

Package ‘MuMIn’

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Description Model selection and model averaging based on information criteria (AICc and alike).

License GPL-2

Depends R (>= 2.13.0)

Imports stats

Suggests

stats4, nlme, mgcv (>= 1.7.5), lme4 (>= 0.999375-16), gamm4, MASS, nnet, survival, geepack

Enhances

aod, coxme, glmmML, MCMCglmm, pscl, spdep, unmarked, ordinal, gee, splm, logistf, caper

LazyLoad yes

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MuMIn-package	<i>Multi-model inference</i>
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Description

The package **MuMIn** contains functions to streamline information-theoretic model selection and carry out model averaging based on the information criteria.

Details

The collection of functions includes:

[dredge](#) performs automated model selection with subsets of the supplied ‘global’ model, and optional choices of other model properties (such as different link functions). The set of models may be generated either with ‘all possible’ combinations, or tailored according to the conditions specified.

[pdredge](#) does the same, but can parallelize model fitting process using a cluster.

[model.sel](#) creates a model selection table from hand-picked models.

[model.avg](#) calculates model averaged parameters, with standard errors and confidence intervals.

[AICc](#) calculates second-order Akaike information criterion.

For a complete list of functions, use `library(help = "MuMIn")`.

By default, AIC_c is used to rank the models and to obtain model selection probabilities, though any other information criteria can be utilised. At least the following ones are currently implemented in R: [AIC](#) and [BIC](#) in package **stats**, and [QAIC](#), [QAICc](#), [ICOMP](#), [CAICF](#), and [Mallows' Cp](#) in **MuMIn**. There is also [DIC](#) extractor for MCMC models, and [QIC](#) for GEE.

Most of R's common modelling functions are supported, for a full inventory see [list of supported models](#).

Author(s)

Kamil Bartoń

References

Burnham, K. P. and Anderson, D. R (2002) *Model selection and multimodel inference: a practical information-theoretic approach*. 2nd ed. New York, Springer-Verlag.

See Also

[AIC](#), [step](#) or [stepAIC](#) for stepwise model selection by AIC.

Examples

```
data(Cement)

fm1 <- lm(y ~ ., data = Cement)
ms1 <- dredge(fm1)
plot(ms1)

model.avg(ms1, subset = delta < 4)

confset.95p <- get.models(ms1, cumsum(weight) <= .95)
avgmod.95p <- model.avg(confset.95p)
summary(avgmod.95p)
confint(avgmod.95p)
```

AICc

*Second-order Akaike Information Criterion***Description**

Calculate second-order Akaike information criterion for one or several fitted model objects (AIC_c, AIC for small samples).

Usage

```
AICc(object, ..., k = 2, REML = NULL)
```

Arguments

<code>object</code>	a fitted model object for which there exists a <code>logLik</code> method, or a <code>logLik</code> object.
<code>...</code>	optionally more fitted model objects.
<code>k</code>	the ‘penalty’ per parameter to be used; the default <code>k = 2</code> is the classical AIC.
<code>REML</code>	optional logical value, passed to the <code>logLik</code> method indicating whether the restricted log-likelihood or log-likelihood should be used. The default is to use the method used for model estimation.

Value

If just one object is provided, returns a numeric value with the corresponding AIC_c; if more than one object are provided, returns a `data.frame` with rows corresponding to the objects and columns representing the number of parameters in the model (`df`) and AIC_c.

Note

AIC_c should be used instead AIC when sample size is small in comparison to the number of estimated parameters (Burnham & Anderson 2002 recommend its use when $n/K < 40$).

Author(s)

Kamil Bartoń

References

Burnham, K. P. and Anderson, D. R (2002) *Model selection and multimodel inference: a practical information-theoretic approach*. 2nd ed. New York, Springer-Verlag.

Hurvich, C. M. and Tsai, C.-L. (1989) Regression and time series model selection in small samples, *Biometrika* 76: 297–307.

See Also

Akaike's An Information Criterion: [AIC](#)

Other implementations: [AICc](#) in package **AICcmodavg**, [AICc](#) in package **bbmle** and [aicc](#) in package **glmulti**

Examples

```
#Model-averaging mixed models

library(nlme)
data(Orthodont, package = "nlme")

# Fit model by REML
fm2 <- lme(distance ~ Sex*age, data = Orthodont,
  random = ~ 1|Subject / Sex, method = "REML")

# Model selection: ranking by AICc using ML
ms2 <- dredge(fm2, trace = TRUE, rank = "AICc", REML = FALSE)

(attr(ms2, "rank.call"))

# Get the models (fitted by REML, as in the global model)
fmList <- get.models(ms2, 1:4)

# Because the models originate from 'dredge(..., rank=AICc, REML=FALSE)',
# the default weights in 'model.avg' are ML based:
summary(model.avg(fmList))

# same result
#model.avg(fmList, rank = "AICc", rank.args = list(REML=FALSE))
```

Beetle

Flour beetle mortality data

Description

Mortality of flour beetles (*Tribolium confusum*) due to exposure to gaseous carbon disulfide CS₂, from Bliss (1935).

Usage

```
data(Beetle)
```

Format

Beetle is a data frame with 5 elements.

dose The dose of CS₂ in mg/L

n.tested Number of beetles tested

n.killed Number of beetles killed

Prop A matrix with two columns named **n.killed** and **n.survived**

mortality Observed mortality rate.

Source

Bliss C. I. (1935) The calculation of the dosage-mortality curve. *Annals of Applied Biology*, 22: 134–167.

References

Burnham, K. P. and Anderson, D. R. (2002) *Model selection and multimodel inference: a practical information-theoretic approach*. 2nd ed. New York, Springer-Verlag.

Examples

```
# "Logistic regression example"
# from Burnham & Anderson (2002) chapter 4.11

data(Beetle)

# Fit a global model with all the considered variables
globmod <- glm(Prop ~ dose + I(dose^2) + log(dose) + I(log(dose)^2),
  data = Beetle, family = binomial)

# A logical expression defining the subset of models to use:
# * either log(dose) or dose
# * the quadratic terms can appear only together with linear terms
msubset <- expression(xor(dose, 'log(dose)') & (dose | !'I(dose^2)')
  & ('log(dose)' | !'I(log(dose)^2)'))

# Table 4.6

# Use 'varying' argument to fit models with different link functions
# Note the use of 'alist' rather than 'list' in order to keep the
# 'family' objects unevaluated
varying.link <- list(family = alist(
  logit = binomial("logit"),
  probit = binomial("probit"),
  cloglog = binomial("cloglog")
))
```

```

(ms12 <- dredge(globmod, subset = msubset, varying = varying.link,
  rank = AIC))

# Table 4.7 "models justifiable a priori"
(ms3 <- subset(ms12, has(dose, !'I(dose^2)'))
# The same result, but would fit the models again:
# ms3 <- update(ms12, update(globmod, . ~ dose), subset =,
#   fixed = ~dose)

mod3 <- get.models(ms3, 1:3)

# Table 4.8. Predicted mortality probability at dose 40.

# calculate confidence intervals on logit scale
logit.ci <- function(p, se, quantile = 2) {
  C. <- exp(quantile * se / (p * (1 - p)))
  p / (p + (1 - p) * c(C., 1/C.))
}

mavg3 <- model.avg(mod3, revised.var = FALSE)

pred <- sapply(mod3, predict, newdata = list(dose = 40), se.fit = TRUE,
  type = "response")

# get predictions both from component and averaged models
pred <- lapply(c(component = mod3, list(averaged = mavg3)), predict,
  newdata = list(dose = 40), type = "response", se.fit = TRUE)
# reshape predicted values
pred <- t(sapply(pred, function(x) unlist(x)[1:2]))
colnames(pred) <- c("fit", "se.fit")

# build the table
tab <- cbind(
  c(Weights(ms3), NA),
  pred,
  matrix(logit.ci(pred[, "fit"], pred[, "se.fit"],
    quantile = c(rep(1.96, 3), 2)), ncol = 2)
)
colnames(tab) <- c("Akaike weight", "Predicted(40)", "SE", "Lower CI",
  "Upper CI")
rownames(tab) <- c(as.character(ms3$family), "model averaged")
print(tab, digits = 3, na.print = "")

# Figure 4.3
newdata <- list(dose = seq(min(Beetle$dose), max(Beetle$dose), length.out = 25))

# add model-averaged prediction with CI, using the same method as above
avpred <- predict(mavg3, newdata, se.fit = TRUE, type = "response")

avci <- matrix(logit.ci(avpred$fit, avpred$se.fit, quantile = 2), ncol = 2)

matplot(newdata$dose, sapply(mod3, predict, newdata, type = "response"),
  type = "l", xlab = quote(list("Dose of" ~ CS[2], (mg/L))),
  ylab = "Mortality", col = 2:4, lty = 3, lwd = 1
)
matplot(newdata$dose, cbind(avpred$fit, avci), type = "l", add = TRUE,
  lwd = 1, lty = c(1, 2, 2), col = 1)

```

```
legend("topleft", NULL, c(as.character(ms3$family), expression('averaged'
  %+-% CI)), lty = c(3, 3, 3, 1), col = c(2:4, 1))
```

Cement	<i>Cement hardening data</i>
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Description

Cement hardening data from Woods et al (1939).

