter tail, which fit these kind of data somewhat better. The application of the GL distribution, parameterized in a way similar to this paper, is not a very common one. [Wong and Bian \(2005\)](#) use a GL distribution in a regression model with autocorrelated errors. They use this type of distribution rather than a Student's t-distribution, as to model the fact that these are oftentimes non symmetric and severely left or right skewed. A similar GL distribution is also used in [Tolikas, Koulakiotis, and Brown \(2007\)](#) who analyze extreme risk and value-at-risk in the German stock market, although they do not use a shape parameter. Regarding inflation rates, the GL distribution is — to our best knowledge — only used in context to expected inflation. [Batchelor and Orr \(1988\)](#) use a logistic distribution (not its generalization) to model the distribution of mean expected inflation rates.

⁷more details are provided in section 8.

3.2. Estimation

We apply the general framework, as developed in Zeileis and Hornik (2007), to a more specific model, in this case by means of a GL distribution. An enhancement to the already existing R package **strucchange**, which currently does not support a GL distribution, is provided with this paper. The asymptotic testing theory still holds for this generalization. The first part of the results we are going to present here, i.e., the tests and the graphical illustration of the empirical fluctuation process were developed in Zeileis and Hornik (2007), the second part of the results - the dating procedure and the illustration of the densities fitted for the subsamples (divided by the breaks) - can be found in Zeileis, Shah, and Patnaik (2010).

In order to analyze our data, we first estimate the parameters of the GL-distribution by means of maximum likelihood, which is possible, if we assume that our parameter vector $\phi = (\theta, \sigma, \delta)$ is stable over time t , i.e. we estimate $\hat{\phi} = \underset{\phi}{\operatorname{argmax}} \sum_{t=1}^n \log f(y_t|\phi)$. Our problem now simplifies to the following question: Is the assumption valid or do the ϕ_t change?

$$H_0 : \phi_t = \phi_0 \quad (t = 1, \dots, n)$$

This can be assessed using the empirical scores $s(y_t|\hat{\phi})$ as measures of model deviation.⁸ The empirical scores process $efp(\cdot)$ captures deviations from zero over time and is given by:

$$efp(t) = \hat{V}^{-1/2} n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} s(y_i|\hat{\theta}, \hat{\sigma}, \hat{\delta}) \quad (0 \leq t \leq 1),$$

where $\hat{V}^{-1/2}$ is some consistent estimator for the variance of the scores, which is used to decorrelate the series as to avoid problems due to serial correlation. We then employ a Functional central limit theorem (FCLT) for $efp(\cdot)$ which converges to a 3-dimensional Brownian bridge:

$$efp(\cdot) \xrightarrow{d} W^0(\cdot)$$

⁸for details see section 8

3.3. Test

Actually the $efp(\cdot)$ could be used for aggregated test statistics in various ways. In our paper, we employ the Supremum of LM (supLM) test by [Andrews \(1993\)](#). This test statistic is given by:

$$\sup_{t \in [0.1, 0.9]} \frac{\|efp(t)\|_2^2}{t(1-t)}$$

where the appropriate p-values can be computed using:

$$\sup_{t \in [0.1, 0.9]} \frac{\|W^0(t)\|_2^2}{t(1-t)}$$

The supLM test is the most powerful test for a single abrupt change in unknown time and has the highest power if more than one parameter of the distribution changes over time. Throughout the paper we will use the supLM test.

4. Breakpoint Estimation

After a break is detected via the supLM test, we proceed by estimating the timing of the break. We estimate B breakpoints τ_1, \dots, τ_B via maximization of the full segmented likelihood:

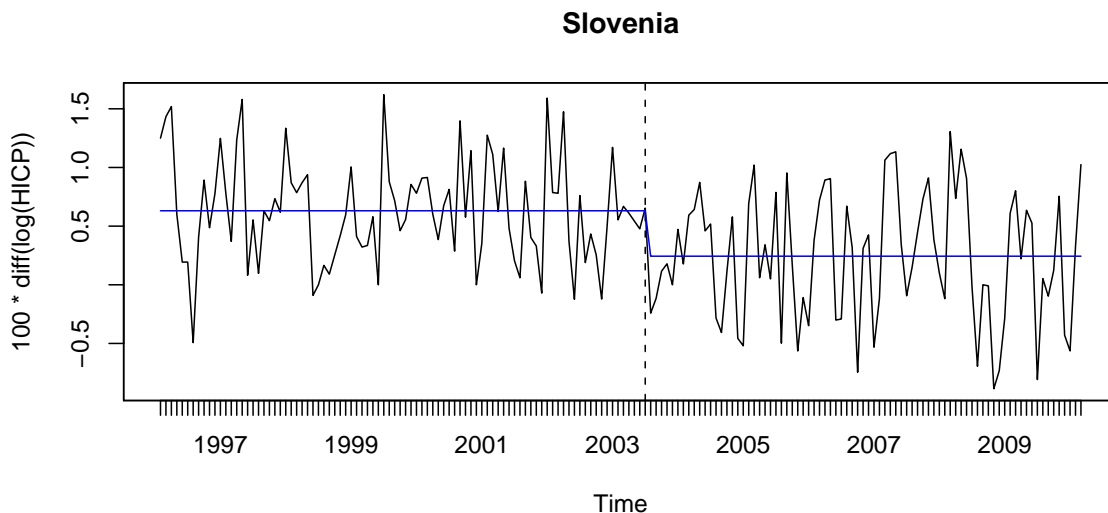
$$\sum_{b=1}^{B+1} \sum_{t=\tau_b+1}^{\tau_b} \log f(y_t | \phi^{(b)})$$

All parameters $\tau_1, \dots, \tau_B, \phi^{(1)}, \dots, \phi^{(B+1)}$ can be estimated jointly using dynamic programming. More details for this estimation technique are provided in [Zeileis *et al.* \(2010\)](#).

