Causal Inference using Generalized Empirical Likelihood Methods with R

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Abstract

To be added

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1 Introduction

In the following, let $X \in \mathbb{R}^q$ be a random q-vector of covariates, $Y(j)$ be the random (potential) outcome when the subject is exposed to the treatment j, where $0 \leq j \leq p$ and $j = 0$ corresponds to the control treatment. We consider Z_0 as an indicator for the control treatment, and in the case when the observational study does not have a well defined control treatment, then we treat Z_0 as the baseline treatment, i.e., the treatment with which we are interested to compare all other treatments. Let further $Z = (Z_j) \in \mathbb{R}^p$ be a random p-vector of treatment indicators (other than the control treatment), with $Z_j = 1$ and $Z_k = 0$ for all $k \neq j$ if the subject receives the treatment j, where $j = 1, \ldots, p$. Since all individuals receive only one treatment, then $\sum_{j=0}^{p} Z_j = 1$, and we only observe $Y = \sum_{j=0}^{p} Y(j) Z_j$. Note that we do not consider here the case of clinical trials with non-compliance where the subjects are assigned to treatments but they may receive other treatments, and thus, the available data consist of only the treatment received and the outcome under the treatment received in addition to the covariates for all subjects.

1.1 Randomized experiments

Let $\theta_1 \in \mathbb{R}$, $\theta_2 = (\theta_{2,1}, \ldots, \theta_{2,p}) \in \mathbb{R}^p$, and $\theta_3 = (\theta_{3,1}, \ldots, \theta_{3,p}) \in \mathbb{R}^p$ be defined as $\theta_1 = \mathrm{E}(Y(0))$, $\theta_{2,j} = [\mathrm{E}(Y(j)) - \mathrm{E}(Y(0))]$, $\theta_{3,j} = \mathrm{E}(Z_j)$, $1 \leq j \leq p$, and let $\theta = (\theta_1, \theta_2, \theta_3) \in \mathbb{R}^{2p+1}$. By the definition of conditional expectation,

$$\mathrm{E}(Y|Z_j = 1) = \mathrm{E}(Y(j)) = \mathrm{E}(YZ_j)/\,\mathrm{E}(Z_j)\,, \quad 1 \leq j \leq p\,. \tag{1}$$

Hence, $\mathrm{E}[(Y - \theta_1 - \theta_2^T Z)Z_j] = 0$ for all $1 \leq j \leq p$, and thus,

$$\mathrm{E}\big((Y - \theta_1 - \theta_2^T Z)Z\big) = 0\,. \tag{2}$$

By the law of total expectation formula,

$$\mathrm{E}(Y) = \sum_{j=0}^{p} \mathrm{E}(Y(j))\,\mathrm{Pr}(Z_j = 1)\,. \tag{3}$$

Hence,

$$\mathrm{E}(Y - \theta_1 - \theta_2^T Z) = 0\,. \tag{4}$$

By the definition of θ_3, $\theta_{3,j} = \mathrm{E}(Z_j)$ for $1 \leq j \leq p$; hence

$$\mathrm{E}(Z - \theta_3) = 0\,. \tag{5}$$

Note that $\mathrm{E}(Z_0) = 1 - \sum_{j=1}^{p} \theta_{3,j}$.

In randomized trials, $(Z_0, Z) \perp\!\!\!\perp X$, which implies that $\mathrm{E}[(Z_j - \theta_{3,j})u(X)] = 0$ for $1 \leq j \leq p$, where $u(X) \in \mathbb{R}^k$ is a k-vector of functions of X. Hence,

$$\mathrm{E}[(Z - \theta_3) \otimes u(X)] = 0\,. \tag{6}$$

To illustrate what $u(x)$ can be, suppose $x = (x_1, x_2) \in \mathbb{R}^2$, then we can define $u(x) = x \in \mathbb{R}^2$, or $u(x) = (x_1, x_2, x_1 x_2) \in \mathbb{R}^3$, or $u(x) = (x_1, x_2, x_1^2, x_2^2, x_1 x_2) \in \mathbb{R}^5$.

