lbfgsb3c: Using the 2011 version of L-BFGSB

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Abstract

In 2011 the authors of the L-BFGSB program published a correction and update to their 1995 code. The latter is the basis of the L-BFGS-B method of the `optim()` function in base-R. The package `lbfgsb3` wrapped the updated code using a `.Fortran` call after removing a very large number of Fortran output statements. Matthew Fidler used this Fortran code and an `Rcpp` interface to produce package `lbfgsb3c` where the function `lbfgsb3c()` returns an object similar to that of base-R `optim()` and that of `optimx::optimr()`. Subsequently, in a fine example of the collaborations that have made **R** so useful, we have merged the functionality of package `lbfgsb3` into `lbfgsb3c`, as explained in this vignette.

Provenance of the R optim::L-BFGS-B and related solvers

The base-R code lbfgsb.c (at writing in R-3.5.2/src/appl/) is commented:

```
/* l-bfgs-b.f -- translated by f2c (version 19991025).

  From ?optim:
  The code for method '"L-BFGS-B"' is based on Fortran code by Zhu,
  Byrd, Lu-Chen and Nocedal obtained from Netlib (file 'opt/lbfgs_bcm.shar')

  The Fortran files contained no copyright information.

  Byrd, R. H., Lu, P., Nocedal, J. and Zhu, C.  (1995) A limited
  memory algorithm for bound constrained optimization.
  \emph{SIAM J. Scientific Computing}, \bold{16}, 1190--1208.
*/
```

The paper R. H. Byrd, Lu, Nocedal, and Zhu (1995a) builds on Lu et al. (1994). There have been a number of other workers who have followed-up on this work, but **R** code and packages seem to have largely stayed with codes derived from these original papers. Though the date of the paper is 1995, the ideas it embodies were around for a decade and a half at least, in particular in Nocedal80 and LiuN89. The definitive Fortran code was published as Zhu et al. (1997). This is available as `toms/778.zip` on www.netlib.org. A side-by-side comparison of the main subroutines in the two downloads from Netlib unfortunately shows a lot of differences. I have not tried to determine if these affect performance or are simply cosmetic.

More seriously perhaps, there were some deficiencies in the code(s), and in 2011 Nocedal's team published a Fortran code with some corrections (Morales and Nocedal (2011)). Since the **R** code predates this, I prepared package `lbfgsb3` (Nash et al. (2015)) to wrap the Fortran code. However, I did not discover any test cases where the `optim::L-BFGS-B` and `lbfgsb3` were different, though I confess to only running some limited tests. There are, in fact, more in this vignette.

In 2016, I was at a Fields Institute optimization conference in Toronto for the 70th birthday of Andy Conn. By sheer serendipity, Nocedal did not attend the conference, but sat down next to me at the conference dinner. When I asked him about the key changes, he said that the most important one was to fix the computation of the machine precision, which was not always correct in the 1995 code. Since **R** gets this number as `.Machine$double.eps`, the offending code is irrelevant.

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Within Morales and Nocedal (2011), there is also reported an improvement in the subspace minimization that is applied in cases of bounds constraints. Since few of the tests I have applied impose such constraints, it is reasonable that I will not have observed performance differences between the base-R `optim` code and my `lbfsgb3` package. More appropriate tests are welcome, and on my agenda.

Besides the ACM TOMS code, there are two related codes from the Northwestern team on NETLIB: http://netlib.org/opt/lbfgs_um.shar is for unconstrained minimization, while http://netlib.org/opt/lbfgs_bcm.shar handles bounds constrained problems. To these are attached references Liu and Nocedal (1989) and R. H. Byrd, Lu, Nocedal, and Zhu (1995b) respectively, most likely reflecting the effort required to implement the constraints.

The unconstrained code has been converted to **C** under the leadership of Naoaki Okazaki (see http://www.chokkan.org/software/liblbfgs/, or the fork at https://github.com/MIRTK/LBFGS). This has been wrapped for **R** as Coppola, Stewart, and Okazaki (2014) as the `lbfgs` package. This can be called from `optimx::optimr()`.

Using Rcpp (see Eddelbuettel and François (2011)) and the Fortran code in package `lbfgs3`, Matthew Fidler developed package `lbfgsb3c` (Fidler et al. (2018)). As this provides a more standard call and return than `lbfgsb3` Fidler and I are unified the two packages as `lbfgsb3x`.

Functions in package `lbfgsb3c`

There is really only one optimizer function in the package, but it may be called by four (4) names:

- `lbfgsb3c()` uses Rcpp (Eddelbuettel (2013), Eddelbuettel and François (2011), Eddelbuettel and Balamuta (2017)) to streamline the call to the underlying Fortran. This is the base function used.
- `lbfgsb3x()` is an alias of `lbfgsb3c()`. We were using this name for a while, and have kept the alias to avoid having to edit test scripts.
- `lbfgsb3`, which imitates a `.Fortran` call of the compiled 2011 Fortran code. The object returned by this routine is NOT equivalent to the object returned by base-R `optim()` or by `optimx::optimr()`. Instead, it includes a structure `info` which contains the detailed diagnostic information of the Fortran code. For most users, this is not of interest, and I only recommend use of this function for those needing to examine how the optimization has been carried out.
- `lbfgsb3f()` is an alias of `lbfsgb3()`.

We recommend using the `lbfsgb3c()` call for most uses.

