SNewton: safeguarded Newton methods for function minimization

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Safeguarded Newton algorithms

So-called **Newton** methods are among the most commonly mentioned in the solution of nonlinear equations or function minimization. However, as discussed in https://en.wikipedia.org/wiki/Newton%27s_method#History, the **Newton** or **Newton-Raphson** method as we know it today was not what either of its supposed originators knew.

This vignette discusses the development of simple safeguarded variants of the Newton method for function minimization in **R**. Note that there are some resources in **R** for solving nonlinear equations by Newton-like methods in the packages **nleqslv** and **pracma**.

The basic approach

If we have a function $f(x)$, with gradient $g(x)$ and second derivative (Hessian) $H(x)$ the first order condition for an extremum (min or max) is

$$g(x) = 0$$

To ensure a minimum, we want

$$H(x) > 0$$

The first order condition leads to a root-finding problem.

It turns out that x need not be a scalar. We can consider it to be a vector of parameters to be determined. This renders $g(x)$ a vector also, and $H(x)$ a matrix. The conditions of optimality then require a zero gradient and positive-definite Hessian.

The Newton approach to such equations is to provide a guess to the root x_try and to then solve the equation

$$H(x_t) * s = -g(x_t)$$

for the search vector s. We update x_t to $x_t + s$ and repeat until we have a very small gradient $g(x_t)$. If $H(x)$ is positive definite, we have a reasonable approximation to a (local) minimum.

Motivations

A particular interest in Newton-like methods its theoretical quadratic convergence. See https://en.wikipedia.org/wiki/Newton%27s_method. That is, the method will converge in one step for a quadratic function $f(x)$, and for "reasonable" functions will converge very rapidly. There are, however, a number of conditions, and practical programs need to include safequards against mis-steps in the iterations.

The principal issues concern the possiblity that $H(x)$ may not be positive definite, at least in some parts of the domain, and that the curvature may be such that a unit step $x_t + s$ does not reduce the function f. We therefore get a number of possible variants of the method when different possible safeguards are applied.

Algorithm possibilities

There are many choices we can make in building a practical code to implement the ideas above. In tandem with the two main issues expressed above, we will consider

- the modification of the solution of the main equation

  $$H(x_t) * s = -g(x_t)$$

  so that a reasonable search vector s is always generated by avoiding Hessian matrices that are not positive definite.

- the selection of a new set of parameters $x_new = x_t + step * s$ so that the function value $f(x_new)$ is less than $f(x_t)$.

The second choice above could be made slightly more stringent so that the Armijo (??ref) condition of sufficient-decrease is met. Adding a curvature requirement gives the Wolfe condisions. See https://en.wikipedia.org/wiki/Wolfe_conditions. The Armijo requirement is generally written

$$f(x_t + step * s) < f(x_t) + c * step * g(x_t)^T * s$$

where c is some number less than 1. Typically \$ c = 1e-4 = 0.0001 \$. Note that the product of gradient times search vector is negative for any reasonable situation, since we are trying to go "downhill".

As a result of the ideas in this section, the code `snewton()` uses a solution of the Newton equations with the Hessian provided (if this is possible, else we stop), along with a backtracking line search. The code `snewtonm` uses a Marquardt stabilization of the Hessian to create

$$Haug = H + 1_n * lambda$$

That is, we add lambda times the unit matrix to H. Then we try the set of parameters found by adding the solution of the Newton equations with $Haug$ in place of H to the current "best" set of parameters. If this new set of parameters has a higher function value than the "best" so far, we increase lambda and try again. Note that we do not need to re-evaluate the gradient or Hessian to do this. Moreover, for some value of lambda, the step is clearly down the gradient (i.e., steepest descents) or we have converged and no progress is possible. This leads to a very compact and elegant code. It is reliable, but may be less efficient than using the un-modified Hessian.

A choice to compute the search vector

The primary concern in solving for s is that the Hessian may not be positive definite. This means that we cannot apply fast and stable methods like the Cholesky decomposition to the matrix. At the time of writing, we use the following approach:

- We attempt to solve

  $$H(x_t) * s = -g(x_t)$$

with **R** directly, and rely on internal checks to catch any cases where the solution fails. We then use `try()` to stop the program in this case.

Choosing the step size

The traditional Newton approach is that the stepsize is taken to be 1. In practice, this can sometimes mean that the function value is not reduced. As an alternative, we can use a simple backtrack search. We start with $step = 1$ (actually the program allows for the element `defstep` of the `control` list to be set to a value other than 1). If the Armijo condition is not met, we replace *step* with $ r * step $ where r is less than 1. Here we suggest `control$stepdec = 0.2`. We repeat until x_t satisfies the Armijo condition or x_t is essentially unchanged by the step.

Here "essentially unchanged" is determined by a test using an offset value, that is, the test

$$(x_t + offset) == (x_t + step * d + offset)$$

where d is the search direction. `control$offset = 100` is used. We could also, and almost equivalently, use the **R** `identical` function.

Examples

These examples are coded as test to the package **snewton**.

A simple example

The following example is trivial, in that the Hessian is a constant matrix, and we achieve convergence immediately.

```
# Try testing calls to see what is transferred (eventually test also ...)
# setup
x0<-c(1,2,3,4)
fnt <- function(x, fscale=10){
  yy <- length(x):1
  val <- sum((yy*x)^2)*fscale
}
grt <- function(x, fscale=10){
  nn <- length(x)
  yy <- nn:1
  #    gg <- rep(NA,nn)
  gg <- 2*(yy^2)*x*fscale
  gg
}

hesst <- function(x, fscale=10){
  nn <- length(x)
  yy <- nn:1
  hh <- diag(2*yy^2*fscale)
  hh
}

library(snewton)
t1 <- snewton(x0, fnt, grt, hesst, control=list(trace=2), fscale=3.0)
```

```
## trace = 2
## Initial function value =  312
```

```
## [1] 1 2 3 4
## Termination test:nf= 1
## current gradient norm = 108
## Iteration  1 :Search vector:[1] -1 -2 -3 -4
## Gradient projection =  -624
## f(xnew)= 0
## end major loop
## Termination test:nf= 2
## current gradient norm = 0
## Small gradient norm
```

```
print(t1)
```

```
## $par
## [1] 0 0 0 0
##
## $value
## [1] 0
##
## $grad
## [1] 0 0 0 0
##
## $Hess
##      [,1] [,2] [,3] [,4]
## [1,]   96    0    0    0
## [2,]    0   54    0    0
## [3,]    0    0   24    0
## [4,]    0    0    0    6
##
## $counts
## $counts$niter
## [1] 2
##
## $counts$nfn
## [1] 2
##
## $counts$ngr
## [1] 2
##
## $counts$nhess
## [1] 1
##
##
## $convcode
## [1] 0
```

```
# we can also use nlm and nlminb
fght <- function(x, fscale=10){
  ## combine f, g and h into single function for nlm
    ff <- fnt(x, fscale)
    gg <- grt(x, fscale)
    hh <- hesst(x, fscale)
    attr(ff, "gradient") <- gg
    attr(ff, "hessian") <- hh
    ff
```

```
}

t1nlm <- nlm(fght, x0, fscale=3.0, hessian=TRUE, print.level=1)
```

```
## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] 1 2 3 4
## Function Value
## [1] 1040
## Gradient:
## [1] 320 360 240  80
##
## iteration = 1
## Parameter:
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
## Function Value
## [1] 1.232595e-29
## Gradient:
## [1] 0.000000e+00 3.996803e-14 3.552714e-14 0.000000e+00
##
## Relative gradient close to zero.
## Current iterate is probably solution.
```

```
print(t1nlm)
```

```
## $minimum
## [1] 1.232595e-29
##
## $estimate
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
##
## $gradient
## [1] 0.000000e+00 3.996803e-14 3.552714e-14 0.000000e+00
##
## $hessian
##              [,1] [,2] [,3]         [,4]
## [1,] 3.200000e+02    0    0 4.235165e-14
## [2,] 0.000000e+00  180    0 0.000000e+00
## [3,] 0.000000e+00    0   80 0.000000e+00
## [4,] 4.235165e-14    0    0 2.000000e+01
##
## $code
## [1] 1
##
## $iterations
## [1] 1
```

```
## BUT ... it looks like nlminb is NOT using a true Newton-type method
t1nlminb <- nlminb(x0, fnt, gradient=grt, hessian=hesst, fscale=3.0, control=list(trace=1))
```

```
##   0:     312.00000:  1.00000  2.00000  3.00000  4.00000
##   1:     31.277896: 0.0946591 0.313486 0.884680  2.50350
##   2: 7.4880156e-31: -1.38778e-17  0.00000 -1.11022e-16 -4.44089e-16
```

```
##   3: 3.6918767e-62: 3.08149e-33  0.00000 2.46519e-32 9.86076e-32
##   4: 1.8202358e-93: -6.84228e-49  0.00000 -5.47382e-48 -2.18953e-47
##   5:8.9744552e-125: 1.51929e-64  0.00000 1.21543e-63 4.86173e-63
##   6:4.4247480e-156: -3.37350e-80  0.00000 -2.69880e-79 -1.07952e-78
##   7:2.1815692e-187: 7.49068e-96  0.00000 5.99255e-95 2.39702e-94
##   8:1.0755967e-218: -1.66327e-111  0.00000 -1.33061e-110 -5.32245e-110
##   9:5.3031010e-250: 3.69319e-127  0.00000 2.95455e-126 1.18182e-125
##  10:2.6146307e-281: -8.20053e-143  0.00000 -6.56043e-142 -2.62417e-141
##  11:1.2891124e-312: 1.82088e-158  0.00000 1.45671e-157 5.82683e-157
##  12:     0.0000000: -4.04317e-174  0.00000 -3.23454e-173 -1.29382e-172
##  13:     0.0000000: -4.04317e-174  0.00000 -3.23454e-173 -1.29382e-172
```

```
print(t1nlminb)
```

```
## $par
## [1] -4.043175e-174   0.000000e+00 -3.234540e-173 -1.293816e-172
##
## $objective
## [1] 0
##
## $convergence
## [1] 0
##
## $iterations
## [1] 13
##
## $evaluations
## function gradient
##       15       13
##
## $message
## [1] "relative convergence (4)"
```

```
# and call them from optimx (i.e., test this gives same results)

library(optimx)
t1nlmo <- optimr(x0, fnt, grt, hess=hesst, method="nlm", fscale=3.0, control=list(trace=1))
```

```
## Unit parameter scaling
## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] 1 2 3 4
## Function Value
## [1] 1040
## Gradient:
## [1] 320 360 240  80
##
## iteration = 1
## Parameter:
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
## Function Value
## [1] 1.232595e-29
## Gradient:
```

```
## [1] 0.000000e+00 3.996803e-14 3.552714e-14 0.000000e+00
##
## Relative gradient close to zero.
## Current iterate is probably solution.
```

```
print(t1nlmo)
```

```
## $convergence
## [1] 0
##
## $value
## [1] 1.232595e-29
##
## $par
## [1] 0.000000e+00 2.220446e-16 4.440892e-16 0.000000e+00
##
## $counts
## [1] NA  1
##
## $message
## [1] "Convergence indicator (code) =  1"
```

```
## FOLLOWING SHOWS UP ERRORS??
tst <- try(t1nlminbo <- optimr(x0, fnt, grt, hessian=hesst, method="nlminb", fscale=3.0, control=list(t
```

```
## Unit parameter scaling
##   0:     312.00000:  1.00000  2.00000  3.00000  4.00000
##   1:     95.955813: -0.557075 0.248290  1.83219  3.61073
##   2:     51.541989: 0.315728 0.0294715  1.11454  3.25716
##   3:     36.151761: -0.496527 -0.0131770 0.397712  2.73344
##   4:     15.857662: 0.221301 0.308983 0.0101056  1.90858
##   5:     5.6451087: 0.00686801 -0.261014 0.325348 0.918905
##   6:     2.6284218: -0.0137242 0.179195 0.0814766 0.746709
##   7:    0.39384020: -0.0599550 -0.00435873 0.0526086 0.250049
##   8:    0.22079172: 0.0341382 -0.000510906 0.0319676 0.225523
##   9:   0.072938228: 0.00709733 0.00847916 0.0382438 0.130420
##  10:   0.051625575: -0.0201629 -0.00984016 0.00152106 0.0991118
##  11:  0.0079613002: -0.00575797 -0.00297347 0.00666463 0.0431979
##  12: 0.00046215706: 0.000813264 0.00185374 -0.00499623 0.00356276
##  13: 4.5405825e-05: -0.000801080 -0.000454321 5.39450e-05 -0.00173156
##  14: 8.6153740e-07: 9.98997e-05 -0.000118763 9.38077e-06 1.43821e-05
##  15: 1.1416255e-07: -1.09926e-05 6.28428e-05 8.45952e-06 -1.70757e-05
##  16: 1.1814039e-09: -2.87417e-06 -2.87063e-06 -1.85946e-06 1.31770e-05
##  17: 6.5537434e-11: 7.11902e-07 -5.23279e-08 3.15833e-07 -3.64874e-06
##  18: 2.3717735e-14: 2.25678e-09 2.46297e-08 5.18943e-09 4.75088e-08
##  19: 5.0615498e-17: -5.26531e-10 -1.02086e-09 -3.95557e-10 -1.55911e-09
##  20: 4.4353627e-21: 8.49409e-12 -4.31436e-12 2.54218e-12 1.14318e-11
##  21: 3.2800917e-24: -1.75429e-13 2.29273e-13 -2.49701e-14 -3.54079e-13
##  22: 1.6457267e-29: -2.51767e-16 -5.62293e-16 -2.15602e-16 1.20003e-15
##  23: 5.7043891e-34: 2.34737e-18 1.87634e-18 1.31560e-18 -7.96082e-18
##  24: 3.3869900e-40: -8.75589e-22 2.31612e-21 5.18664e-22 7.16082e-21
##  25: 2.2802943e-45: 1.25249e-25 -6.02277e-24 -6.41554e-24 -1.63935e-23
##  26: 2.6790444e-50: -2.51874e-27 -5.56166e-27 4.62201e-26 -2.24785e-27
##  27: 2.4360441e-54: 3.95778e-29 1.20710e-28 -3.89614e-28 2.20498e-28
##  28: 3.7594442e-61: -2.97704e-32 -7.48758e-32 -3.02365e-32 -2.38788e-31
```

```
##  29: 3.8487517e-67: 3.29444e-35 5.73233e-35 8.56820e-35 2.28007e-34
##  30: 2.6805234e-75: -5.07665e-39 5.77851e-39 -6.56781e-39 -2.84334e-39
##  31: 9.5203946e-82: 3.40098e-42 -3.82627e-42 5.52080e-44 7.10119e-43
##  32: 9.2490490e-90: -1.95390e-46 7.94860e-47 7.74801e-46 -1.18536e-46
##  33: 2.7852864e-97: 1.06935e-50 1.64612e-50 -1.48312e-49 2.42735e-50
##  34:6.2041713e-107: 7.38775e-55 -9.42238e-55 9.36985e-55 -6.67742e-55
##  35:8.3236608e-116: -3.45037e-59 7.28642e-60 -4.43881e-60 9.02262e-59
##  36:1.1974396e-123: 1.90297e-63 3.91110e-63 3.07684e-64 -1.42533e-62
##  37:1.0147936e-131: 8.21922e-68 -5.26631e-67 -1.99652e-68 8.81420e-67
##  38:8.6173489e-141: -1.01460e-71 1.14575e-71 1.76633e-74 6.62779e-72
##  39:4.1589990e-150: 2.43424e-76 -5.21738e-77 9.03871e-78 -6.42980e-76
##  40:2.0106646e-160: -7.67394e-82 -1.58766e-81 -9.60001e-83 5.90567e-81
##  41:5.5292711e-171: -2.09144e-87 1.20732e-86 2.73266e-88 -2.14698e-86
##  42:1.6188979e-182: 1.58201e-92 -1.23116e-92 2.07748e-93 3.23763e-93
##  43:2.8259611e-193: -5.60882e-98 -4.53753e-98 -3.39553e-98 -1.43953e-97
##  44:5.9022515e-203: 3.57781e-103 9.39990e-103 4.83975e-103 2.95582e-102
##  45:2.0276756e-214: 1.37317e-108 -1.27757e-108 -1.95432e-109 -4.75158e-108
##  46:1.4216054e-225: -5.07833e-114 1.28093e-114 -7.40660e-115 6.65398e-114
##  47:1.5179352e-238: 1.27121e-120 1.17833e-120 1.00773e-120 2.86077e-120
##  48:4.4176738e-251: -1.94221e-127 -6.77404e-127 -1.30624e-126 -1.77963e-126
##  49:1.7660155e-262: -3.79783e-133 -3.56811e-133 3.67673e-132 -1.15773e-132
##  50:1.6392324e-273: 1.76691e-138 3.50784e-138 -8.49700e-138 9.84471e-138
##  51:2.0904081e-286: -5.48383e-145 -1.96413e-144 -1.05454e-144 -5.06954e-144
##  52:5.7398433e-299: 1.46757e-151 8.28441e-151 1.46922e-150 1.99423e-150
##  53:2.1949635e-313: 1.48736e-158 4.50935e-159 -1.27714e-157 6.47993e-158
##  54:     0.0000000: -2.35751e-165 1.54288e-166 8.35366e-165 -2.24188e-164
```

```
## Warning in nlminb(start = spar, objective = efn, gradient = egr, lower =
## slower, : NA/NaN function evaluation
```

```
##  55:     0.0000000: -2.35751e-165 1.54288e-166 8.35366e-165 -2.24188e-164
```

```
if (class(tst) == "try-error"){
    cat("try-error on attempt to run nlminb in optimr()\n")
} else { print(t1nlminb) }
```

```
## try-error on attempt to run nlminb in optimr()
```