Usage

```
data(Cement)
```

Format

Cement is a data frame with 5 variables. x1-x4 are four predictor variables expressed as a percentage of weight.

X1 calcium aluminate

X2 tricalcium silicate

X3 tetracalcium alumino ferrite

X4 dicalcium silicate

y calories of heat evolved per gram of cement after 180 days of hardening.

Source

Woods H., Steinour H.H., Starke H.R. (1932) Effect of composition of Portland cement on heat evolved during hardening. *Industrial & Engineering Chemistry* 24, 1207-1214.

References

Burnham, K. P. and Anderson, D. R (2002) *Model selection and multimodel inference: a practical information-theoretic approach*. 2nd ed. New York, Springer-Verlag.

dredge	<i>Automated model selection</i>
--------	----------------------------------

Description

Generate a set of models with combinations (subsets) of the terms in the global model, with optional rules for model inclusion.

Usage

```
dredge(global.model, beta = FALSE, evaluate = TRUE, rank = "AICc",
       fixed = NULL, m.max = NA, m.min = 0, subset, marg.ex = NULL,
       trace = FALSE, varying, extra, ct.args = NULL, ...)

## S3 method for class 'model.selection'
print(x, abbrev.names = TRUE, warnings = getOption("warn") != -1L, ...)
```

Arguments

<code>global.model</code>	a fitted 'global' model object. See 'Details' for a list of supported types.
<code>beta</code>	logical, should standardized coefficients be returned?
<code>evaluate</code>	whether to evaluate and rank the models. If FALSE, a list of model calls is returned.
<code>rank</code>	optional custom rank function (information criterion) to be used instead AICc, e.g. AIC, QAIC or BIC. See 'Details'.
<code>fixed</code>	optional, either a single sided formula or a character vector giving names of terms to be included in all models.
<code>m.max, m.min</code>	optionally the maximum and minimum number of terms in a single model (excluding the intercept), <code>m.max</code> defaults to the number of terms in <code>global.model</code> .
<code>subset</code>	logical expression describing models to keep in the resulting set. See 'Details'.
<code>marg.ex</code>	a character vector specifying names of variables for which NOT to check for marginality restrictions when generating model formulas. If this argument is set to TRUE, all combinations of terms are used (i.e. no checking). If NA or missing, the exceptions will be found based on the terms of <code>global.model</code> . See 'Details'.
<code>trace</code>	if TRUE, all calls to the fitting function (i.e. updated <code>global.model</code> calls) are printed before actual fitting takes place.
<code>varying</code>	optionally, a named list describing the additional arguments to vary between the generated models. Item names correspond to the arguments, and each item provides a list of choices (i.e. <code>list(arg1 = list(choice1, choice2, ...), ...)</code>). Complex elements in the choice list (such as family objects) should be either named (uniquely) or quoted (unevaluated, e.g. using alist , see quote), otherwise it may produce rather unpleasant effects. See example in Beetle .
<code>extra</code>	optional additional statistics to include in the result, provided as functions, function names or a list of such (best if named or quoted). Similarly as in <code>rank</code> argument, each function must accept fitted model object as an argument and return a (value coercible to a) numeric vector. These can be e.g. additional information criteria or goodness-of-fit statistics. The character strings "R^2" and "adjR^2" are treated in a special way, and will add a likelihood-ratio based R^2 and modified- R^2 respectively to the result (this is more efficient than using r.squaredLR directly).
<code>x</code>	a <code>model.selection</code> object, returned by <code>dredge</code> .
<code>abbrev.names</code>	should printed variable names be abbreviated? (useful with many variables).
<code>warnings</code>	if TRUE, errors and warnings issued during the model fitting are printed below the table (currently, only with <code>pdredge</code>). To permanently remove the warnings, set the object's attribute "warnings" to NULL.

ct.args	optional list of arguments to be passed to <code>coefTable</code> (e.g. dispersion parameter for <code>glm</code> affecting standard errors used in subsequent <code>model averaging</code>).
...	optional arguments for the rank function. Any can be an expression (of <code>model call</code>), in which case any <code>x</code> within it will be substituted with a current model.

Details

Models are fitted one by one through repeated evaluation of modified calls to the `global.model` (in a similar fashion as with `update`). This approach, while robust in that it can be applied to a variety of different model object types is not very efficient and may be time-intensive.

Note that the number of combinations grows exponentially with number of predictor variables (2^N , less when interactions are present, see below). As there can be potentially a large number of models to evaluate, to avoid memory overflow the fitted model objects are not stored in the result. To get (a subset of) the models, use `get.models` on the object returned by `dredge`.

For a list of model types that can be used as a `global.model` see [list of supported models](#). Modelling functions not storing call in their result should be evaluated *via* the wrapper created by `updateable`.

Information criterion: rank is found by a call to `match.fun` and may be specified as a function or a symbol (e.g. a back-quoted name) or a character string specifying a function to be searched for from the environment of the call to `dredge`. The function rank must accept model object as its first argument and always return a scalar. Typical choice for rank would be "AIC", "BIC", or "QAIC".

Interactions: `dredge` by default respects marginality constraints, so “all possible combinations” do not include models containing interactions without their respective main effects and all lower order terms. This behaviour can be altered by `marg.ex` argument, which can be used to allow for simple nested designs. For example, with global model of form `a / (x + z)`, one would use `marg.ex = "a"` and `fixed = "a"`. If `global.model` uses such a formula and `marg.ex` is missing or NA, it will be adjusted automatically.

Subsetting: There are three ways to constrain the resulting set of models: setting limits to the number of terms in a model with `m.max` and `m.min`, binding term(s) to all models with `fixed`, and more complex rules can be applied using argument `subset`. To be included in the selection table, the model formulation must satisfy all these conditions.

`subset` can take either a form of an *expression* or a *matrix*. The latter should be a lower triangular matrix with logical values, where columns and rows correspond to `global.model` terms. Value `subset["a", "b"] == FALSE` will exclude any model containing both *a* and *b*. Values other than FALSE (or 0) are taken as TRUE.

In the form of expression, the argument `subset` acts in a similar fashion to that in the function `subset` for `data.frames`: model terms can be referred to by name as variables in the expression, with the difference being that they are always logical (i.e. TRUE if a term exists in the model).

The expression can contain any of the `global.model` terms (`getAllTerms(global.model)` lists them), as well as names of the varying argument items. Names of `global.model` terms take precedence when identical to names of varying, so to avoid ambiguity varying variables in `subset` expression should be enclosed in `V()` (e.g. `subset = V(family) == "Gamma"` assuming that varying is something like `list(family = c(..., "Gamma"))`).

If element names in varying are missing, the elements themselves are used. Call and symbol elements are represented as character values (via `deparse`), and everything except numeric, logical, character and NULL values is replaced by item numbers (e.g. `varying = list(family = list(..., Gamma))`).

should be referred to as `subset = V(family) == 2`. This can quickly become confusing, therefore it is recommended to use named lists in most cases. `demo(dredge.varying)` provides examples.

The subset expression can also contain variable `'*nvar*'` (needs to be backtick-quoted), which is equal to number of terms in the model (**not** the number of estimated parameters K).

To make inclusion of a variable conditional on presence of some other variable, a function `dc` (“**d**ependency **c**hain”) can be used in the subset expression. `dc` takes any number of variables as arguments, and allows a variable to be included only if all preceding variables are also present (e.g. `subset = dc(a, b, c)` allows for models of form `a`, `a+b` and `a+b+c` but not `b`, `c`, `b+c` or `a+c`).

subset expression can have a form of an unevaluated call, expression object, or a one sided formula. See ‘Examples’. Compound model terms (such as ‘as-is’ expressions within `I()` or smooths in `gam`) should be treated as non-syntactic names and enclosed in back-ticks (e.g. `subset = 's(x, k = 2)' || 'I(log(x))'`, see [Quotes](#)). Mind the spacing, names must match exactly the term names in model’s formula. To simply keep certain terms in all models, use of argument `fixed` is more efficient.

subset expression syntax summary:

<code>a & b</code>	indicates that variables <i>a</i> and <i>b</i> must be present (see ‘ Logical Operators ’)
<code>V(x)</code>	indicates a varying variable <i>x</i>
<code>dc(a,b,c,...)</code>	‘dependency chain’: <i>a</i> is allowed only if <i>b</i> is present, and <i>b</i> only if <i>c</i> is present, etc.
<code>'*nvar*'</code>	number of variables

Missing values: Use of `na.action = na.omit` (R’s default) in `global.model` should be avoided, as it results with sub-models fitted to different data sets, if there are missing values. Warning is given if it is detected.

