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An Analysis of the Distribution of Extreme Share Returns in the UK from 1975 to 2000

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1. INTRODUCTION

Extreme changes in share returns that affect a large number of equities are of particular interest to fund managers, investment analysts and financial regulators because it is very difficult to diversify away the risk associated with extreme movements of the whole market. Longin (1996) highlighted that information about the distribution of returns may also be helpful for margin setting in futures markets and in regulating capital requirements for security firms. His US findings indicated that financial regulators could use information about extremes to limit the amount of investment lost if an extreme event such as a market crash occurred.

This paper seeks to characterise the distribution of extreme returns for an index of UK shares over the period 1975 to 2000. A number of distributions are investigated: Gumbel, Frechet, Weibull, Generalised Extreme Value (GEV), Generalised Pareto,

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Log-Normal and Generalised Logistic (GL). The best fitting distribution is identified and the UK findings of the present paper compared with Longin's US results. In particular, daily returns for the Financial Times (FT) All Share index are obtained and the maxima and minima calculated over weekly, monthly, quarterly, half-yearly, yearly, and finally, 2-yearly selection intervals. Summary statistics for these maxima and minima are then plotted on a statistical distribution map in order to reveal what the likely candidates for the best fitting distribution are. The results from fitting each of these distributions to the extremes of a series of UK index returns show empirically that the minima and maxima follow a GL distribution. This distribution fits the data well for the extremes of the FT All Share index over the period of study. We find that the GL fits better than that of the GEV in the majority of cases, and fits satisfactorily overall. By contrast, the GEV distribution, which enjoys support from classical extreme value theory, fails to provide a satisfactory fit overall for our data set of extremes of UK index returns. The GL has fatter tails than the GEV; thus academics and practitioners should be careful of the choice of which distribution to employ.

The remainder of this article is organised as follows: Section 2 contains the literature review; it discusses why the study of extremes is important, defines what is meant by an extreme and outlines the empirical evidence on the distribution of share returns which appears to exist in practice. Section 3 discusses the data and methodology, describing the theory behind the identification of distributions and the statistical estimation procedures employed in this paper. Section 4 analyses the results while Section 5 concludes.

2. LITERATURE REVIEW

(i) Observed Patterns in Return Distributions

The distribution of equity returns (as distinct from that for extremes of equity returns) plays an important role in the theory and application of financial economics. An assumption that is very common in finance theory is that equity returns are (log-) normally distributed. While studies over long horizons have found returns to be approximately normally distributed, there is now a large and growing body of literature which suggests that over short horizons, equity returns are far from normal; instead they display a significant level of kurtosis and, in many cases, skewness.[1]

Empirical evidence which challenges the normality assumption has steadily mounted over the last few decades. For example, Mandelbrot (1963) argued that price changes could be characterised by a stable Paretian distribution with a characteristic exponent of less than two, thus exhibiting fat tails and an infinite variance.[2] Using 30 stocks of the Dow Jones Industrial Average, Fama (1965) confirmed Mandelbrot's (1963) hypothesis that a stable Paretian distribution with a characteristic exponent which was less than two describes share returns better than a normal distribution. Several distributions other than the normal have also been fitted to stock returns. Smith (1981), Gray and French (1990) and Peiró (1994) tested the logistic distribution. Praetz (1972), Blattberg and Gonedes (1974), Gray and French (1990) and Peiró (1994) used the exponential power distribution. Finally, Kon (1984) fitted several mixtures of normal distributions.

Recently, Harris and Küçüközmen (2001) investigated the empirical distribution of UK and US stock returns. They used Datastream to obtain daily, weekly and monthly observations on the FTSE All Share index for the UK and on the S&P 500 Composite index for the US. The period of their investigation was 1979 to 1999. Therefore, there were 5,479 daily, 1,096 weekly and 252 monthly observations. They modelled these continuously compounded daily, weekly and monthly returns for the UK and the US using two flexible families of distributions. The first was the exponential generalised beta (EGB)[3] distribution, introduced by McDonald and Xu (1995) to model US daily equity returns for individual stocks.[4] The second was the skewed generalised-*t* distribution (SGT),[5] introduced by Theodossiou (1998) This skewed extension of the generalised-*t* distribution was originally proposed by McDonald and Newey (1988).[6] This distribution has been used to model daily returns for a range of financial assets, including aggregate equity returns for the US, Canada and Japan.

Harris and Küçüközmen found that for both the UK and the US, daily equity returns were far from normally distributed, but instead displayed a significant level of leptokurtosis, and that the UK data was significantly skew. They found that, compared to the normal distribution, the EGB and SGT distributions both provided a substantially improved fit to the daily, weekly and monthly returns data from both countries. In the case of daily and weekly returns, the SGT distribution was marginally superior to the EGB. On the other hand, they found that the best fit was achieved for monthly returns by members of the EGB family.[7]

In order to investigate the temporal stability of their findings, Harris and Küçüközmen split their daily returns series into three equal length sub-periods and conducted separate analyses for each sub-period. Their results identified significant changes over time in the fitted distributions. During less volatile periods, a number of distributions, including the student-*t* and the logistic, could capture the empirical characteristics of daily equity returns. However, during the more volatile periods that included extreme movements such as for example, the October 1987 crash, these distributions were unable to account for the very high kurtosis in the data.

(ii) The Importance of the Choice of Distribution to Model Extreme Share Returns

There are several reasons why the nature of a distribution that best describes extreme share returns should be important to academics and practitioners in the UK. First, quantitative assessments of the risk attached to a share or a portfolio will depend critically on the shape of the tails of the distribution of returns. The coefficients of skewness and kurtosis are strongly dependent on the shapes of the tails of a distribution, and hence influence risk assessments for share returns. Skewness is a measure of symmetry (or, more accurately, it is lack of symmetry) and risk-averse investors generally seek to avoid negative skewness where there is a non-zero chance that some large negative returns might arise (Kahneman and Tversky, 1982). In addition, kurtosis is a measure of whether or not the data are peaked or flat relative to the normal distribution. It measures how thin or thick the tails of the distribution actually are. Again, a risk-averse investor will usually prefer a distribution with a low kurtosis (that is, where the possibility of large positive or negative return outcomes is no greater than in the normal distribution).

Second, textbook models of portfolio performance make a number of crucial assumptions about the shape of the distribution, which may or may not be appropriate in practice. For example, the traditional measures of risk, such as the variance or standard deviation, assume that the distribution of returns is symmetrical (i.e. has a skewness of 0) and that only a normal proportion of extreme returns of either sign may occur (i.e. has an excess of kurtosis of 0). In his analysis of US data, Longin (1996) found that the distribution of the returns was slightly negatively skewed (−0.506) and exhibited a large excess of kurtosis (22.057). Such results are not compatible with a normal distribution, and suggest that textbook views of risk need to be re-evaluated; and standard finance models such as the CAPM be further developed, to take account of non-normal distributions.[8] There is of course, a long history of attempts to achieve this development but with little success in breaking through into mainstream finance. One of the earliest and still most satisfactory is that of Kraus and Litzenberger (1976). A more recent work in this area is Hwang and Satchell (1999).

Third, analyses of managerial views of risk which have been ascertained by psychologists and behavioural finance specialists (Helliar et al., 2001), indicate that aspects of the distribution other than the mean and standard deviation are useful for decision making purposes. The findings of these investigations suggest that business individuals focus on the downside of the distribution and are concerned with the magnitude of negative values when evaluating risky outcomes. An analysis of the returns distribution would help in this process, especially if it supplied useful information on the size and the probability of the extreme tail of negative share price changes.

Fourth, Kearns and Pagan (1997) argue that both academics and practitioners should attempt to characterise the exact nature of the distribution of returns rather than assume a particular distribution that may be inappropriate. They suggest that current techniques such as Value at Risk (VaR) analysis need inputs about the actual distribution of returns.[9] VaR is a measure of losses resulting from 'normal' market movements subject to a number of simplifying assumptions used in its calculations; VaR aggregates all of the risks in a portfolio into a single number. The VaR estimation is highly dependent on good predictions of uncommon events or risks, as the VaR is calculated from the lowest portfolio returns. Therefore, any statistical method for VaR estimation has to have the prediction of lower tail events as its primary goal.

Fifth, it is well known, for example, that the hedging procedures employed in portfolio insurance will break down if the affected securities' prices move by more than 4% on any given day (Leland, 1985). Thus, those responsible for implementing the hedging procedures implied by portfolio insurance will be eager to know how often returns are likely to lie beyond this benchmark.