Comparison with optim::L-BFGS-B

The new Fortran package claims better performance on bounds-constrained problems. Below we present two fairly simple tests, which unfortunately do not show any advantage. We welcome examples showing differences, either better or not. Note that we use the call that returns expanded information, but we do not interpret that here. See the documentation in the source Fortran for an explanation of the data returned in object `info`.

```r
# ref BT.RES in Nash and Walker-Smith (1987)
library(lbfgsb3c)
sessionInfo()
```

```
## R version 3.5.3 (2019-03-11)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Linux Mint 19.1
##
## Matrix products: default
## BLAS: /usr/lib/x86_64-linux-gnu/openblas/libblas.so.3
## LAPACK: /usr/lib/x86_64-linux-gnu/libopenblasp-r0.2.20.so
```

```
## 
## locale:
##  [1] LC_CTYPE=en_CA.UTF-8       LC_NUMERIC=C
##  [3] LC_TIME=en_CA.UTF-8        LC_COLLATE=en_CA.UTF-8
##  [5] LC_MONETARY=en_CA.UTF-8    LC_MESSAGES=en_CA.UTF-8
##  [7] LC_PAPER=en_CA.UTF-8       LC_NAME=C
##  [9] LC_ADDRESS=C               LC_TELEPHONE=C
## [11] LC_MEASUREMENT=en_CA.UTF-8 LC_IDENTIFICATION=C
## 
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
## 
## other attached packages:
## [1] lbfgsb3c_2018-3.19
## 
## loaded via a namespace (and not attached):
##  [1] compiler_3.5.3  magrittr_1.5    tools_3.5.3     htmltools_0.3.6
##  [5] yaml_2.2.0      Rcpp_1.0.1      stringi_1.3.1   rmarkdown_1.11
##  [9] knitr_1.22      stringr_1.4.0   xfun_0.5        digest_0.6.18
## [13] evaluate_0.13
```

```r
bt.f<-function(x){
 sum(x*x)
}


bt.g<-function(x){
  gg<-2.0*x
}


bt.badsetup<-function(n){
   x<-rep(0,n)
   lo<-rep(0,n)
   up<-lo # to get arrays set
   bmsk<-rep(1,n)
   bmsk[(trunc(n/2)+1)]<-0
   for (i in 1:n) {
      x[i]<-2.2*i-n
      lo[i]<-1.0*(i-1)*(n-1)/n
      up[i]<-1.0*i*(n+1)/n
   }
   result<-list(x=x, lower=lo, upper=up, bdmsk=bmsk)
}


bt.setup0<-function(n){
   x<-rep(0,n)
   lo<-rep(0,n)
   up<-lower # to get arrays set
   bmsk<-rep(1,n)
   bmsk[(trunc(n/2)+1)]<-0
   for (i in 1:n) {
      lo[i]<-1.0*(i-1)*(n-1)/n
      up[i]<-1.0*i*(n+1)/n
   }
   x<-0.5*(lo+up)
```

```
    result<-list(x=x, lower=lo, upper=up, bdmsk=bmsk)
}
nn <- 4
baddy <- bt.badsetup(nn)
lo <- baddy$lower
up <- baddy$upper
x0 <- baddy$x
baddy
```

```
## $x
## [1] -1.8  0.4  2.6  4.8
##
## $lower
## [1] 0.00 0.75 1.50 2.25
##
## $upper
## [1] 1.25 2.50 3.75 5.00
##
## $bdmsk
## [1] 1 1 0 1
```

```
## optim()
solbad0 <- optim(x0, bt.f, bt.g, lower=lo, upper=up, method="L-BFGS-B", control=list(trace=3))
```

```
## N = 4, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 2 variables are exactly at the bounds
## At iterate     0  f=       30.363  |proj g|=        2.55
##
## iterations 1
## function evaluations 2
## segments explored during Cauchy searches 3
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 4
## norm of the final projected gradient 0
## final function value 7.875
##
## F = 7.875
## final  value 7.875000
## converged
```

```
solbad0
```

```
## $par
## [1] 0.00 0.75 1.50 2.25
##
## $value
## [1] 7.875
##
## $counts
## function gradient
##        2        2
##
## $convergence
## [1] 0
```

```
##
## $message
## [1] "CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL"
```

```
solbad1 <- lbfgsb3(x0, bt.f, bt.g, lower=lo, upper=up, control=list(trace=3))
```

```
##
## =================================================================================
## Before call f=17976931348623157081452742373170435679807056752584499659891747680315726078002853876058
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 0.00 0.75 2.60 4.80
## At iteration 0 f=30.362500 max(abs(g))=0.000000
##
## =================================================================================
## Before call f=30.362500 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.00 0.75 1.50 2.25
## At iteration 2 f=7.875000 max(abs(g))=0.000000
##
## =================================================================================
## Before call f=7.875000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=7.875000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 7 or "CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL"
```

```
solbad1
```

```
## $par
## [1] 0.00 0.75 1.50 2.25
##
## $grad
## [1] 0.0 1.5 3.0 4.5
##
## $value
## [1] 7.875
##
## $counts
## [1] 2 2
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL"
##
## $info
## $info$task
```

```
## [1] "CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL"
##
## $info$itask
## [1] 7
##
## $info$lsave
## [1]  TRUE  TRUE  TRUE FALSE
##
## $info$icsave
## [1] 6
##
## $info$dsave
##  [1]  1.000000e+00  3.036250e+01  2.220446e-09  2.777139e+00  2.220446e-16
##  [6]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [11] -1.477500e+01  1.000000e+00  0.000000e+00  1.000000e+00 -3.020000e+01
## [16]  7.712500e+00 -3.020000e+01 -3.020000e-02 -3.020000e+01 -3.020000e+01
## [21]  3.036250e+01  3.036250e+01  3.036250e+01  0.000000e+00  0.000000e+00
## [26]  0.000000e+00  5.000000e+00  1.000000e+00  2.000000e+00
##
## $info$isave
##  [1]  20  25 100    1  21  41  66  91 116 216 316 320 324 328 332 336    0
## [18]   0   0   0   0   3   0   8   0   0   1   0   0   1   0   0   3   2
## [35]   0   1  -1   0   4   5   0   0   0   1
```

```
## Possible timings
## library(microbenchmark)
## tbad0 <- microbenchmark(optim(x0, bt.f, bt.g, lower=lo, upper=up, method="L-BFGS-B"))
## t3c <- microbenchmark(lbfgsb3(x0, bt.f, bt.g, lower=lo, upper=up))
## Via optimx package
## library(optimx)
## meths <- c("L-BFGS-B", "lbfgsb3") # Note: lbfgsb3c not yet in optimx on CRAN
## allbt0 <- opm(x0, bt.f, bt.g, lower=lo, upper=up, method=meths)
## summary(allbt0, order=value)
```

```
# candlestick function
# J C Nash 2011-2-3
cstick.f<-function(x,alpha=100){
  x<-as.vector(x)
  r2<-crossprod(x)
  f<-as.double(r2+alpha/r2)
  return(f)
}

cstick.g<-function(x,alpha=100){
  x<-as.vector(x)
  r2<-as.numeric(crossprod(x))
  g1<-2*x
  g2 <- (-alpha)*2*x/(r2*r2)
  g<-as.double(g1+g2)
  return(g)
}
library(lbfgsb3c)
nn <- 2
x0 <- c(10,10)
lo <- c(1, 1)
```

```r
up <- c(10,10)
print(x0)
```

```
## [1] 10 10
```

```r
c2o <- optim(x0, cstick.f, cstick.g, lower=lo, upper=up, method="L-BFGS-B", control=list(trace=3))
```

```
## N = 2, M = 5 machine precision = 2.22045e-16
## At X0, 2 variables are exactly at the bounds
## At iterate     0  f=         200.5  |proj g|=             9
## Nonpositive definiteness in Cholesky factorization in formk;
##    refresh the lbfgs memory and restart the iteration.
##
## iterations 8
## function evaluations 14
## segments explored during Cauchy searches 14
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 1.12127e-06
## final function value 20
##
## F = 20
## final  value 20.000000
## converged
```

```r
c2o
```

```
## $par
## [1] 2.236068 2.236068
##
## $value
## [1] 20
##
## $counts
## function gradient
##       14       14
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```

```r
c2l <- lbfgsb3(x0, cstick.f, cstick.g, lower=lo, upper=up, control=list(trace=3))
```

```
##
## ===========================================================================
## Before call f=17976931348623157081452742373170435679807056752584499659891747680315726078002853876058
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 10 10
## At iteration 0 f=200.500000 max(abs(g))=19.950000
##
## ===========================================================================
## Before call f=200.500000 task number 21, or "FG_START"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1 1
## At iteration 2 f=52.000000 max(abs(g))=48.000000
##
## ================================================================================
## Before call f=52.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 7.357616 7.357616
## At iteration 3 f=109.192649 max(abs(g))=14.589699
##
## ================================================================================
## Before call f=109.192649 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=109.192649 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1 1
## At iteration 4 f=52.000000 max(abs(g))=48.000000
##
## ================================================================================
## Before call f=52.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 5.875652 5.875652
## At iteration 5 f=70.494861 max(abs(g))=11.504812
##
## ================================================================================
## Before call f=70.494861 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=70.494861 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1 1
## At iteration 6 f=52.000000 max(abs(g))=48.000000
##
## ================================================================================
## Before call f=52.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 4.932981 4.932981
```

```
## At iteration 7 f=50.723310 max(abs(g))=9.449436
##
## =================================================================================
## Before call f=50.723310 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=50.723310 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1 1
## At iteration 8 f=52.000000 max(abs(g))=48.000000
##
## =================================================================================
## Before call f=52.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 2.727727 2.727727
## At iteration 9 f=21.600973 max(abs(g))=2.991871
##
## =================================================================================
## Before call f=21.600973 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=21.600973 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1 1
## At iteration 10 f=52.000000 max(abs(g))=48.000000
##
## =================================================================================
## Before call f=52.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 2.221436 2.221436
## At iteration 11 f=20.001724 max(abs(g))=0.118221
##
## =================================================================================
## Before call f=20.001724 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=20.001724 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
```