Methods: There are `subset` and `plot` methods, the latter produces a graphical representation of model weights and variable relative importance. Coefficients can be extracted with `coef` or `coefTable`.

Value

`dredge` returns an object of class `model.selection`, being a `data.frame` with models’ coefficients (or presence/NA for factors), *df* - number of parameters, log-likelihood, the information criterion value, delta-IC and *Akaike weight*. Models are ordered by the value of the information criterion specified by rank (lowest on top).

The attribute `"calls"` is a list containing the model calls used (arranged in the same order as the models). Other attributes: `"global"` - the `global.model` object, `"rank"` - the rank function used, `"call"` - the matched call, and `"warnings"` - list of errors and warnings given by the modelling function during the fitting, with model number appended to each. The associated model call can be found with `attr(*, "calls")[["i"]]`, where *i* is the model number.

Note

Users should keep in mind the hazards that a “thoughtless approach” of evaluating all possible models poses. Although this procedure is in certain cases useful and justified, it may result in selecting a spurious “best” model, due to the model selection bias.

“Let the computer find out” is a poor strategy and usually reflects the fact that the researcher did not bother to think clearly about the problem of interest and its scientific setting (Burnham and Anderson, 2002).

Author(s)

Kamil Bartoń

See Also

[pdredge](#) is a parallelized version of this function (uses a cluster).

[get.models](#), [model.avg](#), [model.sel](#) for manual model selection tables.

Possible alternatives: [glmulti](#) in package **glmulti** and [bestglm](#) (**bestglm**). [regsubsets](#) in package **leaps** also performs all-subsets regression.

Examples

```
# Example from Burnham and Anderson (2002), page 100:
data(Cement)
fm1 <- lm(y ~ ., data = Cement)
dd <- dredge(fm1)
subset(dd, delta < 4)

# Visualize the model selection table:
if(require(graphics))
  plot(dd)

# Model average models with delta AICc < 4
model.avg(dd, subset = delta < 4)

#or as a 95% confidence set:
model.avg(dd, subset = cumsum(weight) <= .95) # get averaged coefficients

#'Best' model
summary(get.models(dd, 1))[[1]]

## Not run:
# Examples of using 'subset':
# keep only models containing X3
dredge(fm1, subset = ~ X3) # subset as a formula
dredge(fm1, subset = expression(X3)) # subset as expression object
# the same, but more effective:
dredge(fm1, fixed = "X3")
# exclude models containing both X1 and X2 at the same time
dredge(fm1, subset = !(X1 && X2))
# Fit only models containing either X3 or X4 (but not both);
# include X3 only if X2 is present, and X2 only if X1 is present.
dredge(fm1, subset = dc(X1, X2, X3) && xor(X3, X4))
# the same as above, but without using "dc"
dredge(fm1, subset = (X1 | !X2) && (X2 | !X3) && xor(X3, X4))

# Include only models with up to 2 terms (and intercept)
dredge(fm1, m.max = 2)

## End(Not run)

# Add R^2 and F-statistics, use the 'extra' argument
dredge(fm1, m.max = 1, extra = c("R^2", F = function(x)
```

```
summary(x)$fstatistic[[1]])

# with summary statistics:
dredge(fm1, m.max = 1, extra = list(
  "R^2", "*" = function(x) {
    s <- summary(x)
    c(Rsq = s$r.squared, adjRsq = s$adj.r.squared,
      F = s$fstatistic[[1]])
  })
)

# Add other information criteria (but rank with AICc):
dredge(fm1, m.max = 1, extra = alist(AIC, BIC, ICOMP, Cp))
```

Formula manipulation *Manipulate model formulas*

Description

`simplify.formula` rewrites a formula using shorthand notation. Currently only the factor crossing operator `*` is applied, so that expanded expression such as `a+b+a:b` becomes `a*b`. `expand.formula` does the opposite, additionally expanding other expressions, i.e. all nesting (`/`), grouping and `^`.

Usage

```
simplify.formula(x)
expand.formula(x)
```

Arguments

<code>x</code>	a formula or an object from which it can be extracted (such as a fitted model object).
----------------	--

Author(s)

Kamil Bartoń

See Also

[formula](#)
[delete.response](#), [drop.terms](#), and [reformulate](#)

Examples

```
simplify.formula(y ~ a + b + a:b + (c + b)^2)
simplify.formula(y ~ a + b + a:b + 0)

expand.formula(~ a * b)
```

get.models

*Get models***Description**

Generate a list of fitted model objects from a `model.selection` table. `pget.models` can use parallel computation in a cluster to do that.

Usage

```
get.models(object, subset, ...)
pget.models(object, cluster = NA, subset, ...)
```

Arguments

<code>object</code>	object returned by dredge .
<code>subset</code>	subset of models, an expression evaluated within the model selection table, see the subset method . If it is a character vector, it is interpreted as names of rows to be selected. By default, all model objects are fitted and returned (see ‘Note’).
<code>...</code>	additional arguments to update the models. For example, in <code>lme</code> one may want to use <code>method = "REML"</code> while using "ML" for model selection.
<code>cluster</code>	a cluster object. See pdredge for details.

Value

[list](#) of fitted model objects.

Note

As of version 1.6.3, the default behaviour (if `subset` argument is missing) is to return all the models, rather than a ‘confidence set’ with `delta <= 4`.

Author(s)

Kamil Bartoń

See Also

[dredge](#), [model.avg](#)

Examples

```
# Mixed models:

require(nlme)
fm2 <- lme(distance ~ age + Sex, data = Orthodont,
  random = ~ 1 | Subject, method = "ML")
ms2 <- dredge(fm2)

# Get top-most models, but fitted by REML:
(confset.d4 <- get.models(ms2, subset = delta < 4, method = "REML"))
```

importance	<i>Relative variable importance</i>
------------	-------------------------------------

Description

Sum of ‘Akaike weights’ over all models including the explanatory variable.

Usage

```
importance(x)
```

Arguments

x Either a list of fitted model objects, or a "model.selection" or "averaging" object.

Value

a numeric vector of relative importance values, named as the predictor variables.

Author(s)

Kamil Bartoń

See Also

[Weights](#)

[dredge](#), [model.avg](#), [mod.sel](#)

Examples

```
# Generate some models
data(Cement)
fm1 <- lm(y ~ ., data = Cement)
ms1 <- dredge(fm1)

# Importance can be calculated/extracted from various objects:
importance(ms1)
## Not run:
importance(subset(mod.sel(ms1), delta <= 4))
importance(model.avg(ms1, subset = delta <= 4))
importance(subset(ms1, delta <= 4))
importance(get.models(ms1, delta <= 4))

## End(Not run)

# Re-evaluate the importances according to BIC
# note that re-ranking involves fitting the models again

# 'nobs' is not used here for backwards compatibility
lognobs <- log(length(resid(fm1)))
```

```
importance(subset(mod.sel(ms1, rank = AIC, rank.args = list(k = lognobs)),
  cumsum(weight) <= .95))

# This gives a different result than previous command, because 'subset' is
# applied to the original selection table that is ranked with 'AICc'
importance(model.avg(ms1, rank = AIC, rank.args = list(k = lognobs),
  subset = cumsum(weight) <= .95))
```

Information criteria *Various information criteria*

Description

Calculate Mallows' C_p and Bozdogan's ICOMP and CAICF information criteria.

Extract or calculate Deviance Information Criterion from MCMCglmm and mer object.

Usage

```
Cp(object, ..., dispersion = NULL)
ICOMP(object, ..., REML = NULL)
CAICF(object, ..., REML = NULL)
DIC(object, ...)
```

Arguments

object	a fitted model object (in case of ICOMP and CAICF, logLik and vcov methods must exist for the object). For DIC, an object of class MCMCglmm or mer.
...	optionally more fitted model objects.
dispersion	the dispersion parameter. If NULL, it is inferred from object.
REML	optional logical value, passed to the logLik method indicating whether the restricted log-likelihood or log-likelihood should be used. The default is to use the method used for model estimation.

Details

Mallows' C_p statistic is the residual deviance plus twice the estimate of σ^2 times the residual degrees of freedom. It is closely related to AIC (and a multiple of it if the dispersion is known).

ICOMP (I for informational and COMP for complexity) penalizes the covariance complexity of the model, rather than the number of parameters directly.

CAICF (C is for 'consistent' and F denotes the use of the Fisher information matrix) includes with penalty the natural logarithm of the determinant of the estimated Fisher information matrix.

