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# Extreme Risk and Value-at-Risk in the German Stock Market

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# Extreme Risk and Value-at-Risk in the German Stock Market

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ABSTRACT *Extreme Value Theory methods are used to investigate the distribution of the extreme minima in the German stock market over the period 1973 to 2001. Innovative aspects of this paper include (i) a wide set of distributions considered, (ii) L-moment diagrams employed to identify the most appropriate distribution/s, (iii) 'probability weighted moments' used to estimate the parameters of these distribution/s and (iv) the Anderson–Darling goodness of fit test employed to test the adequacy of fit. The 'generalized logistic' distribution is found to provide adequate descriptions of the extreme minima of the German stock market over the period studied. VaR analysis results show that the EVT methods used in this study can be particularly useful for market risk measurement since they produce estimates that outperform those derived by traditional methods at high confidence levels.*

KEY WORDS: Extreme value theory, value-at-risk, L-moments, probability weighted moments, Anderson–Darling goodness of fit test, generalized extreme value distribution, generalized logistic distribution

## 1. Introduction

The importance of financial risk management has significantly increased since the mid-1970s, which saw both the collapse of the fixed exchange rate system and two oil price crises. These major events led to considerable volatility in the capital markets, which together with the proliferation of the derivatives markets, increased trading volumes and technological advances, led to considerable concerns about the effective measurement and management of financial risk. This tendency was further reinforced by a number of financial catastrophes such as the worldwide stock markets collapse in 1987, the Mexican crisis in 1995, the Asian crisis in 1997, as well as the Orange County, Barings Bank and Long Term Capital Management cases. These made clear the inadequacy of existing risk management tools, the fragility of the financial system as well as the devastating consequences that financial crises can have. Regulators moved to address these issues and in 1996 the Bank for International Settlements (BIS, 1996) amended the *Capital Accord* of 1988 to incorporate market risk and stipulated the use of Value-at-Risk (VaR) models by financial firms when estimating the Capital Adequacy Requirements (CAR). This CAR should aim to cover the

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potential losses that might accrue from an institution's market positions and should therefore help to avoid financial disasters.

VaR tries to answer the question of how much the portfolio can lose with a c% confidence over a certain time period. Statistically, it is defined as one of the lower quantiles of the distribution of returns that is rarely exceeded. There are three traditional methods commonly used in VaR estimation; the variance–covariance (VC), the historical simulation (HS) and the Monte Carlo simulation (MCS). The VC method relies heavily on the assumption that financial returns are normally distributed. However, past and recent research suggests that financial returns distributions have fatter tails than suggested by the normal distribution.[1] The implication is that the probability of large losses is much greater than implied by the normal distribution. In contrast, the HS approach does not impose any assumptions regarding the distribution of returns. Rather it only requires that the distribution of historical returns is a good proxy for the distribution of future returns. However, such an assumption may not be valid since historical data may not reflect current market conditions. In addition, insufficient data may be available to allow for reliable estimates to be calculated, since a large number of tail observations is needed. Moreover, the empirical distribution lower tail converges sharply to zero and therefore, losses higher than those experienced in the past have a probability of zero. MCS can also be employed in VaR analysis but this approach is very sensitive to the statistical model chosen to generate returns and it can be rather demanding in terms of computing power.

Despite the widespread use of VaR in risk management there is no consensus among market practitioners and academics regarding which method is the best.[2] A common feature of the above approaches is that they do not efficiently exploit the information that is conveyed in the lower tail of the financial returns distribution. The danger, therefore, is that traditional VaR models are prone to fail when we need them most; in those cases where a bank or a financial institution may suffer enormous losses because of an extreme fall in share prices.

On the other hand, a special branch of statistics, named extreme value theory (EVT), focuses exclusively on these extremes and their associated probabilities by directly studying the tails of distributions. Although the use of EVT in finance is a recent development, research work that highlights its importance is increasing. Probably the first author to apply EVT in finance was Parkinson (1980). He used simulation techniques to compare variance of return estimates obtained by using all data available and by using extreme returns. He concluded that extreme returns contained important information of particular relevance to the tail behaviour of the data. Another important pioneer in introducing EVT in finance was Longin who investigated the limiting distribution of extremes in the US stock market (Longin, 1996). He fitted the generalized extreme value (GEV) distribution to the extremes collected over selection intervals of various lengths and discovered that the extremes of the S&P500 daily returns could be adequately characterized by the Fréchet distribution; a member of the GEV family. An international application of EVT was given by Jondeau and Rockinger (2003) who analysed a broad dataset containing the daily extreme returns of 27 stock markets; both emerging and developed markets were included. With respect to the developed stock markets, they found that the generalised Pareto (GP) provided a good fit to the empirical data when these are collected as excesses over a high threshold while the GEV, in particular the Fréchet, provided a good fit when the extremes were collected over 20 non-overlapping time intervals. However, although they argued that EVT could be an interesting tool for VaR analysis they found it could not help to predict the S&P500 index large negative daily return of −22.83% in October 1987.

The novel methods of this paper are applied to the one of the largest European stock markets in which relatively few detailed EVT studies have been made. A notable exception is Lux (1996) who used daily returns of all 30 stocks that constituted the DAX index for the period 1988 to 1994 to test the stable Pareto hypothesis. He rejected normality but found that, in most cases, the tails of the return distributions were thinner than expected under the stable Pareto distribution. He then proceeded to investigate the fit that the GEV distribution provided to the German data and found that the Fréchet distribution could not be rejected in most of the cases. More recently, he investigated the limiting extremal behaviour of the DAX index returns by using intra-day data and he reached similar results (Lux, 2001).

The role of EVT as an input in VaR estimation has been examined by Pownall and Koedijk (1999) who used data from Asian stock markets and compared VaR estimates generated by using the normal distribution and the RiskMetrics model[3] of Morgan (1996) with estimates generated by using EVT. They found that the EVT-based VaR significantly outperformed the other two models and they attributed this to the ability of EVT to fit fat-tailed time series. The superiority of EVT-based VaR estimates over methods based on the assumption that financial returns are normally distributed is also illustrated in Bali (2003). He examined whether EVT can lead to improved VaR estimates by using daily observations of the annualised yield of the 3-month, 6-month, 1-year and 10-year US treasury securities for the period 1954 to 1998. He clearly rejected the normality hypothesis and found that both the GEV and GP distributions could lead to very precise VaR results; however, EVT-based VaR estimates were on average 24% to 38% larger than those generated by the normal distribution. Based on this finding he argued that the multiplication factor that the BIS uses to multiply the VaR estimates of banks which use internal models is rather too high and that it should be reduced (also argued by Danielson *et al.*, 1998). In a more recent study, Danielson (2002) used US data to compare daily VaR estimates at the 99% confidence level derived from the VC, HS, GARCH, EWMA and EVT methods. He found that the EVT-based VaR provides more accurate VaR estimates than all the other models.

In the literature regarding EVT a number of similarities exist. First, in most studies the GEV and GP are the only distributions used to fit the extremes and secondly, the parameter estimation method is the maximum likelihood (ML). However, there are reasons to believe that there is scope for improvement and this is what the current paper attempts to do to some degree by employing EVT methods whose use in finance has not yet been fully investigated. Nonetheless, there are a limited number of studies that utilize a similar approach to that adopted in this paper. For example, Gettinby *et al.* (2004) employed EVT to investigate the distribution of extreme share returns in the UK from 1975 to 2000. In their analysis they included the generalized logistic (GL) distribution, in addition to the GEV and GP, and used the L-moments version of the probability weighted moments (PWM) method to obtain parameter estimates. They found that the GL distribution provides an adequate description of both the minima and maxima data. Another notable exception is the study by Da Silva and Mendes (2003) who used the PWM method to estimate the parameters of the limiting distribution of extremes in 10 Asian stock markets; both developing and developed. However, they focused solely on the GEV distribution, which was found to provide an adequate fit to these data. In the VaR analysis they conducted, they found EVT-based VaR estimates to be more accurate than estimates based on the normal distribution; however, they tended to be significantly more conservative. Recently, Tolikas and Brown (2006) considered the GL, GEV and GP distributions and they investigated the distribution of the extreme daily share returns in the Greek stock market. Their results granted further support to the ability of the GL distribution to fit extreme data. Using a VaR example, they illustrated that the GL distribution provides more accurate estimates compared to the GEV and normal distribution.

The first aim of this paper is to describe the distribution of the extreme minima of the daily returns of the German stock market. For that purpose, the GEV, GL and GP distributions are considered. L-moment diagrams are used to identify the distribution/s that best fit the empirical data, the parameters of these distributions are estimated using PWM and the Anderson–Darling test statistic is used to test the goodness of fit of the chosen distribution to the empirical data. The second aim of this paper is to assess whether this EVT approach can be useful for risk measurement purposes by deriving VaR estimates and comparing to those generated by traditional approaches. This paper is set out as follows. Section 2 introduces EVT and details the methodology adopted in this paper. Section 3 describes the data while Section 4 contains the results of the analysis of the extremes. In Section 5, VaR estimates generated by the EVT and traditional approaches are presented and compared. Finally, Section 6 summarizes and concludes the paper.