The Rosenbrock function

```
require(snewton)
#Rosenbrock banana valley function
f <- function(x){
return(100*(x[2] - x[1]*x[1])^2 + (1-x[1])^2)
}
#gradient
gr <- function(x){
return(c(-400*x[1]*(x[2] - x[1]*x[1]) - 2*(1-x[1]), 200*(x[2] - x[1]*x[1])))
}
#Hessian
h <- function(x) {
a11 <- 2 - 400*x[2] + 1200*x[1]*x[1]; a21 <- -400*x[1]
return(matrix(c(a11, a21, a21, 200), 2, 2))
}
```

```r
x0 <- c(-1.2, 1)

xx <- x0

# sink("mbrn1-170408.txt", split=TRUE)
t1 <- snewton(x0, fn=f, gr=gr, hess=h, control=list(trace=2))
```

```
## trace = 2
## Initial function value =  24.2
## [1] -1.2  1.0
## Termination test:nf= 1
## current gradient norm = 215.6
## Iteration  1 :Search vector:[1] 0.0247191 0.3806742
## Gradient projection =  -38.82876
## f(xnew)= 4.731884
## end major loop
## Termination test:nf= 2
## current gradient norm = 4.637816
## Iteration  2 :Search vector:[1]  1.938396 -4.555708
## Gradient projection =  -8.433185
## f(xnew)= 1411.845
## Stepsize now = 0.2
## * f(xnew)= 5.4691
## Stepsize now = 0.04
## * f(xnew)= 4.404888
## end major loop
## Termination test:nf= 5
## current gradient norm = 7.092834
## Iteration  3 :Search vector:[1]  0.9043254 -1.9788390
## Gradient projection =  -3.802796
## f(xnew)= 68.30465
## Stepsize now = 0.2
## * f(xnew)= 3.81876
## end major loop
## Termination test:nf= 7
## current gradient norm = 17.767
## Iteration  4 :Search vector:[1]  0.2229402 -0.3708280
## Gradient projection =  -1.143361
## f(xnew)= 3.116464
## end major loop
## Termination test:nf= 8
## current gradient norm = 17.18405
## Iteration  5 :Search vector:[1]  0.1548325 -0.1651864
## Gradient projection =  -1.018618
## f(xnew)= 2.426322
## end major loop
## Termination test:nf= 9
## current gradient norm = 8.247841
## Iteration  6 :Search vector:[1]  0.2656098 -0.2624113
## Gradient projection =  -0.9325458
## f(xnew)= 2.119506
## end major loop
## Termination test:nf= 10
```

```
## current gradient norm = 14.10971
## Iteration  7 :Search vector:[1] 0.08428336 0.02444600
## Gradient projection =  -1.21009
## f(xnew)= 1.419276
## end major loop
## Termination test:nf= 11
## current gradient norm = 2.916075
## Iteration  8 :Search vector:[1]  0.4912612 -0.1788034
## Gradient projection =  -1.178522
## f(xnew)= 6.31152
## Stepsize now = 0.2
## * f(xnew)= 1.213719
## end major loop
## Termination test:nf= 13
## current gradient norm = 3.06729
## Iteration  9 :Search vector:[1]  0.26822820 -0.03346064
## Gradient projection =  -0.6322948
## f(xnew)= 1.194519
## end major loop
## Termination test:nf= 14
## current gradient norm = 14.38927
## Iteration  10 :Search vector:[1] 0.05346150 0.09090021
## Gradient projection =  -1.123225
## f(xnew)= 0.5925966
## end major loop
## Termination test:nf= 15
## current gradient norm = 1.274764
## Iteration  11 :Search vector:[1] 0.4894752 0.2287292
## Gradient projection =  -0.7547131
## f(xnew)= 5.81843
## Stepsize now = 0.2
## * f(xnew)= 0.4648368
## end major loop
## Termination test:nf= 17
## current gradient norm = 2.373989
## Iteration  12 :Search vector:[1] 0.1989862 0.1426528
## Gradient projection =  -0.2953687
## f(xnew)= 0.3799336
## end major loop
## Termination test:nf= 18
## current gradient norm = 7.919099
## Iteration  13 :Search vector:[1] 0.05296398 0.09548404
## Gradient projection =  -0.3636001
## f(xnew)= 0.1767059
## end major loop
## Termination test:nf= 19
## current gradient norm = 0.5610366
## Iteration  14 :Search vector:[1] 0.2686849 0.3147876
## Gradient projection =  -0.2269612
## f(xnew)= 0.5438857
## Stepsize now = 0.2
## * f(xnew)= 0.1363627
## end major loop
## Termination test:nf= 21
```

```
## current gradient norm = 1.026362
## Iteration  15 :Search vector:[1] 0.1804663 0.2340749
## Gradient projection =  -0.1372565
## f(xnew)= 0.1403758
## Stepsize now = 0.2
## * f(xnew)= 0.1115589
## end major loop
## Termination test:nf= 23
## current gradient norm = 1.081634
## Iteration  16 :Search vector:[1] 0.1583356 0.2177055
## Gradient projection =  -0.1102234
## f(xnew)= 0.09218166
## end major loop
## Termination test:nf= 24
## current gradient norm = 7.968122
## Iteration  17 :Search vector:[1] 0.02847694 0.07227004
## Gradient projection =  -0.1354565
## f(xnew)= 0.0204531
## end major loop
## Termination test:nf= 25
## current gradient norm = 0.1621872
## Iteration  18 :Search vector:[1] 0.1228582 0.2114429
## Gradient projection =  -0.03521596
## f(xnew)= 0.02318036
## Stepsize now = 0.2
## * f(xnew)= 0.0141311
## end major loop
## Termination test:nf= 27
## current gradient norm = 0.2505029
## Iteration  19 :Search vector:[1] 0.09453205 0.16796686
## Gradient projection =  -0.02266352
## f(xnew)= 0.00854653
## end major loop
## Termination test:nf= 28
## current gradient norm = 3.442516
## Iteration  20 :Search vector:[1] 0.008495991 0.025525911
## Gradient projection =  -0.0163739
## f(xnew)= 0.0002310919
## end major loop
## Termination test:nf= 29
## current gradient norm = 0.01443637
## Iteration  21 :Search vector:[1] 0.01496847 0.02955454
## Gradient projection =  -0.0004556213
## f(xnew)= 5.066762e-06
## end major loop
## Termination test:nf= 30
## current gradient norm = 0.08917048
## Iteration  22 :Search vector:[1] 0.0002068225 0.0006376106
## Gradient projection =  -1.012952e-05
## f(xnew)= 8.60774e-11
## end major loop
## Termination test:nf= 31
## current gradient norm = 8.555107e-06
## Iteration  23 :Search vector:[1] 9.267845e-06 1.857829e-05
```

```
## Gradient projection =  -1.721533e-10
## f(xnew)= 7.440484e-19
## end major loop
## Termination test:nf= 32
## current gradient norm = 3.419865e-08
## Iteration  24 :Search vector:[1] 7.928735e-11 2.444678e-10
## Gradient projection =  -1.488097e-18
## f(xnew)= 0
## end major loop
## Termination test:nf= 33
## current gradient norm = 0
## Small gradient norm
```

```
print(t1)
```

```
## $par
## [1] 1 1
##
## $value
## [1] 0
##
## $grad
## [1] 0 0
##
## $Hess
##      [,1] [,2]
## [1,]  802 -400
## [2,] -400  200
##
## $counts
## $counts$niter
## [1] 25
##
## $counts$nfn
## [1] 33
##
## $counts$ngr
## [1] 25
##
## $counts$nhess
## [1] 24
##
##
## $convcode
## [1] 0
```

```
# we can also use nlm and nlminb
fght <- function(x){
  ## combine f, g and h into single function for nlm
    ff <- f(x)
    gg <- gr(x)
    hh <- h(x)
    attr(ff, "gradient") <- gg
    attr(ff, "hessian") <- hh
    ff
```

```
}

## ?? SEEMS NOT TO WORK RIGHT!!
t1nlm <- nlm(fght, x0, hessian=TRUE, print.level=2)
```

```
## iteration = 0
## Step:
## [1] 0 0
## Parameter:
## [1] -1.2  1.0
## Function Value
## [1] 24.2
## Gradient:
## [1] -215.6  -88.0
##
## iteration = 1
## Step:
## [1] 0.11917904 0.04942029
## Parameter:
## [1] -1.080821  1.049420
## Function Value
## [1] 5.740059
## Gradient:
## [1] -55.50222 -23.75073
##
## iteration = 2
## Step:
## [1] 0.04232135 0.00930736
## Parameter:
## [1] -1.038500  1.058728
## Function Value
## [1] 4.194502
## Gradient:
## [1] -12.282717  -3.950757
##
## iteration = 3
## Step:
## [1]  0.011967013 -0.001777633
## Parameter:
## [1] -1.026533  1.056950
## Function Value
## [1] 4.107846
## Gradient:
## [1] -2.7469658  0.6361704
##
## iteration = 4
## Step:
## [1]  0.004223361 -0.004154668
## Parameter:
## [1] -1.022309  1.052795
## Function Value
## [1] 4.095632
## Gradient:
## [1] -0.9044183  1.5358367
```

```
##
## iteration = 5
## Step:
## [1]  0.002676889 -0.004622149
## Parameter:
## [1] -1.019632  1.048173
## Function Value
## [1] 4.086179
## Gradient:
## [1] -0.5630994  1.7046170
##
## iteration = 6
## Step:
## [1]  0.002397440 -0.004721889
## Parameter:
## [1] -1.017235  1.043451
## Function Value
## [1] 4.076779
## Gradient:
## [1] -0.500814  1.736893
##
## iteration = 7
## Step:
## [1]  0.002356612 -0.004753367
## Parameter:
## [1] -1.014878  1.038698
## Function Value
## [1] 4.067338
## Gradient:
## [1] -0.489862  1.744000
##
## iteration = 8
## Step:
## [1]  0.002360087 -0.004772393
## Parameter:
## [1] -1.012518  1.033926
## Function Value
## [1] 4.057855
## Gradient:
## [1] -0.488336  1.746487
##
## iteration = 9
## Step:
## [1]  0.002371890 -0.004789282
## Parameter:
## [1] -1.010146  1.029136
## Function Value
## [1] 4.048328
## Gradient:
## [1] -0.4885411  1.7481386
##
## iteration = 10
## Step:
## [1]  0.002385388 -0.004805938
```

```
## Parameter:
## [1] -1.007761  1.024330
## Function Value
## [1] 4.038757
## Gradient:
## [1] -0.4890652  1.7496494
##
## iteration = 11
## Step:
## [1]  0.002399363 -0.004822717
## Parameter:
## [1] -1.005362  1.019508
## Function Value
## [1] 4.029141
## Gradient:
## [1] -0.4896489  1.7511482
##
## iteration = 12
## Step:
## [1]  0.002413592 -0.004839688
## Parameter:
## [1] -1.002948  1.014668
## Function Value
## [1] 4.01948
## Gradient:
## [1] -0.4902447  1.7526588
##
## iteration = 13
## Step:
## [1]  0.002428041 -0.004856866
## Parameter:
## [1] -1.000520  1.009811
## Function Value
## [1] 4.009773
## Gradient:
## [1] -0.4908439  1.7541859
##
## iteration = 14
## Step:
## [1]  0.002442705 -0.004874260
## Parameter:
## [1] -0.9980772  1.0049368
## Function Value
## [1] 4.000019
## Gradient:
## [1] -0.4914446  1.7557308
##
## iteration = 15
## Step:
## [1]  0.002457589 -0.004891873
## Parameter:
## [1] -0.9956196  1.0000449
## Function Value
## [1] 3.990218
```

```
## Gradient:
## [1] -0.4920467  1.7572939
##
## iteration = 16
## Step:
## [1]  0.002472698 -0.004909711
## Parameter:
## [1] -0.9931469  0.9951352
## Function Value
## [1] 3.980369
## Gradient:
## [1] -0.492650  1.758876
##
## iteration = 17
## Step:
## [1]  0.002488037 -0.004927778
## Parameter:
## [1] -0.9906589  0.9902074
## Function Value
## [1] 3.970471
## Gradient:
## [1] -0.4932543  1.7604765
##
## iteration = 18
## Step:
## [1]  0.002503612 -0.004946080
## Parameter:
## [1] -0.9881553  0.9852614
## Function Value
## [1] 3.960524
## Gradient:
## [1] -0.4938597  1.7620970
##
## iteration = 19
## Step:
## [1]  0.002519428 -0.004964622
## Parameter:
## [1] -0.9856359  0.9802967
## Function Value
## [1] 3.950527
## Gradient:
## [1] -0.4944661  1.7637374
##
## iteration = 20
## Step:
## [1]  0.002535492 -0.004983409
## Parameter:
## [1] -0.9831004  0.9753133
## Function Value
## [1] 3.940479
## Gradient:
## [1] -0.4950733  1.7653983
##
## iteration = 21
```

```
## Step:
## [1]  0.002551809 -0.005002448
## Parameter:
## [1] -0.9805486  0.9703109
## Function Value
## [1] 3.930379
## Gradient:
## [1] -0.4956813  1.7670802
##
## iteration = 22
## Step:
## [1]  0.002568387 -0.005021743
## Parameter:
## [1] -0.9779802  0.9652891
## Function Value
## [1] 3.920227
## Gradient:
## [1] -0.4962899  1.7687835
##
## iteration = 23
## Step:
## [1]  0.002585231 -0.005041300
## Parameter:
## [1] -0.9753949  0.9602478
## Function Value
## [1] 3.910022
## Gradient:
## [1] -0.4968992  1.7705088
##
## iteration = 24
## Step:
## [1]  0.002602350 -0.005061127
## Parameter:
## [1] -0.9727926  0.9551867
## Function Value
## [1] 3.899763
## Gradient:
## [1] -0.497509  1.772257
##
## iteration = 25
## Step:
## [1]  0.002619749 -0.005081228
## Parameter:
## [1] -0.9701728  0.9501055
## Function Value
## [1] 3.889449
## Gradient:
## [1] -0.4981191  1.7740275
##
## iteration = 26
## Step:
## [1]  0.002637437 -0.005101612
## Parameter:
## [1] -0.9675354  0.9450039
```

```
## Function Value
## [1] 3.879079
## Gradient:
## [1] -0.4987295  1.7758220
##
## iteration = 27
## Step:
## [1]  0.002655422 -0.005122284
## Parameter:
## [1] -0.9648800  0.9398816
## Function Value
## [1] 3.868653
## Gradient:
## [1] -0.4993401  1.7776407
##
## iteration = 28
## Step:
## [1]  0.002673710 -0.005143252
## Parameter:
## [1] -0.9622063  0.9347383
## Function Value
## [1] 3.85817
## Gradient:
## [1] -0.4999507  1.7794843
##
## iteration = 29
## Step:
## [1]  0.002692311 -0.005164522
## Parameter:
## [1] -0.9595140  0.9295738
## Function Value
## [1] 3.847628
## Gradient:
## [1] -0.5005611  1.7813534
##
## iteration = 30
## Step:
## [1]  0.002711232 -0.005186104
## Parameter:
## [1] -0.9568027  0.9243877
## Function Value
## [1] 3.837027
## Gradient:
## [1] -0.5011712  1.7832486
##
## iteration = 31
## Step:
## [1]  0.002730484 -0.005208003
## Parameter:
## [1] -0.9540722  0.9191797
## Function Value
## [1] 3.826365
## Gradient:
## [1] -0.5017809  1.7851707
```

```
##
## iteration = 32
## Step:
## [1]  0.002750075 -0.005230229
## Parameter:
## [1] -0.9513222  0.9139495
## Function Value
## [1] 3.815643
## Gradient:
## [1] -0.50239  1.78712
##
## iteration = 33
## Step:
## [1]  0.002770014 -0.005252790
## Parameter:
## [1] -0.9485522  0.9086967
## Function Value
## [1] 3.804858
## Gradient:
## [1] -0.5029984  1.7890982
##
## iteration = 34
## Step:
## [1]  0.002790313 -0.005275694
## Parameter:
## [1] -0.9457618  0.9034210
## Function Value
## [1] 3.794009
## Gradient:
## [1] -0.5036057  1.7911052
##
## iteration = 35
## Step:
## [1]  0.002810981 -0.005298950
## Parameter:
## [1] -0.9429509  0.8981220
## Function Value
## [1] 3.783096
## Gradient:
## [1] -0.5042119  1.7931421
##
## iteration = 36
## Step:
## [1]  0.002832029 -0.005322569
## Parameter:
## [1] -0.9401188  0.8927995
## Function Value
## [1] 3.772118
## Gradient:
## [1] -0.5048167  1.7952097
##
## iteration = 37
## Step:
## [1]  0.002853468 -0.005346559
```

```
## Parameter:
## [1] -0.9372654  0.8874529
## Function Value
## [1] 3.761073
## Gradient:
## [1] -0.5054199  1.7973089
##
## iteration = 38
## Step:
## [1]  0.002875310 -0.005370931
## Parameter:
## [1] -0.9343901  0.8820820
## Function Value
## [1] 3.74996
## Gradient:
## [1] -0.5060212  1.7994406
##
## iteration = 39
## Step:
## [1]  0.002897568 -0.005395695
## Parameter:
## [1] -0.9314925  0.8766863
## Function Value
## [1] 3.738778
## Gradient:
## [1] -0.5066205  1.8016058
##
## iteration = 40
## Step:
## [1]  0.002920253 -0.005420863
## Parameter:
## [1] -0.9285722  0.8712654
## Function Value
## [1] 3.727525
## Gradient:
## [1] -0.5072173  1.8038054
##
## iteration = 41
## Step:
## [1]  0.002943380 -0.005446444
## Parameter:
## [1] -0.9256289  0.8658190
## Function Value
## [1] 3.716201
## Gradient:
## [1] -0.5078115  1.8060404
##
## iteration = 42
## Step:
## [1]  0.002966963 -0.005472452
## Parameter:
## [1] -0.9226619  0.8603465
## Function Value
## [1] 3.704804
```

```
## Gradient:
## [1] -0.5084028  1.8083119
##
## iteration = 43
## Step:
## [1]  0.002991015 -0.005498898
## Parameter:
## [1] -0.9196709  0.8548476
## Function Value
## [1] 3.693332
## Gradient:
## [1] -0.5089907  1.8106211
##
## iteration = 44
## Step:
## [1]  0.003015551 -0.005525796
## Parameter:
## [1] -0.9166553  0.8493218
## Function Value
## [1] 3.681785
## Gradient:
## [1] -0.5095751  1.8129691
##
## iteration = 45
## Step:
## [1]  0.003040588 -0.005553158
## Parameter:
## [1] -0.9136147  0.8437687
## Function Value
## [1] 3.67016
## Gradient:
## [1] -0.5101554  1.8153571
##
## iteration = 46
## Step:
## [1]  0.003066143 -0.005580998
## Parameter:
## [1] -0.9105486  0.8381877
## Function Value
## [1] 3.658457
## Gradient:
## [1] -0.5107313  1.8177865
##
## iteration = 47
## Step:
## [1]  0.003092231 -0.005609332
## Parameter:
## [1] -0.9074564  0.8325783
## Function Value
## [1] 3.646673
## Gradient:
## [1] -0.5113024  1.8202585
##
## iteration = 48
```

```
## Step:
## [1]  0.003118872 -0.005638173
## Parameter:
## [1] -0.9043375  0.8269402
## Function Value
## [1] 3.634808
## Gradient:
## [1] -0.5118683  1.8227745
##
## iteration = 49
## Step:
## [1]  0.003146084 -0.005667538
## Parameter:
## [1] -0.9011914  0.8212726
## Function Value
## [1] 3.622858
## Gradient:
## [1] -0.5124284  1.8253361
##
## iteration = 50
## Step:
## [1]  0.003173887 -0.005697443
## Parameter:
## [1] -0.8980175  0.8155752
## Function Value
## [1] 3.610824
## Gradient:
## [1] -0.5129824  1.8279447
##
## iteration = 51
## Step:
## [1]  0.003202302 -0.005727905
## Parameter:
## [1] -0.8948152  0.8098473
## Function Value
## [1] 3.598702
## Gradient:
## [1] -0.5135295  1.8306019
##
## iteration = 52
## Step:
## [1]  0.003231350 -0.005758943
## Parameter:
## [1] -0.8915839  0.8040883
## Function Value
## [1] 3.586492
## Gradient:
## [1] -0.5140693  1.8333095
##
## iteration = 53
## Step:
## [1]  0.003261055 -0.005790574
## Parameter:
## [1] -0.8883228  0.7982978
```

```
## Function Value
## [1] 3.574191
## Gradient:
## [1] -0.5146012  1.8360693
##
## iteration = 54
## Step:
## [1]  0.003291439 -0.005822819
## Parameter:
## [1] -0.8850314  0.7924750
## Function Value
## [1] 3.561797
## Gradient:
## [1] -0.5151245  1.8388830
##
## iteration = 55
## Step:
## [1]  0.003322530 -0.005855699
## Parameter:
## [1] -0.8817088  0.7866193
## Function Value
## [1] 3.549308
## Gradient:
## [1] -0.5156385  1.8417527
##
## iteration = 56
## Step:
## [1]  0.003354353 -0.005889234
## Parameter:
## [1] -0.8783545  0.7807300
## Function Value
## [1] 3.536723
## Gradient:
## [1] -0.5161424  1.8446804
##
## iteration = 57
## Step:
## [1]  0.003386935 -0.005923448
## Parameter:
## [1] -0.8749676  0.7748066
## Function Value
## [1] 3.524038
## Gradient:
## [1] -0.5166355  1.8476683
##
## iteration = 58
## Step:
## [1]  0.003420307 -0.005958365
## Parameter:
## [1] -0.8715473  0.7688482
## Function Value
## [1] 3.511252
## Gradient:
## [1] -0.5171169  1.8507187
```

```
##
## iteration = 59
## Step:
## [1]  0.003454499 -0.005994008
## Parameter:
## [1] -0.8680928  0.7628542
## Function Value
## [1] 3.498362
## Gradient:
## [1] -0.5175857  1.8538340
##
## iteration = 60
## Step:
## [1]  0.003489544 -0.006030405
## Parameter:
## [1] -0.8646032  0.7568238
## Function Value
## [1] 3.485366
## Gradient:
## [1] -0.5180409  1.8570169
##
## iteration = 61
## Step:
## [1]  0.003525477 -0.006067583
## Parameter:
## [1] -0.8610777  0.7507562
## Function Value
## [1] 3.472262
## Gradient:
## [1] -0.5184815  1.8602699
##
## iteration = 62
## Step:
## [1]  0.003562334 -0.006105572
## Parameter:
## [1] -0.8575154  0.7446506
## Function Value
## [1] 3.459046
## Gradient:
## [1] -0.5189064  1.8635959
##
## iteration = 63
## Step:
## [1]  0.003600152 -0.006144401
## Parameter:
## [1] -0.8539152  0.7385062
## Function Value
## [1] 3.445716
## Gradient:
## [1] -0.5193143  1.8669981
##
## iteration = 64
## Step:
## [1]  0.003638975 -0.006184102
```

```
## Parameter:
## [1] -0.8502763  0.7323221
## Function Value
## [1] 3.432269
## Gradient:
## [1] -0.5197039  1.8704795
##
## iteration = 65
## Step:
## [1]  0.003678843 -0.006224711
## Parameter:
## [1] -0.8465974  0.7260974
## Function Value
## [1] 3.418702
## Gradient:
## [1] -0.5200739  1.8740436
##
## iteration = 66
## Step:
## [1]  0.003719803 -0.006266263
## Parameter:
## [1] -0.8428776  0.7198312
## Function Value
## [1] 3.405012
## Gradient:
## [1] -0.5204227  1.8776940
##
## iteration = 67
## Step:
## [1]  0.003761905 -0.006308796
## Parameter:
## [1] -0.8391157  0.7135224
## Function Value
## [1] 3.391196
## Gradient:
## [1] -0.5207487  1.8814346
##
## iteration = 68
## Step:
## [1]  0.003805198 -0.006352349
## Parameter:
## [1] -0.8353105  0.7071700
## Function Value
## [1] 3.37725
## Gradient:
## [1] -0.5210502  1.8852694
##
## iteration = 69
## Step:
## [1]  0.003849738 -0.006396966
## Parameter:
## [1] -0.8314608  0.7007731
## Function Value
## [1] 3.363171
```

```
## Gradient:
## [1] -0.5213253  1.8892029
##
## iteration = 70
## Step:
## [1]  0.003895584 -0.006442692
## Parameter:
## [1] -0.8275652  0.6943304
## Function Value
## [1] 3.348955
## Gradient:
## [1] -0.521572  1.893240
##
## iteration = 71
## Step:
## [1]  0.003942798 -0.006489574
## Parameter:
## [1] -0.8236224  0.6878408
## Function Value
## [1] 3.334599
## Gradient:
## [1] -0.5217881  1.8973845
##
## iteration = 72
## Step:
## [1]  0.003991445 -0.006537663
## Parameter:
## [1] -0.8196310  0.6813031
## Function Value
## [1] 3.320097
## Gradient:
## [1] -0.5219711  1.9016429
##
## iteration = 73
## Step:
## [1]  0.004041597 -0.006587013
## Parameter:
## [1] -0.8155894  0.6747161
## Function Value
## [1] 3.305447
## Gradient:
## [1] -0.5221186  1.9060205
##
## iteration = 74
## Step:
## [1]  0.004093330 -0.006637682
## Parameter:
## [1] -0.8114960  0.6680784
## Function Value
## [1] 3.290643
## Gradient:
## [1] -0.5222276  1.9105235
##
## iteration = 75
```

```
## Step:
## [1]  0.004146724 -0.006689731
## Parameter:
## [1] -0.8073493  0.6613887
## Function Value
## [1] 3.275681
## Gradient:
## [1] -0.5222953  1.9151582
##
## iteration = 76
## Step:
## [1]  0.004201867 -0.006743224
## Parameter:
## [1] -0.8031474  0.6546455
## Function Value
## [1] 3.260556
## Gradient:
## [1] -0.5223182  1.9199318
##
## iteration = 77
## Step:
## [1]  0.004258851 -0.006798233
## Parameter:
## [1] -0.7988886  0.6478472
## Function Value
## [1] 3.245263
## Gradient:
## [1] -0.5222928  1.9248518
##
## iteration = 78
## Step:
## [1]  0.004317778 -0.006854831
## Parameter:
## [1] -0.7945708  0.6409924
## Function Value
## [1] 3.229796
## Gradient:
## [1] -0.5222152  1.9299264
##
## iteration = 79
## Step:
## [1]  0.004378754 -0.006913098
## Parameter:
## [1] -0.7901921  0.6340793
## Function Value
## [1] 3.21415
## Gradient:
## [1] -0.5220811  1.9351644
##
## iteration = 80
## Step:
## [1]  0.004441897 -0.006973119
## Parameter:
## [1] -0.7857502  0.6271062
```

```
## Function Value
## [1] 3.198318
## Gradient:
## [1] -0.5218858  1.9405752
##
## iteration = 81
## Step:
## [1]  0.004507332 -0.007034988
## Parameter:
## [1] -0.7812428  0.6200712
## Function Value
## [1] 3.182295
## Gradient:
## [1] -0.5216242  1.9461692
##
## iteration = 82
## Step:
## [1]  0.004575195 -0.007098802
## Parameter:
## [1] -0.7766676  0.6129724
## Function Value
## [1] 3.166073
## Gradient:
## [1] -0.5212907  1.9519576
##
## iteration = 83
## Step:
## [1]  0.004645633 -0.007164669
## Parameter:
## [1] -0.7720220  0.6058077
## Function Value
## [1] 3.149646
## Gradient:
## [1] -0.5208791  1.9579526
##
## iteration = 84
## Step:
## [1]  0.004718806 -0.007232703
## Parameter:
## [1] -0.7673032  0.5985750
## Function Value
## [1] 3.133005
## Gradient:
## [1] -0.5203826  1.9641674
##
## iteration = 85
## Step:
## [1]  0.004794887 -0.007303028
## Parameter:
## [1] -0.7625083  0.5912720
## Function Value
## [1] 3.116144
## Gradient:
## [1] -0.5197938  1.9706165
```

```
##
## iteration = 86
## Step:
## [1]  0.004874066 -0.007375779
## Parameter:
## [1] -0.7576342  0.5838962
## Function Value
## [1] 3.099053
## Gradient:
## [1] -0.5191043  1.9773157
##
## iteration = 87
## Step:
## [1]  0.004956550 -0.007451103
## Parameter:
## [1] -0.7526777  0.5764451
## Function Value
## [1] 3.081723
## Gradient:
## [1] -0.5183051  1.9842824
##
## iteration = 88
## Step:
## [1]  0.005042564 -0.007529156
## Parameter:
## [1] -0.7476351  0.5689160
## Function Value
## [1] 3.064144
## Gradient:
## [1] -0.517386  1.991536
##
## iteration = 89
## Step:
## [1]  0.005132356 -0.007610113
## Parameter:
## [1] -0.7425028  0.5613058
## Function Value
## [1] 3.046307
## Gradient:
## [1] -0.5163357  1.9990968
##
## iteration = 90
## Step:
## [1]  0.005226199 -0.007694161
## Parameter:
## [1] -0.7372766  0.5536117
## Function Value
## [1] 3.0282
## Gradient:
## [1] -0.5151417  2.0069887
##
## iteration = 91
## Step:
## [1]  0.005324392 -0.007781508
```

```
## Parameter:
## [1] -0.7319522  0.5458302
## Function Value
## [1] 3.009811
## Gradient:
## [1] -0.5137898  2.0152372
##
## iteration = 92
## Step:
## [1]  0.005427268 -0.007872378
## Parameter:
## [1] -0.7265249  0.5379578
## Function Value
## [1] 2.991128
## Gradient:
## [1] -0.5122644  2.0238710
##
## iteration = 93
## Step:
## [1]  0.005535194 -0.007967020
## Parameter:
## [1] -0.7209897  0.5299908
## Function Value
## [1] 2.972138
## Gradient:
## [1] -0.5105478  2.0329219
##
## iteration = 94
## Step:
## [1]  0.005648577 -0.008065708
## Parameter:
## [1] -0.7153411  0.5219251
## Function Value
## [1] 2.952824
## Gradient:
## [1] -0.5086201  2.0424256
##
## iteration = 95
## Step:
## [1]  0.005767874 -0.008168744
## Parameter:
## [1] -0.7095733  0.5137563
## Function Value
## [1] 2.933172
## Gradient:
## [1] -0.5064589  2.0524221
##
## iteration = 96
## Step:
## [1]  0.005893591 -0.008276464
## Parameter:
## [1] -0.7036797  0.5054799
## Function Value
## [1] 2.913164
```

