

# SNewton: safeguarded Newton methods for function minimization

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## Safeguarded Newton algorithms

So-called **Newton** methods are among the most commonly mentioned in the solution of nonlinear equations or function minimization. However, as discussed in [https://en.wikipedia.org/wiki/Newton%27s\\_method#History](https://en.wikipedia.org/wiki/Newton%27s_method#History), the **Newton** or **Newton-Raphson** method as we know it today was not what either of its supposed originators knew.

This vignette discusses the development of simple safeguarded variants of the Newton method for function minimization in **R**. Note that there are some resources in **R** for solving nonlinear equations by Newton-like methods in the packages **nleqslv** and **pracma**.

## The basic approach

If we have a function  $f(x)$ , with gradient  $g(x)$  and second derivative (Hessian)  $H(x)$  the first order condition for an extremum (min or max) is

$$g(x) = 0$$

To ensure a minimum, we want

$$H(x) > 0$$

The first order condition leads to a root-finding problem.

It turns out that  $x$  need not be a scalar. We can consider it to be a vector of parameters to be determined. This renders  $g(x)$  a vector also, and  $H(x)$  a matrix. The conditions of optimality then require a zero gradient and positive-definite Hessian.

The Newton approach to such equations is to provide a guess to the root  $x_{try}$  and to then solve the equation

$$H(x_t) * s = -g(x_t)$$

for the search vector  $s$ . We update  $x_t$  to  $x_t + s$  and repeat until we have a very small gradient  $g(x_t)$ . If  $H(x)$  is positive definite, we have a reasonable approximation to a (local) minimum.

## Motivations

A particular interest in Newton-like methods is its theoretical quadratic convergence. See [https://en.wikipedia.org/wiki/Newton%27s\\_method](https://en.wikipedia.org/wiki/Newton%27s_method). That is, the method will converge in one step for a quadratic function  $f(x)$ , and for “reasonable” functions will converge very rapidly. There are, however, a number of conditions, and practical programs need to include safeguards against mis-steps in the iterations.

The principal issues concern the possibility that  $H(x)$  may not be positive definite, at least in some parts of the domain, and that the curvature may be such that a unit step  $x_t + s$  does not reduce the function  $f$ . We therefore get a number of possible variants of the method when different possible safeguards are applied.

## Algorithm possibilities

There are many choices we can make in building a practical code to implement the ideas above. In tandem with the two main issues expressed above, we will consider

- the modification of the solution of the main equation

$$H(x_t) * s = -g(x_t)$$

so that a reasonable search vector  $s$  is always generated by avoiding Hessian matrices that are not positive definite.

- the selection of a new set of parameters  $x_{new} = x_t + step * s$  so that the function value  $f(x_{new})$  is less than  $f(x_t)$ .

The second choice above could be made slightly more stringent so that the Armijo (??ref) condition of sufficient-decrease is met. Adding a curvature requirement gives the Wolfe conditions. See [https://en.wikipedia.org/wiki/Wolfe\\_conditions](https://en.wikipedia.org/wiki/Wolfe_conditions). The Armijo requirement is generally written

$$f(x_t + step * s) < f(x_t) + c * step * g(x_t)^T * s$$

where  $c$  is some number less than 1. Typically  $c = 1e-4 = 0.0001$ . Note that the product of gradient times search vector is negative for any reasonable situation, since we are trying to go “downhill”.

As a result of the ideas in this section, the code `snewton()` uses a solution of the Newton equations with the Hessian provided (if this is possible, else we stop), along with a backtracking line search. The code `snewtonm` uses a Marquardt stabilization of the Hessian to create

$$Haug = H + 1_n * lambda$$

That is, we add  $lambda$  times the unit matrix to  $H$ . Then we try the set of parameters found by adding the solution of the Newton equations with  $Haug$  in place of  $H$  to the current “best” set of parameters. If this new set of parameters has a higher function value than the “best” so far, we increase  $lambda$  and try again. Note that we do not need to re-evaluate the gradient or Hessian to do this. Moreover, for some value of  $lambda$ , the step is clearly down the gradient (i.e., steepest descents) or we have converged and no progress is possible. This leads to a very compact and elegant code. It is reliable, but may be less efficient than using the un-modified Hessian.

## A choice to compute the search vector

The primary concern in solving for  $s$  is that the Hessian may not be positive definite. This means that we cannot apply fast and stable methods like the Cholesky decomposition to the matrix. At the time of writing, we use the following approach:

- We attempt to solve

$$H(x_t) * s = -g(x_t)$$

with **R** directly, and rely on internal checks to catch any cases where the solution fails. We then use `try()` to stop the program in this case.

## Choosing the step size

The traditional Newton approach is that the stepsize is taken to be 1. In practice, this can sometimes mean that the function value is not reduced. As an alternative, we can use a simple backtrack search. We start with  $step = 1$  (actually the program allows for the element `defstep` of the `control` list to be set to a value other than 1). If the Armijo condition is not met, we replace  $step$  with  $r * step$  where  $r$  is less than 1. Here we suggest `control$stepdec = 0.2`. We repeat until  $x_t$  satisfies the Armijo condition or  $x_t$  is essentially unchanged by the step.

Here “essentially unchanged” is determined by a test using an offset value, that is, the test

$$(x_t + offset) == (x_t + step * d + offset)$$

where  $d$  is the search direction. `control$offset = 100` is used. We could also, and almost equivalently, use the **R** `identical` function.

## Examples

These examples are coded as test to the package **snewton**.

### A simple example

The following example is trivial, in that the Hessian is a constant matrix, and we achieve convergence immediately.

```
# Try testing calls to see what is transferred (eventually test also ...)
# setup
x0<-c(1,2,3,4)
fnt <- function(x, fscale=10){
  yy <- length(x):1
  val <- sum((yy*x)^2)*fscale
}
grt <- function(x, fscale=10){
  nn <- length(x)
  yy <- nn:1
  # gg <- rep(NA,nn)
  gg <- 2*(yy^2)*x*fscale
  gg
}
hesst <- function(x, fscale=10){
  nn <- length(x)
  yy <- nn:1
  hh <- diag(2*yy^2*fscale)
  hh
}

library(snewton)
t1 <- snewton(x0, fnt, grt, hesst, control=list(trace=2), fscale=3.0)

## trace = 2
## Initial function value = 312
```

```

## [1] 1 2 3 4
## Termination test:nf= 1
## current gradient norm = 108
## Iteration 1 :Search vector:[1] -1 -2 -3 -4
## Gradient projection = -624
## f(xnew)= 0
## end major loop
## Termination test:nf= 2
## current gradient norm = 0
## Small gradient norm

print(t1)

## $par
## [1] 0 0 0 0
##
## $value
## [1] 0
##
## $grad
## [1] 0 0 0 0
##
## $Hess
##      [,1] [,2] [,3] [,4]
## [1,]  96   0   0   0
## [2,]   0  54   0   0
## [3,]   0   0  24   0
## [4,]   0   0   0   6
##
## $counts
## $counts$niter
## [1] 2
##
## $counts$nfn
## [1] 2
##
## $counts$ngr
## [1] 2
##
## $counts$nhess
## [1] 1
##
##
## $convcode
## [1] 0

# we can also use nlm and nlminb
fght <- function(x, fscale=10){
  ## combine f, g and h into single function for nlm
  ff <- fnt(x, fscale)
  gg <- grt(x, fscale)
  hh <- hesst(x, fscale)
  attr(ff, "gradient") <- gg
  attr(ff, "hessian") <- hh
  ff

```

```

}

t1nlm <- nlm(fght, x0, fscale=3.0, hessian=TRUE, print.level=1)

## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] 1 2 3 4
## Function Value
## [1] 1040
## Gradient:
## [1] 320 360 240 80
##
## iteration = 1
## Parameter:
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
## Function Value
## [1] 1.232595e-29
## Gradient:
## [1] 0.000000e+00 3.996803e-14 3.552714e-14 0.000000e+00
##
## Relative gradient close to zero.
## Current iterate is probably solution.

print(t1nlm)

## $minimum
## [1] 1.232595e-29
##
## $estimate
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
##
## $gradient
## [1] 0.000000e+00 3.996803e-14 3.552714e-14 0.000000e+00
##
## $hessian
##           [,1] [,2] [,3] [,4]
## [1,] 3.200000e+02 0 0 4.235165e-14
## [2,] 0.000000e+00 180 0 0.000000e+00
## [3,] 0.000000e+00 0 80 0.000000e+00
## [4,] 4.235165e-14 0 0 2.000000e+01
##
## $code
## [1] 1
##
## $iterations
## [1] 1

## BUT ... it looks like nlminb is NOT using a true Newton-type method
t1nlminb <- nlminb(x0, fnt, gradient=grt, hessian=hesst, fscale=3.0, control=list(trace=1))

## 0: 312.00000: 1.00000 2.00000 3.00000 4.00000
## 1: 31.277896: 0.0946591 0.313486 0.884680 2.50350
## 2: 7.4880156e-31: -1.38778e-17 0.00000 -1.11022e-16 -4.44089e-16

```

```
## 3: 3.6918767e-62: 3.08149e-33 0.00000 2.46519e-32 9.86076e-32
## 4: 1.8202358e-93: -6.84228e-49 0.00000 -5.47382e-48 -2.18953e-47
## 5: 8.9744552e-125: 1.51929e-64 0.00000 1.21543e-63 4.86173e-63
## 6: 4.4247480e-156: -3.37350e-80 0.00000 -2.69880e-79 -1.07952e-78
## 7: 2.1815692e-187: 7.49068e-96 0.00000 5.99255e-95 2.39702e-94
## 8: 1.0755967e-218: -1.66327e-111 0.00000 -1.33061e-110 -5.32245e-110
## 9: 5.3031010e-250: 3.69319e-127 0.00000 2.95455e-126 1.18182e-125
## 10: 2.6146307e-281: -8.20053e-143 0.00000 -6.56043e-142 -2.62417e-141
## 11: 1.2891124e-312: 1.82088e-158 0.00000 1.45671e-157 5.82683e-157
## 12: 0.0000000: -4.04317e-174 0.00000 -3.23454e-173 -1.29382e-172
## 13: 0.0000000: -4.04317e-174 0.00000 -3.23454e-173 -1.29382e-172
```

```
print(t1nlminb)
```

```
## $par
## [1] -4.043175e-174 0.000000e+00 -3.234540e-173 -1.293816e-172
##
## $objective
## [1] 0
##
## $convergence
## [1] 0
##
## $iterations
## [1] 13
##
## $evaluations
## function gradient
## 15 13
##
## $message
## [1] "relative convergence (4)"
```

```
# and call them from optimx (i.e., test this gives same results)
```

```
library(optimx)
t1nlmo <- optimr(x0, fnt, grt, hess=hesst, method="nllm", fscale=3.0, control=list(trace=1))
```

```
## Unit parameter scaling
## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] 1 2 3 4
## Function Value
## [1] 1040
## Gradient:
## [1] 320 360 240 80
##
## iteration = 1
## Parameter:
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
## Function Value
## [1] 1.232595e-29
## Gradient:
```

```

## [1] 0.000000e+00 3.996803e-14 3.552714e-14 0.000000e+00
##
## Relative gradient close to zero.
## Current iterate is probably solution.
print(t1nlmo)

## $convergence
## [1] 0
##
## $value
## [1] 1.232595e-29
##
## $par
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
##
## $counts
## [1] NA 1
##
## $message
## [1] "Convergence indicator (code) = 1"
tst <- try(t1nlminbo <- optimr(x0, fnt, grt, hess=hesst, method="nllminb", fscale=3.0, control=list(trac

## Unit parameter scaling
## 0: 312.00000: 1.00000 2.00000 3.00000 4.00000
## 1: 95.955813: -0.557075 0.248290 1.83219 3.61073
## 2: 51.541989: 0.315728 0.0294715 1.11454 3.25716
## 3: 36.151761: -0.496527 -0.0131770 0.397712 2.73344
## 4: 15.857662: 0.221301 0.308983 0.0101056 1.90858
## 5: 5.6451087: 0.00686801 -0.261014 0.325348 0.918905
## 6: 2.6284218: -0.0137242 0.179195 0.0814766 0.746709
## 7: 0.39384020: -0.0599550 -0.00435873 0.0526086 0.250049
## 8: 0.22079172: 0.0341382 -0.000510906 0.0319676 0.225523
## 9: 0.072938228: 0.00709733 0.00847916 0.0382438 0.130420
## 10: 0.051625575: -0.0201629 -0.00984016 0.00152106 0.0991118
## 11: 0.0079613002: -0.00575797 -0.00297347 0.00666463 0.0431979
## 12: 0.00046215706: 0.000813264 0.00185374 -0.00499623 0.00356276
## 13: 4.5405825e-05: -0.000801080 -0.000454321 5.39450e-05 -0.00173156
## 14: 8.6153740e-07: 9.98997e-05 -0.000118763 9.38077e-06 1.43821e-05
## 15: 1.1416255e-07: -1.09926e-05 6.28428e-05 8.45952e-06 -1.70757e-05
## 16: 1.1814039e-09: -2.87417e-06 -2.87063e-06 -1.85946e-06 1.31770e-05
## 17: 6.5537434e-11: 7.11902e-07 -5.23279e-08 3.15833e-07 -3.64874e-06
## 18: 2.3717735e-14: 2.25678e-09 2.46297e-08 5.18943e-09 4.75088e-08
## 19: 5.0615498e-17: -5.26531e-10 -1.02086e-09 -3.95557e-10 -1.55911e-09
## 20: 4.4353627e-21: 8.49409e-12 -4.31436e-12 2.54218e-12 1.14318e-11
## 21: 3.2800917e-24: -1.75429e-13 2.29273e-13 -2.49701e-14 -3.54079e-13
## 22: 1.6457267e-29: -2.51767e-16 -5.62293e-16 -2.15602e-16 1.20003e-15
## 23: 5.7043891e-34: 2.34737e-18 1.87634e-18 1.31560e-18 -7.96082e-18
## 24: 3.3869900e-40: -8.75589e-22 2.31612e-21 5.18664e-22 7.16082e-21
## 25: 2.2802943e-45: 1.25249e-25 -6.02277e-24 -6.41554e-24 -1.63935e-23
## 26: 2.6790444e-50: -2.51874e-27 -5.56166e-27 4.62201e-26 -2.24785e-27
## 27: 2.4360441e-54: 3.95778e-29 1.20710e-28 -3.89614e-28 2.20498e-28
## 28: 3.7594442e-61: -2.97704e-32 -7.48758e-32 -3.02365e-32 -2.38788e-31
## 29: 3.8487517e-67: 3.29444e-35 5.73233e-35 8.56820e-35 2.28007e-34

```

```

## 30: 2.6805234e-75: -5.07665e-39 5.77851e-39 -6.56781e-39 -2.84334e-39
## 31: 9.5203946e-82: 3.40098e-42 -3.82627e-42 5.52080e-44 7.10119e-43
## 32: 9.2490490e-90: -1.95390e-46 7.94860e-47 7.74801e-46 -1.18536e-46
## 33: 2.7852864e-97: 1.06935e-50 1.64612e-50 -1.48312e-49 2.42735e-50
## 34: 6.2041713e-107: 7.38775e-55 -9.42238e-55 9.36985e-55 -6.67742e-55
## 35: 8.3236608e-116: -3.45037e-59 7.28642e-60 -4.43881e-60 9.02262e-59
## 36: 1.1974396e-123: 1.90297e-63 3.91110e-63 3.07684e-64 -1.42533e-62
## 37: 1.0147936e-131: 8.21922e-68 -5.26631e-67 -1.99652e-68 8.81420e-67
## 38: 8.6173489e-141: -1.01460e-71 1.14575e-71 1.76633e-74 6.62779e-72
## 39: 4.1589990e-150: 2.43424e-76 -5.21738e-77 9.03871e-78 -6.42980e-76
## 40: 2.0106646e-160: -7.67394e-82 -1.58766e-81 -9.60001e-83 5.90567e-81
## 41: 5.5292711e-171: -2.09144e-87 1.20732e-86 2.73266e-88 -2.14698e-86
## 42: 1.6188979e-182: 1.58201e-92 -1.23116e-92 2.07748e-93 3.23763e-93
## 43: 2.8259611e-193: -5.60882e-98 -4.53753e-98 -3.39553e-98 -1.43953e-97
## 44: 5.9022515e-203: 3.57781e-103 9.39990e-103 4.83975e-103 2.95582e-102
## 45: 2.0276756e-214: 1.37317e-108 -1.27757e-108 -1.95432e-109 -4.75158e-108
## 46: 1.4216054e-225: -5.07833e-114 1.28093e-114 -7.40660e-115 6.65398e-114
## 47: 1.5179352e-238: 1.27121e-120 1.17833e-120 1.00773e-120 2.86077e-120
## 48: 4.4176738e-251: -1.94221e-127 -6.77404e-127 -1.30624e-126 -1.77963e-126
## 49: 1.7660155e-262: -3.79783e-133 -3.56811e-133 3.67673e-132 -1.15773e-132
## 50: 1.6392324e-273: 1.76691e-138 3.50784e-138 -8.49700e-138 9.84471e-138
## 51: 2.0904081e-286: -5.48383e-145 -1.96413e-144 -1.05454e-144 -5.06954e-144
## 52: 5.7398433e-299: 1.46757e-151 8.28441e-151 1.46922e-150 1.99423e-150
## 53: 2.1949635e-313: 1.48736e-158 4.50935e-159 -1.27714e-157 6.47993e-158
## 54: 0.0000000: -2.35751e-165 1.54288e-166 8.35366e-165 -2.24188e-164

## Warning in nlminb(start = spar, objective = efn, gradient = egr, lower =
## slower, : NA/NaN function evaluation

## 55: 0.0000000: -2.35751e-165 1.54288e-166 8.35366e-165 -2.24188e-164

if (class(tst) == "try-error"){
  cat("try-error on attempt to run nlminb in optimr()\n")
} else { print(t1nlminb) }

## $par
## [1] -4.043175e-174 0.000000e+00 -3.234540e-173 -1.293816e-172
##
## $objective
## [1] 0
##
## $convergence
## [1] 0
##
## $iterations
## [1] 13
##
## $evaluations
## function gradient
## 15 13
##
## $message
## [1] "relative convergence (4)"

```



## The Rosenbrock function

```
require(snewton)
#Rosenbrock banana valley function
f <- function(x){
  return(100*(x[2] - x[1]*x[1])^2 + (1-x[1])^2)
}
#gradient
gr <- function(x){
  return(c(-400*x[1]*(x[2] - x[1]*x[1]) - 2*(1-x[1]), 200*(x[2] - x[1]*x[1])))
}
#Hessian
h <- function(x) {
  a11 <- 2 - 400*x[2] + 1200*x[1]*x[1]; a21 <- -400*x[1]
  return(matrix(c(a11, a21, a21, 200), 2, 2))
}

x0 <- c(-1.2, 1)

xx <- x0

# sink("mbrn1-170408.txt", split=TRUE)
t1 <- snewton(x0, fn=f, gr=gr, hess=h, control=list(trace=2))

## trace = 2
## Initial function value = 24.2
## [1] -1.2 1.0
## Termination test:nf= 1
## current gradient norm = 215.6
## Iteration 1 :Search vector:[1] 0.0247191 0.3806742
## Gradient projection = -38.82876
## f(xnew)= 4.731884
## end major loop
## Termination test:nf= 2
## current gradient norm = 4.637816
## Iteration 2 :Search vector:[1] 1.938396 -4.555708
## Gradient projection = -8.433185
## f(xnew)= 1411.845
## Step size now = 0.2
## * f(xnew)= 5.4691
## Step size now = 0.04
## * f(xnew)= 4.404888
## end major loop
## Termination test:nf= 5
## current gradient norm = 7.092834
## Iteration 3 :Search vector:[1] 0.9043254 -1.9788390
## Gradient projection = -3.802796
## f(xnew)= 68.30465
## Step size now = 0.2
## * f(xnew)= 3.81876
## end major loop
## Termination test:nf= 7
## current gradient norm = 17.767
## Iteration 4 :Search vector:[1] 0.2229402 -0.3708280
```

```

## Gradient projection = -1.143361
## f(xnew)= 3.116464
## end major loop
## Termination test:nf= 8
## current gradient norm = 17.18405
## Iteration 5 :Search vector:[1] 0.1548325 -0.1651864
## Gradient projection = -1.018618
## f(xnew)= 2.426322
## end major loop
## Termination test:nf= 9
## current gradient norm = 8.247841
## Iteration 6 :Search vector:[1] 0.2656098 -0.2624113
## Gradient projection = -0.9325458
## f(xnew)= 2.119506
## end major loop
## Termination test:nf= 10
## current gradient norm = 14.10971
## Iteration 7 :Search vector:[1] 0.08428336 0.02444600
## Gradient projection = -1.21009
## f(xnew)= 1.419276
## end major loop
## Termination test:nf= 11
## current gradient norm = 2.916075
## Iteration 8 :Search vector:[1] 0.4912612 -0.1788034
## Gradient projection = -1.178522
## f(xnew)= 6.31152
## Stepsize now = 0.2
## * f(xnew)= 1.213719
## end major loop
## Termination test:nf= 13
## current gradient norm = 3.06729
## Iteration 9 :Search vector:[1] 0.26822820 -0.03346064
## Gradient projection = -0.6322948
## f(xnew)= 1.194519
## end major loop
## Termination test:nf= 14
## current gradient norm = 14.38927
## Iteration 10 :Search vector:[1] 0.05346150 0.09090021
## Gradient projection = -1.123225
## f(xnew)= 0.5925966
## end major loop
## Termination test:nf= 15
## current gradient norm = 1.274764
## Iteration 11 :Search vector:[1] 0.4894752 0.2287292
## Gradient projection = -0.7547131
## f(xnew)= 5.81843
## Stepsize now = 0.2
## * f(xnew)= 0.4648368
## end major loop
## Termination test:nf= 17
## current gradient norm = 2.373989
## Iteration 12 :Search vector:[1] 0.1989862 0.1426528
## Gradient projection = -0.2953687
## f(xnew)= 0.3799336

```

```

## end major loop
## Termination test:nf= 18
## current gradient norm = 7.919099
## Iteration 13 :Search vector:[1] 0.05296398 0.09548404
## Gradient projection = -0.3636001
## f(xnew)= 0.1767059
## end major loop
## Termination test:nf= 19
## current gradient norm = 0.5610366
## Iteration 14 :Search vector:[1] 0.2686849 0.3147876
## Gradient projection = -0.2269612
## f(xnew)= 0.5438857
## Stepsize now = 0.2
## * f(xnew)= 0.1363627
## end major loop
## Termination test:nf= 21
## current gradient norm = 1.026362
## Iteration 15 :Search vector:[1] 0.1804663 0.2340749
## Gradient projection = -0.1372565
## f(xnew)= 0.1403758
## Stepsize now = 0.2
## * f(xnew)= 0.1115589
## end major loop
## Termination test:nf= 23
## current gradient norm = 1.081634
## Iteration 16 :Search vector:[1] 0.1583356 0.2177055
## Gradient projection = -0.1102234
## f(xnew)= 0.09218166
## end major loop
## Termination test:nf= 24
## current gradient norm = 7.968122
## Iteration 17 :Search vector:[1] 0.02847694 0.07227004
## Gradient projection = -0.1354565
## f(xnew)= 0.0204531
## end major loop
## Termination test:nf= 25
## current gradient norm = 0.1621872
## Iteration 18 :Search vector:[1] 0.1228582 0.2114429
## Gradient projection = -0.03521596
## f(xnew)= 0.02318036
## Stepsize now = 0.2
## * f(xnew)= 0.0141311
## end major loop
## Termination test:nf= 27
## current gradient norm = 0.2505029
## Iteration 19 :Search vector:[1] 0.09453205 0.16796686
## Gradient projection = -0.02266352
## f(xnew)= 0.00854653
## end major loop
## Termination test:nf= 28
## current gradient norm = 3.442516
## Iteration 20 :Search vector:[1] 0.008495991 0.025525911
## Gradient projection = -0.0163739
## f(xnew)= 0.0002310919

```

```

## end major loop
## Termination test:nf= 29
## current gradient norm = 0.01443637
## Iteration 21 :Search vector:[1] 0.01496847 0.02955454
## Gradient projection = -0.0004556213
## f(xnew)= 5.066762e-06
## end major loop
## Termination test:nf= 30
## current gradient norm = 0.08917048
## Iteration 22 :Search vector:[1] 0.0002068225 0.0006376106
## Gradient projection = -1.012952e-05
## f(xnew)= 8.60774e-11
## end major loop
## Termination test:nf= 31
## current gradient norm = 8.555107e-06
## Iteration 23 :Search vector:[1] 9.267845e-06 1.857829e-05
## Gradient projection = -1.721533e-10
## f(xnew)= 7.440484e-19
## end major loop
## Termination test:nf= 32
## current gradient norm = 3.419865e-08
## Iteration 24 :Search vector:[1] 7.928735e-11 2.444678e-10
## Gradient projection = -1.488097e-18
## f(xnew)= 0
## end major loop
## Termination test:nf= 33
## current gradient norm = 0
## Small gradient norm

```

```
print(t1)
```

```

## $par
## [1] 1 1
##
## $value
## [1] 0
##
## $grad
## [1] 0 0
##
## $Hess
##      [,1] [,2]
## [1,] 802 -400
## [2,] -400 200
##
## $counts
## $counts$niter
## [1] 25
##
## $counts$nf
## [1] 33
##
## $counts$ngr
## [1] 25
##

```

```

## $counts$nhess
## [1] 24
##
##
## $convcode
## [1] 0

# we can also use nlm and nlmnb
fght <- function(x){
  ## combine f, g and h into single function for nlm
  ff <- f(x)
  gg <- gr(x)
  hh <- h(x)
  attr(ff, "gradient") <- gg
  attr(ff, "hessian") <- hh
  ff
}

## ?? SEEMS NOT TO WORK RIGHT!!
tlnlm <- nlm(fght, x0, hessian=TRUE, print.level=2)

## iteration = 0
## Step:
## [1] 0 0
## Parameter:
## [1] -1.2 1.0
## Function Value
## [1] 24.2
## Gradient:
## [1] -215.6 -88.0
##
## iteration = 1
## Step:
## [1] 0.11917904 0.04942029
## Parameter:
## [1] -1.080821 1.049420
## Function Value
## [1] 5.740059
## Gradient:
## [1] -55.50222 -23.75073
##
## iteration = 2
## Step:
## [1] 0.04232135 0.00930736
## Parameter:
## [1] -1.038500 1.058728
## Function Value
## [1] 4.194502
## Gradient:
## [1] -12.282717 -3.950757
##
## iteration = 3
## Step:
## [1] 0.011967013 -0.001777633
## Parameter:

```

```

## [1] -1.026533  1.056950
## Function Value
## [1] 4.107846
## Gradient:
## [1] -2.7469658  0.6361704
##
## iteration = 4
## Step:
## [1] 0.004223361 -0.004154668
## Parameter:
## [1] -1.022309  1.052795
## Function Value
## [1] 4.095632
## Gradient:
## [1] -0.9044183  1.5358367
##
## iteration = 5
## Step:
## [1] 0.002676889 -0.004622149
## Parameter:
## [1] -1.019632  1.048173
## Function Value
## [1] 4.086179
## Gradient:
## [1] -0.5630994  1.7046170
##
## iteration = 6
## Step:
## [1] 0.002397440 -0.004721889
## Parameter:
## [1] -1.017235  1.043451
## Function Value
## [1] 4.076779
## Gradient:
## [1] -0.500814  1.736893
##
## iteration = 7
## Step:
## [1] 0.002356612 -0.004753367
## Parameter:
## [1] -1.014878  1.038698
## Function Value
## [1] 4.067338
## Gradient:
## [1] -0.489862  1.744000
##
## iteration = 8
## Step:
## [1] 0.002360087 -0.004772393
## Parameter:
## [1] -1.012518  1.033926
## Function Value
## [1] 4.057855
## Gradient:

```

```

## [1] -0.488336  1.746487
##
## iteration = 9
## Step:
## [1]  0.002371890 -0.004789282
## Parameter:
## [1] -1.010146  1.029136
## Function Value
## [1] 4.048328
## Gradient:
## [1] -0.4885411  1.7481386
##
## iteration = 10
## Step:
## [1]  0.002385388 -0.004805938
## Parameter:
## [1] -1.007761  1.024330
## Function Value
## [1] 4.038757
## Gradient:
## [1] -0.4890652  1.7496494
##
## iteration = 11
## Step:
## [1]  0.002399363 -0.004822717
## Parameter:
## [1] -1.005362  1.019508
## Function Value
## [1] 4.029141
## Gradient:
## [1] -0.4896489  1.7511482
##
## iteration = 12
## Step:
## [1]  0.002413592 -0.004839688
## Parameter:
## [1] -1.002948  1.014668
## Function Value
## [1] 4.01948
## Gradient:
## [1] -0.4902447  1.7526588
##
## iteration = 13
## Step:
## [1]  0.002428041 -0.004856866
## Parameter:
## [1] -1.000520  1.009811
## Function Value
## [1] 4.009773
## Gradient:
## [1] -0.4908439  1.7541859
##
## iteration = 14
## Step:

```

```

## [1] 0.002442705 -0.004874260
## Parameter:
## [1] -0.9980772 1.0049368
## Function Value
## [1] 4.000019
## Gradient:
## [1] -0.4914446 1.7557308
##
## iteration = 15
## Step:
## [1] 0.002457589 -0.004891873
## Parameter:
## [1] -0.9956196 1.0000449
## Function Value
## [1] 3.990218
## Gradient:
## [1] -0.4920467 1.7572939
##
## iteration = 16
## Step:
## [1] 0.002472698 -0.004909711
## Parameter:
## [1] -0.9931469 0.9951352
## Function Value
## [1] 3.980369
## Gradient:
## [1] -0.492650 1.758876
##
## iteration = 17
## Step:
## [1] 0.002488037 -0.004927778
## Parameter:
## [1] -0.9906589 0.9902074
## Function Value
## [1] 3.970471
## Gradient:
## [1] -0.4932543 1.7604765
##
## iteration = 18
## Step:
## [1] 0.002503612 -0.004946080
## Parameter:
## [1] -0.9881553 0.9852614
## Function Value
## [1] 3.960524
## Gradient:
## [1] -0.4938597 1.7620970
##
## iteration = 19
## Step:
## [1] 0.002519428 -0.004964622
## Parameter:
## [1] -0.9856359 0.9802967
## Function Value

```



```

## [1] 3.950527
## Gradient:
## [1] -0.4944661 1.7637374
##
## iteration = 20
## Step:
## [1] 0.002535492 -0.004983409
## Parameter:
## [1] -0.9831004 0.9753133
## Function Value
## [1] 3.940479
## Gradient:
## [1] -0.4950733 1.7653983
##
## iteration = 21
## Step:
## [1] 0.002551809 -0.005002448
## Parameter:
## [1] -0.9805486 0.9703109
## Function Value
## [1] 3.930379
## Gradient:
## [1] -0.4956813 1.7670802
##
## iteration = 22
## Step:
## [1] 0.002568387 -0.005021743
## Parameter:
## [1] -0.9779802 0.9652891
## Function Value
## [1] 3.920227
## Gradient:
## [1] -0.4962899 1.7687835
##
## iteration = 23
## Step:
## [1] 0.002585231 -0.005041300
## Parameter:
## [1] -0.9753949 0.9602478
## Function Value
## [1] 3.910022
## Gradient:
## [1] -0.4968992 1.7705088
##
## iteration = 24
## Step:
## [1] 0.002602350 -0.005061127
## Parameter:
## [1] -0.9727926 0.9551867
## Function Value
## [1] 3.899763
## Gradient:
## [1] -0.497509 1.772257
##

```

```

## iteration = 25
## Step:
## [1] 0.002619749 -0.005081228
## Parameter:
## [1] -0.9701728 0.9501055
## Function Value
## [1] 3.889449
## Gradient:
## [1] -0.4981191 1.7740275
##
## iteration = 26
## Step:
## [1] 0.002637437 -0.005101612
## Parameter:
## [1] -0.9675354 0.9450039
## Function Value
## [1] 3.879079
## Gradient:
## [1] -0.4987295 1.7758220
##
## iteration = 27
## Step:
## [1] 0.002655422 -0.005122284
## Parameter:
## [1] -0.9648800 0.9398816
## Function Value
## [1] 3.868653
## Gradient:
## [1] -0.4993401 1.7776407
##
## iteration = 28
## Step:
## [1] 0.002673710 -0.005143252
## Parameter:
## [1] -0.9622063 0.9347383
## Function Value
## [1] 3.85817
## Gradient:
## [1] -0.4999507 1.7794843
##
## iteration = 29
## Step:
## [1] 0.002692311 -0.005164522
## Parameter:
## [1] -0.9595140 0.9295738
## Function Value
## [1] 3.847628
## Gradient:
## [1] -0.5005611 1.7813534
##
## iteration = 30
## Step:
## [1] 0.002711232 -0.005186104
## Parameter:

```

```

## [1] -0.9568027  0.9243877
## Function Value
## [1] 3.837027
## Gradient:
## [1] -0.5011712  1.7832486
##
## iteration = 31
## Step:
## [1] 0.002730484 -0.005208003
## Parameter:
## [1] -0.9540722  0.9191797
## Function Value
## [1] 3.826365
## Gradient:
## [1] -0.5017809  1.7851707
##
## iteration = 32
## Step:
## [1] 0.002750075 -0.005230229
## Parameter:
## [1] -0.9513222  0.9139495
## Function Value
## [1] 3.815643
## Gradient:
## [1] -0.50239  1.78712
##
## iteration = 33
## Step:
## [1] 0.002770014 -0.005252790
## Parameter:
## [1] -0.9485522  0.9086967
## Function Value
## [1] 3.804858
## Gradient:
## [1] -0.5029984  1.7890982
##
## iteration = 34
## Step:
## [1] 0.002790313 -0.005275694
## Parameter:
## [1] -0.9457618  0.9034210
## Function Value
## [1] 3.794009
## Gradient:
## [1] -0.5036057  1.7911052
##
## iteration = 35
## Step:
## [1] 0.002810981 -0.005298950
## Parameter:
## [1] -0.9429509  0.8981220
## Function Value
## [1] 3.783096
## Gradient:

```

```

## [1] -0.5042119  1.7931421
##
## iteration = 36
## Step:
## [1]  0.002832029 -0.005322569
## Parameter:
## [1] -0.9401188  0.8927995
## Function Value
## [1] 3.772118
## Gradient:
## [1] -0.5048167  1.7952097
##
## iteration = 37
## Step:
## [1]  0.002853468 -0.005346559
## Parameter:
## [1] -0.9372654  0.8874529
## Function Value
## [1] 3.761073
## Gradient:
## [1] -0.5054199  1.7973089
##
## iteration = 38
## Step:
## [1]  0.002875310 -0.005370931
## Parameter:
## [1] -0.9343901  0.8820820
## Function Value
## [1] 3.74996
## Gradient:
## [1] -0.5060212  1.7994406
##
## iteration = 39
## Step:
## [1]  0.002897568 -0.005395695
## Parameter:
## [1] -0.9314925  0.8766863
## Function Value
## [1] 3.738778
## Gradient:
## [1] -0.5066205  1.8016058
##
## iteration = 40
## Step:
## [1]  0.002920253 -0.005420863
## Parameter:
## [1] -0.9285722  0.8712654
## Function Value
## [1] 3.727525
## Gradient:
## [1] -0.5072173  1.8038054
##
## iteration = 41
## Step:

```

```

## [1] 0.002943380 -0.005446444
## Parameter:
## [1] -0.9256289 0.8658190
## Function Value
## [1] 3.716201
## Gradient:
## [1] -0.5078115 1.8060404
##
## iteration = 42
## Step:
## [1] 0.002966963 -0.005472452
## Parameter:
## [1] -0.9226619 0.8603465
## Function Value
## [1] 3.704804
## Gradient:
## [1] -0.5084028 1.8083119
##
## iteration = 43
## Step:
## [1] 0.002991015 -0.005498898
## Parameter:
## [1] -0.9196709 0.8548476
## Function Value
## [1] 3.693332
## Gradient:
## [1] -0.5089907 1.8106211
##
## iteration = 44
## Step:
## [1] 0.003015551 -0.005525796
## Parameter:
## [1] -0.9166553 0.8493218
## Function Value
## [1] 3.681785
## Gradient:
## [1] -0.5095751 1.8129691
##
## iteration = 45
## Step:
## [1] 0.003040588 -0.005553158
## Parameter:
## [1] -0.9136147 0.8437687
## Function Value
## [1] 3.67016
## Gradient:
## [1] -0.5101554 1.8153571
##
## iteration = 46
## Step:
## [1] 0.003066143 -0.005580998
## Parameter:
## [1] -0.9105486 0.8381877
## Function Value

```

```

## [1] 3.658457
## Gradient:
## [1] -0.5107313  1.8177865
##
## iteration = 47
## Step:
## [1] 0.003092231 -0.005609332
## Parameter:
## [1] -0.9074564  0.8325783
## Function Value
## [1] 3.646673
## Gradient:
## [1] -0.5113024  1.8202585
##
## iteration = 48
## Step:
## [1] 0.003118872 -0.005638173
## Parameter:
## [1] -0.9043375  0.8269402
## Function Value
## [1] 3.634808
## Gradient:
## [1] -0.5118683  1.8227745
##
## iteration = 49
## Step:
## [1] 0.003146084 -0.005667538
## Parameter:
## [1] -0.9011914  0.8212726
## Function Value
## [1] 3.622858
## Gradient:
## [1] -0.5124284  1.8253361
##
## iteration = 50
## Step:
## [1] 0.003173887 -0.005697443
## Parameter:
## [1] -0.8980175  0.8155752
## Function Value
## [1] 3.610824
## Gradient:
## [1] -0.5129824  1.8279447
##
## iteration = 51
## Step:
## [1] 0.003202302 -0.005727905
## Parameter:
## [1] -0.8948152  0.8098473
## Function Value
## [1] 3.598702
## Gradient:
## [1] -0.5135295  1.8306019
##

```

```

## iteration = 52
## Step:
## [1] 0.003231350 -0.005758943
## Parameter:
## [1] -0.8915839 0.8040883
## Function Value
## [1] 3.586492
## Gradient:
## [1] -0.5140693 1.8333095
##
## iteration = 53
## Step:
## [1] 0.003261055 -0.005790574
## Parameter:
## [1] -0.8883228 0.7982978
## Function Value
## [1] 3.574191
## Gradient:
## [1] -0.5146012 1.8360693
##
## iteration = 54
## Step:
## [1] 0.003291439 -0.005822819
## Parameter:
## [1] -0.8850314 0.7924750
## Function Value
## [1] 3.561797
## Gradient:
## [1] -0.5151245 1.8388830
##
## iteration = 55
## Step:
## [1] 0.003322530 -0.005855699
## Parameter:
## [1] -0.8817088 0.7866193
## Function Value
## [1] 3.549308
## Gradient:
## [1] -0.5156385 1.8417527
##
## iteration = 56
## Step:
## [1] 0.003354353 -0.005889234
## Parameter:
## [1] -0.8783545 0.7807300
## Function Value
## [1] 3.536723
## Gradient:
## [1] -0.5161424 1.8446804
##
## iteration = 57
## Step:
## [1] 0.003386935 -0.005923448
## Parameter:

```

```

## [1] -0.8749676  0.7748066
## Function Value
## [1] 3.524038
## Gradient:
## [1] -0.5166355  1.8476683
##
## iteration = 58
## Step:
## [1] 0.003420307 -0.005958365
## Parameter:
## [1] -0.8715473  0.7688482
## Function Value
## [1] 3.511252
## Gradient:
## [1] -0.5171169  1.8507187
##
## iteration = 59
## Step:
## [1] 0.003454499 -0.005994008
## Parameter:
## [1] -0.8680928  0.7628542
## Function Value
## [1] 3.498362
## Gradient:
## [1] -0.5175857  1.8538340
##
## iteration = 60
## Step:
## [1] 0.003489544 -0.006030405
## Parameter:
## [1] -0.8646032  0.7568238
## Function Value
## [1] 3.485366
## Gradient:
## [1] -0.5180409  1.8570169
##
## iteration = 61
## Step:
## [1] 0.003525477 -0.006067583
## Parameter:
## [1] -0.8610777  0.7507562
## Function Value
## [1] 3.472262
## Gradient:
## [1] -0.5184815  1.8602699
##
## iteration = 62
## Step:
## [1] 0.003562334 -0.006105572
## Parameter:
## [1] -0.8575154  0.7446506
## Function Value
## [1] 3.459046
## Gradient:

```



```

## [1] -0.5189064  1.8635959
##
## iteration = 63
## Step:
## [1]  0.003600152 -0.006144401
## Parameter:
## [1] -0.8539152  0.7385062
## Function Value
## [1] 3.445716
## Gradient:
## [1] -0.5193143  1.8669981
##
## iteration = 64
## Step:
## [1]  0.003638975 -0.006184102
## Parameter:
## [1] -0.8502763  0.7323221
## Function Value
## [1] 3.432269
## Gradient:
## [1] -0.5197039  1.8704795
##
## iteration = 65
## Step:
## [1]  0.003678843 -0.006224711
## Parameter:
## [1] -0.8465974  0.7260974
## Function Value
## [1] 3.418702
## Gradient:
## [1] -0.5200739  1.8740436
##
## iteration = 66
## Step:
## [1]  0.003719803 -0.006266263
## Parameter:
## [1] -0.8428776  0.7198312
## Function Value
## [1] 3.405012
## Gradient:
## [1] -0.5204227  1.8776940
##
## iteration = 67
## Step:
## [1]  0.003761905 -0.006308796
## Parameter:
## [1] -0.8391157  0.7135224
## Function Value
## [1] 3.391196
## Gradient:
## [1] -0.5207487  1.8814346
##
## iteration = 68
## Step:

```

```

## [1] 0.003805198 -0.006352349
## Parameter:
## [1] -0.8353105 0.7071700
## Function Value
## [1] 3.37725
## Gradient:
## [1] -0.5210502 1.8852694
##
## iteration = 69
## Step:
## [1] 0.003849738 -0.006396966
## Parameter:
## [1] -0.8314608 0.7007731
## Function Value
## [1] 3.363171
## Gradient:
## [1] -0.5213253 1.8892029
##
## iteration = 70
## Step:
## [1] 0.003895584 -0.006442692
## Parameter:
## [1] -0.8275652 0.6943304
## Function Value
## [1] 3.348955
## Gradient:
## [1] -0.521572 1.893240
##
## iteration = 71
## Step:
## [1] 0.003942798 -0.006489574
## Parameter:
## [1] -0.8236224 0.6878408
## Function Value
## [1] 3.334599
## Gradient:
## [1] -0.5217881 1.8973845
##
## iteration = 72
## Step:
## [1] 0.003991445 -0.006537663
## Parameter:
## [1] -0.8196310 0.6813031
## Function Value
## [1] 3.320097
## Gradient:
## [1] -0.5219711 1.9016429
##
## iteration = 73
## Step:
## [1] 0.004041597 -0.006587013
## Parameter:
## [1] -0.8155894 0.6747161
## Function Value

```

```

## [1] 3.305447
## Gradient:
## [1] -0.5221186  1.9060205
##
## iteration = 74
## Step:
## [1] 0.004093330 -0.006637682
## Parameter:
## [1] -0.8114960  0.6680784
## Function Value
## [1] 3.290643
## Gradient:
## [1] -0.5222276  1.9105235
##
## iteration = 75
## Step:
## [1] 0.004146724 -0.006689731
## Parameter:
## [1] -0.8073493  0.6613887
## Function Value
## [1] 3.275681
## Gradient:
## [1] -0.5222953  1.9151582
##
## iteration = 76
## Step:
## [1] 0.004201867 -0.006743224
## Parameter:
## [1] -0.8031474  0.6546455
## Function Value
## [1] 3.260556
## Gradient:
## [1] -0.5223182  1.9199318
##
## iteration = 77
## Step:
## [1] 0.004258851 -0.006798233
## Parameter:
## [1] -0.7988886  0.6478472
## Function Value
## [1] 3.245263
## Gradient:
## [1] -0.5222928  1.9248518
##
## iteration = 78
## Step:
## [1] 0.004317778 -0.006854831
## Parameter:
## [1] -0.7945708  0.6409924
## Function Value
## [1] 3.229796
## Gradient:
## [1] -0.5222152  1.9299264
##

```

```

## iteration = 79
## Step:
## [1] 0.004378754 -0.006913098
## Parameter:
## [1] -0.7901921 0.6340793
## Function Value
## [1] 3.21415
## Gradient:
## [1] -0.5220811 1.9351644
##
## iteration = 80
## Step:
## [1] 0.004441897 -0.006973119
## Parameter:
## [1] -0.7857502 0.6271062
## Function Value
## [1] 3.198318
## Gradient:
## [1] -0.5218858 1.9405752
##
## iteration = 81
## Step:
## [1] 0.004507332 -0.007034988
## Parameter:
## [1] -0.7812428 0.6200712
## Function Value
## [1] 3.182295
## Gradient:
## [1] -0.5216242 1.9461692
##
## iteration = 82
## Step:
## [1] 0.004575195 -0.007098802
## Parameter:
## [1] -0.7766676 0.6129724
## Function Value
## [1] 3.166073
## Gradient:
## [1] -0.5212907 1.9519576
##
## iteration = 83
## Step:
## [1] 0.004645633 -0.007164669
## Parameter:
## [1] -0.7720220 0.6058077
## Function Value
## [1] 3.149646
## Gradient:
## [1] -0.5208791 1.9579526
##
## iteration = 84
## Step:
## [1] 0.004718806 -0.007232703
## Parameter:

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## [1] -0.7673032  0.5985750
## Function Value
## [1] 3.133005
## Gradient:
## [1] -0.5203826  1.9641674
##
## iteration = 85
## Step:
## [1] 0.004794887 -0.007303028
## Parameter:
## [1] -0.7625083  0.5912720
## Function Value
## [1] 3.116144
## Gradient:
## [1] -0.5197938  1.9706165
##
## iteration = 86
## Step:
## [1] 0.004874066 -0.007375779
## Parameter:
## [1] -0.7576342  0.5838962
## Function Value
## [1] 3.099053
## Gradient:
## [1] -0.5191043  1.9773157
##
## iteration = 87
## Step:
## [1] 0.004956550 -0.007451103
## Parameter:
## [1] -0.7526777  0.5764451
## Function Value
## [1] 3.081723
## Gradient:
## [1] -0.5183051  1.9842824
##
## iteration = 88
## Step:
## [1] 0.005042564 -0.007529156
## Parameter:
## [1] -0.7476351  0.5689160
## Function Value
## [1] 3.064144
## Gradient:
## [1] -0.517386  1.991536
##
## iteration = 89
## Step:
## [1] 0.005132356 -0.007610113
## Parameter:
## [1] -0.7425028  0.5613058
## Function Value
## [1] 3.046307
## Gradient:

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## [1] -0.5163357  1.9990968
##
## iteration = 90
## Step:
## [1]  0.005226199 -0.007694161
## Parameter:
## [1] -0.7372766  0.5536117
## Function Value
## [1] 3.0282
## Gradient:
## [1] -0.5151417  2.0069887
##
## iteration = 91
## Step:
## [1]  0.005324392 -0.007781508
## Parameter:
## [1] -0.7319522  0.5458302
## Function Value
## [1] 3.009811
## Gradient:
## [1] -0.5137898  2.0152372
##
## iteration = 92
## Step:
## [1]  0.005427268 -0.007872378
## Parameter:
## [1] -0.7265249  0.5379578
## Function Value
## [1] 2.991128
## Gradient:
## [1] -0.5122644  2.0238710
##
## iteration = 93
## Step:
## [1]  0.005535194 -0.007967020
## Parameter:
## [1] -0.7209897  0.5299908
## Function Value
## [1] 2.972138
## Gradient:
## [1] -0.5105478  2.0329219
##
## iteration = 94
## Step:
## [1]  0.005648577 -0.008065708
## Parameter:
## [1] -0.7153411  0.5219251
## Function Value
## [1] 2.952824
## Gradient:
## [1] -0.5086201  2.0424256
##
## iteration = 95
## Step:

```

```

## [1] 0.005767874 -0.008168744
## Parameter:
## [1] -0.7095733 0.5137563
## Function Value
## [1] 2.933172
## Gradient:
## [1] -0.5064589 2.0524221
##
## iteration = 96
## Step:
## [1] 0.005893591 -0.008276464
## Parameter:
## [1] -0.7036797 0.5054799
## Function Value
## [1] 2.913164
## Gradient:
## [1] -0.5040386 2.0629563
##
## iteration = 97
## Step:
## [1] 0.006026301 -0.008389241
## Parameter:
## [1] -0.6976534 0.4970906
## Function Value
## [1] 2.892781
## Gradient:
## [1] -0.5013302 2.0740791
##
## iteration = 98
## Step:
## [1] 0.006166646 -0.008507489
## Parameter:
## [1] -0.6914867 0.4885831
## Function Value
## [1] 2.872004
## Gradient:
## [1] -0.4983006 2.0858483
##
## iteration = 99
## Step:
## [1] 0.006315353 -0.008631675
## Parameter:
## [1] -0.6851714 0.4799515
## Function Value
## [1] 2.85081
## Gradient:
## [1] -0.4949118 2.0983297
##
## iteration = 100
## Parameter:
## [1] -0.6786981 0.4711891
## Function Value
## [1] 2.829175
## Gradient:

```

```
## [1] -0.4911201  2.1115987
##
## Iteration limit exceeded.  Algorithm failed.
print(t1nlm)

## $minimum
## [1] 2.829175
##
## $estimate
## [1] -0.6786981  0.4711891
##
## $gradient
## [1] -0.4911201  2.1115987
##
## $hessian
##          [,1]      [,2]
## [1,] 366.1188 271.4593
## [2,] 271.4593 200.0000
##
## $code
## [1] 4
##
## $iterations
## [1] 100

## BUT ... it looks like nlminb is NOT using a true Newton-type method
t1nlminb <- nlminb(x0, f, gradient=gr, hessian=h, control=list(trace=1))

##  0:      24.200000: -1.20000  1.00000
##  1:      4.7318843: -1.17528  1.38067
##  2:      4.1185053: -1.02313  1.03084
##  3:      3.4877941: -0.851666  0.701020
##  4:      2.8656106: -0.651455  0.387204
##  5:      2.2659017: -0.455737  0.169391
##  6:      1.7378734: -0.287660  0.0544980
##  7:      1.3374723: -0.0940265 -0.0286528
##  8:      0.95926083: 0.0347014 -0.0153667
##  9:      0.73171551: 0.161329  0.00919067
## 10:      0.55413441: 0.353365  0.0879888
## 11:      0.32780370: 0.430570  0.179430
## 12:      0.24068818: 0.519227  0.259826
## 13:      0.17104970: 0.591831  0.343595
## 14:      0.11873673: 0.660666  0.430489
## 15:      0.078218534: 0.725043  0.520572
## 16:      0.047818423: 0.785832  0.613116
## 17:      0.025947706: 0.843482  0.707654
## 18:      0.010811755: 0.932337  0.861356
## 19:      0.0017636164: 0.958572  0.918173
## 20:      0.00023632061: 0.987247  0.973799
## 21:      4.8951105e-06: 0.998131  0.996148
## 22:      2.9806009e-09: 0.999957  0.999910
## 23:      1.1784043e-15:  1.00000  1.00000
## 24:      1.9486097e-28:  1.00000  1.00000
## 25:      0.0000000:  1.00000  1.00000
```



```

print(t1nlminb)

## $par
## [1] 1 1
##
## $objective
## [1] 0
##
## $convergence
## [1] 0
##
## $iterations
## [1] 25
##
## $evaluations
## function gradient
##      33      26
##
## $message
## [1] "X-convergence (3)"

# and call them from optimx (i.e., test this gives same results)

library(optimx)
t1nlmo <- optimr(x0, f, gr, hess=h, method="nlm", control=list(trace=1))

## Unit parameter scaling
## iteration = 0
## Step:
## [1] 0 0
## Parameter:
## [1] -1.2 1.0
## Function Value
## [1] 24.2
## Gradient:
## [1] -215.6 -88.0
##
## iteration = 1
## Step:
## [1] 0.11917904 0.04942029
## Parameter:
## [1] -1.080821 1.049420
## Function Value
## [1] 5.740059
## Gradient:
## [1] -55.50222 -23.75073
##
## iteration = 2
## Step:
## [1] 0.04232135 0.00930736
## Parameter:
## [1] -1.038500 1.058728
## Function Value
## [1] 4.194502

```

```

## Gradient:
## [1] -12.282717 -3.950757
##
## iteration = 3
## Step:
## [1] 0.011967013 -0.001777633
## Parameter:
## [1] -1.026533 1.056950
## Function Value
## [1] 4.107846
## Gradient:
## [1] -2.7469658 0.6361704
##
## iteration = 4
## Step:
## [1] 0.004223361 -0.004154668
## Parameter:
## [1] -1.022309 1.052795
## Function Value
## [1] 4.095632
## Gradient:
## [1] -0.9044183 1.5358367
##
## iteration = 5
## Step:
## [1] 0.002676889 -0.004622149
## Parameter:
## [1] -1.019632 1.048173
## Function Value
## [1] 4.086179
## Gradient:
## [1] -0.5630994 1.7046170
##
## iteration = 6
## Step:
## [1] 0.002397440 -0.004721889
## Parameter:
## [1] -1.017235 1.043451
## Function Value
## [1] 4.076779
## Gradient:
## [1] -0.500814 1.736893
##
## iteration = 7
## Step:
## [1] 0.002356612 -0.004753367
## Parameter:
## [1] -1.014878 1.038698
## Function Value
## [1] 4.067338
## Gradient:
## [1] -0.489862 1.744000
##
## iteration = 8

```

```

## Step:
## [1] 0.002360087 -0.004772393
## Parameter:
## [1] -1.012518 1.033926
## Function Value
## [1] 4.057855
## Gradient:
## [1] -0.488336 1.746487
##
## iteration = 9
## Step:
## [1] 0.002371890 -0.004789282
## Parameter:
## [1] -1.010146 1.029136
## Function Value
## [1] 4.048328
## Gradient:
## [1] -0.4885411 1.7481386
##
## iteration = 10
## Step:
## [1] 0.002385388 -0.004805938
## Parameter:
## [1] -1.007761 1.024330
## Function Value
## [1] 4.038757
## Gradient:
## [1] -0.4890652 1.7496494
##
## iteration = 11
## Step:
## [1] 0.002399363 -0.004822717
## Parameter:
## [1] -1.005362 1.019508
## Function Value
## [1] 4.029141
## Gradient:
## [1] -0.4896489 1.7511482
##
## iteration = 12
## Step:
## [1] 0.002413592 -0.004839688
## Parameter:
## [1] -1.002948 1.014668
## Function Value
## [1] 4.01948
## Gradient:
## [1] -0.4902447 1.7526588
##
## iteration = 13
## Step:
## [1] 0.002428041 -0.004856866
## Parameter:
## [1] -1.000520 1.009811

```

```

## Function Value
## [1] 4.009773
## Gradient:
## [1] -0.4908439  1.7541859
##
## iteration = 14
## Step:
## [1] 0.002442705 -0.004874260
## Parameter:
## [1] -0.9980772  1.0049368
## Function Value
## [1] 4.000019
## Gradient:
## [1] -0.4914446  1.7557308
##
## iteration = 15
## Step:
## [1] 0.002457589 -0.004891873
## Parameter:
## [1] -0.9956196  1.0000449
## Function Value
## [1] 3.990218
## Gradient:
## [1] -0.4920467  1.7572939
##
## iteration = 16
## Step:
## [1] 0.002472698 -0.004909711
## Parameter:
## [1] -0.9931469  0.9951352
## Function Value
## [1] 3.980369
## Gradient:
## [1] -0.492650  1.758876
##
## iteration = 17
## Step:
## [1] 0.002488037 -0.004927778
## Parameter:
## [1] -0.9906589  0.9902074
## Function Value
## [1] 3.970471
## Gradient:
## [1] -0.4932543  1.7604765
##
## iteration = 18
## Step:
## [1] 0.002503612 -0.004946080
## Parameter:
## [1] -0.9881553  0.9852614
## Function Value
## [1] 3.960524
## Gradient:
## [1] -0.4938597  1.7620970

```

```

##
## iteration = 19
## Step:
## [1] 0.002519428 -0.004964622
## Parameter:
## [1] -0.9856359 0.9802967
## Function Value
## [1] 3.950527
## Gradient:
## [1] -0.4944661 1.7637374
##
## iteration = 20
## Step:
## [1] 0.002535492 -0.004983409
## Parameter:
## [1] -0.9831004 0.9753133
## Function Value
## [1] 3.940479
## Gradient:
## [1] -0.4950733 1.7653983
##
## iteration = 21
## Step:
## [1] 0.002551809 -0.005002448
## Parameter:
## [1] -0.9805486 0.9703109
## Function Value
## [1] 3.930379
## Gradient:
## [1] -0.4956813 1.7670802
##
## iteration = 22
## Step:
## [1] 0.002568387 -0.005021743
## Parameter:
## [1] -0.9779802 0.9652891
## Function Value
## [1] 3.920227
## Gradient:
## [1] -0.4962899 1.7687835
##
## iteration = 23
## Step:
## [1] 0.002585231 -0.005041300
## Parameter:
## [1] -0.9753949 0.9602478
## Function Value
## [1] 3.910022
## Gradient:
## [1] -0.4968992 1.7705088
##
## iteration = 24
## Step:
## [1] 0.002602350 -0.005061127

```

```

## Parameter:
## [1] -0.9727926  0.9551867
## Function Value
## [1] 3.899763
## Gradient:
## [1] -0.497509  1.772257
##
## iteration = 25
## Step:
## [1] 0.002619749 -0.005081228
## Parameter:
## [1] -0.9701728  0.9501055
## Function Value
## [1] 3.889449
## Gradient:
## [1] -0.4981191  1.7740275
##
## iteration = 26
## Step:
## [1] 0.002637437 -0.005101612
## Parameter:
## [1] -0.9675354  0.9450039
## Function Value
## [1] 3.879079
## Gradient:
## [1] -0.4987295  1.7758220
##
## iteration = 27
## Step:
## [1] 0.002655422 -0.005122284
## Parameter:
## [1] -0.9648800  0.9398816
## Function Value
## [1] 3.868653
## Gradient:
## [1] -0.4993401  1.7776407
##
## iteration = 28
## Step:
## [1] 0.002673710 -0.005143252
## Parameter:
## [1] -0.9622063  0.9347383
## Function Value
## [1] 3.85817
## Gradient:
## [1] -0.4999507  1.7794843
##
## iteration = 29
## Step:
## [1] 0.002692311 -0.005164522
## Parameter:
## [1] -0.9595140  0.9295738
## Function Value
## [1] 3.847628

```

```

## Gradient:
## [1] -0.5005611  1.7813534
##
## iteration = 30
## Step:
## [1]  0.002711232 -0.005186104
## Parameter:
## [1] -0.9568027  0.9243877
## Function Value
## [1] 3.837027
## Gradient:
## [1] -0.5011712  1.7832486
##
## iteration = 31
## Step:
## [1]  0.002730484 -0.005208003
## Parameter:
## [1] -0.9540722  0.9191797
## Function Value
## [1] 3.826365
## Gradient:
## [1] -0.5017809  1.7851707
##
## iteration = 32
## Step:
## [1]  0.002750075 -0.005230229
## Parameter:
## [1] -0.9513222  0.9139495
## Function Value
## [1] 3.815643
## Gradient:
## [1] -0.50239  1.78712
##
## iteration = 33
## Step:
## [1]  0.002770014 -0.005252790
## Parameter:
## [1] -0.9485522  0.9086967
## Function Value
## [1] 3.804858
## Gradient:
## [1] -0.5029984  1.7890982
##
## iteration = 34
## Step:
## [1]  0.002790313 -0.005275694
## Parameter:
## [1] -0.9457618  0.9034210
## Function Value
## [1] 3.794009
## Gradient:
## [1] -0.5036057  1.7911052
##
## iteration = 35

```

```

## Step:
## [1] 0.002810981 -0.005298950
## Parameter:
## [1] -0.9429509 0.8981220
## Function Value
## [1] 3.783096
## Gradient:
## [1] -0.5042119 1.7931421
##
## iteration = 36
## Step:
## [1] 0.002832029 -0.005322569
## Parameter:
## [1] -0.9401188 0.8927995
## Function Value
## [1] 3.772118
## Gradient:
## [1] -0.5048167 1.7952097
##
## iteration = 37
## Step:
## [1] 0.002853468 -0.005346559
## Parameter:
## [1] -0.9372654 0.8874529
## Function Value
## [1] 3.761073
## Gradient:
## [1] -0.5054199 1.7973089
##
## iteration = 38
## Step:
## [1] 0.002875310 -0.005370931
## Parameter:
## [1] -0.9343901 0.8820820
## Function Value
## [1] 3.74996
## Gradient:
## [1] -0.5060212 1.7994406
##
## iteration = 39
## Step:
## [1] 0.002897568 -0.005395695
## Parameter:
## [1] -0.9314925 0.8766863
## Function Value
## [1] 3.738778
## Gradient:
## [1] -0.5066205 1.8016058
##
## iteration = 40
## Step:
## [1] 0.002920253 -0.005420863
## Parameter:
## [1] -0.9285722 0.8712654

```



```

## Function Value
## [1] 3.727525
## Gradient:
## [1] -0.5072173  1.8038054
##
## iteration = 41
## Step:
## [1] 0.002943380 -0.005446444
## Parameter:
## [1] -0.9256289  0.8658190
## Function Value
## [1] 3.716201
## Gradient:
## [1] -0.5078115  1.8060404
##
## iteration = 42
## Step:
## [1] 0.002966963 -0.005472452
## Parameter:
## [1] -0.9226619  0.8603465
## Function Value
## [1] 3.704804
## Gradient:
## [1] -0.5084028  1.8083119
##
## iteration = 43
## Step:
## [1] 0.002991015 -0.005498898
## Parameter:
## [1] -0.9196709  0.8548476
## Function Value
## [1] 3.693332
## Gradient:
## [1] -0.5089907  1.8106211
##
## iteration = 44
## Step:
## [1] 0.003015551 -0.005525796
## Parameter:
## [1] -0.9166553  0.8493218
## Function Value
## [1] 3.681785
## Gradient:
## [1] -0.5095751  1.8129691
##
## iteration = 45
## Step:
## [1] 0.003040588 -0.005553158
## Parameter:
## [1] -0.9136147  0.8437687
## Function Value
## [1] 3.67016
## Gradient:
## [1] -0.5101554  1.8153571

```

```

##
## iteration = 46
## Step:
## [1] 0.003066143 -0.005580998
## Parameter:
## [1] -0.9105486 0.8381877
## Function Value
## [1] 3.658457
## Gradient:
## [1] -0.5107313 1.8177865
##
## iteration = 47
## Step:
## [1] 0.003092231 -0.005609332
## Parameter:
## [1] -0.9074564 0.8325783
## Function Value
## [1] 3.646673
## Gradient:
## [1] -0.5113024 1.8202585
##
## iteration = 48
## Step:
## [1] 0.003118872 -0.005638173
## Parameter:
## [1] -0.9043375 0.8269402
## Function Value
## [1] 3.634808
## Gradient:
## [1] -0.5118683 1.8227745
##
## iteration = 49
## Step:
## [1] 0.003146084 -0.005667538
## Parameter:
## [1] -0.9011914 0.8212726
## Function Value
## [1] 3.622858
## Gradient:
## [1] -0.5124284 1.8253361
##
## iteration = 50
## Step:
## [1] 0.003173887 -0.005697443
## Parameter:
## [1] -0.8980175 0.8155752
## Function Value
## [1] 3.610824
## Gradient:
## [1] -0.5129824 1.8279447
##
## iteration = 51
## Step:
## [1] 0.003202302 -0.005727905

```

```

## Parameter:
## [1] -0.8948152  0.8098473
## Function Value
## [1] 3.598702
## Gradient:
## [1] -0.5135295  1.8306019
##
## iteration = 52
## Step:
## [1] 0.003231350 -0.005758943
## Parameter:
## [1] -0.8915839  0.8040883
## Function Value
## [1] 3.586492
## Gradient:
## [1] -0.5140693  1.8333095
##
## iteration = 53
## Step:
## [1] 0.003261055 -0.005790574
## Parameter:
## [1] -0.8883228  0.7982978
## Function Value
## [1] 3.574191
## Gradient:
## [1] -0.5146012  1.8360693
##
## iteration = 54
## Step:
## [1] 0.003291439 -0.005822819
## Parameter:
## [1] -0.8850314  0.7924750
## Function Value
## [1] 3.561797
## Gradient:
## [1] -0.5151245  1.8388830
##
## iteration = 55
## Step:
## [1] 0.003322530 -0.005855699
## Parameter:
## [1] -0.8817088  0.7866193
## Function Value
## [1] 3.549308
## Gradient:
## [1] -0.5156385  1.8417527
##
## iteration = 56
## Step:
## [1] 0.003354353 -0.005889234
## Parameter:
## [1] -0.8783545  0.7807300
## Function Value
## [1] 3.536723

```

```

## Gradient:
## [1] -0.5161424  1.8446804
##
## iteration = 57
## Step:
## [1]  0.003386935 -0.005923448
## Parameter:
## [1] -0.8749676  0.7748066
## Function Value
## [1] 3.524038
## Gradient:
## [1] -0.5166355  1.8476683
##
## iteration = 58
## Step:
## [1]  0.003420307 -0.005958365
## Parameter:
## [1] -0.8715473  0.7688482
## Function Value
## [1] 3.511252
## Gradient:
## [1] -0.5171169  1.8507187
##
## iteration = 59
## Step:
## [1]  0.003454499 -0.005994008
## Parameter:
## [1] -0.8680928  0.7628542
## Function Value
## [1] 3.498362
## Gradient:
## [1] -0.5175857  1.8538340
##
## iteration = 60
## Step:
## [1]  0.003489544 -0.006030405
## Parameter:
## [1] -0.8646032  0.7568238
## Function Value
## [1] 3.485366
## Gradient:
## [1] -0.5180409  1.8570169
##
## iteration = 61
## Step:
## [1]  0.003525477 -0.006067583
## Parameter:
## [1] -0.8610777  0.7507562
## Function Value
## [1] 3.472262
## Gradient:
## [1] -0.5184815  1.8602699
##
## iteration = 62