Finally, there is a growing financial literature in the US showing that the GEV distribution may provide a more appropriate model of the distribution of US returns. The GEV encompasses the Gumbel, Frechet and Weibull distributions (see Appendix B) and hence can be used to model a range of distributional shapes. To date, very little work has been undertaken using UK data and the present paper seeks to remedy this deficiency. Kotz and Nadarajah (1999) give an excellent account of these distributions.

(iii) The Nature of Extreme Events

Extreme events can occur in a wide range of settings, from natural phenomena such as earthquakes, winds, rainfall, tides, floods, and droughts, to human induced phenomena like air quality, share price movements, materials fracture, athletic performances and horse racing. An extreme is an outstanding event, which does not occur frequently, and may have catastrophic consequences; for example, volcanic eruptions, earthquakes, tidal waves, floods, stock-market crashes or collapses of engineering structures.

The earliest recorded application of extreme value theory was by astronomers for rejecting outlying observations. Fuller (1914) was probably the first to publish a paper that described an application of extreme values in flood flows. In 1920, Griffith applied extreme value theory to discuss the phenomena of rupture and flow in solids. In the 1940s and 1950s, Gumbel played a pioneering role in developing a strong statistical methodology for extreme value theory and in bringing to the attention of engineers and statisticians the possibility of using extreme value theory to derive certain distributions which previously had been treated empirically.[10] Gumbel (1941) analysed flood flow data, and in later works applied extreme value theory to the plotting of flood discharges, the estimation of flood levels and the forecasting of floods (Gumbel, 1944, 1945 and 1949).

The study of extreme events in finance, such as, for example the market Crash in 1987 or the 1997 Asian Crisis, has grown in importance over recent years. Both investors and risk managers have become more concerned with events occurring under extreme market conditions. In the field of finance, Longin (1996) has defined an extreme as:

> The lowest daily return (the minimum) or the highest daily return (the maximum) of the stock market index over a given period (Longin, 1996, p. 384).

Therefore, this notion of what constitutes an extreme in finance is different from the definition of an extreme when studying certain types of physical phenomena; in finance, as in flood frequency analysis, an extreme is selected from the data rather than based on some external occurrence. Longin (1996) chose this strategy as it was very difficult to match extreme price movements with rational explanations. Danielsson and De Vries (1997) supported this approach since they argued that extreme values, by definition, occur infrequently, and therefore will not exhibit time dependence or be related to a particular level of volatility. In an earlier paper, Cutler et al. (1989) arrived at a similar conclusion. They analysed large daily price movements for the period from 1928 to 1987 to determine if they were related to the arrival of new information. They concluded that some extreme returns were not actually associated with major news stories.

(iv) The Distribution of Extreme Returns in Finance

The distribution of extreme stock returns has been a longstanding concern of academics and practitioners although increasing interest in this area is mainly due to the development of advanced statistical techniques, and to the availability of higher frequency data-sets over significant periods of time. In the past, relatively little attention had been paid to extreme share price movements as compared to that devoted to expected returns, volatility, or correlations. There have been a number of notable exceptions to this generalisation, however. For example, Parkinson (1980) found that extremes contained information that were useful for the computation of the variance, while Jansen and De Vries (1991) used extremes to investigate the fatness of the distribution tails. More recently, Longin (1996) has studied the asymptotic distribution of extreme share returns in the US for daily observations from 1885 to 1990 on the NYSE. He concluded that the distribution of maxima and minima could be characterised by a Frechet distribution for daily, weekly and monthly returns, implying that there was a greater incidence of extreme returns than that associated with the Log-Normal distribution.[11] As the time span over which returns were cumulated was increased, the goodness of fit of a Frechet distribution to the data improved. In a follow-up investigation over a shorter time span, Longin (2000) found that a Frechet distribution again fitted his US data set best.

To date, very little work has been conducted on this topic in the UK. One exception is a study by Lambert and Lindsey (1999), which analysed the daily evolution of the price of Abbey National Plc shares over a 10-week period. Using regression models based on possibly non-symmetric stable distributions, they concluded that it was the skewness of the distribution, rather than the location and scale parameters, that varied over time.[12] Another notable European exception is Lux (2000). By applying a number of recent innovative techniques to the statistical analysis of extreme values in the German stock market, and using the Hill (1975) estimator, he was able to obtain tail indices below 2 and concluded that there was strong support for asset returns to be fat tailed with finite variance. A similar conclusion was also reached in a more recent study by Lux (2001), when he investigated the intra-daily data from the Frankfurt Stock Exchange.

What emerges from an analysis of the literature, therefore, is that share returns, and particularly extreme values of these returns, are not normally distributed. The next section describes how the data may be analysed to determine if alternative distributions fit.

3. DATA AND METHOD

(i) The L-Moment Technique for Fitting Distributions

The oldest technique for fitting distributions to observed data is known as the method of moments, in which a parametric distribution is fitted to a data-set by equating the sample moments to those of the fitted distribution. The primary advantages of the method of moments are computational ease and conceptual simplicity. Although long established in statistics, moment-based methods are not always satisfactory (see Pearson, 1963). In small samples, the numerical values of sample moments can be very different from those of the probability distribution from which the sample was drawn. In addition, estimates of distribution parameters obtained by the method of moments may be less precise than those obtained by other estimation procedures such as the method of maximum likelihood (MLE). Bowman and Shenton (1985) present a brief account of the development of the method of moments and its applications. The Generalised Method of Moments (GMM) can encompass the MLE approach as a special case and overcomes many of the problems identified with the classical method when analysing large data sets.[13] An alternative approach, known as L-moments, was developed by Hosking (1990) to overcome the difficulties arising in parameter estimation for extreme value distributions from the relatively small data sets available to hydrologists studying flood frequency analysis.

L-moments are analogous to ordinary moments in that their purpose is to summarise theoretical probability distributions and observed samples. They are expectations of certain linear combinations of order statistics; hence they are actually linear functions of the data and known as the name of L-moment. L-moments are becoming popular tools for solving various problems related to parameter estimation and distribution identification (see Hosking, 1990 and 1999; Vogel and Fennessey, 1993; Sankarasubramanian and Srinivasan, 1999; and Hosking and Wallis, 1997).[14] The L-moments provide measures of location, dispersion, skewness and kurtosis, as well as other aspects of the shape of the probability distributions or data samples.

The theoretical values of the first four L-moments are given as:

$$\lambda_1 = EX \tag{1}$$

$$\lambda_2 = \frac{1}{2}E[X_{2:2} - X_{1:2}] \tag{2}$$

$$\lambda_3 = \frac{1}{3}E[X_{3:3} - 2X_{2:3} + X_{1:3}] \tag{3}$$

$$\lambda_4 = \frac{1}{4}E[X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}] \tag{4}$$

where $X_{k:n}$ is the kth order statistic of a random sample of size n. L-moments are directly and linearly related to probability weighted moments (see Appendix A). Sample values of the L-moments can be obtained via these relationships, using plotting position estimates of the probability weighted moments.

The theoretical values of L-moments, denoted as λ_r, and of L-moment ratios, denoted by τ_r, are useful quantities for summarising a probability distribution. The main features of a probability distribution are summarised by the following four measures: the mean or location, λ_1; the scale, λ_2; the L-skewness, τ_3; and the L-kurtosis τ_4.

L-moment ratios analogous to the conventional statistical moment ratios include:

$$\mathrm{L} - \mathrm{skewness}(\tau_3) = \lambda_3/\lambda_2 \tag{5}$$

$$\mathrm{L} - \mathrm{kurtosis}(\tau_4) = \lambda_4/\lambda_2. \tag{6}$$

In Hosking's statistical distribution map, the sample L-kurtosis is plotted against the sample L-skewness for a data-set. The identification of a distribution appropriate to this data-set is then made by selecting the distribution whose theoretical L-kurtosis and L-skewness curve passes closest to it. A two parameter distribution corresponds to a single point on this map (e.g. a Gumbel or Normal distribution); a three parameter distribution to a curve (e.g. the GEV or GL distribution); and finally, a four parameter distribution (e.g. the Generalised Lambda) refers to an area on the statistical distribution map. Hosking (1986) has shown that the sample L-moment ratios equivalent to skewness and kurtosis can take any of the feasible values of the population L-moment ratios.[15]

L-moments have a number of advantages over conventional moments. First, a distribution may be specified by its L-moments, even if some of its conventional moments do not exist (Hosking, 1990). Second, L-moments have theoretical advantages over conventional moments as they are able to characterise a wider range of distributions (Hosking, 1986). Third, they are more robust to outliers than conventional moments, and enable more secure inferences to be made from small samples about an underlying probability distribution (Hosking, 1990). Fourth, parameter estimates obtained from L-moments are sometimes more accurate in small samples than even the maximum likelihood estimates (Hosking, 1990). Finally, as L-moments are always linear combinations of the ranked observations, they are subject to less bias than ordinary moments. This is because the ordinary *r*th moment estimator involves the *r*th power of the deviations between the observations and their mean, which gives greater weight to the observations farther from the mean, and therefore may result in substantial bias and variance (Vogel and Fennessey, 1993).