```
## Gradient:
## [1] -0.5040386  2.0629563
##
## iteration = 97
## Step:
## [1]  0.006026301 -0.008389241
## Parameter:
## [1] -0.6976534  0.4970906
## Function Value
## [1] 2.892781
## Gradient:
## [1] -0.5013302  2.0740791
##
## iteration = 98
## Step:
## [1]  0.006166646 -0.008507489
## Parameter:
## [1] -0.6914867  0.4885831
## Function Value
## [1] 2.872004
## Gradient:
## [1] -0.4983006  2.0858483
##
## iteration = 99
## Step:
## [1]  0.006315353 -0.008631675
## Parameter:
## [1] -0.6851714  0.4799515
## Function Value
## [1] 2.85081
## Gradient:
## [1] -0.4949118  2.0983297
##
## iteration = 100
## Parameter:
## [1] -0.6786981  0.4711891
## Function Value
## [1] 2.829175
## Gradient:
## [1] -0.4911201  2.1115987
##
## Iteration limit exceeded.  Algorithm failed.
```

```
print(t1nlm)
```

```
## $minimum
## [1] 2.829175
##
## $estimate
## [1] -0.6786981  0.4711891
##
## $gradient
## [1] -0.4911201  2.1115987
##
## $hessian
```

```
##          [,1]     [,2]
## [1,] 366.1188 271.4593
## [2,] 271.4593 200.0000
##
## $code
## [1] 4
##
## $iterations
## [1] 100
```

```
## BUT ... it looks like nlminb is NOT using a true Newton-type method
t1nlminb <- nlminb(x0, f, gradient=gr, hessian=h, control=list(trace=1))
```

```
##   0:     24.200000: -1.20000  1.00000
##   1:     4.7318843: -1.17528  1.38067
##   2:     4.1185053: -1.02313  1.03084
##   3:     3.4877941: -0.851666 0.701020
##   4:     2.8656106: -0.651455 0.387204
##   5:     2.2659017: -0.455737 0.169391
##   6:     1.7378734: -0.287660 0.0544980
##   7:     1.3374723: -0.0940265 -0.0286528
##   8:    0.95926083: 0.0347014 -0.0153667
##   9:    0.73171551: 0.161329 0.00919067
##  10:    0.55413441: 0.353365 0.0879888
##  11:    0.32780370: 0.430570 0.179430
##  12:    0.24068818: 0.519227 0.259826
##  13:    0.17104970: 0.591831 0.343595
##  14:    0.11873673: 0.660666 0.430489
##  15:   0.078218534: 0.725043 0.520572
##  16:   0.047818423: 0.785832 0.613116
##  17:   0.025947706: 0.843482 0.707654
##  18:   0.010811755: 0.932337 0.861356
##  19:  0.0017636164: 0.958572 0.918173
##  20: 0.00023632061: 0.987247 0.973799
##  21: 4.8951105e-06: 0.998131 0.996148
##  22: 2.9806009e-09: 0.999957 0.999910
##  23: 1.1784043e-15:  1.00000  1.00000
##  24: 1.9486097e-28:  1.00000  1.00000
##  25:     0.0000000:  1.00000  1.00000
```

```
print(t1nlminb)
```

```
## $par
## [1] 1 1
##
## $objective
## [1] 0
##
## $convergence
## [1] 0
##
## $iterations
## [1] 25
##
## $evaluations
```

```
## function gradient
##       33       26
##
## $message
## [1] "X-convergence (3)"
```

```
# and call them from optimx (i.e., test this gives same results)

library(optimx)
t1nlmo <- optimr(x0, f, gr, hess=h, method="nlm", control=list(trace=1))
```

```
## Unit parameter scaling
## iteration = 0
## Step:
## [1] 0 0
## Parameter:
## [1] -1.2  1.0
## Function Value
## [1] 24.2
## Gradient:
## [1] -215.6  -88.0
##
## iteration = 1
## Step:
## [1] 0.11917904 0.04942029
## Parameter:
## [1] -1.080821  1.049420
## Function Value
## [1] 5.740059
## Gradient:
## [1] -55.50222 -23.75073
##
## iteration = 2
## Step:
## [1] 0.04232135 0.00930736
## Parameter:
## [1] -1.038500  1.058728
## Function Value
## [1] 4.194502
## Gradient:
## [1] -12.282717  -3.950757
##
## iteration = 3
## Step:
## [1]  0.011967013 -0.001777633
## Parameter:
## [1] -1.026533  1.056950
## Function Value
## [1] 4.107846
## Gradient:
## [1] -2.7469658  0.6361704
##
## iteration = 4
## Step:
## [1]  0.004223361 -0.004154668
```

```
## Parameter:
## [1] -1.022309  1.052795
## Function Value
## [1] 4.095632
## Gradient:
## [1] -0.9044183  1.5358367
##
## iteration = 5
## Step:
## [1]  0.002676889 -0.004622149
## Parameter:
## [1] -1.019632  1.048173
## Function Value
## [1] 4.086179
## Gradient:
## [1] -0.5630994  1.7046170
##
## iteration = 6
## Step:
## [1]  0.002397440 -0.004721889
## Parameter:
## [1] -1.017235  1.043451
## Function Value
## [1] 4.076779
## Gradient:
## [1] -0.500814  1.736893
##
## iteration = 7
## Step:
## [1]  0.002356612 -0.004753367
## Parameter:
## [1] -1.014878  1.038698
## Function Value
## [1] 4.067338
## Gradient:
## [1] -0.489862  1.744000
##
## iteration = 8
## Step:
## [1]  0.002360087 -0.004772393
## Parameter:
## [1] -1.012518  1.033926
## Function Value
## [1] 4.057855
## Gradient:
## [1] -0.488336  1.746487
##
## iteration = 9
## Step:
## [1]  0.002371890 -0.004789282
## Parameter:
## [1] -1.010146  1.029136
## Function Value
## [1] 4.048328
```

```
## Gradient:
## [1] -0.4885411  1.7481386
##
## iteration = 10
## Step:
## [1]  0.002385388 -0.004805938
## Parameter:
## [1] -1.007761  1.024330
## Function Value
## [1] 4.038757
## Gradient:
## [1] -0.4890652  1.7496494
##
## iteration = 11
## Step:
## [1]  0.002399363 -0.004822717
## Parameter:
## [1] -1.005362  1.019508
## Function Value
## [1] 4.029141
## Gradient:
## [1] -0.4896489  1.7511482
##
## iteration = 12
## Step:
## [1]  0.002413592 -0.004839688
## Parameter:
## [1] -1.002948  1.014668
## Function Value
## [1] 4.01948
## Gradient:
## [1] -0.4902447  1.7526588
##
## iteration = 13
## Step:
## [1]  0.002428041 -0.004856866
## Parameter:
## [1] -1.000520  1.009811
## Function Value
## [1] 4.009773
## Gradient:
## [1] -0.4908439  1.7541859
##
## iteration = 14
## Step:
## [1]  0.002442705 -0.004874260
## Parameter:
## [1] -0.9980772  1.0049368
## Function Value
## [1] 4.000019
## Gradient:
## [1] -0.4914446  1.7557308
##
## iteration = 15
```

```
## Step:
## [1]  0.002457589 -0.004891873
## Parameter:
## [1] -0.9956196  1.0000449
## Function Value
## [1] 3.990218
## Gradient:
## [1] -0.4920467  1.7572939
##
## iteration = 16
## Step:
## [1]  0.002472698 -0.004909711
## Parameter:
## [1] -0.9931469  0.9951352
## Function Value
## [1] 3.980369
## Gradient:
## [1] -0.492650  1.758876
##
## iteration = 17
## Step:
## [1]  0.002488037 -0.004927778
## Parameter:
## [1] -0.9906589  0.9902074
## Function Value
## [1] 3.970471
## Gradient:
## [1] -0.4932543  1.7604765
##
## iteration = 18
## Step:
## [1]  0.002503612 -0.004946080
## Parameter:
## [1] -0.9881553  0.9852614
## Function Value
## [1] 3.960524
## Gradient:
## [1] -0.4938597  1.7620970
##
## iteration = 19
## Step:
## [1]  0.002519428 -0.004964622
## Parameter:
## [1] -0.9856359  0.9802967
## Function Value
## [1] 3.950527
## Gradient:
## [1] -0.4944661  1.7637374
##
## iteration = 20
## Step:
## [1]  0.002535492 -0.004983409
## Parameter:
## [1] -0.9831004  0.9753133
```

```
## Function Value
## [1] 3.940479
## Gradient:
## [1] -0.4950733  1.7653983
##
## iteration = 21
## Step:
## [1]  0.002551809 -0.005002448
## Parameter:
## [1] -0.9805486  0.9703109
## Function Value
## [1] 3.930379
## Gradient:
## [1] -0.4956813  1.7670802
##
## iteration = 22
## Step:
## [1]  0.002568387 -0.005021743
## Parameter:
## [1] -0.9779802  0.9652891
## Function Value
## [1] 3.920227
## Gradient:
## [1] -0.4962899  1.7687835
##
## iteration = 23
## Step:
## [1]  0.002585231 -0.005041300
## Parameter:
## [1] -0.9753949  0.9602478
## Function Value
## [1] 3.910022
## Gradient:
## [1] -0.4968992  1.7705088
##
## iteration = 24
## Step:
## [1]  0.002602350 -0.005061127
## Parameter:
## [1] -0.9727926  0.9551867
## Function Value
## [1] 3.899763
## Gradient:
## [1] -0.497509  1.772257
##
## iteration = 25
## Step:
## [1]  0.002619749 -0.005081228
## Parameter:
## [1] -0.9701728  0.9501055
## Function Value
## [1] 3.889449
## Gradient:
## [1] -0.4981191  1.7740275
```

```
##
## iteration = 26
## Step:
## [1]  0.002637437 -0.005101612
## Parameter:
## [1] -0.9675354  0.9450039
## Function Value
## [1] 3.879079
## Gradient:
## [1] -0.4987295  1.7758220
##
## iteration = 27
## Step:
## [1]  0.002655422 -0.005122284
## Parameter:
## [1] -0.9648800  0.9398816
## Function Value
## [1] 3.868653
## Gradient:
## [1] -0.4993401  1.7776407
##
## iteration = 28
## Step:
## [1]  0.002673710 -0.005143252
## Parameter:
## [1] -0.9622063  0.9347383
## Function Value
## [1] 3.85817
## Gradient:
## [1] -0.4999507  1.7794843
##
## iteration = 29
## Step:
## [1]  0.002692311 -0.005164522
## Parameter:
## [1] -0.9595140  0.9295738
## Function Value
## [1] 3.847628
## Gradient:
## [1] -0.5005611  1.7813534
##
## iteration = 30
## Step:
## [1]  0.002711232 -0.005186104
## Parameter:
## [1] -0.9568027  0.9243877
## Function Value
## [1] 3.837027
## Gradient:
## [1] -0.5011712  1.7832486
##
## iteration = 31
## Step:
## [1]  0.002730484 -0.005208003
```

```
## Parameter:
## [1] -0.9540722  0.9191797
## Function Value
## [1] 3.826365
## Gradient:
## [1] -0.5017809  1.7851707
##
## iteration = 32
## Step:
## [1]  0.002750075 -0.005230229
## Parameter:
## [1] -0.9513222  0.9139495
## Function Value
## [1] 3.815643
## Gradient:
## [1] -0.50239  1.78712
##
## iteration = 33
## Step:
## [1]  0.002770014 -0.005252790
## Parameter:
## [1] -0.9485522  0.9086967
## Function Value
## [1] 3.804858
## Gradient:
## [1] -0.5029984  1.7890982
##
## iteration = 34
## Step:
## [1]  0.002790313 -0.005275694
## Parameter:
## [1] -0.9457618  0.9034210
## Function Value
## [1] 3.794009
## Gradient:
## [1] -0.5036057  1.7911052
##
## iteration = 35
## Step:
## [1]  0.002810981 -0.005298950
## Parameter:
## [1] -0.9429509  0.8981220
## Function Value
## [1] 3.783096
## Gradient:
## [1] -0.5042119  1.7931421
##
## iteration = 36
## Step:
## [1]  0.002832029 -0.005322569
## Parameter:
## [1] -0.9401188  0.8927995
## Function Value
## [1] 3.772118
```

```
## Gradient:
## [1] -0.5048167  1.7952097
##
## iteration = 37
## Step:
## [1]  0.002853468 -0.005346559
## Parameter:
## [1] -0.9372654  0.8874529
## Function Value
## [1] 3.761073
## Gradient:
## [1] -0.5054199  1.7973089
##
## iteration = 38
## Step:
## [1]  0.002875310 -0.005370931
## Parameter:
## [1] -0.9343901  0.8820820
## Function Value
## [1] 3.74996
## Gradient:
## [1] -0.5060212  1.7994406
##
## iteration = 39
## Step:
## [1]  0.002897568 -0.005395695
## Parameter:
## [1] -0.9314925  0.8766863
## Function Value
## [1] 3.738778
## Gradient:
## [1] -0.5066205  1.8016058
##
## iteration = 40
## Step:
## [1]  0.002920253 -0.005420863
## Parameter:
## [1] -0.9285722  0.8712654
## Function Value
## [1] 3.727525
## Gradient:
## [1] -0.5072173  1.8038054
##
## iteration = 41
## Step:
## [1]  0.002943380 -0.005446444
## Parameter:
## [1] -0.9256289  0.8658190
## Function Value
## [1] 3.716201
## Gradient:
## [1] -0.5078115  1.8060404
##
## iteration = 42
```

```
## Step:
## [1]  0.002966963 -0.005472452
## Parameter:
## [1] -0.9226619  0.8603465
## Function Value
## [1] 3.704804
## Gradient:
## [1] -0.5084028  1.8083119
##
## iteration = 43
## Step:
## [1]  0.002991015 -0.005498898
## Parameter:
## [1] -0.9196709  0.8548476
## Function Value
## [1] 3.693332
## Gradient:
## [1] -0.5089907  1.8106211
##
## iteration = 44
## Step:
## [1]  0.003015551 -0.005525796
## Parameter:
## [1] -0.9166553  0.8493218
## Function Value
## [1] 3.681785
## Gradient:
## [1] -0.5095751  1.8129691
##
## iteration = 45
## Step:
## [1]  0.003040588 -0.005553158
## Parameter:
## [1] -0.9136147  0.8437687
## Function Value
## [1] 3.67016
## Gradient:
## [1] -0.5101554  1.8153571
##
## iteration = 46
## Step:
## [1]  0.003066143 -0.005580998
## Parameter:
## [1] -0.9105486  0.8381877
## Function Value
## [1] 3.658457
## Gradient:
## [1] -0.5107313  1.8177865
##
## iteration = 47
## Step:
## [1]  0.003092231 -0.005609332
## Parameter:
## [1] -0.9074564  0.8325783
```

```
## Function Value
## [1] 3.646673
## Gradient:
## [1] -0.5113024  1.8202585
##
## iteration = 48
## Step:
## [1]  0.003118872 -0.005638173
## Parameter:
## [1] -0.9043375  0.8269402
## Function Value
## [1] 3.634808
## Gradient:
## [1] -0.5118683  1.8227745
##
## iteration = 49
## Step:
## [1]  0.003146084 -0.005667538
## Parameter:
## [1] -0.9011914  0.8212726
## Function Value
## [1] 3.622858
## Gradient:
## [1] -0.5124284  1.8253361
##
## iteration = 50
## Step:
## [1]  0.003173887 -0.005697443
## Parameter:
## [1] -0.8980175  0.8155752
## Function Value
## [1] 3.610824
## Gradient:
## [1] -0.5129824  1.8279447
##
## iteration = 51
## Step:
## [1]  0.003202302 -0.005727905
## Parameter:
## [1] -0.8948152  0.8098473
## Function Value
## [1] 3.598702
## Gradient:
## [1] -0.5135295  1.8306019
##
## iteration = 52
## Step:
## [1]  0.003231350 -0.005758943
## Parameter:
## [1] -0.8915839  0.8040883
## Function Value
## [1] 3.586492
## Gradient:
## [1] -0.5140693  1.8333095
```

```
##
## iteration = 53
## Step:
## [1]  0.003261055 -0.005790574
## Parameter:
## [1] -0.8883228  0.7982978
## Function Value
## [1] 3.574191
## Gradient:
## [1] -0.5146012  1.8360693
##
## iteration = 54
## Step:
## [1]  0.003291439 -0.005822819
## Parameter:
## [1] -0.8850314  0.7924750
## Function Value
## [1] 3.561797
## Gradient:
## [1] -0.5151245  1.8388830
##
## iteration = 55
## Step:
## [1]  0.003322530 -0.005855699
## Parameter:
## [1] -0.8817088  0.7866193
## Function Value
## [1] 3.549308
## Gradient:
## [1] -0.5156385  1.8417527
##
## iteration = 56
## Step:
## [1]  0.003354353 -0.005889234
## Parameter:
## [1] -0.8783545  0.7807300
## Function Value
## [1] 3.536723
## Gradient:
## [1] -0.5161424  1.8446804
##
## iteration = 57
## Step:
## [1]  0.003386935 -0.005923448
## Parameter:
## [1] -0.8749676  0.7748066
## Function Value
## [1] 3.524038
## Gradient:
## [1] -0.5166355  1.8476683
##
## iteration = 58
## Step:
## [1]  0.003420307 -0.005958365
```

```
## Parameter:
## [1] -0.8715473  0.7688482
## Function Value
## [1] 3.511252
## Gradient:
## [1] -0.5171169  1.8507187
##
## iteration = 59
## Step:
## [1]  0.003454499 -0.005994008
## Parameter:
## [1] -0.8680928  0.7628542
## Function Value
## [1] 3.498362
## Gradient:
## [1] -0.5175857  1.8538340
##
## iteration = 60
## Step:
## [1]  0.003489544 -0.006030405
## Parameter:
## [1] -0.8646032  0.7568238
## Function Value
## [1] 3.485366
## Gradient:
## [1] -0.5180409  1.8570169
##
## iteration = 61
## Step:
## [1]  0.003525477 -0.006067583
## Parameter:
## [1] -0.8610777  0.7507562
## Function Value
## [1] 3.472262
## Gradient:
## [1] -0.5184815  1.8602699
##
## iteration = 62
## Step:
## [1]  0.003562334 -0.006105572
## Parameter:
## [1] -0.8575154  0.7446506
## Function Value
## [1] 3.459046
## Gradient:
## [1] -0.5189064  1.8635959
##
## iteration = 63
## Step:
## [1]  0.003600152 -0.006144401
## Parameter:
## [1] -0.8539152  0.7385062
## Function Value
## [1] 3.445716
```

```
## Gradient:
## [1] -0.5193143  1.8669981
##
## iteration = 64
## Step:
## [1]  0.003638975 -0.006184102
## Parameter:
## [1] -0.8502763  0.7323221
## Function Value
## [1] 3.432269
## Gradient:
## [1] -0.5197039  1.8704795
##
## iteration = 65
## Step:
## [1]  0.003678843 -0.006224711
## Parameter:
## [1] -0.8465974  0.7260974
## Function Value
## [1] 3.418702
## Gradient:
## [1] -0.5200739  1.8740436
##
## iteration = 66
## Step:
## [1]  0.003719803 -0.006266263
## Parameter:
## [1] -0.8428776  0.7198312
## Function Value
## [1] 3.405012
## Gradient:
## [1] -0.5204227  1.8776940
##
## iteration = 67
## Step:
## [1]  0.003761905 -0.006308796
## Parameter:
## [1] -0.8391157  0.7135224
## Function Value
## [1] 3.391196
## Gradient:
## [1] -0.5207487  1.8814346
##
## iteration = 68
## Step:
## [1]  0.003805198 -0.006352349
## Parameter:
## [1] -0.8353105  0.7071700
## Function Value
## [1] 3.37725
## Gradient:
## [1] -0.5210502  1.8852694
##
## iteration = 69
```

```
## Step:
## [1]  0.003849738 -0.006396966
## Parameter:
## [1] -0.8314608  0.7007731
## Function Value
## [1] 3.363171
## Gradient:
## [1] -0.5213253  1.8892029
##
## iteration = 70
## Step:
## [1]  0.003895584 -0.006442692
## Parameter:
## [1] -0.8275652  0.6943304
## Function Value
## [1] 3.348955
## Gradient:
## [1] -0.521572  1.893240
##
## iteration = 71
## Step:
## [1]  0.003942798 -0.006489574
## Parameter:
## [1] -0.8236224  0.6878408
## Function Value
## [1] 3.334599
## Gradient:
## [1] -0.5217881  1.8973845
##
## iteration = 72
## Step:
## [1]  0.003991445 -0.006537663
## Parameter:
## [1] -0.8196310  0.6813031
## Function Value
## [1] 3.320097
## Gradient:
## [1] -0.5219711  1.9016429
##
## iteration = 73
## Step:
## [1]  0.004041597 -0.006587013
## Parameter:
## [1] -0.8155894  0.6747161
## Function Value
## [1] 3.305447
## Gradient:
## [1] -0.5221186  1.9060205
##
## iteration = 74
## Step:
## [1]  0.004093330 -0.006637682
## Parameter:
## [1] -0.8114960  0.6680784
```

```
## Function Value
## [1] 3.290643
## Gradient:
## [1] -0.5222276  1.9105235
##
## iteration = 75
## Step:
## [1]  0.004146724 -0.006689731
## Parameter:
## [1] -0.8073493  0.6613887
## Function Value
## [1] 3.275681
## Gradient:
## [1] -0.5222953  1.9151582
##
## iteration = 76
## Step:
## [1]  0.004201867 -0.006743224
## Parameter:
## [1] -0.8031474  0.6546455
## Function Value
## [1] 3.260556
## Gradient:
## [1] -0.5223182  1.9199318
##
## iteration = 77
## Step:
## [1]  0.004258851 -0.006798233
## Parameter:
## [1] -0.7988886  0.6478472
## Function Value
## [1] 3.245263
## Gradient:
## [1] -0.5222928  1.9248518
##
## iteration = 78
## Step:
## [1]  0.004317778 -0.006854831
## Parameter:
## [1] -0.7945708  0.6409924
## Function Value
## [1] 3.229796
## Gradient:
## [1] -0.5222152  1.9299264
##
## iteration = 79
## Step:
## [1]  0.004378754 -0.006913098
## Parameter:
## [1] -0.7901921  0.6340793
## Function Value
## [1] 3.21415
## Gradient:
## [1] -0.5220811  1.9351644
```

```
##
## iteration = 80
## Step:
## [1]  0.004441897 -0.006973119
## Parameter:
## [1] -0.7857502  0.6271062
## Function Value
## [1] 3.198318
## Gradient:
## [1] -0.5218858  1.9405752
##
## iteration = 81
## Step:
## [1]  0.004507332 -0.007034988
## Parameter:
## [1] -0.7812428  0.6200712
## Function Value
## [1] 3.182295
## Gradient:
## [1] -0.5216242  1.9461692
##
## iteration = 82
## Step:
## [1]  0.004575195 -0.007098802
## Parameter:
## [1] -0.7766676  0.6129724
## Function Value
## [1] 3.166073
## Gradient:
## [1] -0.5212907  1.9519576
##
## iteration = 83
## Step:
## [1]  0.004645633 -0.007164669
## Parameter:
## [1] -0.7720220  0.6058077
## Function Value
## [1] 3.149646
## Gradient:
## [1] -0.5208791  1.9579526
##
## iteration = 84
## Step:
## [1]  0.004718806 -0.007232703
## Parameter:
## [1] -0.7673032  0.5985750
## Function Value
## [1] 3.133005
## Gradient:
## [1] -0.5203826  1.9641674
##
## iteration = 85
## Step:
## [1]  0.004794887 -0.007303028
```

```
## Parameter:
## [1] -0.7625083  0.5912720
## Function Value
## [1] 3.116144
## Gradient:
## [1] -0.5197938  1.9706165
##
## iteration = 86
## Step:
## [1]  0.004874066 -0.007375779
## Parameter:
## [1] -0.7576342  0.5838962
## Function Value
## [1] 3.099053
## Gradient:
## [1] -0.5191043  1.9773157
##
## iteration = 87
## Step:
## [1]  0.004956550 -0.007451103
## Parameter:
## [1] -0.7526777  0.5764451
## Function Value
## [1] 3.081723
## Gradient:
## [1] -0.5183051  1.9842824
##
## iteration = 88
## Step:
## [1]  0.005042564 -0.007529156
## Parameter:
## [1] -0.7476351  0.5689160
## Function Value
## [1] 3.064144
## Gradient:
## [1] -0.517386  1.991536
##
## iteration = 89
## Step:
## [1]  0.005132356 -0.007610113
## Parameter:
## [1] -0.7425028  0.5613058
## Function Value
## [1] 3.046307
## Gradient:
## [1] -0.5163357  1.9990968
##
## iteration = 90
## Step:
## [1]  0.005226199 -0.007694161
## Parameter:
## [1] -0.7372766  0.5536117
## Function Value
## [1] 3.0282
```

```
## Gradient:
## [1] -0.5151417  2.0069887
##
## iteration = 91
## Step:
## [1]  0.005324392 -0.007781508
## Parameter:
## [1] -0.7319522  0.5458302
## Function Value
## [1] 3.009811
## Gradient:
## [1] -0.5137898  2.0152372
##
## iteration = 92
## Step:
## [1]  0.005427268 -0.007872378
## Parameter:
## [1] -0.7265249  0.5379578
## Function Value
## [1] 2.991128
## Gradient:
## [1] -0.5122644  2.0238710
##
## iteration = 93
## Step:
## [1]  0.005535194 -0.007967020
## Parameter:
## [1] -0.7209897  0.5299908
## Function Value
## [1] 2.972138
## Gradient:
## [1] -0.5105478  2.0329219
##
## iteration = 94
## Step:
## [1]  0.005648577 -0.008065708
## Parameter:
## [1] -0.7153411  0.5219251
## Function Value
## [1] 2.952824
## Gradient:
## [1] -0.5086201  2.0424256
##
## iteration = 95
## Step:
## [1]  0.005767874 -0.008168744
## Parameter:
## [1] -0.7095733  0.5137563
## Function Value
## [1] 2.933172
## Gradient:
## [1] -0.5064589  2.0524221
##
## iteration = 96
```

```
## Step:
## [1]  0.005893591 -0.008276464
## Parameter:
## [1] -0.7036797  0.5054799
## Function Value
## [1] 2.913164
## Gradient:
## [1] -0.5040386  2.0629563
##
## iteration = 97
## Step:
## [1]  0.006026301 -0.008389241
## Parameter:
## [1] -0.6976534  0.4970906
## Function Value
## [1] 2.892781
## Gradient:
## [1] -0.5013302  2.0740791
##
## iteration = 98
## Step:
## [1]  0.006166646 -0.008507489
## Parameter:
## [1] -0.6914867  0.4885831
## Function Value
## [1] 2.872004
## Gradient:
## [1] -0.4983006  2.0858483
##
## iteration = 99
## Step:
## [1]  0.006315353 -0.008631675
## Parameter:
## [1] -0.6851714  0.4799515
## Function Value
## [1] 2.85081
## Gradient:
## [1] -0.4949118  2.0983297
##
## iteration = 100
## Parameter:
## [1] -0.6786981  0.4711891
## Function Value
## [1] 2.829175
## Gradient:
## [1] -0.4911201  2.1115987
##
## Iteration limit exceeded.  Algorithm failed.
```

```
print(t1nlmo)
```

```
## $convergence
## [1] 1
##
## $value
```

```
## [1] 2.829175
##
## $par
## [1] -0.6786981  0.4711891
##
## $counts
## [1]  NA 100
##
## $message
## [1] "Convergence indicator (code) =  4"
```

```
## FOLLOWING SHOWS UP ERRORS??
tst <- try(t1nlminbo <- optimr(x0, f, gr, hessian=h, method="nlminb", control=list(trace=1)))
```

```
## Unit parameter scaling
##   0:     24.200000: -1.20000  1.00000
##   1:     11.298090: -0.916319  1.11579
##   2:     3.6832653: -0.878971 0.811671
##   3:     3.0264111: -0.738569 0.539330
##   4:     2.8738710: -0.669080 0.417996
##   5:     2.7399461: -0.596536 0.312149
##   6:     2.4721420: -0.512267 0.219383
##   7:     1.9715170: -0.394019 0.138449
##   8:     1.7942340: -0.285988 0.0443101
##   9:     1.4393081: -0.153222 -0.00959680
##  10:     1.1968650: -0.0785139 0.0245146
##  11:     1.0016961: -0.000696349 -0.00174001
##  12:    0.87823307: 0.0808212 -0.0117281
##  13:    0.80844393: 0.162768 -0.00629170
##  14:    0.68863014: 0.203045 0.0180988
##  15:    0.50226244: 0.313088 0.0805843
##  16:    0.37094000: 0.392027 0.157302
##  17:    0.30110961: 0.484979 0.216267
##  18:    0.24838461: 0.525122 0.260628
##  19:    0.15712889: 0.604556 0.362744
##  20:    0.14975086: 0.693216 0.456961
##  21:   0.097345475: 0.690984 0.473152
##  22:   0.074919430: 0.728091 0.526978
##  23:   0.050655822: 0.789126 0.614854
##  24:   0.033264540: 0.845784 0.705613
##  25:   0.013501068: 0.884534 0.781102
##  26:  0.0075988869: 0.927000 0.854564
##  27:  0.0029619374: 0.954465 0.908023
##  28: 0.00077922601: 0.972116 0.945142
##  29: 0.00019764489: 0.991268 0.981510
##  30: 2.4278303e-05: 0.996174 0.992053
##  31: 3.4539879e-07: 0.999626 0.999207
##  32: 3.3528188e-09: 0.999947 0.999895
##  33: 2.3909570e-10:  1.00001  1.00002
##  34: 2.0297517e-16:  1.00000  1.00000
##  35: 4.2918160e-22:  1.00000  1.00000
```

```
if (class(tst) == "try-error"){
    cat("try-error on attempt to run nlminb in optimr()\n")
} else { print(t1nlminb) }
```

```
## try-error on attempt to run nlminb in optimr()
# sink()
```