## 2. Extreme Value Theory

EVT is the statistical study of the extremal behaviour of random variables and its role is to develop procedures that are scientifically rational for describing and estimating their extreme behaviour. This paper follows Longin (1996) in defining extremes as the minimum (or the maximum) of the daily (or weekly, monthly or larger time periods) logarithmic returns of an asset over a given period (also known as the selection interval).[4] To illustrate this point, let us denote the time series of an index daily log-returns with the variable $Y_1,\ Y_2, \ldots, Y_n$. If the length of the selection interval is $m$, we divide the series into non-overlapping time intervals of length $m$. The time series of the extreme minima will be $X_1 = \min(Y_1, \ldots, Y_m),\ X_2 = \min(Y_{m+1}, \ldots, Y_{2m}), \ldots, X_{n/m} = \min(Y_{n-m}, \ldots, Y_n)$. The statistical problem is then to find a probability distribution that adequately describes the behaviour of $X_1, X_2, \ldots, X_{n/m}$.

Under the assumption that returns are independent and identically distributed (iid), Gnedenko (1943) showed that the limiting distribution of the extremes ought to be the GEV. Although the GEV enjoys theoretical support there is overwhelming evidence that financial returns exhibit heteroscedasticity and serial correlation. Kearns and Pagan (1997) used simulation techniques to show that the shape parameter estimates can be exaggerated when the iid assumption is violated. However, Leadbetter *et al.* (1983) showed that EVT is valid for data structures with weak dependence. In addition, Lux (2001) used the DAX index intraday returns for the period 1988 to 1995 and Jondeau and Rockinger (2003) used the DAX index daily returns for the period 1969 to 1998 to analyse the extremes of behaviour in the German stock market. They repeated their analyses by accounting for volatility patterns and autocorrelation in the time series of the returns and found that their results did not significantly change. Therefore, the iid assumption was relaxed.[5] It was also elected to consider the GL and other distributions, in addition to the GEV, accepting a trade off between being theoretically correct and empirically convincing.

### *2.1 Methodology*

Using EVT to estimate VaR involves a number of steps. First, the length of the minima selection period must be chosen. Secondly, distributions that are likely to model adequately the empirical extreme minima returns should be identified. Thirdly, the parameters of these distributions should be estimated and the goodness of fit of these distributions to the data should be tested to choose the one that best fits the empirical data. Finally, VaR estimates can be calculated as certain lower quantiles of the distribution of extremes. In the following paragraphs these steps are analytically presented.

The length $m$ of the selection intervals over which extremes are defined obviously determines the number of minima that are available for analysis. A longer interval will result in fewer data points leading to a lower level of efficiency when estimating the distribution's parameters. The choice is to some extent arbitrary and in order to investigate the sensitivity of parameter estimates to the interval length extremes identified over weekly, monthly and semi-annual time spans are used; these intervals are defined to be 5, 20 and 120 trading days long, respectively. In addition, the behaviour of the extremes distribution over time aggregation is studied by dividing (i) the series of weekly extremes into 10 and 30 sub-periods, (ii) the series of monthly extremes into 2, 4 and 10 sub-periods and (iii) the series of semi-annual extremes into 2 sub-periods.

Having collected the extremes the next step is to consider distributions which would adequately model the empirical data. This involves choosing distributions that would be able to reproduce the variability of the data; in other words to describe fat- tailed distributions. Deriving inspiration from work in flood frequency analysis it was decided to concentrate on the GEV, GL and GP distributions (details can be found in the Appendix); a set of three-parameter distributions. The parameters are usually referred to as location ($\alpha$), scale ($\beta$) and shape ($\kappa$). The first parameter is analogous to the mean; higher values imply larger extremes, in absolute terms. The second is analogous to the standard deviation and high values imply that the distribution of extremes is widely dispersed. The third governs the shape of the distribution and it is probably the most important parameter since larger values, in absolute terms, correspond to fatter-tailed distributions and imply a higher preponderance of extreme observations. In a financial context, therefore, high shape parameter values could be interpreted as an indicator that large negative returns may arise or even as an increased probability of a crash in the near future.

Identification of the most appropriate distributions is accomplished with the aid of L-moments. These are linear combinations of ordered data and in an analogous manner to the conventional moments they provide a set of summary statistics for probability distributions. For any random variable $X$, which has a finite mean, the $r$th L-moment $\lambda_r$ is defined as (Hosking, 1990)

$$\lambda_r \equiv r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} EX_{(r-k:r)}, \quad r = 1, 2, \ldots \tag{1}$$

where $EX_{(r-k:r)}$ is the expectation of the $(r-k)$th extreme-order statistic. The first two such statistics ($\lambda_1$ and $\lambda_2$) are measures of location and scale. In addition, the first two L-moment ratios ($\tau_3$ and $\tau_4$) defined by $\tau_r = \lambda_r/\lambda_2$, for $r \geq 3$, are measures of skewness and kurtosis, respectively.[6] The identification of the distributions that fit the data well is aided by plotting the estimated L-moment ratios, $\tau_3$ and $\tau_4$, on an L-moment diagram and choosing the distribution whose L-skewness and L-kurtosis curve is closest to the plotted point. This is particularly useful since the suitability of many distributions can be assessed on just one diagram.[7] The main advantage of the L-moments is that being linear combinations of the ordered data they are more robust to the presence of outliers than conventional moments.[8]

Having selected a distribution the next step is to estimate its parameters using the method of PWM. This method involves estimating parameters by equating sample moments to those of the chosen distribution. A tractable definition of PWM is supplied by Hosking (1986). Suppose that $X$ is a random variable with a finite mean and a distribution function $F$. Then:

$$\alpha_r = E[X\{1 - F(X)\}^r], \quad r = 0, 1, \ldots \tag{2}$$

where $E[X\{1 - F(X)\}]$ is the expectation of the quantile function of the random variable $[1 - F(X)]$. For small samples, the PWM method is considered to be more efficient than the ML, a particularly important attribute since by definition extremes are rare; even long observational periods can provide relatively few data points if, for example, annual minima are modelled.[9]

Although, PWM may be sensitive to outliers, Hosking (1986) demonstrated that there exist linear relationships between the PWM and the more robust L-moments, as exemplified for the first four L-moments in the following equations:

$$\lambda_1 = EX = \alpha_0 \tag{3}$$

$$\lambda_2 = \frac{1}{2}E(X_{2:2} - X_{1:2}) = \alpha_0 - 2\alpha_1 \tag{4}$$

$$\lambda_3 = \frac{1}{3}E(X_{3:3}2X_{2:3} + X_{1:3}) = \alpha_0 - 6\alpha_1 + 6\alpha_2 \tag{5}$$

$$\lambda_4 = \frac{1}{4}E(X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}) = \alpha_0 - 12\alpha_1 + 30\alpha_2 - 20\alpha_3 \tag{6}$$

After fitting a distribution, it is important to assess how good the fit is. Anderson and Darling (1954) defined a class of test statistics of the form:

$$\int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \phi(x) dF(x) \tag{7}$$

where, $F_n(x)$ is the empirical distribution function of a random variable $X$, $F(x)$ is the cumulative distribution function of $X$, $n$ is the number of observations and $\phi(x)$ is a weight function. When $\phi(x) = [F(x)(1 - F(x))]^{-1}$ the test is known as the AD test and it mainly focuses on measuring discrepancies in both tails; since if either $F(x)$ or $[1 - F(x)]$ is small $\phi(x)$ is large. Stephens (1976) and D'Agostino and Stephens (1986) have demonstrated that in small samples the AD test is the most powerful among a wide set of available tests.

Finally, once the empirical distribution of extremes has been adequately modelled, VaR estimates for the daily returns distribution can be derived from estimates of lower quantiles of the extremes distribution. Recalling that VaR is a quantile of the returns distribution, certain quantiles of the distribution of extremes can be considered as VaR estimates. However, as extremes are collected over non-overlapping periods of a certain length, the confidence level (or the probability of a return not exceeding VaR) needs to be adjusted to convert the EVT-based VaR to a VaR figure that corresponds to the returns distribution of the desired frequency. For the sake of argument assume that the frequency of returns is daily and denote $p_{ext}$ the probability that an extreme return, collected over a period of $T$ daily returns, will not exceed VaR, and $p$ the probability that a daily return will not exceed VaR. Then the probability that a daily return will not exceed the VaR is $p_{ext} = 1 - (1 - p)^T$. The estimated parameters of the extremes distribution can then be used to estimate daily VaR figures at different confidence levels by using the quantile function equation of a distribution.

VaR models can only be useful as far as their forecasts are sufficiently accurate and this is why any VaR model should be validated. Backtesting is the technique of systematically comparing the VaR forecasts with the actual returns by using historical data. Thus, the number of times that the VaR forecasts are violated by the actual returns can be counted and this serves as an indication of how well calibrated a VaR model is. The basic idea is that if a model is perfectly specified then the number of reported violations over a time period should be in line with the confidence level.[10] This idea is central in the BIS recommendations since, in the absence of a VaR model validation method, financial institutions might have an incentive to underestimate the market risk they face, thereby assigning too little capital as a CAR. The statistical test we employ is the test proposed by Christoffersen (1998) which assesses whether the VaR forecasts overestimate or underestimate risk and in addition whether the VaR violations occur in clusters.