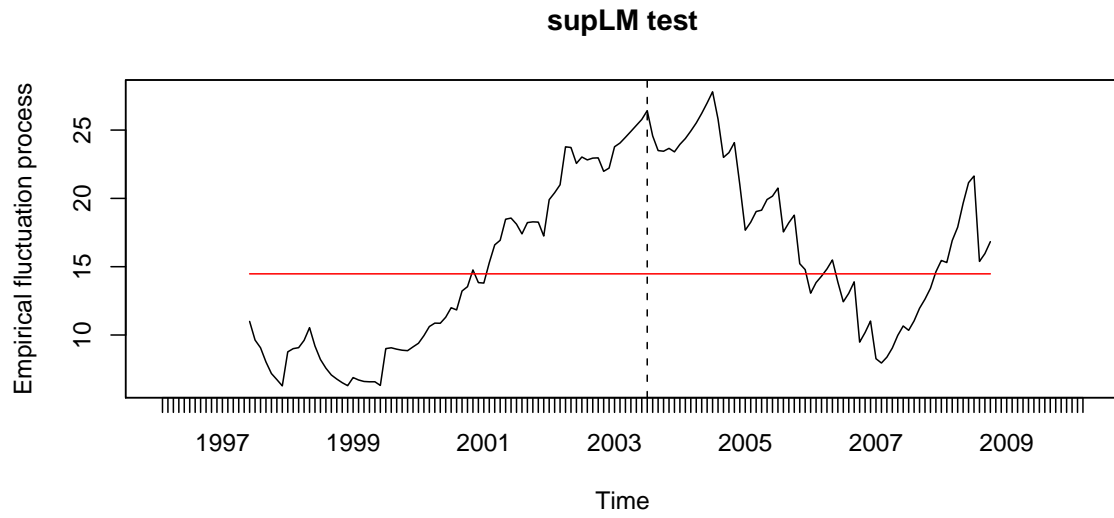
4.1. An Example

To illustrate our methods, we will go through all the steps of our analysis by means of an example. For this purpose, we have chosen the case of Slovenia. The economic reason why we suppose to find changing dynamics can be outlined in the following way: In the first phase of the consolidation after the crackup of Yugoslavia and the short war of 1991, Slovenia experienced very high inflation rates, with yearly growth rates as high as 90% in August 1990. But gradually Slovenia was able to reduce the inflation rate towards much lower values. This first phase is not captured in our data, since it starts as late as Jan. 1996. For the period up to today, we would expect one or two breaks from an economic point of view. The efforts to reach the Maastricht criteria as soon as possible are likely to lead to a further lowering of the mean and possibly also of the variance of the inflation rate. Another break point might possibly be found towards the introduction of the euro coins themselves in 2007.

Actually when we look at the data, this is precisely what we find, a breakpoint (represented by the dotted line) at Aug. 2003, with a clear lowering of the mean (represented by the straight blue lines).

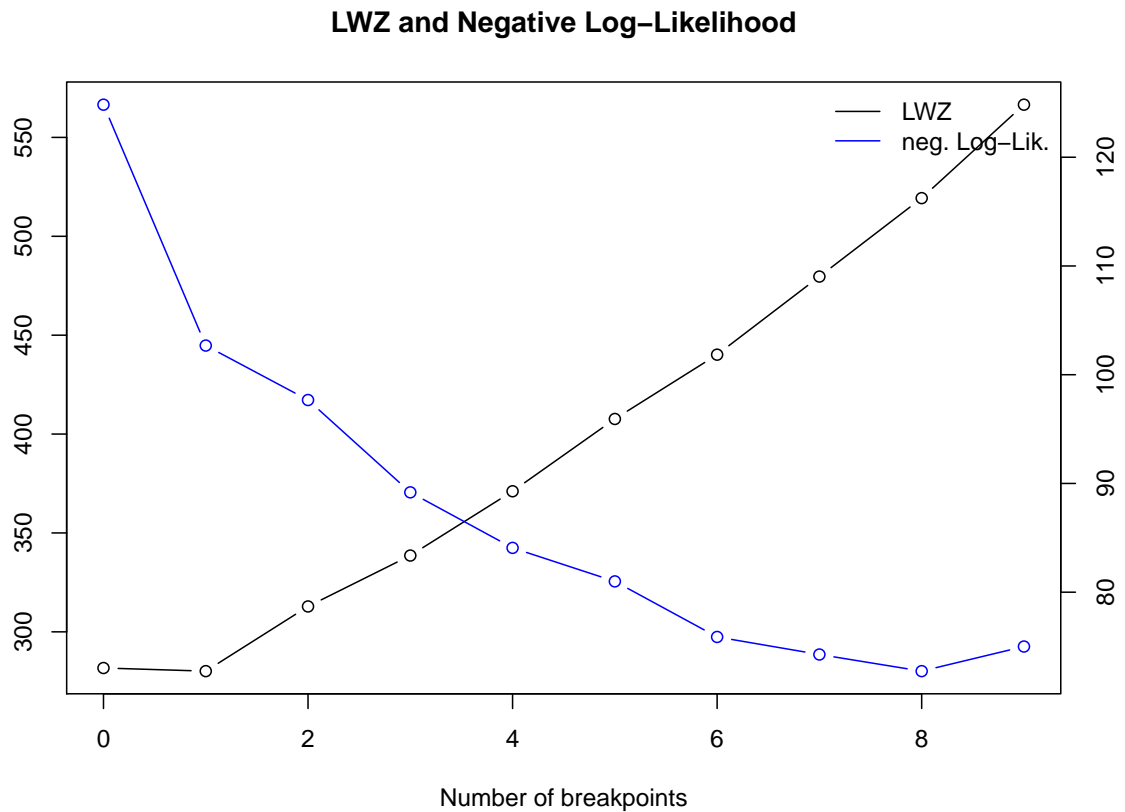


The supLM test graph suggests one break around 2003 and possibly another towards the end of the sample. This second break point is not estimated, since we restricted a regime towards



a length of at least 24 month.

To be as precise as possible with the detection of break points, we also include a graph showing the values of the LWZ (Liu, Wu and Zidek) criteria as developed in [Liu, Wu, and Zidek \(1997\)](#). Whereas the supLM test suggest at least one break, the LWZ criterion favors no break. But, since the decline in the negative log-likelihood from 0 breaks to 1 is the strongest, we concluded that one break point actually is the best alternative.



Slovenia did achieve its goals by two very important measures. First, it liberalized the financial markets in 2003 and – most important – it restricted the year-on-year growth of the monetary base M1 severely starting in 2003. Combined, these actions were enough to ensure Slovenian participation in the euro-zone. The [Intitute of Macroeconomic Analysis and Developement \(2007\)](#) explains this somewhat differently by the cooperation of both the government and the Bank of Slovenia, which resulted in a careful handling of administered prices, a counter-cyclical adjustment of excise duties on fuels and moderate wage increases based on social agreements. According to them, due to theses measures, the effect of the euro introduction on the Slovenian inflation rate was 0.24 percentage points.