```

```

## Step:
## [1] 0.003562334 -0.006105572
## Parameter:
## [1] -0.8575154 0.7446506
## Function Value
## [1] 3.459046
## Gradient:
## [1] -0.5189064 1.8635959
##
## iteration = 63
## Step:
## [1] 0.003600152 -0.006144401
## Parameter:
## [1] -0.8539152 0.7385062
## Function Value
## [1] 3.445716
## Gradient:
## [1] -0.5193143 1.8669981
##
## iteration = 64
## Step:
## [1] 0.003638975 -0.006184102
## Parameter:
## [1] -0.8502763 0.7323221
## Function Value
## [1] 3.432269
## Gradient:
## [1] -0.5197039 1.8704795
##
## iteration = 65
## Step:
## [1] 0.003678843 -0.006224711
## Parameter:
## [1] -0.8465974 0.7260974
## Function Value
## [1] 3.418702
## Gradient:
## [1] -0.5200739 1.8740436
##
## iteration = 66
## Step:
## [1] 0.003719803 -0.006266263
## Parameter:
## [1] -0.8428776 0.7198312
## Function Value
## [1] 3.405012
## Gradient:
## [1] -0.5204227 1.8776940
##
## iteration = 67
## Step:
## [1] 0.003761905 -0.006308796
## Parameter:
## [1] -0.8391157 0.7135224

```

```

## Function Value
## [1] 3.391196
## Gradient:
## [1] -0.5207487  1.8814346
##
## iteration = 68
## Step:
## [1] 0.003805198 -0.006352349
## Parameter:
## [1] -0.8353105  0.7071700
## Function Value
## [1] 3.37725
## Gradient:
## [1] -0.5210502  1.8852694
##
## iteration = 69
## Step:
## [1] 0.003849738 -0.006396966
## Parameter:
## [1] -0.8314608  0.7007731
## Function Value
## [1] 3.363171
## Gradient:
## [1] -0.5213253  1.8892029
##
## iteration = 70
## Step:
## [1] 0.003895584 -0.006442692
## Parameter:
## [1] -0.8275652  0.6943304
## Function Value
## [1] 3.348955
## Gradient:
## [1] -0.521572  1.893240
##
## iteration = 71
## Step:
## [1] 0.003942798 -0.006489574
## Parameter:
## [1] -0.8236224  0.6878408
## Function Value
## [1] 3.334599
## Gradient:
## [1] -0.5217881  1.8973845
##
## iteration = 72
## Step:
## [1] 0.003991445 -0.006537663
## Parameter:
## [1] -0.8196310  0.6813031
## Function Value
## [1] 3.320097
## Gradient:
## [1] -0.5219711  1.9016429

```

```

##
## iteration = 73
## Step:
## [1] 0.004041597 -0.006587013
## Parameter:
## [1] -0.8155894 0.6747161
## Function Value
## [1] 3.305447
## Gradient:
## [1] -0.5221186 1.9060205
##
## iteration = 74
## Step:
## [1] 0.004093330 -0.006637682
## Parameter:
## [1] -0.8114960 0.6680784
## Function Value
## [1] 3.290643
## Gradient:
## [1] -0.5222276 1.9105235
##
## iteration = 75
## Step:
## [1] 0.004146724 -0.006689731
## Parameter:
## [1] -0.8073493 0.6613887
## Function Value
## [1] 3.275681
## Gradient:
## [1] -0.5222953 1.9151582
##
## iteration = 76
## Step:
## [1] 0.004201867 -0.006743224
## Parameter:
## [1] -0.8031474 0.6546455
## Function Value
## [1] 3.260556
## Gradient:
## [1] -0.5223182 1.9199318
##
## iteration = 77
## Step:
## [1] 0.004258851 -0.006798233
## Parameter:
## [1] -0.7988886 0.6478472
## Function Value
## [1] 3.245263
## Gradient:
## [1] -0.5222928 1.9248518
##
## iteration = 78
## Step:
## [1] 0.004317778 -0.006854831

```

```

## Parameter:
## [1] -0.7945708  0.6409924
## Function Value
## [1] 3.229796
## Gradient:
## [1] -0.5222152  1.9299264
##
## iteration = 79
## Step:
## [1] 0.004378754 -0.006913098
## Parameter:
## [1] -0.7901921  0.6340793
## Function Value
## [1] 3.21415
## Gradient:
## [1] -0.5220811  1.9351644
##
## iteration = 80
## Step:
## [1] 0.004441897 -0.006973119
## Parameter:
## [1] -0.7857502  0.6271062
## Function Value
## [1] 3.198318
## Gradient:
## [1] -0.5218858  1.9405752
##
## iteration = 81
## Step:
## [1] 0.004507332 -0.007034988
## Parameter:
## [1] -0.7812428  0.6200712
## Function Value
## [1] 3.182295
## Gradient:
## [1] -0.5216242  1.9461692
##
## iteration = 82
## Step:
## [1] 0.004575195 -0.007098802
## Parameter:
## [1] -0.7766676  0.6129724
## Function Value
## [1] 3.166073
## Gradient:
## [1] -0.5212907  1.9519576
##
## iteration = 83
## Step:
## [1] 0.004645633 -0.007164669
## Parameter:
## [1] -0.7720220  0.6058077
## Function Value
## [1] 3.149646

```



```

## Gradient:
## [1] -0.5208791  1.9579526
##
## iteration = 84
## Step:
## [1]  0.004718806 -0.007232703
## Parameter:
## [1] -0.7673032  0.5985750
## Function Value
## [1] 3.133005
## Gradient:
## [1] -0.5203826  1.9641674
##
## iteration = 85
## Step:
## [1]  0.004794887 -0.007303028
## Parameter:
## [1] -0.7625083  0.5912720
## Function Value
## [1] 3.116144
## Gradient:
## [1] -0.5197938  1.9706165
##
## iteration = 86
## Step:
## [1]  0.004874066 -0.007375779
## Parameter:
## [1] -0.7576342  0.5838962
## Function Value
## [1] 3.099053
## Gradient:
## [1] -0.5191043  1.9773157
##
## iteration = 87
## Step:
## [1]  0.004956550 -0.007451103
## Parameter:
## [1] -0.7526777  0.5764451
## Function Value
## [1] 3.081723
## Gradient:
## [1] -0.5183051  1.9842824
##
## iteration = 88
## Step:
## [1]  0.005042564 -0.007529156
## Parameter:
## [1] -0.7476351  0.5689160
## Function Value
## [1] 3.064144
## Gradient:
## [1] -0.517386  1.991536
##
## iteration = 89

```

```

## Step:
## [1] 0.005132356 -0.007610113
## Parameter:
## [1] -0.7425028 0.5613058
## Function Value
## [1] 3.046307
## Gradient:
## [1] -0.5163357 1.9990968
##
## iteration = 90
## Step:
## [1] 0.005226199 -0.007694161
## Parameter:
## [1] -0.7372766 0.5536117
## Function Value
## [1] 3.0282
## Gradient:
## [1] -0.5151417 2.0069887
##
## iteration = 91
## Step:
## [1] 0.005324392 -0.007781508
## Parameter:
## [1] -0.7319522 0.5458302
## Function Value
## [1] 3.009811
## Gradient:
## [1] -0.5137898 2.0152372
##
## iteration = 92
## Step:
## [1] 0.005427268 -0.007872378
## Parameter:
## [1] -0.7265249 0.5379578
## Function Value
## [1] 2.991128
## Gradient:
## [1] -0.5122644 2.0238710
##
## iteration = 93
## Step:
## [1] 0.005535194 -0.007967020
## Parameter:
## [1] -0.7209897 0.5299908
## Function Value
## [1] 2.972138
## Gradient:
## [1] -0.5105478 2.0329219
##
## iteration = 94
## Step:
## [1] 0.005648577 -0.008065708
## Parameter:
## [1] -0.7153411 0.5219251

```

```

## Function Value
## [1] 2.952824
## Gradient:
## [1] -0.5086201 2.0424256
##
## iteration = 95
## Step:
## [1] 0.005767874 -0.008168744
## Parameter:
## [1] -0.7095733 0.5137563
## Function Value
## [1] 2.933172
## Gradient:
## [1] -0.5064589 2.0524221
##
## iteration = 96
## Step:
## [1] 0.005893591 -0.008276464
## Parameter:
## [1] -0.7036797 0.5054799
## Function Value
## [1] 2.913164
## Gradient:
## [1] -0.5040386 2.0629563
##
## iteration = 97
## Step:
## [1] 0.006026301 -0.008389241
## Parameter:
## [1] -0.6976534 0.4970906
## Function Value
## [1] 2.892781
## Gradient:
## [1] -0.5013302 2.0740791
##
## iteration = 98
## Step:
## [1] 0.006166646 -0.008507489
## Parameter:
## [1] -0.6914867 0.4885831
## Function Value
## [1] 2.872004
## Gradient:
## [1] -0.4983006 2.0858483
##
## iteration = 99
## Step:
## [1] 0.006315353 -0.008631675
## Parameter:
## [1] -0.6851714 0.4799515
## Function Value
## [1] 2.85081
## Gradient:
## [1] -0.4949118 2.0983297

```

```

##
## iteration = 100
## Parameter:
## [1] -0.6786981  0.4711891
## Function Value
## [1] 2.829175
## Gradient:
## [1] -0.4911201  2.1115987
##
## Iteration limit exceeded.  Algorithm failed.
print(t1nlmo)

## $convergence
## [1] 1
##
## $value
## [1] 2.829175
##
## $par
## [1] -0.6786981  0.4711891
##
## $counts
## [1] NA 100
##
## $message
## [1] "Convergence indicator (code) = 4"
## FOLLOWING SHOWS UP ERRORS??
tst <- try(t1nlminbo <- optimr(x0, f, gr, hess=h, method="nllminb", control=list(trace=1)))

## Unit parameter scaling
## 0:      24.200000: -1.20000  1.00000
## 1:      11.298090: -0.916319  1.11579
## 2:      3.6832653: -0.878971  0.811671
## 3:      3.0264111: -0.738569  0.539330
## 4:      2.8738710: -0.669080  0.417996
## 5:      2.7399461: -0.596536  0.312149
## 6:      2.4721420: -0.512267  0.219383
## 7:      1.9715170: -0.394019  0.138449
## 8:      1.7942340: -0.285988  0.0443101
## 9:      1.4393081: -0.153222 -0.00959680
## 10:     1.1968650: -0.0785139  0.0245146
## 11:     1.0016961: -0.000696349 -0.00174001
## 12:     0.87823307: 0.0808212 -0.0117281
## 13:     0.80844393: 0.162768 -0.00629170
## 14:     0.68863014: 0.203045  0.0180988
## 15:     0.50226244: 0.313088  0.0805843
## 16:     0.37094000: 0.392027  0.157302
## 17:     0.30110961: 0.484979  0.216267
## 18:     0.24838461: 0.525122  0.260628
## 19:     0.15712889: 0.604556  0.362744
## 20:     0.14975086: 0.693216  0.456961
## 21:     0.097345475: 0.690984  0.473152
## 22:     0.074919430: 0.728091  0.526978

```

```
## 23: 0.050655822: 0.789126 0.614854
## 24: 0.033264540: 0.845784 0.705613
## 25: 0.013501068: 0.884534 0.781102
## 26: 0.0075988869: 0.927000 0.854564
## 27: 0.0029619374: 0.954465 0.908023
## 28: 0.00077922601: 0.972116 0.945142
## 29: 0.00019764489: 0.991268 0.981510
## 30: 2.4278303e-05: 0.996174 0.992053
## 31: 3.4539879e-07: 0.999626 0.999207
## 32: 3.3528188e-09: 0.999947 0.999895
## 33: 2.3909570e-10: 1.00001 1.00002
## 34: 2.0297517e-16: 1.00000 1.00000
## 35: 4.2918160e-22: 1.00000 1.00000

if (class(tst) == "try-error"){
  cat("try-error on attempt to run nlminb in optimr()\n")
} else { print(t1nlminb) }

## $par
## [1] 1 1
##
## $objective
## [1] 0
##
## $convergence
## [1] 0
##
## $iterations
## [1] 25
##
## $evaluations
## function gradient
##      33      26
##
## $message
## [1] "X-convergence (3)"

# sink()
```

## The Wood function

?? Note that we have NOT found the minimum for the Wood function.

```
#Example: Wood function
#
wood.f <- function(x){
  res <- 100*(x[1]^2-x[2])^2+(1-x[1])^2+90*(x[3]^2-x[4])^2+(1-x[3])^2+
    10.1*((1-x[2])^2+(1-x[4])^2)+19.8*(1-x[2])*(1-x[4])
  return(res)
}

#gradient:
wood.g <- function(x){
  g1 <- 400*x[1]^3-400*x[1]*x[2]+2*x[1]-2
  g2 <- -200*x[1]^2+220.2*x[2]+19.8*x[4]-40
  g3 <- 360*x[3]^3-360*x[3]*x[4]+2*x[3]-2
```

```

g4 <- -180*x[3]^2+200.2*x[4]+19.8*x[2]-40
return(c(g1,g2,g3,g4))
}
#hessian:
wood.h <- function(x){
  h11 <- 1200*x[1]^2-400*x[2]+2;    h12 <- -400*x[1]; h13 <- h14 <- 0
  h22 <- 220.2; h23 <- 0;    h24 <- 19.8
  h33 <- 1080*x[3]^2-360*x[4]+2;    h34 <- -360*x[3]
  h44 <- 200.2
  H <- matrix(c(h11,h12,h13,h14,h12,h22,h23,h24,
                h13,h23,h33,h34,h14,h24,h34,h44),ncol=4)
  return(H)
}

wood.fgh <- function(x){
  fval <- wood.f(x)
  gval <- wood.g(x)
  hval <- wood.h(x)
  attr(fval,"gradient") <- gval
  attr(fval,"hessian")<- hval
  fval
}

#####
x0 <- c(-3,-1,-3,-1) # Wood standard start

library(snewton)
cat("This FAILS to find minimum\n")

## This FAILS to find minimum
wd <- snewton(x0, fn=wood.f, gr=wood.g, hess=wood.h, control=list(trace=2))

## trace = 2
## Initial function value = 19192
## [1] -3 -1 -3 -1
## Termination test:nf= 1
## current gradient norm = 12008
## Iteration 1 :Search vector:[1] 0.3031804 7.1764779 0.3366407 6.8648041
## Gradient projection = -35111.91
## f(xnew)= 1291.439
## end major loop
## Termination test:nf= 2
## current gradient norm = 1190.066
## Iteration 2 :Search vector:[1] 0.8004660 -3.5411163 0.7835229 -3.3003803
## Gradient projection = -1748.969
## f(xnew)= 295.9513
## end major loop
## Termination test:nf= 3
## current gradient norm = 734.5958
## Iteration 3 :Search vector:[1] 0.3717359 -0.6307749 0.4048288 -0.7541032
## Gradient projection = -376.8133
## f(xnew)= 67.68559
## end major loop

```

```

## Termination test:nf= 4
## current gradient norm = 200.1226
## Iteration 4 :Search vector:[1] 0.3094376 -0.6813323 0.2912302 -0.5577128
## Gradient projection = -84.58005
## f(xnew)= 17.33661
## end major loop
## Termination test:nf= 5
## current gradient norm = 78.99793
## Iteration 5 :Search vector:[1] 0.1493372 -0.2198873 0.1651241 -0.2534905
## Gradient projection = -14.90304
## f(xnew)= 8.689077
## end major loop
## Termination test:nf= 6
## current gradient norm = 18.16952
## Iteration 6 :Search vector:[1] 0.06562338 -0.09986141 0.06090227 -0.07716611
## Gradient projection = -1.467156
## f(xnew)= 7.892798
## end major loop
## Termination test:nf= 7
## current gradient norm = 2.773634
## Iteration 7 :Search vector:[1] 0.0036594560 -0.0003950329 0.0170860102 -0.0260521194
## Gradient projection = -0.03184003
## f(xnew)= 7.876516
## end major loop
## Termination test:nf= 8
## current gradient norm = 0.1399245
## Iteration 8 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10

```

```

## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 27
## current gradient norm = 0.1399245
## Iteration 9 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04
## * f(xnew)= 7.876557
## StepSize now = 0.008
## * f(xnew)= 7.876524
## StepSize now = 0.0016
## * f(xnew)= 7.876518
## StepSize now = 0.00032
## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08
## * f(xnew)= 7.876516
## StepSize now = 4.096e-09
## * f(xnew)= 7.876516
## StepSize now = 8.192e-10
## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop

```



```

## Termination test:nf= 46
## current gradient norm = 0.1399245
## Iteration 10 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 65
## current gradient norm = 0.1399245
## Iteration 11 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016

```

```

## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 84
## current gradient norm = 0.1399245
## Iteration 12 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07

```

```

## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 103
## current gradient norm = 0.1399245
## Iteration 13 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12

```

```

## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 122
## current gradient norm = 0.1399245
## Iteration 14 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04
## * f(xnew)= 7.876557
## StepSize now = 0.008
## * f(xnew)= 7.876524
## StepSize now = 0.0016
## * f(xnew)= 7.876518
## StepSize now = 0.00032
## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08
## * f(xnew)= 7.876516
## StepSize now = 4.096e-09
## * f(xnew)= 7.876516
## StepSize now = 8.192e-10
## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## StepSize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 142
## current gradient norm = 0.1399245
## Iteration 15 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892

```

```

## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 161
## current gradient norm = 0.1399245
## Iteration 16 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05

```

```

## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 180
## current gradient norm = 0.1399245
## Iteration 17 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09

```

```

## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 199
## current gradient norm = 0.1399245
## Iteration 18 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13

```

```

## * f(xnew)= 7.876516
## StepSize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 219
## current gradient norm = 0.1399245
## Iteration 19 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04
## * f(xnew)= 7.876557
## StepSize now = 0.008
## * f(xnew)= 7.876524
## StepSize now = 0.0016
## * f(xnew)= 7.876518
## StepSize now = 0.00032
## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08
## * f(xnew)= 7.876516
## StepSize now = 4.096e-09
## * f(xnew)= 7.876516
## StepSize now = 8.192e-10
## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 238
## current gradient norm = 0.1399245
## Iteration 20 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04

```



```

## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 257
## current gradient norm = 0.1399245
## Iteration 21 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06

```

```

## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 277
## current gradient norm = 0.1399245
## Iteration 22 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10

```

```

## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 296
## current gradient norm = 0.1399245
## Iteration 23 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04
## * f(xnew)= 7.876557
## StepSize now = 0.008
## * f(xnew)= 7.876524
## StepSize now = 0.0016
## * f(xnew)= 7.876518
## StepSize now = 0.00032
## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08
## * f(xnew)= 7.876516
## StepSize now = 4.096e-09
## * f(xnew)= 7.876516
## StepSize now = 8.192e-10
## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## StepSize now = 5.24288e-14

```

```

## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 316
## current gradient norm = 0.1399245
## Iteration 24 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 335
## current gradient norm = 0.1399245
## Iteration 25 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008

```

```

## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 354
## current gradient norm = 0.1399245
## Iteration 26 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07

```

```

## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 373
## current gradient norm = 0.1399245
## Iteration 27 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11

```

```

## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 392
## current gradient norm = 0.1399245
## Iteration 28 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04
## * f(xnew)= 7.876557
## StepSize now = 0.008
## * f(xnew)= 7.876524
## StepSize now = 0.0016
## * f(xnew)= 7.876518
## StepSize now = 0.00032
## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08
## * f(xnew)= 7.876516
## StepSize now = 4.096e-09
## * f(xnew)= 7.876516
## StepSize now = 8.192e-10
## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 411
## current gradient norm = 0.1399245
## Iteration 29 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892

```

```

## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 431
## current gradient norm = 0.1399245
## Iteration 30 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032

```



```

## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08
## * f(xnew)= 7.876516
## StepSize now = 4.096e-09
## * f(xnew)= 7.876516
## StepSize now = 8.192e-10
## * f(xnew)= 7.876516
## StepSize now = 1.6384e-10
## * f(xnew)= 7.876516
## StepSize now = 3.2768e-11
## * f(xnew)= 7.876516
## StepSize now = 6.5536e-12
## * f(xnew)= 7.876516
## StepSize now = 1.31072e-12
## * f(xnew)= 7.876516
## StepSize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 450
## current gradient norm = 0.1399245
## Iteration 31 :Search vector:[1] 0.03262892 -0.06483154 -0.03403094 0.06423790
## Gradient projection = 0.001054892
## f(xnew)= 7.87719
## StepSize now = 0.2
## * f(xnew)= 7.876706
## StepSize now = 0.04
## * f(xnew)= 7.876557
## StepSize now = 0.008
## * f(xnew)= 7.876524
## StepSize now = 0.0016
## * f(xnew)= 7.876518
## StepSize now = 0.00032
## * f(xnew)= 7.876516
## StepSize now = 6.4e-05
## * f(xnew)= 7.876516
## StepSize now = 1.28e-05
## * f(xnew)= 7.876516
## StepSize now = 2.56e-06
## * f(xnew)= 7.876516
## StepSize now = 5.12e-07
## * f(xnew)= 7.876516
## StepSize now = 1.024e-07
## * f(xnew)= 7.876516
## StepSize now = 2.048e-08

```

```

## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## No progress in linesearch!
print(wd)

## $par
## [1] -0.9965602  1.0031106 -0.9406650  0.8958990
##
## $value
## [1] 7.876516
##
## $grad
## [1] -0.015475627 -0.002678324 -0.139924452 -0.052547714
##
## $Hess
##      [,1]      [,2]      [,3]      [,4]
## [1,] 792.5144 398.6241  0.0000  0.0000
## [2,] 398.6241 220.2000  0.0000 19.8000
## [3,]  0.0000  0.0000 635.1150 338.6394
## [4,]  0.0000 19.8000 338.6394 200.2000
##
## $counts
## $counts$niter
## [1] 31
##
## $counts$nf
## [1] 470
##
## $counts$ngr
## [1] 31
##
## $counts$nhess
## [1] 31
##
##
## $convcode
## [1] 93

```

```
wdm <- snewtonm(x0, fn=wood.f, gr=wood.g, hess=wood.h, control=list(trace=2))
```

```
## trace = 2
## Start snewtonm  f0= 19192  at  [1] -3 -1 -3 -1
## Iteration  1  fbest= 19192
## lambda = 0.0001220703  fval= 1291.437
## Iteration  2  fbest= 1291.437
## lambda = 1.831055e-05  fval= 295.9512
## Iteration  3  fbest= 295.9512
## lambda = 2.746582e-06  fval= 67.68561
## Iteration  4  fbest= 67.68561
## lambda = 4.119873e-07  fval= 17.33662
## Iteration  5  fbest= 17.33662
## lambda = 6.17981e-08  fval= 8.689077
## Iteration  6  fbest= 8.689077
## lambda = 9.269714e-09  fval= 7.892798
## Iteration  7  fbest= 7.892798
## lambda = 1.390457e-09  fval= 7.876516
## Iteration  8  fbest= 7.876516
## lambda = 2.085686e-10  fval= 7.87719
## lambda = 0.001220703  fval= 7.877199
## lambda = 0.01220703  fval= 7.877309
## lambda = 0.1220703  fval= 8.086469
## lambda = 1.220703  fval= 7.876389
## Iteration 13  fbest= 7.876389
## lambda = 0.1831055  fval= 7.874695
## Iteration 14  fbest= 7.874695
## lambda = 0.02746582  fval= 7.86493
## Iteration 15  fbest= 7.86493
## lambda = 0.004119873  fval= 7.960279
## lambda = 0.04119873  fval= 7.880975
## lambda = 0.4119873  fval= 7.852834
## Iteration 18  fbest= 7.852834
## lambda = 0.0617981  fval= 2687428
## lambda = 0.617981  fval= 7.83364
## Iteration 20  fbest= 7.83364
## lambda = 0.09269714  fval= 10.04037
## lambda = 0.9269714  fval= 7.80479
## Iteration 22  fbest= 7.80479
## lambda = 0.1390457  fval= 187.1183
## lambda = 1.390457  fval= 7.760677
## Iteration 24  fbest= 7.760677
## lambda = 0.2085686  fval= 39.79228
## lambda = 2.085686  fval= 7.694405
## Iteration 26  fbest= 7.694405
## lambda = 0.3128529  fval= 71.47948
## lambda = 3.128529  fval= 7.593398
## Iteration 28  fbest= 7.593398
## lambda = 0.4692793  fval= 29.76551
## lambda = 4.692793  fval= 7.440723
## Iteration 30  fbest= 7.440723
## lambda = 0.7039189  fval= 15.78816
## lambda = 7.039189  fval= 7.220427
## Iteration 32  fbest= 7.220427
```

```

## lambda = 1.055878 fval= 9.872978
## lambda = 10.55878 fval= 6.945088
## Iteration 34 fbest= 6.945088
## lambda = 1.583818 fval= 8.791386
## lambda = 15.83818 fval= 6.677653
## Iteration 36 fbest= 6.677653
## lambda = 2.375726 fval= 8.252204
## lambda = 23.75726 fval= 6.477078
## Iteration 38 fbest= 6.477078
## lambda = 3.56359 fval= 6.868378
## lambda = 35.6359 fval= 6.345119
## Iteration 40 fbest= 6.345119
## lambda = 5.345384 fval= 6.009281
## Iteration 41 fbest= 6.009281
## lambda = 0.8018077 fval= 5.126601
## Iteration 42 fbest= 5.126601
## lambda = 0.1202711 fval= 14.61183
## lambda = 1.202711 fval= 5.0219
## Iteration 44 fbest= 5.0219
## lambda = 0.1804067 fval= 3.691554
## Iteration 45 fbest= 3.691554
## lambda = 0.02706101 fval= 5.793891
## lambda = 0.2706101 fval= 4.183769
## lambda = 2.706101 fval= 3.31558
## Iteration 48 fbest= 3.31558
## lambda = 0.4059151 fval= 3.195488
## Iteration 49 fbest= 3.195488
## lambda = 0.06088727 fval= 2.456016
## Iteration 50 fbest= 2.456016
## lambda = 0.00913309 fval= 19.57353
## lambda = 0.0913309 fval= 15.43651
## lambda = 0.913309 fval= 3.729664
## lambda = 9.13309 fval= 2.268927
## Iteration 54 fbest= 2.268927
## lambda = 1.369964 fval= 5.355945
## lambda = 13.69964 fval= 2.115327
## Iteration 56 fbest= 2.115327
## lambda = 2.054945 fval= 10.1671
## lambda = 20.54945 fval= 1.988546
## Iteration 58 fbest= 1.988546
## lambda = 3.082418 fval= 10.44886
## lambda = 30.82418 fval= 1.889195
## Iteration 60 fbest= 1.889195
## lambda = 4.623627 fval= 5.077595
## lambda = 46.23627 fval= 1.815587
## Iteration 62 fbest= 1.815587
## lambda = 6.935441 fval= 2.080748
## lambda = 69.35441 fval= 1.763332
## Iteration 64 fbest= 1.763332
## lambda = 10.40316 fval= 1.492527
## Iteration 65 fbest= 1.492527
## lambda = 1.560474 fval= 1.124371
## Iteration 66 fbest= 1.124371
## lambda = 0.2340711 fval= 1.485749

```

```

## lambda = 2.340711 fval= 0.9137214
## Iteration 68 fbest= 0.9137214
## lambda = 0.3511067 fval= 0.5280731
## Iteration 69 fbest= 0.5280731
## lambda = 0.052666 fval= 1.525743
## lambda = 0.52666 fval= 0.6554124
## lambda = 5.2666 fval= 0.3958719
## Iteration 72 fbest= 0.3958719
## lambda = 0.78999 fval= 0.2303101
## Iteration 73 fbest= 0.2303101
## lambda = 0.1184985 fval= 0.08047485
## Iteration 74 fbest= 0.08047485
## lambda = 0.01777478 fval= 0.03236792
## Iteration 75 fbest= 0.03236792
## lambda = 0.002666216 fval= 0.001857496
## Iteration 76 fbest= 0.001857496
## lambda = 0.0003999324 fval= 4.536306e-05
## Iteration 77 fbest= 4.536306e-05
## lambda = 5.998987e-05 fval= 6.363907e-09
## Iteration 78 fbest= 6.363907e-09
## lambda = 8.99848e-06 fval= 6.401252e-16
## Iteration 79 fbest= 6.401252e-16
## lambda = 1.349772e-06 fval= 6.530659e-28
## Iteration 80 fbest= 6.530659e-28
## lambda = 2.024658e-07 fval= 1.139283e-27
## lambda = 0.001220703 fval= 1.139283e-27
## lambda = 0.01220703 fval= 1.115725e-27
## lambda = 0.1220703 fval= 1.056216e-27
## lambda = 1.220703 fval= 7.994871e-28
## lambda = 12.20703 fval= 6.583661e-28
## lambda = 122.0703 fval= 6.519368e-28
## Iteration 87 fbest= 6.519368e-28
## lambda = 18.31055 fval= 6.257627e-28
## Iteration 88 fbest= 6.257627e-28
## lambda = 2.746582 fval= 3.868623e-28
## Iteration 89 fbest= 3.868623e-28
## lambda = 0.4119873 fval= 1.082712e-29
## Iteration 90 fbest= 1.082712e-29
## lambda = 0.0617981 fval= 8.313115e-29
## lambda = 0.617981 fval= 8.072759e-29
## lambda = 6.17981 fval= 4.990655e-29
## lambda = 61.7981 fval= 3.035142e-29
## Null step
## Finished

print(wdm)

## $xs
## [1] 1 1 1 1
##
## $fv
## [1] 1.082712e-29
##
## $grd
## [1] -5.462297e-14 -7.105427e-15 -5.817569e-14 0.000000e+00