## 3. Data Description

The dataset consists of 7257 daily logarithmic returns collected from Datastream[11] and covers the 29 year period from 2 January 1973 to 28 December 2001. Table 1 contains descriptive statistics of the DAX-DS index daily returns for the whole period as well as for the two sub-periods 1973 to 1987.5 and 1987.5 to 2001. The time series of the DAX-DS index had a daily mean return of 0.03% and a daily standard deviation of 0.98% while the minimum daily return was −12.14% and the maximum 5.91%. Surprisingly, the minimum daily return did not occur during the international stock markets collapse in 1987 but 3 weeks before the fall of the Berlin wall on 9 November 1989. According to Schnusenberg (2000), the euphoria prevailing at the time of German unification due to the prospective business opportunities, initially affected the market in a positive manner. However, the political risk involved due to any potential intervention of the Soviet Union after unification, negatively affected German and American shareholders, driving the stock market downwards. Additionally, it is also interesting to note that although the UK and US markets fell by more than 20% on 19 October 1987, the German stock market fell by only 6.84%. This is probably because (i) the German market is considered to be more 'domestic' than the UK and US markets, (ii) German investors traditionally prefer bonds to equities, and (iii) structural and operational differences (Pike *et al.*, 1991).

The DAX-DS daily returns exhibited a negative skewness of −0.769 and a kurtosis of 9.121. These values imply a great preponderance of large negative daily returns and indicate deviations from normality; also confirmed by the Shapiro–Wilk test statistic. It is also noteworthy that the

**Table 1.** Descriptive statistics for DAX-DS daily returns

| | *n* | *m* | Mean (%) | St.Dev (%) | Min (%) | Max (%) | Skewness | Kurtosis | *SW* |
|---|---|---|---|---|---|---|---|---|---|
| 1973–2001 | | | | | | | | | |
| Daily | 7257 | | 0.03 | 0.98 | −12.14 | 5.91 | −0.769* | 9.121* | 0.964* |
| *Minima* | | | | | | | | | |
| Weekly | 1451 | 5 | −0.96 | 0.96 | −12.14 | 1.11 | −3.392* | 22.783* | 0.869* |
| Monthly | 363 | 20 | −1.69 | 1.27 | −12.14 | −0.36 | −3.370* | 18.340* | 0.843* |
| Semi-annual | 61 | 120 | −3.03 | 2.04 | −12.14 | −1.01 | −2.358* | 6.958* | 0.867* |
| 1973–1987.5 | | | | | | | | | |
| Daily | 3616 | | 0.02 | 0.77 | −5.06 | 4.49 | −0.258* | 2.572* | 0.987* |
| *Minima* | | | | | | | | | |
| Weekly | 723 | 5 | −0.79 | 0.66 | −5.06 | 0.69 | −1.690* | 4.759* | 0.938* |
| Monthly | 181 | 20 | −1.38 | 0.76 | −5.06 | −0.36 | −1.603* | 3.275* | 0.929* |
| Semi-annual | 30 | 120 | −2.20 | 0.95 | −5.06 | −1.01 | −1.165* | 1.288 | 0.946* |
| 1987.5–2001 | | | | | | | | | |
| Daily | 3641 | | 0.03 | 1.15 | −12.14 | 5.91 | −0.871* | 8.567* | 0.962* |
| *Minima* | | | | | | | | | |
| Weekly | 728 | 5 | −1.13 | 1.16 | −12.14 | 1.11 | −3.223* | 18.424* | 0.864* |
| Monthly | 182 | 20 | −2.00 | 1.56 | −12.14 | −0.50 | −2.977* | 12.855* | 0.848* |
| Semi-annual | 31 | 120 | −3.83 | 2.46 | −12.14 | −1.28 | −1.805* | 3.511* | 0.898* |

This table includes descriptive statistics for the DAX-DS index daily returns weekly, monthly and semi-annual minima over the period 1973 to 2001 and the two sub-periods 1973 to 1987.5 and 1987.5 to 2001. *n* denotes the number of observations, *m* denoted the length of the extremes selection interval, St.Dev denotes the standard deviation of returns, the minimum and maximum daily returns are indicated as Min and Max and *SW* indicates the Shapiro–Wilk test which examines the hypothesis that the daily returns and the daily returns minima are normally distributed.
*indicates statistical significance at the 5% level.

**Table 2.** Frequency of the DAX-DS index large negative daily returns

| Threshold | $< \mu - 2\sigma$ | $< \mu - 3\sigma$ | $< \mu - 4\sigma$ |
|---|---|---|---|
| 1973–2001 | | | |
| Total | 188 | 63 | 26 |
| Expected on assumption of normality | 165 | 10 | 0 |
| In cluster | 132 | 35 | 15 |
| 1973–1987.5 | | | |
| Total | 97 | 30 | 8 |
| Expected on assumption of normality | 82 | 5 | 0 |
| In cluster | 72 | 13 | 0 |
| 1987.5–2001 | | | |
| Total | 97 | 33 | 15 |
| Expected on assumption of normality | 83 | 5 | 0 |
| In cluster | 69 | 21 | 8 |

This table includes the total number of daily returns which exceed four thresholds defined as the mean ($\mu$) minus two, three and four standard deviations ($\sigma$). It also includes the expected frequency under the assumption that daily returns are normally distributed. Additionally, the row named *In cluster* includes the number of daily returns followed or preceded by another daily return within a time period of two trading weeks which also exceeded the corresponding threshold.



second sub-period was significantly more volatile than the first one. Additionally, the minimum daily return was larger, in absolute terms, as was the kurtosis value. Not surprisingly, normality was also rejected for both sub-periods. In Table 2 we focus more on the left tail of the distribution of the DAX-DS index daily returns by examining the frequency of the daily returns which exceeded three thresholds defined as $\mu - 2\sigma$, $\mu - 3\sigma$ and $\mu - 4\sigma$, where $\mu$ is the overall daily mean and $\sigma$ is the overall daily standard deviation for the whole period and the two sub-periods. It can be noticed that the number of total daily returns which exceeded the three thresholds were always larger than the numbers expected under the assumption of normality.

What should also be important for investors is whether these excess returns tended to cluster over time. This was examined by considering the frequency of excess returns when these were followed or preceded by another excess return within a time period of two trading weeks. Naturally, clustering was more severe for the case of daily returns which were lower than $\mu - 2\sigma$. It can also be noted that large negative daily returns tended to cluster more in the second sub-period than in the first for the two thresholds $\mu - \sigma$ and $\mu - 4\sigma$. The results indicated that investors assuming a normal distribution would tend to underestimate the risk involved when investing in the German stock market. The consequences could be severe for the value of a portfolio if we consider that returns lower than $\mu - 3\sigma$ or $\mu - 4\sigma$ are likely to be very large.[12]

In general, the descriptive statistics indicate that the distribution of daily returns was different in the two sub-periods examined. This comes as no surprise since the second sub-period contains significant international and domestic economic and political events, which affected the German stock market. For example, the stock markets crash[13] in 1987, the stressed market conditions due to the political uncertainty during and before the German unification[14] in 1989, the Gulf crisis[15] in 1990, the coup against the Soviet leader Michael Gorbachev, which severely affected the German stock market because of the fears that German banks were greatly exposed to risk due to the huge amounts they had lent to the Soviet Union[16] and finally, the period 1997 to 2001 which contains a number of international financial crises (Asian, Latin American and Russian)[17] and the effects of the terrorist attack on the USA in 2001, which negatively affected all stock markets.[18]

## 4. Analysis of the Extremes in the German Stock Market

Weekly, monthly and semi-annual minima were collected over the 29 year period under examination. Descriptive statistics for the minima of different selection periods are presented in Table 1 where it becomes apparent that as larger selection periods were chosen, the mean minimum return tended to increase, in absolute terms. It can also be noticed that the magnitude of the minima was larger in the second sub-period than in the first one, for all selection intervals. Normality was also rejected for all selection intervals and sub-periods. Subsection 4.1 describes the identification of the appropriate distribution/s and subsection 4.2 details the estimation of parameters and the goodness of fit test.

### *4.1 Identifying the Distribution of the Extreme Minimum Daily Returns*

The signed and squared L-skewness (*sign* ($\tau_3^2$)) and L-kurtosis ($\tau_4$) were calculated for each sub-period and plotted on an L-moments diagram.[19] In the same diagram the relationship between the theoretical signed and squared L-skewness and L-kurtosis of different distributions were also plotted. Figure 1 contains the $\tau_3^2$ and $\tau_4$ for the series of the DAX-DS weekly minima divided into 30 sub-periods. From a first look, it seems that one could exclude all the distributions except the GL and the GEV. This is because the points of the ${\tau_3}^2$ and $\tau_4$, for each of the 30 sub-periods of the weekly minima series, are mainly dispersed around the theoretical curves of the GL and the GEV distributions. The corresponding L-moment plots were also generated for different sub-divisions of the weekly, monthly and semi-annual series of minima and similar patterns appeared in all of these diagrams.[20] Although, the L-moment diagrams do not provide any additional support in favour of either the GL or the GEV, the visual evidence suggests that the analysis should focus only on these two distributions. However, in order to choose between the GL and the GEV, further analysis is required and a more formal test of goodness of fit of these two distributions should be applied.