Let $T = (X, Z, Y) \in \mathbb{R}^{p+q+1}$ denote a generic random variable distributed according to a distribution on $\mathbb{R}^{p+q+1}$[1]. Therefore, Equations (2) and (4) to (6) imply that the parameter of interest θ^0 satisfies the following moment conditions:

$$\mathrm{E}(g(T, \theta^0)) = 0\,, \tag{7}$$

where $\theta^0 = (\theta_1^0, \theta_2^0, \theta_3^0) \in \mathbb{R}^{2p+1}$ and $g(t, \theta)$ is defined as

$$g(t; \theta) = \begin{pmatrix} y - \theta_1 - \theta_2^T z \\ (y - \theta_1 - \theta_2^T z) z \\ z - \theta_3 \\ (z - \theta_3) \otimes u(x) \end{pmatrix}\,, \quad t = (x, z, y) \in \mathbb{R}^{p+q+1}\,. \tag{8}$$

The ACE of the treatment j is given by $\tau_j^0 = \theta_{2,j}^0$, for $1 \leq j \leq p$.

The package offers a way to estimate θ^0 using the generalized method of moments (GEL). Using the primal form of GEL, the estimator of θ^0 is defined as:

$$\hat{\theta} = \mathrm{argmin}_{\theta \in \mathbb{R}^{2p+1}} \min_{p \in \mathbb{P}^n} \left\{ D_\gamma(p, n^{-1} 1_n) : \sum_{i=1}^n p_i g(T_i; \theta) = 0 \right\}, \tag{9}$$

where

$$\mathbb{P}^n = \left\{ p = (p_i) \in \mathbb{R}^n : \sum_{i=1}^n p_i = 1\,, p_i \geq 0 \right\},$$

and $D_\gamma(p, n^{-1} 1_n)$ is the power divergence discrepancy function (Newey and Smith, 2004):

$$D_\gamma(p, n^{-1} 1_n) = \sum_{i=1}^n \frac{(np_i)^{\gamma+1} - 1}{n\gamma(\gamma + 1)}\,.$$

In particular, $\gamma = -1$ corresponds to the empirical likelihood (EL), $\gamma = 0$ corresponds to the exponential tilting (ET), $\gamma = 1$ corresponds to the Euclidean empirical likelihood (EEL) estimator also known as the continuously updated GMM estimator (CUE), and $\gamma = -1/2$ corresponds to the Hellinger distance (HD) used by Kitamura et al. (2013). Newey and Smith (2004) present the GEL method in its dual form, which is the following saddle point problem:

$$\hat{\theta} = \mathrm{argmin}_{\theta \in \mathbb{R}^{2p+1}} \max_{\lambda \in \mathbb{R}^{1+p(2+k)}} \sum_{i=1}^n \rho_\gamma\big(\lambda^T g(T_i; \theta)\big)\,, \tag{10}$$

where

$$\rho_\gamma(v) = -\frac{(1 + \gamma v)^{(\gamma+1)/\gamma}}{\gamma + 1}\,.$$

In particular $\rho_{-1}(v) = \log(1 - v)$ for EL, $\rho_0(v) = -\exp(v)$ for ET, $\rho_1(v) = -1/2 - v - v^2/2$ for EEL, and $\rho_{-1/2}(v) = -2/(1 - v/2)$ for HD. Using the dual form, the estimated probability weights from the primal problem are defined as:

$$\hat{p}_i(\theta, \lambda) = \frac{\rho'_\gamma(\lambda^T g(T_i; \theta))}{\sum_{j=1}^n \rho'_\gamma(\lambda^T g(T_i; \theta))}\,, \tag{11}$$

where $\rho'_\gamma(v)$ is the first order derivative of $\rho_\gamma(v)$.