5. Results

The results as we show them here are grouped according to patterns, which we think have some common properties. We present the breakdates and the parameter values of the segments as well as the dates of entry into the ERM/ERM II or the date of euro introduction. To give a guideline for interpretation, we provide a short overview over the history of the EMU. It basically consisted of 3 stages: stage I (1990-1994) as a phase of liberalization, stage II (1995-1998) a phase of convergence and stage III (1999-2001) the transition period, which ended with the introduction of the euro as legal tender and started with the Exchange Rate Mechanism II (ERM II) for Non-members and the irrevocable fixing of the exchange rates of the Member States.

Belgium and Luxembourg							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
Belgium	Feb 1991–Aug 2007	0.131	0.067	0.091	Mar 1979	–	Jan 1999
	Sep 2007–Mar 2010	0.135	0.196	0.297			
Luxembourg	Feb 1995–Dec 1998	0.088	0.013	0.261	Mar 1979	–	Jan 1999
	Jan 1999–Mar 2010	0.222	0.239	–0.811			
The Eastern Countries							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
CzechRepublic	Feb 1995–Jul 1998	0.697	0.336	1.139	–	–	–
	Aug 1998–Mar 2010	0.182	0.216	0.990			
Estonia	Feb 1996–Mar 1998	0.865	0.420	0.404	–	Jun 2004	Jan 2011
	Apr 1998–Mar 2010	0.333	0.206	0.802			
Hungary	Feb 1995–May 1998	1.606	1.024	0.878	–	–	–
	Jun 1998–Mar 2010	0.507	0.316	0.709			
Poland	Feb 1996–May 2001	0.855	0.421	0.668	–	–	–
	Jun 2001–Mar 2010	0.202	0.123	–0.315			
Slovakia	Feb 1995–Apr 1997	0.480	0.058	1.139	–	Nov 2005	Jan 2009
	May 1997–Feb 2004	0.587	0.442	1.140			
	Mar 2004–Mar 2010	0.186	0.089	1.139			
Slovenia	Feb 1996–Jul 2003	0.631	0.211	0.588	–	Jun 2004	Jan 2007
	Aug 2003–Mar 2010	0.244	0.344	0.143			
Italy and Spain							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
Italy	Feb 1990–May 1996	0.414	0.041	0.963	Mar 1979	–	Jan 1999
	Jun 1996–Dec 2000	0.168	0.020	0.726			
	Jan 2001–Mar 2010	0.200	0.106	0.021			
Spain	Feb 1992–May 1996	0.372	0.070	1.139	Jun 1986	–	Jan 1999
	Jun 1996–Dec 2000	0.200	0.040	0.019			
	Jan 2001–Mar 2010	0.223	0.342	–0.362			
Ireland							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
Ireland	Feb 1995–Mar 2008	0.255	0.205	–0.696	Mar 1979	–	Jan 1999
	Apr 2008–Mar 2010	–0.131	0.184	–0.995			
No change countries							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
Greece	Feb 1995–Feb 2001	0.400	1.505	0.040	Mar 1998	Jan 1999	Jan 2001
	Mar 2001–Mar 2010	0.355	0.182	0.507			
Netherlands	Feb 1990–Mar 2010	0.185	0.293	0.598	Mar 1979	–	Jan 1999

Finland	Feb 1990–Mar 2010	0.165	0.132	0.280	Oct 1996	–	Jan 1999
Non participating countries							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
Sweden	Feb 1990–Jan 1993	0.475	0.572	1.139	–	–	–
	Feb 1993–Mar 2010	0.155	0.185	0.534			
United Kingdom	Feb 1990–Apr 1992	0.570	0.387	1.139	–	–	–
	May 1992–Mar 2010	0.162	0.149	–1.265			
The other countries							
Country	Segment	Mean	Variance	Skewness	ERM	ERM II	Euro
Austria	Feb 1990–Sep 2007	0.163	0.057	0.606	Jan 1995	–	Jan 1999
	Oct 2007–Mar 2010	0.173	0.163	0.169			
Denmark	Feb 1990–Jun 2000	0.166	0.091	–0.742	Mar 1979	Jan 1999	–
	Jul 2000–Mar 2010	0.168	0.188	1.047			
France	Feb 1990–Dec 2004	0.159	0.058	0.196	Mar 1979	–	Jan 1999
	Jan 2005–Mar 2010	0.150	0.131	–0.794			
Germany	Feb 1995–May 2000	0.088	0.060	0.922	Mar 1979	–	Jan 1999
	Jun 2000–Dec 2004	0.140	0.164	0.992			
	Jan 2005–Mar 2010	0.142	0.184	–0.662			
Portugal	Feb 1990–May 1992	0.884	0.185	1.139	Apr 1992	–	Jan 1999
	Jun 1992–Mar 2010	0.345	0.105	0.422			

The first distinct group of countries consists of Belgium and Luxembourg. This may come as no surprise, since both countries formed a monetary union prior to the start of the ERM. In the past, they enjoyed very stable inflation rates with low values of both mean and variance, this changed around the timing of the irrevocable fixing of the exchange rates. But in the case of Belgium and Luxembourg, what we actually observe is an alternation due to the definition of the HICP. According to their national statistical offices, these changes can be traced back to the inclusion of sales (winter and summer) in January 1999 for Belgium and January 2000 for Luxembourg which was demanded by a Commission regulation, details may be found in: [Official Journal of the European Communities \(2010\)](#) This lead to a 6 month seasonality pattern (January and July) afterwards (meaning after the breakpoints induced by sales inclusion), since sales periods in Belgium and Luxembourg are regulated by law and have to occur in the first two weeks of these months and therefor we observe this sharp price decreases in both month. Interestingly enough, both these price decreases are immediately followed by increases in February and August of even higher magnitude.

The second of these groups are the Eastern European countries: the Czech Republic, Estonia, Hungary, Poland, Slovakia and Slovenia. In almost all of these countries, the mean and the variance of their inflation rates declined in the later part of the 90ies. Most of them experienced a break in 1997/1998, with Poland, Slovakia and Slovenia somewhat later.

Another combination of countries with – this time – identical results are the two biggest southern European countries, Italy and Spain. Both of them experienced high rates of inflation in the early 90ies. Afterwards, in the phase leading up to the monetary union, mean and variance were significantly reduced. This trend changed drastically soon after the fixing of the exchange rates leading to higher mean and much higher variances then ever before since the beginning of the HICP series.

Italy is quite a nice example to explain the usefulness of the skewness parameter. If we focus on the first two periods, with skewness lower than 1 (which indicates that the distribution is skewed to the right) we find that positive shocks are not normally neutralized by negative shocks. This changes in the last phase with skewness negative, indicating that the GL is now skewed to the left and that positive shocks will more likely be followed by negative shocks.

Ireland can be considered a rather special case. It is the only economy where the effects of the ongoing financial crisis of 2008 are visible. We find a structural break in 2008 following the strong contraction triggered by the bursting of the Irish housing bubble and a beginning deflation.