### *4.2 Parameter Estimates and Goodness of Fit Test*

The whole sample and the different sub-periods of the weekly minima were fitted by the GL and GEV distributions and the parameters were estimated by the PWM method for all of these cases. Parameter estimates for both distributions and the AD goodness of fit test $p$-values are reported in Table 3. When the whole sample was fitted by the GL and GEV distributions the AD $p$-values were low indicating that both distributions cannot provide an adequately fit. This is probably because the extremes distribution has changed over this 29 year period; thus, the data were generated by a mixture of distributions and a single distribution was not likely to provide a good fit. However, when the whole sample was divided into sub-periods both the GL and GEV distributions improved their ability to provide an adequate description. For example when ten sub-periods were used, both distributions fitted the weekly minima adequately in seven of them; however, the GL fitted the minima better than the GEV distribution in the seven sub-periods. When 30 sub-periods were used GL provided an adequate fit in 27 sub-periods and the GEV in 26 sub-periods. The GL fitted better in 22 sub-periods and the GEV in only eight of them. The AD

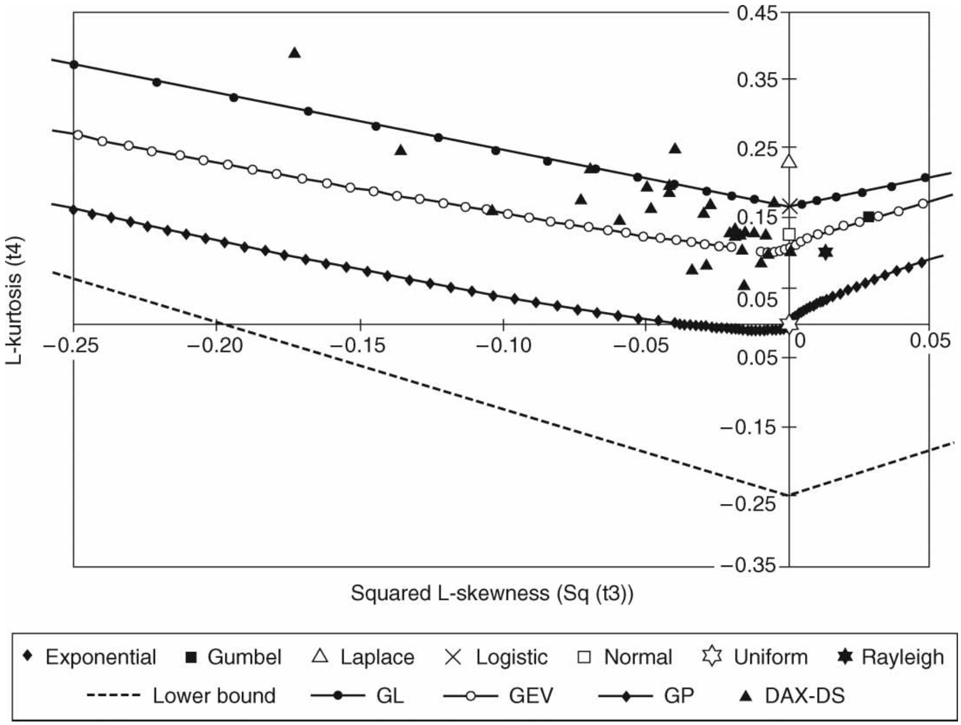


**Figure 1.** L-moments ratios diagram for the DAX-DS weekly minima. *Notes*: This diagram illustrates the L-moments ratios points for the DAX-DS index daily returns weekly minima, divided into 30 sub-periods, over the period 1973 to 2001. The plots of the signed squared L-skewness and L-kurtosis are mainly concentrated around the theoretical curves of the GL and the GEV distributions indicating that these two distributions are likely to fit adequately the empirical data

$p$-values for the GL distribution ranged from a low of 0.001 to a high of 0.935 and for the GEV distribution from a low of 0.000 to a high of 0.771.

In terms of the shape parameter estimates, it is interesting to note that the shape parameter for the GEV distribution sometimes takes a positive value; a positive value implies the Weibull distribution while a negative one implies the Fréchet. This finding is at variance with previous studies of developed stock markets (Longin, 1996 in the USA and Gettinby *et al.*, 2004 in the UK) where there were no sign changes in the shape parameter of the fitted GEV distribution. Figure 2 illustrates the behaviour over time of the shape parameter for the GEV and GL distributions; the most noticeable feature being the change from relative stability in the first half to a situation of greater variability in the second half. Clearly, one could talk about two different regimes in terms of the extremes behaviour where in the second one the behaviour of the shape parameter for both distributions implies that the probability of a large negative daily return or even a stock market crash was higher than in the first one. It is also interesting to note the impact of the global turbulence in stock markets at the end of 1987 where the shape value for the GEV changed from 0.050 to −0.224 and for the GL from 0.138 to 0.322 (periods 15 and 16), the impact of the negative market sentiment in Germany prior to the German unification where the shape value changed from 0.324 to −0.350 for the GEV and from −0.220 to 0.416 for the GL (periods 17 and 18) and the impact of the coup against the Soviet leader, which led to the maximum shape values for both the

**Table 3.** Weekly minima GEV and GL PWM parameter estimates and AD $p$-values

| | | GEV estimates | | | | GL estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sub-periods ($s$) | $N$ | $\beta_s$ | $\alpha_s$ | $\kappa_s$ | $p$-value | $\beta_s$ | $\alpha_s$ | $\kappa_s$ | $p$-value | Better fit |
| $s = 1$ | | | | | | | | | | |
| 1. (2/1/73–28/12/01) | 1451 | 0.005 | 0.005 | −0.205 | 0.000 | −0.007 | 0.004 | 0.308 | 0.021 | GL |
| $s = 10$ | | | | | | | | | | |
| 1. (02/01/73–26/11/75) | 145 | 0.006 | 0.006 | 0.038 | 0.702 | −0.009 | 0.004 | 0.146 | 0.914 | GL |
| 2. (27/11/75–18/10/78) | 145 | 0.004 | 0.003 | 0.053 | 0.745 | −0.005 | 0.002 | 0.136 | 0.756 | GL |
| 3. (19/10/78–17/09/81) | 145 | 0.005 | 0.003 | 0.065 | 0.557 | −0.006 | 0.002 | 0.129 | 0.805 | GL |
| 4. (18/09/81–09/08/84) | 145 | 0.005 | 0.004 | −0.014 | 0.686 | −0.007 | 0.003 | 0.179 | 0.757 | GL |
| 5. (10/08/84–13/07/87) | 145 | 0.007 | 0.007 | −0.081 | 0.232 | −0.009 | 0.005 | 0.223 | 0.069 | GEV |
| 6. (14/07/87–01/06/90) | 145 | 0.005 | 0.006 | −0.342 | 0.000 | −0.008 | 0.005 | 0.410 | 0.013 | GL |
| 7. (05/06/90–30/04/93) | 145 | 0.005 | 0.005 | −0.290 | 0.002 | −0.007 | 0.004 | 0.371 | 0.039 | GL |
| 8. (03/05/93–14/03/96) | 145 | 0.005 | 0.004 | −0.055 | 0.432 | −0.007 | 0.003 | 0.206 | 0.258 | GEV |
| 9. (15/03/96–10/02/99) | 145 | 0.007 | 0.007 | −0.190 | 0.011 | −0.010 | 0.005 | 0.298 | 0.034 | GL |
| 10. (11/02/99–28/12/01) | 146 | 0.010 | 0.008 | −0.016 | 0.526 | −0.013 | 0.005 | 0.180 | 0.379 | GEV |
| $s = 30$ | | | | | | | | | | |
| 1. (02/01/73–12/12/73) | 48 | 0.008 | 0.006 | 0.070 | 0.764 | −0.010 | 0.004 | 0.126 | 0.904 | GL |
| 2. (13/12/73–29/11/74) | 48 | 0.007 | 0.006 | 0.111 | 0.694 | −0.009 | 0.003 | 0.101 | 0.443 | GEV |
| 3. (02/12/74–18/11/75) | 48 | 0.004 | 0.005 | −0.047 | 0.062 | −0.006 | 0.004 | 0.200 | 0.551 | GL |
| 4. (20/11/75–03/11/76) | 48 | 0.006 | 0.003 | 0.093 | 0.771 | −0.007 | 0.002 | 0.111 | 0.915 | GL |
| 5. (04/11/76–18/10/77) | 48 | 0.003 | 0.003 | 0.004 | 0.744 | −0.005 | 0.002 | 0.167 | 0.935 | GL |
| 6. (19/10/77–04/10/78) | 48 | 0.003 | 0.003 | 0.125 | 0.738 | −0.004 | 0.002 | 0.092 | 0.818 | GL |
| 7. (05/10/78–19/09/79) | 48 | 0.004 | 0.003 | 0.159 | 0.070 | −0.006 | 0.002 | 0.072 | 0.379 | GL |
| 8. (20/09/79–08/09/80) | 48 | 0.005 | 0.004 | 0.064 | 0.701 | −0.006 | 0.002 | 0.130 | 0.547 | GEV |
| 9. (09/09/80–27/08/81) | 48 | 0.004 | 0.003 | −0.004 | 0.117 | −0.006 | 0.002 | 0.172 | 0.324 | GL |
| 10. (28/08/81–13/08/82) | 48 | 0.005 | 0.003 | 0.042 | 0.758 | −0.006 | 0.002 | 0.143 | 0.866 | GL |
| 11. (16/08/82–28/07/83) | 48 | 0.005 | 0.005 | −0.053 | 0.391 | −0.007 | 0.003 | 0.205 | 0.799 | GL |
| 12. (29/07/83–11/07/84) | 48 | 0.005 | 0.004 | 0.051 | 0.682 | −0.007 | 0.002 | 0.138 | 0.664 | GEV |
| 13. (12/07/84–01/07/85) | 48 | 0.004 | 0.004 | 0.059 | 0.439 | −0.006 | 0.002 | 0.132 | 0.508 | GL |
| 14. (02/07/85–19/06/86) | 48 | 0.008 | 0.008 | −0.022 | 0.201 | −0.011 | 0.005 | 0.184 | 0.112 | GEV |