Value

If just one object is provided, the functions return a numeric value with the corresponding IC; otherwise a data.frame with rows corresponding to the objects is returned.

References

- Mallows, C. L. (1973) Some comments on *Cp*. *Technometrics* 15: 661–675.
- Bozdogan, H. and Haughton, D.M.A. (1998) Information complexity criteria for regression models. *Comp. Stat. & Data Analysis* 28: 51–76.
- Anderson, D. R. and Burnham, K. P. (1999) Understanding information criteria for selection among capture-recapture or ring recovery models. *Bird Study* 46: 14–21.
- Spiegelhalter, D.J., Best, N.G., Carlin, B.R., van der Linde, A. (2002) Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society Series B-Statistical Methodology* 64: 583–616.

See Also

[AIC](#) and [BIC](#) in [stats](#), [AICc](#). [QIC](#) for GEE model selection. [extractDIC](#) in package [arm](#), on which the (non-visible) method `extractDIC.mer` used by `DIC` is based.

Model utilities

Model utility functions

Description

These functions extract or calculate various values from provided fitted model objects(s). They are mainly meant for internal use, but may be also useful for end-users.

`beta.weights` computes standardized coefficients (beta weights) for a model;

`coeffs` extracts model coefficients;

`getAllTerms` extracts independent variable names from a model object;

`coefTable` extracts a table of coefficients, standard errors and associated degrees of freedom when possible;

`model.names` generates shorthand (alpha)numeric names for one or several fitted models;

Usage

```
beta.weights(model)
```

```
coeffs(model)
```

```
getAllTerms(x, ...)
```

```
## S3 method for class 'terms'
```

```
getAllTerms(x, offset = TRUE, intercept = FALSE, ...)
```

```
coefTable(model, ...)
```

```
## S3 method for class 'lme'
```

```
coefTable(model, adjustSigma, ...)
```

```
## S3 method for class 'gee'
```

```
coefTable(model, ..., type = c("naive", "robust"))
```

```
model.names(object, ..., labels = NULL, use.letters = FALSE)
```


Arguments

model	a fitted model object.
object	a fitted model object or a list of such objects.
x	a fitted model object or a formula.
offset	should 'offset' terms be included?
intercept	should terms names include the intercept?
labels	optionally, a character vector with names of all the terms, e.g. from a global model. <code>model.names</code> enumerates the model terms in order of their appearance in the list and in the models. So, changing the order of the models would lead to different names. The argument 'labels' can be used to prevent this happening.
...	for <code>model.names</code> , more fitted model objects. For <code>coefTable</code> arguments that are passed to appropriate summary method (e.g. dispersion parameter for <code>glm</code> may be used here). In other functions often not used.
use.letters	logical, whether letters should be used instead of numeric codes.
type	for GEE models, the type of covariance estimator to calculate returned standard errors on. Either "naive" or "robust" ('sandwich').
adjustSigma	See summary.lme .

Details

The functions `coeffs`, `getAllTerms` and `coefTable` provide interface between the model object and `model.avg` (and `dredge`). Custom methods can be written to provide support for additional classes of models.

Note

`coeffs`'s value is in most cases identical to that returned by [coef](#), the only difference being it returns fixed effects' coefficients for mixed models, and the value is always a named numeric vector.

Use of `tTable` is deprecated in favour of `coefTable`.

Author(s)

Kamil Bartoń

model.avg

Model averaging

Description

Model averaging based on an information criterion.

Usage

```
model.avg(object, ..., revised.var = TRUE)

## Default S3 method:
model.avg(object, ..., beta = FALSE, rank = NULL, rank.args = NULL,
          revised.var = TRUE, dispersion = NULL, ct.args = NULL)

## S3 method for class 'model.selection'
model.avg(object, subset, fit = FALSE, ..., revised.var = TRUE)
```

Arguments

<code>object</code>	a fitted model object or a list of such objects, or a <code>model.selection</code> object. See ‘Details’.
<code>...</code>	for default method, more fitted model objects. Otherwise, arguments that are passed to the default method.
<code>beta</code>	logical, should standardized coefficients be returned?
<code>rank</code>	optionally, a custom rank function (information criterion) to use instead of AICc, e.g. BIC or QAIC, may be omitted if <code>object</code> is a model list returned by <code>get.models</code> or a <code>model.selection</code> object. See ‘Details’.
<code>rank.args</code>	optional list of arguments for the rank function. If one is an expression, an <code>x</code> within it is substituted with a current model.
<code>revised.var</code>	logical, indicating whether to use revised formula for standard errors. See par.avg .
<code>dispersion</code>	the dispersion parameter for the family used. See summary.glm . This is used currently only with <code>glm</code> , is silently ignored otherwise.
<code>ct.args</code>	optional list of arguments to be passed to coefTable (besides dispersion).
<code>subset</code>	see subset method for <code>model.selection</code> object.
<code>fit</code>	if TRUE, the component models are fitted using <code>get.models</code> . See ‘Details’.

Details

`model.avg` may be used either with a list of models, or directly with a `model.selection` object (e.g. returned by `dredge`). In the latter case, the models from the model selection table are not evaluated unless the argument `fit` is set to TRUE or some additional arguments are present (such as `rank` or `dispersion`). This results in much faster calculation, but has certain drawbacks, because the fitted component model objects are not stored, and some methods (e.g. `predict`, `fitted`, `model.matrix` or `vcov`) would not be available with the returned object. Otherwise, `get.models` is called prior to averaging, and `...` are passed to it.

For a list of model types that are accepted see [list of supported models](#).

`rank` is found by a call to [match.fun](#) and typically is specified as a function or a symbol (e.g. a back-quoted name) or a character string specifying a function to be searched for from the environment of the call to `lapply`. `rank` must be a function able to accept `model` as a first argument and must always return a scalar.

Several standard methods for fitted model objects exist for class averaging, including [summary](#), [predict](#), `coef`, `confint`, [formula](#),

[vcov](#). The `coef` method accepts argument `full`, if set to `TRUE` the full model-averaged coefficients are returned, rather than subset-averaged ones. `logLik` returns a list of [logLik](#) objects for the component models.

Value

An object of class averaging is a list with components:

<code>summary</code>	a <code>data.frame</code> with log-likelihood, IC, Delta(IC) and Akaike weights for the component models.
<code>coef.shrinkage</code>	a vector of full model-averaged coefficients, see ‘Note’.
<code>coefArray</code>	an array of component models’ coefficients, their standard errors, and degrees of freedom.
<code>term.codes</code>	names of the terms with numerical codes used in the summary.
<code>avg.model</code>	the model averaged parameters. A <code>data.frame</code> containing averaged coefficients, unconditional standard error, adjusted SE (if <i>dfs</i> are available) and z-values (coefficient and SE) and significance (assuming a normal error distribution).
<code>importance</code>	relative importance of the predictor variables (including interactions), calculated as a sum of the <i>Akaike weights</i> over all of the models in which the parameter of interest appears.
<code>term.names</code>	character vector giving names of all terms in the model.
<code>x, formula</code>	the model matrix and formula corresponding to the one that would be used in a single model. <code>formula</code> contains only the averaged coefficients.
<code>call</code>	the matched call.

In addition, the object has following attributes:

<code>modelList</code>	a list of component model objects.
<code>beta</code>	logical, were standardized coefficients used?
<code>revised.var</code>	if <code>TRUE</code> , the standard errors were calculated with the revised formula (See par.avg).

Note

The ‘subset’ (or ‘conditional’) average only averages over the models where the parameter appears. An alternative, the ‘full’ average assumes that a variable is included in every model, but in some models the corresponding coefficient is set to zero. Unlike the ‘subset average’, it does not have a tendency of biasing the value away from zero. It is, however, an unresolved issue how the variance of this estimate should be calculated, therefore the standard errors and confidence interval are returned only for the subset-averaged coefficients (as from version $\geq 1.5.0$ argument `method` is no longer accepted).

Averaging models with different contrasts for the same factor would yield nonsense results, currently no checking for contrast consistency is done.

From version 1.0.1, `print` method provides only a concise output (similarly as for `lm`). To print a full summary of the results use `summary` function. Confidence intervals can be obtained with [confint](#).

Author(s)

Kamil Bartoń

References

Burnham, K. P. and Anderson, D. R (2002) *Model selection and multimodel inference: a practical information-theoretic approach*. 2nd ed. New York, Springer-Verlag.

See Also

See [par.avg](#) for more details of model averaged parameter calculation.

[dredge](#), [get.models](#)

[AICc](#) has examples of averaging models fitted by REML.

[modavg](#) in package **AICcmodavg**, and [coef.glmulti](#) in package **glmulti** also perform model averaging.