The Wood function

?? Note that we have NOT found the minimum for the Wood function.

```
#Example: Wood function
#
wood.f <- function(x){
  res <- 100*(x[1]^2-x[2])^2+(1-x[1])^2+90*(x[3]^2-x[4])^2+(1-x[3])^2+
    10.1*((1-x[2])^2+(1-x[4])^2)+19.8*(1-x[2])*(1-x[4])
  return(res)
}
#gradient:
wood.g <- function(x){
  g1 <- 400*x[1]^3-400*x[1]*x[2]+2*x[1]-2
  g2 <- -200*x[1]^2+220.2*x[2]+19.8*x[4]-40
  g3 <- 360*x[3]^3-360*x[3]*x[4]+2*x[3]-2
  g4 <- -180*x[3]^2+200.2*x[4]+19.8*x[2]-40
  return(c(g1,g2,g3,g4))
}
#hessian:
wood.h <- function(x){
  h11 <- 1200*x[1]^2-400*x[2]+2;    h12 <- -400*x[1]; h13 <- h14 <- 0
  h22 <- 220.2; h23 <- 0;    h24 <- 19.8
  h33 <- 1080*x[3]^2-360*x[4]+2;    h34 <- -360*x[3]
  h44 <- 200.2
  H <- matrix(c(h11,h12,h13,h14,h12,h22,h23,h24,
                h13,h23,h33,h34,h14,h24,h34,h44),ncol=4)
  return(H)
}

wood.fgh <- function(x){
      fval <- wood.f(x)
      gval <- wood.g(x)
      hval <- wood.h(x)
      attr(fval,"gradient") <- gval
      attr(fval,"hessian")<- hval
      fval
}

######################################################
x0 <- c(-3,-1,-3,-1) # Wood standard start

library(snewton)
cat("This FAILS to find minimum\n")
```

```
## This FAILS to find minimum
```

```
wd <- snewton(x0, fn=wood.f, gr=wood.g, hess=wood.h, control=list(trace=2))
```

```
## trace = 2
## Initial function value =  19192
```

```
## [1] -3 -1 -3 -1
## Termination test:nf= 1
## current gradient norm = 12008
## Iteration  1 :Search vector:[1] 0.3031804 7.1764779 0.3366407 6.8648041
## Gradient projection =  -35111.91
## f(xnew)= 1291.439
## end major loop
## Termination test:nf= 2
## current gradient norm = 1190.066
## Iteration  2 :Search vector:[1]  0.8004660 -3.5411163  0.7835229 -3.3003803
## Gradient projection =  -1748.969
## f(xnew)= 295.9513
## end major loop
## Termination test:nf= 3
## current gradient norm = 734.5958
## Iteration  3 :Search vector:[1]  0.3717359 -0.6307749  0.4048288 -0.7541032
## Gradient projection =  -376.8133
## f(xnew)= 67.68559
## end major loop
## Termination test:nf= 4
## current gradient norm = 200.1226
## Iteration  4 :Search vector:[1]  0.3094376 -0.6813323  0.2912302 -0.5577128
## Gradient projection =  -84.58005
## f(xnew)= 17.33661
## end major loop
## Termination test:nf= 5
## current gradient norm = 78.99793
## Iteration  5 :Search vector:[1]  0.1493372 -0.2198873  0.1651241 -0.2534905
## Gradient projection =  -14.90304
## f(xnew)= 8.689077
## end major loop
## Termination test:nf= 6
## current gradient norm = 18.16952
## Iteration  6 :Search vector:[1]  0.06562338 -0.09986141  0.06090227 -0.07716611
## Gradient projection =  -1.467156
## f(xnew)= 7.892798
## end major loop
## Termination test:nf= 7
## current gradient norm = 2.773634
## Iteration  7 :Search vector:[1]  0.0036594560 -0.0003950329  0.0170860102 -0.0260521194
## Gradient projection =  -0.03184003
## f(xnew)= 7.876516
## end major loop
## Termination test:nf= 8
## current gradient norm = 0.1399245
## Iteration  8 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
```

```
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 27
## current gradient norm = 0.1399245
## Iteration  9 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
```

```
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 46
## current gradient norm = 0.1399245
## Iteration  10 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
```

```
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 65
## current gradient norm = 0.1399245
## Iteration  11 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 84
## current gradient norm = 0.1399245
## Iteration  12 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
```

```
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 103
## current gradient norm = 0.1399245
## Iteration  13 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
```

```
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 122
## current gradient norm = 0.1399245
## Iteration  14 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
```

```
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 142
## current gradient norm = 0.1399245
## Iteration  15 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
```

```
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 161
## current gradient norm = 0.1399245
## Iteration  16 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 180
## current gradient norm = 0.1399245
## Iteration  17 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
```

```
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 199
## current gradient norm = 0.1399245
## Iteration  18 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
```

```
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 219
## current gradient norm = 0.1399245
## Iteration  19 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
```

```
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 238
## current gradient norm = 0.1399245
## Iteration  20 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 257
```

```
## current gradient norm = 0.1399245
## Iteration  21 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 277
## current gradient norm = 0.1399245
## Iteration  22 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
```

```
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 296
## current gradient norm = 0.1399245
## Iteration  23 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
```

```
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 316
## current gradient norm = 0.1399245
## Iteration  24 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
```

```
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 335
## current gradient norm = 0.1399245
## Iteration  25 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 354
## current gradient norm = 0.1399245
## Iteration  26 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
```

```
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 373
## current gradient norm = 0.1399245
## Iteration  27 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
```

```
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 392
## current gradient norm = 0.1399245
## Iteration  28 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
```

```
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 411
## current gradient norm = 0.1399245
## Iteration  29 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
```

```
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 431
## current gradient norm = 0.1399245
## Iteration  30 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## end major loop
## Termination test:nf= 450
## current gradient norm = 0.1399245
## Iteration  31 :Search vector:[1]  0.03262892 -0.06483154 -0.03403094  0.06423790
## Gradient projection =  0.001054892
## f(xnew)= 7.87719
## Stepsize now = 0.2
## * f(xnew)= 7.876706
```

```
## Stepsize now = 0.04
## * f(xnew)= 7.876557
## Stepsize now = 0.008
## * f(xnew)= 7.876524
## Stepsize now = 0.0016
## * f(xnew)= 7.876518
## Stepsize now = 0.00032
## * f(xnew)= 7.876516
## Stepsize now = 6.4e-05
## * f(xnew)= 7.876516
## Stepsize now = 1.28e-05
## * f(xnew)= 7.876516
## Stepsize now = 2.56e-06
## * f(xnew)= 7.876516
## Stepsize now = 5.12e-07
## * f(xnew)= 7.876516
## Stepsize now = 1.024e-07
## * f(xnew)= 7.876516
## Stepsize now = 2.048e-08
## * f(xnew)= 7.876516
## Stepsize now = 4.096e-09
## * f(xnew)= 7.876516
## Stepsize now = 8.192e-10
## * f(xnew)= 7.876516
## Stepsize now = 1.6384e-10
## * f(xnew)= 7.876516
## Stepsize now = 3.2768e-11
## * f(xnew)= 7.876516
## Stepsize now = 6.5536e-12
## * f(xnew)= 7.876516
## Stepsize now = 1.31072e-12
## * f(xnew)= 7.876516
## Stepsize now = 2.62144e-13
## * f(xnew)= 7.876516
## Stepsize now = 5.24288e-14
## * f(xnew)= 7.876516
## No progress in linesearch!
```

```
print(wd)
```

```
## $par
## [1] -0.9965602  1.0031106 -0.9406650  0.8958990
##
## $value
## [1] 7.876516
##
## $grad
## [1] -0.015475627 -0.002678324 -0.139924452 -0.052547714
##
## $Hess
##          [,1]     [,2]     [,3]     [,4]
## [1,] 792.5144 398.6241   0.0000   0.0000
## [2,] 398.6241 220.2000   0.0000  19.8000
## [3,]   0.0000   0.0000 635.1150 338.6394
## [4,]   0.0000  19.8000 338.6394 200.2000
```

```
##
## $counts
## $counts$niter
## [1] 31
##
## $counts$nfn
## [1] 470
##
## $counts$ngr
## [1] 31
##
## $counts$nhess
## [1] 31
##
##
## $convcode
## [1] 93
```

```
wdm <- snewtonm(x0, fn=wood.f, gr=wood.g, hess=wood.h, control=list(trace=2))
```

```
## trace = 2
##  Start snewtonm   f0= 19192   at  [1] -3 -1 -3 -1
## Iteration  1  fbest= 19192
##  lambda = 0.0001220703   fval= 1291.437
## Iteration  2  fbest= 1291.437
##  lambda = 1.831055e-05   fval= 295.9512
## Iteration  3  fbest= 295.9512
##  lambda = 2.746582e-06   fval= 67.68561
## Iteration  4  fbest= 67.68561
##  lambda = 4.119873e-07   fval= 17.33662
## Iteration  5  fbest= 17.33662
##  lambda = 6.17981e-08   fval= 8.689077
## Iteration  6  fbest= 8.689077
##  lambda = 9.269714e-09   fval= 7.892798
## Iteration  7  fbest= 7.892798
##  lambda = 1.390457e-09   fval= 7.876516
## Iteration  8  fbest= 7.876516
##  lambda = 2.085686e-10   fval= 7.87719
##  lambda = 0.001220703   fval= 7.877199
##  lambda = 0.01220703   fval= 7.877309
##  lambda = 0.1220703   fval= 8.086469
##  lambda = 1.220703   fval= 7.876389
## Iteration  13  fbest= 7.876389
##  lambda = 0.1831055   fval= 7.874695
## Iteration  14  fbest= 7.874695
##  lambda = 0.02746582   fval= 7.86493
## Iteration  15  fbest= 7.86493
##  lambda = 0.004119873   fval= 7.960279
##  lambda = 0.04119873   fval= 7.880975
##  lambda = 0.4119873   fval= 7.852834
## Iteration  18  fbest= 7.852834
##  lambda = 0.0617981   fval= 2687428
##  lambda = 0.617981   fval= 7.83364
## Iteration  20  fbest= 7.83364
##  lambda = 0.09269714   fval= 10.04037
```

```
##  lambda = 0.9269714   fval= 7.80479
## Iteration  22  fbest= 7.80479
##  lambda = 0.1390457   fval= 187.1183
##  lambda = 1.390457   fval= 7.760677
## Iteration  24  fbest= 7.760677
##  lambda = 0.2085686   fval= 39.79228
##  lambda = 2.085686   fval= 7.694405
## Iteration  26  fbest= 7.694405
##  lambda = 0.3128529   fval= 71.47948
##  lambda = 3.128529   fval= 7.593398
## Iteration  28  fbest= 7.593398
##  lambda = 0.4692793   fval= 29.76551
##  lambda = 4.692793   fval= 7.440723
## Iteration  30  fbest= 7.440723
##  lambda = 0.7039189   fval= 15.78816
##  lambda = 7.039189   fval= 7.220427
## Iteration  32  fbest= 7.220427
##  lambda = 1.055878   fval= 9.872978
##  lambda = 10.55878   fval= 6.945088
## Iteration  34  fbest= 6.945088
##  lambda = 1.583818   fval= 8.791386
##  lambda = 15.83818   fval= 6.677653
## Iteration  36  fbest= 6.677653
##  lambda = 2.375726   fval= 8.252204
##  lambda = 23.75726   fval= 6.477078
## Iteration  38  fbest= 6.477078
##  lambda = 3.56359   fval= 6.868378
##  lambda = 35.6359   fval= 6.345119
## Iteration  40  fbest= 6.345119
##  lambda = 5.345384   fval= 6.009281
## Iteration  41  fbest= 6.009281
##  lambda = 0.8018077   fval= 5.126601
## Iteration  42  fbest= 5.126601
##  lambda = 0.1202711   fval= 14.61183
##  lambda = 1.202711   fval= 5.0219
## Iteration  44  fbest= 5.0219
##  lambda = 0.1804067   fval= 3.691554
## Iteration  45  fbest= 3.691554
##  lambda = 0.02706101   fval= 5.793891
##  lambda = 0.2706101   fval= 4.183769
##  lambda = 2.706101   fval= 3.31558
## Iteration  48  fbest= 3.31558
##  lambda = 0.4059151   fval= 3.195488
## Iteration  49  fbest= 3.195488
##  lambda = 0.06088727   fval= 2.456016
## Iteration  50  fbest= 2.456016
##  lambda = 0.00913309   fval= 19.57353
##  lambda = 0.0913309   fval= 15.43651
##  lambda = 0.913309   fval= 3.729664
##  lambda = 9.13309   fval= 2.268927
## Iteration  54  fbest= 2.268927
##  lambda = 1.369964   fval= 5.355945
##  lambda = 13.69964   fval= 2.115327
## Iteration  56  fbest= 2.115327
```

```
##  lambda = 2.054945   fval= 10.1671
##  lambda = 20.54945   fval= 1.988546
## Iteration  58  fbest= 1.988546
##  lambda = 3.082418   fval= 10.44886
##  lambda = 30.82418   fval= 1.889195
## Iteration  60  fbest= 1.889195
##  lambda = 4.623627   fval= 5.077595
##  lambda = 46.23627   fval= 1.815587
## Iteration  62  fbest= 1.815587
##  lambda = 6.935441   fval= 2.080748
##  lambda = 69.35441   fval= 1.763332
## Iteration  64  fbest= 1.763332
##  lambda = 10.40316   fval= 1.492527
## Iteration  65  fbest= 1.492527
##  lambda = 1.560474   fval= 1.124371
## Iteration  66  fbest= 1.124371
##  lambda = 0.2340711   fval= 1.485749
##  lambda = 2.340711   fval= 0.9137214
## Iteration  68  fbest= 0.9137214
##  lambda = 0.3511067   fval= 0.5280731
## Iteration  69  fbest= 0.5280731
##  lambda = 0.052666   fval= 1.525743
##  lambda = 0.52666   fval= 0.6554124
##  lambda = 5.2666   fval= 0.3958719
## Iteration  72  fbest= 0.3958719
##  lambda = 0.78999   fval= 0.2303101
## Iteration  73  fbest= 0.2303101
##  lambda = 0.1184985   fval= 0.08047485
## Iteration  74  fbest= 0.08047485
##  lambda = 0.01777478   fval= 0.03236792
## Iteration  75  fbest= 0.03236792
##  lambda = 0.002666216   fval= 0.001857496
## Iteration  76  fbest= 0.001857496
##  lambda = 0.0003999324   fval= 4.536306e-05
## Iteration  77  fbest= 4.536306e-05
##  lambda = 5.998987e-05   fval= 6.363907e-09
## Iteration  78  fbest= 6.363907e-09
##  lambda = 8.99848e-06   fval= 6.401252e-16
## Iteration  79  fbest= 6.401252e-16
##  lambda = 1.349772e-06   fval= 6.530659e-28
## Iteration  80  fbest= 6.530659e-28
##  lambda = 2.024658e-07   fval= 1.139283e-27
##  lambda = 0.001220703   fval= 1.139283e-27
##  lambda = 0.01220703   fval= 1.115725e-27
##  lambda = 0.1220703   fval= 1.056216e-27
##  lambda = 1.220703   fval= 7.994871e-28
##  lambda = 12.20703   fval= 6.583661e-28
##  lambda = 122.0703   fval= 6.519368e-28
## Iteration  87  fbest= 6.519368e-28
##  lambda = 18.31055   fval= 6.257627e-28
## Iteration  88  fbest= 6.257627e-28
##  lambda = 2.746582   fval= 3.868623e-28
## Iteration  89  fbest= 3.868623e-28
##  lambda = 0.4119873   fval= 1.082712e-29
```

```
## Iteration  90  fbest= 1.082712e-29
##  lambda = 0.0617981   fval= 8.313115e-29
##  lambda = 0.617981   fval= 8.072759e-29
##  lambda = 6.17981   fval= 4.990655e-29
##  lambda = 61.7981   fval= 3.035142e-29
## Null step
## Finished
```

```
print(wdm)
```

```
## $xs
## [1] 1 1 1 1
##
## $fv
## [1] 1.082712e-29
##
## $grd
## [1] -5.462297e-14 -7.105427e-15 -5.817569e-14  0.000000e+00
##
## $Hess
##      [,1]   [,2] [,3]   [,4]
## [1,]  802 -400.0    0    0.0
## [2,] -400  220.2    0   19.8
## [3,]    0    0.0  722 -360.0
## [4,]    0   19.8 -360  200.2
##
## $counts
## $counts$niter
## [1] 95
##
## $counts$nfn
## [1] 94
##
## $counts$ngr
## [1] 53
##
## $counts$nhess
## [1] 53
```

```
cat("\n\n nlm() gives similar results\n")
```

```
##
##
##  nlm() gives similar results
```

```
t1nlm <- nlm(wood.fgh, x0, print.level=2)
```

```
## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] -3 -1 -3 -1
## Function Value
## [1] 19192
## Gradient:
## [1] -12008  -2080 -10808  -1880
```

```
##
## iteration = 1
## Step:
## [1] 0.9116709 0.7367972 0.9020511 0.7516950
## Parameter:
## [1] -2.0883291 -0.2632028 -2.0979489 -0.2483050
## Function Value
## [1] 4166.416
## Gradient:
## [1] -3869.0187  -975.0974 -3517.9318  -887.1722
##
## iteration = 2
## Step:
## [1] 0.5765246 0.5075403 0.5704409 0.5190481
## Parameter:
## [1] -1.5118046  0.2443374 -1.5275080  0.2707431
## Function Value
## [1] 834.2691
## Gradient:
## [1] -1239.3913  -437.9468 -1139.2505  -400.9499
##
## iteration = 3
## Step:
## [1] 0.3452070 0.3077087 0.3415096 0.3149447
## Parameter:
## [1] -1.1665976  0.5520461 -1.1859985  0.5856878
## Function Value
## [1] 142.99
## Gradient:
## [1] -381.7993 -179.0328 -354.8650 -165.0014
##
## iteration = 4
## Step:
## [1] 0.1855520 0.1499399 0.1833595 0.1541052
## Parameter:
## [1] -0.9810455  0.7019861 -1.0026390  0.7397929
## Function Value
## [1] 24.17926
## Gradient:
## [1] -106.17301  -63.26483  -99.83461  -58.94541
##
## iteration = 5
## Step:
## [1] 0.07782451 0.05872218 0.07664496 0.06140066
## Parameter:
## [1] -0.9032210  0.7607082 -0.9259940  0.8011936
## Function Value
## [1] 9.839749
## Gradient:
## [1] -23.71342 -19.79005 -22.61047 -18.88270
##
## iteration = 6
## Step:
## [1] 0.01753078 0.02499409 0.01707308 0.02682737
```

```
## Parameter:
## [1] -0.8856902  0.7857023 -0.9089209  0.8280210
## Function Value
## [1] 8.69256
## Gradient:
## [1] -3.326713 -7.482969 -3.201469 -7.378002
##
## iteration = 7
## Step:
## [1] -0.002949427  0.015908253 -0.003260912  0.017258585
## Parameter:
## [1] -0.8886397  0.8016106 -0.9121818  0.8452796
## Function Value
## [1] 8.500379
## Gradient:
## [1]  0.4633586 -4.6849032  0.5115861 -4.6767719
##
## iteration = 8
## Step:
## [1] -0.006478477  0.012986001 -0.006839623  0.014092239
## Parameter:
## [1] -0.8951181  0.8145966 -0.9190215  0.8593718
## Function Value
## [1] 8.374769
## Gradient:
## [1]  0.9933129 -3.8575663  1.0490214 -3.8528361
##
## iteration = 9
## Step:
## [1] -0.006355528  0.011342611 -0.006713964  0.012301691
## Parameter:
## [1] -0.9014737  0.8259392 -0.9257354  0.8716735
## Function Value
## [1] 8.275711
## Gradient:
## [1]  0.9872768 -3.4000080  1.0433216 -3.3948677
##
## iteration = 10
## Step:
## [1] -0.005715703  0.010053796 -0.006042570  0.010901894
## Parameter:
## [1] -0.9071894  0.8359930 -0.9317780  0.8825754
## Function Value
## [1] 8.196865
## Gradient:
## [1]  0.9031660 -3.0378609  0.9550868 -3.0335914
##
## iteration = 11
## Step:
## [1] -0.005075092  0.008946817 -0.005366382  0.009701843
## Parameter:
## [1] -0.9122645  0.8449398 -0.9371444  0.8922772
## Function Value
## [1] 8.133955
```

```
## Gradient:
## [1]  0.8146503 -2.7224545  0.8616142 -2.7194185
##
## iteration = 12
## Step:
## [1] -0.004503371  0.007976157 -0.004762011  0.008650385
## Parameter:
## [1] -0.9167678  0.8529160 -0.9419064  0.9009276
## Function Value
## [1] 8.083658
## Gradient:
## [1]  0.7329608 -2.4421895  0.7752177 -2.4403344
##
## iteration = 13
## Step:
## [1] -0.004001282  0.007119844 -0.004231240  0.007723143
## Parameter:
## [1] -0.9207691  0.8600358 -0.9461376  0.9086507
## Function Value
## [1] 8.043374
## Gradient:
## [1]  0.6591984 -2.1919825  0.6972101 -2.1911665
##
## iteration = 14
## Step:
## [1] -0.003560435  0.006362120 -0.003765379  0.006902861
## Parameter:
## [1] -0.9243296  0.8663979 -0.9499030  0.9155536
## Function Value
## [1] 8.011061
## Gradient:
## [1]  0.5928602 -1.9682377  0.6270863 -1.9683199
##
## iteration = 15
## Step:
## [1] -0.003172450  0.005690158 -0.003355552  0.006175565
## Parameter:
## [1] -0.9275020  0.8720881 -0.9532586  0.9217292
## Function Value
## [1] 7.985107
## Gradient:
## [1]  0.5332412 -1.7679572  0.5640969 -1.7688150
##
## iteration = 16
## Step:
## [1] -0.002830125  0.005093157 -0.002994097  0.005529510
## Parameter:
## [1] -0.9303321  0.8771812 -0.9562527  0.9272587
## Function Value
## [1] 7.964235
## Gradient:
## [1]  0.4796636 -1.5885404  0.5075167 -1.5900697
##
## iteration = 17
```