(*continued*)

**Table 3.** Continued

| | GEV estimates | | | | | GL estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sub-periods ($s$) | $N$ | $\beta_s$ | $\alpha_s$ | $\kappa_s$ | $p$-value | $\beta_s$ | $\alpha_s$ | $\kappa_s$ | $p$-value | Better fit |
| 15. (20/06/86–04/06/87) | 48 | 0.009 | 0.008 | 0.050 | 0.767 | −0.013 | 0.005 | 0.138 | 0.886 | GL |
| 16. (05/06/87–19/05/88) | 48 | 0.009 | 0.010 | −0.224 | 0.229 | −0.013 | 0.007 | 0.322 | 0.203 | GEV |
| 17. (20/05/88–03/05/89) | 48 | 0.005 | 0.004 | 0.324 | 0.248 | −0.007 | 0.002 | −0.022 | 0.267 | GL |
| 18. (05/05/89–18/04/90) | 48 | 0.004 | 0.005 | −0.350 | 0.002 | −0.007 | 0.004 | 0.416 | 0.029 | GL |
| 19. (19/04/90–08/04/91) | 48 | 0.009 | 0.008 | −0.076 | 0.771 | −0.012 | 0.005 | 0.220 | 0.910 | GL |
| 20. (09/04/91–24/03/92) | 48 | 0.004 | 0.003 | −0.442 | 0.000 | −0.005 | 0.002 | 0.487 | 0.001 | GL |
| 21. (25/03/92–10/03/93) | 48 | 0.004 | 0.004 | −0.054 | 0.272 | −0.006 | 0.003 | 0.205 | 0.620 | GL |
| 22. (11/03/93–21/02/94) | 48 | 0.004 | 0.005 | −0.001 | 0.170 | −0.006 | 0.003 | 0.170 | 0.120 | GEV |
| 23. (22/02/94–02/02/95) | 48 | 0.007 | 0.004 | −0.082 | 0.257 | −0.008 | 0.003 | 0.224 | 0.643 | GL |
| 24. (03/02/95–08/01/96) | 48 | 0.004 | 0.004 | −0.150 | 0.732 | −0.006 | 0.003 | 0.270 | 0.818 | GL |
| 25. (19/01/96–06/01/97) | 48 | 0.005 | 0.003 | −0.288 | 0.303 | −0.006 | 0.002 | 0.369 | 0.466 | GL |
| 26. (07/01/97–17/12/97) | 48 | 0.007 | 0.008 | −0.082 | 0.157 | −0.011 | 0.006 | 0.224 | 0.307 | GL |
| 27. (18/12/97–03/12/98) | 48 | 0.010 | 0.010 | −0.111 | 0.032 | −0.014 | 0.007 | 0.243 | 0.094 | GL |
| 28. (04/12/98–17/11/99) | 48 | 0.009 | 0.008 | 0.130 | 0.507 | −0.012 | 0.005 | 0.089 | 0.395 | GEV |
| 29. (18/11/99–26/10/00) | 48 | 0.010 | 0.008 | 0.069 | 0.103 | −0.013 | 0.005 | 0.126 | 0.047 | GEV |
| 30. (27/10/00–28/12/01) | 59 | 0.011 | 0.008 | −0.141 | 0.526 | −0.015 | 0.006 | 0.264 | 0.903 | GL |

This table includes the PWM parameter estimates and the Anderson–Darling goodness of fit test $p$-values for the GEV fitted to the reverse weekly minima and for the GL fitted to the weekly minima over the period 1986 to 2001. The GEV distribution is fitted to the reverse weekly minima since results that hold for a random variable $X_s$ generated by the GEV distribution can be extended for the reverse variable $-X_s$ (see Longin, 1996). $N$ denotes the number of extreme observations in each sub−period (this is equal to $m/s$, where $m$ is the total number of weekly extremes and $s$ is the number of sub−periods into which the series is divided), and $\beta_s$, $\alpha_s$ and $\kappa_s$ denote the location, scale and shape parameters, respectively.

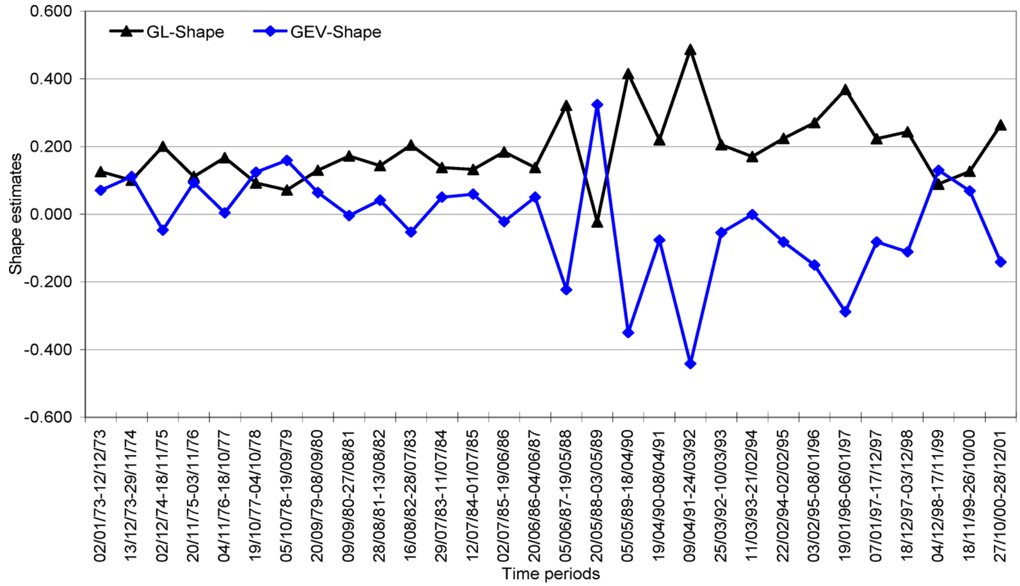


**Figure 2.** Shape parameter estimates for the GEV and GL distributions for the case of weekly minima divided into 30 sub-periods. *Notes*: This diagram illustrates the behaviour of the shape parameter for both the GEV and GL distributions when these are fitted to the series of weekly extremes divided into 30 sub-periods.

GEV (−0.442) and GL (0.487) distributions. The volatile behaviour of the shape parameter for both the GEV and GL distributions during the second part of the whole period is expected to have a significant effect upon VaR estimates. One would expect VaR estimates to be higher when the shape parameter values were higher. This would naturally lead to larger capital adequacy reserves for financial firms if they were to be protected against large negative movements in the market place.

The behaviour of the extremes under time aggregation was then examined by fitting the whole series as well as the series of sub-periods of the monthly and semi-annual extremes. Results of the monthly minima fitted by both GL and GEV distributions are presented in Table 4. When the monthly extremes of the whole period were fitted the GEV provided an adequate fit with a $p$-value of 0.148, while the fit that the GL provided was not acceptable. However, when sub-periods of the monthly minima were fitted, the GL improved its ability to fit the extremes; for example, in the case of two sub-periods, the GEV fitted better than the GL only in the first one. In the case of four sub-periods, the GL fitted the monthly extremes adequately in all sub-periods and better than the GEV in three of them. On the other hand, the GEV failed to fit the second sub-period adequately. Additionally, when the monthly minima were divided into 10 sub-periods the GL fitted all of them adequately with $p$-values ranked from a low of 0.065 to a high of 0.935. Although, the GL and GEV each fitted five of the ten sub-periods better than the other, it is the GEV that fails to fit adequately in two of them. In terms of the shape parameter values, the sign for the case of the GEV shifts from positive in sub-periods which contain fewer large negative daily returns to negative in sub-periods which contain larger extremes.

Again, one can see the impact of the stock markets crash in 1987 and the negative sentiment in market before German unification. For example, in sub-period 6 the shape parameters for both the GEV and GL take their maximum, in absolute terms, values of −0.526 and 0.556, respectively.