[1]Notice that we omit Z_0 from T because its value is implied by Z through $Z_0 = 1 - \sum_{j=1}^p Z_j$.

1.2 Observational studies

When the treatment (group) assignment is not random, we can still use the GEL as a weighting method. GEL is used as a way to re-weight the probability of each observation so that our sample is as if it had been generated by a randomized experiment. The parameter of interest θ^0 satisfies the following moment conditions:

$$\mathrm{E}_0(g(T,\theta^0)) = 0\,, \tag{12}$$

where θ^0 is as in Section 1.1, and $g(t,\theta)$ is defined as

$$g(t;\theta) = \begin{pmatrix} y-\theta_1-\theta_2^T z \\ (y-\theta_1-\theta_2^T z)z \\ z-\theta_3 \\ (z-\theta_3)\otimes u(x) \\ u(x)-u_0 \end{pmatrix}\,, \quad t=(x,z,y)\in\mathbb{R}^{p+q+1}\,, \tag{13}$$

where u_0 is the expected value of $u(X)$ for a target population. Note that while the first three moment conditions (under E_0) identify the parameters, the fourth moment condition makes Z "almost independent" of X as $k\to\infty$. The last condition is what differentiates randomized experiments from observational studies. It imposes moments of X to match the ones from a given target population. The choice of u_0 is driven by the type of causal effect we are interested in. We will present the different options in the next section.

2 Estimating the causal effect

The package is based on the "gmm4" package (Chaussé, 2019). As for the methods included in the package, there are two ways to proceed. We can create a model object and use the *modelFit* method to estimate it, or we can directly estimate a model using a general function. We first present the first approach because it helps better understand the structure of the package.

2.1 An S4 class object for causal inference

To illustrate the methods, we consider the experiment analyzed first by Lalonde (1986) and used later by Dehejia and Wahba (1999, 2002). The objective of the original paper was to measure the effect of a training program on the real income. The dependent variable is the real income in 1978 and the covariates used for matching the treated group to the control are age, education, 1975 real income and dummy variables for race, marital status, and academic achievement.

First, we load the package and the dataset:

```
library(causalGel)
data(nsw)
## We express income in thousands for better stability
nsw$re78 <- nsw$re78/1000
nsw$re75 <- nsw$re75/1000
```

The model class, is "causalGel" which inherits directly from "functionGel" class. The constructor is the causalModel() function. The arguments are:

- *g*: A formula that defines the regression of the outome on the treatment indicator. For our dataset, the variable "treat" is the indicator, and "re78" is the outcome. The formula is therefore:

```
g <- re78~treat
```

- *balm*: A formula or a data.frame representing $u(X)$. For example, if we want to balance 1975 income, age, education and race, we would use the following:

```
balm <- ~age+ed+black+hisp+re75
```

- *theta0*: An optional starting value to be passed to the numerical algorithm.
- *momType*: This is the main argument to determine which type of causal effect we want to estimate. The options are:
  - "ACE": This one is for estimating the average causal effect. The moment function $g(t;\theta)$ is defined by Equation (13), and μ_0 is defined as:

$$\mu_0 = \frac{1}{n}\sum_{i=1}^n u(X_i)$$

  - "ACT": This is for the causal effect on the treated. In that case, the argument "ACTmom" determines which of the treatment groups we are refering to. The moment function $g(t;\theta)$ is defined by Equation (13), and μ_0 is defined as:

$$\mu_0 = \frac{1}{n_j}\sum_{i=1}^n Z_{ji}u(X_i)\,,$$

    where $n_j = \sum_{i=1}^n Z_{ji}$, and j is the value of "ACTmom".
  - "ACC": This is for the causal effect on the control. The moment function $g(t;\theta)$ is defined by Equation (13), and μ_0 is defined as:

$$\mu_0 = \frac{1}{n_0}\sum_{i=1}^n Z_{0i}u(X_i)\,,$$

    where $n_0 = \sum_{i=1}^n Z_{0i}$.
  - "uncondBal": This is used to estimate the average causal effect in randomized trials. The moment function $g(t;\theta)$ is defined by Equation (8). In the case of observational data, it is not recommended because the moments are balanced, but represent estimates of the moments of an undefined population.
  - "fixedMom": The causal effect of a target population for which $\mathrm{E}(\mu(X)$ is known is estimated. The moment function $g(t;\theta)$ is defined by Equation (13), and μ_0 is set to "popMom", which is another argument of causalModel() (see below).
- *popMom*: A $k\times 1$ vector, representing $\mathrm{E}(\mu(X))$. If provided, *momType* is automatically set to "popMom".
- *gelType*: The type of GEL method. The options are "EL" (the default), "ET", "EEL" and "HD", as defined above. The exponentially tilted empirical likelihood

(ETEL) and exponentially tilted Hellinger distance (ETHD) are also available. The last available method is "REEL" which is the restricted EEL. The solution is obtained by restricting the EEL implied probability, defined in Equation (11), to be non-negative.

- *rhoFct*: An optional $\rho(v)$ function if the desired GEL method is not available in the package (see the vignette of gmm4 for more details).
- *data*: A data.frame with all the variables needed to evaluate the formulas g and *balm*.

The following are three different models:

```
ace <- causalModel(g, balm, nsw, momType="ACE", gelType="EL")
act <- causalModel(g, balm, nsw, momType="ACT", gelType="EL")
aceRT <- causalModel(g, balm, nsw, momType="uncondBal", gelType="EL")
```

The third one is the ACE assuming randomized assignments. A print method for that class summarizes the model:

```
ace

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 13
## Number of balancing covariates: 5
## Sample size:  722
```

The option "printBalCov" allows us to see the balancing covariates:

```
print(act, printBalCov=TRUE)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Causal effect on the treated
## Number of treatments: 1
## Number of moment conditions: 13
## Number of balancing covariates: 5
## Sample size:  722
## Balancing covariates:
##   age, ed, black
##  hisp, re75
```

To add powers and interactions, we can follow the usual rules for formula. Here is an example:

```
balm2 <- ~age*ed+black+hisp+re75+I(re75^2)
ace2 <- causalModel(g, balm2, nsw, momType="ACE", gelType="EL")
print(ace2, printBalCov=TRUE)
```

```
## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 17
## Number of balancing covariates: 7
## Sample size:  722
## Balancing covariates:
##   age, ed, black
##  hisp, re75, I(re75^2)
##  age:ed
```

2.2 The *modelFit* method and the "causalGelfit" object

It simply calls the method for "functionGel", and creates a "causalGelfit" class object. The simpliest way to use it is to only provide the model object. The *print* allows to show the $\hat{\lambda}$ and model.

```
fit1 <- modelFit(ace)
print(fit1, model=FALSE, lambda=TRUE)

## Convergence Theta:  0
## Convergence Lambda:  0
## coefficients:
##       control   causalEffect  probTreatment
##     5.0945926      0.8223392      0.4113576
## lambdas:
##       control   causalEffect  probTreatment      treat_age       treat_ed
## -5.171057e-07   1.432392e-06   5.289036e-01  -3.288704e-03  -5.962977e-02
##   treat_black     treat_hisp     treat_re75            age             ed
##  1.635445e-01   2.907282e-01   8.043465e-04  -5.643722e-05   4.119271e-03
##         black           hisp           re75
## -5.736834e-03   3.484319e-03  -8.729253e-04
```

The coefficients are labeled as "control" for θ_1, "causalEffect" for θ_2, and "probTreatment" for θ_3. The following are the existing methods for "causalGelfit" objects. Since "causalGelfit" contains a "gelfit" object, most methods are for GEL fit objects.