(ii) Statistical Estimation and Testing Procedures

The distribution of extremes depends on a number of parameters, (usually three) which may include shape, scale and location and which must be estimated from the sample data.[16] Consequently, the estimates will be subject to sampling error. Thus, a method of fitting the parameters must be chosen to minimise these errors. A method that is suitable for one distribution may not be efficient for another distribution. More importantly, a method which is efficient in estimating the parameters may not be efficient in predicting quantiles of the distribution (Al-Baidhani and Sinclair, 1987). A number of estimation methods have been proposed in the literature, but the method used in this paper is that of Probability Weighted Moments (PWMs).

A mathematical definition of PWMs is given in Appendix A. The use of PWMs in parameter estimation originated in hydrology where the recording of annual maximum flood heights has generally been of relatively short duration, and data sets of between 15 and 45 observations are not uncommon. In flood frequency analysis, PWMs is regarded as the best method for parameter estimation. The main advantage of PWMs over conventional moments is that PWMs are linear functions of the ordered data and therefore suffer less from the effects of sampling variability. PWMs are more robust to outliers in the data than conventional moments and they frequently yield more efficient parameter estimates than their conventional moment counterparts (Landwehr et al., 1979; and Hosking, 1986). However, the PWM method does have a number of limitations. One of the main limitations is that estimation of the PWMs and hence the parameters depends on the choice of plotting position. Careful choice of plotting positions is therefore important. Several authors, including Cunnane (1978) and Harter (1984) have pointed out the merits of using an unbiased plotting position, i.e. one that produces an unbiased estimate of the PWMs. Sinclair and Ahmad (1988) devised a plotting position that results in estimates of shape parameter that are also location-invariant, i.e. such that the estimate of the shape parameter is not affected by a linear transformation of the data. We have used the location-invariant plotting position in estimating the PWMs.[17]

To test the goodness of fit of the theoretical distribution to a given data set we use the Anderson Darling test for a number of reasons. Firstly, Stephens (1976) demonstrated that this test can be more powerful than the traditional Pearson Chi-Squared goodness of fit test for small sample sizes.[18] Secondly, d'Agostino and Stephens (1986) also showed that, among the class of Empirical Distribution Function (EDF), tests available the Anderson Darling test is one of the most powerful. Thirdly, the test statistic measures the squared difference between the observed and the theoretical distribution function and incorporates a weight function that gives greater emphasis to both tails of the distribution. So it is equally effective for assessing the fit of a distribution for both the maxima and minima.[19] Fourthly, another reason for using the Anderson Darling test statistic is that it is relatively straight-forward to calculate. The combination of these traits makes the Anderson Darling an attractive statistic for assessing the goodness of fit of distributions to small samples of extreme returns.

For the Anderson Darling test statistic (A^2) large values indicate a bad fit and small values indicate a good fit; the closer the values are to zero, the better the fit. The distribution of A^2 for a finite sample depends upon (i) the particular distribution whose fit is being tested, (ii) the estimation method employed and (iii) which parameters are estimated. Ahmad (1988) generated a table of critical values of the Anderson-Darling test for the GEV distribution when all three parameters are estimated by PWM.

(iii) Data

Using Datastream, 6,784 daily observations of the FT All Share index from January 1975 to December 2000 were collected. Logarithmic returns were calculated using the identity of:

$$R_{i,t} = \mathrm{LN}(P_{i,t}/P_{i,t-1}) \tag{7}$$

where $R_{i,t}$ is the return on the index for period t, $P_{i,t}$ is the price of the index at the end of period t, and $P_{i,t-1}$ is the price of the index at the end of the period $t-1$.[20, 21] From this data the daily return series was calculated and the minima (or maxima) over successive weekly, monthly, quarterly, half-yearly, yearly and 2-yearly selection intervals were then determined. The weekly minimum (maximum) is defined as the largest negative (positive) daily return in the stock market during that week. Table 1 shows the descriptive statistics for the daily returns as well as for the minima and maxima over the different selection intervals: weekly, monthly, quarterly, half-yearly, yearly and 2-yearly time spans. Specifically, the number of observations, the minimum, the maximum, the mean and the standard deviation are reported. In addition, the coefficients of skewness and kurtosis are shown together with their standard errors.

An analysis of the summary statistics (Table 1) for the returns series and the extreme values reveals a number of interesting findings. Firstly, the daily returns over the whole sample period have a positive mean of 0.07% and a standard deviation of

Table 1

Descriptive Statistics for Extremes of Daily Returns Over Various Selection Intervals for the Period 1975 to 2000

Selection Interval	*N*	*Min.*	*Max.*	*Mean*	*S.D.*	*Skew.* *(S.E.)*	*Kurt.* *(S.E.)*	*J-B*
Daily	6,784	−0.1191	0.0894	0.0007	0.0097	−0.220 (0.030)	11.087 (0.059)	18,540.65
Weekly Min.	1,352	−0.1191	0.0100	−0.0091	0.0085	−3.191 (0.067)	26.455 (0.133)	33,285.51
Monthly Min.	312	−0.1191	−0.0046	−0.0163	0.0103	−4.306 (0.138)	33.933 (0.275)	13,403.10
Quarterly Min.	104	−0.1191	−0.0098	−0.0217	0.0129	−4.831 (0.237)	32.815 (0.469)	4,256.65
Half-Yearly Min.	52	−0.1191	−0.0120	−0.0256	0.0164	−4.106 (0.330)	21.301 (0.650)	871.763
Yearly Min.	26	−0.1191	−0.0174	−0.0323	0.0207	−3.343 (0.456)	12.929 (0.887)	155.23
2-Yearly Min.	13	−0.1191	−0.0189	−0.0366	0.0282	−2.472 (0.616)	6.406 (1.191)	19.53
Weekly Max.	1,352	−0.0092	0.0894	0.0102	0.0082	2.950 (0.067)	17.859 (0.133)	14,399.16
Monthly Max.	312	0.0055	0.0894	0.0164	0.0097	3.492 (0.138)	18.747 (0.275)	3,857.52
Quarterly Max.	104	0.0092	0.0894	0.0215	0.0118	2.855 (0.237)	11.438 (0.469)	449.80

Half-Yearly Max.	52	0.0104	0.0894	0.0258	0.0138	2.505	8.425	118.16
						(0.330)	(0.650)	
Yearly Max.	26	0.0126	0.0894	0.0307	0.0162	2.293	6.271	34.37
						(0.456)	(0.887)	
2-Yearly Max.	13	0.0171	0.0894	0.0348	0.0197	2.089	4.668	10.97
						(0.616)	(1.191)	

Notes:
This table shows the descriptive statistics for daily returns as well as the minima and maxima over the different selection intervals: weekly, monthly, quarterly, half-yearly, yearly and 2-yearly. The number of observations, the minimum, the maximum, the mean and standard deviation are reported. The raw coefficients of skewness and kurtosis are shown with their standard errors. *N* is the number of observations, *S.D.* denotes standard deviation, *Skew*. denotes skewness, *Kurt*. denotes excess of kurtosis and *S.E.* denotes standard error. The standard errors shown are those which apply to small as well as large samples, see the paper by Urzua (1996), and can also be found in the textbook by Sokal and Rolf (1981). *J-B* denotes the test statistic for the Jarque-Bera (1980) test, i.e. the sum of squares of the standardized skewness and kurtosis. If the data were a random sample from a normally distributed population, the Jarque-Bera statistic would be asymptotically distributed as a Chi-squared variate with two degrees of freedom.

0.97%. However, this mean figure masks a wide spread of values in the data. The daily returns vary from a low of −11.91% during the Crash of 1987, to a high of 8.94% in January 1975. The normality assumption is strongly rejected for this data since the Jarque-Bera statistic takes the value 18,540.65 which greatly exceeds 9.21, the 99% quantile of the Chi-squared distribution with two degrees of freedom.