```
## Step:
## [1] -0.002527390  0.004561911 -0.002674558  0.004954699
## Parameter:
## [1] -0.9328595  0.8817432 -0.9589272  0.9322134
## Function Value
## [1] 7.947433
## Gradient:
## [1]  0.4315101 -1.4277069  0.4566857 -1.4298200
##
## iteration = 18
## Step:
## [1] -0.002259118  0.004088520 -0.002391484  0.004442553
## Parameter:
## [1] -0.9351186  0.8858317 -0.9613187  0.9366559
## Function Value
## [1] 7.933894
## Gradient:
## [1]  0.3882254 -1.2834487  0.4110108 -1.2860707
##
## iteration = 19
## Step:
## [1] -0.002020955  0.003666169 -0.002140251  0.003985678
## Parameter:
## [1] -0.9371396  0.8894978 -0.9634590  0.9406416
## Function Value
## [1] 7.922976
## Gradient:
## [1]  0.3493109 -1.1539919  0.3699607 -1.1570590
##
## iteration = 20
## Step:
## [1] -0.001809184  0.003288947 -0.001916914  0.003577664
## Parameter:
## [1] -0.9389488  0.8927868 -0.9653759  0.9442193
## Function Value
## [1] 7.914165
## Gradient:
## [1]  0.3143196 -1.0377658  0.3330597 -1.0412235
##
## iteration = 21
## Step:
## [1] -0.001620614  0.002951713 -0.001718090  0.003212935
## Parameter:
## [1] -0.9405694  0.8957385 -0.9670940  0.9474322
## Function Value
## [1] 7.90705
## Gradient:
## [1]  0.2828509 -0.9333771  0.2998820 -0.9371784
##
## iteration = 22
## Step:
## [1] -0.001452492  0.002649974 -0.001540861  0.002886621
## Parameter:
## [1] -0.9420219  0.8983885 -0.9686348  0.9503188
```

```
## Function Value
## [1] 7.901301
## Gradient:
## [1]  0.2545455 -0.8395874  0.2700463 -0.8436915
##
## iteration = 23
## Step:
## [1] -0.001302432  0.002379790 -0.001382703  0.002594456
## Parameter:
## [1] -0.9433243  0.9007683 -0.9700175  0.9529133
## Function Value
## [1] 7.896654
## Gradient:
## [1]  0.2290818 -0.7552944  0.2432114 -0.7596662
##
## iteration = 24
## Step:
## [1] -0.001168361  0.002137702 -0.001241420  0.002332689
## Parameter:
## [1] -0.9444927  0.9029060 -0.9712589  0.9552460
## Function Value
## [1] 7.892895
## Gradient:
## [1]  0.2061710 -0.6795158  0.2190714 -0.6841244
##
## iteration = 25
## Step:
## [1] -0.001048470  0.001920659 -0.001115098  0.002098019
## Parameter:
## [1] -0.9455411  0.9048266 -0.9723740  0.9573440
## Function Value
## [1] 7.889854
## Gradient:
## [1]  0.1855546 -0.6113746  0.1973525 -0.6161934
##
## iteration = 26
## Step:
## [1] -0.0009411735  0.0017259680 -0.0010020619  0.0018875279
## Parameter:
## [1] -0.9464823  0.9065526 -0.9733761  0.9592315
## Function Value
## [1] 7.887392
## Gradient:
## [1]  0.1670005 -0.5500879  0.1778092 -0.5550933
##
## iteration = 27
## Step:
## [1] -0.0008450808  0.0015512445 -0.0009008417  0.0016986351
## Parameter:
## [1] -0.9473274  0.9081038 -0.9742769  0.9609302
## Function Value
## [1] 7.885399
## Gradient:
## [1]  0.1503004 -0.4949553  0.1602211 -0.5001268
```

```
##
## iteration = 28
## Step:
## [1] -0.0007589677  0.0013943750 -0.0008101442  0.0015290525
## Parameter:
## [1] -0.9480864  0.9094982 -0.9750871  0.9624592
## Function Value
## [1] 7.883785
## Gradient:
## [1]  0.1352675 -0.4453502  0.1443910 -0.4506697
##
## iteration = 29
## Step:
## [1] -0.0006817541  0.0012534820 -0.0007288287  0.0013767484
## Parameter:
## [1] -0.9487681  0.9107517 -0.9758159  0.9638360
## Function Value
## [1] 7.882477
## Gradient:
## [1]  0.1217341 -0.4007116  0.1301415 -0.4061631
##
## iteration = 30
## Step:
## [1] -0.0006124854  0.0011268958 -0.0006558873  0.0012399159
## Parameter:
## [1] -0.9493806  0.9118786 -0.9764718  0.9650759
## Function Value
## [1] 7.881417
## Gradient:
## [1]  0.1095495 -0.3605364  0.1173135 -0.3661059
##
## iteration = 31
## Step:
## [1] -0.0005503158  0.0010131294 -0.0005904275  0.0011169461
## Parameter:
## [1] -0.9499309  0.9128917 -0.9770622  0.9661928
## Function Value
## [1] 7.880559
## Gradient:
## [1]  0.09857832 -0.32437405  0.10576421 -0.33004899
##
## iteration = 32
## Step:
## [1] -0.0004944952  0.0009108570 -0.0005316577  0.0010064045
## Parameter:
## [1] -0.9504254  0.9138026 -0.9775939  0.9671992
## Function Value
## [1] 7.879862
## Gradient:
## [1]  0.08869905 -0.29181992  0.09536535 -0.29758930
##
## iteration = 33
## Step:
## [1] -0.0004443567  0.0008188950 -0.0004788748  0.0009070108
```

```
## Parameter:
## [1] -0.9508698  0.9146215 -0.9780728  0.9681062
## Function Value
## [1] 7.879298
## Gradient:
## [1]  0.07980241 -0.26251108  0.08600161 -0.26836511
##
## iteration = 34
## Step:
## [1] -0.0003993074  0.0007361861 -0.0004314530  0.0008176213
## Parameter:
## [1] -0.9512691  0.9153577 -0.9785042  0.9689239
## Function Value
## [1] 7.87884
## Gradient:
## [1]  0.07179017 -0.23612164  0.07756940 -0.24205160
##
## iteration = 35
## Step:
## [1] -0.0003588187  0.0006617846 -0.0003888351  0.0007372134
## Parameter:
## [1] -0.9516279  0.9160194 -0.9788931  0.9696611
## Function Value
## [1] 7.878469
## Gradient:
## [1]  0.06457399 -0.21235885  0.06997558 -0.21835702
##
## iteration = 36
## Step:
## [1] -0.0003224194  0.0005948446 -0.0003505244  0.0006648718
## Parameter:
## [1] -0.9519503  0.9166143 -0.9792436  0.9703259
## Function Value
## [1] 7.878168
## Gradient:
## [1]  0.05807447 -0.19095972  0.06313643 -0.19701921
##
## iteration = 37
## Step:
## [1] -0.0002896886  0.0005346081 -0.0003160773  0.0005997772
## Parameter:
## [1] -0.9522400  0.9171489 -0.9795597  0.9709257
## Function Value
## [1] 7.877923
## Gradient:
## [1]  0.05222015 -0.17168788  0.05697664 -0.17780255
##
## iteration = 38
## Step:
## [1] -0.0002602503  0.0004803963 -0.0002850973  0.0005411952
## Parameter:
## [1] -0.9525003  0.9176293 -0.9798447  0.9714669
## Function Value
## [1] 7.877725
```

```
## Gradient:
## [1]  0.04694676 -0.15433082  0.05142848 -0.16049519
##
## iteration = 39
## Step:
## [1] -0.0002337683  0.0004316002 -0.0002572303  0.0004884677
## Parameter:
## [1] -0.9527340  0.9180609 -0.9801020  0.9719554
## Function Value
## [1] 7.877564
## Gradient:
## [1]  0.04219647 -0.13869745  0.04643103 -0.14490666
##
## iteration = 40
## Step:
## [1] -0.0002099415  0.0003876740 -0.0002321590  0.0004410041
## Parameter:
## [1] -0.9529440  0.9184486 -0.9803341  0.9723964
## Function Value
## [1] 7.877434
## Gradient:
## [1]  0.03791725 -0.12461594  0.04192945 -0.13086560
##
## iteration = 41
## Step:
## [1] -0.0001885004  0.0003481274 -0.0002095994  0.0003982746
## Parameter:
## [1] -0.9531325  0.9187967 -0.9805437  0.9727947
## Function Value
## [1] 7.877328
## Gradient:
## [1]  0.03406226 -0.11193167  0.03787442 -0.11821790
##
## iteration = 42
## Step:
## [1] -0.0001692034  0.0003125205 -0.0001892971  0.0003598034
## Parameter:
## [1] -0.9533017  0.9191092 -0.9807330  0.9731545
## Function Value
## [1] 7.877242
## Gradient:
## [1]  0.03058934 -0.10050556  0.03422152 -0.10682487
##
## iteration = 43
## Step:
## [1] -0.0001518338  0.0002804581 -0.0001710239  0.0003251634
## Parameter:
## [1] -0.9534535  0.9193897 -0.9809041  0.9734796
## Function Value
## [1] 7.877172
## Gradient:
## [1]  0.02746054 -0.09021245  0.03093080 -0.09656171
##
## iteration = 44
```

```
## Step:
## [1] -0.0001361975  0.0002515852 -0.0001545751  0.0002939707
## Parameter:
## [1] -0.9535897  0.9196413 -0.9810586  0.9737736
## Function Value
## [1] 7.877115
## Gradient:
## [1]  0.02464170 -0.08093968  0.02796628 -0.08731610
##
## iteration = 45
## Step:
## [1] -0.0001221200  0.0002255827 -0.0001397672  0.0002658804
## Parameter:
## [1] -0.9537118  0.9198668 -0.9811984  0.9740395
## Function Value
## [1] 7.877069
## Gradient:
## [1]  0.02210205 -0.07258586  0.02529556 -0.07898695
##
## iteration = 46
## Step:
## [1] -0.0001094447  0.0002021638 -0.0001264351  0.0002405825
## Parameter:
## [1] -0.9538213  0.9200690 -0.9813248  0.9742801
## Function Value
## [1] 7.877032
## Gradient:
## [1]  0.01981390 -0.06505973  0.02288949 -0.07148325
##
## iteration = 47
## Step:
## [1] -9.803093e-05  1.810706e-04 -1.144309e-04  2.177981e-04
## Parameter:
## [1] -0.9539193  0.9202501 -0.9814393  0.9744979
## Function Value
## [1] 7.877002
## Gradient:
## [1]  0.01775230 -0.05827911  0.02072180 -0.06472304
##
## iteration = 48
## Step:
## [1] -8.775243e-05  1.620711e-04 -1.036215e-04  1.972765e-04
## Parameter:
## [1] -0.9540070  0.9204122 -0.9815429  0.9746951
## Function Value
## [1] 7.876977
## Gradient:
## [1]  0.01589478 -0.05217001  0.01876885 -0.05863257
##
## iteration = 49
## Step:
## [1] -7.849563e-05  1.449568e-04 -9.388733e-05  1.787923e-04
## Parameter:
## [1] -0.9540855  0.9205571 -0.9816368  0.9748739
```

```
## Function Value
## [1] 7.876957
## Gradient:
## [1]  0.01422113 -0.04666582  0.01700935 -0.05314538
##
## iteration = 50
## Step:
## [1] -7.015842e-05  1.295399e-04 -8.512093e-05  1.621426e-04
## Parameter:
## [1] -0.9541557  0.9206866 -0.9817219  0.9750361
## Function Value
## [1] 7.876941
## Gradient:
## [1]  0.01271311 -0.04170654  0.01542411 -0.04820166
##
## iteration = 51
## Step:
## [1] -6.264904e-05  1.156516e-04 -7.722567e-05  1.471448e-04
## Parameter:
## [1] -0.9542184  0.9208023 -0.9817991  0.9751832
## Function Value
## [1] 7.876928
## Gradient:
## [1]  0.01135431 -0.03723815  0.01399587 -0.04374754
##
## iteration = 52
## Step:
## [1] -5.588494e-05  1.031398e-04 -7.011467e-05  1.336346e-04
## Parameter:
## [1] -0.9542742  0.9209054 -0.9818692  0.9753169
## Function Value
## [1] 7.876917
## Gradient:
## [1]  0.01012996 -0.03321200  0.01270906 -0.03973448
##
## iteration = 53
## Step:
## [1] -4.979185e-05  9.186780e-05 -6.370975e-05  1.214641e-04
## Parameter:
## [1] -0.9543240  0.9209973 -0.9819329  0.9754383
## Function Value
## [1] 7.876908
## Gradient:
## [1]  0.009026736 -0.029584242  0.011549679 -0.036118777
##
## iteration = 54
## Step:
## [1] -4.430297e-05  8.171235e-05 -5.794061e-05  1.105004e-04
## Parameter:
## [1] -0.9543683  0.9210790 -0.9819909  0.9755488
## Function Value
## [1] 7.876901
## Gradient:
## [1]  0.008032641 -0.026315422  0.010505094 -0.032861072
```

```
##
## iteration = 55
## Step:
## [1] -3.935818e-05  7.256263e-05 -5.274397e-05  1.006234e-04
## Parameter:
## [1] -0.9544077  0.9211516 -0.9820436  0.9756494
## Function Value
## [1] 7.876895
## Gradient:
## [1]  0.007136870 -0.023369977  0.009563939 -0.029925902
##
## iteration = 56
## Step:
## [1] -3.490339e-05  6.431882e-05 -4.806288e-05  9.172539e-05
## Parameter:
## [1] -0.9544426  0.9212159 -0.9820917  0.9757412
## Function Value
## [1] 7.876891
## Gradient:
## [1]  0.006329690 -0.020715882  0.008715969 -0.027281325
##
## iteration = 57
## Step:
## [1] -3.088991e-05  5.689103e-05 -4.384610e-05  8.370920e-05
## Parameter:
## [1] -0.9544735  0.9212728 -0.9821355  0.9758249
## Function Value
## [1] 7.876887
## Gradient:
## [1]  0.005602331 -0.018324283  0.007951959 -0.024898566
##
## iteration = 58
## Step:
## [1] -2.727389e-05  5.019833e-05 -4.004750e-05  7.648736e-05
## Parameter:
## [1] -0.9545008  0.9213230 -0.9821756  0.9759014
## Function Value
## [1] 7.876884
## Gradient:
## [1]  0.004946890 -0.016169189  0.007263594 -0.022751704
##
## iteration = 59
## Step:
## [1] -2.401587e-05  4.416784e-05 -3.662555e-05  6.998109e-05
## Parameter:
## [1] -0.9545248  0.9213672 -0.9822122  0.9759713
## Function Value
## [1] 7.876881
## Gradient:
## [1]  0.004356251 -0.014227187  0.006643386 -0.020817386
##
## iteration = 60
## Step:
## [1] -2.108033e-05  3.873392e-05 -3.354285e-05  6.411946e-05
```

```
## Parameter:
## [1] -0.9545459  0.9214059 -0.9822458  0.9760355
## Function Value
## [1] 7.876879
## Gradient:
## [1]  0.003823999 -0.012477182  0.006084587 -0.019074574
##
## iteration = 61
## Step:
## [1] -1.843528e-05  3.383744e-05 -3.076575e-05  5.883857e-05
## Parameter:
## [1] -0.9545643  0.9214397 -0.9822765  0.9760943
## Function Value
## [1] 7.876877
## Gradient:
## [1]  0.003344358 -0.010900169  0.005581120 -0.017504313
##
## iteration = 62
## Step:
## [1] -1.605189e-05  2.942515e-05 -2.826394e-05  5.408088e-05
## Parameter:
## [1] -0.9545803  0.9214692 -0.9823048  0.9761484
## Function Value
## [1] 7.876876
## Gradient:
## [1]  0.002912120 -0.009479027  0.005127508 -0.016089527
##
## iteration = 63
## Step:
## [1] -1.390424e-05  2.544908e-05 -2.601010e-05  4.979455e-05
## Parameter:
## [1] -0.9545942  0.9214946 -0.9823308  0.9761982
## Function Value
## [1] 7.876875
## Gradient:
## [1]  0.002522596 -0.008198330  0.004718816 -0.014814831
##
## iteration = 64
## Step:
## [1] -1.196895e-05  2.186604e-05 -2.397965e-05  4.593290e-05
## Parameter:
## [1] -0.9546062  0.9215165 -0.9823548  0.9762441
## Function Value
## [1] 7.876874
## Gradient:
## [1]  0.002171559 -0.007044181  0.004350599 -0.013666362
##
## iteration = 65
## Step:
## [1] -1.022498e-05  1.863710e-05 -2.215046e-05  4.245386e-05
## Parameter:
## [1] -0.9546164  0.9215351 -0.9823769  0.9762866
## Function Value
## [1] 7.876873
```

```
## Gradient:
## [1]  0.001855202 -0.006004056  0.004018852 -0.012631632
##
## iteration = 66
## Step:
## [1] -8.653371e-06  1.572720e-05 -2.050257e-05  3.931955e-05
## Parameter:
## [1] -0.9546251  0.9215508 -0.9823974  0.9763259
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001570093 -0.005066674  0.003719966 -0.011699386
##
## iteration = 67
## Step:
## [1] -7.237055e-06  1.310476e-05 -1.901803e-05  3.649584e-05
## Parameter:
## [1] -0.9546323  0.9215639 -0.9824164  0.9763624
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001313142 -0.004221869  0.003450689 -0.010859486
##
## iteration = 68
## Step:
## [1] -5.960649e-06  1.074131e-05 -1.768066e-05  3.395197e-05
## Parameter:
## [1] -0.9546383  0.9215747 -0.9824341  0.9763963
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.001081562 -0.003460482  0.003208092 -0.010102797
##
## iteration = 69
## Step:
## [1] -4.810300e-06  8.611225e-06 -1.647587e-05  3.166025e-05
## Parameter:
## [1] -0.9546431  0.9215833 -0.9824506  0.9764280
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.0008728447 -0.0027742610  0.0029895339 -0.0094210856
##
## iteration = 70
## Step:
## [1] -3.773526e-06  6.691408e-06 -1.539054e-05  2.959571e-05
## Parameter:
## [1] -0.9546469  0.9215900 -0.9824660  0.9764576
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0006847275 -0.0021557690  0.0027926375 -0.0088069365
##
## iteration = 71
```

```
## Step:
## [1] -2.839083e-06  4.961048e-06 -1.441284e-05  2.773588e-05
## Parameter:
## [1] -0.9546497  0.9215949 -0.9824804  0.9764853
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.000515173 -0.001598306  0.002615259 -0.008253666
##
## iteration = 72
## Step:
## [1] -1.996843e-06  3.401398e-06 -1.353211e-05  2.606049e-05
## Parameter:
## [1] -0.9546517  0.9215983 -0.9824939  0.9765114
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0003623451 -0.0010958356  0.0024554681 -0.0077552527
##
## iteration = 73
## Step:
## [1] -1.237679e-06  1.995567e-06 -1.273876e-05  2.455130e-05
## Parameter:
## [1] -0.9546529  0.9216003 -0.9825067  0.9765359
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0002245894 -0.0006429174  0.0023115243 -0.0073062704
##
## iteration = 74
## Step:
## [1] -5.533700e-07  7.283349e-07 -1.202413e-05  2.319185e-05
## Parameter:
## [1] -0.9546535  0.9216011 -0.9825187  0.9765591
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0001004150 -0.0002346499  0.0021818599 -0.0069018300
##
## iteration = 75
## Step:
## [1]  6.349210e-08 -4.140111e-07 -1.138043e-05  2.196734e-05
## Parameter:
## [1] -0.9546534  0.9216007 -0.9825301  0.9765811
## Function Value
## [1] 7.876869
## Gradient:
## [1] -1.152136e-05  1.333833e-04  2.065062e-03 -6.537526e-03
##
## iteration = 76
## Step:
## [1]  6.195811e-07 -1.443826e-06 -1.080065e-05  2.086440e-05
## Parameter:
## [1] -0.9546528  0.9215992 -0.9825409  0.9766020
```

```
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0001124301  0.0004651620  0.0019598589 -0.0062093889
##
## iteration = 77
## Step:
## [1]  1.120910e-06 -2.372243e-06 -1.027846e-05  1.987102e-05
## Parameter:
## [1] -0.9546517  0.9215968 -0.9825512  0.9766218
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002034020  0.0007642722  0.0018651024 -0.0059138415
##
## iteration = 78
## Step:
## [1]  1.572897e-06 -3.209292e-06 -9.808155e-06  1.897636e-05
## Parameter:
## [1] -0.9546501  0.9215936 -0.9825610  0.9766408
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002854198  0.0010339451  0.0017797595 -0.0056476608
##
## iteration = 79
## Step:
## [1]  1.980424e-06 -3.964012e-06 -9.384605e-06  1.817064e-05
## Parameter:
## [1] -0.9546481  0.9215897 -0.9825704  0.9766590
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0003593692  0.0012770923  0.0017028990 -0.0054079425
##
## iteration = 80
## Step:
## [1]  2.347888e-06 -4.644546e-06 -9.003184e-06  1.744507e-05
## Parameter:
## [1] -0.9546458  0.9215850 -0.9825794  0.9766764
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.000426048  0.001496337  0.001633682 -0.005192070
##
## iteration = 81
## Step:
## [1]  2.679254e-06 -5.258234e-06 -8.659724e-06  1.679173e-05
## Parameter:
## [1] -0.9546431  0.9215798 -0.9825880  0.9766932
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0004861755  0.0016940444  0.0015713518 -0.0049976836
```

```
##
## iteration = 82
## Step:
## [1]  2.978091e-06 -5.811686e-06 -8.350468e-06  1.620346e-05
## Parameter:
## [1] -0.9546401  0.9215740 -0.9825964  0.9767094
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0005403996  0.0018723437  0.0015152278 -0.0048226599
##
## iteration = 83
## Step:
## [1]  3.247619e-06 -6.310861e-06 -8.072033e-06  1.567383e-05
## Parameter:
## [1] -0.9546369  0.9215676 -0.9826044  0.9767251
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000589304  0.002033155  0.001464696 -0.004665084
##
## iteration = 84
## Step:
## [1]  3.490735e-06 -6.761128e-06 -7.821371e-06  1.519705e-05
## Parameter:
## [1] -0.9546334  0.9215609 -0.9826123  0.9767403
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000633415  0.002178207  0.001419203 -0.004523229
##
## iteration = 85
## Step:
## [1]  3.710052e-06 -7.167323e-06 -7.595732e-06  1.476787e-05
## Parameter:
## [1] -0.9546297  0.9215537 -0.9826199  0.9767551
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0006732066  0.0023090593  0.0013782505 -0.0043955420
##
## iteration = 86
## Step:
## [1]  3.907924e-06 -7.533807e-06 -7.392643e-06  1.438160e-05
## Parameter:
## [1] -0.9546258  0.9215462 -0.9826272  0.9767694
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000709106  0.002427116  0.001341389 -0.004280621
##
## iteration = 87
## Step:
## [1]  4.086473e-06 -7.864506e-06 -7.209873e-06  1.403400e-05
```

```
## Parameter:
## [1] -0.9546217  0.9215383 -0.9826345  0.9767835
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0007414982  0.0025336428  0.0013082150 -0.0041772039
##
## iteration = 88
## Step:
## [1]  4.247610e-06 -8.162960e-06 -7.045411e-06  1.372122e-05
## Parameter:
## [1] -0.9546174  0.9215301 -0.9826415  0.9767972
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000770730  0.002629780  0.001278363 -0.004084153