```
## [1] 2.240681 2.240681
## At iteration 12 f=20.000170 max(abs(g))=0.036788
##
## ===============================================================================
## Before call f=20.000170 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=20.000170 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 2.236113 2.236113
## At iteration 13 f=20.000000 max(abs(g))=0.000362
##
## ===============================================================================
## Before call f=20.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=20.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 2.236068 2.236068
## At iteration 14 f=20.000000 max(abs(g))=0.000001
##
## ===============================================================================
## Before call f=20.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=20.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
```

```
c2l
```

```
## $par
## [1] 2.236068 2.236068
##
## $grad
## [1] -1.121272e-06 -1.121272e-06
##
## $value
## [1] 20
##
## $counts
## [1] 14 14
##
## $convergence
## [1] 0
```

```
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
##
## $info
## $info$task
## [1] "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
##
## $info$itask
## [1] 8
##
## $info$lsave
## [1] FALSE  TRUE  TRUE  TRUE
##
## $info$icsave
## [1] 6
##
## $info$dsave
##  [1]  7.975088e+00  2.000000e+01  2.220446e-09  6.427782e-05  2.220446e-16
##  [6]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [11]  1.019265e-10  2.719644e+04  1.121272e-06  1.000000e+00 -3.295018e-08
## [16]  4.131638e-09 -3.295018e-08 -3.295018e-11 -3.295018e-08 -3.295018e-08
## [21]  2.000000e+01  2.000000e+01  2.000000e+01  0.000000e+00  0.000000e+00
## [26]  0.000000e+00  5.000000e+00  2.719644e+04  5.439289e+04
##
## $info$isave
##  [1]  10  25 100   1  11  21  46  71  96 196 296 298 300 302 304 306   0
## [18]   0   0   0   0  14   0   8   0   0   1   3   3   8   3   0   1  14
## [35]   0   1   0   2   0   3   0   0   0   2
## meths <- c("L-BFGS-B", "lbfgsb3c", "Rvmmin", "Rcgmin", "Rtnmin")
## require(optimx)

## cstick2a <- opm(x0, cstick.f, cstick.g, method=meths, upper=up, lower=lo, control=list(kkt=FALSE))
## print(summary(cstick2a, par.select=1:2, order=value))
lo <- c(4, 4)
c2ob <- optim(x0, cstick.f, cstick.g, lower=lo, upper=up, method="L-BFGS-B", control=list(trace=3))

## N = 2, M = 5 machine precision = 2.22045e-16
## At X0, 2 variables are exactly at the bounds
## At iterate     0  f=         200.5  |proj g|=              6
##
## iterations 1
## function evaluations 2
## segments explored during Cauchy searches 2
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 2
## norm of the final projected gradient 0
## final function value 35.125
##
## F = 35.125
## final  value 35.125000
## converged
```

```
c2ob
```

```
## $par
## [1] 4 4
##
## $value
## [1] 35.125
##
## $counts
## function gradient
##        2        2
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL"
```

```
c2lb <- lbfgsb3(x0, cstick.f, cstick.g, lower=lo, upper=up, control=list(trace=3))
```

```
##
## ================================================================================
## Before call f=179769313486231570814527423731704356798070567525844996598917476803157260780028538760589
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 10 10
## At iteration 0 f=200.500000 max(abs(g))=19.950000
##
## ================================================================================
## Before call f=200.500000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 4 4
## At iteration 2 f=35.125000 max(abs(g))=7.218750
##
## ================================================================================
## Before call f=35.125000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=35.125000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 7 or "CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL"
```

```
c2lb
```

```
## $par
## [1] 4 4
##
## $grad
## [1] 7.21875 7.21875
##
## $value
```

```
## [1] 35.125
##
## $counts
## [1] 2 2
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL"
##
## $info
## $info$task
## [1] "CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL"
##
## $info$itask
## [1] 7
##
## $info$lsave
## [1] FALSE  TRUE  TRUE FALSE
##
## $info$icsave
## [1] 6
##
## $info$dsave
##  [1]  1.000000e+00  2.005000e+02  2.220446e-09  8.485281e+00  2.220446e-16
##  [6]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [11] -8.662500e+01  1.000000e+00  0.000000e+00  1.000000e+00 -2.394000e+02
## [16]  7.200000e+01 -2.394000e+02 -2.394000e-01 -2.394000e+02 -2.394000e+02
## [21]  2.005000e+02  2.005000e+02  2.005000e+02  0.000000e+00  0.000000e+00
## [26]  0.000000e+00  5.000000e+00  1.000000e+00  2.000000e+00
##
## $info$isave
##  [1]  10  25 100   1  11  21  46  71  96 196 296 298 300 302 304 306   0
## [18]   0   0   0   0   2   0   8   0   0   1   0   0   1   0   0   2   2
## [35]   0   1  -1   0   2   3   0   0   0   1
## cstick2b <- opm(x0, cstick.f, cstick.g, method=meths, upper=up, lower=lo, control=list(kkt=FALSE))
## print(summary(cstick2b, par.select=1:2, order=value))

nn <- 100
x0 <- rep(10, nn)
up <- rep(10, nn)
lo <- rep(1e-4, nn)
cco <- optim(x0, cstick.f, cstick.g, lower=lo, upper=up, method="L-BFGS-B", control=list(trace=3))

## N = 100, M = 5 machine precision = 2.22045e-16
## At X0, 100 variables are exactly at the bounds
## At iterate     0  f=          10000  |proj g|=        9.9999
## Nonpositive definiteness in Cholesky factorization in formk;
##    refresh the lbfgs memory and restart the iteration.
## At iterate    10  f =         20.004  |proj g|=        0.024942
##
## iterations 13
## function evaluations 23
```

```
## segments explored during Cauchy searches 311
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 1.75206e-08
## final function value 20
##
## F = 20
## final  value 20.000000
## converged
```