Another interesting group consists of 3 economically rather different countries. In none of them do we find any evidence of a structural change. Whereas the Netherlands and Finland are both on the lower side of the inflation margin, Greece exhibits some very strange behaviour. It has by far the highest variance of all series and seems to be completely different from all the others.

The last group with clear patterns consists of the two countries that decided not to be part of any European Monetary System⁹, the United Kingdom and Sweden. Both have a clear break in the early 90ies, and both of these breaks can be traced back to crisis, the currency crisis of the United Kingdom cumulating into the “Black Wednesday” in 1992 and the run on the Swedish currency in the same year. No clear interpretation is available for: Austria, Denmark, France, Germany and Portugal.

Busetti *et al.* (2007) find 3 clusters in their paper, if they include all the 12 countries in question (Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Luxembourg, Spain, Greece, Portugal, Ireland). The first 5 countries form the “low inflation club”, Italy, the Netherlands and Luxembourg the “medium inflation club”, whereas the other four countries form a high inflation club. We do not find much similarity between our cluster (which focuses on the timing of the break and the change in the parameters) and what they find via aggregation into inflation clubs. They also find that from 1980—1997 there was convergence of inflation rates, but afterwards there is some diverging behavior. This does not quite go along with our findings, that do not indicate stronger divergence after 1997 by means of break point detection.

⁹If we ignore the short participating time of the United Kingdom in the EMS.

6. Discussion

To clarify some issues, let's take a look at the data aggregated in just one graph. If more countries have the same inflation rate in a given month, the lines are overlapped and thus get darker.¹⁰ We do see that many of these countries follow a rather similar path. One event clearly visible is the financial crises and the short deflationary phase starting somewhere around the autumn of 2007 and lasting a few months. However, this was not enough for most of the countries – with the exception of Ireland – to trigger a structural break in inflation rate dynamics.

One interesting aspect we would like to focus on is the clear trend in means. What we have plotted here actually is a rolling mean, where the mean is calculated using the first moment of a fitted GL distribution. It is clearly visible that these rates converged towards some common mean (graphically emphasized by the redder lines) which, from 2005 onwards, comes rather close to the ECB target rate (which would translate to a monthly increase of roughly 2 per mill). So concerning the mean of the inflation rates, we see a clear convergence pattern. However, this is not the case for variance, where we find clear evidence of an increase. This of course does make some economic sense, since the convergence criteria and the ECB focus on mean inflation only and do not take volatility into account.

¹⁰We would see one black line if all countries would show the same month-on-month increase at any one time.

7. Conclusion

We were able to outline changes in the dynamics of the inflation rates in various countries across Europe. However, we fail to find any clear pattern relating to the euro introduction, but find convincing evidence, that the zeal of the Eastern European countries to join the European currency at the earliest possible date, clearly helped in reducing mean and variance of their inflation rates.

Considering the severe increase in overall volatility of the European inflation rates, it should be pointed out however, that most of these increases could possibly be traced back to the strong seasonality in our time series. The economic reason for this however remains a riddle and should be subject to future research.

8. Appendix

8.1. GL distribution

The moments are:

$$E(y) = \theta + \sigma(\gamma(\delta) - \gamma(1)) \quad (1)$$

$$Var(y) = \sigma^2(\gamma'(\delta) + \gamma'(1)) \quad (2)$$

$$Skew(y) = \frac{\gamma''(\delta) - \gamma''(1)}{(\gamma'(\delta) + \gamma'(1))^{3/2}} \quad (3)$$

where $\gamma(\cdot)$ is the digamma function and its derivatives.

The log-likelihood is:

$$\begin{aligned} \log f(y|\theta, \sigma, \delta) &= \log(\delta) - \log(\sigma) \\ &\quad - \frac{1}{\sigma}(y - \theta) - (\delta + 1) \\ &\quad \times \log(1 + \exp^{-\frac{y-\theta}{\sigma}}) \end{aligned} \quad (4)$$

The resulting score function, where the scores are the derivatives of the log-likelihood function, has 3 components $(s_\theta, s_\sigma, s_\delta)$, with $\tilde{y} = \exp^{-\frac{y-\theta}{\sigma}}$. These are used later on for the construction of the test statistics and are a means of measuring the estimation error in a maximum likelihood framework.

$$s_\theta(y|\theta, \sigma, \delta) = \frac{\delta \log f(y|\theta, \sigma, \delta)}{\delta \theta} \quad (5)$$

$$= \frac{1}{\sigma} - (\delta + 1) \cdot \frac{\frac{1}{\sigma} \tilde{y}}{(1 + \tilde{y})}$$

$$s_\sigma(y|\theta, \sigma, \delta) = \frac{\delta \log f(y|\theta, \sigma, \delta)}{\delta \sigma} \quad (6)$$

$$= -\frac{1}{\sigma} + \frac{1}{\sigma^2}(y - \theta) - (\delta + 1) \times \frac{\frac{1}{\sigma^2}(y - \theta)\tilde{y}}{(1 + \tilde{y})}$$