**Table 4.** Monthly minima GEV and GL PWM parameter estimates and AD $p$-values

| | | GEV estimates | | | | GL estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sub-periods ($s$) | N | $\beta_s$ | $\alpha_s$ | $\kappa_s$ | $p$-value | $\beta_s$ | $\alpha_s$ | $\kappa_s$ | $p$-value | Better fit |
| $s = 1$ | | | | | | | | | | |
| 1. (02/01/73–28/12/01) | 363 | 0.011 | 0.006 | −0.281 | 0.148 | −0.014 | 0.005 | 0.364 | 0.038 | GEV |
| $s = 2$ | | | | | | | | | | |
| 1. (02/01/73–03/08/87) | 182 | 0.010 | 0.005 | −0.166 | 0.337 | −0.012 | 0.003 | 0.281 | 0.183 | GEV |
| 2. (04/08/87–28/12/01) | 181 | 0.013 | 0.007 | −0.286 | 0.031 | −0.016 | 0.006 | 0.367 | 0.045 | GL |
| $s = 4$ | | | | | | | | | | |
| 1. (02/01/73–14/04/80) | 91 | 0.009 | 0.004 | −0.145 | 0.695 | −0.011 | 0.003 | 0.266 | 0.333 | GEV |
| 2. (15/04/80–03/08/87) | 91 | 0.011 | 0.005 | −0.158 | 0.028 | −0.013 | 0.004 | 0.275 | 0.068 | GL |
| 3. (04/08/87–08/11/94) | 91 | 0.011 | 0.006 | −0.432 | 0.452 | −0.014 | 0.005 | 0.480 | 0.517 | GL |
| 4. (09/11/94–28/12/01) | 90 | 0.015 | 0.009 | −0.103 | 0.157 | −0.019 | 0.006 | 0.238 | 0.159 | GL |
| $s = 10$ | | | | | | | | | | |
| 1. (02/01/73–18/11/75) | 36 | 0.012 | 0.005 | −0.079 | 0.605 | −0.014 | 0.004 | 0.222 | 0.433 | GEV |
| 2. (20/11/75–04/10/78) | 36 | 0.008 | 0.003 | −0.038 | 0.659 | −0.009 | 0.002 | 0.195 | 0.495 | GEV |
| 3. (05/10/78–27/08/81) | 36 | 0.009 | 0.003 | 0.070 | 0.588 | −0.010 | 0.002 | 0.126 | 0.366 | GEV |
| 4. (28/08/81–11/07/84) | 36 | 0.011 | 0.004 | −0.072 | 0.007 | −0.012 | 0.003 | 0.217 | 0.161 | GL |
| 5. (12/07/84–04/06/87) | 36 | 0.015 | 0.008 | 0.006 | 0.328 | −0.018 | 0.005 | 0.166 | 0.177 | GEV |
| 6. (05/06/87–18/04/90) | 36 | 0.012 | 0.006 | −0.526 | 0.209 | −0.015 | 0.005 | 0.556 | 0.288 | GL |
| 7. (19/04/90–10/03/93) | 36 | 0.011 | 0.006 | −0.383 | 0.034 | −0.014 | 0.005 | 0.441 | 0.065 | GL |
| 8. (11/03/93–18/01/96) | 36 | 0.011 | 0.005 | 0.022 | 0.163 | −0.013 | 0.003 | 0.156 | 0.105 | GEV |
| 9. (19/01/96–03/12/98) | 36 | 0.015 | 0.010 | −0.127 | 0.051 | −0.019 | 0.007 | 0.254 | 0.062 | GL |
| 10. (04/12/98–28/12/01) | 39 | 0.019 | 0.007 | −0.149 | 0.617 | −0.022 | 0.005 | 0.270 | 0.935 | GL |

This table includes the PWM parameter estimates and the Anderson-Darling goodness of fit test $p$-values for the GEV fitted to the reverse monthly minima (see Table 3 note) and the GL fitted to the monthly minima over the period 1973 to 2001. $N$ denotes the number of extreme observations in each sub-period (this is equal to $m/s$, where $m$ is the total number of monthly extremes and $s$ is the number of sub-periods into which the series is divided) and $\beta_s$, $\alpha_s$ and $\kappa_s$ denote the location, scale and shape parameters, respectively.

**Table 5.** Semi-annual minima GEV and GL PWM parameter estimates and AD $p$-values

| | GEV estimates | | | | | GL estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sub-periods ($s$) | N | $\beta_n$ | $\alpha_n$ | $\kappa_n$ | $p$-value | $\beta_n$ | $\alpha_n$ | $\kappa_n$ | $p$-value | Better fit |
| $s = 1$ | | | | | | | | | | |
| 1. (02/01/73–28/12/01) | 61 | 0.021 | 0.010 | −0.291 | 0.309 | −0.025 | 0.008 | 0.371 | 0.324 | GL |
| $s = 2$ | | | | | | | | | | |
| 1. (02/01/73–24/11/87) | 31 | 0.018 | 0.007 | −0.229 | 0.259 | −0.020 | 0.005 | 0.326 | 0.245 | GEV |
| 2. (25/11/87–28/12/01) | 30 | 0.026 | 0.012 | −0.285 | 0.611 | −0.031 | 0.009 | 0.367 | 0.730 | GL |

This table includes the PWM parameter estimates and the Anderson-Darling goodness of fit test $p$-values for the GEV fitted to the reverse semi-annual minima (see Table 3 note) and the GL fitted to the semi-annual minima over the period 1973 to 2001. $N$ denotes the number of extreme observations in each sub-period (this is equal to $m/s$, where $m$ is the total number of semi-annual extremes and $s$ is the number of sub-periods into which the series is divided) and $\beta_s$, $\alpha_s$ and $\kappa_s$ denote the location, scale and shape parameters, respectively.



Finally, when the semi-annual minima of the DAX-DS index daily returns were fitted, both the GEV and GL distributions provided an adequate fit in the case of the whole series and in the case of two sub-periods (Table 5). The GL distribution fitted the whole series better than the GEV, with a $p$-value of 0.324 and also fitted the second sub-period better with a rather high $p$-value of 0.730.

In summary, when compared with the GEV, the GL provides a better fit to the extreme minima of the DAX-DS index daily returns over the period 1973 to 2001 since it performs better in more sub-periods and across different selection intervals. However, it should also be noted that there were cases where the GEV outperformed the GL distribution. Additionally, it seems that the nature of extremes is changing over time since the behaviour of the shape parameter for both distributions changes across different sub-periods.[21] In particular, volatile sub-periods which contained large negative daily returns tended to result to higher shape values than sub-periods of low volatility which contained fewer and smaller negative daily returns.

## 5. Estimating and Comparing Value-at-Risk

In this section the usefulness of EVT methods for market risk measurement purposes is assessed by employing VaR analysis. In particular, VaR estimates for the daily returns distribution at different confidence levels were derived using lower quantiles of the distribution of extremes. For that purpose, both the GEV and GL distributions were considered and the estimated parameters of the weekly extremes distribution (static approach)[22] were used. In addition, inspired by the indication that the distribution of extremes is time variant, an attempt was made to incorporate the time-varying behaviour of the parameters in the VaR estimation[23] by using a moving window (MW). This has the advantage that the number of parameters to be estimated remains the same while not adding any further computational difficulties. Finally, for comparison reasons, VaR estimates generated by traditional methods such as the VC, HS and the MCS based on the normal distribution were also derived. For all these methods 250 historical daily returns were used for the calculation of parameters used in the VaR estimation while for the HS 1000 past daily returns were also used since it is impossible to derive VaR estimates at very high confidence levels by using only 250 past daily returns. In order to examine the performance of each approach the results were backtested over the two time periods from 2/1/87 to 30/12/91 and from 2/1/97 to

28/12/01; these periods contain some of the largest negative daily returns in the German stock market.[24]

The results are summarized in Table 6. Focusing first on the time period 2/1/87 to 2/1/92, the rather poor results of the VC method came as no surprise. Although at lower confidence levels this method overestimated risk, as one moved further into the tails of the empirical distribution risk was systematically underestimated. For example, at the 99.75% and 99.90% confidence levels, one would expect to have three and one VaR violations, respectively, however the best forecasts resulted in 13 and 11, respectively. These 10 unexpected violations would have had a large impact on the value of a portfolio since they were large negative daily returns.[25] The HS method performed better especially at high confidence levels. However, the forecasting accuracy in the lower tail occurred when 1000 past periods were used to model the empirical distribution and this was at the expense of accuracy at lower confidence levels. The performance of the MCS method is, in general, disappointing with the exception of the 95% confidence level where it is just acceptable. The inability of the traditional models to capture the extreme negative daily returns is rather serious because the unexpected violations at high confidence levels could have a large impact on the performance of a portfolio.

Similarly, the performance of the VaR estimates based on EVT when the static approach is used is very poor with the exception of the GL distribution at the 99.90% confidence level. The likely reason is that during this volatile period the parameters of the extremes distributions were changing frequently over time and thus, they could not rigorously reflect current market conditions. On the other hand, the use of a MW seems to have the potential to mitigate this problem to some degree. Indeed, it can be noticed that the EVT approach provides results that are very accurate from the 99.50% confidence level and higher. For example, at the 99.50%, 99.75% and 99.90% confidence level, one would expect to have six, three and one VaR violations, respectively, from a perfectly accurate model. The use of the static approach and the GL distribution fitted to the weekly minima divided into 30 sub-periods resulted in 19, 13 and two violations, respectively, while the use of a MW of 50 weekly minima resulted in nine, four and one violations, respectively. In addition, the GL distribution performed better than the GEV and this was because the GL is a fatter tailed distribution and therefore, it can accommodate larger extremes better than the GEV.