```
cco
```

```
## $par
##   [1] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [8] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [15] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [22] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [29] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [36] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [43] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [50] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [57] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [64] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [71] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [78] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [85] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [92] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##  [99] 0.3162278 0.3162278
##
## $value
## [1] 20
##
## $counts
## function gradient
##       23       23
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```

```
ccl <- lbfgsb3(x0, cstick.f, cstick.g, lower=lo, upper=up, control=list(trace=3))
```

```
##
## ====================================================================================
## Before call f=1797693134862315708145274237317043567980705675258449965989174768031572607800285387605895
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
##   [1] 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
##  [24] 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
##  [47] 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
##  [70] 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
##  [93] 10 10 10 10 10 10 10 10
## At iteration 0 f=10000.010000 max(abs(g))=19.999980
```

```
## 
## ==========================================================================
## Before call f=10000.010000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##     [1] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [12] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [23] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [34] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [45] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [56] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [67] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [78] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [89] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [100] 1e-04
## At iteration 2 f=100000000.000001 max(abs(g))=19999999999.999786
## 
## ==========================================================================
## Before call f=100000000.000001 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##     [1] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##     [8] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [15] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [22] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [29] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [36] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [43] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [50] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [57] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [64] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [71] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [78] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [85] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [92] 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217 6.666217
##    [99] 6.666217 6.666217
## At iteration 3 f=4443.867047 max(abs(g))=13.332366
## 
## ==========================================================================
## Before call f=4443.867047 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ==========================================================================
## Before call f=4443.867047 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##     [1] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [12] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [23] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##    [34] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
```

```
## [45] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [56] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [67] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [78] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [89] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [100] 1e-04
## At iteration 4 f=100000000.000001 max(abs(g))=19999999999.999786
##
## ===============================================================================
## Before call f=100000000.000001 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##    [8] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [15] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [22] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [29] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [36] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [43] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [50] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [57] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [64] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [71] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [78] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [85] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [92] 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046 4.444046
##   [99] 4.444046 4.444046
## At iteration 5 f=1975.005455 max(abs(g))=8.887865
##
## ===============================================================================
## Before call f=1975.005455 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=1975.005455 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [12] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [23] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [34] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [45] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [56] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [67] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [78] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
##   [89] 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04 1e-04
## [100] 1e-04
## At iteration 6 f=100000000.000001 max(abs(g))=19999999999.999786
##
## ===============================================================================
## Before call f=100000000.000001 task number 20, or "FG_LNSRCH"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##    [8] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [15] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [22] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [29] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [36] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [43] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [50] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [57] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [64] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [71] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [78] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [85] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [92] 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704 2.962704
##   [99] 2.962704 2.962704
## At iteration 7 f=877.875251 max(abs(g))=5.924638
##
## ================================================================================
## Before call f=877.875251 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=877.875251 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##    [6] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [11] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [16] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [21] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [26] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [31] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [36] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [41] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [46] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [51] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [56] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [61] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [66] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [71] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [76] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [81] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [86] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [91] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
##   [96] 0.0009255641 0.0009255641 0.0009255641 0.0009255641 0.0009255641
## At iteration 8 f=1167312.048755 max(abs(g))=25223796.150889
##
## ================================================================================
## Before call f=1167312.048755 task number 20, or "FG_LNSRCH"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##    [8] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [15] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [22] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [29] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [36] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [43] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [50] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [57] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [64] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [71] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [78] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [85] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [92] 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486 1.974486
##   [99] 1.974486 1.974486
## At iteration 9 f=390.116136 max(abs(g))=3.946375
##
## ================================================================================
## Before call f=390.116136 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=390.116136 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##    [6] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [11] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [16] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [21] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [26] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [31] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [36] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [41] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [46] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [51] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [56] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [61] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [66] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [71] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [76] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [81] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [86] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [91] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
##   [96] 0.003123492 0.003123492 0.003123492 0.003123492 0.003123492
## At iteration 10 f=102498.874093 max(abs(g))=656309.404764
##
## ================================================================================
## Before call f=102498.874093 task number 20, or "FG_LNSRCH"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##    [8] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [15] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [22] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [29] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [36] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [43] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [50] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [57] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [64] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [71] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [78] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [85] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [92] 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159 1.314159
##   [99] 1.314159 1.314159
## At iteration 11 f=173.280478 max(abs(g))=2.619506
##
## ================================================================================
## Before call f=173.280478 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=173.280478 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##    [7] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [13] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [19] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [25] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [31] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [37] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [43] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [49] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [55] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [61] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [67] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [73] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [79] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [85] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [91] 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003 0.01054003
##   [97] 0.01054003 0.01054003 0.01054003 0.01054003
## At iteration 12 f=9001.546538 max(abs(g))=17080.648379
##
## ================================================================================
## Before call f=9001.546538 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
```

```
##    [1] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##    [8] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [15] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [22] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [29] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [36] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [43] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [50] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [57] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [64] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [71] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [78] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [85] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [92] 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362 0.8692362
##   [99] 0.8692362 0.8692362
## At iteration 13 f=76.880658 max(abs(g))=1.708020
##
## ===========================================================================
## Before call f=76.880658 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===========================================================================
## Before call f=76.880658 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##    [8] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [15] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [22] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [29] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [36] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [43] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [50] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [57] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [64] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [71] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [78] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [85] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [92] 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013 0.0355013
##   [99] 0.0355013 0.0355013
## At iteration 14 f=793.561220 max(abs(g))=446.918351
##
## ===========================================================================
## Before call f=793.561220 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##    [8] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [15] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [22] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [29] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
```

```
##   [36] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [43] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [50] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [57] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [64] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [71] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [78] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [85] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [92] 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149 0.5626149
##   [99] 0.5626149 0.5626149
## At iteration 15 f=34.812759 max(abs(g))=1.012926
##
## ==============================================================================
## Before call f=34.812759 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=34.812759 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##    [8] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [15] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [22] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [29] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [36] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [43] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [50] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [57] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [64] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [71] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [78] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [85] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [92] 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916 0.1157916
##   [99] 0.1157916 0.1157916
## At iteration 16 f=75.924777 max(abs(g))=12.650870
##
## ==============================================================================
## Before call f=75.924777 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##    [8] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [15] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [22] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [29] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [36] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [43] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [50] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [57] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
##   [64] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
```

```
## [71] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
## [78] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
## [85] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
## [92] 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455 0.3715455
## [99] 0.3715455 0.3715455
## At iteration 17 f=21.048563 max(abs(g))=0.353154
##
## ================================================================================
## Before call f=21.048563 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=21.048563 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [8] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [15] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [22] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [29] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [36] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [43] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [50] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [57] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [64] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [71] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [78] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [85] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [92] 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722 0.2692722
## [99] 0.2692722 0.2692722
## At iteration 18 f=21.042426 max(abs(g))=0.485823
##
## ================================================================================
## Before call f=21.042426 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [8] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [15] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [22] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [29] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [36] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [43] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [50] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [57] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [64] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [71] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [78] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [85] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [92] 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951 0.3284951
## [99] 0.3284951 0.3284951
```

```
## At iteration 19 f=20.057968 max(abs(g))=0.092777
##
## ==============================================================================
## Before call f=20.057968 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=20.057968 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##    [8] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [15] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [22] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [29] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [36] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [43] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [50] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [57] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [64] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [71] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [78] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [85] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [92] 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555 0.3131555
##   [99] 0.3131555 0.3131555
## At iteration 20 f=20.003813 max(abs(g))=0.024942
##
## ==============================================================================
## Before call f=20.003813 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=20.003813 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##    [8] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [15] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [22] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [29] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [36] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [43] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [50] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [57] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [64] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [71] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [78] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [85] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [92] 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056 0.3164056
##   [99] 0.3164056 0.3164056
```

```
## At iteration 21 f=20.000013 max(abs(g))=0.001422
##
## ================================================================================
## Before call f=20.000013 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=20.000013 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##    [8] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [15] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [22] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [29] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [36] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [43] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [50] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [57] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [64] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [71] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [78] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [85] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [92] 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304 0.3162304
##   [99] 0.3162304 0.3162304
## At iteration 22 f=20.000000 max(abs(g))=0.000021
##
## ================================================================================
## Before call f=20.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=20.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##    [1] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##    [8] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [15] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [22] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [29] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [36] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [43] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [50] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [57] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [64] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [71] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [78] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [85] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [92] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [99] 0.3162278 0.3162278
```

```
## At iteration 23 f=20.000000 max(abs(g))=0.000000
##
## ================================================================================
## Before call f=20.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=20.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
```