Second, the mean values for the minima returns are all negative for the different selection intervals. The absolute size of the mean increases with the length of selection-interval, from a value of 0.91% for weekly time intervals to 3.66% for 2-yearly intervals. Similarly, the standard deviation increases for the minimum returns as the selection-interval gets longer, recording a low of 0.85% for the weekly minima values to a high of 2.82% for the 2-yearly minima values. For each selection interval the data are non-normal, as the excess of kurtosis values are all significantly greater than 0.

Third, the mean values for the maxima are positive for all selection intervals. Again the mean value increases with the length of selection interval, from a low of 1.02% for weekly intervals to a high of 3.48% for the 2-yearly intervals. The standard deviation for the maxima also increases from 0.82% for weekly intervals to 1.97% for the 2-yearly selection interval. Again, the skewness and kurtosis statistics indicate that the distribution of the maxima over all selection-intervals is non-normal.

Finally, the observed distributions of the minimum values are fatter tailed than for their maximum counterparts. According to the descriptive figures, the minima exhibit a negative skewness while the maxima are positively skewed. However, the distributions of the maxima and minima are not simply mirror images of each other; both show different values for standard deviation, skewness and kurtosis at each selection interval.

4. RESULTS

The values of τ_3 (L-skewness) and τ_4 (L-kurtosis) for different distributions were estimated and then plotted to generate an L-moment ratio diagram. Figure 1 shows the ratio for the maxima, while Figure 2 displays the same ratio for the minima

Figure 1

L-moment Ratio Diagrams for Weekly Maxima

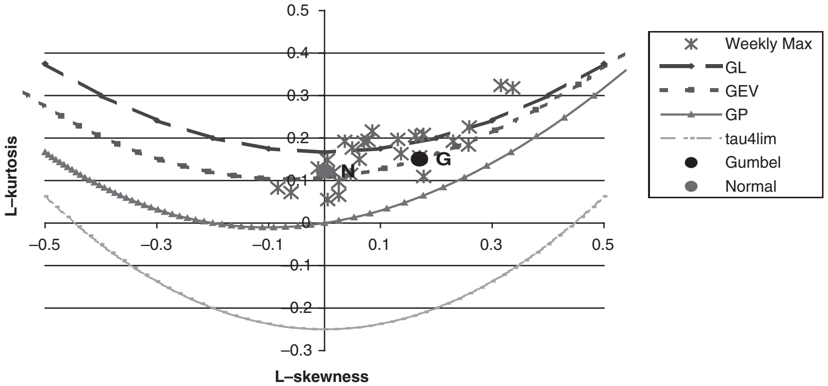
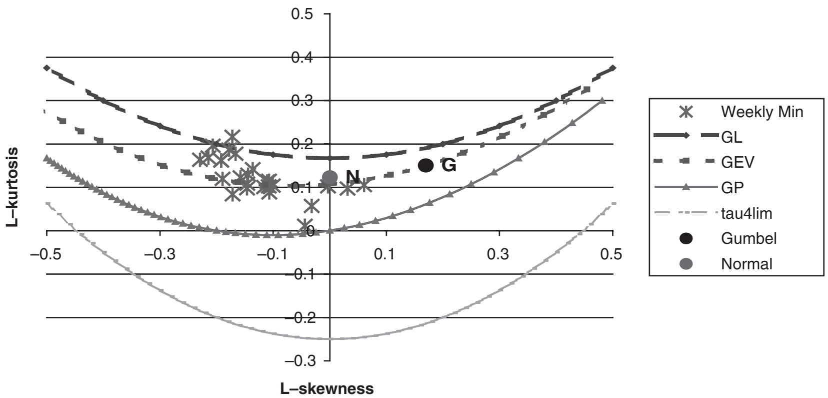


Figure 2

L-moment Ratio Diagrams for Weekly Minima



Notes:
Figures 1 and 2 show the L-skewness and L-kurtosis for the various distributions for weekly maxima and minima respectively.
The distributions plotted: GL = generalised logistic, GEV = generalised extreme value, GP = generalised Pareto, tau4lim = lower bound for all distributions, G = Gumbel and N = Normal.

values. The diagrams show the relationship between sample estimates of both L-skewness and L-kurtosis, computed from the weekly maxima and minima of daily returns of the FT All

Share index over the period 1975 to 2000. For comparison purposes, the curves indicate the theoretical relationships between L-skewness and L-kurtosis for the GEV, GL, GP, the Normal and the Gumbel distributions; while the lower bound of the L-kurtosis for all distributions is represented by 'tau4lim'.

One of the main purposes of these diagrams is to determine which distribution best characterises the minima and maxima datasets. A visual inspection of these diagrams reveals that the GEV or GL distribution fits both the weekly maxima and minima well. This is because the weekly maxima and minima are mainly congregated around the theoretical curve for these distributions. The Generalised Pareto distribution can be ruled out as both the weekly maximum and minimum data do not straddle the theoretical curve for this distribution. However, further statistical analysis is required in order to determine which of the GEV and the GL distributions best fit the data. Therefore, parameter estimates and goodness of fit statistics are calculated for these distributions and the results shown in Tables 2 and 3.

(i) Fitting GEV and GL to Maxima and Minima of Daily Returns

Table 2 facilitates a comparison of the parameter estimates for the GEV distribution fitted by PWM, to extremes of daily returns over certain selection intervals; this analysis is provided over the whole period from 1975–2000 and in different sub-periods. The distributions of maxima and minima are summarised over selection intervals which varied in length from 1 quarter to 1 year and for sub-periods of 6.5 or 13 years duration. An Anderson Darling test was then employed to provide a measure of the goodness of fit to each data set.

In analysing the minimal returns, the shape parameter takes positive values for all selection intervals and all sub-periods over the whole time frame; these values vary from a low of 0.50 to a high of 1.15, and indicate that the minima returns are negatively skewed. For the period from June 1981 to December 1987, the scale parameter had a particularly large value of 0.0136; this result is hardly surprising since the sub-period included the Crash of 1987. As one would expect, the location parameter is negative for the minimal returns and its behaviour

Table 2

Parameter Estimates and Goodness of Fit Tests for the GEV Distribution, Based on Extremes of Daily Returns

GEV Selection Interval	*Period or Sub-period*	*N*	*Shape*	*Scale*	*Location*	*AD*	*p-value*
Panel A: Minimal Returns							
Quarterly	Jan 1975–June 1981	26	0.87	0.0099	−0.024	0.357	0.152
	June 1981–Dec 1987	26	1.07	0.0136	−0.022	1.438	0.012
	Jan 1988–June 1994	26	0.76	0.0085	−0.021	0.162	0.669
	June 1994–Dec 2000	26	0.50	0.0076	−0.022	0.283	0.292
Half-Yearly	Jan 1975–Dec 1987	26	0.99	0.0165	−0.029	1.261	0.004
	Jan 1988–Dec 2000	26	0.57	0.0086	−0.024	0.152	0.694
Yearly	Jan 1975–Dec 2000	26	1.15	0.0161	−0.031	0.444	0.066
Panel B: Maximal Returns							
Quarterly	Jan 1975–June 1981	26	−0.41	0.0056	0.020	0.257	0.360
	June 1981–Dec 1987	26	−0.28	0.0050	0.016	0.117	0.706
	Jan 1988–June 1994	26	−0.27	0.0052	0.015	0.236	0.423
	June 1994–Dec 2000	26	−0.02	0.0056	0.015	0.400	0.102
Half-Yearly	Jan 1975–Dec 1987	26	−0.34	0.0069	0.021	0.292	0.270
	Jan 1988–Dec 2000	26	−0.11	0.0066	0.019	0.522	0.030
Yearly	Jan 1975–Dec 2000	26	−0.30	0.0075	0.023	0.303	0.247

Notes:
This table shows a comparison of parameter estimates and goodness of fit for the GEV distribution fitted by PWM to extremes of daily returns over various selection intervals both for the whole 26-year period and over certain sub-periods. *N* is the number of observations, *AD* denotes the Anderson-Darling statistic and *p-value* denotes the probability of such a fit being obtained in a random sample from a GEV distribution.

is relatively stable across the sub-periods within each selection interval. Overall, the A^2 test statistic suggests that the GEV fits adequately in 5 out of the 7 cases. The fact that the *p*-value is greater than 0.05 in all but two instances does not allow us to reject the null hypothesis that the distribution is approximated by the GEV.