```

```

##
## $Hess
##      [,1] [,2] [,3] [,4]
## [1,] 802 -400.0  0   0.0
## [2,] -400 220.2  0  19.8
## [3,]  0   0.0 722 -360.0
## [4,]  0  19.8 -360 200.2
##
## $counts
## $counts$niter
## [1] 95
##
## $counts$nfn
## [1] 94
##
## $counts$ngr
## [1] 53
##
## $counts$nhess
## [1] 53
cat("\n\n nlm() gives similar results\n")

##
##
## nlm() gives similar results
t1nlm <- nlm(wood.fgh, x0, print.level=2)

## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] -3 -1 -3 -1
## Function Value
## [1] 19192
## Gradient:
## [1] -12008 -2080 -10808 -1880
##
## iteration = 1
## Step:
## [1] 0.9116709 0.7367972 0.9020511 0.7516950
## Parameter:
## [1] -2.0883291 -0.2632028 -2.0979489 -0.2483050
## Function Value
## [1] 4166.416
## Gradient:
## [1] -3869.0187 -975.0974 -3517.9318 -887.1722
##
## iteration = 2
## Step:
## [1] 0.5765246 0.5075403 0.5704409 0.5190481
## Parameter:
## [1] -1.5118046 0.2443374 -1.5275080 0.2707431
## Function Value

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## [1] 834.2691
## Gradient:
## [1] -1239.3913 -437.9468 -1139.2505 -400.9499
##
## iteration = 3
## Step:
## [1] 0.3452070 0.3077087 0.3415096 0.3149447
## Parameter:
## [1] -1.1665976 0.5520461 -1.1859985 0.5856878
## Function Value
## [1] 142.99
## Gradient:
## [1] -381.7993 -179.0328 -354.8650 -165.0014
##
## iteration = 4
## Step:
## [1] 0.1855520 0.1499399 0.1833595 0.1541052
## Parameter:
## [1] -0.9810455 0.7019861 -1.0026390 0.7397929
## Function Value
## [1] 24.17926
## Gradient:
## [1] -106.17301 -63.26483 -99.83461 -58.94541
##
## iteration = 5
## Step:
## [1] 0.07782451 0.05872218 0.07664496 0.06140066
## Parameter:
## [1] -0.9032210 0.7607082 -0.9259940 0.8011936
## Function Value
## [1] 9.839749
## Gradient:
## [1] -23.71342 -19.79005 -22.61047 -18.88270
##
## iteration = 6
## Step:
## [1] 0.01753078 0.02499409 0.01707308 0.02682737
## Parameter:
## [1] -0.8856902 0.7857023 -0.9089209 0.8280210
## Function Value
## [1] 8.69256
## Gradient:
## [1] -3.326713 -7.482969 -3.201469 -7.378002
##
## iteration = 7
## Step:
## [1] -0.002949427 0.015908253 -0.003260912 0.017258585
## Parameter:
## [1] -0.8886397 0.8016106 -0.9121818 0.8452796
## Function Value
## [1] 8.500379
## Gradient:
## [1] 0.4633586 -4.6849032 0.5115861 -4.6767719
##

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## iteration = 8
## Step:
## [1] -0.006478477  0.012986001 -0.006839623  0.014092239
## Parameter:
## [1] -0.8951181  0.8145966 -0.9190215  0.8593718
## Function Value
## [1] 8.374769
## Gradient:
## [1] 0.9933129 -3.8575663  1.0490214 -3.8528361
##
## iteration = 9
## Step:
## [1] -0.006355528  0.011342611 -0.006713964  0.012301691
## Parameter:
## [1] -0.9014737  0.8259392 -0.9257354  0.8716735
## Function Value
## [1] 8.275711
## Gradient:
## [1] 0.9872768 -3.4000080  1.0433216 -3.3948677
##
## iteration = 10
## Step:
## [1] -0.005715703  0.010053796 -0.006042570  0.010901894
## Parameter:
## [1] -0.9071894  0.8359930 -0.9317780  0.8825754
## Function Value
## [1] 8.196865
## Gradient:
## [1] 0.9031660 -3.0378609  0.9550868 -3.0335914
##
## iteration = 11
## Step:
## [1] -0.005075092  0.008946817 -0.005366382  0.009701843
## Parameter:
## [1] -0.9122645  0.8449398 -0.9371444  0.8922772
## Function Value
## [1] 8.133955
## Gradient:
## [1] 0.8146503 -2.7224545  0.8616142 -2.7194185
##
## iteration = 12
## Step:
## [1] -0.004503371  0.007976157 -0.004762011  0.008650385
## Parameter:
## [1] -0.9167678  0.8529160 -0.9419064  0.9009276
## Function Value
## [1] 8.083658
## Gradient:
## [1] 0.7329608 -2.4421895  0.7752177 -2.4403344
##
## iteration = 13
## Step:
## [1] -0.004001282  0.007119844 -0.004231240  0.007723143
## Parameter:

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## [1] -0.9207691  0.8600358 -0.9461376  0.9086507
## Function Value
## [1] 8.043374
## Gradient:
## [1]  0.6591984 -2.1919825  0.6972101 -2.1911665
##
## iteration = 14
## Step:
## [1] -0.003560435  0.006362120 -0.003765379  0.006902861
## Parameter:
## [1] -0.9243296  0.8663979 -0.9499030  0.9155536
## Function Value
## [1] 8.011061
## Gradient:
## [1]  0.5928602 -1.9682377  0.6270863 -1.9683199
##
## iteration = 15
## Step:
## [1] -0.003172450  0.005690158 -0.003355552  0.006175565
## Parameter:
## [1] -0.9275020  0.8720881 -0.9532586  0.9217292
## Function Value
## [1] 7.985107
## Gradient:
## [1]  0.5332412 -1.7679572  0.5640969 -1.7688150
##
## iteration = 16
## Step:
## [1] -0.002830125  0.005093157 -0.002994097  0.005529510
## Parameter:
## [1] -0.9303321  0.8771812 -0.9562527  0.9272587
## Function Value
## [1] 7.964235
## Gradient:
## [1]  0.4796636 -1.5885404  0.5075167 -1.5900697
##
## iteration = 17
## Step:
## [1] -0.002527390  0.004561911 -0.002674558  0.004954699
## Parameter:
## [1] -0.9328595  0.8817432 -0.9589272  0.9322134
## Function Value
## [1] 7.947433
## Gradient:
## [1]  0.4315101 -1.4277069  0.4566857 -1.4298200
##
## iteration = 18
## Step:
## [1] -0.002259118  0.004088520 -0.002391484  0.004442553
## Parameter:
## [1] -0.9351186  0.8858317 -0.9613187  0.9366559
## Function Value
## [1] 7.933894
## Gradient:

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## [1] 0.3882254 -1.2834487 0.4110108 -1.2860707
##
## iteration = 19
## Step:
## [1] -0.002020955 0.003666169 -0.002140251 0.003985678
## Parameter:
## [1] -0.9371396 0.8894978 -0.9634590 0.9406416
## Function Value
## [1] 7.922976
## Gradient:
## [1] 0.3493109 -1.1539919 0.3699607 -1.1570590
##
## iteration = 20
## Step:
## [1] -0.001809184 0.003288947 -0.001916914 0.003577664
## Parameter:
## [1] -0.9389488 0.8927868 -0.9653759 0.9442193
## Function Value
## [1] 7.914165
## Gradient:
## [1] 0.3143196 -1.0377658 0.3330597 -1.0412235
##
## iteration = 21
## Step:
## [1] -0.001620614 0.002951713 -0.001718090 0.003212935
## Parameter:
## [1] -0.9405694 0.8957385 -0.9670940 0.9474322
## Function Value
## [1] 7.90705
## Gradient:
## [1] 0.2828509 -0.9333771 0.2998820 -0.9371784
##
## iteration = 22
## Step:
## [1] -0.001452492 0.002649974 -0.001540861 0.002886621
## Parameter:
## [1] -0.9420219 0.8983885 -0.9686348 0.9503188
## Function Value
## [1] 7.901301
## Gradient:
## [1] 0.2545455 -0.8395874 0.2700463 -0.8436915
##
## iteration = 23
## Step:
## [1] -0.001302432 0.002379790 -0.001382703 0.002594456
## Parameter:
## [1] -0.9433243 0.9007683 -0.9700175 0.9529133
## Function Value
## [1] 7.896654
## Gradient:
## [1] 0.2290818 -0.7552944 0.2432114 -0.7596662
##
## iteration = 24
## Step:

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## [1] -0.001168361  0.002137702 -0.001241420  0.002332689
## Parameter:
## [1] -0.9444927  0.9029060 -0.9712589  0.9552460
## Function Value
## [1] 7.892895
## Gradient:
## [1] 0.2061710 -0.6795158  0.2190714 -0.6841244
##
## iteration = 25
## Step:
## [1] -0.001048470  0.001920659 -0.001115098  0.002098019
## Parameter:
## [1] -0.9455411  0.9048266 -0.9723740  0.9573440
## Function Value
## [1] 7.889854
## Gradient:
## [1] 0.1855546 -0.6113746  0.1973525 -0.6161934
##
## iteration = 26
## Step:
## [1] -0.0009411735  0.0017259680 -0.0010020619  0.0018875279
## Parameter:
## [1] -0.9464823  0.9065526 -0.9733761  0.9592315
## Function Value
## [1] 7.887392
## Gradient:
## [1] 0.1670005 -0.5500879  0.1778092 -0.5550933
##
## iteration = 27
## Step:
## [1] -0.0008450808  0.0015512445 -0.0009008417  0.0016986351
## Parameter:
## [1] -0.9473274  0.9081038 -0.9742769  0.9609302
## Function Value
## [1] 7.885399
## Gradient:
## [1] 0.1503004 -0.4949553  0.1602211 -0.5001268
##
## iteration = 28
## Step:
## [1] -0.0007589677  0.0013943750 -0.0008101442  0.0015290525
## Parameter:
## [1] -0.9480864  0.9094982 -0.9750871  0.9624592
## Function Value
## [1] 7.883785
## Gradient:
## [1] 0.1352675 -0.4453502  0.1443910 -0.4506697
##
## iteration = 29
## Step:
## [1] -0.0006817541  0.0012534820 -0.0007288287  0.0013767484
## Parameter:
## [1] -0.9487681  0.9107517 -0.9758159  0.9638360
## Function Value

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## [1] 7.882477
## Gradient:
## [1] 0.1217341 -0.4007116 0.1301415 -0.4061631
##
## iteration = 30
## Step:
## [1] -0.0006124854 0.0011268958 -0.0006558873 0.0012399159
## Parameter:
## [1] -0.9493806 0.9118786 -0.9764718 0.9650759
## Function Value
## [1] 7.881417
## Gradient:
## [1] 0.1095495 -0.3605364 0.1173135 -0.3661059
##
## iteration = 31
## Step:
## [1] -0.0005503158 0.0010131294 -0.0005904275 0.0011169461
## Parameter:
## [1] -0.9499309 0.9128917 -0.9770622 0.9661928
## Function Value
## [1] 7.880559
## Gradient:
## [1] 0.09857832 -0.32437405 0.10576421 -0.33004899
##
## iteration = 32
## Step:
## [1] -0.0004944952 0.0009108570 -0.0005316577 0.0010064045
## Parameter:
## [1] -0.9504254 0.9138026 -0.9775939 0.9671992
## Function Value
## [1] 7.879862
## Gradient:
## [1] 0.08869905 -0.29181992 0.09536535 -0.29758930
##
## iteration = 33
## Step:
## [1] -0.0004443567 0.0008188950 -0.0004788748 0.0009070108
## Parameter:
## [1] -0.9508698 0.9146215 -0.9780728 0.9681062
## Function Value
## [1] 7.879298
## Gradient:
## [1] 0.07980241 -0.26251108 0.08600161 -0.26836511
##
## iteration = 34
## Step:
## [1] -0.0003993074 0.0007361861 -0.0004314530 0.0008176213
## Parameter:
## [1] -0.9512691 0.9153577 -0.9785042 0.9689239
## Function Value
## [1] 7.87884
## Gradient:
## [1] 0.07179017 -0.23612164 0.07756940 -0.24205160
##

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## iteration = 35
## Step:
## [1] -0.0003588187  0.0006617846 -0.0003888351  0.0007372134
## Parameter:
## [1] -0.9516279  0.9160194 -0.9788931  0.9696611
## Function Value
## [1] 7.878469
## Gradient:
## [1] 0.06457399 -0.21235885  0.06997558 -0.21835702
##
## iteration = 36
## Step:
## [1] -0.0003224194  0.0005948446 -0.0003505244  0.0006648718
## Parameter:
## [1] -0.9519503  0.9166143 -0.9792436  0.9703259
## Function Value
## [1] 7.878168
## Gradient:
## [1] 0.05807447 -0.19095972  0.06313643 -0.19701921
##
## iteration = 37
## Step:
## [1] -0.0002896886  0.0005346081 -0.0003160773  0.0005997772
## Parameter:
## [1] -0.9522400  0.9171489 -0.9795597  0.9709257
## Function Value
## [1] 7.877923
## Gradient:
## [1] 0.05222015 -0.17168788  0.05697664 -0.17780255
##
## iteration = 38
## Step:
## [1] -0.0002602503  0.0004803963 -0.0002850973  0.0005411952
## Parameter:
## [1] -0.9525003  0.9176293 -0.9798447  0.9714669
## Function Value
## [1] 7.877725
## Gradient:
## [1] 0.04694676 -0.15433082  0.05142848 -0.16049519
##
## iteration = 39
## Step:
## [1] -0.0002337683  0.0004316002 -0.0002572303  0.0004884677
## Parameter:
## [1] -0.9527340  0.9180609 -0.9801020  0.9719554
## Function Value
## [1] 7.877564
## Gradient:
## [1] 0.04219647 -0.13869745  0.04643103 -0.14490666
##
## iteration = 40
## Step:
## [1] -0.0002099415  0.0003876740 -0.0002321590  0.0004410041
## Parameter:

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## [1] -0.9529440  0.9184486 -0.9803341  0.9723964
## Function Value
## [1] 7.877434
## Gradient:
## [1]  0.03791725 -0.12461594  0.04192945 -0.13086560
##
## iteration = 41
## Step:
## [1] -0.0001885004  0.0003481274 -0.0002095994  0.0003982746
## Parameter:
## [1] -0.9531325  0.9187967 -0.9805437  0.9727947
## Function Value
## [1] 7.877328
## Gradient:
## [1]  0.03406226 -0.11193167  0.03787442 -0.11821790
##
## iteration = 42
## Step:
## [1] -0.0001692034  0.0003125205 -0.0001892971  0.0003598034
## Parameter:
## [1] -0.9533017  0.9191092 -0.9807330  0.9731545
## Function Value
## [1] 7.877242
## Gradient:
## [1]  0.03058934 -0.10050556  0.03422152 -0.10682487
##
## iteration = 43
## Step:
## [1] -0.0001518338  0.0002804581 -0.0001710239  0.0003251634
## Parameter:
## [1] -0.9534535  0.9193897 -0.9809041  0.9734796
## Function Value
## [1] 7.877172
## Gradient:
## [1]  0.02746054 -0.09021245  0.03093080 -0.09656171
##
## iteration = 44
## Step:
## [1] -0.0001361975  0.0002515852 -0.0001545751  0.0002939707
## Parameter:
## [1] -0.9535897  0.9196413 -0.9810586  0.9737736
## Function Value
## [1] 7.877115
## Gradient:
## [1]  0.02464170 -0.08093968  0.02796628 -0.08731610
##
## iteration = 45
## Step:
## [1] -0.0001221200  0.0002255827 -0.0001397672  0.0002658804
## Parameter:
## [1] -0.9537118  0.9198668 -0.9811984  0.9740395
## Function Value
## [1] 7.877069
## Gradient:

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## [1] 0.02210205 -0.07258586 0.02529556 -0.07898695
##
## iteration = 46
## Step:
## [1] -0.0001094447 0.0002021638 -0.0001264351 0.0002405825
## Parameter:
## [1] -0.9538213 0.9200690 -0.9813248 0.9742801
## Function Value
## [1] 7.877032
## Gradient:
## [1] 0.01981390 -0.06505973 0.02288949 -0.07148325
##
## iteration = 47
## Step:
## [1] -9.803093e-05 1.810706e-04 -1.144309e-04 2.177981e-04
## Parameter:
## [1] -0.9539193 0.9202501 -0.9814393 0.9744979
## Function Value
## [1] 7.877002
## Gradient:
## [1] 0.01775230 -0.05827911 0.02072180 -0.06472304
##
## iteration = 48
## Step:
## [1] -8.775243e-05 1.620711e-04 -1.036215e-04 1.972765e-04
## Parameter:
## [1] -0.9540070 0.9204122 -0.9815429 0.9746951
## Function Value
## [1] 7.876977
## Gradient:
## [1] 0.01589478 -0.05217001 0.01876885 -0.05863257
##
## iteration = 49
## Step:
## [1] -7.849563e-05 1.449568e-04 -9.388733e-05 1.787923e-04
## Parameter:
## [1] -0.9540855 0.9205571 -0.9816368 0.9748739
## Function Value
## [1] 7.876957
## Gradient:
## [1] 0.01422113 -0.04666582 0.01700935 -0.05314538
##
## iteration = 50
## Step:
## [1] -7.015842e-05 1.295399e-04 -8.512093e-05 1.621426e-04
## Parameter:
## [1] -0.9541557 0.9206866 -0.9817219 0.9750361
## Function Value
## [1] 7.876941
## Gradient:
## [1] 0.01271311 -0.04170654 0.01542411 -0.04820166
##
## iteration = 51
## Step:

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## [1] -6.264904e-05  1.156516e-04 -7.722567e-05  1.471448e-04
## Parameter:
## [1] -0.9542184  0.9208023 -0.9817991  0.9751832
## Function Value
## [1] 7.876928
## Gradient:
## [1] 0.01135431 -0.03723815  0.01399587 -0.04374754
##
## iteration = 52
## Step:
## [1] -5.588494e-05  1.031398e-04 -7.011467e-05  1.336346e-04
## Parameter:
## [1] -0.9542742  0.9209054 -0.9818692  0.9753169
## Function Value
## [1] 7.876917
## Gradient:
## [1] 0.01012996 -0.03321200  0.01270906 -0.03973448
##
## iteration = 53
## Step:
## [1] -4.979185e-05  9.186780e-05 -6.370975e-05  1.214641e-04
## Parameter:
## [1] -0.9543240  0.9209973 -0.9819329  0.9754383
## Function Value
## [1] 7.876908
## Gradient:
## [1] 0.009026736 -0.029584242  0.011549679 -0.036118777
##
## iteration = 54
## Step:
## [1] -4.430297e-05  8.171235e-05 -5.794061e-05  1.105004e-04
## Parameter:
## [1] -0.9543683  0.9210790 -0.9819909  0.9755488
## Function Value
## [1] 7.876901
## Gradient:
## [1] 0.008032641 -0.026315422  0.010505094 -0.032861072
##
## iteration = 55
## Step:
## [1] -3.935818e-05  7.256263e-05 -5.274397e-05  1.006234e-04
## Parameter:
## [1] -0.9544077  0.9211516 -0.9820436  0.9756494
## Function Value
## [1] 7.876895
## Gradient:
## [1] 0.007136870 -0.023369977  0.009563939 -0.029925902
##
## iteration = 56
## Step:
## [1] -3.490339e-05  6.431882e-05 -4.806288e-05  9.172539e-05
## Parameter:
## [1] -0.9544426  0.9212159 -0.9820917  0.9757412
## Function Value

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## [1] 7.876891
## Gradient:
## [1] 0.006329690 -0.020715882 0.008715969 -0.027281325
##
## iteration = 57
## Step:
## [1] -3.088991e-05 5.689103e-05 -4.384610e-05 8.370920e-05
## Parameter:
## [1] -0.9544735 0.9212728 -0.9821355 0.9758249
## Function Value
## [1] 7.876887
## Gradient:
## [1] 0.005602331 -0.018324283 0.007951959 -0.024898566
##
## iteration = 58
## Step:
## [1] -2.727389e-05 5.019833e-05 -4.004750e-05 7.648736e-05
## Parameter:
## [1] -0.9545008 0.9213230 -0.9821756 0.9759014
## Function Value
## [1] 7.876884
## Gradient:
## [1] 0.004946890 -0.016169189 0.007263594 -0.022751704
##
## iteration = 59
## Step:
## [1] -2.401587e-05 4.416784e-05 -3.662555e-05 6.998109e-05
## Parameter:
## [1] -0.9545248 0.9213672 -0.9822122 0.9759713
## Function Value
## [1] 7.876881
## Gradient:
## [1] 0.004356251 -0.014227187 0.006643386 -0.020817386
##
## iteration = 60
## Step:
## [1] -2.108033e-05 3.873392e-05 -3.354285e-05 6.411946e-05
## Parameter:
## [1] -0.9545459 0.9214059 -0.9822458 0.9760355
## Function Value
## [1] 7.876879
## Gradient:
## [1] 0.003823999 -0.012477182 0.006084587 -0.019074574
##
## iteration = 61
## Step:
## [1] -1.843528e-05 3.383744e-05 -3.076575e-05 5.883857e-05
## Parameter:
## [1] -0.9545643 0.9214397 -0.9822765 0.9760943
## Function Value
## [1] 7.876877
## Gradient:
## [1] 0.003344358 -0.010900169 0.005581120 -0.017504313
##

```

```

## iteration = 62
## Step:
## [1] -1.605189e-05  2.942515e-05 -2.826394e-05  5.408088e-05
## Parameter:
## [1] -0.9545803  0.9214692 -0.9823048  0.9761484
## Function Value
## [1] 7.876876
## Gradient:
## [1] 0.002912120 -0.009479027  0.005127508 -0.016089527
##
## iteration = 63
## Step:
## [1] -1.390424e-05  2.544908e-05 -2.601010e-05  4.979455e-05
## Parameter:
## [1] -0.9545942  0.9214946 -0.9823308  0.9761982
## Function Value
## [1] 7.876875
## Gradient:
## [1] 0.002522596 -0.008198330  0.004718816 -0.014814831
##
## iteration = 64
## Step:
## [1] -1.196895e-05  2.186604e-05 -2.397965e-05  4.593290e-05
## Parameter:
## [1] -0.9546062  0.9215165 -0.9823548  0.9762441
## Function Value
## [1] 7.876874
## Gradient:
## [1] 0.002171559 -0.007044181  0.004350599 -0.013666362
##
## iteration = 65
## Step:
## [1] -1.022498e-05  1.863710e-05 -2.215046e-05  4.245386e-05
## Parameter:
## [1] -0.9546164  0.9215351 -0.9823769  0.9762866
## Function Value
## [1] 7.876873
## Gradient:
## [1] 0.001855202 -0.006004056  0.004018852 -0.012631632
##
## iteration = 66
## Step:
## [1] -8.653371e-06  1.572720e-05 -2.050257e-05  3.931955e-05
## Parameter:
## [1] -0.9546251  0.9215508 -0.9823974  0.9763259
## Function Value
## [1] 7.876872
## Gradient:
## [1] 0.001570093 -0.005066674  0.003719966 -0.011699386
##
## iteration = 67
## Step:
## [1] -7.237055e-06  1.310476e-05 -1.901803e-05  3.649584e-05
## Parameter:

```

```

## [1] -0.9546323  0.9215639 -0.9824164  0.9763624
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001313142 -0.004221869  0.003450689 -0.010859486
##
## iteration = 68
## Step:
## [1] -5.960649e-06  1.074131e-05 -1.768066e-05  3.395197e-05
## Parameter:
## [1] -0.9546383  0.9215747 -0.9824341  0.9763963
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.001081562 -0.003460482  0.003208092 -0.010102797
##
## iteration = 69
## Step:
## [1] -4.810300e-06  8.611225e-06 -1.647587e-05  3.166025e-05
## Parameter:
## [1] -0.9546431  0.9215833 -0.9824506  0.9764280
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.0008728447 -0.0027742610  0.0029895339 -0.0094210856
##
## iteration = 70
## Step:
## [1] -3.773526e-06  6.691408e-06 -1.539054e-05  2.959571e-05
## Parameter:
## [1] -0.9546469  0.9215900 -0.9824660  0.9764576
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0006847275 -0.0021557690  0.0027926375 -0.0088069365
##
## iteration = 71
## Step:
## [1] -2.839083e-06  4.961048e-06 -1.441284e-05  2.773588e-05
## Parameter:
## [1] -0.9546497  0.9215949 -0.9824804  0.9764853
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.000515173 -0.001598306  0.002615259 -0.008253666
##
## iteration = 72
## Step:
## [1] -1.996843e-06  3.401398e-06 -1.353211e-05  2.606049e-05
## Parameter:
## [1] -0.9546517  0.9215983 -0.9824939  0.9765114
## Function Value
## [1] 7.87687
## Gradient:

```

```

## [1] 0.0003623451 -0.0010958356 0.0024554681 -0.0077552527
##
## iteration = 73
## Step:
## [1] -1.237679e-06 1.995567e-06 -1.273876e-05 2.455130e-05
## Parameter:
## [1] -0.9546529 0.9216003 -0.9825067 0.9765359
## Function Value
## [1] 7.87687
## Gradient:
## [1] 0.0002245894 -0.0006429174 0.0023115243 -0.0073062704
##
## iteration = 74
## Step:
## [1] -5.533700e-07 7.283349e-07 -1.202413e-05 2.319185e-05
## Parameter:
## [1] -0.9546535 0.9216011 -0.9825187 0.9765591
## Function Value
## [1] 7.87687
## Gradient:
## [1] 0.0001004150 -0.0002346499 0.0021818599 -0.0069018300
##
## iteration = 75
## Step:
## [1] 6.349210e-08 -4.140111e-07 -1.138043e-05 2.196734e-05
## Parameter:
## [1] -0.9546534 0.9216007 -0.9825301 0.9765811
## Function Value
## [1] 7.876869
## Gradient:
## [1] -1.152136e-05 1.333833e-04 2.065062e-03 -6.537526e-03
##
## iteration = 76
## Step:
## [1] 6.195811e-07 -1.443826e-06 -1.080065e-05 2.086440e-05
## Parameter:
## [1] -0.9546528 0.9215992 -0.9825409 0.9766020
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0001124301 0.0004651620 0.0019598589 -0.0062093889
##
## iteration = 77
## Step:
## [1] 1.120910e-06 -2.372243e-06 -1.027846e-05 1.987102e-05
## Parameter:
## [1] -0.9546517 0.9215968 -0.9825512 0.9766218
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002034020 0.0007642722 0.0018651024 -0.0059138415
##
## iteration = 78
## Step:

```

```

## [1] 1.572897e-06 -3.209292e-06 -9.808155e-06 1.897636e-05
## Parameter:
## [1] -0.9546501 0.9215936 -0.9825610 0.9766408
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002854198 0.0010339451 0.0017797595 -0.0056476608
##
## iteration = 79
## Step:
## [1] 1.980424e-06 -3.964012e-06 -9.384605e-06 1.817064e-05
## Parameter:
## [1] -0.9546481 0.9215897 -0.9825704 0.9766590
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0003593692 0.0012770923 0.0017028990 -0.0054079425
##
## iteration = 80
## Step:
## [1] 2.347888e-06 -4.644546e-06 -9.003184e-06 1.744507e-05
## Parameter:
## [1] -0.9546458 0.9215850 -0.9825794 0.9766764
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.000426048 0.001496337 0.001633682 -0.005192070
##
## iteration = 81
## Step:
## [1] 2.679254e-06 -5.258234e-06 -8.659724e-06 1.679173e-05
## Parameter:
## [1] -0.9546431 0.9215798 -0.9825880 0.9766932
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0004861755 0.0016940444 0.0015713518 -0.0049976836
##
## iteration = 82
## Step:
## [1] 2.978091e-06 -5.811686e-06 -8.350468e-06 1.620346e-05
## Parameter:
## [1] -0.9546401 0.9215740 -0.9825964 0.9767094
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0005403996 0.0018723437 0.0015152278 -0.0048226599
##
## iteration = 83
## Step:
## [1] 3.247619e-06 -6.310861e-06 -8.072033e-06 1.567383e-05
## Parameter:
## [1] -0.9546369 0.9215676 -0.9826044 0.9767251
## Function Value

```

```

## [1] 7.876868
## Gradient:
## [1] -0.000589304  0.002033155  0.001464696 -0.004665084
##
## iteration = 84
## Step:
## [1] 3.490735e-06 -6.761128e-06 -7.821371e-06 1.519705e-05
## Parameter:
## [1] -0.9546334  0.9215609 -0.9826123  0.9767403
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000633415  0.002178207  0.001419203 -0.004523229
##
## iteration = 85
## Step:
## [1] 3.710052e-06 -7.167323e-06 -7.595732e-06 1.476787e-05
## Parameter:
## [1] -0.9546297  0.9215537 -0.9826199  0.9767551
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0006732066  0.0023090593  0.0013782505 -0.0043955420
##
## iteration = 86
## Step:
## [1] 3.907924e-06 -7.533807e-06 -7.392643e-06 1.438160e-05
## Parameter:
## [1] -0.9546258  0.9215462 -0.9826272  0.9767694
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000709106  0.002427116  0.001341389 -0.004280621
##
## iteration = 87
## Step:
## [1] 4.086473e-06 -7.864506e-06 -7.209873e-06 1.403400e-05
## Parameter:
## [1] -0.9546217  0.9215383 -0.9826345  0.9767835
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0007414982  0.0025336428  0.0013082150 -0.0041772039
##
## iteration = 88
## Step:
## [1] 4.247610e-06 -8.162960e-06 -7.045411e-06 1.372122e-05
## Parameter:
## [1] -0.9546174  0.9215301 -0.9826415  0.9767972
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000770730  0.002629780  0.001278363 -0.004084153
##

```

```

## iteration = 89
## Step:
## [1] 4.393056e-06 -8.432358e-06 -6.897446e-06 1.343985e-05
## Parameter:
## [1] -0.9546131 0.9215217 -0.9826484 0.9768106
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000797114 0.002716555 0.001251504 -0.004000443
##
## iteration = 90
## Step:
## [1] 4.524363e-06 -8.675573e-06 -6.764348e-06 1.318675e-05
## Parameter:
## [1] -0.9546085 0.9215130 -0.9826552 0.9768238
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008209319 0.0027948936 0.0012273423 -0.0039251507
##
## iteration = 91
## Step:
## [1] 4.642931e-06 -8.895197e-06 -6.644645e-06 1.295915e-05
## Parameter:
## [1] -0.9546039 0.9215041 -0.9826618 0.9768368
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008424374 0.0028656307 0.0012056113 -0.0038574434
##
## iteration = 92
## Step:
## [1] 4.750018e-06 -9.093561e-06 -6.537012e-06 1.275452e-05
## Parameter:
## [1] -0.9545991 0.9214951 -0.9826683 0.9768495
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008618593 0.0029295178 0.0011860705 -0.0037965711
##
## iteration = 93
## Step:
## [1] 4.846761e-06 -9.272768e-06 -6.440257e-06 1.257058e-05
## Parameter:
## [1] -0.9545943 0.9214858 -0.9826748 0.9768621
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0008794036 0.0029872324 0.0011685032 -0.0037418580
##
## iteration = 94
## Step:
## [1] 4.934182e-06 -9.434714e-06 -6.353302e-06 1.240530e-05
## Parameter:

```

```

## [1] -0.9545894  0.9214763 -0.9826811  0.9768745
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000895256  0.003039385  0.001152714 -0.003692695
##
## iteration = 95
## Step:
## [1] 5.013203e-06 -9.581105e-06 -6.275179e-06  1.225682e-05
## Parameter:
## [1] -0.9545843  0.9214668 -0.9826874  0.9768868
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000909584  0.003086526  0.001138528 -0.003648534
##
## iteration = 96
## Step:
## [1] 5.084656e-06 -9.713480e-06 -6.205014e-06  1.212348e-05
## Parameter:
## [1] -0.9545793  0.9214571 -0.9826936  0.9768989
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009225381  0.0031291509  0.0011257852 -0.0036088788
##
## iteration = 97
## Step:
## [1] 5.149289e-06 -9.833224e-06 -6.142021e-06  1.200379e-05
## Parameter:
## [1] -0.9545741  0.9214472 -0.9826998  0.9769109
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009342544  0.0031677063  0.0011143439 -0.0035732853
##
## iteration = 98
## Step:
## [1] 5.207775e-06 -9.941589e-06 -6.085489e-06  1.189640e-05
## Parameter:
## [1] -0.9545689  0.9214373 -0.9827058  0.9769228
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009448553  0.0032025948  0.0011040749 -0.0035413514
##
## iteration = 99
## Step:
## [1] 5.260724e-06 -1.003970e-05 -6.034780e-06  1.180009e-05
## Parameter:
## [1] -0.9545636  0.9214272 -0.9827119  0.9769346
## Function Value
## [1] 7.876867
## Gradient:

```



```
## [1] -0.0009544511  0.0032341791  0.0010948625 -0.0035127154
##
## iteration = 100
## Parameter:
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009631411  0.0032627860  0.0010866021 -0.0034870511
##
## Iteration limit exceeded.  Algorithm failed.
```

```
print(t1nlm)
```

```
## $minimum
## [1] 7.876867
##
## $estimate
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
##
## $gradient
## [1] -0.0009631411  0.0032627860  0.0010866021 -0.0034870511
##
## $code
## [1] 4
##
## $iterations
## [1] 100
```

```
## BUT ... it looks like nlminb is NOT using a true Newton-type method
t1nlminb <- nlminb(x0, wood.f, gradient=wood.g, hess=wood.h, control=list(trace=1))
```

```
## 0:      19192.000: -3.00000 -1.00000 -3.00000 -1.00000
## 1:      7844.8110: -2.34188 -0.814182 -2.36728 -0.827234
## 2:      1216.1133: -1.68443 0.393908 -1.69404 0.313883
## 3:      63.588133: -1.45306 1.71905 -1.44613 1.65143
## 4:      16.494685: -1.19893 1.31181 -1.19548 1.29854
## 5:      8.5754053: -1.04247 1.05440 -1.02968 1.02406
## 6:      7.8893787: -0.984105 0.972760 -0.971950 0.949965
## 7:      7.8769236: -0.977607 0.965750 -0.960225 0.933163
## 8:      7.8762403: -1.00695 1.02307 -0.929896 0.875292
## 9:      7.8736524: -1.06370 1.13800 -0.867196 0.759942
## 10:     7.8639782: -1.11594 1.25222 -0.798496 0.645011
## 11:     7.8548631: -1.13591 1.29937 -0.765236 0.596982
## 12:     7.8409865: -1.21945 1.48926 -0.637546 0.403740
## 13:     7.7765044: -1.25895 1.59258 -0.545841 0.302611
## 14:     7.7110154: -1.29377 1.68164 -0.448823 0.206860
## 15:     7.6479639: -1.36018 1.85456 -0.237404 0.0279485
## 16:     7.3414193: -1.36613 1.87495 -0.131228 0.0197128
## 17:     7.1459204: -1.36939 1.88395 -0.0220586 0.00419477
## 18:     6.8499981: -1.37092 1.88810 0.199369 0.00469949
## 19:     6.2503936: -1.33300 1.78413 0.360771 0.116495
## 20:     5.7947116: -1.29523 1.68459 0.513971 0.245855
## 21:     5.3563474: -1.24407 1.55349 0.629984 0.387008
## 22:     4.7857877: -1.13048 1.27366 0.840203 0.665704
```

```
## 23:      4.0497195: -0.992126 0.973269 0.983984 0.954618
## 24:      3.5179818: -0.841522 0.692665 1.12885 1.25650
## 25:      3.0488024: -0.658857 0.407011 1.24823 1.54704
## 26:      2.6325343: -0.500037 0.229669 1.31902 1.73685
## 27:      2.3059389: -0.307769 0.0630001 1.38020 1.90295
## 28:      1.9350808: -0.174234 0.0178386 1.39533 1.94834
## 29:      1.6534354: -0.0296107 -0.0129818 1.40343 1.97124
## 30:      1.5716752: 0.234877 -0.00937673 1.40482 1.97512
## 31:      0.94438690: 0.306481 0.0937164 1.36921 1.87497
## 32:      0.74178204: 0.438677 0.174107 1.34892 1.82029
## 33:      0.53974471: 0.526746 0.269899 1.31329 1.72450
## 34:      0.40832179: 0.737124 0.500274 1.22372 1.49026
## 35:      0.12697466: 0.792936 0.627489 1.16789 1.36128
## 36:      0.058109411: 0.874374 0.757557 1.11579 1.24255
## 37:      0.014841605: 0.942386 0.883727 1.05723 1.11444
## 38:      0.0015171247: 0.983721 0.966104 1.01742 1.03352
## 39: 2.9694247e-05: 0.997648 0.995118 1.00255 1.00487
## 40: 1.6239267e-08: 0.999951 0.999896 1.00006 1.00010
## 41: 5.1701724e-15: 1.00000 1.00000 1.00000 1.00000
## 42: 2.3217039e-28: 1.00000 1.00000 1.00000 1.00000
## 43: 1.8414972e-30: 1.00000 1.00000 1.00000 1.00000
```

```
print(t1nlminb)
```

```
## $par
## [1] 1 1 1 1
##
## $objective
## [1] 1.841497e-30
##
## $convergence
## [1] 0
##
## $iterations
## [1] 43
##
## $evaluations
## function gradient
##      54      44
##
## $message
## [1] "X-convergence (3)"
```

```
# and call them from optimx (i.e., test this gives same results)
```

```
library(optimx)
t1nlmo <- optimr(x0, wood.f, wood.g, hess=wood.h, method="nlm", control=list(trace=1))
```

```
## Unit parameter scaling
## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] -3 -1 -3 -1
## Function Value
```

```

## [1] 19192
## Gradient:
## [1] -12008 -2080 -10808 -1880
##
## iteration = 1
## Step:
## [1] 0.9116709 0.7367972 0.9020511 0.7516950
## Parameter:
## [1] -2.0883291 -0.2632028 -2.0979489 -0.2483050
## Function Value
## [1] 4166.416
## Gradient:
## [1] -3869.0187 -975.0974 -3517.9318 -887.1722
##
## iteration = 2
## Step:
## [1] 0.5765246 0.5075403 0.5704409 0.5190481
## Parameter:
## [1] -1.5118046 0.2443374 -1.5275080 0.2707431
## Function Value
## [1] 834.2691
## Gradient:
## [1] -1239.3913 -437.9468 -1139.2505 -400.9499
##
## iteration = 3
## Step:
## [1] 0.3452070 0.3077087 0.3415096 0.3149447
## Parameter:
## [1] -1.1665976 0.5520461 -1.1859985 0.5856878
## Function Value
## [1] 142.99
## Gradient:
## [1] -381.7993 -179.0328 -354.8650 -165.0014
##
## iteration = 4
## Step:
## [1] 0.1855520 0.1499399 0.1833595 0.1541052
## Parameter:
## [1] -0.9810455 0.7019861 -1.0026390 0.7397929
## Function Value
## [1] 24.17926
## Gradient:
## [1] -106.17301 -63.26483 -99.83461 -58.94541
##
## iteration = 5
## Step:
## [1] 0.07782451 0.05872218 0.07664496 0.06140066
## Parameter:
## [1] -0.9032210 0.7607082 -0.9259940 0.8011936
## Function Value
## [1] 9.839749
## Gradient:
## [1] -23.71342 -19.79005 -22.61047 -18.88270
##

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## iteration = 6
## Step:
## [1] 0.01753078 0.02499409 0.01707308 0.02682737
## Parameter:
## [1] -0.8856902 0.7857023 -0.9089209 0.8280210
## Function Value
## [1] 8.69256
## Gradient:
## [1] -3.326713 -7.482969 -3.201469 -7.378002
##
## iteration = 7
## Step:
## [1] -0.002949427 0.015908253 -0.003260912 0.017258585
## Parameter:
## [1] -0.8886397 0.8016106 -0.9121818 0.8452796
## Function Value
## [1] 8.500379
## Gradient:
## [1] 0.4633586 -4.6849032 0.5115861 -4.6767719
##
## iteration = 8
## Step:
## [1] -0.006478477 0.012986001 -0.006839623 0.014092239
## Parameter:
## [1] -0.8951181 0.8145966 -0.9190215 0.8593718
## Function Value
## [1] 8.374769
## Gradient:
## [1] 0.9933129 -3.8575663 1.0490214 -3.8528361
##
## iteration = 9
## Step:
## [1] -0.006355528 0.011342611 -0.006713964 0.012301691
## Parameter:
## [1] -0.9014737 0.8259392 -0.9257354 0.8716735
## Function Value
## [1] 8.275711
## Gradient:
## [1] 0.9872768 -3.4000080 1.0433216 -3.3948677
##
## iteration = 10
## Step:
## [1] -0.005715703 0.010053796 -0.006042570 0.010901894
## Parameter:
## [1] -0.9071894 0.8359930 -0.9317780 0.8825754
## Function Value
## [1] 8.196865
## Gradient:
## [1] 0.9031660 -3.0378609 0.9550868 -3.0335914
##
## iteration = 11
## Step:
## [1] -0.005075092 0.008946817 -0.005366382 0.009701843
## Parameter:

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## [1] -0.9122645  0.8449398 -0.9371444  0.8922772
## Function Value
## [1] 8.133955
## Gradient:
## [1]  0.8146503 -2.7224545  0.8616142 -2.7194185
##
## iteration = 12
## Step:
## [1] -0.004503371  0.007976157 -0.004762011  0.008650385
## Parameter:
## [1] -0.9167678  0.8529160 -0.9419064  0.9009276
## Function Value
## [1] 8.083658
## Gradient:
## [1]  0.7329608 -2.4421895  0.7752177 -2.4403344
##
## iteration = 13
## Step:
## [1] -0.004001282  0.007119844 -0.004231240  0.007723143
## Parameter:
## [1] -0.9207691  0.8600358 -0.9461376  0.9086507
## Function Value
## [1] 8.043374
## Gradient:
## [1]  0.6591984 -2.1919825  0.6972101 -2.1911665
##
## iteration = 14
## Step:
## [1] -0.003560435  0.006362120 -0.003765379  0.006902861
## Parameter:
## [1] -0.9243296  0.8663979 -0.9499030  0.9155536
## Function Value
## [1] 8.011061
## Gradient:
## [1]  0.5928602 -1.9682377  0.6270863 -1.9683199
##
## iteration = 15
## Step:
## [1] -0.003172450  0.005690158 -0.003355552  0.006175565
## Parameter:
## [1] -0.9275020  0.8720881 -0.9532586  0.9217292
## Function Value
## [1] 7.985107
## Gradient:
## [1]  0.5332412 -1.7679572  0.5640969 -1.7688150
##
## iteration = 16
## Step:
## [1] -0.002830125  0.005093157 -0.002994097  0.005529510
## Parameter:
## [1] -0.9303321  0.8771812 -0.9562527  0.9272587
## Function Value
## [1] 7.964235
## Gradient:

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## [1] 0.4796636 -1.5885404 0.5075167 -1.5900697
##
## iteration = 17
## Step:
## [1] -0.002527390 0.004561911 -0.002674558 0.004954699
## Parameter:
## [1] -0.9328595 0.8817432 -0.9589272 0.9322134
## Function Value
## [1] 7.947433
## Gradient:
## [1] 0.4315101 -1.4277069 0.4566857 -1.4298200
##
## iteration = 18
## Step:
## [1] -0.002259118 0.004088520 -0.002391484 0.004442553
## Parameter:
## [1] -0.9351186 0.8858317 -0.9613187 0.9366559
## Function Value
## [1] 7.933894
## Gradient:
## [1] 0.3882254 -1.2834487 0.4110108 -1.2860707
##
## iteration = 19
## Step:
## [1] -0.002020955 0.003666169 -0.002140251 0.003985678
## Parameter:
## [1] -0.9371396 0.8894978 -0.9634590 0.9406416
## Function Value
## [1] 7.922976
## Gradient:
## [1] 0.3493109 -1.1539919 0.3699607 -1.1570590
##
## iteration = 20
## Step:
## [1] -0.001809184 0.003288947 -0.001916914 0.003577664
## Parameter:
## [1] -0.9389488 0.8927868 -0.9653759 0.9442193
## Function Value
## [1] 7.914165
## Gradient:
## [1] 0.3143196 -1.0377658 0.3330597 -1.0412235
##
## iteration = 21
## Step:
## [1] -0.001620614 0.002951713 -0.001718090 0.003212935
## Parameter:
## [1] -0.9405694 0.8957385 -0.9670940 0.9474322
## Function Value
## [1] 7.90705
## Gradient:
## [1] 0.2828509 -0.9333771 0.2998820 -0.9371784
##
## iteration = 22
## Step:

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## [1] -0.001452492  0.002649974 -0.001540861  0.002886621
## Parameter:
## [1] -0.9420219  0.8983885 -0.9686348  0.9503188
## Function Value
## [1] 7.901301
## Gradient:
## [1] 0.2545455 -0.8395874  0.2700463 -0.8436915
##
## iteration = 23
## Step:
## [1] -0.001302432  0.002379790 -0.001382703  0.002594456
## Parameter:
## [1] -0.9433243  0.9007683 -0.9700175  0.9529133
## Function Value
## [1] 7.896654
## Gradient:
## [1] 0.2290818 -0.7552944  0.2432114 -0.7596662
##
## iteration = 24
## Step:
## [1] -0.001168361  0.002137702 -0.001241420  0.002332689
## Parameter:
## [1] -0.9444927  0.9029060 -0.9712589  0.9552460
## Function Value
## [1] 7.892895
## Gradient:
## [1] 0.2061710 -0.6795158  0.2190714 -0.6841244
##
## iteration = 25
## Step:
## [1] -0.001048470  0.001920659 -0.001115098  0.002098019
## Parameter:
## [1] -0.9455411  0.9048266 -0.9723740  0.9573440
## Function Value
## [1] 7.889854
## Gradient:
## [1] 0.1855546 -0.6113746  0.1973525 -0.6161934
##
## iteration = 26
## Step:
## [1] -0.0009411735  0.0017259680 -0.0010020619  0.0018875279
## Parameter:
## [1] -0.9464823  0.9065526 -0.9733761  0.9592315
## Function Value
## [1] 7.887392
## Gradient:
## [1] 0.1670005 -0.5500879  0.1778092 -0.5550933
##
## iteration = 27
## Step:
## [1] -0.0008450808  0.0015512445 -0.0009008417  0.0016986351
## Parameter:
## [1] -0.9473274  0.9081038 -0.9742769  0.9609302
## Function Value

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## [1] 7.885399
## Gradient:
## [1] 0.1503004 -0.4949553 0.1602211 -0.5001268
##
## iteration = 28
## Step:
## [1] -0.0007589677 0.0013943750 -0.0008101442 0.0015290525
## Parameter:
## [1] -0.9480864 0.9094982 -0.9750871 0.9624592
## Function Value
## [1] 7.883785
## Gradient:
## [1] 0.1352675 -0.4453502 0.1443910 -0.4506697
##
## iteration = 29
## Step:
## [1] -0.0006817541 0.0012534820 -0.0007288287 0.0013767484
## Parameter:
## [1] -0.9487681 0.9107517 -0.9758159 0.9638360
## Function Value
## [1] 7.882477
## Gradient:
## [1] 0.1217341 -0.4007116 0.1301415 -0.4061631
##
## iteration = 30
## Step:
## [1] -0.0006124854 0.0011268958 -0.0006558873 0.0012399159
## Parameter:
## [1] -0.9493806 0.9118786 -0.9764718 0.9650759
## Function Value
## [1] 7.881417
## Gradient:
## [1] 0.1095495 -0.3605364 0.1173135 -0.3661059
##
## iteration = 31
## Step:
## [1] -0.0005503158 0.0010131294 -0.0005904275 0.0011169461
## Parameter:
## [1] -0.9499309 0.9128917 -0.9770622 0.9661928
## Function Value
## [1] 7.880559
## Gradient:
## [1] 0.09857832 -0.32437405 0.10576421 -0.33004899
##
## iteration = 32
## Step:
## [1] -0.0004944952 0.0009108570 -0.0005316577 0.0010064045
## Parameter:
## [1] -0.9504254 0.9138026 -0.9775939 0.9671992
## Function Value
## [1] 7.879862
## Gradient:
## [1] 0.08869905 -0.29181992 0.09536535 -0.29758930
##

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## iteration = 33
## Step:
## [1] -0.0004443567  0.0008188950 -0.0004788748  0.0009070108
## Parameter:
## [1] -0.9508698  0.9146215 -0.9780728  0.9681062
## Function Value
## [1] 7.879298
## Gradient:
## [1] 0.07980241 -0.26251108  0.08600161 -0.26836511
##
## iteration = 34
## Step:
## [1] -0.0003993074  0.0007361861 -0.0004314530  0.0008176213
## Parameter:
## [1] -0.9512691  0.9153577 -0.9785042  0.9689239
## Function Value
## [1] 7.87884
## Gradient:
## [1] 0.07179017 -0.23612164  0.07756940 -0.24205160
##
## iteration = 35
## Step:
## [1] -0.0003588187  0.0006617846 -0.0003888351  0.0007372134
## Parameter:
## [1] -0.9516279  0.9160194 -0.9788931  0.9696611
## Function Value
## [1] 7.878469
## Gradient:
## [1] 0.06457399 -0.21235885  0.06997558 -0.21835702
##
## iteration = 36
## Step:
## [1] -0.0003224194  0.0005948446 -0.0003505244  0.0006648718
## Parameter:
## [1] -0.9519503  0.9166143 -0.9792436  0.9703259
## Function Value
## [1] 7.878168
## Gradient:
## [1] 0.05807447 -0.19095972  0.06313643 -0.19701921
##
## iteration = 37
## Step:
## [1] -0.0002896886  0.0005346081 -0.0003160773  0.0005997772
## Parameter:
## [1] -0.9522400  0.9171489 -0.9795597  0.9709257
## Function Value
## [1] 7.877923
## Gradient:
## [1] 0.05222015 -0.17168788  0.05697664 -0.17780255
##
## iteration = 38
## Step:
## [1] -0.0002602503  0.0004803963 -0.0002850973  0.0005411952
## Parameter:

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## [1] -0.9525003  0.9176293 -0.9798447  0.9714669
## Function Value
## [1] 7.877725
## Gradient:
## [1]  0.04694676 -0.15433082  0.05142848 -0.16049519
##
## iteration = 39
## Step:
## [1] -0.0002337683  0.0004316002 -0.0002572303  0.0004884677
## Parameter:
## [1] -0.9527340  0.9180609 -0.9801020  0.9719554
## Function Value
## [1] 7.877564
## Gradient:
## [1]  0.04219647 -0.13869745  0.04643103 -0.14490666
##
## iteration = 40
## Step:
## [1] -0.0002099415  0.0003876740 -0.0002321590  0.0004410041
## Parameter:
## [1] -0.9529440  0.9184486 -0.9803341  0.9723964
## Function Value
## [1] 7.877434
## Gradient:
## [1]  0.03791725 -0.12461594  0.04192945 -0.13086560
##
## iteration = 41
## Step:
## [1] -0.0001885004  0.0003481274 -0.0002095994  0.0003982746
## Parameter:
## [1] -0.9531325  0.9187967 -0.9805437  0.9727947
## Function Value
## [1] 7.877328
## Gradient:
## [1]  0.03406226 -0.11193167  0.03787442 -0.11821790
##
## iteration = 42
## Step:
## [1] -0.0001692034  0.0003125205 -0.0001892971  0.0003598034
## Parameter:
## [1] -0.9533017  0.9191092 -0.9807330  0.9731545
## Function Value
## [1] 7.877242
## Gradient:
## [1]  0.03058934 -0.10050556  0.03422152 -0.10682487
##
## iteration = 43
## Step:
## [1] -0.0001518338  0.0002804581 -0.0001710239  0.0003251634
## Parameter:
## [1] -0.9534535  0.9193897 -0.9809041  0.9734796
## Function Value
## [1] 7.877172
## Gradient:

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## [1] 0.02746054 -0.09021245 0.03093080 -0.09656171
##
## iteration = 44
## Step:
## [1] -0.0001361975 0.0002515852 -0.0001545751 0.0002939707
## Parameter:
## [1] -0.9535897 0.9196413 -0.9810586 0.9737736
## Function Value
## [1] 7.877115
## Gradient:
## [1] 0.02464170 -0.08093968 0.02796628 -0.08731610
##
## iteration = 45
## Step:
## [1] -0.0001221200 0.0002255827 -0.0001397672 0.0002658804
## Parameter:
## [1] -0.9537118 0.9198668 -0.9811984 0.9740395
## Function Value
## [1] 7.877069
## Gradient:
## [1] 0.02210205 -0.07258586 0.02529556 -0.07898695
##
## iteration = 46
## Step:
## [1] -0.0001094447 0.0002021638 -0.0001264351 0.0002405825
## Parameter:
## [1] -0.9538213 0.9200690 -0.9813248 0.9742801
## Function Value
## [1] 7.877032
## Gradient:
## [1] 0.01981390 -0.06505973 0.02288949 -0.07148325
##
## iteration = 47
## Step:
## [1] -9.803093e-05 1.810706e-04 -1.144309e-04 2.177981e-04
## Parameter:
## [1] -0.9539193 0.9202501 -0.9814393 0.9744979
## Function Value
## [1] 7.877002
## Gradient:
## [1] 0.01775230 -0.05827911 0.02072180 -0.06472304
##
## iteration = 48
## Step:
## [1] -8.775243e-05 1.620711e-04 -1.036215e-04 1.972765e-04
## Parameter:
## [1] -0.9540070 0.9204122 -0.9815429 0.9746951
## Function Value
## [1] 7.876977
## Gradient:
## [1] 0.01589478 -0.05217001 0.01876885 -0.05863257
##
## iteration = 49
## Step:

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## [1] -7.849563e-05  1.449568e-04 -9.388733e-05  1.787923e-04
## Parameter:
## [1] -0.9540855  0.9205571 -0.9816368  0.9748739
## Function Value
## [1] 7.876957
## Gradient:
## [1] 0.01422113 -0.04666582  0.01700935 -0.05314538
##
## iteration = 50
## Step:
## [1] -7.015842e-05  1.295399e-04 -8.512093e-05  1.621426e-04
## Parameter:
## [1] -0.9541557  0.9206866 -0.9817219  0.9750361
## Function Value
## [1] 7.876941
## Gradient:
## [1] 0.01271311 -0.04170654  0.01542411 -0.04820166
##
## iteration = 51
## Step:
## [1] -6.264904e-05  1.156516e-04 -7.722567e-05  1.471448e-04
## Parameter:
## [1] -0.9542184  0.9208023 -0.9817991  0.9751832
## Function Value
## [1] 7.876928
## Gradient:
## [1] 0.01135431 -0.03723815  0.01399587 -0.04374754
##
## iteration = 52
## Step:
## [1] -5.588494e-05  1.031398e-04 -7.011467e-05  1.336346e-04
## Parameter:
## [1] -0.9542742  0.9209054 -0.9818692  0.9753169
## Function Value
## [1] 7.876917
## Gradient:
## [1] 0.01012996 -0.03321200  0.01270906 -0.03973448
##
## iteration = 53
## Step:
## [1] -4.979185e-05  9.186780e-05 -6.370975e-05  1.214641e-04
## Parameter:
## [1] -0.9543240  0.9209973 -0.9819329  0.9754383
## Function Value
## [1] 7.876908
## Gradient:
## [1] 0.009026736 -0.029584242  0.011549679 -0.036118777
##
## iteration = 54
## Step:
## [1] -4.430297e-05  8.171235e-05 -5.794061e-05  1.105004e-04
## Parameter:
## [1] -0.9543683  0.9210790 -0.9819909  0.9755488
## Function Value

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## [1] 7.876901
## Gradient:
## [1] 0.008032641 -0.026315422 0.010505094 -0.032861072
##
## iteration = 55
## Step:
## [1] -3.935818e-05 7.256263e-05 -5.274397e-05 1.006234e-04
## Parameter:
## [1] -0.9544077 0.9211516 -0.9820436 0.9756494
## Function Value
## [1] 7.876895
## Gradient:
## [1] 0.007136870 -0.023369977 0.009563939 -0.029925902
##
## iteration = 56
## Step:
## [1] -3.490339e-05 6.431882e-05 -4.806288e-05 9.172539e-05
## Parameter:
## [1] -0.9544426 0.9212159 -0.9820917 0.9757412
## Function Value
## [1] 7.876891
## Gradient:
## [1] 0.006329690 -0.020715882 0.008715969 -0.027281325
##
## iteration = 57
## Step:
## [1] -3.088991e-05 5.689103e-05 -4.384610e-05 8.370920e-05
## Parameter:
## [1] -0.9544735 0.9212728 -0.9821355 0.9758249
## Function Value
## [1] 7.876887
## Gradient:
## [1] 0.005602331 -0.018324283 0.007951959 -0.024898566
##
## iteration = 58
## Step:
## [1] -2.727389e-05 5.019833e-05 -4.004750e-05 7.648736e-05
## Parameter:
## [1] -0.9545008 0.9213230 -0.9821756 0.9759014
## Function Value
## [1] 7.876884
## Gradient:
## [1] 0.004946890 -0.016169189 0.007263594 -0.022751704
##
## iteration = 59
## Step:
## [1] -2.401587e-05 4.416784e-05 -3.662555e-05 6.998109e-05
## Parameter:
## [1] -0.9545248 0.9213672 -0.9822122 0.9759713
## Function Value
## [1] 7.876881
## Gradient:
## [1] 0.004356251 -0.014227187 0.006643386 -0.020817386
##

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## iteration = 60
## Step:
## [1] -2.108033e-05  3.873392e-05 -3.354285e-05  6.411946e-05
## Parameter:
## [1] -0.9545459  0.9214059 -0.9822458  0.9760355
## Function Value
## [1] 7.876879
## Gradient:
## [1] 0.003823999 -0.012477182  0.006084587 -0.019074574
##
## iteration = 61
## Step:
## [1] -1.843528e-05  3.383744e-05 -3.076575e-05  5.883857e-05
## Parameter:
## [1] -0.9545643  0.9214397 -0.9822765  0.9760943
## Function Value
## [1] 7.876877
## Gradient:
## [1] 0.003344358 -0.010900169  0.005581120 -0.017504313
##
## iteration = 62
## Step:
## [1] -1.605189e-05  2.942515e-05 -2.826394e-05  5.408088e-05
## Parameter:
## [1] -0.9545803  0.9214692 -0.9823048  0.9761484
## Function Value
## [1] 7.876876
## Gradient:
## [1] 0.002912120 -0.009479027  0.005127508 -0.016089527
##
## iteration = 63
## Step:
## [1] -1.390424e-05  2.544908e-05 -2.601010e-05  4.979455e-05
## Parameter:
## [1] -0.9545942  0.9214946 -0.9823308  0.9761982
## Function Value
## [1] 7.876875
## Gradient:
## [1] 0.002522596 -0.008198330  0.004718816 -0.014814831
##
## iteration = 64
## Step:
## [1] -1.196895e-05  2.186604e-05 -2.397965e-05  4.593290e-05
## Parameter:
## [1] -0.9546062  0.9215165 -0.9823548  0.9762441
## Function Value
## [1] 7.876874
## Gradient:
## [1] 0.002171559 -0.007044181  0.004350599 -0.013666362
##
## iteration = 65
## Step:
## [1] -1.022498e-05  1.863710e-05 -2.215046e-05  4.245386e-05
## Parameter:

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## [1] -0.9546164  0.9215351 -0.9823769  0.9762866
## Function Value
## [1] 7.876873
## Gradient:
## [1]  0.001855202 -0.006004056  0.004018852 -0.012631632
##
## iteration = 66
## Step:
## [1] -8.653371e-06  1.572720e-05 -2.050257e-05  3.931955e-05
## Parameter:
## [1] -0.9546251  0.9215508 -0.9823974  0.9763259
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001570093 -0.005066674  0.003719966 -0.011699386
##
## iteration = 67
## Step:
## [1] -7.237055e-06  1.310476e-05 -1.901803e-05  3.649584e-05
## Parameter:
## [1] -0.9546323  0.9215639 -0.9824164  0.9763624
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001313142 -0.004221869  0.003450689 -0.010859486
##
## iteration = 68
## Step:
## [1] -5.960649e-06  1.074131e-05 -1.768066e-05  3.395197e-05
## Parameter:
## [1] -0.9546383  0.9215747 -0.9824341  0.9763963
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.001081562 -0.003460482  0.003208092 -0.010102797
##
## iteration = 69
## Step:
## [1] -4.810300e-06  8.611225e-06 -1.647587e-05  3.166025e-05
## Parameter:
## [1] -0.9546431  0.9215833 -0.9824506  0.9764280
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.0008728447 -0.0027742610  0.0029895339 -0.0094210856
##
## iteration = 70
## Step:
## [1] -3.773526e-06  6.691408e-06 -1.539054e-05  2.959571e-05
## Parameter:
## [1] -0.9546469  0.9215900 -0.9824660  0.9764576
## Function Value
## [1] 7.87687
## Gradient:

```

```

## [1] 0.0006847275 -0.0021557690 0.0027926375 -0.0088069365
##
## iteration = 71
## Step:
## [1] -2.839083e-06 4.961048e-06 -1.441284e-05 2.773588e-05
## Parameter:
## [1] -0.9546497 0.9215949 -0.9824804 0.9764853
## Function Value
## [1] 7.87687
## Gradient:
## [1] 0.000515173 -0.001598306 0.002615259 -0.008253666
##
## iteration = 72
## Step:
## [1] -1.996843e-06 3.401398e-06 -1.353211e-05 2.606049e-05
## Parameter:
## [1] -0.9546517 0.9215983 -0.9824939 0.9765114
## Function Value
## [1] 7.87687
## Gradient:
## [1] 0.0003623451 -0.0010958356 0.0024554681 -0.0077552527
##
## iteration = 73
## Step:
## [1] -1.237679e-06 1.995567e-06 -1.273876e-05 2.455130e-05
## Parameter:
## [1] -0.9546529 0.9216003 -0.9825067 0.9765359
## Function Value
## [1] 7.87687
## Gradient:
## [1] 0.0002245894 -0.0006429174 0.0023115243 -0.0073062704
##
## iteration = 74
## Step:
## [1] -5.533700e-07 7.283349e-07 -1.202413e-05 2.319185e-05
## Parameter:
## [1] -0.9546535 0.9216011 -0.9825187 0.9765591
## Function Value
## [1] 7.87687
## Gradient:
## [1] 0.0001004150 -0.0002346499 0.0021818599 -0.0069018300
##
## iteration = 75
## Step:
## [1] 6.349210e-08 -4.140111e-07 -1.138043e-05 2.196734e-05
## Parameter:
## [1] -0.9546534 0.9216007 -0.9825301 0.9765811
## Function Value
## [1] 7.876869
## Gradient:
## [1] -1.152136e-05 1.333833e-04 2.065062e-03 -6.537526e-03
##
## iteration = 76
## Step:

```



```

## [1] 6.195811e-07 -1.443826e-06 -1.080065e-05 2.086440e-05
## Parameter:
## [1] -0.9546528 0.9215992 -0.9825409 0.9766020
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0001124301 0.0004651620 0.0019598589 -0.0062093889
##
## iteration = 77
## Step:
## [1] 1.120910e-06 -2.372243e-06 -1.027846e-05 1.987102e-05
## Parameter:
## [1] -0.9546517 0.9215968 -0.9825512 0.9766218
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002034020 0.0007642722 0.0018651024 -0.0059138415
##
## iteration = 78
## Step:
## [1] 1.572897e-06 -3.209292e-06 -9.808155e-06 1.897636e-05
## Parameter:
## [1] -0.9546501 0.9215936 -0.9825610 0.9766408
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002854198 0.0010339451 0.0017797595 -0.0056476608
##
## iteration = 79
## Step:
## [1] 1.980424e-06 -3.964012e-06 -9.384605e-06 1.817064e-05
## Parameter:
## [1] -0.9546481 0.9215897 -0.9825704 0.9766590
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0003593692 0.0012770923 0.0017028990 -0.0054079425
##
## iteration = 80
## Step:
## [1] 2.347888e-06 -4.644546e-06 -9.003184e-06 1.744507e-05
## Parameter:
## [1] -0.9546458 0.9215850 -0.9825794 0.9766764
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.000426048 0.001496337 0.001633682 -0.005192070
##
## iteration = 81
## Step:
## [1] 2.679254e-06 -5.258234e-06 -8.659724e-06 1.679173e-05
## Parameter:
## [1] -0.9546431 0.9215798 -0.9825880 0.9766932
## Function Value

```

```

## [1] 7.876869
## Gradient:
## [1] -0.0004861755  0.0016940444  0.0015713518 -0.0049976836
##
## iteration = 82
## Step:
## [1] 2.978091e-06 -5.811686e-06 -8.350468e-06  1.620346e-05
## Parameter:
## [1] -0.9546401  0.9215740 -0.9825964  0.9767094
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0005403996  0.0018723437  0.0015152278 -0.0048226599
##
## iteration = 83
## Step:
## [1] 3.247619e-06 -6.310861e-06 -8.072033e-06  1.567383e-05
## Parameter:
## [1] -0.9546369  0.9215676 -0.9826044  0.9767251
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000589304  0.002033155  0.001464696 -0.004665084
##
## iteration = 84
## Step:
## [1] 3.490735e-06 -6.761128e-06 -7.821371e-06  1.519705e-05
## Parameter:
## [1] -0.9546334  0.9215609 -0.9826123  0.9767403
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000633415  0.002178207  0.001419203 -0.004523229
##
## iteration = 85
## Step:
## [1] 3.710052e-06 -7.167323e-06 -7.595732e-06  1.476787e-05
## Parameter:
## [1] -0.9546297  0.9215537 -0.9826199  0.9767551
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0006732066  0.0023090593  0.0013782505 -0.0043955420
##
## iteration = 86
## Step:
## [1] 3.907924e-06 -7.533807e-06 -7.392643e-06  1.438160e-05
## Parameter:
## [1] -0.9546258  0.9215462 -0.9826272  0.9767694
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000709106  0.002427116  0.001341389 -0.004280621
##

```

```

## iteration = 87
## Step:
## [1] 4.086473e-06 -7.864506e-06 -7.209873e-06 1.403400e-05
## Parameter:
## [1] -0.9546217 0.9215383 -0.9826345 0.9767835
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0007414982 0.0025336428 0.0013082150 -0.0041772039
##
## iteration = 88
## Step:
## [1] 4.247610e-06 -8.162960e-06 -7.045411e-06 1.372122e-05
## Parameter:
## [1] -0.9546174 0.9215301 -0.9826415 0.9767972
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000770730 0.002629780 0.001278363 -0.004084153
##
## iteration = 89
## Step:
## [1] 4.393056e-06 -8.432358e-06 -6.897446e-06 1.343985e-05
## Parameter:
## [1] -0.9546131 0.9215217 -0.9826484 0.9768106
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000797114 0.002716555 0.001251504 -0.004000443
##
## iteration = 90
## Step:
## [1] 4.524363e-06 -8.675573e-06 -6.764348e-06 1.318675e-05
## Parameter:
## [1] -0.9546085 0.9215130 -0.9826552 0.9768238
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008209319 0.0027948936 0.0012273423 -0.0039251507
##
## iteration = 91
## Step:
## [1] 4.642931e-06 -8.895197e-06 -6.644645e-06 1.295915e-05
## Parameter:
## [1] -0.9546039 0.9215041 -0.9826618 0.9768368
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008424374 0.0028656307 0.0012056113 -0.0038574434
##
## iteration = 92
## Step:
## [1] 4.750018e-06 -9.093561e-06 -6.537012e-06 1.275452e-05
## Parameter:

```

```

## [1] -0.9545991  0.9214951 -0.9826683  0.9768495
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008618593  0.0029295178  0.0011860705 -0.0037965711
##
## iteration = 93
## Step:
## [1] 4.846761e-06 -9.272768e-06 -6.440257e-06  1.257058e-05
## Parameter:
## [1] -0.9545943  0.9214858 -0.9826748  0.9768621
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0008794036  0.0029872324  0.0011685032 -0.0037418580
##
## iteration = 94
## Step:
## [1] 4.934182e-06 -9.434714e-06 -6.353302e-06  1.240530e-05
## Parameter:
## [1] -0.9545894  0.9214763 -0.9826811  0.9768745
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000895256  0.003039385  0.001152714 -0.003692695
##
## iteration = 95
## Step:
## [1] 5.013203e-06 -9.581105e-06 -6.275179e-06  1.225682e-05
## Parameter:
## [1] -0.9545843  0.9214668 -0.9826874  0.9768868
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000909584  0.003086526  0.001138528 -0.003648534
##
## iteration = 96
## Step:
## [1] 5.084656e-06 -9.713480e-06 -6.205014e-06  1.212348e-05
## Parameter:
## [1] -0.9545793  0.9214571 -0.9826936  0.9768989
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009225381  0.0031291509  0.0011257852 -0.0036088788
##
## iteration = 97
## Step:
## [1] 5.149289e-06 -9.833224e-06 -6.142021e-06  1.200379e-05
## Parameter:
## [1] -0.9545741  0.9214472 -0.9826998  0.9769109
## Function Value
## [1] 7.876867
## Gradient:

```

```
## [1] -0.0009342544  0.0031677063  0.0011143439 -0.0035732853
##
## iteration = 98
## Step:
## [1] 5.207775e-06 -9.941589e-06 -6.085489e-06  1.189640e-05
## Parameter:
## [1] -0.9545689  0.9214373 -0.9827058  0.9769228
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009448553  0.0032025948  0.0011040749 -0.0035413514
##
## iteration = 99
## Step:
## [1] 5.260724e-06 -1.003970e-05 -6.034780e-06  1.180009e-05
## Parameter:
## [1] -0.9545636  0.9214272 -0.9827119  0.9769346
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009544511  0.0032341791  0.0010948625 -0.0035127154
##
## iteration = 100
## Parameter:
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009631411  0.0032627860  0.0010866021 -0.0034870511
##
## Iteration limit exceeded.  Algorithm failed.
```

```
print(t1nlmo)
```

```
## $convergence
## [1] 1
##
## $value
## [1] 7.876867
##
## $par
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
##
## $counts
## [1] NA 100
##
## $message
## [1] "Convergence indicator (code) = 4"
```

```
tst<-try(t1nlminbo <- optimr(x0, wood.f, wood.g, hess=wood.h, method="nllminb", control=list(trace=1)))
```

```
## Unit parameter scaling
## 0:      19192.000: -3.000000 -1.000000 -3.000000 -1.000000
## 1:      7427.8901: -2.26768 -0.873148 -2.34086 -0.885346
## 2:      2311.9042: -1.74107 -0.585584 -1.59870 -0.586687
```

```

## 3: 729.03082: -0.890037 -0.154631 -1.48501 -0.309027
## 4: 71.909337: -0.662552 0.00461052 -0.584618 0.0259220
## 5: 45.347493: -0.496228 0.812202 -0.597210 0.591584
## 6: 40.726120: -0.781428 0.331672 -0.911765 1.35892
## 7: 38.214601: -0.616770 0.466804 -1.29242 1.11452
## 8: 9.0237120: -0.632614 0.459113 -1.06180 1.20941
## 9: 8.2132107: -0.730131 0.520690 -1.08662 1.20729
## 10: 8.1312801: -0.712954 0.540234 -1.10168 1.20856
## 11: 7.9975970: -0.730221 0.552271 -1.09958 1.22996
## 12: 7.9277653: -0.744706 0.569886 -1.11342 1.24388
## 13: 7.8789361: -0.755791 0.588373 -1.11841 1.26427
## 14: 7.8572896: -0.769023 0.602168 -1.12872 1.28509
## 15: 7.8542867: -0.758659 0.588755 -1.13933 1.30758
## 16: 7.8516697: -0.747519 0.572011 -1.14845 1.32802
## 17: 7.8407298: -0.701838 0.502829 -1.18349 1.40792
## 18: 7.8347831: -0.675646 0.464352 -1.20184 1.45170
## 19: 7.8269048: -0.647600 0.425840 -1.21863 1.49492
## 20: 7.8180840: -0.618136 0.387587 -1.23640 1.53702
## 21: 7.7885058: -0.552793 0.312287 -1.26851 1.61892
## 22: 7.7821195: -0.419328 0.160287 -1.33332 1.77893
## 23: 7.5970425: -0.307065 0.0941103 -1.35111 1.82958
## 24: 7.5804254: -0.208693 0.0202352 -1.37462 1.89421
## 25: 7.3437276: -0.0780988 -0.00556015 -1.35638 1.86918
## 26: 7.1581672: -0.0194280 0.0186872 -1.34933 1.82889
## 27: 7.0299891: 0.0547616 0.00526381 -1.35282 1.83147
## 28: 6.9376955: 0.129979 0.00128356 -1.35077 1.82643
## 29: 6.8290238: 0.203941 0.0135397 -1.34645 1.81845
## 30: 6.6127919: 0.271070 0.0466648 -1.34054 1.81041
## 31: 6.1309123: 0.402538 0.151936 -1.32765 1.78307
## 32: 6.0145341: 0.536980 0.241710 -1.30999 1.72983
## 33: 5.5405642: 0.616386 0.365849 -1.28116 1.64778
## 34: 5.2319328: 0.795053 0.584208 -1.20685 1.45258
## 35: 4.5260478: 0.890916 0.783724 -1.10587 1.19953
## 36: 4.2748887: 1.01190 0.991796 -1.00274 0.965731
## 37: 3.8118699: 1.08143 1.18900 -0.870747 0.716572
## 38: 3.4183212: 1.15499 1.32928 -0.799149 0.611139
## 39: 3.1274174: 1.21586 1.47840 -0.710683 0.531683
## 40: 2.9319507: 1.27451 1.60801 -0.636964 0.411656
## 41: 2.8074922: 1.32360 1.73246 -0.554527 0.287616
## 42: 2.6979772: 1.32786 1.76497 -0.514976 0.238953
## 43: 2.5978103: 1.34096 1.80616 -0.473380 0.201421
## 44: 2.3504274: 1.38633 1.93240 -0.341213 0.0937173
## 45: 2.1879401: 1.38800 1.95384 -0.238752 0.0837674
## 46: 1.9738919: 1.40950 2.00599 -0.163369 0.0369160
## 47: 1.9313844: 1.44936 2.10143 -0.00276268 -0.0511139
## 48: 1.8029335: 1.42563 2.02275 -0.0940048 0.00894612
## 49: 1.7285050: 1.41878 2.01719 -0.0661541 -0.00273086
## 50: 1.6366521: 1.41533 2.00942 0.0310760 -0.0322593
## 51: 1.5496975: 1.40481 1.97844 0.0339044 -0.0185982
## 52: 1.4051372: 1.38420 1.91570 0.101330 -0.000593362
## 53: 1.2768628: 1.36993 1.87268 0.183317 0.0207516
## 54: 1.0456071: 1.33870 1.78169 0.375868 0.104726
## 55: 0.68801294: 1.31481 1.71351 0.459776 0.220743
## 56: 0.52234496: 1.29010 1.64819 0.573486 0.309683

```

```
## 57: 0.40368085: 1.26199 1.58808 0.657749 0.403783
## 58: 0.31496884: 1.23861 1.55128 0.682889 0.474259
## 59: 0.24192757: 1.22004 1.50114 0.726901 0.526257
## 60: 0.17455784: 1.18712 1.41509 0.791930 0.611852
## 61: 0.12430840: 1.15221 1.32010 0.839019 0.685985
## 62: 0.092133333: 1.13812 1.28426 0.843130 0.706236
## 63: 0.053032095: 1.10146 1.20252 0.890392 0.788802
## 64: 0.013424283: 1.05583 1.11085 0.947014 0.893435
## 65: 0.0036272623: 1.03077 1.06089 0.971812 0.944280
## 66: 0.0019038445: 1.00653 1.01008 0.998354 0.993783
## 67: 0.00020234604: 1.00160 1.00299 0.999171 0.999539
## 68: 5.8035620e-05: 1.00048 1.00066 1.00056 1.00139
## 69: 1.3800484e-05: 0.999916 0.999665 1.00068 1.00135
## 70: 5.7617897e-07: 0.999894 0.999763 1.00021 1.00036
## 71: 1.1827891e-08: 0.999977 0.999952 1.00003 1.00005
## 72: 4.1006268e-11: 0.999999 0.999998 1.00000 1.00000
## 73: 8.2725368e-14: 1.00000 1.00000 1.00000 1.00000
## 74: 3.9750565e-16: 1.00000 1.00000 1.00000 1.00000
## 75: 3.9451721e-18: 1.00000 1.00000 1.00000 1.00000
```

```
if (class(tst) == "try-error"){
  cat("try-error on attempt to run nlminb in optimr()\n")
} else { print(t1nlminb) }
```

```
## $par
## [1] 1 1 1 1
##
## $objective
## [1] 1.841497e-30
##
## $convergence
## [1] 0
##
## $iterations
## [1] 43
##
## $evaluations
## function gradient
##      54      44
##
## $message
## [1] "X-convergence (3)"
```

```
# sink()
```

## A generalized Rosenbrock function

There are several generalizations of the Rosenbrock function ([??ref](#))

```
# genrosa function code -- attempts to match the rosenbrock at gs=100 and x=c(-1.2,1)
genrosa.f<- function(x, gs=NULL){ # objective function
## One generalization of the Rosenbrock banana valley function (n parameters)
  n <- length(x)
  if(is.null(gs)) { gs=100.0 }
  # Note do not at 1.0 so min at 0
```

```

    fval<-sum (gs*(x[1:(n-1)]^2 - x[2:n])^2 + (x[1:(n-1)] - 1)^2)
}

genrosa.g <- function(x, gs=NULL){
# vectorized gradient for genrose.f
# Ravi Varadhan 2009-04-03
  n <- length(x)
  if(is.null(gs)) { gs=100.0 }
  gg <- as.vector(rep(0, n))
  tn <- 2:n
  tn1 <- tn - 1
  z1 <- x[tn] - x[tn1]^2
  z2 <- 1 - x[tn1]
  # f = gs*z1*z1 + z2*z2
  gg[tn] <- 2 * (gs * z1)
  gg[tn1] <- gg[tn1] - 4 * gs * x[tn1] * z1 - 2 * z2
  return(gg)
}

genrosa.h <- function(x, gs=NULL) { ## compute Hessian
  if(is.null(gs)) { gs=100.0 }
  n <- length(x)
  hh<-matrix(rep(0, n*n),n,n)
  for (i in 2:n) {
    z1<-x[i]-x[i-1]*x[i-1]
    # z2<-1.0 - x[i-1]
    hh[i,i]<-hh[i,i]+2.0*(gs+1.0)
    hh[i-1,i-1]<-hh[i-1,i-1]-4.0*gs*z1-4.0*gs*x[i-1]*(-2.0*x[i-1])
    hh[i,i-1]<-hh[i,i-1]-4.0*gs*x[i-1]
    hh[i-1,i]<-hh[i-1,i]-4.0*gs*x[i-1]
  }
  return(hh)
}

require(snewton)
cat("Generalized Rosenbrock tests\n")

## Generalized Rosenbrock tests
cat("original function")

## original function
x0 <- c(-1.2, 1)
solorig <- snewton(x0, genrosa.f, genrosa.g, genrosa.h)

## trace = 1
## Initial function value = 24.2
## nf= 1
## Iteration 1 :nf= 2
## Iteration 2 :nf= 3
## Iteration 3 :nf= 4
## Iteration 4 :nf= 6
## Iteration 5 :nf= 8
## Iteration 6 :nf= 10

```



```
## Iteration 7 :nf= 12
## Iteration 8 :nf= 14
## Iteration 9 :nf= 16
## Iteration 10 :nf= 18
## Iteration 11 :nf= 20
## Iteration 12 :nf= 22
## Iteration 13 :nf= 24
## Iteration 14 :nf= 26
## Iteration 15 :nf= 28
## Iteration 16 :nf= 30
## Iteration 17 :nf= 32
## Iteration 18 :nf= 33
## Iteration 19 :nf= 34
## Iteration 20 :nf= 36
## Iteration 21 :nf= 37
## Iteration 22 :nf= 38
## Iteration 23 :nf= 39
## Iteration 24 :nf= 40
## Iteration 25 :nf= 41
## Iteration 26 :nf= 42
## Iteration 27 :nf= 43
## Iteration 28 :nf= 44
## Iteration 29 :nf= 45
## Iteration 30 :nf= 46
## Iteration 31 :nf= 47
## Iteration 32 :nf= 48
## Iteration 33 :nf= 49
## Iteration 34 :nf= 50
## Iteration 35 :nf= 51
## Iteration 36 :nf= 52
## Iteration 37 :nf= 53
## Iteration 38 :nf= 54
## Iteration 39 :nf= 55
## Iteration 40 :nf= 56
## Iteration 41 :nf= 57
## Iteration 42 :nf= 58
## Iteration 43 :nf= 59
## Iteration 44 :nf= 60
## Iteration 45 :nf= 61
## Iteration 46 :nf= 62
## Iteration 47 :nf= 63
## Iteration 48 :nf= 64
## Iteration 49 :nf= 65
## Iteration 50 :nf= 66
## Iteration 51 :nf= 67
## Iteration 52 :nf= 68
## Iteration 53 :nf= 69
## Iteration 54 :nf= 70
## Iteration 55 :nf= 71
## Iteration 56 :nf= 72
## Iteration 57 :nf= 73
## Iteration 58 :nf= 74
## Iteration 59 :nf= 75
## Iteration 60 :nf= 76
```

```
## Iteration 61 :nf= 77
## Iteration 62 :nf= 78
## Iteration 63 :nf= 79
## Iteration 64 :nf= 80
## Iteration 65 :nf= 81
## Iteration 66 :nf= 82
## Iteration 67 :nf= 83
## Iteration 68 :nf= 84
## Iteration 69 :nf= 85
## Iteration 70 :nf= 86
## Iteration 71 :nf= 87
## Iteration 72 :nf= 88
## Iteration 73 :nf= 89
## Iteration 74 :nf= 90
## Iteration 75 :nf= 91
## Iteration 76 :nf= 92
## Iteration 77 :nf= 93
## Iteration 78 :nf= 94
## Iteration 79 :nf= 95
## Iteration 80 :nf= 96
## Iteration 81 :nf= 97
## Iteration 82 :nf= 98
## Iteration 83 :nf= 99
## Iteration 84 :nf= 100
## Iteration 85 :nf= 101
## Iteration 86 :nf= 102
## Iteration 87 :nf= 103
## Iteration 88 :nf= 104
## Iteration 89 :nf= 105
## Iteration 90 :nf= 106
## Iteration 91 :nf= 107
## Iteration 92 :nf= 108
## Iteration 93 :nf= 109
## Iteration 94 :nf= 110
## Iteration 95 :nf= 111
## Iteration 96 :nf= 112
## Iteration 97 :nf= 113
## Iteration 98 :nf= 114
## Iteration 99 :nf= 115
## Iteration 100 :nf= 116
## Iteration 101 :nf= 117
## Iteration 102 :nf= 118
## Iteration 103 :nf= 119
## Iteration 104 :nf= 120
## Iteration 105 :nf= 121
## Iteration 106 :nf= 122
## Iteration 107 :nf= 123
## Iteration 108 :nf= 124
## Iteration 109 :nf= 125
## Iteration 110 :nf= 126
## Iteration 111 :nf= 127
## Iteration 112 :nf= 128
## Iteration 113 :nf= 129
## Iteration 114 :nf= 130
```

```

## Iteration 115 :nf= 131
## Iteration 116 :nf= 132
## Iteration 117 :nf= 133
## Iteration 118 :nf= 134
## Iteration 119 :nf= 135
## Iteration 120 :nf= 136
## Iteration 121 :nf= 137
## Iteration 122 :nf= 138
## Iteration 123 :nf= 139
## Iteration 124 :nf= 140
## Iteration 125 :nf= 141
## Iteration 126 :nf= 142
## Iteration 127 :nf= 143
## Iteration 128 :No progress before linesearch!
## No progress in linesearch!

```

```
print(solorig)
```

```

## $par
## [1] 1 1
##
## $value
## [1] 2.972526e-28
##
## $grad
## [1] 5.462297e-14 -4.440892e-14
##
## $Hess
##      [,1] [,2]
## [1,] 800 -400
## [2,] -400 202
##
## $counts
## $counts$niter
## [1] 128
##
## $counts$nf
## [1] 144
##
## $counts$ngr
## [1] 128
##
## $counts$nhess
## [1] 128
##
##
## $convcode
## [1] 93

```

```
print(eigen(solorig$Hess)$values)
```

```
## [1] 1000.400641 1.599359
```

```
solorigm <- newtonm(x0, genrosa.f, genrosa.g, genrosa.h)
```

```
## trace = 1
```

```

## Start snwtonm  f0= 24.2  at  [1] -1.2  1.0
## Iteration  1  fbest= 24.2
##  lambda = 0.0001220703  fval= 4.691037
## Iteration  2  fbest= 4.691037
##  lambda = 1.831055e-05  fval= 4.537878
## Iteration  3  fbest= 4.537878
##  lambda = 2.746582e-06  fval= 3.033359
## Iteration  4  fbest= 3.033359
##  lambda = 4.119873e-07  fval= 7.883925
##  lambda = 0.001220703  fval= 7.870029
##  lambda = 0.01220703  fval= 7.746736
##  lambda = 0.1220703  fval= 6.67461
##  lambda = 1.220703  fval= 2.997128
## Iteration  9  fbest= 2.997128
##  lambda = 0.1831055  fval= 1.799511
## Iteration 10  fbest= 1.799511
##  lambda = 0.02746582  fval= 48.82691
##  lambda = 0.2746582  fval= 32.02697
##  lambda = 2.746582  fval= 2.798801
##  lambda = 27.46582  fval= 1.621835
## Iteration 14  fbest= 1.621835
##  lambda = 4.119873  fval= 4.540856
##  lambda = 41.19873  fval= 1.499646
## Iteration 16  fbest= 1.499646
##  lambda = 6.17981  fval= 2.268889
##  lambda = 61.7981  fval= 1.418034
## Iteration 18  fbest= 1.418034
##  lambda = 9.269714  fval= 1.203653
## Iteration 19  fbest= 1.203653
##  lambda = 1.390457  fval= 0.7529179
## Iteration 20  fbest= 0.7529179
##  lambda = 0.2085686  fval= 8.235142
##  lambda = 2.085686  fval= 1.485472
##  lambda = 20.85686  fval= 0.6414404
## Iteration 23  fbest= 0.6414404
##  lambda = 3.128529  fval= 0.6430264
##  lambda = 31.28529  fval= 0.5779681
## Iteration 25  fbest= 0.5779681
##  lambda = 4.692793  fval= 0.4683426
## Iteration 26  fbest= 0.4683426
##  lambda = 0.7039189  fval= 0.2656592
## Iteration 27  fbest= 0.2656592
##  lambda = 0.1055878  fval= 0.5121154
##  lambda = 1.055878  fval= 0.2218384
## Iteration 29  fbest= 0.2218384
##  lambda = 0.1583818  fval= 0.09357798
## Iteration 30  fbest= 0.09357798
##  lambda = 0.02375726  fval= 0.06471772
## Iteration 31  fbest= 0.06471772
##  lambda = 0.00356359  fval= 0.02374038
## Iteration 32  fbest= 0.02374038
##  lambda = 0.0005345384  fval= 0.01191497
## Iteration 33  fbest= 0.01191497
##  lambda = 8.018077e-05  fval= 0.006286775

```

```

## Iteration 34 fbest= 0.006286775
## lambda = 1.202711e-05 fval= 0.003343837
## Iteration 35 fbest= 0.003343837
## lambda = 1.804067e-06 fval= 0.001801541
## Iteration 36 fbest= 0.001801541
## lambda = 2.706101e-07 fval= 0.0009795606
## Iteration 37 fbest= 0.0009795606
## lambda = 4.059151e-08 fval= 0.0005369973
## Iteration 38 fbest= 0.0005369973
## lambda = 6.088727e-09 fval= 0.0002963302
## Iteration 39 fbest= 0.0002963302
## lambda = 9.13309e-10 fval= 0.0001643687
## Iteration 40 fbest= 0.0001643687
## lambda = 1.369964e-10 fval= 9.153421e-05
## Iteration 41 fbest= 9.153421e-05
## lambda = 2.054945e-11 fval= 5.112784e-05
## Iteration 42 fbest= 5.112784e-05
## lambda = 3.082418e-12 fval= 2.862351e-05
## Iteration 43 fbest= 2.862351e-05
## lambda = 4.623627e-13 fval= 1.605228e-05
## Iteration 44 fbest= 1.605228e-05
## lambda = 6.935441e-14 fval= 9.013925e-06
## Iteration 45 fbest= 9.013925e-06
## lambda = 1.040316e-14 fval= 5.066584e-06
## Iteration 46 fbest= 5.066584e-06
## lambda = 1.560474e-15 fval= 2.849938e-06
## Iteration 47 fbest= 2.849938e-06
## lambda = 2.340711e-16 fval= 1.603966e-06
## Iteration 48 fbest= 1.603966e-06
## lambda = 3.511067e-17 fval= 9.030983e-07
## Iteration 49 fbest= 9.030983e-07
## lambda = 5.2666e-18 fval= 5.086395e-07
## Iteration 50 fbest= 5.086395e-07
## lambda = 7.8999e-19 fval= 2.865409e-07
## Iteration 51 fbest= 2.865409e-07
## lambda = 1.184985e-19 fval= 1.614506e-07
## Iteration 52 fbest= 1.614506e-07
## lambda = 1.777478e-20 fval= 9.098079e-08
## Iteration 53 fbest= 9.098079e-08
## lambda = 2.666216e-21 fval= 5.127467e-08
## Iteration 54 fbest= 5.127467e-08
## lambda = 3.999324e-22 fval= 2.889937e-08
## Iteration 55 fbest= 2.889937e-08
## lambda = 5.998987e-23 fval= 1.628914e-08
## Iteration 56 fbest= 1.628914e-08
## lambda = 8.99848e-24 fval= 9.181762e-09
## Iteration 57 fbest= 9.181762e-09
## lambda = 1.349772e-24 fval= 5.175683e-09
## Iteration 58 fbest= 5.175683e-09
## lambda = 2.024658e-25 fval= 2.917558e-09
## Iteration 59 fbest= 2.917558e-09
## lambda = 3.036987e-26 fval= 1.644671e-09
## Iteration 60 fbest= 1.644671e-09
## lambda = 4.555481e-27 fval= 9.271383e-10

```

```

## Iteration 61 fbest= 9.271383e-10
## lambda = 6.833221e-28 fval= 5.22654e-10
## Iteration 62 fbest= 5.22654e-10
## lambda = 1.024983e-28 fval= 2.94637e-10
## Iteration 63 fbest= 2.94637e-10
## lambda = 1.537475e-29 fval= 1.660974e-10
## Iteration 64 fbest= 1.660974e-10
## lambda = 2.306212e-30 fval= 9.363539e-11
## Iteration 65 fbest= 9.363539e-11
## lambda = 3.459318e-31 fval= 5.278599e-11
## Iteration 66 fbest= 5.278599e-11
## lambda = 5.188977e-32 fval= 2.975763e-11
## Iteration 67 fbest= 2.975763e-11
## lambda = 7.783466e-33 fval= 1.677563e-11
## Iteration 68 fbest= 1.677563e-11
## lambda = 1.16752e-33 fval= 9.457142e-12
## Iteration 69 fbest= 9.457142e-12
## lambda = 1.75128e-34 fval= 5.331402e-12
## Iteration 70 fbest= 5.331402e-12
## lambda = 2.62692e-35 fval= 3.005545e-12
## Iteration 71 fbest= 3.005545e-12
## lambda = 3.940379e-36 fval= 1.694359e-12
## Iteration 72 fbest= 1.694359e-12
## lambda = 5.910569e-37 fval= 9.551853e-13
## Iteration 73 fbest= 9.551853e-13
## lambda = 8.865854e-38 fval= 5.384806e-13
## Iteration 74 fbest= 5.384806e-13
## lambda = 1.329878e-38 fval= 3.035656e-13
## Iteration 75 fbest= 3.035656e-13
## lambda = 1.994817e-39 fval= 1.711335e-13
## Iteration 76 fbest= 1.711335e-13
## lambda = 2.992226e-40 fval= 9.647566e-14
## Iteration 77 fbest= 9.647566e-14
## lambda = 4.488338e-41 fval= 5.438767e-14
## Iteration 78 fbest= 5.438767e-14
## lambda = 6.732508e-42 fval= 3.066078e-14
## Iteration 79 fbest= 3.066078e-14
## lambda = 1.009876e-42 fval= 1.728486e-14
## Iteration 80 fbest= 1.728486e-14
## lambda = 1.514814e-43 fval= 9.744257e-15
## Iteration 81 fbest= 9.744257e-15
## lambda = 2.272221e-44 fval= 5.493277e-15
## Iteration 82 fbest= 5.493277e-15
## lambda = 3.408332e-45 fval= 3.096808e-15
## Iteration 83 fbest= 3.096808e-15
## lambda = 5.112498e-46 fval= 1.745811e-15
## Iteration 84 fbest= 1.745811e-15
## lambda = 7.668747e-47 fval= 9.841922e-16
## Iteration 85 fbest= 9.841922e-16
## lambda = 1.150312e-47 fval= 5.548336e-16
## Iteration 86 fbest= 5.548336e-16
## lambda = 1.725468e-48 fval= 3.127847e-16
## Iteration 87 fbest= 3.127847e-16
## lambda = 2.588202e-49 fval= 1.763309e-16

```

```

## Iteration 88 fbest= 1.763309e-16
## lambda = 3.882303e-50 fval= 9.940568e-17
## Iteration 89 fbest= 9.940568e-17
## lambda = 5.823455e-51 fval= 5.603947e-17
## Iteration 90 fbest= 5.603947e-17
## lambda = 8.735182e-52 fval= 3.159198e-17
## Iteration 91 fbest= 3.159198e-17
## lambda = 1.310277e-52 fval= 1.780983e-17
## Iteration 92 fbest= 1.780983e-17
## lambda = 1.965416e-53 fval= 1.00402e-17
## Iteration 93 fbest= 1.00402e-17
## lambda = 2.948124e-54 fval= 5.660116e-18
## Iteration 94 fbest= 5.660116e-18
## lambda = 4.422186e-55 fval= 3.190863e-18
## Iteration 95 fbest= 3.190863e-18
## lambda = 6.633279e-56 fval= 1.798833e-18
## Iteration 96 fbest= 1.798833e-18
## lambda = 9.949918e-57 fval= 1.014084e-18
## Iteration 97 fbest= 1.014084e-18
## lambda = 1.492488e-57 fval= 5.716848e-19
## Iteration 98 fbest= 5.716848e-19
## lambda = 2.238732e-58 fval= 3.222845e-19
## Iteration 99 fbest= 3.222845e-19
## lambda = 3.358097e-59 fval= 1.816864e-19
## Iteration 100 fbest= 1.816864e-19
## lambda = 5.037146e-60 fval= 1.024248e-19
## Iteration 101 fbest= 1.024248e-19
## lambda = 7.555719e-61 fval= 5.774147e-20
## Iteration 102 fbest= 5.774147e-20
## lambda = 1.133358e-61 fval= 3.255148e-20
## Iteration 103 fbest= 3.255148e-20
## lambda = 1.700037e-62 fval= 1.835074e-20
## Iteration 104 fbest= 1.835074e-20
## lambda = 2.550055e-63 fval= 1.034513e-20
## Iteration 105 fbest= 1.034513e-20
## lambda = 3.825083e-64 fval= 5.832015e-21
## Iteration 106 fbest= 5.832015e-21
## lambda = 5.737624e-65 fval= 3.287771e-21
## Iteration 107 fbest= 3.287771e-21
## lambda = 8.606437e-66 fval= 1.853462e-21
## Iteration 108 fbest= 1.853462e-21
## lambda = 1.290965e-66 fval= 1.044879e-21
## Iteration 109 fbest= 1.044879e-21
## lambda = 1.936448e-67 fval= 5.890438e-22
## Iteration 110 fbest= 5.890438e-22
## lambda = 2.904672e-68 fval= 3.320702e-22
## Iteration 111 fbest= 3.320702e-22
## lambda = 4.357009e-69 fval= 1.87203e-22
## Iteration 112 fbest= 1.87203e-22
## lambda = 6.535513e-70 fval= 1.055345e-22
## Iteration 113 fbest= 1.055345e-22
## lambda = 9.803269e-71 fval= 5.949423e-23
## Iteration 114 fbest= 5.949423e-23
## lambda = 1.47049e-71 fval= 3.353935e-23

```

```

## Iteration 115 fbest= 3.353935e-23
## lambda = 2.205736e-72 fval= 1.890777e-23
## Iteration 116 fbest= 1.890777e-23
## lambda = 3.308603e-73 fval= 1.065893e-23
## Iteration 117 fbest= 1.065893e-23
## lambda = 4.962905e-74 fval= 6.008859e-24
## Iteration 118 fbest= 6.008859e-24
## lambda = 7.444357e-75 fval= 3.387352e-24
## Iteration 119 fbest= 3.387352e-24
## lambda = 1.116654e-75 fval= 1.909542e-24
## Iteration 120 fbest= 1.909542e-24
## lambda = 1.67498e-76 fval= 1.076397e-24
## Iteration 121 fbest= 1.076397e-24
## lambda = 2.512471e-77 fval= 6.067524e-25
## Iteration 122 fbest= 6.067524e-25
## lambda = 3.768706e-78 fval= 3.420209e-25
## Iteration 123 fbest= 3.420209e-25
## lambda = 5.653059e-79 fval= 1.928249e-25
## Iteration 124 fbest= 1.928249e-25
## lambda = 8.479588e-80 fval= 1.087071e-25
## Iteration 125 fbest= 1.087071e-25
## lambda = 1.271938e-80 fval= 6.126275e-26
## Iteration 126 fbest= 6.126275e-26
## lambda = 1.907907e-81 fval= 3.452499e-26
## Iteration 127 fbest= 3.452499e-26
## lambda = 2.861861e-82 fval= 1.946164e-26
## Iteration 128 fbest= 1.946164e-26
## lambda = 4.292792e-83 fval= 1.097479e-26
## Iteration 129 fbest= 1.097479e-26
## lambda = 6.439187e-84 fval= 6.180836e-27
## Iteration 130 fbest= 6.180836e-27
## lambda = 9.658781e-85 fval= 3.482081e-27
## Iteration 131 fbest= 3.482081e-27
## lambda = 1.448817e-85 fval= 1.963573e-27
## Iteration 132 fbest= 1.963573e-27
## lambda = 2.173226e-86 fval= 1.106883e-27
## Iteration 133 fbest= 1.106883e-27
## lambda = 3.259839e-87 fval= 6.240013e-28
## Iteration 134 fbest= 6.240013e-28
## lambda = 4.889758e-88 fval= 3.491203e-28
## Iteration 135 fbest= 3.491203e-28
## lambda = 7.334637e-89 fval= 1.969194e-28
## Iteration 136 fbest= 1.969194e-28
## lambda = 1.100196e-89 fval= 1.112417e-28
## Iteration 137 fbest= 1.112417e-28
## lambda = 1.650293e-90 fval= 6.336772e-29
## Iteration 138 fbest= 6.336772e-29
## lambda = 2.47544e-91 fval= 3.717507e-29
## Iteration 139 fbest= 3.717507e-29
## lambda = 3.71316e-92 fval= 1.972152e-29
## Iteration 140 fbest= 1.972152e-29
## lambda = 5.56974e-93 fval= 1.109336e-29
## Iteration 141 fbest= 1.109336e-29
## lambda = 8.35461e-94 fval= 7.198356e-30

```



```

## Iteration 142 fbest= 7.198356e-30
## lambda = 1.253191e-94 fval= 3.5622e-30
## Iteration 143 fbest= 3.5622e-30
## lambda = 1.879787e-95 fval= 2.083086e-30
## Iteration 144 fbest= 2.083086e-30
## lambda = 2.819681e-96 fval= 1.232595e-30
## Iteration 145 fbest= 1.232595e-30
## lambda = 4.229521e-97 fval= 1.836567e-30
## lambda = 0.001220703 fval= 1.836567e-30
## lambda = 0.01220703 fval= 1.836567e-30
## lambda = 0.1220703 fval= 2.021456e-30
## lambda = 1.220703 fval= 2.230997e-30
## lambda = 12.20703 fval= 2.46519e-30
## Null step
## Finished

print(solorigm)

## $xs
## [1] 1 1
##
## $fv
## [1] 1.232595e-30
##
## $grd
## [1] -2.220446e-15 0.000000e+00
##
## $Hess
##      [,1] [,2]
## [1,] 800 -400
## [2,] -400 202
##
## $counts
## $counts$niter
## [1] 152
##
## $counts$nfn
## [1] 151
##
## $counts$ngr
## [1] 132
##
## $counts$nhess
## [1] 132

print(eigen(solorigm$Hess)$values)

## [1] 1000.400641 1.599359

cat("Start with 50 values of pi and scale factor 10\n")

## Start with 50 values of pi and scale factor 10
x0 <- rep(pi, 50)
sol50pi <- snewton(x0, genrosa.f, genrosa.g, genrosa.h, gs=10)

## trace = 1

```

```
## Initial function value = 22405.14
## nf= 1
## Iteration 1 :nf= 2
## Iteration 2 :nf= 3
## Iteration 3 :nf= 4
## Iteration 4 :nf= 5
## Iteration 5 :nf= 6
## Iteration 6 :nf= 7
## Iteration 7 :nf= 8
## Iteration 8 :nf= 9
## Iteration 9 :nf= 10
## Iteration 10 :nf= 11
## Iteration 11 :nf= 12
## Iteration 12 :nf= 13
## Iteration 13 :nf= 14
## Iteration 14 :nf= 15
## Iteration 15 :nf= 16
## Iteration 16 :nf= 17
## Iteration 17 :nf= 18
## Iteration 18 :nf= 19
## Iteration 19 :nf= 20
## Iteration 20 :nf= 21
## Iteration 21 :nf= 22
## Iteration 22 :nf= 23
## Iteration 23 :nf= 24
## Iteration 24 :nf= 25
## Iteration 25 :nf= 26
## Iteration 26 :nf= 27
## Iteration 27 :nf= 28
## Iteration 28 :nf= 29
## Iteration 29 :nf= 30
## Iteration 30 :nf= 31
## Iteration 31 :nf= 32
## Iteration 32 :nf= 33
## Iteration 33 :nf= 34
## Iteration 34 :nf= 35
## Iteration 35 :nf= 36
## Iteration 36 :nf= 37
## Iteration 37 :nf= 38
## Iteration 38 :nf= 39
## Iteration 39 :nf= 40
## Iteration 40 :nf= 41
## Iteration 41 :nf= 42
## Iteration 42 :nf= 43
## Iteration 43 :nf= 44
## Iteration 44 :nf= 45
## Iteration 45 :nf= 46
## Iteration 46 :nf= 47
## Iteration 47 :nf= 48
## Iteration 48 :nf= 49
## Iteration 49 :nf= 50
## Iteration 50 :nf= 51
## Iteration 51 :nf= 52
## Iteration 52 :nf= 53
```

```
## Iteration 53 :nf= 54
## Iteration 54 :nf= 55
## Iteration 55 :nf= 56
## Iteration 56 :nf= 57
## Iteration 57 :nf= 58
## Iteration 58 :nf= 59
## Iteration 59 :nf= 60
## Iteration 60 :nf= 61
## Iteration 61 :nf= 62
## Iteration 62 :nf= 63
## Iteration 63 :nf= 64
## Iteration 64 :nf= 65
## Iteration 65 :nf= 66
## Iteration 66 :nf= 67
## Iteration 67 :nf= 68
## Iteration 68 :nf= 69
## Iteration 69 :nf= 70
## Iteration 70 :nf= 71
## Iteration 71 :nf= 72
## Iteration 72 :nf= 73
## Iteration 73 :nf= 74
## Iteration 74 :nf= 75
## Iteration 75 :nf= 76
## Iteration 76 :nf= 77
## Iteration 77 :nf= 78
## Iteration 78 :nf= 79
## Iteration 79 :nf= 80
## Iteration 80 :nf= 81
## Iteration 81 :nf= 82
## Iteration 82 :nf= 83
## Iteration 83 :nf= 84
## Iteration 84 :nf= 85
## Iteration 85 :nf= 86
## Iteration 86 :nf= 87
## Iteration 87 :nf= 88
## Iteration 88 :nf= 89
## Iteration 89 :nf= 90
## Iteration 90 :nf= 91
## Iteration 91 :nf= 92
## Iteration 92 :nf= 93
## Iteration 93 :nf= 94
## Iteration 94 :nf= 95
## Iteration 95 :nf= 96
## Iteration 96 :nf= 97
## Iteration 97 :nf= 98
## Iteration 98 :nf= 99
## Iteration 99 :nf= 100
## Iteration 100 :nf= 101
## Iteration 101 :nf= 102
## Iteration 102 :nf= 103
## Iteration 103 :nf= 104
## Iteration 104 :nf= 105
## Iteration 105 :nf= 106
## Iteration 106 :nf= 107
```

```

## Iteration 107 :nf= 108
## Iteration 108 :nf= 109
## Iteration 109 :nf= 110
## Iteration 110 :nf= 111
## Iteration 111 :nf= 112
## Iteration 112 :nf= 113
## Iteration 113 :nf= 114
## Iteration 114 :nf= 115
## Iteration 115 :nf= 116
## Iteration 116 :nf= 117
## Iteration 117 :nf= 118
## Iteration 118 :nf= 119
## Iteration 119 :nf= 120
## Iteration 120 :nf= 121
## Iteration 121 :nf= 122
## Iteration 122 :nf= 123
## Iteration 123 :nf= 124
## Iteration 124 :nf= 125
## Iteration 125 :nf= 126
## Iteration 126 :nf= 127
## Iteration 127 :nf= 128
## Iteration 128 :nf= 129
## Iteration 129 :nf= 130
## Iteration 130 :nf= 131
## Iteration 131 :nf= 132
## Iteration 132 :nf= 133
## Iteration 133 :nf= 134
## Iteration 134 :nf= 135
## Iteration 135 :nf= 136
## Iteration 136 :nf= 137
## Iteration 137 :nf= 138
## Iteration 138 :nf= 139
## Iteration 139 :nf= 140
## Iteration 140 :nf= 141
## Iteration 141 :nf= 142
## Iteration 142 :nf= 143
## Iteration 143 :nf= 144
## Iteration 144 :nf= 145
## Iteration 145 :No progress before linesearch!
## No progress in linesearch!