##
## iteration = 89
## Step:
## [1]  4.393056e-06 -8.432358e-06 -6.897446e-06  1.343985e-05
## Parameter:
## [1] -0.9546131  0.9215217 -0.9826484  0.9768106
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000797114  0.002716555  0.001251504 -0.004000443
##
## iteration = 90
## Step:
## [1]  4.524363e-06 -8.675573e-06 -6.764348e-06  1.318675e-05
## Parameter:
## [1] -0.9546085  0.9215130 -0.9826552  0.9768238
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008209319  0.0027948936  0.0012273423 -0.0039251507
##
## iteration = 91
## Step:
## [1]  4.642931e-06 -8.895197e-06 -6.644645e-06  1.295915e-05
## Parameter:
## [1] -0.9546039  0.9215041 -0.9826618  0.9768368
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008424374  0.0028656307  0.0012056113 -0.0038574434
##
## iteration = 92
## Step:
## [1]  4.750018e-06 -9.093561e-06 -6.537012e-06  1.275452e-05
## Parameter:
## [1] -0.9545991  0.9214951 -0.9826683  0.9768495
## Function Value
## [1] 7.876868
```

```
## Gradient:
## [1] -0.0008618593  0.0029295178  0.0011860705 -0.0037965711
##
## iteration = 93
## Step:
## [1]  4.846761e-06 -9.272768e-06 -6.440257e-06  1.257058e-05
## Parameter:
## [1] -0.9545943  0.9214858 -0.9826748  0.9768621
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0008794036  0.0029872324  0.0011685032 -0.0037418580
##
## iteration = 94
## Step:
## [1]  4.934182e-06 -9.434714e-06 -6.353302e-06  1.240530e-05
## Parameter:
## [1] -0.9545894  0.9214763 -0.9826811  0.9768745
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000895256  0.003039385  0.001152714 -0.003692695
##
## iteration = 95
## Step:
## [1]  5.013203e-06 -9.581105e-06 -6.275179e-06  1.225682e-05
## Parameter:
## [1] -0.9545843  0.9214668 -0.9826874  0.9768868
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000909584  0.003086526  0.001138528 -0.003648534
##
## iteration = 96
## Step:
## [1]  5.084656e-06 -9.713480e-06 -6.205014e-06  1.212348e-05
## Parameter:
## [1] -0.9545793  0.9214571 -0.9826936  0.9768989
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009225381  0.0031291509  0.0011257852 -0.0036088788
##
## iteration = 97
## Step:
## [1]  5.149289e-06 -9.833224e-06 -6.142021e-06  1.200379e-05
## Parameter:
## [1] -0.9545741  0.9214472 -0.9826998  0.9769109
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009342544  0.0031677063  0.0011143439 -0.0035732853
##
## iteration = 98
```

```
## Step:
## [1]  5.207775e-06 -9.941589e-06 -6.085489e-06  1.189640e-05
## Parameter:
## [1] -0.9545689  0.9214373 -0.9827058  0.9769228
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009448553  0.0032025948  0.0011040749 -0.0035413514
##
## iteration = 99
## Step:
## [1]  5.260724e-06 -1.003970e-05 -6.034780e-06  1.180009e-05
## Parameter:
## [1] -0.9545636  0.9214272 -0.9827119  0.9769346
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009544511  0.0032341791  0.0010948625 -0.0035127154
##
## iteration = 100
## Parameter:
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009631411  0.0032627860  0.0010866021 -0.0034870511
##
## Iteration limit exceeded.  Algorithm failed.
```

```
print(t1nlm)
```

```
## $minimum
## [1] 7.876867
##
## $estimate
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
##
## $gradient
## [1] -0.0009631411  0.0032627860  0.0010866021 -0.0034870511
##
## $code
## [1] 4
##
## $iterations
## [1] 100
```

```
## BUT ... it looks like nlminb is NOT using a true Newton-type method
t1nlminb <- nlminb(x0, wood.f, gradient=wood.g, hessian=wood.h, control=list(trace=1))
```

```
##   0:     19192.000: -3.00000 -1.00000 -3.00000 -1.00000
##   1:     7844.8110: -2.34188 -0.814182 -2.36728 -0.827234
##   2:     1216.1133: -1.68443 0.393908 -1.69404 0.313883
##   3:     63.588133: -1.45306  1.71905 -1.44613  1.65143
##   4:     16.494685: -1.19893  1.31181 -1.19548  1.29854
##   5:     8.5754053: -1.04247  1.05440 -1.02968  1.02406
```

```
##   6:     7.8893787: -0.984105 0.972760 -0.971950 0.949965
##   7:     7.8769236: -0.977607 0.965750 -0.960225 0.933163
##   8:     7.8762403: -1.00695  1.02307 -0.929896 0.875292
##   9:     7.8736524: -1.06370  1.13800 -0.867196 0.759942
##  10:     7.8639782: -1.11594  1.25222 -0.798496 0.645011
##  11:     7.8548631: -1.13591  1.29937 -0.765236 0.596982
##  12:     7.8409865: -1.21945  1.48926 -0.637546 0.403740
##  13:     7.7765044: -1.25895  1.59258 -0.545841 0.302611
##  14:     7.7110154: -1.29377  1.68164 -0.448823 0.206860
##  15:     7.6479639: -1.36018  1.85456 -0.237404 0.0279485
##  16:     7.3414193: -1.36613  1.87495 -0.131228 0.0197128
##  17:     7.1459204: -1.36939  1.88395 -0.0220586 0.00419477
##  18:     6.8499981: -1.37092  1.88810 0.199369 0.00469949
##  19:     6.2503936: -1.33300  1.78413 0.360771 0.116495
##  20:     5.7947116: -1.29523  1.68459 0.513971 0.245855
##  21:     5.3563474: -1.24407  1.55349 0.629984 0.387008
##  22:     4.7857877: -1.13048  1.27366 0.840203 0.665704
##  23:     4.0497195: -0.992126 0.973269 0.983984 0.954618
##  24:     3.5179818: -0.841522 0.692665  1.12885  1.25650
##  25:     3.0488024: -0.658857 0.407011  1.24823  1.54704
##  26:     2.6325343: -0.500037 0.229669  1.31902  1.73685
##  27:     2.3059389: -0.307769 0.0630001  1.38020  1.90295
##  28:     1.9350808: -0.174234 0.0178386  1.39533  1.94834
##  29:     1.6534354: -0.0296107 -0.0129818  1.40343  1.97124
##  30:     1.5716752: 0.234877 -0.00937673  1.40482  1.97512
##  31:    0.94438690: 0.306481 0.0937164  1.36921  1.87497
##  32:    0.74178204: 0.438677 0.174107  1.34892  1.82029
##  33:    0.53974471: 0.526746 0.269899  1.31329  1.72450
##  34:    0.40832179: 0.737124 0.500274  1.22372  1.49026
##  35:    0.12697466: 0.792936 0.627489  1.16789  1.36128
##  36:   0.058109411: 0.874374 0.757557  1.11579  1.24255
##  37:   0.014841605: 0.942386 0.883727  1.05723  1.11444
##  38:  0.0015171247: 0.983721 0.966104  1.01742  1.03352
##  39: 2.9694247e-05: 0.997648 0.995118  1.00255  1.00487
##  40: 1.6239267e-08: 0.999951 0.999896  1.00006  1.00010
##  41: 5.1701724e-15:  1.00000  1.00000  1.00000  1.00000
##  42: 2.3217039e-28:  1.00000  1.00000  1.00000  1.00000
##  43: 1.8414972e-30:  1.00000  1.00000  1.00000  1.00000
```

```
print(t1nlminb)
```

```
## $par
## [1] 1 1 1 1
##
## $objective
## [1] 1.841497e-30
##
## $convergence
## [1] 0
##
## $iterations
## [1] 43
##
## $evaluations
## function gradient
```

```
##       54       44
##
## $message
## [1] "X-convergence (3)"
```

```
# and call them from optimx (i.e., test this gives same results)

library(optimx)
t1nlmo <- optimr(x0, wood.f, wood.g, hess=wood.h, method="nlm", control=list(trace=1))
```

```
## Unit parameter scaling
## iteration = 0
## Step:
## [1] 0 0 0 0
## Parameter:
## [1] -3 -1 -3 -1
## Function Value
## [1] 19192
## Gradient:
## [1] -12008  -2080 -10808  -1880
##
## iteration = 1
## Step:
## [1] 0.9116709 0.7367972 0.9020511 0.7516950
## Parameter:
## [1] -2.0883291 -0.2632028 -2.0979489 -0.2483050
## Function Value
## [1] 4166.416
## Gradient:
## [1] -3869.0187  -975.0974 -3517.9318  -887.1722
##
## iteration = 2
## Step:
## [1] 0.5765246 0.5075403 0.5704409 0.5190481
## Parameter:
## [1] -1.5118046  0.2443374 -1.5275080  0.2707431
## Function Value
## [1] 834.2691
## Gradient:
## [1] -1239.3913  -437.9468 -1139.2505  -400.9499
##
## iteration = 3
## Step:
## [1] 0.3452070 0.3077087 0.3415096 0.3149447
## Parameter:
## [1] -1.1665976  0.5520461 -1.1859985  0.5856878
## Function Value
## [1] 142.99
## Gradient:
## [1] -381.7993 -179.0328 -354.8650 -165.0014
##
## iteration = 4
## Step:
## [1] 0.1855520 0.1499399 0.1833595 0.1541052
## Parameter:
```

```
## [1] -0.9810455  0.7019861 -1.0026390  0.7397929
## Function Value
## [1] 24.17926
## Gradient:
## [1] -106.17301  -63.26483  -99.83461  -58.94541
##
## iteration = 5
## Step:
## [1] 0.07782451 0.05872218 0.07664496 0.06140066
## Parameter:
## [1] -0.9032210  0.7607082 -0.9259940  0.8011936
## Function Value
## [1] 9.839749
## Gradient:
## [1] -23.71342 -19.79005 -22.61047 -18.88270
##
## iteration = 6
## Step:
## [1] 0.01753078 0.02499409 0.01707308 0.02682737
## Parameter:
## [1] -0.8856902  0.7857023 -0.9089209  0.8280210
## Function Value
## [1] 8.69256
## Gradient:
## [1] -3.326713 -7.482969 -3.201469 -7.378002
##
## iteration = 7
## Step:
## [1] -0.002949427  0.015908253 -0.003260912  0.017258585
## Parameter:
## [1] -0.8886397  0.8016106 -0.9121818  0.8452796
## Function Value
## [1] 8.500379
## Gradient:
## [1]  0.4633586 -4.6849032  0.5115861 -4.6767719
##
## iteration = 8
## Step:
## [1] -0.006478477  0.012986001 -0.006839623  0.014092239
## Parameter:
## [1] -0.8951181  0.8145966 -0.9190215  0.8593718
## Function Value
## [1] 8.374769
## Gradient:
## [1]  0.9933129 -3.8575663  1.0490214 -3.8528361
##
## iteration = 9
## Step:
## [1] -0.006355528  0.011342611 -0.006713964  0.012301691
## Parameter:
## [1] -0.9014737  0.8259392 -0.9257354  0.8716735
## Function Value
## [1] 8.275711
## Gradient:
```

```
## [1]  0.9872768 -3.4000080  1.0433216 -3.3948677
##
## iteration = 10
## Step:
## [1] -0.005715703  0.010053796 -0.006042570  0.010901894
## Parameter:
## [1] -0.9071894  0.8359930 -0.9317780  0.8825754
## Function Value
## [1] 8.196865
## Gradient:
## [1]  0.9031660 -3.0378609  0.9550868 -3.0335914
##
## iteration = 11
## Step:
## [1] -0.005075092  0.008946817 -0.005366382  0.009701843
## Parameter:
## [1] -0.9122645  0.8449398 -0.9371444  0.8922772
## Function Value
## [1] 8.133955
## Gradient:
## [1]  0.8146503 -2.7224545  0.8616142 -2.7194185
##
## iteration = 12
## Step:
## [1] -0.004503371  0.007976157 -0.004762011  0.008650385
## Parameter:
## [1] -0.9167678  0.8529160 -0.9419064  0.9009276
## Function Value
## [1] 8.083658
## Gradient:
## [1]  0.7329608 -2.4421895  0.7752177 -2.4403344
##
## iteration = 13
## Step:
## [1] -0.004001282  0.007119844 -0.004231240  0.007723143
## Parameter:
## [1] -0.9207691  0.8600358 -0.9461376  0.9086507
## Function Value
## [1] 8.043374
## Gradient:
## [1]  0.6591984 -2.1919825  0.6972101 -2.1911665
##
## iteration = 14
## Step:
## [1] -0.003560435  0.006362120 -0.003765379  0.006902861
## Parameter:
## [1] -0.9243296  0.8663979 -0.9499030  0.9155536
## Function Value
## [1] 8.011061
## Gradient:
## [1]  0.5928602 -1.9682377  0.6270863 -1.9683199
##
## iteration = 15
## Step:
```

```
## [1] -0.003172450  0.005690158 -0.003355552  0.006175565
## Parameter:
## [1] -0.9275020  0.8720881 -0.9532586  0.9217292
## Function Value
## [1] 7.985107
## Gradient:
## [1]  0.5332412 -1.7679572  0.5640969 -1.7688150
##
## iteration = 16
## Step:
## [1] -0.002830125  0.005093157 -0.002994097  0.005529510
## Parameter:
## [1] -0.9303321  0.8771812 -0.9562527  0.9272587
## Function Value
## [1] 7.964235
## Gradient:
## [1]  0.4796636 -1.5885404  0.5075167 -1.5900697
##
## iteration = 17
## Step:
## [1] -0.002527390  0.004561911 -0.002674558  0.004954699
## Parameter:
## [1] -0.9328595  0.8817432 -0.9589272  0.9322134
## Function Value
## [1] 7.947433
## Gradient:
## [1]  0.4315101 -1.4277069  0.4566857 -1.4298200
##
## iteration = 18
## Step:
## [1] -0.002259118  0.004088520 -0.002391484  0.004442553
## Parameter:
## [1] -0.9351186  0.8858317 -0.9613187  0.9366559
## Function Value
## [1] 7.933894
## Gradient:
## [1]  0.3882254 -1.2834487  0.4110108 -1.2860707
##
## iteration = 19
## Step:
## [1] -0.002020955  0.003666169 -0.002140251  0.003985678
## Parameter:
## [1] -0.9371396  0.8894978 -0.9634590  0.9406416
## Function Value
## [1] 7.922976
## Gradient:
## [1]  0.3493109 -1.1539919  0.3699607 -1.1570590
##
## iteration = 20
## Step:
## [1] -0.001809184  0.003288947 -0.001916914  0.003577664
## Parameter:
## [1] -0.9389488  0.8927868 -0.9653759  0.9442193
## Function Value
```

```
## [1] 7.914165
## Gradient:
## [1]  0.3143196 -1.0377658  0.3330597 -1.0412235
##
## iteration = 21
## Step:
## [1] -0.001620614  0.002951713 -0.001718090  0.003212935
## Parameter:
## [1] -0.9405694  0.8957385 -0.9670940  0.9474322
## Function Value
## [1] 7.90705
## Gradient:
## [1]  0.2828509 -0.9333771  0.2998820 -0.9371784
##
## iteration = 22
## Step:
## [1] -0.001452492  0.002649974 -0.001540861  0.002886621
## Parameter:
## [1] -0.9420219  0.8983885 -0.9686348  0.9503188
## Function Value
## [1] 7.901301
## Gradient:
## [1]  0.2545455 -0.8395874  0.2700463 -0.8436915
##
## iteration = 23
## Step:
## [1] -0.001302432  0.002379790 -0.001382703  0.002594456
## Parameter:
## [1] -0.9433243  0.9007683 -0.9700175  0.9529133
## Function Value
## [1] 7.896654
## Gradient:
## [1]  0.2290818 -0.7552944  0.2432114 -0.7596662
##
## iteration = 24
## Step:
## [1] -0.001168361  0.002137702 -0.001241420  0.002332689
## Parameter:
## [1] -0.9444927  0.9029060 -0.9712589  0.9552460
## Function Value
## [1] 7.892895
## Gradient:
## [1]  0.2061710 -0.6795158  0.2190714 -0.6841244
##
## iteration = 25
## Step:
## [1] -0.001048470  0.001920659 -0.001115098  0.002098019
## Parameter:
## [1] -0.9455411  0.9048266 -0.9723740  0.9573440
## Function Value
## [1] 7.889854
## Gradient:
## [1]  0.1855546 -0.6113746  0.1973525 -0.6161934
##
```

```
## iteration = 26
## Step:
## [1] -0.0009411735  0.0017259680 -0.0010020619  0.0018875279
## Parameter:
## [1] -0.9464823  0.9065526 -0.9733761  0.9592315
## Function Value
## [1] 7.887392
## Gradient:
## [1]  0.1670005 -0.5500879  0.1778092 -0.5550933
##
## iteration = 27
## Step:
## [1] -0.0008450808  0.0015512445 -0.0009008417  0.0016986351
## Parameter:
## [1] -0.9473274  0.9081038 -0.9742769  0.9609302
## Function Value
## [1] 7.885399
## Gradient:
## [1]  0.1503004 -0.4949553  0.1602211 -0.5001268
##
## iteration = 28
## Step:
## [1] -0.0007589677  0.0013943750 -0.0008101442  0.0015290525
## Parameter:
## [1] -0.9480864  0.9094982 -0.9750871  0.9624592
## Function Value
## [1] 7.883785
## Gradient:
## [1]  0.1352675 -0.4453502  0.1443910 -0.4506697
##
## iteration = 29
## Step:
## [1] -0.0006817541  0.0012534820 -0.0007288287  0.0013767484
## Parameter:
## [1] -0.9487681  0.9107517 -0.9758159  0.9638360
## Function Value
## [1] 7.882477
## Gradient:
## [1]  0.1217341 -0.4007116  0.1301415 -0.4061631
##
## iteration = 30
## Step:
## [1] -0.0006124854  0.0011268958 -0.0006558873  0.0012399159
## Parameter:
## [1] -0.9493806  0.9118786 -0.9764718  0.9650759
## Function Value
## [1] 7.881417
## Gradient:
## [1]  0.1095495 -0.3605364  0.1173135 -0.3661059
##
## iteration = 31
## Step:
## [1] -0.0005503158  0.0010131294 -0.0005904275  0.0011169461
## Parameter:
```

```
## [1] -0.9499309  0.9128917 -0.9770622  0.9661928
## Function Value
## [1] 7.880559
## Gradient:
## [1]  0.09857832 -0.32437405  0.10576421 -0.33004899
##
## iteration = 32
## Step:
## [1] -0.0004944952  0.0009108570 -0.0005316577  0.0010064045
## Parameter:
## [1] -0.9504254  0.9138026 -0.9775939  0.9671992
## Function Value
## [1] 7.879862
## Gradient:
## [1]  0.08869905 -0.29181992  0.09536535 -0.29758930
##
## iteration = 33
## Step:
## [1] -0.0004443567  0.0008188950 -0.0004788748  0.0009070108
## Parameter:
## [1] -0.9508698  0.9146215 -0.9780728  0.9681062
## Function Value
## [1] 7.879298
## Gradient:
## [1]  0.07980241 -0.26251108  0.08600161 -0.26836511
##
## iteration = 34
## Step:
## [1] -0.0003993074  0.0007361861 -0.0004314530  0.0008176213
## Parameter:
## [1] -0.9512691  0.9153577 -0.9785042  0.9689239
## Function Value
## [1] 7.87884
## Gradient:
## [1]  0.07179017 -0.23612164  0.07756940 -0.24205160
##
## iteration = 35
## Step:
## [1] -0.0003588187  0.0006617846 -0.0003888351  0.0007372134
## Parameter:
## [1] -0.9516279  0.9160194 -0.9788931  0.9696611
## Function Value
## [1] 7.878469
## Gradient:
## [1]  0.06457399 -0.21235885  0.06997558 -0.21835702
##
## iteration = 36
## Step:
## [1] -0.0003224194  0.0005948446 -0.0003505244  0.0006648718
## Parameter:
## [1] -0.9519503  0.9166143 -0.9792436  0.9703259
## Function Value
## [1] 7.878168
## Gradient:
```

```
## [1]  0.05807447 -0.19095972  0.06313643 -0.19701921
##
## iteration = 37
## Step:
## [1] -0.0002896886  0.0005346081 -0.0003160773  0.0005997772
## Parameter:
## [1] -0.9522400  0.9171489 -0.9795597  0.9709257
## Function Value
## [1] 7.877923
## Gradient:
## [1]  0.05222015 -0.17168788  0.05697664 -0.17780255
##
## iteration = 38
## Step:
## [1] -0.0002602503  0.0004803963 -0.0002850973  0.0005411952
## Parameter:
## [1] -0.9525003  0.9176293 -0.9798447  0.9714669
## Function Value
## [1] 7.877725
## Gradient:
## [1]  0.04694676 -0.15433082  0.05142848 -0.16049519
##
## iteration = 39
## Step:
## [1] -0.0002337683  0.0004316002 -0.0002572303  0.0004884677
## Parameter:
## [1] -0.9527340  0.9180609 -0.9801020  0.9719554
## Function Value
## [1] 7.877564
## Gradient:
## [1]  0.04219647 -0.13869745  0.04643103 -0.14490666
##
## iteration = 40
## Step:
## [1] -0.0002099415  0.0003876740 -0.0002321590  0.0004410041
## Parameter:
## [1] -0.9529440  0.9184486 -0.9803341  0.9723964
## Function Value
## [1] 7.877434
## Gradient:
## [1]  0.03791725 -0.12461594  0.04192945 -0.13086560
##
## iteration = 41
## Step:
## [1] -0.0001885004  0.0003481274 -0.0002095994  0.0003982746
## Parameter:
## [1] -0.9531325  0.9187967 -0.9805437  0.9727947
## Function Value
## [1] 7.877328
## Gradient:
## [1]  0.03406226 -0.11193167  0.03787442 -0.11821790
##
## iteration = 42
## Step:
```

```
## [1] -0.0001692034  0.0003125205 -0.0001892971  0.0003598034
## Parameter:
## [1] -0.9533017  0.9191092 -0.9807330  0.9731545
## Function Value
## [1] 7.877242
## Gradient:
## [1]  0.03058934 -0.10050556  0.03422152 -0.10682487
##
## iteration = 43
## Step:
## [1] -0.0001518338  0.0002804581 -0.0001710239  0.0003251634
## Parameter:
## [1] -0.9534535  0.9193897 -0.9809041  0.9734796
## Function Value
## [1] 7.877172
## Gradient:
## [1]  0.02746054 -0.09021245  0.03093080 -0.09656171
##
## iteration = 44
## Step:
## [1] -0.0001361975  0.0002515852 -0.0001545751  0.0002939707
## Parameter:
## [1] -0.9535897  0.9196413 -0.9810586  0.9737736
## Function Value
## [1] 7.877115
## Gradient:
## [1]  0.02464170 -0.08093968  0.02796628 -0.08731610
##
## iteration = 45
## Step:
## [1] -0.0001221200  0.0002255827 -0.0001397672  0.0002658804
## Parameter:
## [1] -0.9537118  0.9198668 -0.9811984  0.9740395
## Function Value
## [1] 7.877069
## Gradient:
## [1]  0.02210205 -0.07258586  0.02529556 -0.07898695
##
## iteration = 46
## Step:
## [1] -0.0001094447  0.0002021638 -0.0001264351  0.0002405825
## Parameter:
## [1] -0.9538213  0.9200690 -0.9813248  0.9742801
## Function Value
## [1] 7.877032
## Gradient:
## [1]  0.01981390 -0.06505973  0.02288949 -0.07148325
##
## iteration = 47
## Step:
## [1] -9.803093e-05  1.810706e-04 -1.144309e-04  2.177981e-04
## Parameter:
## [1] -0.9539193  0.9202501 -0.9814393  0.9744979
## Function Value
```

```