An examination of the maximal returns in Table 2 also reveals a number of interesting results. For example, the shape parameter is negative for all selection intervals and all the sub-periods examined, suggesting that these maxima returns are skewed to the right. The scale parameter is relatively stable over the sub-periods, and takes larger values for the longer selection intervals. For example, it varies from around

Table 3

Parameter Estimates and Goodness of Fit Tests for the GL Distribution, Based on Extremes of Daily Returns

Selection Interval	*Period or Sub-period*	*N*	*Shape*	*Scale*	*Location*	*AD*	*p-value*
Panel A: Minimal Returns							
Quarterly	Jan 1975–June 1981	26	0.36	0.0039	−0.021	0.194	0.769
	June 1981–Dec 1987	26	0.53	0.0041	−0.017	0.424	0.189
	Jan 1988–June 1994	26	0.24	0.0038	−0.018	0.153	0.886
	June 1994–Dec 2000	26	0.11	0.0038	−0.020	0.302	0.421
Half-Yearly	Jan 1975–Dec 1987	26	0.51	0.0052	−0.023	0.731	0.026
	Jan 1988–Dec 2000	26	0.15	0.0042	−0.022	0.329	0.354
Yearly	Jan 1975–Dec 2000	26	0.49	0.0054	−0.026	0.092	0.850
Panel B: Maximal Returns							
Quarterly	Jan 1975–June 1981	26	−0.46	0.0047	0.022	0.248	0.581
	June 1981–Dec 1987	26	−0.36	0.0039	0.018	0.110	0.922
	Jan 1988–June 1994	26	−0.35	0.0040	0.017	0.266	0.526
	June 1994–Dec 2000	26	−0.18	0.0037	0.017	0.550	0.082
Half-Yearly	Jan 1975–Dec 1987	26	−0.41	0.0055	0.024	0.303	0.416
	Jan 1988–Dec 2000	26	−0.24	0.0046	0.021	0.443	0.166
Yearly	Jan 1975–Dec 2000	26	−0.38	0.0058	0.026	0.268	0.519

Notes:
This table shows a comparison of parameter estimates and goodness of fit for the GL distribution fitted by PWM to extremes of daily returns over various selection intervals both for the whole 26-year period and over certain sub-periods. *N* is the number of observations, *AD* denotes the Anderson-Darling statistic and *p-value* denotes the probability of such a fit being obtained in a random sample from a GL distribution.

0.0054 for the quarterly maxima to a high of 0.0075 for the annual maxima. In addition, as expected the location parameter is positive, and relatively stable. Finally, an analysis of the A^2 test statistic suggests that GEV fits the maxima values adequately in 6 out of the 7 cases. Only for the sub-period from January 1988 to December 2000 is the p-value less than 0.05, allowing us to reject the null hypothesis that the GEV distribution is appropriate. An obvious difference between the results for minima and maxima is that the greater dispersion of minima is revealed in their larger values for the whole scale parameter.

Table 3 summarises the fitting of the GL (see Appendix B) distribution. As before, the shape parameter for the minima of daily returns is positive. However, this parameter is relatively variable ranging from a low of 0.11 to a high of 0.53 for the sub-period that contains the October 1987 Crash. By contrast, the scale parameter is more stable over the different selection intervals and sub-periods, but takes its largest value of 0.0052 in the sub-period from January 1975 to December 1987. As one would expect, the location parameter is negative for the minimal returns and is relatively stable over the selection intervals and sub-periods studied. Finally, the A^2 test statistic indicates that, overall, the fit is adequate in 6 out of the 7 cases.

Turning to the statistics for the maximal returns, the shape parameter is seen to be negative for all the different selection intervals and in each of the various sub-periods, as well as for the whole period. This distribution therefore suggests that extreme share price changes are positively skewed. The scale parameter is relatively constant over the selection-intervals, and is larger for the extremes arising from longer selection intervals. The location parameter is positive and relatively stable. Finally the GL fits the maxima adequately in all 7 cases.

In comparing Tables 2 and 3 a number of similarities emerge. Both fitted distributions have positive shape parameters for the minima and negative for the maxima. The two fitted distributions have relatively stable scale parameters but there is less variation among the parameters for the GL distribution for both the maxima and minima data values. As expected, in both distributions, the location parameter for minima values are negative and for maxima values are positive. Overall the GEV fits satisfactorily in 11 out of the 14 instances, while the GL fits well in 12 cases.

(ii) The Effect of Choice of Return Frequency and of Selection Interval

Table 4 provides a comparison of goodness of fit to extremes of data for the whole 26-year period between the GL and GEV distributions; the *p*-values of the Anderson Darling statistic are used. The table shows results for daily, weekly and monthly returns over a variety of selection intervals. The results of this table build on the findings of Table 2 and Table 3 in that different data frequencies are employed, thus enabling us to examine whether data frequency (daily, weekly and monthly returns) influence the findings.

According to this table, the GL provides an adequate fit for the maxima over all selection intervals when daily returns are

Table 4

A Comparison of the GL and GEV Distributions for Different Return Frequencies

	Minima			*Maxima*			
Selection Interval	*GL*	*GEV*	*Better Fit*	*GL*	*GEV*	*Better Fit*	*N*
Panel A: Daily Returns							
Weekly	0.003*	0.000	GL	0.280	0.000	GL	1,352
Monthly	0.008*	0.000	GL	0.120	0.001	GL	312
Quarterly	0.467	0.045	GL	0.480	0.340	GL	104
Half Year	0.721	0.020	GL	0.253	0.047	GL	52
Annual	0.850	0.066	GL	0.519	0.246	GL	26
2-Yearly	0.047	0.067	GEV	0.525	0.350	GL	13
Panel B: Weekly Returns							
Monthly	0.418	0.000	GL	0.775	0.151	GL	312
Quarterly	0.171	0.008	GL	0.583	0.534	GL	104
Half Year	0.799	0.003	GL	0.196	0.255	GEV	52
Annual	0.378	0.012	GL	0.318	0.113	GL	26
2-Yearly	0.379	0.569	GEV	0.007	0.000	GL	13
Panel C: Monthly Returns							
Quarterly	0.082	0.033	GL	0.382	0.306	GL	104
Half Year	0.004	0.051	GEV	0.557	0.290	GL	52
Annual	0.619	0.002	GL	0.015*	0.001	GL	26
2-Yearly	0.307	0.040	GL	0.000*	0.000	GL	13

Notes:
This table shows the average p-values of the Anderson-Darling statistic for the GL and GEV distributions fitted to extremes of daily, weekly and monthly returns for the 26-year sample period. N denotes the number of observations. An * indicates that the GL fits better than the GEV. However, neither are an adequate fit because they do not have a p-value >0.05.

used. It fits better than the GEV in all but one instance for the minima.[22] In addition, the GL fits the minima adequately in three of the six selection intervals. For weekly returns, the GL fits adequately in nine out of the ten cases, and fits better than the GEV in all but two instances. Finally, for monthly returns, it is only in one case (half-yearly selection intervals for minima) that the GEV fits better than the GL. Overall, the analysis of this table indicates that the GL is a more appropriate distribution than the GEV for characterising the extremes of daily, weekly and monthly returns. Specifically, when minima are estimated annually, the GL fits adequately for daily, weekly and monthly returns while the GEV does not. Therefore, when estimating the downside risk the GL appears to be the more useful model for investors.

The Anderson Darling tests and their corresponding *p*-values for the GEV and GL distributions for the extremes of daily returns for 26 one-year sub-periods are shown in Table 5. The selection interval is one month and the number of observations is 12 in each case. Overall, the *p*-value is 4.34E-06 and, consequently, the GEV appears to be a bad fit for the monthly minima. In analysing the *p*-values for each sub-period, the GEV fits the data in 21 out of the 26 cases.[23] When the GEV is fitted to the maxima values, the overall *p*-value is 0.0005, which is an improvement on the minima, but is still not a particularly good fit. For the different sub-periods, the GEV fits adequately in 22 out of the 26 cases.

The results of the A^2 tests for the GL distribution suggests that the data conform more closely to a generalised logistic distribution; the portmanteau *p*-value of 0.0081 is higher than for the GEV, although the overall fit is still not an adequate one. In 22 sub-periods out of 26, the GL does fit the data for the minima.[24] However, when the portmanteau test is applied to the results for the maxima, the *p*-value of 0.120 confirms that the GL distribution fits satisfactorily. Furthermore, separate examination of the various sub-periods shows that the GL fails to fit in only one out of the 26 cases.