- *vcov*: It computes the covariance matrix of $\hat{\theta}$ and $\hat{\lambda}$ in a list. The list contains other information used by other methods. We don't often need to run the method, but if needed, the covariance matrix of $\hat{\theta}$ is

  ```
  vcov(fit1)$vcov_par

  ##
  ##   7.481449e-02 -7.392353e-02 4.311762e-06
  ##  -7.392353e-02  2.242136e-01 2.762385e-05
  ##   4.311762e-06  2.762385e-05 3.353775e-04
  ```

  By default, the covariance matrix is robust to misspecification, which is what we should used in observational studies. For randomized trials, we can set the argument "robToMiss" to FALSE, because it is not needed.

- *confint* The method computes a confidence interval. By default, it is a Wald type of confidence:

```
confint(fit1)

##
##  Wald type confidence interval
##                   0.025   0.975
## control          4.5585  5.6307
## causalEffect    -0.1057  1.7504
## probTreatment    0.3755  0.4473
```

It is also possible to get an interval based on the inversion of the likelihoo ratio. The empirical likelihood confidence is:

```
confint(fit1, 2, type="invLR")

##
##  Confidence interval based on the inversion of the LR test
##                   0.025  0.975
## causalEffect  -0.07155  1.808
```

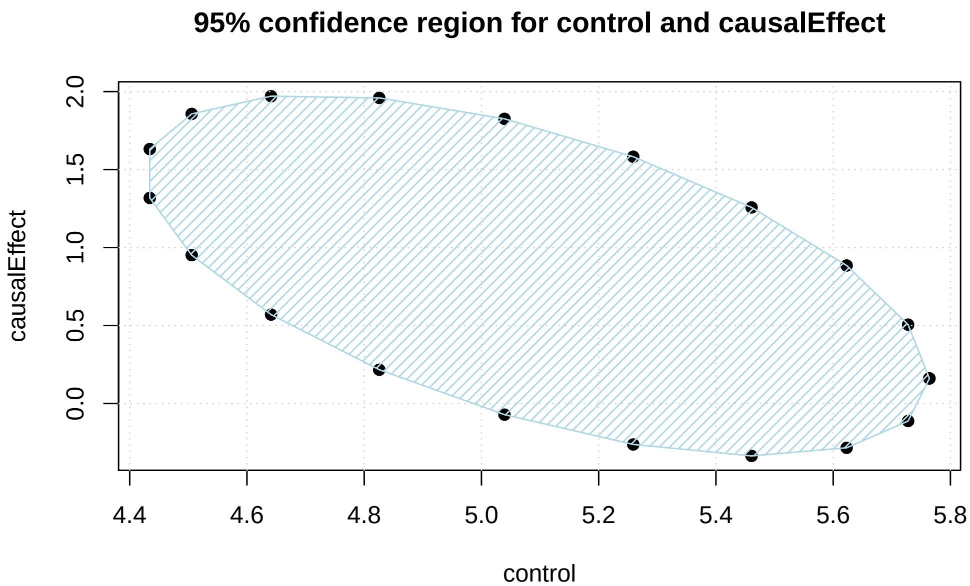
Confidence regions are also possible using a pair of coefficients:

```
cr <- confint(fit1, 1:2, area=TRUE)
cr

## Wald type confidence region
## ***************************
## Level: 0.95
## Number of points: 20
## Variables:
##  Range for control: [4.434, 5.764]
##  Range for causalEffect: [-0.3362, 1.971]
```

This is an object of class "mconfint" for which a *plot* method exists:

```
plot(cr, col="lightblue", density=20)
```