## [1] 7.877002
## Gradient:
## [1]  0.01775230 -0.05827911  0.02072180 -0.06472304
##
## iteration = 48
## Step:
## [1] -8.775243e-05  1.620711e-04 -1.036215e-04  1.972765e-04
## Parameter:
## [1] -0.9540070  0.9204122 -0.9815429  0.9746951
## Function Value
## [1] 7.876977
## Gradient:
## [1]  0.01589478 -0.05217001  0.01876885 -0.05863257
##
## iteration = 49
## Step:
## [1] -7.849563e-05  1.449568e-04 -9.388733e-05  1.787923e-04
## Parameter:
## [1] -0.9540855  0.9205571 -0.9816368  0.9748739
## Function Value
## [1] 7.876957
## Gradient:
## [1]  0.01422113 -0.04666582  0.01700935 -0.05314538
##
## iteration = 50
## Step:
## [1] -7.015842e-05  1.295399e-04 -8.512093e-05  1.621426e-04
## Parameter:
## [1] -0.9541557  0.9206866 -0.9817219  0.9750361
## Function Value
## [1] 7.876941
## Gradient:
## [1]  0.01271311 -0.04170654  0.01542411 -0.04820166
##
## iteration = 51
## Step:
## [1] -6.264904e-05  1.156516e-04 -7.722567e-05  1.471448e-04
## Parameter:
## [1] -0.9542184  0.9208023 -0.9817991  0.9751832
## Function Value
## [1] 7.876928
## Gradient:
## [1]  0.01135431 -0.03723815  0.01399587 -0.04374754
##
## iteration = 52
## Step:
## [1] -5.588494e-05  1.031398e-04 -7.011467e-05  1.336346e-04
## Parameter:
## [1] -0.9542742  0.9209054 -0.9818692  0.9753169
## Function Value
## [1] 7.876917
## Gradient:
## [1]  0.01012996 -0.03321200  0.01270906 -0.03973448
##
```

```
## iteration = 53
## Step:
## [1] -4.979185e-05  9.186780e-05 -6.370975e-05  1.214641e-04
## Parameter:
## [1] -0.9543240  0.9209973 -0.9819329  0.9754383
## Function Value
## [1] 7.876908
## Gradient:
## [1]  0.009026736 -0.029584242  0.011549679 -0.036118777
##
## iteration = 54
## Step:
## [1] -4.430297e-05  8.171235e-05 -5.794061e-05  1.105004e-04
## Parameter:
## [1] -0.9543683  0.9210790 -0.9819909  0.9755488
## Function Value
## [1] 7.876901
## Gradient:
## [1]  0.008032641 -0.026315422  0.010505094 -0.032861072
##
## iteration = 55
## Step:
## [1] -3.935818e-05  7.256263e-05 -5.274397e-05  1.006234e-04
## Parameter:
## [1] -0.9544077  0.9211516 -0.9820436  0.9756494
## Function Value
## [1] 7.876895
## Gradient:
## [1]  0.007136870 -0.023369977  0.009563939 -0.029925902
##
## iteration = 56
## Step:
## [1] -3.490339e-05  6.431882e-05 -4.806288e-05  9.172539e-05
## Parameter:
## [1] -0.9544426  0.9212159 -0.9820917  0.9757412
## Function Value
## [1] 7.876891
## Gradient:
## [1]  0.006329690 -0.020715882  0.008715969 -0.027281325
##
## iteration = 57
## Step:
## [1] -3.088991e-05  5.689103e-05 -4.384610e-05  8.370920e-05
## Parameter:
## [1] -0.9544735  0.9212728 -0.9821355  0.9758249
## Function Value
## [1] 7.876887
## Gradient:
## [1]  0.005602331 -0.018324283  0.007951959 -0.024898566
##
## iteration = 58
## Step:
## [1] -2.727389e-05  5.019833e-05 -4.004750e-05  7.648736e-05
## Parameter:
```

```
## [1] -0.9545008  0.9213230 -0.9821756  0.9759014
## Function Value
## [1] 7.876884
## Gradient:
## [1]  0.004946890 -0.016169189  0.007263594 -0.022751704
##
## iteration = 59
## Step:
## [1] -2.401587e-05  4.416784e-05 -3.662555e-05  6.998109e-05
## Parameter:
## [1] -0.9545248  0.9213672 -0.9822122  0.9759713
## Function Value
## [1] 7.876881
## Gradient:
## [1]  0.004356251 -0.014227187  0.006643386 -0.020817386
##
## iteration = 60
## Step:
## [1] -2.108033e-05  3.873392e-05 -3.354285e-05  6.411946e-05
## Parameter:
## [1] -0.9545459  0.9214059 -0.9822458  0.9760355
## Function Value
## [1] 7.876879
## Gradient:
## [1]  0.003823999 -0.012477182  0.006084587 -0.019074574
##
## iteration = 61
## Step:
## [1] -1.843528e-05  3.383744e-05 -3.076575e-05  5.883857e-05
## Parameter:
## [1] -0.9545643  0.9214397 -0.9822765  0.9760943
## Function Value
## [1] 7.876877
## Gradient:
## [1]  0.003344358 -0.010900169  0.005581120 -0.017504313
##
## iteration = 62
## Step:
## [1] -1.605189e-05  2.942515e-05 -2.826394e-05  5.408088e-05
## Parameter:
## [1] -0.9545803  0.9214692 -0.9823048  0.9761484
## Function Value
## [1] 7.876876
## Gradient:
## [1]  0.002912120 -0.009479027  0.005127508 -0.016089527
##
## iteration = 63
## Step:
## [1] -1.390424e-05  2.544908e-05 -2.601010e-05  4.979455e-05
## Parameter:
## [1] -0.9545942  0.9214946 -0.9823308  0.9761982
## Function Value
## [1] 7.876875
## Gradient:
```

```
## [1]  0.002522596 -0.008198330  0.004718816 -0.014814831
##
## iteration = 64
## Step:
## [1] -1.196895e-05  2.186604e-05 -2.397965e-05  4.593290e-05
## Parameter:
## [1] -0.9546062  0.9215165 -0.9823548  0.9762441
## Function Value
## [1] 7.876874
## Gradient:
## [1]  0.002171559 -0.007044181  0.004350599 -0.013666362
##
## iteration = 65
## Step:
## [1] -1.022498e-05  1.863710e-05 -2.215046e-05  4.245386e-05
## Parameter:
## [1] -0.9546164  0.9215351 -0.9823769  0.9762866
## Function Value
## [1] 7.876873
## Gradient:
## [1]  0.001855202 -0.006004056  0.004018852 -0.012631632
##
## iteration = 66
## Step:
## [1] -8.653371e-06  1.572720e-05 -2.050257e-05  3.931955e-05
## Parameter:
## [1] -0.9546251  0.9215508 -0.9823974  0.9763259
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001570093 -0.005066674  0.003719966 -0.011699386
##
## iteration = 67
## Step:
## [1] -7.237055e-06  1.310476e-05 -1.901803e-05  3.649584e-05
## Parameter:
## [1] -0.9546323  0.9215639 -0.9824164  0.9763624
## Function Value
## [1] 7.876872
## Gradient:
## [1]  0.001313142 -0.004221869  0.003450689 -0.010859486
##
## iteration = 68
## Step:
## [1] -5.960649e-06  1.074131e-05 -1.768066e-05  3.395197e-05
## Parameter:
## [1] -0.9546383  0.9215747 -0.9824341  0.9763963
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.001081562 -0.003460482  0.003208092 -0.010102797
##
## iteration = 69
## Step:
```

```
## [1] -4.810300e-06  8.611225e-06 -1.647587e-05  3.166025e-05
## Parameter:
## [1] -0.9546431  0.9215833 -0.9824506  0.9764280
## Function Value
## [1] 7.876871
## Gradient:
## [1]  0.0008728447 -0.0027742610  0.0029895339 -0.0094210856
##
## iteration = 70
## Step:
## [1] -3.773526e-06  6.691408e-06 -1.539054e-05  2.959571e-05
## Parameter:
## [1] -0.9546469  0.9215900 -0.9824660  0.9764576
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0006847275 -0.0021557690  0.0027926375 -0.0088069365
##
## iteration = 71
## Step:
## [1] -2.839083e-06  4.961048e-06 -1.441284e-05  2.773588e-05
## Parameter:
## [1] -0.9546497  0.9215949 -0.9824804  0.9764853
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.000515173 -0.001598306  0.002615259 -0.008253666
##
## iteration = 72
## Step:
## [1] -1.996843e-06  3.401398e-06 -1.353211e-05  2.606049e-05
## Parameter:
## [1] -0.9546517  0.9215983 -0.9824939  0.9765114
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0003623451 -0.0010958356  0.0024554681 -0.0077552527
##
## iteration = 73
## Step:
## [1] -1.237679e-06  1.995567e-06 -1.273876e-05  2.455130e-05
## Parameter:
## [1] -0.9546529  0.9216003 -0.9825067  0.9765359
## Function Value
## [1] 7.87687
## Gradient:
## [1]  0.0002245894 -0.0006429174  0.0023115243 -0.0073062704
##
## iteration = 74
## Step:
## [1] -5.533700e-07  7.283349e-07 -1.202413e-05  2.319185e-05
## Parameter:
## [1] -0.9546535  0.9216011 -0.9825187  0.9765591
## Function Value
```

```
## [1] 7.87687
## Gradient:
## [1]  0.0001004150 -0.0002346499  0.0021818599 -0.0069018300
##
## iteration = 75
## Step:
## [1]  6.349210e-08 -4.140111e-07 -1.138043e-05  2.196734e-05
## Parameter:
## [1] -0.9546534  0.9216007 -0.9825301  0.9765811
## Function Value
## [1] 7.876869
## Gradient:
## [1] -1.152136e-05  1.333833e-04  2.065062e-03 -6.537526e-03
##
## iteration = 76
## Step:
## [1]  6.195811e-07 -1.443826e-06 -1.080065e-05  2.086440e-05
## Parameter:
## [1] -0.9546528  0.9215992 -0.9825409  0.9766020
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0001124301  0.0004651620  0.0019598589 -0.0062093889
##
## iteration = 77
## Step:
## [1]  1.120910e-06 -2.372243e-06 -1.027846e-05  1.987102e-05
## Parameter:
## [1] -0.9546517  0.9215968 -0.9825512  0.9766218
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002034020  0.0007642722  0.0018651024 -0.0059138415
##
## iteration = 78
## Step:
## [1]  1.572897e-06 -3.209292e-06 -9.808155e-06  1.897636e-05
## Parameter:
## [1] -0.9546501  0.9215936 -0.9825610  0.9766408
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0002854198  0.0010339451  0.0017797595 -0.0056476608
##
## iteration = 79
## Step:
## [1]  1.980424e-06 -3.964012e-06 -9.384605e-06  1.817064e-05
## Parameter:
## [1] -0.9546481  0.9215897 -0.9825704  0.9766590
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0003593692  0.0012770923  0.0017028990 -0.0054079425
##
```

```
## iteration = 80
## Step:
## [1]  2.347888e-06 -4.644546e-06 -9.003184e-06  1.744507e-05
## Parameter:
## [1] -0.9546458  0.9215850 -0.9825794  0.9766764
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.000426048  0.001496337  0.001633682 -0.005192070
##
## iteration = 81
## Step:
## [1]  2.679254e-06 -5.258234e-06 -8.659724e-06  1.679173e-05
## Parameter:
## [1] -0.9546431  0.9215798 -0.9825880  0.9766932
## Function Value
## [1] 7.876869
## Gradient:
## [1] -0.0004861755  0.0016940444  0.0015713518 -0.0049976836
##
## iteration = 82
## Step:
## [1]  2.978091e-06 -5.811686e-06 -8.350468e-06  1.620346e-05
## Parameter:
## [1] -0.9546401  0.9215740 -0.9825964  0.9767094
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0005403996  0.0018723437  0.0015152278 -0.0048226599
##
## iteration = 83
## Step:
## [1]  3.247619e-06 -6.310861e-06 -8.072033e-06  1.567383e-05
## Parameter:
## [1] -0.9546369  0.9215676 -0.9826044  0.9767251
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000589304  0.002033155  0.001464696 -0.004665084
##
## iteration = 84
## Step:
## [1]  3.490735e-06 -6.761128e-06 -7.821371e-06  1.519705e-05
## Parameter:
## [1] -0.9546334  0.9215609 -0.9826123  0.9767403
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000633415  0.002178207  0.001419203 -0.004523229
##
## iteration = 85
## Step:
## [1]  3.710052e-06 -7.167323e-06 -7.595732e-06  1.476787e-05
## Parameter:
```

```
## [1] -0.9546297  0.9215537 -0.9826199  0.9767551
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0006732066  0.0023090593  0.0013782505 -0.0043955420
##
## iteration = 86
## Step:
## [1]  3.907924e-06 -7.533807e-06 -7.392643e-06  1.438160e-05
## Parameter:
## [1] -0.9546258  0.9215462 -0.9826272  0.9767694
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000709106  0.002427116  0.001341389 -0.004280621
##
## iteration = 87
## Step:
## [1]  4.086473e-06 -7.864506e-06 -7.209873e-06  1.403400e-05
## Parameter:
## [1] -0.9546217  0.9215383 -0.9826345  0.9767835
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0007414982  0.0025336428  0.0013082150 -0.0041772039
##
## iteration = 88
## Step:
## [1]  4.247610e-06 -8.162960e-06 -7.045411e-06  1.372122e-05
## Parameter:
## [1] -0.9546174  0.9215301 -0.9826415  0.9767972
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000770730  0.002629780  0.001278363 -0.004084153
##
## iteration = 89
## Step:
## [1]  4.393056e-06 -8.432358e-06 -6.897446e-06  1.343985e-05
## Parameter:
## [1] -0.9546131  0.9215217 -0.9826484  0.9768106
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.000797114  0.002716555  0.001251504 -0.004000443
##
## iteration = 90
## Step:
## [1]  4.524363e-06 -8.675573e-06 -6.764348e-06  1.318675e-05
## Parameter:
## [1] -0.9546085  0.9215130 -0.9826552  0.9768238
## Function Value
## [1] 7.876868
## Gradient:
```

```
## [1] -0.0008209319  0.0027948936  0.0012273423 -0.0039251507
##
## iteration = 91
## Step:
## [1]  4.642931e-06 -8.895197e-06 -6.644645e-06  1.295915e-05
## Parameter:
## [1] -0.9546039  0.9215041 -0.9826618  0.9768368
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008424374  0.0028656307  0.0012056113 -0.0038574434
##
## iteration = 92
## Step:
## [1]  4.750018e-06 -9.093561e-06 -6.537012e-06  1.275452e-05
## Parameter:
## [1] -0.9545991  0.9214951 -0.9826683  0.9768495
## Function Value
## [1] 7.876868
## Gradient:
## [1] -0.0008618593  0.0029295178  0.0011860705 -0.0037965711
##
## iteration = 93
## Step:
## [1]  4.846761e-06 -9.272768e-06 -6.440257e-06  1.257058e-05
## Parameter:
## [1] -0.9545943  0.9214858 -0.9826748  0.9768621
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0008794036  0.0029872324  0.0011685032 -0.0037418580
##
## iteration = 94
## Step:
## [1]  4.934182e-06 -9.434714e-06 -6.353302e-06  1.240530e-05
## Parameter:
## [1] -0.9545894  0.9214763 -0.9826811  0.9768745
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000895256  0.003039385  0.001152714 -0.003692695
##
## iteration = 95
## Step:
## [1]  5.013203e-06 -9.581105e-06 -6.275179e-06  1.225682e-05
## Parameter:
## [1] -0.9545843  0.9214668 -0.9826874  0.9768868
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.000909584  0.003086526  0.001138528 -0.003648534
##
## iteration = 96
## Step:
```

```
## [1]  5.084656e-06 -9.713480e-06 -6.205014e-06  1.212348e-05
## Parameter:
## [1] -0.9545793  0.9214571 -0.9826936  0.9768989
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009225381  0.0031291509  0.0011257852 -0.0036088788
##
## iteration = 97
## Step:
## [1]  5.149289e-06 -9.833224e-06 -6.142021e-06  1.200379e-05
## Parameter:
## [1] -0.9545741  0.9214472 -0.9826998  0.9769109
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009342544  0.0031677063  0.0011143439 -0.0035732853
##
## iteration = 98
## Step:
## [1]  5.207775e-06 -9.941589e-06 -6.085489e-06  1.189640e-05
## Parameter:
## [1] -0.9545689  0.9214373 -0.9827058  0.9769228
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009448553  0.0032025948  0.0011040749 -0.0035413514
##
## iteration = 99
## Step:
## [1]  5.260724e-06 -1.003970e-05 -6.034780e-06  1.180009e-05
## Parameter:
## [1] -0.9545636  0.9214272 -0.9827119  0.9769346
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009544511  0.0032341791  0.0010948625 -0.0035127154
##
## iteration = 100
## Parameter:
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
## Function Value
## [1] 7.876867
## Gradient:
## [1] -0.0009631411  0.0032627860  0.0010866021 -0.0034870511
##
## Iteration limit exceeded.  Algorithm failed.
```

```
print(t1nlmo)
```

```
## $convergence
## [1] 1
##
## $value
## [1] 7.876867
```

```
##
## $par
## [1] -0.9545583  0.9214171 -0.9827179  0.9769463
##
## $counts
## [1]  NA 100
##
## $message
## [1] "Convergence indicator (code) =  4"
```

```
## FOLLOWING SHOWS UP ERRORS??
tst<-try(t1nlminbo <- optimr(x0, wood.f, wood.g, hessian=wood.h, method="nlminb", control=list(trace=1))
```

```
## Unit parameter scaling
##   0:     19192.000: -3.00000 -1.00000 -3.00000 -1.00000
##   1:     7427.8901: -2.26768 -0.873148 -2.34086 -0.885346
##   2:     2311.9042: -1.74107 -0.585584 -1.59870 -0.586687
##   3:     729.03082: -0.890037 -0.154631 -1.48501 -0.309027
##   4:     71.909337: -0.662552 0.00461052 -0.584618 0.0259220
##   5:     45.347493: -0.496228 0.812202 -0.597210 0.591584
##   6:     40.726120: -0.781428 0.331672 -0.911765  1.35892
##   7:     38.214601: -0.616770 0.466804 -1.29242  1.11452
##   8:     9.0237120: -0.632614 0.459113 -1.06180  1.20941
##   9:     8.2132107: -0.730131 0.520690 -1.08662  1.20729
##  10:     8.1312801: -0.712954 0.540234 -1.10168  1.20856
##  11:     7.9975970: -0.730221 0.552271 -1.09958  1.22996
##  12:     7.9277653: -0.744706 0.569886 -1.11342  1.24388
##  13:     7.8789361: -0.755791 0.588373 -1.11841  1.26427
##  14:     7.8572896: -0.769023 0.602168 -1.12872  1.28509
##  15:     7.8542867: -0.758659 0.588755 -1.13933  1.30758
##  16:     7.8516697: -0.747519 0.572011 -1.14845  1.32802
##  17:     7.8407298: -0.701838 0.502829 -1.18349  1.40792
##  18:     7.8347831: -0.675646 0.464352 -1.20184  1.45170
##  19:     7.8269048: -0.647600 0.425840 -1.21863  1.49492
##  20:     7.8180840: -0.618136 0.387587 -1.23640  1.53702
##  21:     7.7885058: -0.552793 0.312287 -1.26851  1.61892
##  22:     7.7821195: -0.419328 0.160287 -1.33332  1.77893
##  23:     7.5970425: -0.307065 0.0941103 -1.35111  1.82958
##  24:     7.5804254: -0.208693 0.0202352 -1.37462  1.89421
##  25:     7.3437276: -0.0780988 -0.00556015 -1.35638  1.86918
##  26:     7.1581672: -0.0194280 0.0186872 -1.34933  1.82889
##  27:     7.0299891: 0.0547616 0.00526381 -1.35282  1.83147
##  28:     6.9376955: 0.129979 0.00128356 -1.35077  1.82643
##  29:     6.8290238: 0.203941 0.0135397 -1.34645  1.81845
##  30:     6.6127919: 0.271070 0.0466648 -1.34054  1.81041
##  31:     6.1309123: 0.402538 0.151936 -1.32765  1.78307
##  32:     6.0145341: 0.536980 0.241710 -1.30999  1.72983
##  33:     5.5405642: 0.616386 0.365849 -1.28116  1.64778
##  34:     5.2319328: 0.795053 0.584208 -1.20685  1.45258
##  35:     4.5260478: 0.890916 0.783724 -1.10587  1.19953
##  36:     4.2748887:  1.01190 0.991796 -1.00274 0.965731
##  37:     3.8118699:  1.08143  1.18900 -0.870747 0.716572
##  38:     3.4183212:  1.15499  1.32928 -0.799149 0.611139
##  39:     3.1274174:  1.21586  1.47840 -0.710683 0.531683
##  40:     2.9319507:  1.27451  1.60801 -0.636964 0.411656
```

```
##  41:     2.8074922:  1.32360  1.73246 -0.554527 0.287616
##  42:     2.6979772:  1.32786  1.76497 -0.514976 0.238953
##  43:     2.5978103:  1.34096  1.80616 -0.473380 0.201421
##  44:     2.3504274:  1.38633  1.93240 -0.341213 0.0937173
##  45:     2.1879401:  1.38800  1.95384 -0.238752 0.0837674
##  46:     1.9738919:  1.40950  2.00599 -0.163369 0.0369160
##  47:     1.9313844:  1.44936  2.10143 -0.00276268 -0.0511139
##  48:     1.8029335:  1.42563  2.02275 -0.0940048 0.00894612
##  49:     1.7285050:  1.41878  2.01719 -0.0661541 -0.00273086
##  50:     1.6366521:  1.41533  2.00942 0.0310760 -0.0322593
##  51:     1.5496975:  1.40481  1.97844 0.0339044 -0.0185982
##  52:     1.4051372:  1.38420  1.91570 0.101330 -0.000593362
##  53:     1.2768628:  1.36993  1.87268 0.183317 0.0207516
##  54:     1.0456071:  1.33870  1.78169 0.375868 0.104726
##  55:    0.68801294:  1.31481  1.71351 0.459776 0.220743
##  56:    0.52234496:  1.29010  1.64819 0.573486 0.309683
##  57:    0.40368085:  1.26199  1.58808 0.657749 0.403783
##  58:    0.31496884:  1.23861  1.55128 0.682889 0.474259
##  59:    0.24192757:  1.22004  1.50114 0.726901 0.526257
##  60:    0.17455784:  1.18712  1.41509 0.791930 0.611852
##  61:    0.12430840:  1.15221  1.32010 0.839019 0.685985
##  62:   0.092133333:  1.13812  1.28426 0.843130 0.706236
##  63:   0.053032095:  1.10146  1.20252 0.890392 0.788802
##  64:   0.013424283:  1.05583  1.11085 0.947014 0.893435
##  65:  0.0036272623:  1.03077  1.06089 0.971812 0.944280
##  66:  0.0019038445:  1.00653  1.01008 0.998354 0.993783
##  67: 0.00020234604:  1.00160  1.00299 0.999171 0.999539
##  68: 5.8035620e-05:  1.00048  1.00066  1.00056  1.00139
##  69: 1.3800484e-05: 0.999916 0.999665  1.00068  1.00135
##  70: 5.7617897e-07: 0.999894 0.999763  1.00021  1.00036
##  71: 1.1827891e-08: 0.999977 0.999952  1.00003  1.00005
##  72: 4.1006268e-11: 0.999999 0.999998  1.00000  1.00000
##  73: 8.2725368e-14:  1.00000  1.00000  1.00000  1.00000
##  74: 3.9750565e-16:  1.00000  1.00000  1.00000  1.00000
##  75: 3.9451721e-18:  1.00000  1.00000  1.00000  1.00000
```

```
if (class(tst) == "try-error"){
    cat("try-error on attempt to run nlminb in optimr()\n")
} else { print(t1nlminb) }
```

```
## try-error on attempt to run nlminb in optimr()
```

```
# sink()
```