In comparing the two distributions, a number of interesting points emerge. First for the minima, neither distribution fits particularly well although the GL achieves better results, fitting 22 out of the 26 cases, compared with 21 out of the 26 instances for the GEV. Indeed, the GL outperforms the GEV in 19 out of the 26 cases. The probability of getting 19 or more successes in 26 trials with the probability of success set to 0.5 is 0.015, suggesting that it is not just a matter of chance that the GL fits better than the GEV. For the maxima, the GL also fits better than the GEV; it has an overall *p*-value of 0.120 compared to 0.001 for the GEV. It also outperforms the GEV in 22 out of the 26 cases. The probability of this happening by chance is again fairly small (0.00027). Overall, from this simple analysis the GL can be seen to fit better than the GEV for monthly intervals.

Table 5

An Analysis of the Fit of the GL and GEV Distributions Over Time

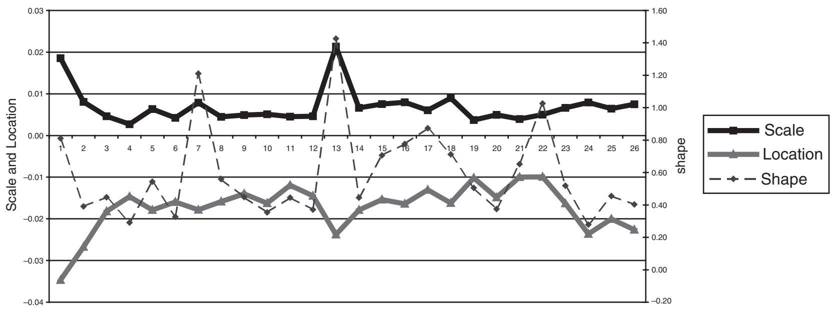
	Minima				*Maxima*			
Sub-period	*GEV AD*	*p-value*	*GL AD*	*p-value*	*GEV AD*	*p-value*	*GL AD*	*p-value*
1	0.353	0.130	0.467	0.119	0.452	0.047	0.500	0.093
2	0.228	0.392	0.275	0.467	0.269	0.282	0.331	0.320
3	0.159	0.596	0.190	0.754	0.390	0.090	0.388	0.213
4	0.241	0.354	0.310	0.371	0.363	0.118	0.298	0.401
5	0.533	0.019	0.781	0.009	0.166	0.581	0.154	0.862
6	0.218	0.423	0.304	0.386	0.568	0.012	0.653	0.027
7	0.332	0.159	0.197	0.732	0.208	0.454	0.236	0.593
8	0.139	0.628	0.203	0.709	0.204	0.468	0.205	0.704
9	0.365	0.115	0.459	0.126	0.249	0.332	0.192	0.749
10	0.501	0.027	0.584	0.048	0.350	0.133	0.461	0.125
11	0.135	0.630	0.155	0.858	0.270	0.280	0.294	0.414
12	0.307	0.201	0.373	0.237	0.330	0.162	0.440	0.146
13	0.311	0.193	0.197	0.732	0.431	0.059	0.446	0.139
14	0.539	0.017	0.760	0.010	0.463	0.042	0.337	0.308
15	0.694	0.002	0.360	0.262	0.271	0.277	0.288	0.428
16	0.343	0.143	0.456	0.129	0.219	0.422	0.260	0.514
17	1.111	0.000	0.475	0.112	0.206	0.459	0.287	0.431
18	0.374	0.105	0.504	0.090	0.365	0.115	0.401	0.195
19	0.202	0.474	0.178	0.794	0.202	0.474	0.229	0.617
20	0.257	0.312	0.326	0.332	0.217	0.427	0.275	0.468
21	0.238	0.363	0.342	0.297	0.400	0.081	0.438	0.147
22	0.446	0.050	0.380	0.226	0.178	0.546	0.187	0.764
23	0.151	0.613	0.201	0.719	0.416	0.069	0.529	0.074
24	0.318	0.181	0.399	0.197	0.381	0.099	0.432	0.155
25	0.352	0.131	0.526	0.076	0.149	0.617	0.180	0.787
26	0.312	0.192	0.337	0.308	0.470	0.039	0.315	0.358
Overall		4.3E-06*		0.00809*		0.0005*		0.12019*

Notes:
This table shows the Anderson Darling test statistic (AD) and associated *p*-values for the GEV and GL Distributions fitted by PWM to monthly extremes of daily returns for 26 one-year sub-periods.
* The *p*-value for this portmanteau test is obtained by summing −2ln of each Anderson-Darling *p*-value, and using the inverse of the Chi-Squared cumulative distribution function with 52 degrees of freedom.

Figures 3 and 4 show the behaviour over time of the parameter estimates for monthly minima of daily returns. In particular, Figure 3 shows the parameter estimates for the GEV distribution fitted by PWM to extremes of daily returns for 26 one year sub-periods for the minima. The selection interval is

Figure 3

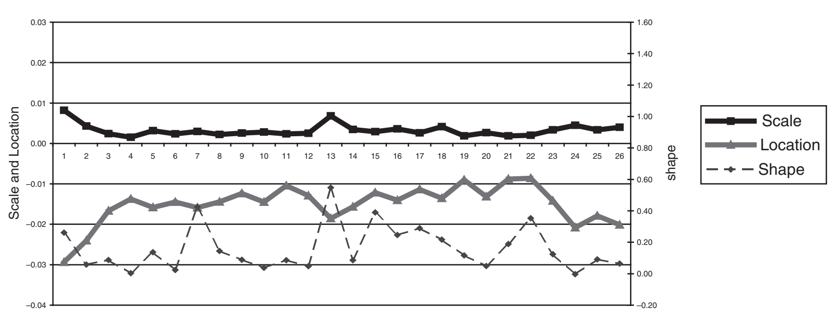
Comparison of Parameter Estimates for the GEV Distribution Fitted by PWM to Extremes of Daily Returns for 26 One-year Sub-periods for the Minimum



one month, hence the number of observations is 12 in each case. From this figure, we can observe that the scale and shape parameter are positive while the location parameter is negative. One important observation is that the scale and shape both peak in 1987, the year of the stock market Crash; this year has the largest negative daily return in the study. Figure 4 shows the corresponding information for the GL distribution. The scale and shape parameter are positive and the location parameter negative. For the GL, the shape parameter peaks in 1987, again

Figure 4

Comparison of Parameter Estimates for the GL Distribution Fitted by PWM to Extremes of Daily Returns for 26 One-year Sub-periods for the Minimum



strengthening the argument that there is a relationship between large falls and the shape parameter.

It is important to compare the GEV and GL for the minima, as investors and practitioners are more concerned with large falls in the market rather than the sizeable increases. As Figures 3 and 4 illustrate, the shape and the scale parameter values of both distributions are positive, while the location parameter exhibits a negative value in both. There are a number of important differences between the GEV and GL for the minima. For example, the GL shape parameter exhibits four peaks (1981, 1987, 1989 and 1996) compared to the GEV with only three peaks (1981, 1987 and 1996). Also, the peaks in all three parameters for the GEV are greater than that of the corresponding peaks of the GL.

Figures 5 and 6 show the parameter estimates for the GEV and GL distribution respectively for the monthly maxima. For the GEV (Figure 5), the scale and location parameters are both positive, although the shape parameter is very volatile varying from negative to positive values and peaking in 2000. Figure 6 shows the maxima of the GL distribution. In this figure it is the scale and location parameter that are positive across the study. The shape parameter is the most volatile parameter, peaking in 2000. A noteworthy difference between the GEV and GL for maxima returns is the tendency for the GL distribution to

Figure 5

Comparison of Parameter Estimates for the GEV Distribution Fitted by PWM to Extremes of Daily Returns for 26 One-year Sub-periods for the Maximum

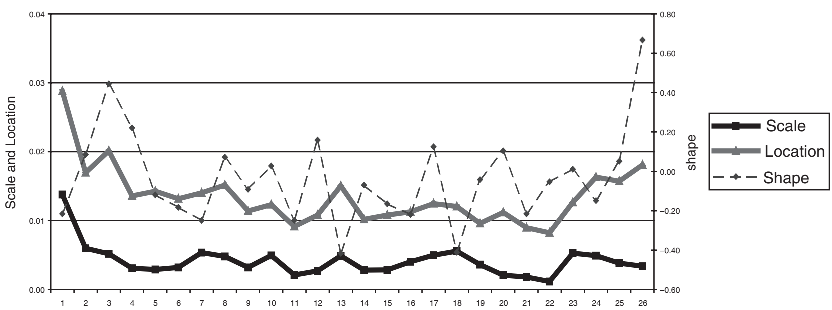


Figure 6

Comparison of Parameter Estimates for the GL Distribution Fitted by PWM to Extremes of Daily Returns for 26 One-year Sub-periods for the Maximum

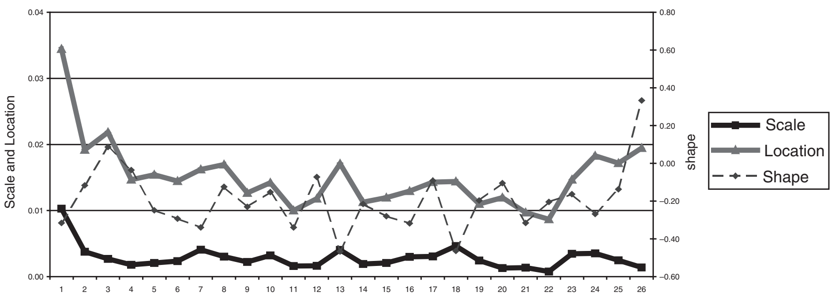


exhibit much less variation in the shape parameter than the GEV distribution. Hence the GL distribution is to be preferred for quantile estimation purposes.