- *summary*: The method creates a “summaryGel” with its own *print* method. It returns an output similar to the summary method of “lm” objects.

```
summary(fit1)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 13
## Number of balancing covariates: 5
## Sample size:  722
## Convergence Theta: 0
## Convergence Lambda: 0
## Average |Sum of pt*gt()]|: 3.1458e-16
## |Sum of pt - 1|: 0
##
## coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## control       5.094593   0.273522 18.6259  < 2e-16 ***
## causalEffect  0.822339   0.473512  1.7367  0.08244 .
## probTreatment 0.411358   0.018313 22.4622  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Lambdas:
##                  Estimate  Std. Error  t value Pr(>|t|)
## control       -5.1711e-07  2.5083e-08 -20.6155   <2e-16 ***
## causalEffect   1.4324e-06  5.8736e-08  24.3871   <2e-16 ***
## probTreatment  5.2890e-01  6.0880e-01   0.8688   0.3850
## treat_age     -3.2887e-03  1.1630e-02  -0.2828   0.7773
## treat_ed      -5.9630e-02  4.4931e-02  -1.3271   0.1845
```

```
## treat_black    1.6354e-01  2.6588e-01   0.6151   0.5385
## treat_hisp     2.9073e-01  3.4583e-01   0.8407   0.4005
## treat_re75     8.0435e-04  1.5495e-02   0.0519   0.9586
## age           -5.6437e-05  7.1538e-04  -0.0789   0.9371
## ed             4.1193e-03  4.4968e-03   0.9160   0.3596
## black         -5.7368e-03  1.1078e-02  -0.5178   0.6046
## hisp           3.4843e-03  1.8667e-02   0.1867   0.8519
## re75          -8.7293e-04  9.3098e-04  -0.9376   0.3484
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##  Test E(g)=0
##       Statistics  df   pvalue
## LR:       3.0180  10  0.98100
## LM:       3.0169  10  0.98102
##  J:       3.0177  10  0.98100
```

2.3 Other useful methods

Instead of creating a new model with different balancing moments $\mu(X)$, it is possible to use the method "[" to subset the existing $\mu(X)$. We can think of a model object as being a two dimensional array, the first dimension being the balancing moments, and the second being the observations. Consider the following

```
ace <- causalModel(re78~treat, ~(age+black+ed)*(age+black+ed) + I(age^2) + I(ed^2),
                   data=nsw)
print(ace, TRUE)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 19
## Number of balancing covariates: 8
## Sample size:  722
## Balancing covariates:
##   age, black, ed
##  I(age^2), I(ed^2), age:black
##  age:ed, black:ed
```

Suppose we want to removed the squared components:

```
print(ace[-c(3,4)], TRUE)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 15
## Number of balancing covariates: 6
```

```
## Sample size:  722
## Balancing covariates:
##   age, black, I(ed^2)
##  age:black, age:ed, black:ed
```

Or remove interactions

```
print(ace[1:5], TRUE)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 13
## Number of balancing covariates: 5
## Sample size:  722
## Balancing covariates:
##   age, black, ed
##  I(age^2), I(ed^2)
```

We can use a subset by add a second argument:

```
print(ace[,1:100], TRUE)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 19
## Number of balancing covariates: 8
## Sample size:  100
## Balancing covariates:
##   age, black, ed
##  I(age^2), I(ed^2), age:black
##  age:ed, black:ed

print(ace[1:3,nsw$re75>0], TRUE)

## Causal Model using GEL Methods
## *******************************
## GEL Type:  EL
## Model type: Average causal effect
## Number of treatments: 1
## Number of moment conditions: 9
## Number of balancing covariates: 3
## Sample size:  433
## Balancing covariates:
##   age, black, ed
```

An easy way to re-estimate a new model specified by "[", is to use the method for "causalGelfit" objects:

	Model 1	Model 2	Model 3
control	5.1263***	5.1627***	5.1013***
	(0.2795)	(0.2972)	(0.2777)
causalEffect	0.7713	0.6855	0.4535
	(0.4798)	(0.4876)	(0.7817)
probTreatment	0.4114***	0.4120***	0.1500***
	(0.0183)	(0.0184)	(0.0160)
Num. obs.	722	716	500
Num. Bal. Cov.	8	8	3

*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$

Table 1: Statistical models

```
fit <- modelFit(ace)
fit2 <- fit[,nsw$age<48]
fit3 <- fit[1:3,1:500]
```

The results are shown in Table 1, which is constructed using the "texreg" package of Leifeld (2013). The code for the *extract* method is in the appendix.