Examples

```
# Example from Burnham and Anderson (2002), page 100:
library(MuMIn)
data(Cement)
fm1 <- lm(y ~ ., data = Cement)
(ms1 <- dredge(fm1))

#models with delta.aicc < 4
summary(model.avg(ms1, subset = delta < 4))

#or as a 95% confidence set:
avgmod.95p <- model.avg(ms1, cumsum(weight) <= .95)
confint(avgmod.95p)

## Not run:
# The same result, but re-fitting the models via 'get.models'
confset.95p <- get.models(ms1, cumsum(weight) <= .95)
model.avg(confset.95p)

# Force re-fitting the component models
model.avg(ms1, cumsum(weight) <= .95, fit = TRUE)
# Models are also fitted if additional arguments are given
model.avg(ms1, cumsum(weight) <= .95, rank = "AIC")

## End(Not run)

## Not run:
# using BIC (Schwarz's Bayesian criterion) to rank the models
BIC <- function(x) AIC(x, k = log(length(residuals(x))))
model.avg(confset.95p, rank = BIC)
# the same result, using AIC directly, with argument k
# 'x' in a quoted 'rank' argument is substituted with a model object
# (in this case it does not make much sense as the number of observations is
# common to all models)
model.avg(confset.95p, rank = AIC, rank.args = alist(k = log(length(residuals(x)))))

## End(Not run)
```

model.sel	<i>model selection table</i>
-----------	------------------------------

Description

Build a model selection table.

Usage

```
model.sel(object, ...)

## S3 method for class 'model.selection'
model.sel(object, rank = NULL, rank.args = NULL, ...)
## Default S3 method:
model.sel(object, ..., rank = NULL, rank.args = NULL)
```

Arguments

object	A fitted model object, a list of such objects, or a "model.selection" object.
...	More fitted model objects.
rank	Optional, custom rank function (information criterion) to use instead of AICc, e.g. QAIC or BIC, may be omitted if object is a model list returned by <code>get.models</code> .
rank.args	Optional list of arguments for the rank function. If one is an expression, an x within it is substituted with a current model.

Value

An object of class "model.selection" with columns containing useful information about each model: the coefficients, df, log-likelihood, the value of the information criterion used, Delta(IC) and 'Akaike weight'. If any arguments differ between the modelling function calls, the result will include additional columns showing them (except for formulas and some other arguments).

Author(s)

Kamil Bartoń

See Also

[dredge](#), [AICc](#), [list of supported models](#).

Possible alternatives: [ICtab](#) (in package **bbmle**), or [aictab](#) (**AICcmodavg**).

Examples

```
data(Cement)
Cement$X1 <- cut(Cement$X1, 3)
Cement$X2 <- cut(Cement$X2, 2)
```

```

fm1 <- glm(formula = y ~ X1 + X2 * X3, data = Cement)
fm2 <- update(fm1, . ~ . - X1 - X2)
fm3 <- update(fm1, . ~ . - X2 - X3)

## ranked with AICc by default
(msAICc <- model.sel(fm1, fm2, fm3))

## ranked with BIC
model.sel(fm1, fm2, fm3, rank = AIC, rank.args = alist(k = log(nobs(x))))
# or
# model.sel(msAICc, rank = AIC, rank.args = alist(k = log(nobs(x))))
# or
# update(msAICc, rank = AIC, rank.args = alist(k = log(nobs(x))))

```

MuMIn-models

List of supported models

Description

List of model classes accepted by `model.avg`, `model.sel`, and `dredge`.

Details

Fitted model objects that can be used with model selection and model averaging functions include those returned by:

- `lm`, `glm` (package **stats**);
- `rlm`, `glm.nb` and `polr` (**MASS**);
- `multinom` (**nnet**);
- `lme`, `gls` (**nlme**);
- `lmer`, `glmer` (**lme4**);
- `gam`, `gamm*` (**mgcv**);
- `gamm4*` (**gamm4**);
- `glmmML` (**glmmML**);
- `glmmadmb` (**glmmADMB** from R-Forge);
- `hurdle`, `zeroinfl` (**pscl**);
- `negbin`, `betabin` (class `glimML`, package **aod**);
- `sarlm`, `spautolm` (**spdep**);
- `spml` (if fitted by ML, **splm**);
- `coxph`, `survreg` (**survival**);
- `coxme`, `lmeKin` (**coxme**);
- `rq` (**quantreg**);
- `clm` and `clmm` (**ordinal**);
- `logistf` (**logistf**);
- `crunch*`, `pgls` (**caper**);

- functions from package **unmarked** (within the class `unmarkedFit`);
- `mark` and related functions (class `mark` from package **RMark**). Note currently `dredge` can only manipulate formula element of the argument `model.parameters`, keeping its other elements intact.

Generalized Estimation Equation model implementations: `geeglm` from package **geepack**, `gee` from **gee**, and `yags` from **yags** (from R-Forge) can be used with `QIC` as the selection criterion.

`MCMCglmm`* models (package **MCMCglmm**) with `DIC` as the rank function are accepted by `model.sel` and `dredge`.

Other classes are also likely to be supported, in particular if they inherit from one of the above classes. In general, the models averaged with `model.avg` may belong to different types (e.g. `glm` and `gam`), provided they use the same data and response, and if it is valid to do so. This applies also to constructing model selection tables with `model.sel`.

Note

* In order to use `gamm4`, `gamm4`, `crunch` or `MCMCglmm` with `dredge`, an `updateable` wrapper for these functions should be created.

See Also

`model.avg`, `model.sel` and `dredge`.

`par.avg`

Parameter averaging

Description

Average a single model coefficient based on provided weights. It is mostly intended for internal use.

Usage

```
par.avg(x, se, weight, df = NULL, level = 1 - alpha, alpha = 0.05,
        revised.var = TRUE, adjusted = TRUE)
```

Arguments

<code>x</code>	vector of parameters.
<code>se</code>	vector of standard errors.
<code>weight</code>	vector of weights.
<code>df</code>	(optional) vector of degrees of freedom.
<code>alpha, level</code>	significance level for calculating confidence intervals.
<code>revised.var</code>	logical, should the revised formula for standard errors be used? See ‘Details’.
<code>adjusted</code>	logical, should the inflated standard errors be calculated? See ‘Details’.

Details

Unconditional standard errors are square root of the variance estimator, calculated either according to the original equation in Burnham and Anderson (2002, equation 4.7), or a newer, revised formula from Burnham and Anderson (2004, equation 4) (if `revised.var = TRUE`, this is the default). If `adjusted = TRUE` (the default) and degrees of freedom are given, the adjusted standard error estimator and confidence intervals with improved coverage are returned (see Burnham and Anderson 2002, section 4.3.3).

Value

`par.avg` returns a vector with named elements:

Coefficient	model coefficients,
SE	unconditional standard error,
Adjusted SE	adjusted standard error,
Lower CI, Upper CI	unconditional confidence intervals.

Author(s)

Kamil Bartoń

References

Burnham, K. P. and Anderson, D. R. (2002) *Model selection and multimodel inference: a practical information-theoretic approach*. 2nd ed.

Burnham, K. P. and Anderson, D. R. (2004). *Multimodel inference - understanding AIC and BIC in model selection*. *Sociological Methods & Research* 33(2): 261-304.

See Also

[model.avg](#) for model averaging.

pdredge

Automated model selection using parallel computation

Description

Parallelized version of dredge.

Usage

```
pdredge(global.model, cluster = NA, beta = FALSE, evaluate = TRUE, rank = "AICc",
        fixed = NULL, m.max = NA, m.min = 0, subset, marg.ex = NULL, trace = FALSE,
        varying, extra, ct.args = NULL, check = FALSE, ...)
```


Arguments

`global.model`, `beta`, `evaluate`, `rank`
 see [dredge](#).

`fixed`, `m.max`, `m.min`, `subset`, `marg.ex`, `varying`, `extra`, `ct.args`, ...
 see [dredge](#).

`trace` displays the generated calls, but may not work as expected since the models are evaluated in batches rather than one by one.

`cluster` either a valid cluster object, or NA for a single threaded execution.

`check` either integer or logical value controlling how much checking for existence and correctness of dependencies is done on the cluster nodes. See ‘Details’.

Details

All the dependencies for fitting the `global.model`, including the data and any objects the modelling function will use must be exported into the cluster worker nodes (e.g. *via* `clusterExport`). The required packages must be also loaded therein (e.g. *via* `clusterEvalQ(..., library(package))`), before the cluster is used by `pdredge`.

If `check` is TRUE or positive, `pdredge` tries to check whether all the variables and functions used in the call to `global.model` are present in the cluster nodes’ `.GlobalEnv` before proceeding further. This causes false errors if some arguments of the model call (other than `subset`) would be evaluated in data environment. In that case using `check = FALSE` (the default) is desirable.

