Generalized Empirical Likelihood with R

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Abstract

This an extention of the gmmS4 vignette, to explain how to use the package for generalized empirical likelihood estimation.

Contents

1 A very brief review of the GEL method

We present how to use the package to estimate models by the Generalized Empirical Likelihood method (GEL) (see Newey and Smith (2004) for the iid case and Anatolyev (2005) for weakly dependent processes). We assume that the reader has read the gmmS4 vignette in which many classes and methods needed are defined. We first describe the method without going into too much details. The author can refer to the above papers for a detailed description, or Chaussé (2010) who explains GEL estimation using the gmm (Chausse (2019))package.

The estimation is based on the following moment conditions

$$\mathrm{E}[g_i(\theta)] = 0,$$

For the iid case, the estimator is defined as the solution to either

$$\hat{\theta} = \arg\min_{\theta, p_i} \sum_{i=1}^{n} h_n(p_i)$$

subject to,

$$\sum_{i=1}^{n} p_i g_i(\theta) = 0$$

and

$$\sum_{i=1}^{n} p_i = 1,$$

where $h_n(p_i)$ belongs to the following Cressie-Read family of discrepancies:

$$h_n(p_i) = \frac{[\gamma(\gamma+1)]^{-1}[(np_i)^{\gamma+1} - 1]}{n},$$

or

$$\hat{\theta} = \arg\min_{\theta} \left[\max_{\lambda} \frac{1}{n} \sum_{i=1}^{n} \rho\left(\lambda' g_i(\theta)\right) \right] \tag{1}$$

The first is the primal and the second is the dual problem, the latter being preferred in general to define GEL estimators. The vector λ is the Lagrange multiplier associated with the first constraint in the primal problem. Its estimator plays an important role in testing the validity of the moment conditions. $\rho(v)$ is a strictly concave function normalized so that $\rho'(0) = \rho''(0) = -1$. It can be shown that $\rho(v) = \ln(1-v)$ corresponds to Empirical Likelihood (EL) of Owen (2001) , $\rho(v) = -\exp(v)$ to the Exponential Tilting (ET) of Kitamura and Stutzer (1997), and $\rho(v) = (-v - v^2/2)$ to the Continuous Updated GMM estimator (CUE) of Hansen et al. (1996). In the context of GEL, the CUE is also known at the Euclidean Empirical Likelihood (EEL), because it corresponds to $h_n(p_i)$ being the Euclidean distance.

If we relax the iid assumption, the problem is identical, but the moment function must be smoothed using a kernel method. Smith (2001) proposes to replace $g_i(\theta)$ by:

$$g_i^w(\theta) = \sum_{s=-m}^{m} w(s) g_{i-s}(\theta)$$

where $w(s)$ are kernel based weights that sum to one (see also Kitamura and Stutzer (1997) and Smith (2001).

2 An S4 class object for GEL models

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Appendix

A Some extra codes