ccl

```
## $par
##    [1] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##    [8] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [15] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [22] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [29] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [36] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [43] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [50] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [57] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [64] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [71] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [78] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [85] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [92] 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278 0.3162278
##   [99] 0.3162278 0.3162278
##
## $grad
##    [1] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##    [6] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [11] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [16] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [21] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [26] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [31] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [36] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [41] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [46] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [51] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [56] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [61] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [66] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [71] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [76] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [81] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [86] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [91] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##   [96] -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08 -1.752061e-08
##
## $value
## [1] 20
```

```
##
## $counts
## [1] 23 23
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
##
## $info
## $info$task
## [1] "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
##
## $info$itask
## [1] 8
##
## $info$lsave
## [1] FALSE  TRUE  TRUE  TRUE
##
## $info$icsave
## [1] 6
##
## $info$dsave
##  [1]  7.993158e+00  2.000000e+01  2.220446e-09  2.598193e-05  2.220446e-16
##  [6]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [11]  4.552192e-12  1.216732e+05  1.752061e-08  1.000000e+00 -5.395867e-09
## [16]  6.750607e-10 -5.395867e-09 -5.395867e-12 -5.395867e-09 -5.395867e-09
## [21]  2.000000e+01  2.000000e+01  2.000000e+01  0.000000e+00  0.000000e+00
## [26]  0.000000e+00  5.000000e+00  1.216732e+05  2.433463e+05
##
## $info$isave
##  [1]  500    25   100     1   501  1001  1026  1051  1076  1176  1276  1376  1476  1576
## [15] 1676  1776     0     0     0     0     0   311     0     8     0     0     1     5
## [29]    5    13    10     0     1    23     0     1     0   100     0   101     0     0
## [43]    0     2
```

```
## cstickc0 <- opm(x0, cstick.f, cstick.g, method=meths, upper=up, lower=lo, control=list(kkt=FALSE))
## print(summary(cstickc0, par.select=1:5, order=value))
## lo <- rep(1, nn)
## cstickca <- opm(x0, cstick.f, cstick.g, method=meths, upper=up, lower=lo, control=list(kkt=FALSE))
## print(summary(cstickca, par.select=1:5, order=value))
## lo <- rep(4, nn)
## cstickcb <- opm(x0, cstick.f, cstick.g, method=meths, upper=up, lower=lo, control=list(kkt=FALSE))
## print(summary(cstickcb, par.select=1:5, order=value))
```

```
# require(funconstrain) ## not in CRAN, so explicit inclusion of this function
# exrosen <- ex_rosen()
# exrosenf <- exrosen$fn
exrosenf <- function (par) {
    n <- length(par)
    if (n%%2 != 0) {
        stop("Extended Rosenbrock: n must be even")
    }
    fsum <- 0
```

```r
    for (i in 1:(n/2)) {
        p2 <- 2 * i
        p1 <- p2 - 1
        f_p1 <- 10 * (par[p2] - par[p1]^2)
        f_p2 <- 1 - par[p1]
        fsum <- fsum + f_p1 * f_p1 + f_p2 * f_p2
    }
    fsum
}
# exroseng <- exrosen$gr
exroseng <- function (par) {
    n <- length(par)
    if (n%%2 != 0) {
        stop("Extended Rosenbrock: n must be even")
    }
    grad <- rep(0, n)
    for (i in 1:(n/2)) {
        p2 <- 2 * i
        p1 <- p2 - 1
        xx <- par[p1] * par[p1]
        yx <- par[p2] - xx
        f_p1 <- 10 * yx
        f_p2 <- 1 - par[p1]
        grad[p1] <- grad[p1] - 400 * par[p1] * yx - 2 * f_p2
        grad[p2] <- grad[p2] + 200 * yx
    }
    grad
}

exrosenx0 <- function (n = 20) {
    if (n%%2 != 0) {
        stop("Extended Rosenbrock: n must be even")
    }
    rep(c(-1.2, 1), n/2)
}

## meths <- c("L-BFGS-B", "lbfgsb3c", "Rvmmin", "Rcgmin", "Rtnmin")
## require(optimx)
for (n in seq(2,12, by=2)) {
  cat("ex_rosen try for n=",n,"\n")
  x0 <- exrosenx0(n)
  lo <- rep(.5, n)
  up <- rep(3, n)
  print(x0)
  eo <- optim(x0, exrosenf, exroseng, lower=lo, upper=up, method="L-BFGS-B", control=list(trace=3))
  eo
  el <- lbfgsb3(x0, exrosenf, exroseng, lower=lo, upper=up, control=list(trace=3))
  el
##   erfg <- opm(x0, exrosenf, exroseng, method=meths, lower=lo, upper=up)
##   print(summary(erfg, par.select=1:2, order=value))
}

## ex_rosen try for n= 2
## [1] -1.2  1.0
```