If `check` is TRUE or greater than one, `pdredge` will compare the `global.model` updated at the cluster nodes with the one given as argument.

Value

See [dredge](#).

Author(s)

Kamil Bartoń

See Also

`makeCluster` and other cluster related functions in packages **parallel** or **snow**.

Examples

```
# One of these packages is required:
## Not run: require(parallel) || require(snow)

# From example(Beetle)
data(Beetle)

Beetle100 <- Beetle[sample(nrow(Beetle), 100, replace = TRUE),]

fm1 <- glm(Prop ~ dose + I(dose^2) + log(dose) + I(log(dose)^2),
  data = Beetle100, family = binomial)

msubset <- expression(xor(dose, 'log(dose)') & (dose | !'I(dose^2)'))
```

```

      & ('log(dose)' | !'I(log(dose)^2)')
    varying.link <- list(family = alist(logit = binomial("logit"),
      probit = binomial("probit"), cloglog = binomial("cloglog") ))

# Set up the cluster
clusterType <- if(length(find.package("snow", quiet = TRUE))) "SOCK" else "PSOCK"
clust <- try(makeCluster(getOption("cl.cores", 2), type = clusterType))

clusterExport(clust, "Beetle100")

# noticeable gain only when data has about 3000 rows (Windows 2-core machine)
print(system.time(dredge(fm1, subset = msubset, varying = varying.link)))
print(system.time(pdredge(fm1, cluster = FALSE, subset = msubset,
  varying = varying.link)))
print(system.time(pdd <- pdredge(fm1, cluster = clust, subset = msubset,
  varying = varying.link)))

print(pdd)

## Not run:
# Time consuming example with 'unmarked' model, based on example(pcount).
# Having enough patience you can run this with 'demo(pdredge.pcount)'.
library(unmarked)
data(mallard)
mallardUMF <- unmarkedFramePCount(mallard.y, siteCovs = mallard.site,
  obsCovs = mallard.obs)
(ufm.mallard <- pcount(~ ivel + date + I(date^2) ~ length + elev + forest,
  mallardUMF, K = 30))
clusterEvalQ(clust, library(unmarked))
clusterExport(clust, "mallardUMF")

# 'stats4' is needed for AIC to work with unmarkedFit objects but is not
# loaded automatically with 'unmarked'.
require(stats4)
invisible(clusterCall(clust, "library", "stats4", character.only = TRUE))

#system.time(print(pdd1 <- pdredge(ufm.mallard,
#  subset = 'p(date)' | !'p(I(date^2))', rank = AIC)))

system.time(print(pdd2 <- pdredge(ufm.mallard, clust,
  subset = 'p(date)' | !'p(I(date^2))', rank = AIC, extra = "adjR^2")))

# best models and null model
subset(pdd2, delta < 2 | df == min(df))

# Compare with the model selection table from unmarked
# the statistics should be identical:
models <- pget.models(pdd2, clust, delta < 2 | df == min(df))

modSel(fitList(fits = structure(models, names = model.names(models),
  labels = getAllTerms(ufm.mallard))), nullmod = "(Null)")

## End(Not run)

stopCluster(clust)

```

predict.averaging	<i>Predict method for the averaged model</i>
-------------------	--

Description

Model-averaged predictions with optional standard errors.

Usage

```
## S3 method for class 'averaging'
predict(object, newdata = NULL, se.fit = FALSE,
        interval = NULL, type = NA, backtransform = FALSE, full = TRUE, ...)
```

Arguments

object	An object returned by <code>model.avg</code> .
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are used.
se.fit	logical, indicates if standard errors should be returned. This has any effect only if the predict methods for each of the component models support it.
interval	Currently not used.
type	The type of predictions to return (see documentation for <code>predict</code> appropriate for the class of used component models). If omitted, the default type is used. See 'Details'.
backtransform	If TRUE, the averaged predictions are back-transformed from link scale to response scale. This makes sense provided that all component models use the same family, and the prediction from each of the component models is calculated on the link scale (as specified by type. For <code>glm</code> , use <code>type = "link"</code>). See 'Details'.
full	If TRUE, the full model averaged coefficients are used (only if <code>se.fit = FALSE</code> and the component objects are a result of <code>lm</code>).
...	Arguments to be passed to respective predict method (e.g. <code>level</code> for lme model).

Details

If all the component models are ordinary linear models, the prediction can be made either with the full averaged coefficients (the argument `full = TRUE` this is the default) or subset-averaged coefficients. Otherwise the prediction is obtained by calling `predict` on each component model and weighted averaging the results, which corresponds to the assumption that all predictors are present in all models, but those not estimated are equal zero. See 'Note' in [model.avg](#). Predictions from component models with standard errors are passed to `par.avg` and averaged in the same way as the coefficients.

Predictions on the response scale from generalized models can be calculated by averaging predictions of each model on the link scale, followed by inverse transformation (this is achieved with `type = "link"` and `backtransform = TRUE`). This is only possible if all component models use the same family and link function. Alternatively, predictions from each model on response scale may be averaged (with `type = "response"` and `backtransform = FALSE`). Note that this leads to results differing from those calculated with the former method. See also [predict.glm](#).

Value

If `se.fit = FALSE`, a vector of predictions, otherwise a list with components: `fit` containing the predictions, and `se.fit` with the estimated standard errors.

Note

This method relies on availability of the predict methods for the component model classes (except when all component models are of class `lm`).

The package **MuMIn** includes predict methods for `lme`, `gls` and `lmer` (**lme4**), all of which can calculate standard errors of the predictions (with `se.fit = TRUE`). The former two enhance the original predict methods from package **nlme**, and with `se.fit = FALSE` they return identical result. **MuMIn**'s versions are always used in averaged model predictions (so it is possible to predict with standard errors), but from within global environment they will be found only if **MuMIn** is before **nlme** on the [search list](#) (or directly extracted from namespace as `MuMIn::predict.lme`).

predict method for mer models currently can only calculate values on the outermost level (equivalent to `level = 0` in `predict.lme`).

Author(s)

Kamil Barton

See Also

[model.avg](#), and [par.avg](#) for details of model-averaged parameter calculation.

[predict.lme](#), [predict.gls](#)

Examples

```
require(graphics)

# Example from Burnham and Anderson (2002), page 100:
data(Cement)
fm1 <- lm(y ~ X1 + X2 + X3 + X4, data = Cement)

ms1 <- dredge(fm1)
confset.95p <- get.models(ms1, subset = cumsum(weight) <= .95)
avgm <- model.avg(confset.95p)

nseq <- function(x, len = length(x)) seq(min(x, na.rm = TRUE),
    max(x, na.rm=TRUE), length = len)

# New predictors: X1 along the range of original data, other
# variables held constant at their means
newdata <- as.data.frame(lapply(lapply(Cement[1:4], mean), rep, 25))
newdata$X1 <- nseq(Cement$X1, nrow(newdata))

n <- length(confset.95p)

# Predictions from each of the models in a set, and with averaged coefficients
pred <- data.frame(
  model = sapply(confset.95p, predict, newdata = newdata),
  averaged.subset = predict(avgm, newdata, full = FALSE),
  averaged.full = predict(avgm, newdata, full = TRUE)
```

```

)

opal <- palette(c(topo.colors(n), "black", "red", "orange"))
matplot(newdata$X1, pred, type = "l",
lwd = c(rep(2,n),3,3), lty = 1,
      xlab = "X1", ylab = "y", col=1:7)

# For comparison, prediction obtained by averaging predictions of the component
# models
pred.se <- predict(avgm, newdata, se.fit = TRUE)
y <- pred.se$fit
ci <- pred.se$se.fit * 2
matplot(newdata$X1, cbind(y, y - ci, y + ci), add = TRUE, type="l",
lty = 2, col = n + 3, lwd = 3)

legend("topleft",
      legend=c(lapply(confset.95p, formula),
        paste(c("subset", "full"), "averaged")), "averaged predictions + CI"),
      lty = 1, lwd = c(rep(2,n),3,3,3), cex = .75, col=1:8)

palette(opal)

```

QAIC	<i>Quasi AIC or AICc</i>
------	--------------------------

Description

Calculate a modification of Akaike's Information Criterion for overdispersed count data (or its version corrected for small sample, "quasi-AIC_c"), for one or several fitted model objects.

Usage

```

QAIC(object, ..., chat, k = 2)
QAICc(object, ..., chat, k = 2)

```

Arguments

object	a fitted model object.
...	optionally, more fitted model objects.
chat	\hat{c} , the variance inflation factor.
k	the 'penalty' per parameter.

Value

If only one object is provided, returns a numeric value with the corresponding QAIC or QAIC_c; otherwise returns a data.frame with rows corresponding to the objects.