(iii) Comparison of Results with US Findings

In a recent US study, Longin (2000) examined the asymptotic distribution of extreme S & P 500 index returns. The database he used consisted of daily returns on the S & P index over the period January 1962 to December 1993.[25] There are a number of similarities and differences between the present study and Longin's investigation. In terms of similarities, both studies attempt to characterise the distribution of extreme share returns for a market index that contains a large number of shares, over a similar time period that includes the October 1987 Crash; both indices are for developed stock markets. In each study, the distributions of both the minima and the maxima of daily returns are investigated over a number of selection intervals that include weekly, monthly, quarterly and half-yearly. The distributions fitted to the data sets include the Gumbel, Frechet and Weibull, either as separate distributions in Longin's study or as special cases of the GEV distribution in the present study.

The FT All Share Index is based on a larger number of shares but relates to a smaller volume stock-market than the S & P 500.

In addition, Longin's time period of 32 years is slightly longer than the 26 years of the present study and encompasses nearly three business cycles rather than the two cycles covered here. In this study, an additional distribution, the GL, is fitted, because it caters for fatter tailed data sets than its GEV counterpart. Longin estimates the parameters using the method of maximum likelihood, confines his attention to a series of at least 63 observations, and uses Sherman's (1957) test statistic to evaluate the goodness of fit. Longin reverses the sign of the minima before fitting a distribution. By contrast, the authors of the present study estimate the parameters using the probability weighted moment method, allowing distributions to be fitted to data sets with as few as 12 observations. Also, the Anderson-Darling goodness of fit statistic which is used here is more powerful in the case of small samples and is particularly appropriate for studying extremes because it gives greater weight to the fit in the tails of the distribution. Because we use generalised types of distributions there is no need to reverse the sign of the minima. In this study we also explore a wider range of combinations of selection intervals and we divide the data-set into sub-periods to explore the inter-temporal stability of the parameters of the fitted distributions.

The largest fall in Longin's US daily return series, occurred in the Crash of 1987 (−22.90%). It was in this same period that the largest fall was recorded in our UK-based study (−11.91%). The US market decline was therefore more extreme in this instance.

Out of his list of three possible distributions, Longin found the best fitting distribution for the minima and for the maxima of daily returns over all four selection intervals (weekly, monthly, quarterly and half-yearly) and for the complete period of the study to be the Frechet. He reported that for the minima and for the maxima, the Frechet distribution fitted adequately except for the weekly selection interval, and that for the minima the Gumbel distribution could be rejected in all four cases. However, for the maxima of daily returns the Gumbel distribution could not be rejected for the quarterly and half-yearly selection intervals.

The results of this UK study indicate that the best fitting distribution for the minima and for the maxima of daily returns over all four selection intervals (weekly, monthly, quarterly and half-yearly) and for the complete period of the study is the GL. It fitted adequately for minima over quarterly and half-yearly selection intervals and fitted adequately for maxima over all selection intervals. On the other hand, the GEV did not characterise the minima values in an appropriate fashion for any of these selection intervals, and fitted adequately for the maxima only in the case of the quarterly selection interval.

One of the more important similarities between the studies is that, as the selection interval gets longer, the scale parameter increases. This finding therefore strengthens Longin's conclusion that negative extremes are more dispersed when the return series is split into longer time horizons. Again, within both studies the absolute value of the location parameter was seen to increase as the selection interval got longer. On the other hand, both studies found that the value of the shape parameter did not change much with the length of selection interval.

5. CONCLUSION AND FUTURE RESEARCH

This study investigates the extreme price movements of the UK stock market using daily returns over the period 1975 to 2000. The statistical distributions that can account for the behaviour of minimal and maximal returns over a given period are estimated using extreme value theory.

The distributions initially considered were the Gumbel, Frechet, Weibull, GEV, Generalised Pareto, Log-Normal and the GL. Plotting the weekly maxima and minima on a statistical distribution map identified the likely distributions as the GEV and GL. Subsequent data analysis and assessment of the goodness of fit of these distributions revealed that the best fitting distribution for the UK for the period 1975 to 2000 was the Generalised Logistic. Moreover, the GL fitted adequately in the majority of combinations of selection interval and sub-period. Although theoretically the GEV distribution ought to provide a satisfactory fit to the data, our results suggest that it fails to do so in practice. This failure may be due to inter-temporal variation in the distribution of returns on the FT All Share index selected over a variety of periods. The identification of a distribution that does actually fit the data is an interesting and important finding.

The choice of the GL rather than the GEV distribution to model the behaviour of extreme returns has important implications for investors who wish to assess the risk of a portfolio, and for financial regulators who employ VaR because the GL has fatter tails than the GEV. For example, based on overall estimates obtained from the UK data in this study, the once in a career (40-year) low fitted return for the GL is 130% of that for the GEV, and the 100-year low for a fitted return for the GL is about 145% of the corresponding GEV derived figure.[26]

One avenue for future research would be to fit these distributions to other markets such as the US, and Japanese markets and also a more volatile market for example the Turkish market using the techniques employed in this paper. The reasoning behind this is to determine if one distribution fits all the countries or whether the most appropriate distribution changes from country to country. A greater challenge would be to attempt to account for changes in the parameter values of the fitted distributions over annual sub-periods using economic variables for the countries concerned. This might allow forecast values of economic variables to suggest how the parameter values of the fitted distribution might change in future, thereby making it feasible to estimate VaR based on a future rather than a past risk distribution.

It has been pointed out that the application of non-parametric techniques might well constitute a useful area for future research since such techniques clearly have the potential to address some of the problems associated with the distributional form studied in the current paper. The extension from considering separate univariate distributions for the minima and maxima to employing an appropriate bivariate distribution to describe their joint behaviour would be particularly advantageous.

APPENDIX A

Definitions of Probability Weighted Moments and L-moments

(i) Definition of the rth Probability Weighted Moment for the Random Variable X with Distribution Function F(X)

$$\beta_r = E[X\{F(X)\}^r],\ r = 0, 1, \ldots,$$

(ii) Plotting Position Based Estimator for the rth Probability Weighted Moment β_r

$$b_r = \frac{1}{n}\sum_{i=1}^{n} p_{i:n}^{r} x_{i:n}$$

where $x_{i:n}$ is the ith order statistic of a sample of size n, and

$$p_{i:n} = \frac{i+\gamma}{n+\delta}$$

is a plotting position with constants γ and δ, whose values can be chosen to optimise some aspect of the estimation for a specific distribution.

(iii) Relationships between L-moments and Probability Weighted Moments

$$\lambda_1 = \beta_0$$

$$\lambda_2 = 2\beta_1 - \beta_0$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0.$$

APPENDIX B

Distribution for Extremes

(i) The Generalised Extreme Value (GEV) Distribution

The Generalised Extreme Value (GEV) distribution was first introduced by Jenkinson (1955) and recommended for use by UK hydrologists the Natural Environmental Research Council (1975). It combines into a single form the three possible types of limiting distribution, namely the Gumbel, Frechet and Weibull distribution, for extreme values. It has a probability density function (pdf) $f(x)$, cumulative density function (cdf) $F(x)$ and an inverse distribution $x(F)$, as seen respectively below:

$$f(x) = \alpha^{-1} e^{-(1-k)y} e^{-e^{-y}}$$

$$F(x) = e^{-e^{-y}}$$

where:

$$\begin{aligned} y &= -k^{-1} \log\{1 - k(x-\xi)/\alpha\} && k \neq 0 \\ &= (x-\xi)/\alpha && k = 0. \end{aligned}$$

The inverse distribution function has the form:

$$\begin{aligned} x(F) &= \xi + \alpha\left\{1 - (-\log F)^k\right\}/k && k \neq 0 \\ &= \xi - \alpha \log(-\log F) && k = 0. \end{aligned}$$

(ii) Weibull Distribution

This is defined by:

$$F(x) = 1 - e[-\{(x-\lambda)/\beta\}^{\gamma}] \quad \lambda \leq x \leq \infty.$$

The Weibull distribution is a reversed GEV distribution with parameters:

$$k = 1/\gamma, \alpha = \beta/\gamma \text{ and } \xi = \lambda + \beta.$$

In this ξ and α are location and scale parameters, respectively, and k is the shape parameter of the distribution.