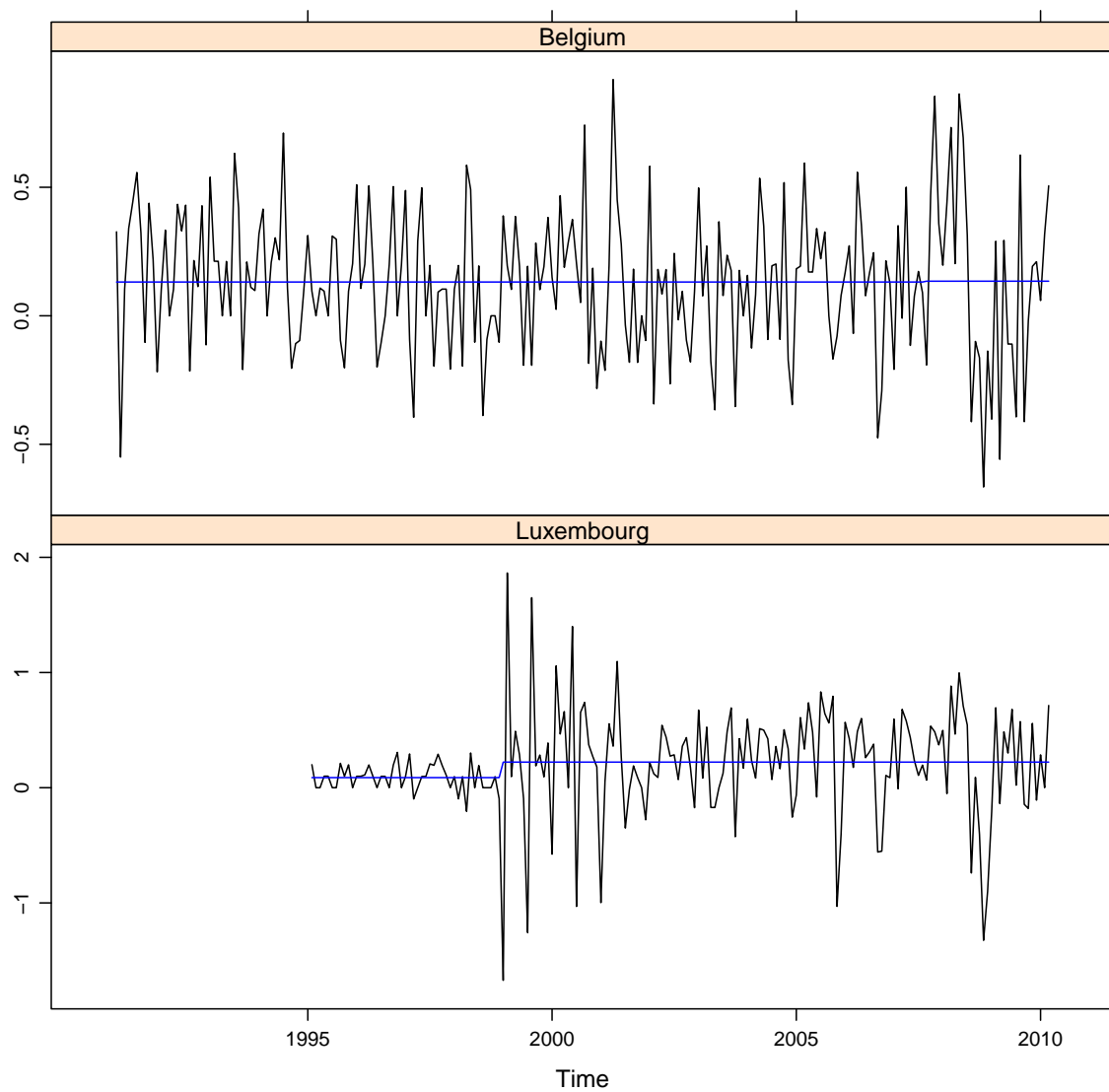
$$s_\delta(y|\theta, \sigma, \delta) = \frac{\delta \log f(y|\theta, \sigma, \delta)}{\delta \delta} \quad (7)$$

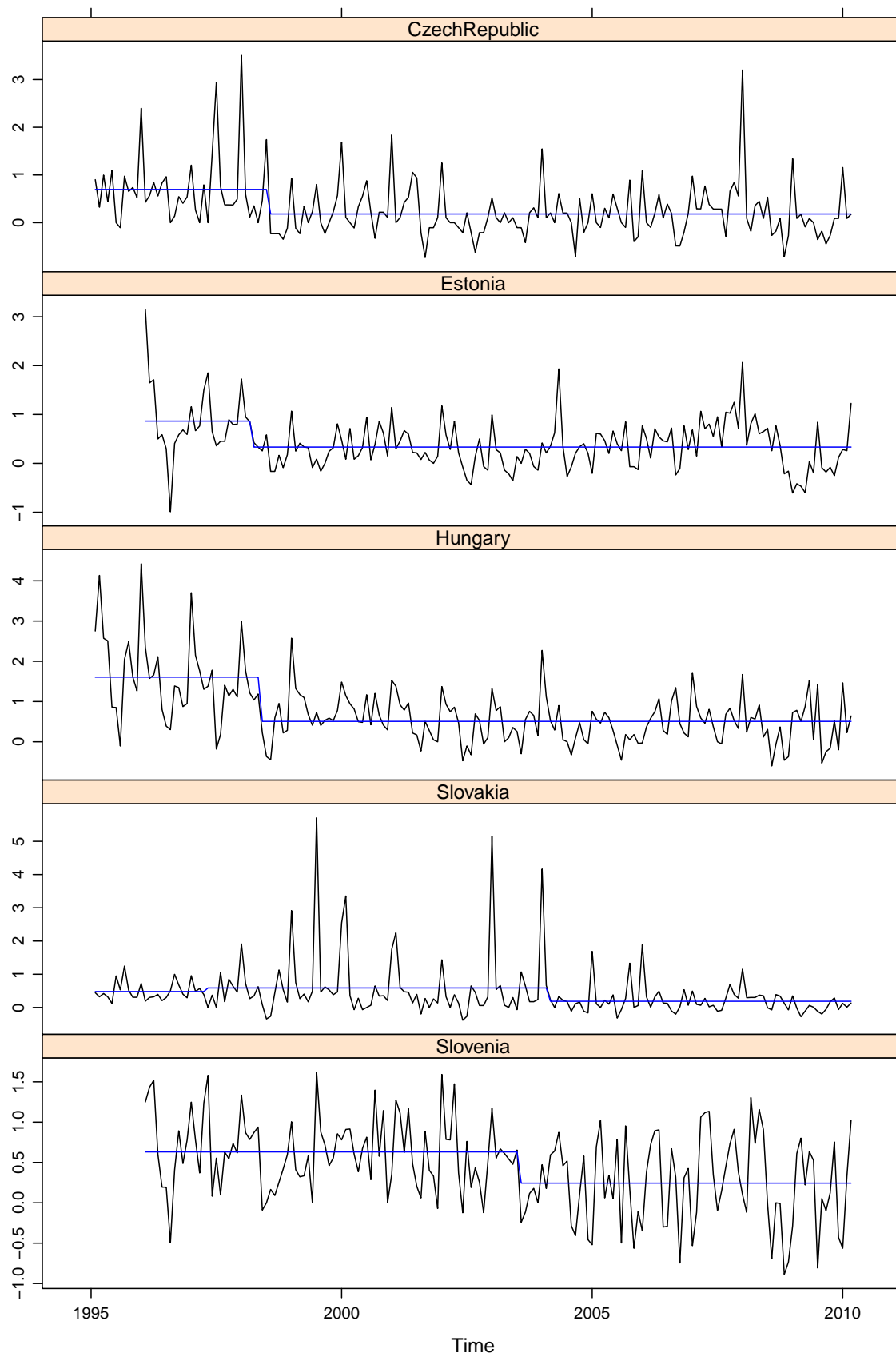
$$= \frac{1}{\delta} - \log(1 + \tilde{y})$$