27

```
## N = 2, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 1 variables are exactly at the bounds
## At iterate     0  f=         56.5  |proj g|=         2.5
## At iterate    10  f =    2.473e-06  |proj g|=      0.021128
##
## iterations 13
## function evaluations 19
## segments explored during Cauchy searches 14
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 9.85451e-06
## final function value 4.63785e-11
##
## F = 4.63785e-11
## final  value 0.000000
## converged
##
## =================================================================================
## Before call f=17976931348623157081452742373170435679807056752584499659891747680315726078002853876058
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 0.5 1.0
## At iteration 0 f=56.500000 max(abs(g))=150.000000
##
## =================================================================================
## Before call f=56.500000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 3.0 0.5
## At iteration 2 f=7229.000000 max(abs(g))=1700.000000
##
## =================================================================================
## Before call f=7229.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.8802684 0.9239463
## At iteration 3 f=2.236637 max(abs(g))=29.814773
##
## =================================================================================
## Before call f=2.236637 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=2.236637 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.1176910 0.9323731
## At iteration 4 f=10.053887 max(abs(g))=63.372032
```

```
## 
## ================================================================================
## Before call f=10.053887 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9579399 0.9267031
## At iteration 5 f=0.009967 max(abs(g))=1.810862
## 
## ================================================================================
## Before call f=0.009967 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ================================================================================
## Before call f=0.009967 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.963677 0.927100
## At iteration 6 f=0.001567 max(abs(g))=0.314675
## 
## ================================================================================
## Before call f=0.001567 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ================================================================================
## Before call f=0.001567 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629480 0.9270802
## At iteration 7 f=0.001376 max(abs(g))=0.001478
## 
## ================================================================================
## Before call f=0.001376 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ================================================================================
## Before call f=0.001376 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629685 0.9271132
## At iteration 8 f=0.001375 max(abs(g))=0.001048
## 
## ================================================================================
## Before call f=0.001375 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9630504 0.9272453
```

```
## At iteration 9 f=0.001370 max(abs(g))=0.011159
##
## ===============================================================================
## Before call f=0.001370 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9633784 0.9277737
## At iteration 10 f=0.001352 max(abs(g))=0.051670
##
## ===============================================================================
## Before call f=0.001352 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.001352 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9679509 0.9358966
## At iteration 11 f=0.001134 max(abs(g))=0.206451
##
## ===============================================================================
## Before call f=0.001134 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.001134 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9756907 0.9504056
## At iteration 12 f=0.000836 max(abs(g))=0.313355
##
## ===============================================================================
## Before call f=0.000836 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.000836 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9861682 0.9709040
## At iteration 13 f=0.000455 max(abs(g))=0.324762
##
## ===============================================================================
## Before call f=0.000455 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
```

```
## ================================================================================
## Before call f=0.000455 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9950881 0.9893577
## At iteration 14 f=0.000095 max(abs(g))=0.168543
##
## ================================================================================
## Before call f=0.000095 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000095 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9985471 0.9970361
## At iteration 15 f=0.000002 max(abs(g))=0.012034
##
## ================================================================================
## Before call f=0.000002 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000002 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9998188 0.9996336
## At iteration 16 f=0.000000 max(abs(g))=0.000800
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000009 1.000015
## At iteration 17 f=0.000000 max(abs(g))=0.000530
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9996797 0.9998104
## At iteration 18 f=0.000020 max(abs(g))=0.090157
##
## ================================================================================
## Before call f=0.000020 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000007 1.000014
## At iteration 19 f=0.000000 max(abs(g))=0.000006
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
## ex_rosen try for n= 4
## [1] -1.2  1.0 -1.2  1.0
## N = 4, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 2 variables are exactly at the bounds
## At iterate     0  f=           113  |proj g|=          2.5
## At iterate    10  f =   4.9459e-06  |proj g|=      0.021128
##
## iterations 13
## function evaluations 19
## segments explored during Cauchy searches 16
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 9.85451e-06
## final function value 9.2757e-11
##
## F = 9.2757e-11
## final  value 0.000000
## converged
##
## ================================================================================
## Before call f=179769313486231570814527423731704356798070567525844996598917476803157260780028538760589
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 0.5 1.0 0.5 1.0
## At iteration 0 f=113.000000 max(abs(g))=150.000000
##
## ================================================================================
## Before call f=113.000000 task number 21, or "FG_START"
## returned from lbfgsb3
```

```
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 3.0 0.5 3.0 0.5
## At iteration 2 f=14458.000000 max(abs(g))=1700.000000
##
## ================================================================================
## Before call f=14458.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.8802684 0.9239463 0.8802684 0.9239463
## At iteration 3 f=4.473275 max(abs(g))=29.814773
##
## ================================================================================
## Before call f=4.473275 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=4.473275 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.1176910 0.9323731 1.1176910 0.9323731
## At iteration 4 f=20.107774 max(abs(g))=63.372032
##
## ================================================================================
## Before call f=20.107774 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9579399 0.9267031 0.9579399 0.9267031
## At iteration 5 f=0.019934 max(abs(g))=1.810862
##
## ================================================================================
## Before call f=0.019934 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.019934 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.963677 0.927100 0.963677 0.927100
## At iteration 6 f=0.003134 max(abs(g))=0.314675
##
## ================================================================================
## Before call f=0.003134 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.003134 task number 1, or "NEW_X"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629480 0.9270802 0.9629480 0.9270802
## At iteration 7 f=0.002753 max(abs(g))=0.001478
##
## ================================================================================
## Before call f=0.002753 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.002753 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629685 0.9271132 0.9629685 0.9271132
## At iteration 8 f=0.002750 max(abs(g))=0.001048
##
## ================================================================================
## Before call f=0.002750 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9630504 0.9272453 0.9630504 0.9272453
## At iteration 9 f=0.002740 max(abs(g))=0.011159
##
## ================================================================================
## Before call f=0.002740 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9633784 0.9277737 0.9633784 0.9277737
## At iteration 10 f=0.002703 max(abs(g))=0.051670
##
## ================================================================================
## Before call f=0.002703 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.002703 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9679509 0.9358966 0.9679509 0.9358966
## At iteration 11 f=0.002267 max(abs(g))=0.206451
##
## ================================================================================
## Before call f=0.002267 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
```

```
## Before call f=0.002267 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9756907 0.9504056 0.9756907 0.9504056
## At iteration 12 f=0.001673 max(abs(g))=0.313355
##
## =============================================================================
## Before call f=0.001673 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.001673 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9861682 0.9709040 0.9861682 0.9709040
## At iteration 13 f=0.000910 max(abs(g))=0.324762
##
## =============================================================================
## Before call f=0.000910 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.000910 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9950881 0.9893577 0.9950881 0.9893577
## At iteration 14 f=0.000190 max(abs(g))=0.168543
##
## =============================================================================
## Before call f=0.000190 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.000190 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9985471 0.9970361 0.9985471 0.9970361
## At iteration 15 f=0.000005 max(abs(g))=0.012034
##
## =============================================================================
## Before call f=0.000005 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.000005 task number 1, or "NEW_X"
## returned from lbfgsb3
```

```
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9998188 0.9996336 0.9998188 0.9996336
## At iteration 16 f=0.000000 max(abs(g))=0.000800
##
## =================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000009 1.000015 1.000009 1.000015
## At iteration 17 f=0.000000 max(abs(g))=0.000530
##
## =================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9996797 0.9998104 0.9996797 0.9998104
## At iteration 18 f=0.000041 max(abs(g))=0.090157
##
## =================================================================================
## Before call f=0.000041 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000007 1.000014 1.000007 1.000014
## At iteration 19 f=0.000000 max(abs(g))=0.000006
##
## =================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
## ex_rosen try for n= 6
## [1] -1.2  1.0 -1.2  1.0 -1.2  1.0
## N = 6, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 3 variables are exactly at the bounds
## At iterate     0  f=         169.5  |proj g|=          2.5
```

```
## At iterate    10  f =    7.4189e-06  |proj g|=        0.021128
##
## iterations 13
## function evaluations 19
## segments explored during Cauchy searches 18
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 9.85451e-06
## final function value 1.39136e-10
##
## F = 1.39136e-10
## final  value 0.000000
## converged
##
## =================================================================================
## Before call f=179769313486231570814527423731704356798070567525844996598917476803157260780028538760589
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 0.5 1.0 0.5 1.0 0.5 1.0