Note

\hat{c} is the dispersion parameter estimated from the global model, and can be calculated by dividing model's deviance by the number of residual degrees of freedom.

In calculation of QAIC, the number of model parameters is increased by 1 to account for estimating the overdispersion parameter. Without overdispersion, $\hat{c} = 1$ and QAIC is equal to AIC.

Note that `glm` does not compute maximum-likelihood estimates in models within the *quasi*- family. In case it is justified, and with a proper caution, a workaround could be used by 'borrowing' the `aic` element from the corresponding 'non-quasi' family (see 'Example').

Author(s)

Kamil Bartoń

See Also

[AICc](#), [quasi](#) family used for models with over-dispersion

Examples

```
# Based on "example(predict.glm)", with one number changed to create
# overdispersion
budworm <- data.frame(
  ldose = rep(0:5, 2), sex = factor(rep(c("M", "F"), c(6, 6))),
  numdead = c(10, 4, 9, 12, 18, 20, 0, 2, 6, 10, 12, 16))
budworm$SF = cbind(numdead = budworm$numdead,
  numalive = 20 - budworm$numdead)

budworm.lg <- glm(SF ~ sex*ldose, data = budworm, family = binomial)
(chat <- deviance(budworm.lg) / df.residual(budworm.lg))

dredge(budworm.lg, rank = "QAIC", chat = chat)
dredge(budworm.lg, rank = "AIC")

## Not run:
# A 'hacked' constructor for quasibinomial family object, that allows for
# ML estimation
x.quasibinomial <- function(...) {
  res <- quasibinomial(...)
  res$aic <- binomial(...)$aic
  res
}
QAIC(update(budworm.lg, family = x.quasibinomial), chat = chat)

## End(Not run)
```

QIC

QIC and quasi-Likelihood for GEE

Description

Calculate quasi-likelihood under the independence model criterion (QIC) for Generalized Estimating Equations.

Usage

```
QIC(object, ..., typeR = FALSE)
QICu(object, ..., typeR = FALSE)
quasiLik(object, ...)
```

Arguments

<code>object</code>	a fitted model object of class <code>gee</code> , <code>geepack</code> or <code>yags</code> .
<code>...</code>	for <code>QIC</code> and <code>QIC_u</code> , optionally more fitted model objects.
<code>typeR</code>	logical, whether to calculate <code>QIC(R)</code> . <code>QIC(R)</code> is based on quasi-likelihood of a working correlation R model. Defaults to <code>FALSE</code> , and <code>QIC(I)</code> based on independence model is returned.

Value

If just one object is provided, returns a numeric value with the corresponding QIC; if more than one object are provided, returns a `data.frame` with rows corresponding to the objects and one column representing QIC or `QICu`.

Note

This implementation is based partly on (revised) code from packages **yags** (R-Forge) and **ape**. The functions are still in experimental stage and should be used with caution.

Author(s)

Kamil Bartoń

References

Pan W. (2001) Akaike's Information Criterion in Generalized Estimating Equations. *Biometrics* 57: 120-125

Hardin J. W., Hilbe, J. M. (2003) *Generalized Estimating Equations*. Chapman & Hall/CRC

See Also

Methods exist for [gee](#) (package **gee**), [geeglm](#) (**geepack**), and [yags](#) (**yags** on R-Forge). `yags` and `compar.gee` from package **ape** both provide QIC values.

Examples

```
library(geepack)
data(ohio)

fm1 <- geeglm(resp ~ age * smoke, id = id, data = ohio,
  family = binomial, corstr = "exchangeable", scale.fix = TRUE)
fm2 <- update(fm1, corstr = "ar1")
fm3 <- update(fm1, corstr = "unstructured")

model.sel(fm1, fm2, fm3, rank = QIC)

## Not run:
# same result:
```

```
dredge(fm1, m.min = 3, rank = QIC, varying = list(
  corstr = list("exchangeable", "unstructured", "ar1")
))

## End(Not run)
```

r.squaredGLMM

Pseudo-R-squared for Generalized Mixed-Effect models

Description

Calculate a conditional and marginal coefficient of determination for Generalized mixed-effect models (R_{GLMM}^2).

Usage

```
r.squaredGLMM(x, nullfx = NULL)
```

Arguments

x a fitted linear model object.

nullfx optionally, a fitted *null* model including only intercept and all the random effects of the reference model.

Details

For mixed-effects models, R^2 can be categorized into two types: marginal and conditional. **Marginal R^2** represents the variance explained by fixed factors, and is defined as:

$$R_{GLMM(m)}^2 = \frac{\sigma_f^2}{\sigma_f^2 + \sum_{l=1}^u \sigma_l^2 + \sigma_\epsilon^2}$$

Conditional R^2 is interpreted as variance explained by both fixed and random factors (i.e. the entire model), and is calculated according to the equation:

$$R_{GLMM(c)}^2 = \frac{\sigma_f^2 + \sum_{l=1}^u \sigma_l^2}{\sigma_f^2 + \sum_{l=1}^u \sigma_l^2 + \sigma_\epsilon^2}$$

where σ_f^2 is the variance of the fixed effect components, and $\sum \sigma_l^2$ is the sum of all u variance components (group, individual, etc.), and σ_ϵ^2 is the residual variance.

Value

r.squaredGLMM returns a numeric vector with two values for marginal and conditional R_{GLMM}^2 .

Note

R_{GLMM}^2 can be calculated also for fixed-effect models. In the simplest case of OLS it reduces to $\text{var}(\text{fitted}) / (\text{var}(\text{fitted}) + \text{deviance} / 2)$. Yet, unlike likelihood-ratio based R^2 for OLS, value of this statistic differs from that of the classical R^2 .

Currently methods exist for classes: `mer(Mod)`, `lme`, `glmmML` and `(g)lm`.

See note in [r.squaredLR](#) help page for comment on using R^2 in model selection.

This implementation is based on R code from ‘Supporting Information’ for Nakagawa & Schielzeth (2012).

This function is in experimental stage and should be used with caution. Specifically, conditional R_{GLMM}^2 for Poisson family models cannot be yet calculated (NA is returned).

References

Nakagawa, S, Schielzeth, H. (2012). A general and simple method for obtaining R^2 from Generalized Linear Mixed-effects Models. *Methods in Ecology and Evolution*: (online) doi:10.1111/j.2041-210x.2012.00261.x

See Also

[summary.lm](#), [r.squaredLR](#)

Examples

```
library(lme4)
data(Orthodont, package = "nlme")

fm1 <- lmer(distance ~ Sex * age + (1 | Subject), data = Orthodont)

r.squaredGLMM(fm1)
r.squaredLR(fm1)
r.squaredLR(fm1, null.RE = TRUE)
```

r.squaredLR

Likelihood-ratio based pseudo-R-squared

Description

Calculate a coefficient of determination based on the likelihood-ratio test (R_{LR}^2).

Usage

```
r.squaredLR(x, null = NULL, null.RE = FALSE)

null.fit(x, evaluate = FALSE, RE.keep = FALSE, envir = NULL)
```

Arguments

<code>x</code>	a fitted model object.
<code>null</code>	a fitted <i>null</i> model. If not provided, <code>null.fit</code> will be used to construct it. <code>null.fit</code> 's capabilities are limited to only a few model classes, for others the <i>null</i> model has to be specified manually.
<code>null.RE</code>	logical, should the null model contain random factors? Only used if no <i>null</i> model is given, otherwise omitted, with a warning.
<code>evaluate</code>	if TRUE evaluate the fitted model object else return the call.
<code>RE.keep</code>	if TRUE, the random effects of the original model are included.
<code>envir</code>	the environment in which the <i>null</i> model is to be evaluated, defaults to the environment of the original model's formula.

Details

This statistic is one of the several proposed pseudo-R-squared's for nonlinear regression models. It is based on an improvement from *null* (intercept only) model to the fitted model, and calculated as

$$R_{LR}^2 = 1 - \exp\left(-\frac{2}{n}(\log Lik(x) - \log Lik(0))\right)$$

where $\log Lik(x)$ and $\log Lik(0)$ are the log-likelihoods of the fitted and the *null* model respectively. ML estimates are used for this purpose in when models have been fitted by RESTRICTED ML (by calling `logLik` with argument `REML = FALSE`). Note that the *null* model can include the random factors of the original model, in which case the statistic represents the 'variance explained' by fixed effects.

For OLS models the value is consistent with classical R^2 . In some cases (e.g. in logistic regression), the maximum R_{LR}^2 is less than one. The modification proposed by Nagelkerke (1991) adjusts the R_{LR}^2 to achieve 1 at its maximum: $\bar{R}^2 = R_{LR}^2 / \max(R_{LR}^2)$ where $\max(R_{LR}^2) = 1 - \exp(\frac{2}{n} \log Lik(0))$.