```

```
print(sol50pi)
```

```

## $par
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [36] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
##
## $value
## [1] 6.108742e-29
##
## $grad
## [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [6] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [11] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [16] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00

```

```

## [21] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [26] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [31] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [36] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [41] 0.000000e+00 0.000000e+00 -8.881784e-15 4.884981e-15 9.769963e-15
## [46] -1.199041e-14 7.549517e-15 -1.154632e-14 1.332268e-14 4.440892e-15
##
## $Hess
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
## [1,] 80 -40 0 0 0 0 0 0 0 0 0 0 0
## [2,] -40 102 -40 0 0 0 0 0 0 0 0 0 0
## [3,] 0 -40 102 -40 0 0 0 0 0 0 0 0 0
## [4,] 0 0 -40 102 -40 0 0 0 0 0 0 0 0
## [5,] 0 0 0 -40 102 -40 0 0 0 0 0 0 0
## [6,] 0 0 0 0 -40 102 -40 0 0 0 0 0 0
## [7,] 0 0 0 0 0 -40 102 -40 0 0 0 0 0
## [8,] 0 0 0 0 0 0 -40 102 -40 0 0 0 0
## [9,] 0 0 0 0 0 0 0 -40 102 -40 0 0 0
## [10,] 0 0 0 0 0 0 0 0 -40 102 -40 0 0
## [11,] 0 0 0 0 0 0 0 0 0 -40 102 -40 0
## [12,] 0 0 0 0 0 0 0 0 0 0 -40 102 -40
## [13,] 0 0 0 0 0 0 0 0 0 0 0 -40 102
## [14,] 0 0 0 0 0 0 0 0 0 0 0 0 -40
## [15,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [16,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [17,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [18,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [19,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [20,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [21,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [22,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [23,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [24,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [25,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [26,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [27,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [28,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [29,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [30,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [31,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [32,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [33,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [34,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [35,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [36,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [37,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [38,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [39,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [40,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [41,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [42,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [43,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [44,] 0 0 0 0 0 0 0 0 0 0 0 0 0
## [45,] 0 0 0 0 0 0 0 0 0 0 0 0 0

```

## [46,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [47,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [48,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [49,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [50,]	0	0	0	0	0	0	0	0	0	0	0	0	0
##	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]		
## [1,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [2,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [3,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [4,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [5,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [6,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [7,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [8,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [9,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [10,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [11,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [12,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [13,]	-40	0	0	0	0	0	0	0	0	0	0	0	0
## [14,]	102	-40	0	0	0	0	0	0	0	0	0	0	0
## [15,]	-40	102	-40	0	0	0	0	0	0	0	0	0	0
## [16,]	0	-40	102	-40	0	0	0	0	0	0	0	0	0
## [17,]	0	0	-40	102	-40	0	0	0	0	0	0	0	0
## [18,]	0	0	0	-40	102	-40	0	0	0	0	0	0	0
## [19,]	0	0	0	0	-40	102	-40	0	0	0	0	0	0
## [20,]	0	0	0	0	0	-40	102	-40	0	0	0	0	0
## [21,]	0	0	0	0	0	0	-40	102	-40	0	0	0	0
## [22,]	0	0	0	0	0	0	0	-40	102	-40	0	0	0
## [23,]	0	0	0	0	0	0	0	0	-40	102	-40	0	0
## [24,]	0	0	0	0	0	0	0	0	0	-40	102	-40	0
## [25,]	0	0	0	0	0	0	0	0	0	0	0	-40	0
## [26,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [27,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [28,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [29,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [30,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [31,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [32,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [33,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [34,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [35,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [36,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [37,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [38,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [39,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [40,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [41,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [42,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [43,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [44,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [45,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [46,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [47,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [48,]	0	0	0	0	0	0	0	0	0	0	0	0	0

##	[49,]	0	0	0	0	0	0	0	0	0	0	0
##	[50,]	0	0	0	0	0	0	0	0	0	0	0
##		[,25]	[,26]	[,27]	[,28]	[,29]	[,30]	[,31]	[,32]	[,33]	[,34]	[,35]
##	[1,]	0	0	0	0	0	0	0	0	0	0	0
##	[2,]	0	0	0	0	0	0	0	0	0	0	0
##	[3,]	0	0	0	0	0	0	0	0	0	0	0
##	[4,]	0	0	0	0	0	0	0	0	0	0	0
##	[5,]	0	0	0	0	0	0	0	0	0	0	0
##	[6,]	0	0	0	0	0	0	0	0	0	0	0
##	[7,]	0	0	0	0	0	0	0	0	0	0	0
##	[8,]	0	0	0	0	0	0	0	0	0	0	0
##	[9,]	0	0	0	0	0	0	0	0	0	0	0
##	[10,]	0	0	0	0	0	0	0	0	0	0	0
##	[11,]	0	0	0	0	0	0	0	0	0	0	0
##	[12,]	0	0	0	0	0	0	0	0	0	0	0
##	[13,]	0	0	0	0	0	0	0	0	0	0	0
##	[14,]	0	0	0	0	0	0	0	0	0	0	0
##	[15,]	0	0	0	0	0	0	0	0	0	0	0
##	[16,]	0	0	0	0	0	0	0	0	0	0	0
##	[17,]	0	0	0	0	0	0	0	0	0	0	0
##	[18,]	0	0	0	0	0	0	0	0	0	0	0
##	[19,]	0	0	0	0	0	0	0	0	0	0	0
##	[20,]	0	0	0	0	0	0	0	0	0	0	0
##	[21,]	0	0	0	0	0	0	0	0	0	0	0
##	[22,]	0	0	0	0	0	0	0	0	0	0	0
##	[23,]	0	0	0	0	0	0	0	0	0	0	0
##	[24,]	-40	0	0	0	0	0	0	0	0	0	0
##	[25,]	102	-40	0	0	0	0	0	0	0	0	0
##	[26,]	-40	102	-40	0	0	0	0	0	0	0	0
##	[27,]	0	-40	102	-40	0	0	0	0	0	0	0
##	[28,]	0	0	-40	102	-40	0	0	0	0	0	0
##	[29,]	0	0	0	-40	102	-40	0	0	0	0	0
##	[30,]	0	0	0	0	-40	102	-40	0	0	0	0
##	[31,]	0	0	0	0	0	-40	102	-40	0	0	0
##	[32,]	0	0	0	0	0	0	-40	102	-40	0	0
##	[33,]	0	0	0	0	0	0	0	-40	102	-40	0
##	[34,]	0	0	0	0	0	0	0	0	-40	102	-40
##	[35,]	0	0	0	0	0	0	0	0	0	-40	102
##	[36,]	0	0	0	0	0	0	0	0	0	0	-40
##	[37,]	0	0	0	0	0	0	0	0	0	0	0
##	[38,]	0	0	0	0	0	0	0	0	0	0	0
##	[39,]	0	0	0	0	0	0	0	0	0	0	0
##	[40,]	0	0	0	0	0	0	0	0	0	0	0
##	[41,]	0	0	0	0	0	0	0	0	0	0	0
##	[42,]	0	0	0	0	0	0	0	0	0	0	0
##	[43,]	0	0	0	0	0	0	0	0	0	0	0
##	[44,]	0	0	0	0	0	0	0	0	0	0	0
##	[45,]	0	0	0	0	0	0	0	0	0	0	0
##	[46,]	0	0	0	0	0	0	0	0	0	0	0
##	[47,]	0	0	0	0	0	0	0	0	0	0	0
##	[48,]	0	0	0	0	0	0	0	0	0	0	0
##	[49,]	0	0	0	0	0	0	0	0	0	0	0
##	[50,]	0	0	0	0	0	0	0	0	0	0	0
##		[,36]	[,37]	[,38]	[,39]	[,40]	[,41]	[,42]	[,43]	[,44]	[,45]	[,46]

```

## [1,] 0 0 0 0 0 0 0 0 0 0 0 0
## [2,] 0 0 0 0 0 0 0 0 0 0 0 0
## [3,] 0 0 0 0 0 0 0 0 0 0 0 0
## [4,] 0 0 0 0 0 0 0 0 0 0 0 0
## [5,] 0 0 0 0 0 0 0 0 0 0 0 0
## [6,] 0 0 0 0 0 0 0 0 0 0 0 0
## [7,] 0 0 0 0 0 0 0 0 0 0 0 0
## [8,] 0 0 0 0 0 0 0 0 0 0 0 0
## [9,] 0 0 0 0 0 0 0 0 0 0 0 0
## [10,] 0 0 0 0 0 0 0 0 0 0 0 0
## [11,] 0 0 0 0 0 0 0 0 0 0 0 0
## [12,] 0 0 0 0 0 0 0 0 0 0 0 0
## [13,] 0 0 0 0 0 0 0 0 0 0 0 0
## [14,] 0 0 0 0 0 0 0 0 0 0 0 0
## [15,] 0 0 0 0 0 0 0 0 0 0 0 0
## [16,] 0 0 0 0 0 0 0 0 0 0 0 0
## [17,] 0 0 0 0 0 0 0 0 0 0 0 0
## [18,] 0 0 0 0 0 0 0 0 0 0 0 0
## [19,] 0 0 0 0 0 0 0 0 0 0 0 0
## [20,] 0 0 0 0 0 0 0 0 0 0 0 0
## [21,] 0 0 0 0 0 0 0 0 0 0 0 0
## [22,] 0 0 0 0 0 0 0 0 0 0 0 0
## [23,] 0 0 0 0 0 0 0 0 0 0 0 0
## [24,] 0 0 0 0 0 0 0 0 0 0 0 0
## [25,] 0 0 0 0 0 0 0 0 0 0 0 0
## [26,] 0 0 0 0 0 0 0 0 0 0 0 0
## [27,] 0 0 0 0 0 0 0 0 0 0 0 0
## [28,] 0 0 0 0 0 0 0 0 0 0 0 0
## [29,] 0 0 0 0 0 0 0 0 0 0 0 0
## [30,] 0 0 0 0 0 0 0 0 0 0 0 0
## [31,] 0 0 0 0 0 0 0 0 0 0 0 0
## [32,] 0 0 0 0 0 0 0 0 0 0 0 0
## [33,] 0 0 0 0 0 0 0 0 0 0 0 0
## [34,] 0 0 0 0 0 0 0 0 0 0 0 0
## [35,] -40 0 0 0 0 0 0 0 0 0 0 0
## [36,] 102 -40 0 0 0 0 0 0 0 0 0 0
## [37,] -40 102 -40 0 0 0 0 0 0 0 0 0
## [38,] 0 -40 102 -40 0 0 0 0 0 0 0 0
## [39,] 0 0 -40 102 -40 0 0 0 0 0 0 0
## [40,] 0 0 0 -40 102 -40 0 0 0 0 0 0
## [41,] 0 0 0 0 -40 102 -40 0 0 0 0 0
## [42,] 0 0 0 0 0 -40 102 -40 0 0 0 0
## [43,] 0 0 0 0 0 0 -40 102 -40 0 0 0
## [44,] 0 0 0 0 0 0 0 -40 102 -40 0 0
## [45,] 0 0 0 0 0 0 0 0 -40 102 -40 0
## [46,] 0 0 0 0 0 0 0 0 0 -40 102 0
## [47,] 0 0 0 0 0 0 0 0 0 0 -40 0
## [48,] 0 0 0 0 0 0 0 0 0 0 0 0
## [49,] 0 0 0 0 0 0 0 0 0 0 0 0
## [50,] 0 0 0 0 0 0 0 0 0 0 0 0
##      [,47] [,48] [,49] [,50]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0

```



```

## [4,]      0      0      0      0
## [5,]      0      0      0      0
## [6,]      0      0      0      0
## [7,]      0      0      0      0
## [8,]      0      0      0      0
## [9,]      0      0      0      0
## [10,]     0      0      0      0
## [11,]     0      0      0      0
## [12,]     0      0      0      0
## [13,]     0      0      0      0
## [14,]     0      0      0      0
## [15,]     0      0      0      0
## [16,]     0      0      0      0
## [17,]     0      0      0      0
## [18,]     0      0      0      0
## [19,]     0      0      0      0
## [20,]     0      0      0      0
## [21,]     0      0      0      0
## [22,]     0      0      0      0
## [23,]     0      0      0      0
## [24,]     0      0      0      0
## [25,]     0      0      0      0
## [26,]     0      0      0      0
## [27,]     0      0      0      0
## [28,]     0      0      0      0
## [29,]     0      0      0      0
## [30,]     0      0      0      0
## [31,]     0      0      0      0
## [32,]     0      0      0      0
## [33,]     0      0      0      0
## [34,]     0      0      0      0
## [35,]     0      0      0      0
## [36,]     0      0      0      0
## [37,]     0      0      0      0
## [38,]     0      0      0      0
## [39,]     0      0      0      0
## [40,]     0      0      0      0
## [41,]     0      0      0      0
## [42,]     0      0      0      0
## [43,]     0      0      0      0
## [44,]     0      0      0      0
## [45,]     0      0      0      0
## [46,]    -40      0      0      0
## [47,]    102    -40      0      0
## [48,]    -40    102    -40      0
## [49,]      0    -40    102    -40
## [50,]      0      0    -40     22
##
## $counts
## $counts$niter
## [1] 145
##
## $counts$nfn
## [1] 146

```

```

##
## $counts$ngr
## [1] 145
##
## $counts$nhess
## [1] 145
##
##
## $convcode
## [1] 93

print(eigen(sol50pi$Hess)$values)

## [1] 181.84200 181.36863 180.58176 179.48449 178.08116 176.37730 174.37964
## [8] 172.09607 169.53560 166.70834 163.62545 160.29911 156.74243 152.96948
## [15] 148.99513 144.83509 140.50578 136.02429 131.40832 126.67610 121.84632
## [22] 116.93804 111.97066 106.96381 101.93725 96.91085 91.90447 86.93791
## [29] 82.03080 77.20253 72.47223 67.85859 63.37989 59.05387 54.89766
## [36] 50.92776 47.15992 43.60907 40.28933 37.21385 34.39481 31.84332
## [43] 29.56937 27.58175 25.88797 24.49427 23.40556 22.62547 22.15648
## [50] 2.00000

sol50pim <- snewtonm(x0, genrosa.f, genrosa.g, genrosa.h, gs=10)

## trace = 1
## Start snewtonm f0= 22405.14 at [1] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [8] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [15] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [22] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [29] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [36] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [43] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [50] 3.141593
## Iteration 1 fbest= 22405.14
## lambda = 0.0001220703 fval= 4133.339
## Iteration 2 fbest= 4133.339
## lambda = 1.831055e-05 fval= 731.8323
## Iteration 3 fbest= 731.8323
## lambda = 2.746582e-06 fval= 115.5295
## Iteration 4 fbest= 115.5295
## lambda = 4.119873e-07 fval= 16.41807
## Iteration 5 fbest= 16.41807
## lambda = 6.17981e-08 fval= 4.728525
## Iteration 6 fbest= 4.728525
## lambda = 9.269714e-09 fval= 3.909583
## Iteration 7 fbest= 3.909583
## lambda = 1.390457e-09 fval= 3.643291
## Iteration 8 fbest= 3.643291
## lambda = 2.085686e-10 fval= 3.385992
## Iteration 9 fbest= 3.385992
## lambda = 3.128529e-11 fval= 3.134871
## Iteration 10 fbest= 3.134871
## lambda = 4.692793e-12 fval= 2.89029
## Iteration 11 fbest= 2.89029
## lambda = 7.039189e-13 fval= 2.652637

```

```

## Iteration 12 fbest= 2.652637
## lambda = 1.055878e-13 fval= 2.42233
## Iteration 13 fbest= 2.42233
## lambda = 1.583818e-14 fval= 2.199815
## Iteration 14 fbest= 2.199815
## lambda = 2.375726e-15 fval= 1.98557
## Iteration 15 fbest= 1.98557
## lambda = 3.56359e-16 fval= 1.780102
## Iteration 16 fbest= 1.780102
## lambda = 5.345384e-17 fval= 1.583946
## Iteration 17 fbest= 1.583946
## lambda = 8.018077e-18 fval= 1.397661
## Iteration 18 fbest= 1.397661
## lambda = 1.202711e-18 fval= 1.221824
## Iteration 19 fbest= 1.221824
## lambda = 1.804067e-19 fval= 1.057021
## Iteration 20 fbest= 1.057021
## lambda = 2.706101e-20 fval= 0.9038294
## Iteration 21 fbest= 0.9038294
## lambda = 4.059151e-21 fval= 0.7627985
## Iteration 22 fbest= 0.7627985
## lambda = 6.088727e-22 fval= 0.6344206
## Iteration 23 fbest= 0.6344206
## lambda = 9.13309e-23 fval= 0.5190938
## Iteration 24 fbest= 0.5190938
## lambda = 1.369964e-23 fval= 0.4170763
## Iteration 25 fbest= 0.4170763
## lambda = 2.054945e-24 fval= 0.3284352
## Iteration 26 fbest= 0.3284352
## lambda = 3.082418e-25 fval= 0.2529935
## Iteration 27 fbest= 0.2529935
## lambda = 4.623627e-26 fval= 0.1902851
## Iteration 28 fbest= 0.1902851
## lambda = 6.935441e-27 fval= 0.1395286
## Iteration 29 fbest= 0.1395286
## lambda = 1.040316e-27 fval= 0.09963431
## Iteration 30 fbest= 0.09963431
## lambda = 1.560474e-28 fval= 0.06925367
## Iteration 31 fbest= 0.06925367
## lambda = 2.340711e-29 fval= 0.0468708
## Iteration 32 fbest= 0.0468708
## lambda = 3.511067e-30 fval= 0.03092309
## Iteration 33 fbest= 0.03092309
## lambda = 5.2666e-31 fval= 0.01992538
## Iteration 34 fbest= 0.01992538
## lambda = 7.8999e-32 fval= 0.01257024
## Iteration 35 fbest= 0.01257024
## lambda = 1.184985e-32 fval= 0.007785817
## Iteration 36 fbest= 0.007785817
## lambda = 1.777478e-33 fval= 0.004748192
## Iteration 37 fbest= 0.004748192
## lambda = 2.666216e-34 fval= 0.002858877
## Iteration 38 fbest= 0.002858877
## lambda = 3.999324e-35 fval= 0.001703596

```

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## Iteration 39 fbest= 0.001703596
## lambda = 5.998987e-36 fval= 0.001006829
## Iteration 40 fbest= 0.001006829
## lambda = 8.99848e-37 fval= 0.0005911864
## Iteration 41 fbest= 0.0005911864
## lambda = 1.349772e-37 fval= 0.0003453769
## Iteration 42 fbest= 0.0003453769
## lambda = 2.024658e-38 fval= 0.0002009824
## Iteration 43 fbest= 0.0002009824
## lambda = 3.036987e-39 fval= 0.0001166031
## Iteration 44 fbest= 0.0001166031
## lambda = 4.555481e-40 fval= 6.749228e-05
## Iteration 45 fbest= 6.749228e-05
## lambda = 6.833221e-41 fval= 3.899665e-05
## Iteration 46 fbest= 3.899665e-05
## lambda = 1.024983e-41 fval= 2.250153e-05
## Iteration 47 fbest= 2.250153e-05
## lambda = 1.537475e-42 fval= 1.297025e-05
## Iteration 48 fbest= 1.297025e-05
## lambda = 2.306212e-43 fval= 7.470387e-06
## Iteration 49 fbest= 7.470387e-06
## lambda = 3.459318e-44 fval= 4.300099e-06
## Iteration 50 fbest= 4.300099e-06
## lambda = 5.188977e-45 fval= 2.474097e-06
## Iteration 51 fbest= 2.474097e-06
## lambda = 7.783466e-46 fval= 1.423001e-06
## Iteration 52 fbest= 1.423001e-06
## lambda = 1.16752e-46 fval= 8.182387e-07
## Iteration 53 fbest= 8.182387e-07
## lambda = 1.75128e-47 fval= 4.704014e-07
## Iteration 54 fbest= 4.704014e-07
## lambda = 2.62692e-48 fval= 2.703908e-07
## Iteration 55 fbest= 2.703908e-07
## lambda = 3.940379e-49 fval= 1.554052e-07
## Iteration 56 fbest= 1.554052e-07
## lambda = 5.910569e-50 fval= 8.93103e-08
## Iteration 57 fbest= 8.93103e-08
## lambda = 8.865854e-51 fval= 5.132265e-08
## Iteration 58 fbest= 5.132265e-08
## lambda = 1.329878e-51 fval= 2.949137e-08
## Iteration 59 fbest= 2.949137e-08
## lambda = 1.994817e-52 fval= 1.694589e-08
## Iteration 60 fbest= 1.694589e-08
## lambda = 2.992226e-53 fval= 9.736917e-09
## Iteration 61 fbest= 9.736917e-09
## lambda = 4.488338e-54 fval= 5.594601e-09
## Iteration 62 fbest= 5.594601e-09
## lambda = 6.732508e-55 fval= 3.214472e-09
## Iteration 63 fbest= 3.214472e-09
## lambda = 1.009876e-55 fval= 1.846906e-09
## Iteration 64 fbest= 1.846906e-09
## lambda = 1.514814e-56 fval= 1.061147e-09
## Iteration 65 fbest= 1.061147e-09
## lambda = 2.272221e-57 fval= 6.096822e-10

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## Iteration 66 fbest= 6.096822e-10
## lambda = 3.408332e-58 fval= 3.502911e-10
## Iteration 67 fbest= 3.502911e-10
## lambda = 5.112498e-59 fval= 2.012578e-10
## Iteration 68 fbest= 2.012578e-10
## lambda = 7.668747e-60 fval= 1.156312e-10
## Iteration 69 fbest= 1.156312e-10
## lambda = 1.150312e-60 fval= 6.643493e-11
## Iteration 70 fbest= 6.643493e-11
## lambda = 1.725468e-61 fval= 3.816955e-11
## Iteration 71 fbest= 3.816955e-11
## lambda = 2.588202e-62 fval= 2.192992e-11
## Iteration 72 fbest= 2.192992e-11
## lambda = 3.882303e-63 fval= 1.259959e-11
## Iteration 73 fbest= 1.259959e-11
## lambda = 5.823455e-64 fval= 7.238953e-12
## Iteration 74 fbest= 7.238953e-12
## lambda = 8.735182e-65 fval= 4.159055e-12
## Iteration 75 fbest= 4.159055e-12
## lambda = 1.310277e-65 fval= 2.389535e-12
## Iteration 76 fbest= 2.389535e-12
## lambda = 1.965416e-66 fval= 1.372878e-12
## Iteration 77 fbest= 1.372878e-12
## lambda = 2.948124e-67 fval= 7.887702e-13
## Iteration 78 fbest= 7.887702e-13
## lambda = 4.422186e-68 fval= 4.531781e-13
## Iteration 79 fbest= 4.531781e-13
## lambda = 6.633279e-69 fval= 2.603678e-13
## Iteration 80 fbest= 2.603678e-13
## lambda = 9.949918e-70 fval= 1.49591e-13
## Iteration 81 fbest= 1.49591e-13
## lambda = 1.492488e-70 fval= 8.594562e-14
## Iteration 82 fbest= 8.594562e-14
## lambda = 2.238732e-71 fval= 4.937896e-14
## Iteration 83 fbest= 4.937896e-14
## lambda = 3.358097e-72 fval= 2.837005e-14
## Iteration 84 fbest= 2.837005e-14
## lambda = 5.037146e-73 fval= 1.629965e-14
## Iteration 85 fbest= 1.629965e-14
## lambda = 7.555719e-74 fval= 9.364757e-15
## Iteration 86 fbest= 9.364757e-15
## lambda = 1.133358e-74 fval= 5.380401e-15
## Iteration 87 fbest= 5.380401e-15
## lambda = 1.700037e-75 fval= 3.091241e-15
## Iteration 88 fbest= 3.091241e-15
## lambda = 2.550055e-76 fval= 1.776033e-15
## Iteration 89 fbest= 1.776033e-15
## lambda = 3.825083e-77 fval= 1.020397e-15
## Iteration 90 fbest= 1.020397e-15
## lambda = 5.737624e-78 fval= 5.862559e-16
## Iteration 91 fbest= 5.862559e-16
## lambda = 8.606437e-79 fval= 3.368258e-16
## Iteration 92 fbest= 3.368258e-16
## lambda = 1.290965e-79 fval= 1.93519e-16

```

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## Iteration 93 fbest= 1.93519e-16
## lambda = 1.936448e-80 fval= 1.111838e-16
## Iteration 94 fbest= 1.111838e-16
## lambda = 2.904672e-81 fval= 6.387925e-17
## Iteration 95 fbest= 6.387925e-17
## lambda = 4.357009e-82 fval= 3.6701e-17
## Iteration 96 fbest= 3.6701e-17
## lambda = 6.535513e-83 fval= 2.108609e-17
## Iteration 97 fbest= 2.108609e-17
## lambda = 9.803269e-84 fval= 1.211474e-17
## Iteration 98 fbest= 1.211474e-17
## lambda = 1.47049e-84 fval= 6.96037e-18
## Iteration 99 fbest= 6.96037e-18
## lambda = 2.205736e-85 fval= 3.998991e-18
## Iteration 100 fbest= 3.998991e-18
## lambda = 3.308603e-86 fval= 2.297569e-18
## Iteration 101 fbest= 2.297569e-18
## lambda = 4.962905e-87 fval= 1.320039e-18
## Iteration 102 fbest= 1.320039e-18
## lambda = 7.444357e-88 fval= 7.584114e-19
## Iteration 103 fbest= 7.584114e-19
## lambda = 1.116654e-88 fval= 4.357355e-19
## Iteration 104 fbest= 4.357355e-19
## lambda = 1.67498e-89 fval= 2.503463e-19
## Iteration 105 fbest= 2.503463e-19
## lambda = 2.512471e-90 fval= 1.438332e-19
## Iteration 106 fbest= 1.438332e-19
## lambda = 3.768706e-91 fval= 8.263755e-20
## Iteration 107 fbest= 8.263755e-20
## lambda = 5.653059e-92 fval= 4.747835e-20
## Iteration 108 fbest= 4.747835e-20
## lambda = 8.479588e-93 fval= 2.727809e-20
## Iteration 109 fbest= 2.727809e-20
## lambda = 1.271938e-93 fval= 1.567227e-20
## Iteration 110 fbest= 1.567227e-20
## lambda = 1.907907e-94 fval= 9.004297e-21
## Iteration 111 fbest= 9.004297e-21
## lambda = 2.861861e-95 fval= 5.173295e-21
## Iteration 112 fbest= 5.173295e-21
## lambda = 4.292792e-96 fval= 2.972255e-21
## Iteration 113 fbest= 2.972255e-21
## lambda = 6.439187e-97 fval= 1.707674e-21
## Iteration 114 fbest= 1.707674e-21
## lambda = 9.658781e-98 fval= 9.811258e-22
## Iteration 115 fbest= 9.811258e-22
## lambda = 1.448817e-98 fval= 5.636946e-22
## Iteration 116 fbest= 5.636946e-22
## lambda = 2.173226e-99 fval= 3.238625e-22
## Iteration 117 fbest= 3.238625e-22
## lambda = 3.259839e-100 fval= 1.860713e-22
## Iteration 118 fbest= 1.860713e-22
## lambda = 4.889758e-101 fval= 1.069047e-22
## Iteration 119 fbest= 1.069047e-22
## lambda = 7.334637e-102 fval= 6.142062e-23

```

```

## Iteration 120 fbest= 6.142062e-23
## lambda = 1.100196e-102 fval= 3.528866e-23
## Iteration 121 fbest= 3.528866e-23
## lambda = 1.650293e-103 fval= 2.027416e-23
## Iteration 122 fbest= 2.027416e-23
## lambda = 2.47544e-104 fval= 1.164851e-23
## Iteration 123 fbest= 1.164851e-23
## lambda = 3.71316e-105 fval= 6.69274e-24
## Iteration 124 fbest= 6.69274e-24
## lambda = 5.56974e-106 fval= 3.84531e-24
## Iteration 125 fbest= 3.84531e-24
## lambda = 8.35461e-107 fval= 2.20946e-24
## Iteration 126 fbest= 2.20946e-24
## lambda = 1.253191e-107 fval= 1.26947e-24
## Iteration 127 fbest= 1.26947e-24
## lambda = 1.879787e-108 fval= 7.294277e-25
## Iteration 128 fbest= 7.294277e-25
## lambda = 2.819681e-109 fval= 4.191511e-25
## Iteration 129 fbest= 4.191511e-25
## lambda = 4.229521e-110 fval= 2.407963e-25
## Iteration 130 fbest= 2.407963e-25
## lambda = 6.344282e-111 fval= 1.383034e-25
## Iteration 131 fbest= 1.383034e-25
## lambda = 9.516423e-112 fval= 7.948656e-26
## Iteration 132 fbest= 7.948656e-26
## lambda = 1.427463e-112 fval= 4.565858e-26
## Iteration 133 fbest= 4.565858e-26
## lambda = 2.141195e-113 fval= 2.623978e-26
## Iteration 134 fbest= 2.623978e-26
## lambda = 3.211793e-114 fval= 1.509071e-26
## Iteration 135 fbest= 1.509071e-26
## lambda = 4.817689e-115 fval= 8.669335e-27
## Iteration 136 fbest= 8.669335e-27
## lambda = 7.226534e-116 fval= 4.973768e-27
## Iteration 137 fbest= 4.973768e-27
## lambda = 1.08398e-116 fval= 2.859917e-27
## Iteration 138 fbest= 2.859917e-27
## lambda = 1.62597e-117 fval= 1.644134e-27
## Iteration 139 fbest= 1.644134e-27
## lambda = 2.438955e-118 fval= 9.493941e-28
## Iteration 140 fbest= 9.493941e-28
## lambda = 3.658433e-119 fval= 5.466806e-28
## Iteration 141 fbest= 5.466806e-28
## lambda = 5.487649e-120 fval= 3.137694e-28
## Iteration 142 fbest= 3.137694e-28
## lambda = 8.231473e-121 fval= 1.81931e-28
## Iteration 143 fbest= 1.81931e-28
## lambda = 1.234721e-121 fval= 1.047706e-28
## Iteration 144 fbest= 1.047706e-28
## lambda = 1.852082e-122 fval= 6.108742e-29
## Iteration 145 fbest= 6.108742e-29
## lambda = 2.778122e-123 fval= 3.604108e-29
## Iteration 146 fbest= 3.604108e-29
## lambda = 4.167183e-124 fval= 1.991874e-29

```

```

## Iteration 147 fbest= 1.991874e-29
## lambda = 6.250775e-125 fval= 1.311481e-29
## Iteration 148 fbest= 1.311481e-29
## lambda = 9.376163e-126 fval= 7.937913e-30
## Iteration 149 fbest= 7.937913e-30
## lambda = 1.406424e-126 fval= 4.930381e-30
## Iteration 150 fbest= 4.930381e-30
## lambda = 2.109637e-127 fval= 4.289431e-30
## Iteration 151 fbest= 4.289431e-30
## lambda = 3.164455e-128 fval= 1.528418e-30
## Iteration 152 fbest= 1.528418e-30
## lambda = 4.746682e-129 fval= 2.021456e-30
## lambda = 0.001220703 fval= 2.021456e-30
## lambda = 0.01220703 fval= 2.021456e-30
## lambda = 0.1220703 fval= 2.021456e-30
## lambda = 1.220703 fval= 2.169367e-30
## Null step
## Finished