## At iteration 0 f=169.500000 max(abs(g))=150.000000
##
## =================================================================================
## Before call f=169.500000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 3.0 0.5 3.0 0.5 3.0 0.5
## At iteration 2 f=21687.000000 max(abs(g))=1700.000000
##
## =================================================================================
## Before call f=21687.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684 0.9239463
## At iteration 3 f=6.709912 max(abs(g))=29.814773
##
## =================================================================================
## Before call f=6.709912 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =================================================================================
## Before call f=6.709912 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910 0.9323731
## At iteration 4 f=30.161662 max(abs(g))=63.372032
##
## =================================================================================
## Before call f=30.161662 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
```

```
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399 0.9267031
## At iteration 5 f=0.029901 max(abs(g))=1.810862
##
## ===============================================================================
## Before call f=0.029901 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.029901 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.963677 0.927100 0.963677 0.927100 0.963677 0.927100
## At iteration 6 f=0.004701 max(abs(g))=0.314675
##
## ===============================================================================
## Before call f=0.004701 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.004701 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480 0.9270802
## At iteration 7 f=0.004129 max(abs(g))=0.001478
##
## ===============================================================================
## Before call f=0.004129 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.004129 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685 0.9271132
## At iteration 8 f=0.004125 max(abs(g))=0.001048
##
## ===============================================================================
## Before call f=0.004125 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504 0.9272453
## At iteration 9 f=0.004110 max(abs(g))=0.011159
##
## ===============================================================================
## Before call f=0.004110 task number 20, or "FG_LNSRCH"
```

```
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784 0.9277737
## At iteration 10 f=0.004055 max(abs(g))=0.051670
##
## ===============================================================================
## Before call f=0.004055 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.004055 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509 0.9358966
## At iteration 11 f=0.003401 max(abs(g))=0.206451
##
## ===============================================================================
## Before call f=0.003401 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.003401 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907 0.9504056
## At iteration 12 f=0.002509 max(abs(g))=0.313355
##
## ===============================================================================
## Before call f=0.002509 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.002509 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682 0.9709040
## At iteration 13 f=0.001365 max(abs(g))=0.324762
##
## ===============================================================================
## Before call f=0.001365 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.001365 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
```

```
## computing f and g at prm=
## [1] 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881 0.9893577
## At iteration 14 f=0.000285 max(abs(g))=0.168543
##
## ============================================================================
## Before call f=0.000285 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000285 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471 0.9970361
## At iteration 15 f=0.000007 max(abs(g))=0.012034
##
## ============================================================================
## Before call f=0.000007 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000007 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188 0.9996336
## At iteration 16 f=0.000000 max(abs(g))=0.000800
##
## ============================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000009 1.000015 1.000009 1.000015 1.000009 1.000015
## At iteration 17 f=0.000000 max(abs(g))=0.000530
##
## ============================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797 0.9998104
```

```
## At iteration 18 f=0.000061 max(abs(g))=0.090157
##
## ================================================================================
## Before call f=0.000061 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000007 1.000014 1.000007 1.000014 1.000007 1.000014
## At iteration 19 f=0.000000 max(abs(g))=0.000006
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
## ex_rosen try for n= 8
## [1] -1.2  1.0 -1.2  1.0 -1.2  1.0 -1.2  1.0
## N = 8, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 4 variables are exactly at the bounds
## At iterate      0  f=           226  |proj g|=          2.5
## At iterate     10  f =   9.8919e-06  |proj g|=     0.021128
##
## iterations 14
## function evaluations 20
## segments explored during Cauchy searches 21
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 2.31163e-08
## final function value 1.60731e-16
##
## F = 1.60731e-16
## final  value 0.000000
## converged
##
## ================================================================================
## Before call f=1797693134862315708145274237317043567980705675258449965989174768031572607800285387605890
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
## [1] 0.5 1.0 0.5 1.0 0.5 1.0 0.5 1.0
## At iteration 0 f=226.000000 max(abs(g))=150.000000
##
## ================================================================================
## Before call f=226.000000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 3.0 0.5 3.0 0.5 3.0 0.5 3.0 0.5
## At iteration 2 f=28916.000000 max(abs(g))=1700.000000
```

```
## 
## ===============================================================================
## Before call f=28916.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684
## [8] 0.9239463
## At iteration 3 f=8.946550 max(abs(g))=29.814773
## 
## ===============================================================================
## Before call f=8.946550 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ===============================================================================
## Before call f=8.946550 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910
## [8] 0.9323731
## At iteration 4 f=40.215549 max(abs(g))=63.372032
## 
## ===============================================================================
## Before call f=40.215549 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399
## [8] 0.9267031
## At iteration 5 f=0.039868 max(abs(g))=1.810862
## 
## ===============================================================================
## Before call f=0.039868 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ===============================================================================
## Before call f=0.039868 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.963677 0.927100 0.963677 0.927100 0.963677 0.927100 0.963677 0.927100
## At iteration 6 f=0.006268 max(abs(g))=0.314675
## 
## ===============================================================================
## Before call f=0.006268 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ===============================================================================
## Before call f=0.006268 task number 1, or "NEW_X"
## returned from lbfgsb3
```

```
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480
## [8] 0.9270802
## At iteration 7 f=0.005506 max(abs(g))=0.001478
##
## ================================================================================
## Before call f=0.005506 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.005506 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685
## [8] 0.9271132
## At iteration 8 f=0.005501 max(abs(g))=0.001048
##
## ================================================================================
## Before call f=0.005501 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504
## [8] 0.9272453
## At iteration 9 f=0.005481 max(abs(g))=0.011159
##
## ================================================================================
## Before call f=0.005481 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784
## [8] 0.9277737
## At iteration 10 f=0.005407 max(abs(g))=0.051670
##
## ================================================================================
## Before call f=0.005407 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.005407 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509
## [8] 0.9358966
## At iteration 11 f=0.004535 max(abs(g))=0.206451
##
## ================================================================================
## Before call f=0.004535 task number 20, or "FG_LNSRCH"
```

```
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.004535 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907
## [8] 0.9504056
## At iteration 12 f=0.003346 max(abs(g))=0.313355
##
## ================================================================================
## Before call f=0.003346 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.003346 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682
## [8] 0.9709040
## At iteration 13 f=0.001820 max(abs(g))=0.324762
##
## ================================================================================
## Before call f=0.001820 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.001820 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881
## [8] 0.9893577
## At iteration 14 f=0.000381 max(abs(g))=0.168543
##
## ================================================================================
## Before call f=0.000381 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000381 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471
## [8] 0.9970361
## At iteration 15 f=0.000010 max(abs(g))=0.012034
##
```

```
## ================================================================================
## Before call f=0.000010 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000010 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188
## [8] 0.9996336
## At iteration 16 f=0.000000 max(abs(g))=0.000800
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000009 1.000015 1.000009 1.000015 1.000009 1.000015 1.000009 1.000015
## At iteration 17 f=0.000000 max(abs(g))=0.000530
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797
## [8] 0.9998104
## At iteration 18 f=0.000082 max(abs(g))=0.090157
##
## ================================================================================
## Before call f=0.000082 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1.000007 1.000014 1.000007 1.000014 1.000007 1.000014 1.000007 1.000014
## At iteration 19 f=0.000000 max(abs(g))=0.000006
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
```

```
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 1 1 1 1 1 1 1 1
## At iteration 20 f=0.000000 max(abs(g))=0.000000
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
## ex_rosen try for n= 10
##  [1] -1.2  1.0 -1.2  1.0 -1.2  1.0 -1.2  1.0 -1.2  1.0
## N = 10, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 5 variables are exactly at the bounds
## At iterate     0  f=        282.5  |proj g|=        2.5
## At iterate    10  f =   1.2365e-05  |proj g|=     0.021128
##
## iterations 14
## function evaluations 20
## segments explored during Cauchy searches 23
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 2.31162e-08
## final function value 2.00917e-16
##
## F = 2.00917e-16
## final  value 0.000000
## converged
##
## ================================================================================
## Before call f=17976931348623157081452742373170435679807056752584499659891747680315726078002853876058⁚
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
##  [1] 0.5 1.0 0.5 1.0 0.5 1.0 0.5 1.0 0.5 1.0
## At iteration 0 f=282.500000 max(abs(g))=150.000000
##
## ================================================================================
## Before call f=282.500000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 3.0 0.5 3.0 0.5 3.0 0.5 3.0 0.5 3.0 0.5
## At iteration 2 f=36145.000000 max(abs(g))=1700.000000
##
## ================================================================================
```

```
## Before call f=36145.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684
##  [8] 0.9239463 0.8802684 0.9239463
## At iteration 3 f=11.183187 max(abs(g))=29.814773
##
## ============================================================================
## Before call f=11.183187 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=11.183187 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910
##  [8] 0.9323731 1.1176910 0.9323731
## At iteration 4 f=50.269436 max(abs(g))=63.372032
##
## ============================================================================
## Before call f=50.269436 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399
##  [8] 0.9267031 0.9579399 0.9267031
## At iteration 5 f=0.049836 max(abs(g))=1.810862
##
## ============================================================================
## Before call f=0.049836 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.049836 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.963677 0.927100 0.963677 0.927100 0.963677 0.927100 0.963677
##  [8] 0.927100 0.963677 0.927100
## At iteration 6 f=0.007835 max(abs(g))=0.314675
##
## ============================================================================
## Before call f=0.007835 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.007835 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
```