`null.fit` tries to guess the *null* model call, given the provided fitted model object. This would be usually a `glm`. The function will give an error for an unrecognized class.

Value

`r.squaredLR` returns a value of R_{LR}^2 , and the attribute `"adj.r.squared"` gives the Nagelkerke's modified statistic. Note that this is not the same as nor equivalent to the classical 'adjusted R squared'.

`null.fit` returns the fitted *null* model object (if `evaluate = TRUE`) or an unevaluated call to fit a *null* model.

Note

R^2 is a useful goodness-of-fit measure as it has the interpretation of the proportion of the variance 'explained', but it performs poorly in model selection, and is not suitable for use in the same way as the information criterions.

References

- Cox, D. R. and Snell, E. J. (1989) *The analysis of binary data*, 2nd ed. London, Chapman and Hall
- Magee, L. (1990) R^2 measures based on Wald and likelihood ratio joint significance tests. *Amer. Stat.* 44: 250-253
- Nagelkerke, N. J. D. (1991) A note on a general definition of the coefficient of determination. *Biometrika* 78: 691-692

See Also

[summary.lm](#), [r.squaredGLMM](#)

subset.model.selection

Subsetting model selection table

Description

Return subsets of a model selection table returned by dredge or model.sel.

Usage

```
## S3 method for class 'model.selection'
subset(x, subset, select, recalc.weights = TRUE,
       recalc.delta = FALSE, ...)
## S3 method for class 'model.selection'
x[i, j, recalc.weights = TRUE, recalc.delta = FALSE, ...]
```

Arguments

<code>x</code>	a <code>model.selection</code> object to be subsetting.
<code>subset, select</code>	logical expressions indicating columns and rows to keep. See subset .
<code>i, j</code>	indices specifying elements to extract.
<code>recalc.weights</code>	logical value specifying whether Akaike weights should be normalized across the new set of models to sum to one.
<code>recalc.delta</code>	logical value specifying whether <i>delta</i> [<i>IC</i>] should be calculated for the new set of models (this is not done by default).
<code>...</code>	further arguments passed to [.data.frame] .

Value

A `model.selection` object containing only the selected models (rows). When columns are selected (arguments `select` or `j` are provided), a plain `data.frame` is returned.

Note

Unlike the method for `data.frame`, extracting with only one index (i.e. `x[i]`) will select rows rather than columns.

To select rows according to presence or absence of the variables (rather than their value), a pseudo-function `has` may be used, e.g. `subset(x, has(a, !b))` will select rows with *a* **and** without *b* (this is equivalent to `!is.na(a) & is.na(b)`). `has` can take any number of arguments. Importantly, the `has()` notation cannot be used in the `subset` argument for `dredge`, where the variable names should be given directly, with the same effect.

To select rows where one variable can be present conditional on the presence of other variable(s), the function `dc` (**d**ependency **c**hain) can be used. `dc` takes any number of variables as arguments, and allows a variable to be included only if all the preceding arguments are also included (e.g. `subset = dc(a, b, c)` allows for models of form *a*, *a+b* and *a+b+c* but not *b*, *c*, *b+c* or *a+c*).

Author(s)

Kamil Bartoń

See Also[dredge](#), [subset](#) and [\[.data.frame\]](#) for subsetting and extracting from data.frames.**Examples**

```
data(Cement)
fm1 <- lm(formula = y ~ X1 + X2 + X3 + X4, data = Cement)

# generate models where each variable is included only if the previous
# are included too, e.g. X2 only if X1 is there, and X3 only if X2 and X1
dredge(fm1, subset = dc(X1, X2, X3, X4))

# which is equivalent to
# dredge(fm1, subset = (!X2 | X1) & (!X3 | X2) & (!X4 | X3))

# alternatively, generate "all possible" combinations
ms0 <- dredge(fm1)
# ...and afterwards select the subset of models
subset(ms0, dc(X1, X2, X3, X4))
# which is equivalent to
# subset(ms0, (has(!X2) | has(X1)) & (has(!X3) | has(X2)) & (has(!X4) | has(X3)))

# Different ways of finding a confidence set of models:
# delta(AIC) cutoff
subset(ms0, delta <= 4, recalc.weights = FALSE)
# cumulative sum of Akaike weights
subset(ms0, cumsum(weight) <= .95, recalc.weights = FALSE)
# relative likelihood
subset(ms0, (weight / weight[1]) > (1/8), recalc.weights = FALSE)
```

updateable

*Make a function return updateable result***Description**

Creates a function wrapper that stores a call in the values returned by its argument FUN.

Usage

```
updateable(FUN)
updateable2(FUN, Class)

# updateable wrapper for mgcv::gamm and gamm4::gamm4
uGamm(formula, random = NULL, ..., lme4 = inherits(random, "formula"))
```

Arguments

<code>FUN</code>	function to be modified, found via match.fun .
<code>Class</code>	optional character vector naming class(es) to be set onto the result of <code>FUN</code> (not possible with formal S4 objects).
<code>formula, random, ...</code>	arguments to be passed to <code>gamm</code> or <code>gamm4</code>
<code>lme4</code>	if <code>TRUE</code> , <code>gamm4</code> is called, <code>gamm</code> otherwise.

Details

Most model fitting functions in **R** returns an object that can be updated or re-fitted via [update](#). This is thanks to the `call` stored in the object, which can be used (possibly modified) later on. It is also utilised by `dredge` to generate sub-models.

Some functions (such as `gamm` or `MCMCglmm`) do not provide their result with the `call` element. In this case `updateable` can be used on that function to add it. The resulting wrapper should be used in exactly the same way as the original function.

Value

A function with the same arguments as `FUN`, wrapping a call to `FUN` and adding an element named `call` to its result if possible, or an attribute "`call`" (if the returned value is atomic or a formal S4 object).

Note

`uGamm` sets also an appropriate class onto the result ("`gamm4`" and/or "`gamm`"), which is needed for some generics defined in **MuMIn** to work (note that unlike the functions created by `updateable` it has no formal arguments of the original function. As of version 1.9.2, `MuMIn::gamm` is no longer available.

MuMIn replaces the default method for `getCall` (defined originally in package **stats**), with a function that can extract the call also when it is an [attribute](#) (rather than an element of the object).

Author(s)

Kamil Bartoń

See Also

[update](#), [getCall](#), [getElement](#), [attributes](#)
[gamm](#), [gamm4](#)

Examples

```
# Simple example with cor.test:

# From example(cor.test)
x <- c(44.4, 45.9, 41.9, 53.3, 44.7, 44.1, 50.7, 45.2, 60.1)
y <- c( 2.6,  3.1,  2.5,  5.0,  3.6,  4.0,  5.2,  2.8,  3.8)

ct1 <- cor.test(x, y, method = "kendall", alternative = "greater")
```

```

uCor.test <- updateable(cor.test)

ct2 <- uCor.test(x, y, method = "kendall", alternative = "greater")

getCall(ct1) # --> NULL
getCall(ct2)

#update(ct1, method = "pearson") --> Error
update(ct2, method = "pearson")
update(ct2, alternative = "two.sided")

## predefined wrapper for 'gamm':
library(mgcv)
set.seed(0)
dat <- gamSim(6, n = 100, scale = 5, dist = "normal")

fmm1 <- uGamm(y ~s(x0)+ s(x3) + s(x2), family = gaussian, data = dat,
  random = list(fac = ~1))

getCall(fmm1)
class(fmm1)
###

## Not run:
library(caper)
data(shorebird)
shorebird <- comparative.data(shorebird.tree, shorebird.data, Species)

fm1 <- crunch(Egg.Mass ~ F.Mass * M.Mass, data = shorebird)

uCrunch <- updateable(crunch)

fm2 <- uCrunch(Egg.Mass ~ F.Mass * M.Mass, data = shorebird)

getCall(fm1)
getCall(fm2)
update(fm2) # Error with 'fm1'
dredge(fm2)

## End(Not run)

```

Weights

Akaike weights

Description

Calculate or extract normalized model likelihoods ('Akaike weights').

Usage

```
Weights(x)
```

Arguments

x a numeric vector of information criterion values such as AIC, or objects returned by functions like `AIC`. There are also methods for extracting Akaike weights from a `model.selection` or averaging objects.

Value

a numeric vector of normalized likelihoods.

Author(s)

Kamil Bartoń

See Also

[importance](#)

[weights](#), which extracts fitting weights from model objects.

Examples

```
data(Beetle)

fm1 <- glm(Prop ~ dose, data=Beetle, family=binomial)
fm2 <- update(fm1, . ~ . + I(dose^2))
fm3 <- update(fm1, . ~ log(dose))
fm4 <- update(fm3, . ~ . + I(log(dose)^2))

round(Weights(AICc(fm1, fm2, fm3, fm4)), 3)
```

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