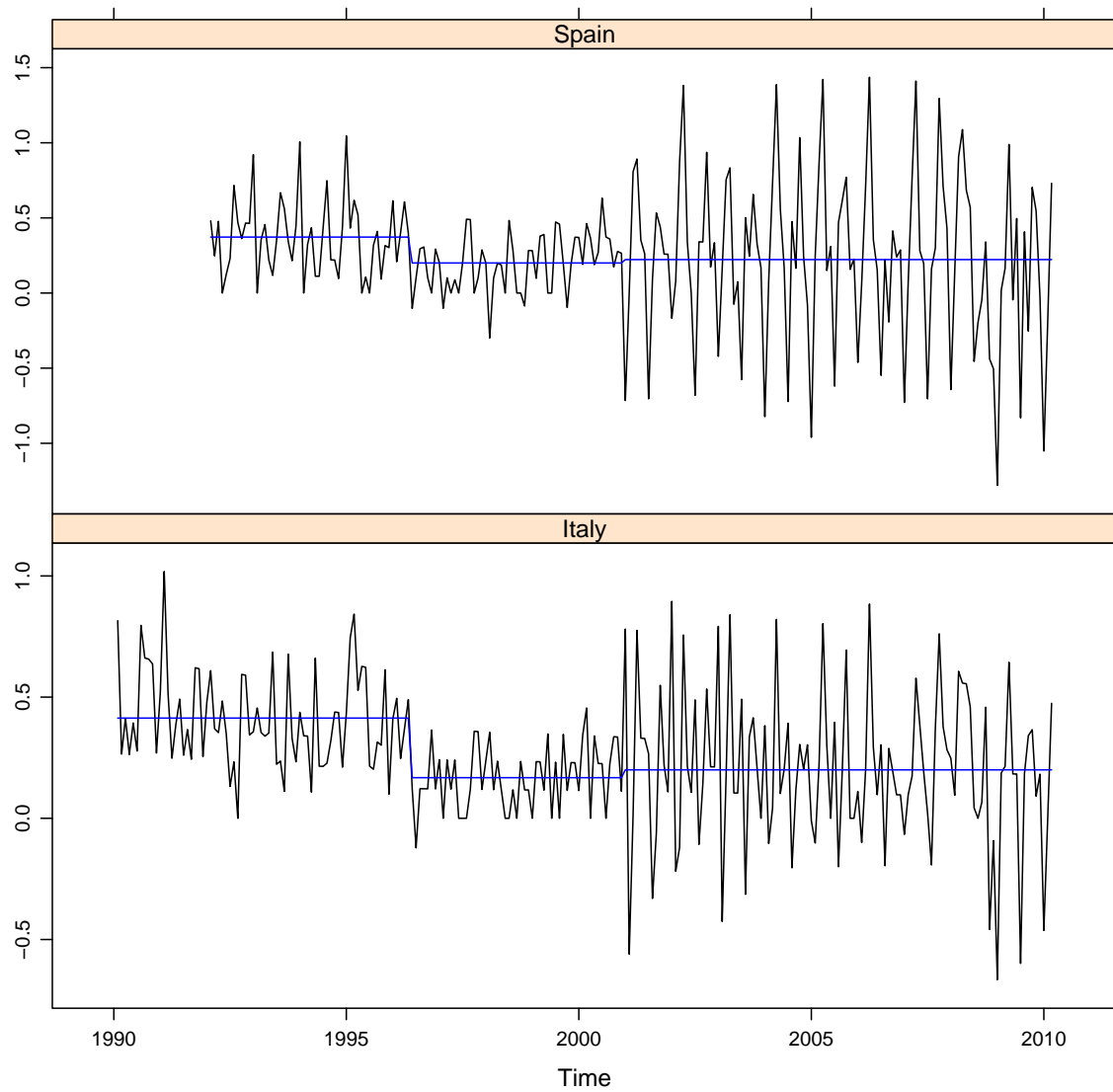
The scores of the parameters $\phi = (\theta, \sigma, \delta)$ sum up to 0: $\sum_{t=1}^n s(y_t|\hat{\phi}) = 0$.