```

```
print(sol50pim)
```

```

## $xs
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [36] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
##
## $fv
## [1] 1.528418e-30
##
## $grd
## [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [6] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [11] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [16] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [21] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [26] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [31] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [36] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [41] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
## [46] -8.881784e-15 4.884981e-15 8.881784e-16 1.776357e-15 0.000000e+00
##
## $Hess
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
## [1,] 80 -40 0 0 0 0 0 0 0 0 0 0 0
## [2,] -40 102 -40 0 0 0 0 0 0 0 0 0 0
## [3,] 0 -40 102 -40 0 0 0 0 0 0 0 0 0
## [4,] 0 0 -40 102 -40 0 0 0 0 0 0 0 0
## [5,] 0 0 0 -40 102 -40 0 0 0 0 0 0 0
## [6,] 0 0 0 0 -40 102 -40 0 0 0 0 0 0
## [7,] 0 0 0 0 0 -40 102 -40 0 0 0 0 0
## [8,] 0 0 0 0 0 0 -40 102 -40 0 0 0 0
## [9,] 0 0 0 0 0 0 0 -40 102 -40 0 0 0
## [10,] 0 0 0 0 0 0 0 0 -40 102 -40 0 0
## [11,] 0 0 0 0 0 0 0 0 0 -40 102 -40 0
## [12,] 0 0 0 0 0 0 0 0 0 0 -40 102 -40
## [13,] 0 0 0 0 0 0 0 0 0 0 0 -40 102

```



## [14,]	0	0	0	0	0	0	0	0	0	0	0	0	-40
## [15,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [16,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [17,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [18,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [19,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [20,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [21,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [22,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [23,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [24,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [25,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [26,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [27,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [28,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [29,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [30,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [31,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [32,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [33,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [34,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [35,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [36,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [37,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [38,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [39,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [40,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [41,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [42,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [43,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [44,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [45,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [46,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [47,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [48,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [49,]	0	0	0	0	0	0	0	0	0	0	0	0	0
## [50,]	0	0	0	0	0	0	0	0	0	0	0	0	0
##	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]		
## [1,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [2,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [3,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [4,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [5,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [6,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [7,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [8,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [9,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [10,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [11,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [12,]	0	0	0	0	0	0	0	0	0	0	0	0	
## [13,]	-40	0	0	0	0	0	0	0	0	0	0	0	
## [14,]	102	-40	0	0	0	0	0	0	0	0	0	0	
## [15,]	-40	102	-40	0	0	0	0	0	0	0	0	0	
## [16,]	0	-40	102	-40	0	0	0	0	0	0	0	0	

## [17,]	0	0	-40	102	-40	0	0	0	0	0	0
## [18,]	0	0	0	-40	102	-40	0	0	0	0	0
## [19,]	0	0	0	0	-40	102	-40	0	0	0	0
## [20,]	0	0	0	0	0	-40	102	-40	0	0	0
## [21,]	0	0	0	0	0	0	-40	102	-40	0	0
## [22,]	0	0	0	0	0	0	0	-40	102	-40	0
## [23,]	0	0	0	0	0	0	0	0	-40	102	-40
## [24,]	0	0	0	0	0	0	0	0	0	-40	102
## [25,]	0	0	0	0	0	0	0	0	0	0	-40
## [26,]	0	0	0	0	0	0	0	0	0	0	0
## [27,]	0	0	0	0	0	0	0	0	0	0	0
## [28,]	0	0	0	0	0	0	0	0	0	0	0
## [29,]	0	0	0	0	0	0	0	0	0	0	0
## [30,]	0	0	0	0	0	0	0	0	0	0	0
## [31,]	0	0	0	0	0	0	0	0	0	0	0
## [32,]	0	0	0	0	0	0	0	0	0	0	0
## [33,]	0	0	0	0	0	0	0	0	0	0	0
## [34,]	0	0	0	0	0	0	0	0	0	0	0
## [35,]	0	0	0	0	0	0	0	0	0	0	0
## [36,]	0	0	0	0	0	0	0	0	0	0	0
## [37,]	0	0	0	0	0	0	0	0	0	0	0
## [38,]	0	0	0	0	0	0	0	0	0	0	0
## [39,]	0	0	0	0	0	0	0	0	0	0	0
## [40,]	0	0	0	0	0	0	0	0	0	0	0
## [41,]	0	0	0	0	0	0	0	0	0	0	0
## [42,]	0	0	0	0	0	0	0	0	0	0	0
## [43,]	0	0	0	0	0	0	0	0	0	0	0
## [44,]	0	0	0	0	0	0	0	0	0	0	0
## [45,]	0	0	0	0	0	0	0	0	0	0	0
## [46,]	0	0	0	0	0	0	0	0	0	0	0
## [47,]	0	0	0	0	0	0	0	0	0	0	0
## [48,]	0	0	0	0	0	0	0	0	0	0	0
## [49,]	0	0	0	0	0	0	0	0	0	0	0
## [50,]	0	0	0	0	0	0	0	0	0	0	0
##	[,25]	[,26]	[,27]	[,28]	[,29]	[,30]	[,31]	[,32]	[,33]	[,34]	[,35]
## [1,]	0	0	0	0	0	0	0	0	0	0	0
## [2,]	0	0	0	0	0	0	0	0	0	0	0
## [3,]	0	0	0	0	0	0	0	0	0	0	0
## [4,]	0	0	0	0	0	0	0	0	0	0	0
## [5,]	0	0	0	0	0	0	0	0	0	0	0
## [6,]	0	0	0	0	0	0	0	0	0	0	0
## [7,]	0	0	0	0	0	0	0	0	0	0	0
## [8,]	0	0	0	0	0	0	0	0	0	0	0
## [9,]	0	0	0	0	0	0	0	0	0	0	0
## [10,]	0	0	0	0	0	0	0	0	0	0	0
## [11,]	0	0	0	0	0	0	0	0	0	0	0
## [12,]	0	0	0	0	0	0	0	0	0	0	0
## [13,]	0	0	0	0	0	0	0	0	0	0	0
## [14,]	0	0	0	0	0	0	0	0	0	0	0
## [15,]	0	0	0	0	0	0	0	0	0	0	0
## [16,]	0	0	0	0	0	0	0	0	0	0	0
## [17,]	0	0	0	0	0	0	0	0	0	0	0
## [18,]	0	0	0	0	0	0	0	0	0	0	0
## [19,]	0	0	0	0	0	0	0	0	0	0	0

## [20,]	0	0	0	0	0	0	0	0	0	0	0
## [21,]	0	0	0	0	0	0	0	0	0	0	0
## [22,]	0	0	0	0	0	0	0	0	0	0	0
## [23,]	0	0	0	0	0	0	0	0	0	0	0
## [24,]	-40	0	0	0	0	0	0	0	0	0	0
## [25,]	102	-40	0	0	0	0	0	0	0	0	0
## [26,]	-40	102	-40	0	0	0	0	0	0	0	0
## [27,]	0	-40	102	-40	0	0	0	0	0	0	0
## [28,]	0	0	-40	102	-40	0	0	0	0	0	0
## [29,]	0	0	0	-40	102	-40	0	0	0	0	0
## [30,]	0	0	0	0	-40	102	-40	0	0	0	0
## [31,]	0	0	0	0	0	-40	102	-40	0	0	0
## [32,]	0	0	0	0	0	0	-40	102	-40	0	0
## [33,]	0	0	0	0	0	0	0	-40	102	-40	0
## [34,]	0	0	0	0	0	0	0	0	-40	102	-40
## [35,]	0	0	0	0	0	0	0	0	0	-40	102
## [36,]	0	0	0	0	0	0	0	0	0	0	-40
## [37,]	0	0	0	0	0	0	0	0	0	0	0
## [38,]	0	0	0	0	0	0	0	0	0	0	0
## [39,]	0	0	0	0	0	0	0	0	0	0	0
## [40,]	0	0	0	0	0	0	0	0	0	0	0
## [41,]	0	0	0	0	0	0	0	0	0	0	0
## [42,]	0	0	0	0	0	0	0	0	0	0	0
## [43,]	0	0	0	0	0	0	0	0	0	0	0
## [44,]	0	0	0	0	0	0	0	0	0	0	0
## [45,]	0	0	0	0	0	0	0	0	0	0	0
## [46,]	0	0	0	0	0	0	0	0	0	0	0
## [47,]	0	0	0	0	0	0	0	0	0	0	0
## [48,]	0	0	0	0	0	0	0	0	0	0	0
## [49,]	0	0	0	0	0	0	0	0	0	0	0
## [50,]	0	0	0	0	0	0	0	0	0	0	0
##	[,36]	[,37]	[,38]	[,39]	[,40]	[,41]	[,42]	[,43]	[,44]	[,45]	[,46]
## [1,]	0	0	0	0	0	0	0	0	0	0	0
## [2,]	0	0	0	0	0	0	0	0	0	0	0
## [3,]	0	0	0	0	0	0	0	0	0	0	0
## [4,]	0	0	0	0	0	0	0	0	0	0	0
## [5,]	0	0	0	0	0	0	0	0	0	0	0
## [6,]	0	0	0	0	0	0	0	0	0	0	0
## [7,]	0	0	0	0	0	0	0	0	0	0	0
## [8,]	0	0	0	0	0	0	0	0	0	0	0
## [9,]	0	0	0	0	0	0	0	0	0	0	0
## [10,]	0	0	0	0	0	0	0	0	0	0	0
## [11,]	0	0	0	0	0	0	0	0	0	0	0
## [12,]	0	0	0	0	0	0	0	0	0	0	0
## [13,]	0	0	0	0	0	0	0	0	0	0	0
## [14,]	0	0	0	0	0	0	0	0	0	0	0
## [15,]	0	0	0	0	0	0	0	0	0	0	0
## [16,]	0	0	0	0	0	0	0	0	0	0	0
## [17,]	0	0	0	0	0	0	0	0	0	0	0
## [18,]	0	0	0	0	0	0	0	0	0	0	0
## [19,]	0	0	0	0	0	0	0	0	0	0	0
## [20,]	0	0	0	0	0	0	0	0	0	0	0
## [21,]	0	0	0	0	0	0	0	0	0	0	0
## [22,]	0	0	0	0	0	0	0	0	0	0	0

```

## [23,] 0 0 0 0 0 0 0 0 0 0 0 0
## [24,] 0 0 0 0 0 0 0 0 0 0 0 0
## [25,] 0 0 0 0 0 0 0 0 0 0 0 0
## [26,] 0 0 0 0 0 0 0 0 0 0 0 0
## [27,] 0 0 0 0 0 0 0 0 0 0 0 0
## [28,] 0 0 0 0 0 0 0 0 0 0 0 0
## [29,] 0 0 0 0 0 0 0 0 0 0 0 0
## [30,] 0 0 0 0 0 0 0 0 0 0 0 0
## [31,] 0 0 0 0 0 0 0 0 0 0 0 0
## [32,] 0 0 0 0 0 0 0 0 0 0 0 0
## [33,] 0 0 0 0 0 0 0 0 0 0 0 0
## [34,] 0 0 0 0 0 0 0 0 0 0 0 0
## [35,] -40 0 0 0 0 0 0 0 0 0 0 0
## [36,] 102 -40 0 0 0 0 0 0 0 0 0 0
## [37,] -40 102 -40 0 0 0 0 0 0 0 0 0
## [38,] 0 -40 102 -40 0 0 0 0 0 0 0 0
## [39,] 0 0 -40 102 -40 0 0 0 0 0 0 0
## [40,] 0 0 0 -40 102 -40 0 0 0 0 0 0
## [41,] 0 0 0 0 -40 102 -40 0 0 0 0 0
## [42,] 0 0 0 0 0 -40 102 -40 0 0 0 0
## [43,] 0 0 0 0 0 0 -40 102 -40 0 0 0
## [44,] 0 0 0 0 0 0 0 -40 102 -40 0 0
## [45,] 0 0 0 0 0 0 0 0 -40 102 -40 0
## [46,] 0 0 0 0 0 0 0 0 0 -40 102 0
## [47,] 0 0 0 0 0 0 0 0 0 0 -40 102
## [48,] 0 0 0 0 0 0 0 0 0 0 0 0
## [49,] 0 0 0 0 0 0 0 0 0 0 0 0
## [50,] 0 0 0 0 0 0 0 0 0 0 0 0
##      [,47] [,48] [,49] [,50]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
## [5,] 0 0 0 0
## [6,] 0 0 0 0
## [7,] 0 0 0 0
## [8,] 0 0 0 0
## [9,] 0 0 0 0
## [10,] 0 0 0 0
## [11,] 0 0 0 0
## [12,] 0 0 0 0
## [13,] 0 0 0 0
## [14,] 0 0 0 0
## [15,] 0 0 0 0
## [16,] 0 0 0 0
## [17,] 0 0 0 0
## [18,] 0 0 0 0
## [19,] 0 0 0 0
## [20,] 0 0 0 0
## [21,] 0 0 0 0
## [22,] 0 0 0 0
## [23,] 0 0 0 0
## [24,] 0 0 0 0
## [25,] 0 0 0 0

```

```

## [26,]      0      0      0      0
## [27,]      0      0      0      0
## [28,]      0      0      0      0
## [29,]      0      0      0      0
## [30,]      0      0      0      0
## [31,]      0      0      0      0
## [32,]      0      0      0      0
## [33,]      0      0      0      0
## [34,]      0      0      0      0
## [35,]      0      0      0      0
## [36,]      0      0      0      0
## [37,]      0      0      0      0
## [38,]      0      0      0      0
## [39,]      0      0      0      0
## [40,]      0      0      0      0
## [41,]      0      0      0      0
## [42,]      0      0      0      0
## [43,]      0      0      0      0
## [44,]      0      0      0      0
## [45,]      0      0      0      0
## [46,]     -40      0      0      0
## [47,]     102     -40      0      0
## [48,]     -40     102     -40      0
## [49,]      0     -40     102     -40
## [50,]      0      0     -40      22
##
## $counts
## $counts$niter
## [1] 158
##
## $counts$nf
## [1] 157
##
## $counts$ngr
## [1] 152
##
## $counts$nhess
## [1] 152

print(eigen(sol50pim$Hess)$values)

## [1] 181.84200 181.36863 180.58176 179.48449 178.08116 176.37730 174.37964
## [8] 172.09607 169.53560 166.70834 163.62545 160.29911 156.74243 152.96948
## [15] 148.99513 144.83509 140.50578 136.02429 131.40832 126.67610 121.84632
## [22] 116.93804 111.97066 106.96381 101.93725 96.91085 91.90447 86.93791
## [29] 82.03080 77.20253 72.47223 67.85859 63.37989 59.05387 54.89766
## [36] 50.92776 47.15992 43.60907 40.28933 37.21385 34.39481 31.84332
## [43] 29.56937 27.58175 25.88797 24.49427 23.40556 22.62547 22.15648
## [50] 2.00000

# ?? do we want to try nlm, nlminb, and optimx versions??

```

## The Hobbs weed infestation problem

??ref

```
## Optimization test function HOBBS
## ?? refs (put in .doc??)
## Nash and Walker-Smith (1987, 1989) ...

hobbs.f<- function(x){ ## Hobbs weeds problem -- function
  if (abs(12*x[3]) > 500) { # check computability
    fbad<- .Machine$double.xmax
    return(fbad)
  }
  res<-hobbs.res(x)
  f<-sum(res*res)
}

hobbs.res<-function(x){ # Hobbs weeds problem -- residual
# This variant uses looping
  if(length(x) != 3) stop("hobbs.res -- parameter vector n!=3")
  y<-c(5.308, 7.24, 9.638, 12.866, 17.069, 23.192, 31.443, 38.558, 50.156, 62.948,
       75.995, 91.972)
  t<-1:12
  if(abs(12*x[3])>50) {
    res<-rep(Inf,12)
  } else {
    res<-x[1]/(1+x[2]*exp(-x[3]*t)) - y
  }
}

hobbs.jac<-function(x){ # Jacobian of Hobbs weeds problem
  jj<-matrix(0.0, 12, 3)
  t<-1:12
  yy<-exp(-x[3]*t)
  zz<-1.0/(1+x[2]*yy)
  jj[t,1] <- zz
  jj[t,2] <- -x[1]*zz*zz*yy
  jj[t,3] <- x[1]*zz*zz*yy*x[2]*t
  return(jj)
}

hobbs.g<-function(x){ # gradient of Hobbs weeds problem
# NOT EFFICIENT TO CALL AGAIN
  jj<-hobbs.jac(x)
  res<-hobbs.res(x)
  gg<-as.vector(2.*t(jj) %*% res)
  return(gg)
}

hobbs.rsd<-function(x) { # Jacobian second derivative
  rsd<-array(0.0, c(12,3,3))
```

```

t<-1:12
yy<-exp(-x[3]*t)
zz<-1.0/(1+x[2]*yy)
rsd[t,1,1]<- 0.0
rsd[t,2,1]<- -yy*zz*zz
rsd[t,1,2]<- -yy*zz*zz
rsd[t,2,2]<- 2.0*x[1]*yy*yy*zz*zz*zz
rsd[t,3,1]<- t*x[2]*yy*zz*zz
rsd[t,1,3]<- t*x[2]*yy*zz*zz
rsd[t,3,2]<- t*x[1]*yy*zz*zz*(1-2*x[2]*yy*zz)
rsd[t,2,3]<- t*x[1]*yy*zz*zz*(1-2*x[2]*yy*zz)
##   rsd[t,3,3]<- 2*t*t*x[1]*x[2]*x[2]*yy*yy*zz*zz*zz
rsd[t,3,3]<- -t*t*x[1]*x[2]*yy*zz*zz*(1-2*yy*zz*x[2])
return(rsd)
}

```

```

hobbs.h <- function(x) { ## compute Hessian
#   cat("Hessian not yet available\n")
#   return(NULL)
H<-matrix(0,3,3)
res<-hobbs.res(x)
jj<-hobbs.jac(x)
rsd<-hobbs.rsd(x)
##   H<-2.0*(t(res) %%% rsd + t(jj) %%% jj)
  for (j in 1:3) {
    for (k in 1:3) {
      for (i in 1:12) {
        H[j,k]<-H[j,k]+res[i]*rsd[i,j,k]
      }
    }
  }
  H<-2*(H + t(jj) %%% jj)
  return(H)
}

```

```

hobbsrsd.tst<-function(x) { # test rsd calculations
  hh<-1e-7 # use this for delta for derivatives
  Ja<-hobbs.jac(x)
  rsd<-hobbs.rsd(x)
  x1<-x+c(hh,0,0)
  x2<-x+c(0,hh,0)
  x3<-x+c(0,0,hh)
  Ja1<-hobbs.jac(x1)
  Ja2<-hobbs.jac(x2)
  Ja3<-hobbs.jac(x3)
  cat("w.r.t. x1 ")
  print(maxard((Ja1-Ja)/hh,rsd[, ,1] ))
  cat("w.r.t. x2 ")
  print(maxard((Ja2-Ja)/hh,rsd[, ,2] ))
  cat("w.r.t. x3 ")
  print(maxard((Ja3-Ja)/hh,rsd[, ,3] ))
}

```

```

}

hobbs.doc <- function() { ## documentation for hobbs
  cat("One generalization of the Rosenbrock banana valley function (n parameters)\n")
  ## How should we do the documentation output?
}

hobbs.setup <- function(n=NULL, dotdat=NULL) {
  # if (is.null(gs) ) { gs<-100.0 } # set the scaling
  # if ( is.null(n) ) {
  #   n <- readline("Order of problem (n):")
  # }
  n<-3 # fixed for Hobbs, as is m=12
  x<-rep(2,n)
  lower<-rep(-100.0, n)
  upper<-rep(100.0, n)
  bdmsk<-rep(1,n)
  if (! is.null(dotdat) ) {
    fargs<-paste("gs=",gs,sep='') # ?? still need to do this nicely
  } else { fargs<-NULL }
  gsu<-list(x=x,lower=lower,upper=upper,bdmsk=bdmsk,fargs=fargs)
  return(gsu)
}

hobbs.fgh <- function(x) { # all 3 for trust method
  stopifnot(is.numeric(x))
  stopifnot(length(x) == 3)
  f<-hobbs.f(x)
  g<-hobbs.g(x)
  B<-hobbs.h(x)
  list(value = f, gradient = g, hessian = B)
}

require(snewton)
x0 <- c(200, 50, .3)
cat("Start for Hobbs:")

## Start for Hobbs:
print(x0)

## [1] 200.0  50.0   0.3

solx0 <- snewton(x0, hobbs.f, hobbs.g, hobbs.h)

## trace = 1
## Initial function value = 158.2324
## nf= 1
## Iteration 1 :nf= 2
## Iteration 2 :nf= 3
## Iteration 3 :nf= 4
## Iteration 4 :nf= 5

```



```
## Iteration 5 :nf= 6
## Iteration 6 :nf= 7
## Iteration 7 :nf= 8
## Iteration 8 :nf= 9
## Iteration 9 :nf= 10
## Iteration 10 :No progress before linesearch!
## No progress in linesearch!
```

```
print(solx0)
```

```
## $par
## [1] 196.1862618 49.0916395 0.3135697
##
## $value
## [1] 2.587277
##
## $grad
## [1] -4.440892e-16 2.109424e-15 -3.979039e-13
##
## $Hess
##           [,1]      [,2]      [,3]
## [1,]  1.265461  -3.256125  1602.105
## [2,] -3.256125   8.627095 -4095.206
## [3,] 1602.105263 -4095.206388 2043434.033
##
## $counts
## $counts$niter
## [1] 10
##
## $counts$nf
## [1] 11
##
## $counts$ngr
## [1] 10
##
## $counts$nhess
## [1] 10
##
##
## $convcode
## [1] 93
```

```
print(eigen(solx0$Hess)$values)
```

```
## [1] 2.043443e+06 4.249248e-01 4.413953e-03
```

```
cat("This test finds a saddle point\n")
```

```
## This test finds a saddle point
```

```
x1s <- c(100, 10, .1)
cat("Start for Hobbs:")
```

```
## Start for Hobbs:
```

```
print(x1s)
```

```
## [1] 100.0 10.0 0.1
```

```
solxls <- snewton(xls, hobbs.f, hobbs.g, hobbs.h, control=list(trace=2))
```

```
## trace = 2
## Initial function value = 10685.29
## [1] 100.0 10.0 0.1
## Termination test:nf= 1
## current gradient norm = 82341.59
## Iteration 1 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 22
## current gradient norm = 82341.59
## Iteration 2 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
```

```

## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 43
## current gradient norm = 82341.59
## Iteration 3 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81

```

```

## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 65
## current gradient norm = 82341.59
## Iteration 4 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18

```

```

## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 87
## current gradient norm = 82341.59
## Iteration 5 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31

```

```

## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 109
## current gradient norm = 82341.59
## Iteration 6 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29

```

```

## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 131
## current gradient norm = 82341.59
## Iteration 7 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29

```

```

## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## Stepsize now = 4.194304e-16
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 154
## current gradient norm = 82341.59
## Iteration 8 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29

```



```

## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 176
## current gradient norm = 82341.59
## Iteration 9 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29

```

```

## end major loop
## Termination test:nf= 197
## current gradient norm = 82341.59
## Iteration 10 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## StepSize now = 0.2
## * f(xnew)= 18092.97
## StepSize now = 0.04
## * f(xnew)= 12450.3
## StepSize now = 0.008
## * f(xnew)= 11045.81
## StepSize now = 0.0016
## * f(xnew)= 10757.64
## StepSize now = 0.00032
## * f(xnew)= 10699.77
## StepSize now = 6.4e-05
## * f(xnew)= 10688.18
## StepSize now = 1.28e-05
## * f(xnew)= 10685.87
## StepSize now = 2.56e-06
## * f(xnew)= 10685.4
## StepSize now = 5.12e-07
## * f(xnew)= 10685.31
## StepSize now = 1.024e-07
## * f(xnew)= 10685.29
## StepSize now = 2.048e-08
## * f(xnew)= 10685.29
## StepSize now = 4.096e-09
## * f(xnew)= 10685.29
## StepSize now = 8.192e-10
## * f(xnew)= 10685.29
## StepSize now = 1.6384e-10
## * f(xnew)= 10685.29
## StepSize now = 3.2768e-11
## * f(xnew)= 10685.29
## StepSize now = 6.5536e-12
## * f(xnew)= 10685.29
## StepSize now = 1.31072e-12
## * f(xnew)= 10685.29
## StepSize now = 2.62144e-13
## * f(xnew)= 10685.29
## StepSize now = 5.24288e-14
## * f(xnew)= 10685.29
## StepSize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 218
## current gradient norm = 82341.59
## Iteration 11 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## StepSize now = 0.2
## * f(xnew)= 18092.97

```

```

## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 239
## current gradient norm = 82341.59
## Iteration 12 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77

```

```

## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## Stepsize now = 4.194304e-16
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 262
## current gradient norm = 82341.59
## Iteration 13 :Search vector:[1] -75.4852409 4.0706426 -0.4183675
## Gradient projection = 45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87

```

```

## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## Stepsize now = 4.194304e-16
## * f(xnew)= 10685.29
## No progress in linesearch!

```

```
print(solx1s)
```

```

## $par
## [1] 100.0 10.0 0.1
##
## $value
## [1] 10685.29
##
## $grad
## [1] -100.9131 783.5327 -82341.5897
##
## $Hess
##          [,1]      [,2]      [,3]
## [1,]  0.7158366  2.050058 -350.4172
## [2,]  2.0500582 -74.869818  774.4752
## [3,] -350.4171783 774.475203 -126055.8038
##
## $counts
## $counts$niter
## [1] 13
##
## $counts$nfn
## [1] 285

```

```

##
## $counts$ngr
## [1] 13
##
## $counts$nhess
## [1] 13
##
##
## $convcode
## [1] 93
print(eigen(solx1s$Hess)$values)

## [1] 1.690081e+00 -7.010902e+01 -1.260615e+05
cat("Following test fails with ERROR -- Why?\n")

## Following test fails with ERROR -- Why?
x1 <- c(1, 1, 1)
cat("Start for Hobbs:")

## Start for Hobbs:
print(x1)

## [1] 1 1 1
ftest <- try(solx1 <- snewton(x1, hobbs.f, hobbs.g, hobbs.h, control=list(trace=2)))

## trace = 2
## Initial function value = 23520.58
## [1] 1 1 1
## Termination test:nf= 1
## current gradient norm = 824.0421
## Iteration 1 :Search vector:[1] 41.42169 15.89584 16.90107
## Gradient projection = -34243.81
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= 22180.36
## end major loop
## Termination test:nf= 4
## current gradient norm = 788.357
## Iteration 2 :Search vector:[1] 34.646760 2.720555 4.942733
## Gradient projection = -27342.91
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= 17274.02
## end major loop
## Termination test:nf= 6
## current gradient norm = 626.2841
## Iteration 3 :Search vector:[1] 24.001827 2.813229 21.382377
## Gradient projection = -14880.99
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf

```

```

## StepSize now = 0.04
## * f(xnew)= 16690.2
## end major loop
## Termination test:nf= 9
## current gradient norm = 601.6369
## Iteration 4 :Search vector:[1] 24.5128581 0.5681561 10.9076966
## Gradient projection = -14683.69
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= 16114.55
## end major loop
## Termination test:nf= 12
## current gradient norm = 577.618
## Iteration 5 :Search vector:[1] 23.7542447 0.2788724 8.2638815
## Gradient projection = -13675.8
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= 16005.58
## end major loop
## Termination test:nf= 16
## current gradient norm = 572.9978
## Iteration 6 :Search vector:[1] 23.5866837 0.2513985 7.9269186
## Gradient projection = -13472.22
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= 15898.23
## end major loop
## Termination test:nf= 20
## current gradient norm = 568.4145
## Iteration 7 :Search vector:[1] 23.4174706 0.2277944 7.6221392
## Gradient projection = -13269.91
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= 15792.5
## end major loop
## Termination test:nf= 24
## current gradient norm = 563.8677
## Iteration 8 :Search vector:[1] 23.2471543 0.2073461 7.3446873
## Gradient projection = -13069.21
## f(xnew)= Inf

```

```

## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= 15771.6
## end major loop
## Termination test:nf= 29
## current gradient norm = 562.9655
## Iteration 9 :Search vector:[1] 23.2130700 0.2036327 7.2924028
## Gradient projection = -13029.39
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= Inf
## StepSize now = 0.00032
## * f(xnew)= 15767.43
## end major loop
## Termination test:nf= 35
## current gradient norm = 562.7854
## Iteration 10 :Search vector:[1] 23.2062534 0.2029037 7.2820653
## Gradient projection = -13021.44
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= Inf
## StepSize now = 0.00032
## * f(xnew)= 15763.27
## end major loop
## Termination test:nf= 41
## current gradient norm = 562.6053
## Iteration 11 :Search vector:[1] 23.1994359 0.2021785 7.2717632
## Gradient projection = -13013.49
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= Inf

```



```

## StepSize now = 0.00032
## * f(xnew)= 15759.1
## end major loop
## Termination test:nf= 47
## current gradient norm = 562.4253
## Iteration 12 :Search vector:[1] 23.192618 0.201457 7.261496
## Gradient projection = -13005.54
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= Inf
## StepSize now = 0.00032
## * f(xnew)= Inf
## StepSize now = 6.4e-05
## * f(xnew)= 15758.27
## end major loop
## Termination test:nf= 54
## current gradient norm = 562.3893
## Iteration 13 :Search vector:[1] 23.1912540 0.2013133 7.2594478
## Gradient projection = -13003.95
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= Inf
## StepSize now = 0.00032
## * f(xnew)= Inf
## StepSize now = 6.4e-05
## * f(xnew)= 15757.44
## end major loop
## Termination test:nf= 61
## current gradient norm = 562.3533
## Iteration 14 :Search vector:[1] 23.1898903 0.2011697 7.2574005
## Gradient projection = -13002.36
## f(xnew)= Inf
## StepSize now = 0.2
## * f(xnew)= Inf
## StepSize now = 0.04
## * f(xnew)= Inf
## StepSize now = 0.008
## * f(xnew)= Inf
## StepSize now = 0.0016
## * f(xnew)= Inf
## StepSize now = 0.00032
## * f(xnew)= Inf

```

```

## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= 15757.27
## end major loop
## Termination test:nf= 69
## current gradient norm = 562.3461
## Iteration 15 :Search vector:[1] 23.189618 0.201141 7.256991
## Gradient projection = -13002.05
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= 15757.11
## end major loop
## Termination test:nf= 77
## current gradient norm = 562.3389
## Iteration 16 :Search vector:[1] 23.1893448 0.2011123 7.2565820
## Gradient projection = -13001.73
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= 15756.94
## end major loop
## Termination test:nf= 85
## current gradient norm = 562.3317
## Iteration 17 :Search vector:[1] 23.1890721 0.2010836 7.2561729
## Gradient projection = -13001.41
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf

```

```

## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= 15756.91
## end major loop
## Termination test:nf= 94
## current gradient norm = 562.3302
## Iteration 18 :Search vector:[1] 23.1890175 0.2010779 7.2560910
## Gradient projection = -13001.35
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= 15756.91
## end major loop
## Termination test:nf= 105
## current gradient norm = 562.3302
## Iteration 19 :Search vector:[1] 23.1890154 0.2010776 7.2560878
## Gradient projection = -13001.35
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf

```

```

## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 116
## current gradient norm = 562.3301
## Iteration 20 :Search vector:[1] 23.1890132 0.2010774 7.2560845
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 128
## current gradient norm = 562.3301
## Iteration 21 :Search vector:[1] 23.1890127 0.2010774 7.2560838
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf

```

```

## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 140
## current gradient norm = 562.3301
## Iteration 22 :Search vector:[1] 23.1890123 0.2010773 7.2560832
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 154
## current gradient norm = 562.3301
## Iteration 23 :Search vector:[1] 23.1890123 0.2010773 7.2560832
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf

```

```

## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 168
## current gradient norm = 562.3301
## Iteration 24 :Search vector:[1] 23.1890123 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9

```

```

## end major loop
## Termination test:nf= 182
## current gradient norm = 562.3301
## Iteration 25 :Search vector:[1] 23.1890123 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 196
## current gradient norm = 562.3301
## Iteration 26 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf

```

```

## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 211
## current gradient norm = 562.3301
## Iteration 27 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 227
## current gradient norm = 562.3301
## Iteration 28 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf

```



```

## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 243
## current gradient norm = 562.3301
## Iteration 29 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf

```

```

## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 260
## current gradient norm = 562.3301
## Iteration 30 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 277

```

```

## current gradient norm = 562.3301
## Iteration 31 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 295
## current gradient norm = 562.3301
## Iteration 32 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf

```

```

## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 314
## current gradient norm = 562.3301
## Iteration 33 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf

```

```

## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 333
## current gradient norm = 562.3301
## Iteration 34 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf

```

```

## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 352
## current gradient norm = 562.3301
## Iteration 35 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 371
## current gradient norm = 562.3301
## Iteration 36 :Search vector:[1] 23.1890122 0.2010773 7.2560831
## Gradient projection = -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf

```

```

## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= Inf
## Stepsize now = 5.24288e-14
## * f(xnew)= Inf
## end major loop
## Termination test:nf= 391
## current gradient norm = Inf
## Iteration 37 :Search vector:[1] NaN NaN NaN
## Gradient projection = NaN

```

```

if (class(ftest) != "try-error") {
  print(solx1)
  print(eigen(solx1$Hess)$values)
}

```

```

# we can also use nlm and nlminb
#??

```

```

# and call them from optimx (i.e., test this gives same results)

```