```
## computing f and g at prm=
## [1] 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480
## [8] 0.9270802 0.9629480 0.9270802
## At iteration 7 f=0.006882 max(abs(g))=0.001478
##
## ===============================================================================
## Before call f=0.006882 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.006882 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685
## [8] 0.9271132 0.9629685 0.9271132
## At iteration 8 f=0.006876 max(abs(g))=0.001048
##
## ===============================================================================
## Before call f=0.006876 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504
## [8] 0.9272453 0.9630504 0.9272453
## At iteration 9 f=0.006851 max(abs(g))=0.011159
##
## ===============================================================================
## Before call f=0.006851 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784
## [8] 0.9277737 0.9633784 0.9277737
## At iteration 10 f=0.006758 max(abs(g))=0.051670
##
## ===============================================================================
## Before call f=0.006758 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ===============================================================================
## Before call f=0.006758 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
## [1] 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509
## [8] 0.9358966 0.9679509 0.9358966
## At iteration 11 f=0.005669 max(abs(g))=0.206451
##
## ===============================================================================
## Before call f=0.005669 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
```

```
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.005669 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907
##  [8] 0.9504056 0.9756907 0.9504056
## At iteration 12 f=0.004182 max(abs(g))=0.313355
##
## =============================================================================
## Before call f=0.004182 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.004182 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682
##  [8] 0.9709040 0.9861682 0.9709040
## At iteration 13 f=0.002275 max(abs(g))=0.324762
##
## =============================================================================
## Before call f=0.002275 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.002275 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881
##  [8] 0.9893577 0.9950881 0.9893577
## At iteration 14 f=0.000476 max(abs(g))=0.168543
##
## =============================================================================
## Before call f=0.000476 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =============================================================================
## Before call f=0.000476 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471
##  [8] 0.9970361 0.9985471 0.9970361
## At iteration 15 f=0.000012 max(abs(g))=0.012034
##
## =============================================================================
```

```
## Before call f=0.000012 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000012 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188
##  [8] 0.9996336 0.9998188 0.9996336
## At iteration 16 f=0.000000 max(abs(g))=0.000800
##
## ============================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 1.000009 1.000015 1.000009 1.000015 1.000009 1.000015 1.000009
##  [8] 1.000015 1.000009 1.000015
## At iteration 17 f=0.000000 max(abs(g))=0.000530
##
## ============================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ============================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797
##  [8] 0.9998104 0.9996797 0.9998104
## At iteration 18 f=0.000102 max(abs(g))=0.090157
##
## ============================================================================
## Before call f=0.000102 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 1.000007 1.000014 1.000007 1.000014 1.000007 1.000014 1.000007
##  [8] 1.000014 1.000007 1.000014
## At iteration 19 f=0.000000 max(abs(g))=0.000006
##
## ============================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
```

```
## 
## ========================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 1 1 1 1 1 1 1 1 1 1
## At iteration 20 f=0.000000 max(abs(g))=0.000000
## 
## ========================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
## 
## ========================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
## ex_rosen try for n= 12
##  [1] -1.2  1.0 -1.2  1.0 -1.2  1.0 -1.2  1.0 -1.2  1.0 -1.2  1.0
## N = 12, M = 5 machine precision = 2.22045e-16
## The initial X is infeasible.  Restart with its projection.
## At X0, 6 variables are exactly at the bounds
## At iterate     0   f=            339   |proj g|=          2.5
## At iterate    10   f =   1.4838e-05   |proj g|=     0.021128
## 
## iterations 14
## function evaluations 20
## segments explored during Cauchy searches 25
## BFGS updates skipped 0
## active bounds at final generalized Cauchy point 0
## norm of the final projected gradient 2.31163e-08
## final function value 2.411e-16
## 
## F = 2.411e-16
## final  value 0.000000
## converged
## 
## ========================================================================
## Before call f=179769313486231570814527423731704356798070567525844996598917476803157260780002853876058
## returned from lbfgsb3
## returned itask is 21 or "FG_START"
## computing f and g at prm=
##  [1] 0.5 1.0 0.5 1.0 0.5 1.0 0.5 1.0 0.5 1.0 0.5 1.0
## At iteration 0 f=339.000000 max(abs(g))=150.000000
## 
## ========================================================================
## Before call f=339.000000 task number 21, or "FG_START"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 3.0 0.5 3.0 0.5 3.0 0.5 3.0 0.5 3.0 0.5 3.0 0.5
## At iteration 2 f=43374.000000 max(abs(g))=1700.000000
## 
```

```
## ================================================================================
## Before call f=43374.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684 0.9239463 0.8802684
##  [8] 0.9239463 0.8802684 0.9239463 0.8802684 0.9239463
## At iteration 3 f=13.419825 max(abs(g))=29.814773
##
## ================================================================================
## Before call f=13.419825 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=13.419825 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910 0.9323731 1.1176910
##  [8] 0.9323731 1.1176910 0.9323731 1.1176910 0.9323731
## At iteration 4 f=60.323323 max(abs(g))=63.372032
##
## ================================================================================
## Before call f=60.323323 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399 0.9267031 0.9579399
##  [8] 0.9267031 0.9579399 0.9267031 0.9579399 0.9267031
## At iteration 5 f=0.059803 max(abs(g))=1.810862
##
## ================================================================================
## Before call f=0.059803 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.059803 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.963677 0.927100 0.963677 0.927100 0.963677 0.927100 0.963677
##  [8] 0.927100 0.963677 0.927100 0.963677 0.927100
## At iteration 6 f=0.009401 max(abs(g))=0.314675
##
## ================================================================================
## Before call f=0.009401 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.009401 task number 1, or "NEW_X"
## returned from lbfgsb3
```

```
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480 0.9270802 0.9629480
##  [8] 0.9270802 0.9629480 0.9270802 0.9629480 0.9270802
## At iteration 7 f=0.008258 max(abs(g))=0.001478
##
## ================================================================================
## Before call f=0.008258 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.008258 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685 0.9271132 0.9629685
##  [8] 0.9271132 0.9629685 0.9271132 0.9629685 0.9271132
## At iteration 8 f=0.008251 max(abs(g))=0.001048
##
## ================================================================================
## Before call f=0.008251 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504 0.9272453 0.9630504
##  [8] 0.9272453 0.9630504 0.9272453 0.9630504 0.9272453
## At iteration 9 f=0.008221 max(abs(g))=0.011159
##
## ================================================================================
## Before call f=0.008221 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784 0.9277737 0.9633784
##  [8] 0.9277737 0.9633784 0.9277737 0.9633784 0.9277737
## At iteration 10 f=0.008110 max(abs(g))=0.051670
##
## ================================================================================
## Before call f=0.008110 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.008110 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509 0.9358966 0.9679509
##  [8] 0.9358966 0.9679509 0.9358966 0.9679509 0.9358966
## At iteration 11 f=0.006802 max(abs(g))=0.206451
##
## ================================================================================
## Before call f=0.006802 task number 20, or "FG_LNSRCH"
```

```
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=0.006802 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907 0.9504056 0.9756907
##  [8] 0.9504056 0.9756907 0.9504056 0.9756907 0.9504056
## At iteration 12 f=0.005019 max(abs(g))=0.313355
##
## ==============================================================================
## Before call f=0.005019 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=0.005019 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682 0.9709040 0.9861682
##  [8] 0.9709040 0.9861682 0.9709040 0.9861682 0.9709040
## At iteration 13 f=0.002730 max(abs(g))=0.324762
##
## ==============================================================================
## Before call f=0.002730 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=0.002730 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881 0.9893577 0.9950881
##  [8] 0.9893577 0.9950881 0.9893577 0.9950881 0.9893577
## At iteration 14 f=0.000571 max(abs(g))=0.168543
##
## ==============================================================================
## Before call f=0.000571 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ==============================================================================
## Before call f=0.000571 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471 0.9970361 0.9985471
##  [8] 0.9970361 0.9985471 0.9970361 0.9985471 0.9970361
## At iteration 15 f=0.000015 max(abs(g))=0.012034
##
```

```
## ================================================================================
## Before call f=0.000015 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000015 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##   [1] 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188 0.9996336 0.9998188
##   [8] 0.9996336 0.9998188 0.9996336 0.9998188 0.9996336
## At iteration 16 f=0.000000 max(abs(g))=0.000800
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##   [1] 1.000009 1.000015 1.000009 1.000015 1.000009 1.000015 1.000009
##   [8] 1.000015 1.000009 1.000015 1.000009 1.000015
## At iteration 17 f=0.000000 max(abs(g))=0.000530
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## ================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##   [1] 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797 0.9998104 0.9996797
##   [8] 0.9998104 0.9996797 0.9998104 0.9996797 0.9998104
## At iteration 18 f=0.000123 max(abs(g))=0.090157
##
## ================================================================================
## Before call f=0.000123 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##   [1] 1.000007 1.000014 1.000007 1.000014 1.000007 1.000014 1.000007
##   [8] 1.000014 1.000007 1.000014 1.000007 1.000014
## At iteration 19 f=0.000000 max(abs(g))=0.000006
##
## ================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
```

```
## returned itask is 1 or "NEW_X"
##
## =====================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 20 or "FG_LNSRCH"
## computing f and g at prm=
##  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1
## At iteration 20 f=0.000000 max(abs(g))=0.000000
##
## =====================================================================================
## Before call f=0.000000 task number 20, or "FG_LNSRCH"
## returned from lbfgsb3
## returned itask is 1 or "NEW_X"
##
## =====================================================================================
## Before call f=0.000000 task number 1, or "NEW_X"
## returned from lbfgsb3
## returned itask is 8 or "CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH"
```

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