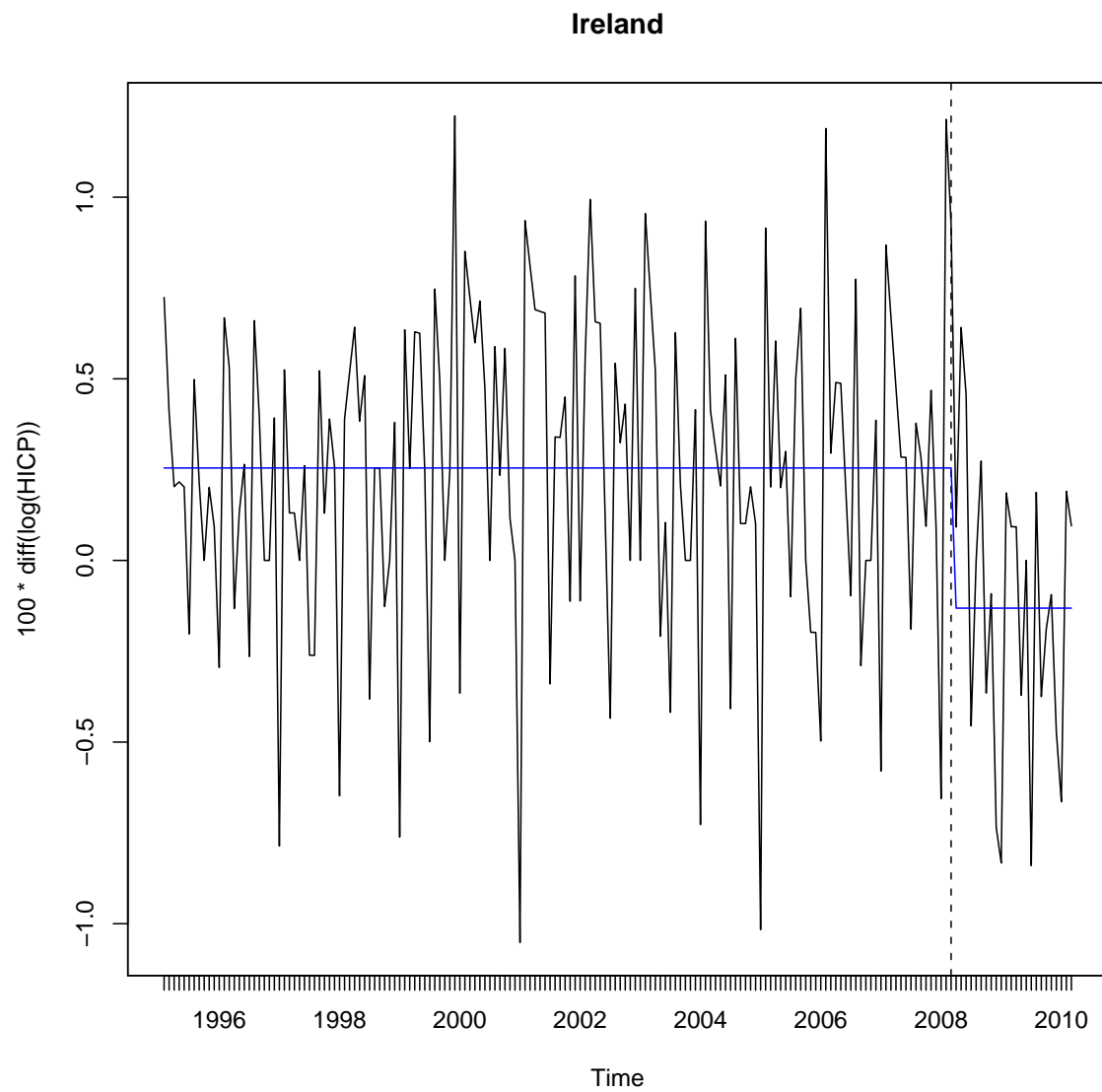
8.2. Graphs

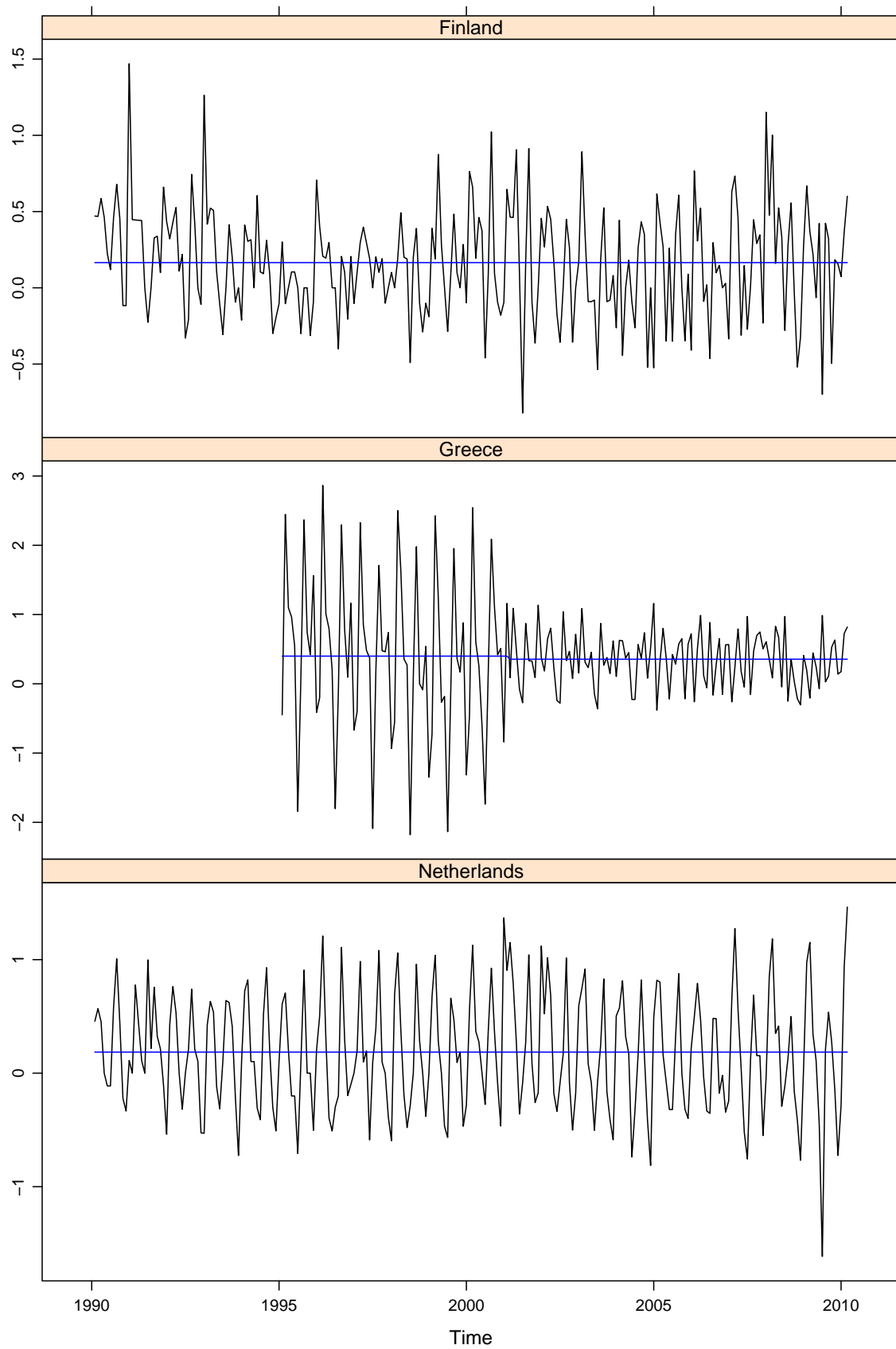
Here we present the actual data where the blue lines in the graphs correspond to the estimated mean of the series.