Details about the convergence are obtained using the *checkConv* method:

```
checkConv(fit)

## Convergence details of the Causal estimation
## ********************************************
## Average causal effect
##
## Convergence of the Lambdas: TRUE
## Convergence of the Coefficients: TRUE
## Achieved moment balancing: TRUE
##
## Moments for each group:
##              treat=0    treat=1
## age        24.520776  24.520776
## black       0.800554   0.800554
## ed         10.267313  10.267313
## I(age^2)  645.110803 645.110803
## I(ed^2)   108.319945 108.319945
## age:black  19.876731  19.876731
## age:ed    252.038781 252.038781
## black:ed    8.265928   8.265928
```

It compares sample moments of $\mu(X)$ for each group, using the estimated implied probabilities. We can then see if the balancing was achieved. As an example, the first column is $[\sum_{i=1}^n \hat{p}_i(1-Z_i)\mu(X_i)]/[\sum_{i=1}^n \hat{p}_i(1-Z_i)]$, and the second column is $[\sum_{i=1}^n \hat{p}_i Z_i\mu(X_i)]/[\sum_{i=1}^n \hat{p}_i Z_i]$, which are respectively estimates of $\mathrm{E}(\mu(X)|Z=0)$ and $\mathrm{E}(\mu(X)|Z=1)$. We can see that the moments are well balanced, at least up to six decimals.

3 The aceGEL function

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A Some extra codes

The following *extract* is used with the "texreg" package of Leifeld (2013) to produce nice latex tables.

```
library(causalGel)
library(texreg)
setMethod("extract", "causalGelfit",
          function(model, includeSpecTest=FALSE,
                   specTest=c("LR","LM","J"), include.nobs=TRUE,
                   include.obj.fcn=TRUE, ...)
              {
                  specTest <- match.arg(specTest)
                  s <- summary(model, ...)
                  wspecTest <- grep(specTest, rownames(s@specTest@test))
                  spec <- modelDims(model@model)
                  coefs <- s@coef
                  names <- rownames(coefs)
                  coef <- coefs[, 1]
                  se <- coefs[, 2]
                  pval <- coefs[, 4]
                  n <- model@model@n
                  gof <- numeric()
                  gof.names <- character()
```

```r
        gof.decimal <- logical()
        if (includeSpecTest) {
            if (spec$k == spec$q)
                {
                    obj.fcn <- NA
                    obj.pv <- NA
                } else {
                    obj.fcn <- s@specTest@test[wspecTest,1]
                    obj.pv <- s@specTest@test[wspecTest,3]
                }
            gof <- c(gof, obj.fcn, obj.pv)
            gof.names <- c(gof.names,
                           paste(specTest,"-test Statistics", sep=""),
                           paste(specTest,"-test p-value", sep=""))
            gof.decimal <- c(gof.decimal, TRUE, TRUE)
        }
        if (include.nobs == TRUE) {
            gof <- c(gof, n)
            gof.names <- c(gof.names, "Num.\\ obs.")
            gof.decimal <- c(gof.decimal, FALSE)
        }
        nbal <- length(model@model@X@balCov)
        gof.names <- c(gof.names, "Num. Bal. Cov.")
        gof <- c(gof, nbal)
        gof.decimal <- c(gof.decimal, FALSE)
        tr <- createTexreg(coef.names = names, coef = coef,
                           se = se, pvalues = pval,
                           gof.names = gof.names, gof = gof,
                           gof.decimal = gof.decimal)
        return(tr)
    })
```