When the period 2/1/97 to 28/12/01 was considered, the results found to be qualitatively similar leading to analogous conclusions. It came as no surprise that the VC method significantly underestimated the tail risk during this period. In particular, at the 99.75% confidence level, one expected three VaR violations but the observed number[26] was 17. The HS method using 250 past daily returns appeared to become more accurate as one moved from lower to higher confidence levels. However, when more past data was used to estimate VaR at high confidence levels, accuracy decreased. The performance of the MCS method was entirely unsatisfactory since at all confidence levels there were a large number of unexpected VaR violations. The performance of the EVT-based VaR using the static approach was very disappointing for both the GEV and GL distributions. The use of a MW considerably improved the results especially deep into the tails of the empirical distribution where both distributions generated the most accurate results of all methods examined. It can also be noted that at the 99.90% confidence level the VaR forecasts based on a MW using the GEV distribution result in a higher $p$-value than when the GL is used. This can be a little deceptive if one considers that the two daily returns that the GEV fails to capture are of size −7.20% (28/10/97, Asian crisis) and −7.21% (11/9/01, terrorist attack on the USA). In this case it would be more prudent to prefer the forecasts of the GL distribution since it provides better protection against such extreme movements.

**Table 6.** Daily VaR backtesting results at various confidence levels

| Confidence level | 90.00% | 95.00% | 97.50% | 99.00% | 99.50% | 99.75% | 99.90% |
|---|---|---|---|---|---|---|---|
| Panel A | | | | | | | |
| Period: 2/1/87-2/1/92 | | | | | | | |
| Number of daily returns: 1248 | | | | | | | |
| Expected violations | 125 | 62 | 31 | 12 | 6 | 3 | 1 |
| VC250 | 81 (0.000) | 51 (0.060) | 29 (0.088) | 21 (0.001) | 16 (0.000) | 13 (0.000) | 11 (0.000) |
| HS250 | 115 (0.000) | 60 (0.086) | 34 (0.181) | 12 (0.259) | 5 (0.858) | – | – |
| HS1000 | 142 (0.000) | 76 (0.000) | 33 (0.169) | 15 (0.002) | 11 (0.003) | 7 (0.015) | 3 (0.412) |
| MCS250 | 91 (0.000) | 54 (0.056) | 27 (0.048) | 19 (0.002) | 15 (0.000) | 13 (0.000) | 11 (0.000) |
| GL static | 149 (0.000) | 94 (0.000) | 54 (0.000) | 29 (0.000) | 19 (0.000) | 13 (0.000) | 2 (0.823) |
| GEV static | 150 (0.000) | 89 (0.000) | 51 (0.000) | 28 (0.000) | 22 (0.000) | 16 (0.000) | 8 (0.000) |
| GL MW | 145 (0.000) | 76 (0.001) | 35 (0.185) | 15 (0.002) | 9 (0.087) | 4 (0.881) | 1 (0.973) |
| GEV MW | 144 (0.000) | 75 (0.000) | 34 (0.181) | 15 (0.002) | 9 (0.087) | 5 (0.607) | 3 (0.412) |
| Panel B | | | | | | | |
| Period: 2/1/97-28/12/01 | | | | | | | |
| Number of daily returns: 1260 | | | | | | | |
| Expected violations | 126 | 63 | 32 | 13 | 6 | 3 | 1 |
| VC250 | 132 (0.003) | 76 (0.079) | 54 (0.000) | 31 (0.000) | 22 (0.000) | 17 (0.000) | 13 (0.000) |
| HS250 | 137 (0.002) | 80 (0.025) | 39 (0.153) | 16 (0.282) | 7 (0.926) | – | – |
| HS1000 | 191 (0.000) | 114 (0.000) | 58 (0.000) | 26 (0.001) | 12 (0.002) | 8 (0.009) | 4 (0.150) |
| MCS250 | 136 (0.000) | 85 (0.016) | 56 (0.000) | 33 (0.000) | 24 (0.000) | 20 (0.000) | 13 (0.000) |
| GL static | 196 (0.000) | 117 (0.000) | 75 (0.000) | 33 (0.000) | 18 (0.000) | 11 (0.000) | 0 (0.283) |
| GEV static | 196 (0.000) | 113 (0.000) | 66 (0.000) | 30 (0.000) | 20 (0.000) | 13 (0.000) | 2 (0.829) |
| GL MW | 171 (0.000) | 106 (0.000) | 58 (0.000) | 24 (0.004) | 13 (0.019) | 6 (0.351) | 0 (0.283) |
| GEV MW | 172 (0.000) | 100 (0.000) | 54 (0.000) | 24 (0.004) | 13 (0.019) | 8 (0.069) | 2 (0.829) |

This table contains the number of times the daily VaR forecasts are violated by the actual daily returns. A well defined VaR model should result in a number which is close to the expected number of violations. For the EVT-based VaR weekly minima are used and estimates are derived using a static and a Moving Window (MW) approach. For the static approach parameter estimates of the GL and GEV distributions fitted in the weekly minima divided into 30 sub-periods are used while for the MW approach the length of the window is 50 weekly minima. The values in parentheses are the $p$-values of the Christoffersen test statistic.

## 6. Conclusions

In this paper the asymptotic distribution of the extreme daily negative movements in the German stock market was investigated. Overall, it was found that the extreme minima of the daily returns can be adequately described by the GL distribution. This is particularly important because EVT, as it is currently applied in finance, considers only the GEV or GP distributions. However it was shown that a fatter tailed distribution, the GL, is a better model for the lower tail of the German stock market extremes and thus, it deserves serious consideration when modelling extreme financial returns. In addition, when the series of extremes were divided into sub-periods it was found that the nature of the extremes distribution changes over time, sometimes considerably, probably reflecting different economic and political regimes or even structural changes in the German stock market.

In terms of the VaR analysis, the empirical results indicated that EVT methods can be helpful in risk measurement when the focus is on the extreme returns which occur with very low probabilities; less than 1%. The only other method that could compete with the EVT-based VaR accuracy at high confidence levels was the HS. However, in order for this method to give good estimates at high confidence levels a large number of data points are needed and this can be a serious constraint. On the other hand, at low confidence levels it appears that the GEV and GL distributions do not offer any significant benefits since other less sophisticated and computationally demanding methods can provide estimates that sometimes tend to be more accurate than those derived by EVT. This is reasonable since the GEV and GL distributions focus on modelling the lower quantiles of the distribution of returns and not its central part. Finally, the results strongly indicated that the use of MWs of extremes instead of non-overlapping sub-periods lead to significant improvements in the accuracy of VaR estimates, since this approach allows the parameters of the extremes distribution to change even with weekly frequency, thus incorporating current market conditions. An interesting direction for future research would be to investigate whether the behaviour of the parameters of the extremes distribution could be explained, modelled and predicted using macroeconomic factors or expectations of the stock markets participants.



## Notes

[1] See Aparicio and Estrada (2001) for a comprehensive review of the literature regarding the empirical distributions of financial returns.

[2] For example, Beder (1995) applied eight commonly used variations of the HS and MCS methods to three hypothetical portfolios and she found that VaR estimates can vary significantly from one method to another; sometimes even by as much as 14 times. Hendricks (1996) evaluated the exponential weighted moving average (EWMA), VC and HS models using 1000 randomly selected foreign exchange portfolios. He found that none of the models used and their possible variations is consistently superior to the others and that the choice of confidence level can have a substantial effect on the VaR estimates. These findings are in line with those of Brooks and Persand (2000) and Marshall and Siegel (1997).

[3] The RiskMetrics model is based on EWMA estimates of volatility.

[4] An alternative way to analyse the behaviour of the extremes is known as the 'peaks over threshold' (POT) method, according to which, extremes are defined as excesses over a threshold (see Davison and Smith, 1990 for a detailed description of the POT). However, financial returns tend to cluster and this could lead to considerably serial dependence in the time series of the extremes. Additionally, there is an issue regarding the choice of the threshold. A low one will result in many central observations entering the sample while a high one will leave so few in the sample that could lead to inaccurate estimates. On the other hand, the definition of extremes adopted in this paper means that some of the minimum extremes will not be large negative returns or they might be even positive returns. Regrettably, there is a decision to be made and in order to avoid or reduce as much as possible the problem of serial dependence in the

time series of extremes it was decided to collect the extremes as the minimum daily returns over non-overlapping time intervals of pre-specified length.

[5] In addition, the series of the data will be divided into sub-periods and moving window techniques will be used to estimate VaR. It is believed that these approaches will help to mitigate the problem with non-iid data since it is likely to capture some of the non-stationary behaviour. Another alternatively would be to fit the tail of the conditional distribution of returns by using an autoregressive volatility model (e.g. GARCH), standardize the returns by the estimated conditional volatility and proceed in EVT analysis. This approach has received attention by McNeil and Frey (2000) and Byström (2004). However, additional parameters have to be estimated which make this approach subject to increased estimation standard error and model risk.

[6] As noted by the editor and the referee the definition of the L-moment ratios is not scale free and that it would be possible to adopt a definition that makes the L-moment ratios scale free. However, in order to be in line with the existing literature it was decided to keep the definition of the L-moment ratios unchanged.

[7] On such a diagram, a three-parameter distribution (e.g. the Weibull) is represented by a curve whereas a two-parameter distribution (e.g. the normal) is represented by a single point.