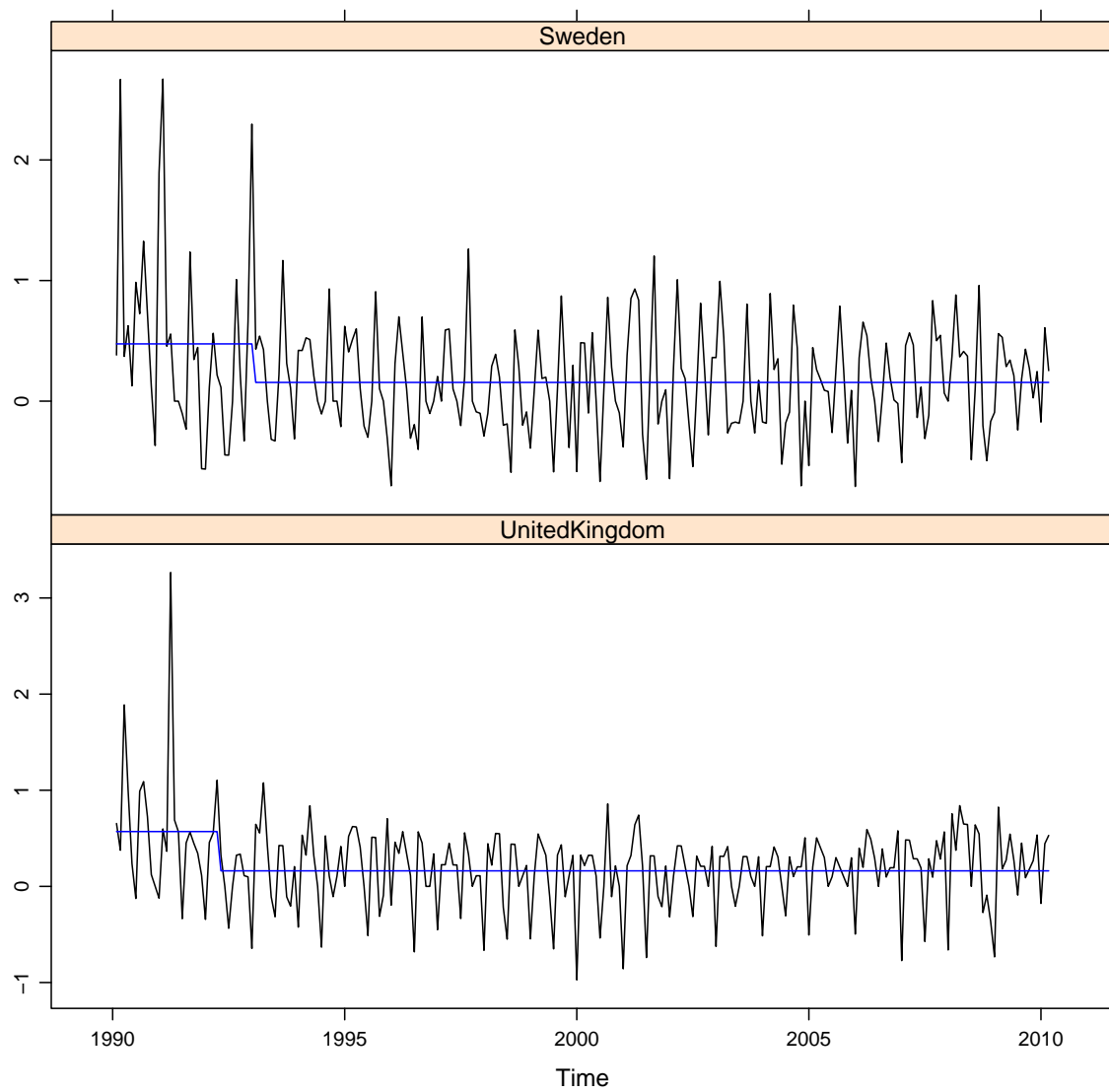


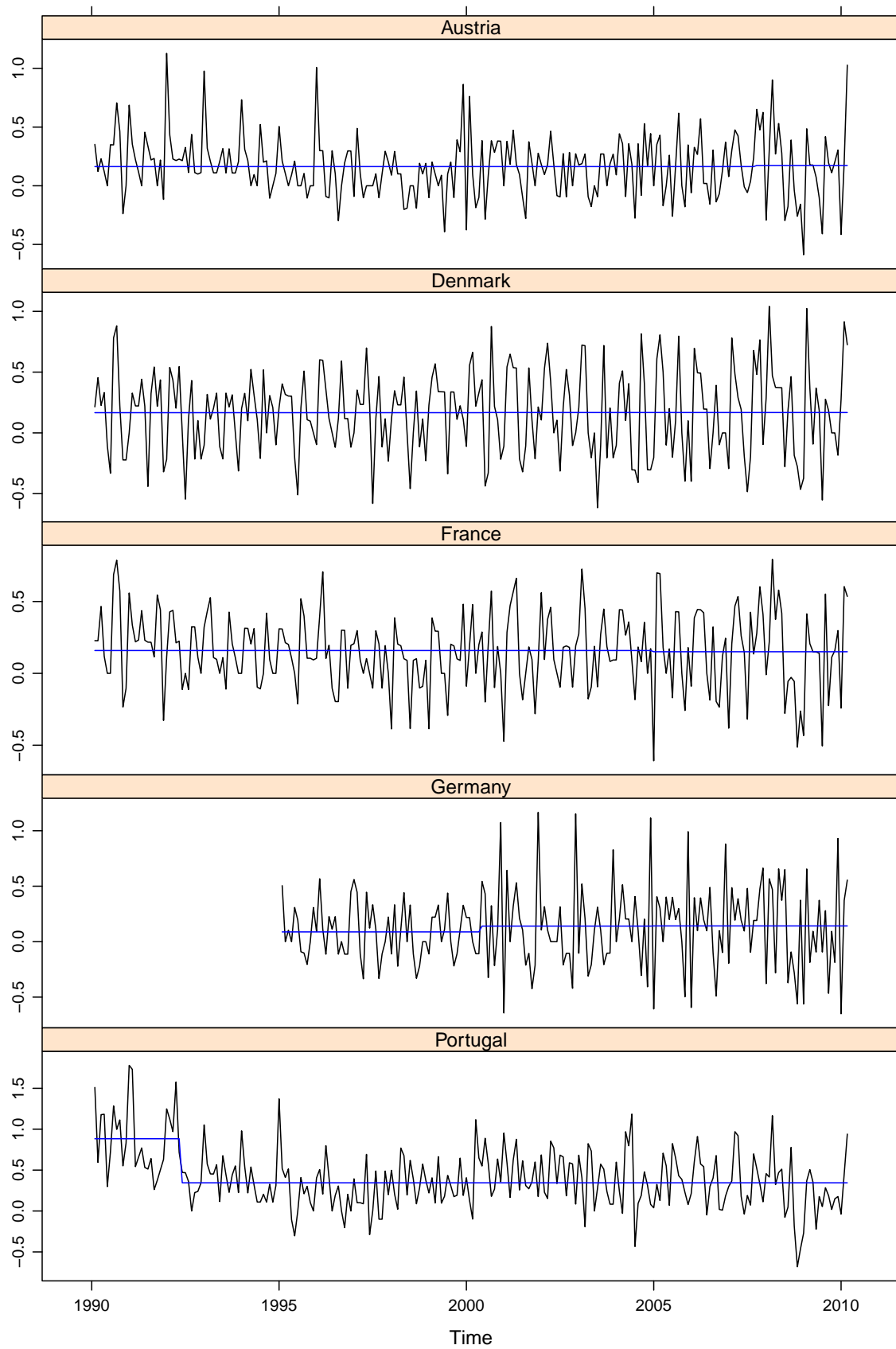












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