(iii) Gumbel Distribution

This is defined by:

$$f(x) = \alpha^{-1} \exp\{-(x-\xi)/\alpha\} \exp[-\exp\{-(x-\xi)/\alpha\}], \quad -\infty < x < \infty.$$

The Gumbel distribution is a two-parameter distribution, i.e. it is a special case of the GEV when the shape parameter takes the value of zero.

(iv) Generalised Logistic

The Generalised Logistic distribution gets its name because of the similarity to the generalised Pareto and the generalised extreme value distributions.

This distribution has a probability density function (pdf), cumulative density function (cdf) and an inverse distribution, as seen respectively below:

$$F(x) = 1/(1 + e^{-y}).$$

From this we can derive the log pdf:

$$\ln f = -\ln\alpha + \frac{(1-k)}{k}\ln\left\{1 - \frac{k}{\alpha}(x-\xi)\right\} - 2\ln\left[1 + \left\{1 - \frac{k}{\alpha}(x-\xi)\right\}^{\frac{1}{k}}\right].$$

This can be written in a simpler form:

$$\ln f = -\ln\alpha - (1-k)y - 2\ln(1 + e^{-y}).$$

The cdf is as follows:

$$F(x) = 1/(1 + e^{-y}),$$

where:

$$\begin{aligned} y &= -k^{-1}\log\{1 - k(x-\xi)/\alpha\} \qquad && k \neq 0, \\ &= (x-\xi)/\alpha && k = 0. \end{aligned}$$

and

$$e^{-y} = \left\{1 - \frac{k}{\alpha}(x-\xi)\right\}^{\frac{1}{k}}$$

$$\begin{gathered} \xi + \alpha/k \leq x < \infty \text{ if } k < 0, \\ -\infty < x < \infty \text{ if } x = 0, \\ -\infty < x \leq \xi + \alpha/k \text{ if } k > 0. \end{gathered}$$

The inverse distribution function has the form:

$$\begin{aligned} x(F) &= \xi + \alpha\left[1 - \{(1-F)/F\}^{k}\right]/k \qquad && k \neq 0 \\ &= \xi + \alpha\log\{(1-F)/F\} && k = 0. \end{aligned}$$

NOTES

1 The earliest evidence of non-normality is reported by Mandelbrot (1963); this evidence was confirmed by Fama (1965).
2 When the characteristic exponent of a stable Paretian distribution is exactly equal to two, then the normal distribution is obtained. Hence the latter is a special case of the former.
3 Examples of the EGB distribution can include the Gumbel and Logistic distributions.
4 The EGB distribution is a flexible five-parameter distribution that allows very diverse levels of skewness and kurtosis and nests more than thirty well known distributions.
5 Examples of the SGT distribution can include the student-*t*, Laplace, Cauchy and Normal distributions.
6 The SGT is also a flexible distribution that allows for very diverse levels of skewness and kurtosis, and nests many other well known distributions.
7 Contrary to this, in an earlier US study, Blattberg and Gonedes (1974) found that the distribution of monthly returns conformed well to the normality hypothesis.
8 Capital Asset Pricing Model.
9 Danielsson and De Vries (1997) define VaR as an amount lost on a portfolio with a given small probability over a fixed number of days. In summary it is a statistical measure of possible portfolio losses.
10 Classical extreme value theory (Gnedenko, 1943; and Galambos 1978 and 1987) shows that, under certain conditions the limiting distributions for the extreme daily returns (either minima or maxima) are the three distributions incorporated in the GEV.
11 The standard procedure in the analysis of share returns data is to make a logarithmic transformation and then apply statistical techniques whose validity is based on the assumption of a normal distribution for the log transformed data. This is equivalent to assuming that the original returns follow the log-normal distribution.
12 Recently, Bensalah (2000) used extreme value theory (EVT) to generate two significant results. First, when the asymptotic distribution of a series of maxima (minima) is modelled, under certain conditions the distribution of the standardised maximum of the series can be shown to converge to the Gumbel, Frechet or Weibull distributions (which are special cases of the GEV distribution). The second significant result that Bensalah (2000) refers to is concerned with modelling the distribution of excessive values over a given threshold. Extreme value theory can be used to show that the limiting distribution is a Generalised Pareto distribution (GPD).
13 Hansen (1982) was a proponent of the use of this technique for large samples encountered in econometrics analysis. An account of the application of GMM in a variety of settings can be found in Mátyás (1999).
14 Further evidence of the growing use of L-moments can be adduced from the increasing frequency of downloads of the software by Cox (1988) i.e. 1 download in calendar year 1999, 6 in 2000, 24 in 2001, and 7 in the first two months of 2002.
15 The studies by Hosking (1990), Vogel and Fennessey (1993) and Sankarasubramanian and Srinivasan (1999) provide strong support for the conclusion that L-moment estimators are superior to conventional moment estimators for most goodness of fit applications in hydrology.

16 Here the location parameter defines either the lower limit (for maxima) or the upper limit (for minima). The shape parameter serves to model both skewness and kurtosis, for example the L-skewness of the GL distribution is $-k$ of the shape parameter. The scale parameter is analogous to the standard deviation.

17 The authors also estimated the parameters of the GEV and GL distributions using maximum likelihood. In each case there is little difference between the PWM and the ML estimates. Therefore it seems likely that we have selected appropriate values for the coefficients in the plotting position.

18 An advantage of the Pearson Chi-Squared is that tables relating critical values to degrees of freedom are readily available.

19 By comparison the Pearson Chi-Squared CDF compares the observed and expected frequencies of observations in a range of intervals.

20 One reason for using changes in the natural logarithmic (LN) price rather than simple price changes is outlined in Fama (1965) an article which argued that the change in logarithmic price is the yield, with continuous compounding, from holding the share for that period. In addition there are both theoretical and empirical reasons why logarithmic returns are preferable to discrete returns as Strong (1992) has remarked. Theoretically, logarithmic returns are analytically more tractable when linking together sub-periods returns to form returns over longer intervals. Empirically, logarithmic returns are more likely to be normally distributed and therefore satisfy the assumptions of standard statistical techniques more fully.

21 Dividend payments were not included when returns were estimated for the shares in the FT All Share index. We believe that the systematic bias caused by this omission is not critical, for the following reasons. Firstly, the index is made up of a large number of shares so we would expect the dividend payments to be fairly well spread throughout the year, rather than to exert a large influence on the index price on a particular day. Secondly, the levels of dividends enjoyed by investors in UK companies have been modest during the period of the study, Thirdly, changes in dividends have been less volatile than the changes in the returns. Dimson and Marsh (2001) gives details of the spread of dividend payments by UK companies, and provides evidence of some clustering. Fourthly, the results of a simulation in which analyses similar to those in the paper were carried out on data that did include dividends, albeit for a shorter 11-year time period, strongly suggest that omission of dividends will not have a material effect on the conclusions. In fact the overlap between confidence intervals for the parameters of the GEV distribution for monthly minima of daily returns with and without dividends over the 11 years averaged 97.6% for the shape, 97.6% for the scale and 95.8% for the location. Fifthly, in the simulation described above, the goodness of fit of both the GEV and the GL distribution, measured by the Anderson-Darling statistic, decreased when dividends were included in the returns, but the GL maintained its advantage over the GEV.

22 Conclusions from this and all other p-value tests are drawn at the 5% significance level.

23 The tension between the above statement mainly arises from a badly fitting episode in sub-periods 14 to 17 inclusive. Omission of these four sets of

values would give an overall *p*-value of 0.007 and thus adequate fit in 18 out of 22 sub-periods.

24 There are two particularly badly fitting sub-periods namely 5 and 14. If they were to be omitted, the overall *p*-value would be adequate (*p*-value of 0.100).

25 Although the time period is different (a 32-year period) it is the closest match to a comparative US and UK study.

26 The returns are measured on a continuously compounded (logarithmic) basis.

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