[8] The calculation of conventional moments, like skewness and kurtosis, involves third and fourth powers. Thus, greater weight is given to outliers, which can lead to considerable bias and variance.

[9] Hosking *et al.* (1985) showed that for the GEV distribution, parameters and quantiles made using the L-moments version of the PWM method are estimated with at least 70% efficiency. For example, when the shape parameter of the GEV is −0.2, the asymptotic bias of the 0.01 quantile estimated by the PWM and ML methods is found to be −0.2 and 1.6, respectively. They also demonstrated that for shape parameter values in the range −0.5 to 0.5 and samples of up to 100 observations, PWM estimates have lower root-mean-square error than estimates generated by the ML method. Similar results are reported in the literature for the GP distribution (Hosking and Wallis, 1987; Roótzen and Tajvidi, 1997).

[10] For example if we test a VaR model at the 95% confidence level we expect that the actual returns will be larger than the VaR forecasts only 5% of the time (e.g. if we use 1000 past daily returns we expect to record 50 VaR violations).

[11] The corresponding Datastream code is TOTMKBD and the index is composed of 250 of the most heavily traded shares that aim to cover 70–80% of the total market capitalization. Prices take account of capital changes and in order to retain some familiarity this index is denoted as DAX-DS. The main advantage of using this index instead of the DAX is that it is available for a much longer period.

[12] For example the eight daily returns which occurred in the cluster and exceeded the $\mu - 4\sigma$ threshold, occurred on 19/10/87 (−6.84%), 26/10/87 (−5.12%), 28/10/87 (−4.66%), 9/11/87 (−5.87%), 10/11/87 (−6.57%), 1/10/98 (−5.33%), 2/10/98 (−5.30%), 11/9/01 (−7.21%) and 20/9/01 (−5.54%).

[13] This period contains the large negative daily returns of −6.84% on 19/10/87, −4.28% on 20/10/87, −5.12% on 26/10/87, −4.66% on 28/10/87 and −6.57% on 10/11/87.

[14] This period contains the largest negative daily return of −12.14% on 16/10/89.

[15] This period contains the large negative daily returns of −5.00% on 6/8/90, −3.08% on 17/8/90, −4.40% on 21/8/90 and −3.19% on 23/8/90.

[16] This period contains the large negative daily return of −9.29% on 19/8/91.

[17] These period contains the large negative daily returns of −3.97% on 23/10/97, −3.60% on 27/10/97, −7.21% on 28/10/97, −3.69% on 11/8/98, −5.24% on 21/8/98, −4.97% on 10/9/98, −3.30% on 17/9/98, −3.29% on 21/9/98, −5.33% on 1/10/98, −5.30% on 2/10/98 and −3.91% on 8/10/98.

[18] This period contains the large negative daily returns of −7.21% on 11/9/01, −4.12% on 14/9/01, −3.04% on 19/9/01 and −5.54% on 20/9/01.

[19] The signed and squared L-skewness, instead of the L-skewness, is used because it almost restores linearity in the L-moments ratios diagram and thus, provides a clearer view of the distribution location.

[20] In the interest of brevity these diagrams are not included in the paper; however, they are available from the authors upon request.

[21] Similar indications for the behaviour of extremes in the German stock market were obtained by Lux (2001). This characteristic of the extremes has also been noted by McNeil and Frey (2000) and Pownall and Koedijk (1999).

[22] The same VaR analysis was also carried out by using the series of monthly and semi-annual extremes. However, the series of weekly extremes found to give the best results and therefore, only these are reported. For the traditional VaR methods 250, 500, 1000 and 1500 past daily returns were used. However, to make the comparison fair, the best results from these methods were also used.

[23] There have been attempts to take into account the time varying distributional characteristics of the extremes by using autoregressive processes (McNeil and Frey, 2000; Pownall and Koedijk, 1999) or quantile regression techniques

(Engle and Manganelli, 2004). However, these approaches introduce yet more parameters in to the modelling procedure and this is likely to result in larger estimation errors and possibly even more inaccurate VaR estimates.

[24] The first period was a volatile period for the German stock market because of the effects of the stock markets' crash in 1987, the turbulence due to the political uncertainty surrounding German unification, the Gulf crisis and the coup against the Soviet leader Michael Gorbachev. The daily standard deviation was 1.24%, skewness was −1.353 and kurtosis was 13.489. Normality was firmly rejected by the Shapiro–Wilk test. The second period was even more volatile with a daily standard deviation of 1.40%, a skewness of −0.444 and a kurtosis of 1.963. This period contains the turbulence during the Asian and Russian crises and the negative market sentiment after the events of 11 September 2001 that drove the stock markets downwards. In addition, these time periods contain 1248 and 1260 daily returns, respectively, sizes which can be considered adequate for statistical evaluation.

[25] At the 99.75% confidence level, the 10 unexpected VaR violations occurred in 28/1/87 (−5.06%), 19/10/87 (−6.84%), 26/10/87 (−5.12%), 28/10/87 (−4.66%), 9/11/87 (−5.86%), 10/11/87 (−6.57%), 16/10/89 (−12.14%), 6/8/90 (−5.00%), 21/8/90 (−4.40%) and 19/8/91 (−9.29%).

[26] For example, the 14 largest unexpected VaR violations occurred in 23/10/97 (−3.97%), 27/10/97 (−3.60%), 28/10/97 (−7.20%), 21/8/98 (−5.23%), 10/9/98 (−4.96%), 1/10/98 (−5.33), 2/10/98 (−5.30%), 13/1/99 (−4.52%), 31/1/00 (−3.64%), 14/3/01 (−3.53%), 22/3/01 (−4.32%), 11/9/01 (−7.21%), 14/9/01 (−4.12%) and 20/9/01 (−5.47%).

## Appendix

The GEV, GL and GP are three parameter distributions which have the following PDFs, CDFs and quantile functions $X(F)$. The parameters $\alpha$ and $\beta$ are called scale and location respectively while the parameter $\kappa$ is called the shape parameter and it determines the type of the distribution.

| | Generalized extreme value | Generalized logistic | Generalized Pareto |
|---|---|---|---|
| PDF | $f(x) = \alpha^{-1} e^{-(1-\kappa)y} e^{-e^{-y}}$<br>where<br>$y = \begin{cases} -\kappa^{-1} \log\{1 - \kappa(x-\beta)/\alpha\}, & \kappa \neq 0 \\ (x-\beta)/\alpha, & \kappa = 0 \end{cases}$<br>Subject to the restrictions<br>$\begin{cases} \beta + \alpha/\kappa \leq x < \infty, & \kappa < 0 \\ -\infty < x < \infty, & \kappa = 0 \\ -\infty < x \leq \beta + \alpha/\kappa, & \kappa > 0 \end{cases}$ | $f(x) = \alpha^{-1} e^{-(1-\kappa)y} / (1 + e^{-y})^2$<br>where<br>$y = \begin{cases} -\kappa^{-1} \log\{1 - \kappa(x-\beta)/\alpha\}, & \kappa \neq 0 \\ (x-\beta)/\alpha, & \kappa = 0 \end{cases}$<br>Subject to the restrictions<br>$\begin{cases} \beta + \alpha/\kappa \leq x < \infty, & \kappa < 0 \\ -\infty < x < \infty, & \kappa = 0 \\ -\infty < x \leq \beta + \alpha/\kappa, & \kappa > 0 \end{cases}$ | $f(x) = \alpha^{-1} e^{-(1-\kappa)y}$<br>where<br>$y = \begin{cases} -\kappa^{-1} \log\{1 - \kappa(x-\beta)/\alpha\}, & \kappa \neq 0 \\ (x-\beta)/\alpha, & \kappa = 0 \end{cases}$<br>Subject to the restrictions<br>$\begin{cases} 0 \leq x < \infty, & \kappa \leq 0 \\ 0 \leq x \leq \alpha/\kappa, & \kappa > 0 \end{cases}$ |
| CDF | $F(x) = e^{-e^{-y}}$ | $F(x) = 1/(1 + e^{-y})$ | $F(x) = 1 - e^{-y}$ |
| $X(F)$ | $X(F) = \begin{cases} \beta + \alpha\{1 - (-\log F)^{\kappa}\}/\kappa, & \kappa \neq 0 \\ \beta - \alpha \log(-\log F), & \kappa = 0 \end{cases}$<br>The Weibull distribution is the special case of the GEV when $\kappa > 0$, the Gumbel distribution is the special case for $\kappa = 0$ and the Fréchet distribution is the special case for $\kappa < 0$. | $X(F) = \begin{cases} \beta + \alpha[1 - \{(1-F)/F\}^{\kappa}]/\kappa, & \kappa \neq 0 \\ \beta - \alpha \log\{(1-F)/F\}, & \kappa = 0 \end{cases}$<br>The logistic distribution is the special case of the GL when $\kappa = 0$. | $X(F) = \begin{cases} \beta + \alpha\{1 - (1-F)^{\kappa}\}/\kappa, & \kappa \neq 0 \\ \beta - \alpha \log(1-F), & \kappa = 0 \end{cases}$<br>The exponential and the uniform distributions are the special case of the GP when $\kappa = 0$ and $\kappa = 1$, respectively, on the interval $0 \leq x \leq \alpha$. |

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