A generalized Rosenbrock function

There are several generalizations of the Rosenbrock function (??ref)

```
# genrosa function code -- attempts to match the rosenbrock at gs=100 and x=c(-1.2,1)
genrosa.f<- function(x, gs=NULL){ # objective function
## One generalization of the Rosenbrock banana valley function (n parameters)
    n <- length(x)
        if(is.null(gs)) { gs=100.0 }
        # Note do not at 1.0 so min at 0
    fval<-sum (gs*(x[1:(n-1)]^2 - x[2:n])^2 + (x[1:(n-1)] - 1)^2)
```

```
}

genrosa.g <- function(x, gs=NULL){
# vectorized gradient for genrose.f
# Ravi Varadhan 2009-04-03
    n <- length(x)
        if(is.null(gs)) { gs=100.0 }
    gg <- as.vector(rep(0, n))
    tn <- 2:n
    tn1 <- tn - 1
    z1 <- x[tn] - x[tn1]^2
    z2 <- 1 - x[tn1]
        # f = gs*z1*z1 + z2*z2
    gg[tn] <- 2 * (gs * z1)
    gg[tn1] <- gg[tn1] - 4 * gs * x[tn1] * z1 - 2 *z2
    return(gg)
}

genrosa.h <- function(x, gs=NULL) { ## compute Hessian
   if(is.null(gs)) { gs=100.0 }
    n <- length(x)
    hh<-matrix(rep(0, n*n),n,n)
    for (i in 2:n) {
        z1<-x[i]-x[i-1]*x[i-1]
#       z2<-1.0 - x[i-1]
                hh[i,i]<-hh[i,i]+2.0*(gs+1.0)
                hh[i-1,i-1]<-hh[i-1,i-1]-4.0*gs*z1-4.0*gs*x[i-1]*(-2.0*x[i-1])
                hh[i,i-1]<-hh[i,i-1]-4.0*gs*x[i-1]
                hh[i-1,i]<-hh[i-1,i]-4.0*gs*x[i-1]
    }
        return(hh)
}

require(snewton)
cat("Generalized Rosenbrock tests\n")
```

```
## Generalized Rosenbrock tests
```

```
cat("original function")
```

```
## original function
```

```
x0 <- c(-1.2, 1)
solorig <- snewton(x0, genrosa.f, genrosa.g, genrosa.h)
```

```
## trace = 1
## Initial function value =  24.2
## nf= 1
## Iteration  1 :nf= 2
## Iteration  2 :nf= 3
## Iteration  3 :nf= 4
## Iteration  4 :nf= 6
## Iteration  5 :nf= 8
## Iteration  6 :nf= 10
## Iteration  7 :nf= 12
```

```
## Iteration  8 :nf= 14
## Iteration  9 :nf= 16
## Iteration  10 :nf= 18
## Iteration  11 :nf= 20
## Iteration  12 :nf= 22
## Iteration  13 :nf= 24
## Iteration  14 :nf= 26
## Iteration  15 :nf= 28
## Iteration  16 :nf= 30
## Iteration  17 :nf= 32
## Iteration  18 :nf= 33
## Iteration  19 :nf= 34
## Iteration  20 :nf= 36
## Iteration  21 :nf= 37
## Iteration  22 :nf= 38
## Iteration  23 :nf= 39
## Iteration  24 :nf= 40
## Iteration  25 :nf= 41
## Iteration  26 :nf= 42
## Iteration  27 :nf= 43
## Iteration  28 :nf= 44
## Iteration  29 :nf= 45
## Iteration  30 :nf= 46
## Iteration  31 :nf= 47
## Iteration  32 :nf= 48
## Iteration  33 :nf= 49
## Iteration  34 :nf= 50
## Iteration  35 :nf= 51
## Iteration  36 :nf= 52
## Iteration  37 :nf= 53
## Iteration  38 :nf= 54
## Iteration  39 :nf= 55
## Iteration  40 :nf= 56
## Iteration  41 :nf= 57
## Iteration  42 :nf= 58
## Iteration  43 :nf= 59
## Iteration  44 :nf= 60
## Iteration  45 :nf= 61
## Iteration  46 :nf= 62
## Iteration  47 :nf= 63
## Iteration  48 :nf= 64
## Iteration  49 :nf= 65
## Iteration  50 :nf= 66
## Iteration  51 :nf= 67
## Iteration  52 :nf= 68
## Iteration  53 :nf= 69
## Iteration  54 :nf= 70
## Iteration  55 :nf= 71
## Iteration  56 :nf= 72
## Iteration  57 :nf= 73
## Iteration  58 :nf= 74
## Iteration  59 :nf= 75
## Iteration  60 :nf= 76
## Iteration  61 :nf= 77
```

```
## Iteration  62 :nf= 78
## Iteration  63 :nf= 79
## Iteration  64 :nf= 80
## Iteration  65 :nf= 81
## Iteration  66 :nf= 82
## Iteration  67 :nf= 83
## Iteration  68 :nf= 84
## Iteration  69 :nf= 85
## Iteration  70 :nf= 86
## Iteration  71 :nf= 87
## Iteration  72 :nf= 88
## Iteration  73 :nf= 89
## Iteration  74 :nf= 90
## Iteration  75 :nf= 91
## Iteration  76 :nf= 92
## Iteration  77 :nf= 93
## Iteration  78 :nf= 94
## Iteration  79 :nf= 95
## Iteration  80 :nf= 96
## Iteration  81 :nf= 97
## Iteration  82 :nf= 98
## Iteration  83 :nf= 99
## Iteration  84 :nf= 100
## Iteration  85 :nf= 101
## Iteration  86 :nf= 102
## Iteration  87 :nf= 103
## Iteration  88 :nf= 104
## Iteration  89 :nf= 105
## Iteration  90 :nf= 106
## Iteration  91 :nf= 107
## Iteration  92 :nf= 108
## Iteration  93 :nf= 109
## Iteration  94 :nf= 110
## Iteration  95 :nf= 111
## Iteration  96 :nf= 112
## Iteration  97 :nf= 113
## Iteration  98 :nf= 114
## Iteration  99 :nf= 115
## Iteration  100 :nf= 116
## Iteration  101 :nf= 117
## Iteration  102 :nf= 118
## Iteration  103 :nf= 119
## Iteration  104 :nf= 120
## Iteration  105 :nf= 121
## Iteration  106 :nf= 122
## Iteration  107 :nf= 123
## Iteration  108 :nf= 124
## Iteration  109 :nf= 125
## Iteration  110 :nf= 126
## Iteration  111 :nf= 127
## Iteration  112 :nf= 128
## Iteration  113 :nf= 129
## Iteration  114 :nf= 130
## Iteration  115 :nf= 131
```

```
## Iteration  116 :nf= 132
## Iteration  117 :nf= 133
## Iteration  118 :nf= 134
## Iteration  119 :nf= 135
## Iteration  120 :nf= 136
## Iteration  121 :nf= 137
## Iteration  122 :nf= 138
## Iteration  123 :nf= 139
## Iteration  124 :nf= 140
## Iteration  125 :nf= 141
## Iteration  126 :nf= 142
## Iteration  127 :nf= 143
## Iteration  128 :No progress before linesearch!
## No progress in linesearch!
```

```
print(solorig)
```

```
## $par
## [1] 1 1
##
## $value
## [1] 2.972526e-28
##
## $grad
## [1]  5.462297e-14 -4.440892e-14
##
## $Hess
##      [,1] [,2]
## [1,]  800 -400
## [2,] -400  202
##
## $counts
## $counts$niter
## [1] 128
##
## $counts$nfn
## [1] 144
##
## $counts$ngr
## [1] 128
##
## $counts$nhess
## [1] 128
##
##
## $convcode
## [1] 93
```

```
print(eigen(solorig$Hess)$values)
```

```
## [1] 1000.400641    1.599359
```

```
solorigm <- snewtonm(x0, genrosa.f, genrosa.g, genrosa.h)
```

```
## trace = 1
##  Start snewtonm   f0= 24.2   at  [1] -1.2  1.0
```

```
## Iteration  1  fbest= 24.2
##  lambda = 0.0001220703   fval= 4.691037
## Iteration  2  fbest= 4.691037
##  lambda = 1.831055e-05   fval= 4.537878
## Iteration  3  fbest= 4.537878
##  lambda = 2.746582e-06   fval= 3.033359
## Iteration  4  fbest= 3.033359
##  lambda = 4.119873e-07   fval= 7.883925
##  lambda = 0.001220703   fval= 7.870029
##  lambda = 0.01220703   fval= 7.746736
##  lambda = 0.1220703   fval= 6.67461
##  lambda = 1.220703   fval= 2.997128
## Iteration  9  fbest= 2.997128
##  lambda = 0.1831055   fval= 1.799511
## Iteration  10  fbest= 1.799511
##  lambda = 0.02746582   fval= 48.82691
##  lambda = 0.2746582   fval= 32.02697
##  lambda = 2.746582   fval= 2.798801
##  lambda = 27.46582   fval= 1.621835
## Iteration  14  fbest= 1.621835
##  lambda = 4.119873   fval= 4.540856
##  lambda = 41.19873   fval= 1.499646
## Iteration  16  fbest= 1.499646
##  lambda = 6.17981   fval= 2.268889
##  lambda = 61.7981   fval= 1.418034
## Iteration  18  fbest= 1.418034
##  lambda = 9.269714   fval= 1.203653
## Iteration  19  fbest= 1.203653
##  lambda = 1.390457   fval= 0.7529179
## Iteration  20  fbest= 0.7529179
##  lambda = 0.2085686   fval= 8.235142
##  lambda = 2.085686   fval= 1.485472
##  lambda = 20.85686   fval= 0.6414404
## Iteration  23  fbest= 0.6414404
##  lambda = 3.128529   fval= 0.6430264
##  lambda = 31.28529   fval= 0.5779681
## Iteration  25  fbest= 0.5779681
##  lambda = 4.692793   fval= 0.4683426
## Iteration  26  fbest= 0.4683426
##  lambda = 0.7039189   fval= 0.2656592
## Iteration  27  fbest= 0.2656592
##  lambda = 0.1055878   fval= 0.5121154
##  lambda = 1.055878   fval= 0.2218384
## Iteration  29  fbest= 0.2218384
##  lambda = 0.1583818   fval= 0.09357798
## Iteration  30  fbest= 0.09357798
##  lambda = 0.02375726   fval= 0.06471772
## Iteration  31  fbest= 0.06471772
##  lambda = 0.00356359   fval= 0.02374038
## Iteration  32  fbest= 0.02374038
##  lambda = 0.0005345384   fval= 0.01191497
## Iteration  33  fbest= 0.01191497
##  lambda = 8.018077e-05   fval= 0.006286775
## Iteration  34  fbest= 0.006286775
```

```
##  lambda = 1.202711e-05   fval= 0.003343837
## Iteration  35  fbest= 0.003343837
##  lambda = 1.804067e-06   fval= 0.001801541
## Iteration  36  fbest= 0.001801541
##  lambda = 2.706101e-07   fval= 0.0009795606
## Iteration  37  fbest= 0.0009795606
##  lambda = 4.059151e-08   fval= 0.0005369973
## Iteration  38  fbest= 0.0005369973
##  lambda = 6.088727e-09   fval= 0.0002963302
## Iteration  39  fbest= 0.0002963302
##  lambda = 9.13309e-10   fval= 0.0001643687
## Iteration  40  fbest= 0.0001643687
##  lambda = 1.369964e-10   fval= 9.153421e-05
## Iteration  41  fbest= 9.153421e-05
##  lambda = 2.054945e-11   fval= 5.112784e-05
## Iteration  42  fbest= 5.112784e-05
##  lambda = 3.082418e-12   fval= 2.862351e-05
## Iteration  43  fbest= 2.862351e-05
##  lambda = 4.623627e-13   fval= 1.605228e-05
## Iteration  44  fbest= 1.605228e-05
##  lambda = 6.935441e-14   fval= 9.013925e-06
## Iteration  45  fbest= 9.013925e-06
##  lambda = 1.040316e-14   fval= 5.066584e-06
## Iteration  46  fbest= 5.066584e-06
##  lambda = 1.560474e-15   fval= 2.849938e-06
## Iteration  47  fbest= 2.849938e-06
##  lambda = 2.340711e-16   fval= 1.603966e-06
## Iteration  48  fbest= 1.603966e-06
##  lambda = 3.511067e-17   fval= 9.030983e-07
## Iteration  49  fbest= 9.030983e-07
##  lambda = 5.2666e-18   fval= 5.086395e-07
## Iteration  50  fbest= 5.086395e-07
##  lambda = 7.8999e-19   fval= 2.865409e-07
## Iteration  51  fbest= 2.865409e-07
##  lambda = 1.184985e-19   fval= 1.614506e-07
## Iteration  52  fbest= 1.614506e-07
##  lambda = 1.777478e-20   fval= 9.098079e-08
## Iteration  53  fbest= 9.098079e-08
##  lambda = 2.666216e-21   fval= 5.127467e-08
## Iteration  54  fbest= 5.127467e-08
##  lambda = 3.999324e-22   fval= 2.889937e-08
## Iteration  55  fbest= 2.889937e-08
##  lambda = 5.998987e-23   fval= 1.628914e-08
## Iteration  56  fbest= 1.628914e-08
##  lambda = 8.99848e-24   fval= 9.181762e-09
## Iteration  57  fbest= 9.181762e-09
##  lambda = 1.349772e-24   fval= 5.175683e-09
## Iteration  58  fbest= 5.175683e-09
##  lambda = 2.024658e-25   fval= 2.917558e-09
## Iteration  59  fbest= 2.917558e-09
##  lambda = 3.036987e-26   fval= 1.644671e-09
## Iteration  60  fbest= 1.644671e-09
##  lambda = 4.555481e-27   fval= 9.271383e-10
## Iteration  61  fbest= 9.271383e-10
```

```
##  lambda = 6.833221e-28   fval= 5.22654e-10
## Iteration  62  fbest= 5.22654e-10
##  lambda = 1.024983e-28   fval= 2.94637e-10
## Iteration  63  fbest= 2.94637e-10
##  lambda = 1.537475e-29   fval= 1.660974e-10
## Iteration  64  fbest= 1.660974e-10
##  lambda = 2.306212e-30   fval= 9.363539e-11
## Iteration  65  fbest= 9.363539e-11
##  lambda = 3.459318e-31   fval= 5.278599e-11
## Iteration  66  fbest= 5.278599e-11
##  lambda = 5.188977e-32   fval= 2.975763e-11
## Iteration  67  fbest= 2.975763e-11
##  lambda = 7.783466e-33   fval= 1.677563e-11
## Iteration  68  fbest= 1.677563e-11
##  lambda = 1.16752e-33   fval= 9.457142e-12
## Iteration  69  fbest= 9.457142e-12
##  lambda = 1.75128e-34   fval= 5.331402e-12
## Iteration  70  fbest= 5.331402e-12
##  lambda = 2.62692e-35   fval= 3.005545e-12
## Iteration  71  fbest= 3.005545e-12
##  lambda = 3.940379e-36   fval= 1.694359e-12
## Iteration  72  fbest= 1.694359e-12
##  lambda = 5.910569e-37   fval= 9.551853e-13
## Iteration  73  fbest= 9.551853e-13
##  lambda = 8.865854e-38   fval= 5.384806e-13
## Iteration  74  fbest= 5.384806e-13
##  lambda = 1.329878e-38   fval= 3.035656e-13
## Iteration  75  fbest= 3.035656e-13
##  lambda = 1.994817e-39   fval= 1.711335e-13
## Iteration  76  fbest= 1.711335e-13
##  lambda = 2.992226e-40   fval= 9.647566e-14
## Iteration  77  fbest= 9.647566e-14
##  lambda = 4.488338e-41   fval= 5.438767e-14
## Iteration  78  fbest= 5.438767e-14
##  lambda = 6.732508e-42   fval= 3.066078e-14
## Iteration  79  fbest= 3.066078e-14
##  lambda = 1.009876e-42   fval= 1.728486e-14
## Iteration  80  fbest= 1.728486e-14
##  lambda = 1.514814e-43   fval= 9.744257e-15
## Iteration  81  fbest= 9.744257e-15
##  lambda = 2.272221e-44   fval= 5.493277e-15
## Iteration  82  fbest= 5.493277e-15
##  lambda = 3.408332e-45   fval= 3.096808e-15
## Iteration  83  fbest= 3.096808e-15
##  lambda = 5.112498e-46   fval= 1.745811e-15
## Iteration  84  fbest= 1.745811e-15
##  lambda = 7.668747e-47   fval= 9.841922e-16
## Iteration  85  fbest= 9.841922e-16
##  lambda = 1.150312e-47   fval= 5.548336e-16
## Iteration  86  fbest= 5.548336e-16
##  lambda = 1.725468e-48   fval= 3.127847e-16
## Iteration  87  fbest= 3.127847e-16
##  lambda = 2.588202e-49   fval= 1.763309e-16
## Iteration  88  fbest= 1.763309e-16
```

```
##  lambda = 3.882303e-50   fval= 9.940568e-17
## Iteration  89  fbest= 9.940568e-17
##  lambda = 5.823455e-51   fval= 5.603947e-17
## Iteration  90  fbest= 5.603947e-17
##  lambda = 8.735182e-52   fval= 3.159198e-17
## Iteration  91  fbest= 3.159198e-17
##  lambda = 1.310277e-52   fval= 1.780983e-17
## Iteration  92  fbest= 1.780983e-17
##  lambda = 1.965416e-53   fval= 1.00402e-17
## Iteration  93  fbest= 1.00402e-17
##  lambda = 2.948124e-54   fval= 5.660116e-18
## Iteration  94  fbest= 5.660116e-18
##  lambda = 4.422186e-55   fval= 3.190863e-18
## Iteration  95  fbest= 3.190863e-18
##  lambda = 6.633279e-56   fval= 1.798833e-18
## Iteration  96  fbest= 1.798833e-18
##  lambda = 9.949918e-57   fval= 1.014084e-18
## Iteration  97  fbest= 1.014084e-18
##  lambda = 1.492488e-57   fval= 5.716848e-19
## Iteration  98  fbest= 5.716848e-19
##  lambda = 2.238732e-58   fval= 3.222845e-19
## Iteration  99  fbest= 3.222845e-19
##  lambda = 3.358097e-59   fval= 1.816864e-19
## Iteration  100  fbest= 1.816864e-19
##  lambda = 5.037146e-60   fval= 1.024248e-19
## Iteration  101  fbest= 1.024248e-19
##  lambda = 7.555719e-61   fval= 5.774147e-20
## Iteration  102  fbest= 5.774147e-20
##  lambda = 1.133358e-61   fval= 3.255148e-20
## Iteration  103  fbest= 3.255148e-20
##  lambda = 1.700037e-62   fval= 1.835074e-20
## Iteration  104  fbest= 1.835074e-20
##  lambda = 2.550055e-63   fval= 1.034513e-20
## Iteration  105  fbest= 1.034513e-20
##  lambda = 3.825083e-64   fval= 5.832015e-21
## Iteration  106  fbest= 5.832015e-21
##  lambda = 5.737624e-65   fval= 3.287771e-21
## Iteration  107  fbest= 3.287771e-21
##  lambda = 8.606437e-66   fval= 1.853462e-21
## Iteration  108  fbest= 1.853462e-21
##  lambda = 1.290965e-66   fval= 1.044879e-21
## Iteration  109  fbest= 1.044879e-21
##  lambda = 1.936448e-67   fval= 5.890438e-22
## Iteration  110  fbest= 5.890438e-22
##  lambda = 2.904672e-68   fval= 3.320702e-22
## Iteration  111  fbest= 3.320702e-22
##  lambda = 4.357009e-69   fval= 1.87203e-22
## Iteration  112  fbest= 1.87203e-22
##  lambda = 6.535513e-70   fval= 1.055345e-22
## Iteration  113  fbest= 1.055345e-22
##  lambda = 9.803269e-71   fval= 5.949423e-23
## Iteration  114  fbest= 5.949423e-23
##  lambda = 1.47049e-71   fval= 3.353935e-23
## Iteration  115  fbest= 3.353935e-23
```

```
##  lambda = 2.205736e-72   fval= 1.890777e-23
## Iteration  116  fbest= 1.890777e-23
##  lambda = 3.308603e-73   fval= 1.065893e-23
## Iteration  117  fbest= 1.065893e-23
##  lambda = 4.962905e-74   fval= 6.008859e-24
## Iteration  118  fbest= 6.008859e-24
##  lambda = 7.444357e-75   fval= 3.387352e-24
## Iteration  119  fbest= 3.387352e-24
##  lambda = 1.116654e-75   fval= 1.909542e-24
## Iteration  120  fbest= 1.909542e-24
##  lambda = 1.67498e-76   fval= 1.076397e-24
## Iteration  121  fbest= 1.076397e-24
##  lambda = 2.512471e-77   fval= 6.067524e-25
## Iteration  122  fbest= 6.067524e-25
##  lambda = 3.768706e-78   fval= 3.420209e-25
## Iteration  123  fbest= 3.420209e-25
##  lambda = 5.653059e-79   fval= 1.928249e-25
## Iteration  124  fbest= 1.928249e-25
##  lambda = 8.479588e-80   fval= 1.087071e-25
## Iteration  125  fbest= 1.087071e-25
##  lambda = 1.271938e-80   fval= 6.126275e-26
## Iteration  126  fbest= 6.126275e-26
##  lambda = 1.907907e-81   fval= 3.452499e-26
## Iteration  127  fbest= 3.452499e-26
##  lambda = 2.861861e-82   fval= 1.946164e-26
## Iteration  128  fbest= 1.946164e-26
##  lambda = 4.292792e-83   fval= 1.097479e-26
## Iteration  129  fbest= 1.097479e-26
##  lambda = 6.439187e-84   fval= 6.180836e-27
## Iteration  130  fbest= 6.180836e-27
##  lambda = 9.658781e-85   fval= 3.482081e-27
## Iteration  131  fbest= 3.482081e-27
##  lambda = 1.448817e-85   fval= 1.963573e-27
## Iteration  132  fbest= 1.963573e-27
##  lambda = 2.173226e-86   fval= 1.106883e-27
## Iteration  133  fbest= 1.106883e-27
##  lambda = 3.259839e-87   fval= 6.240013e-28
## Iteration  134  fbest= 6.240013e-28
##  lambda = 4.889758e-88   fval= 3.491203e-28
## Iteration  135  fbest= 3.491203e-28
##  lambda = 7.334637e-89   fval= 1.969194e-28
## Iteration  136  fbest= 1.969194e-28
##  lambda = 1.100196e-89   fval= 1.112417e-28
## Iteration  137  fbest= 1.112417e-28
##  lambda = 1.650293e-90   fval= 6.336772e-29
## Iteration  138  fbest= 6.336772e-29
##  lambda = 2.47544e-91   fval= 3.717507e-29
## Iteration  139  fbest= 3.717507e-29
##  lambda = 3.71316e-92   fval= 1.972152e-29
## Iteration  140  fbest= 1.972152e-29
##  lambda = 5.56974e-93   fval= 1.109336e-29
## Iteration  141  fbest= 1.109336e-29
##  lambda = 8.35461e-94   fval= 7.198356e-30
## Iteration  142  fbest= 7.198356e-30
```

```
##  lambda = 1.253191e-94   fval= 3.5622e-30
## Iteration  143  fbest= 3.5622e-30
##  lambda = 1.879787e-95   fval= 2.083086e-30
## Iteration  144  fbest= 2.083086e-30
##  lambda = 2.819681e-96   fval= 1.232595e-30
## Iteration  145  fbest= 1.232595e-30
##  lambda = 4.229521e-97   fval= 1.836567e-30
##  lambda = 0.001220703   fval= 1.836567e-30
##  lambda = 0.01220703   fval= 1.836567e-30
##  lambda = 0.1220703   fval= 2.021456e-30
##  lambda = 1.220703   fval= 2.230997e-30
##  lambda = 12.20703   fval= 2.46519e-30
## Null step
## Finished
```

```
print(solorigm)
```

```
## $xs
## [1] 1 1
##
## $fv
## [1] 1.232595e-30
##
## $grd
## [1] -2.220446e-15  0.000000e+00
##
## $Hess
##      [,1] [,2]
## [1,]  800 -400
## [2,] -400  202
##
## $counts
## $counts$niter
## [1] 152
##
## $counts$nfn
## [1] 151
##
## $counts$ngr
## [1] 132
##
## $counts$nhess
## [1] 132
```

```
print(eigen(solorigm$Hess)$values)
```

```
## [1] 1000.400641    1.599359
```

```
cat("Start with 50 values of pi and scale factor 10\n")
```

```
## Start with 50 values of pi and scale factor 10
```

```
x0 <- rep(pi, 50)
sol50pi <- snewton(x0, genrosa.f, genrosa.g, genrosa.h, gs=10)
```

```
## trace = 1
## Initial function value =  22405.14
```

```
## nf= 1
## Iteration  1 :nf= 2
## Iteration  2 :nf= 3
## Iteration  3 :nf= 4
## Iteration  4 :nf= 5
## Iteration  5 :nf= 6
## Iteration  6 :nf= 7
## Iteration  7 :nf= 8
## Iteration  8 :nf= 9
## Iteration  9 :nf= 10
## Iteration  10 :nf= 11
## Iteration  11 :nf= 12
## Iteration  12 :nf= 13
## Iteration  13 :nf= 14
## Iteration  14 :nf= 15
## Iteration  15 :nf= 16
## Iteration  16 :nf= 17
## Iteration  17 :nf= 18
## Iteration  18 :nf= 19
## Iteration  19 :nf= 20
## Iteration  20 :nf= 21
## Iteration  21 :nf= 22
## Iteration  22 :nf= 23
## Iteration  23 :nf= 24
## Iteration  24 :nf= 25
## Iteration  25 :nf= 26
## Iteration  26 :nf= 27
## Iteration  27 :nf= 28
## Iteration  28 :nf= 29
## Iteration  29 :nf= 30
## Iteration  30 :nf= 31
## Iteration  31 :nf= 32
## Iteration  32 :nf= 33
## Iteration  33 :nf= 34
## Iteration  34 :nf= 35
## Iteration  35 :nf= 36
## Iteration  36 :nf= 37
## Iteration  37 :nf= 38
## Iteration  38 :nf= 39
## Iteration  39 :nf= 40
## Iteration  40 :nf= 41
## Iteration  41 :nf= 42
## Iteration  42 :nf= 43
## Iteration  43 :nf= 44
## Iteration  44 :nf= 45
## Iteration  45 :nf= 46
## Iteration  46 :nf= 47
## Iteration  47 :nf= 48
## Iteration  48 :nf= 49
## Iteration  49 :nf= 50
## Iteration  50 :nf= 51
## Iteration  51 :nf= 52
## Iteration  52 :nf= 53
## Iteration  53 :nf= 54
```

```
## Iteration  54 :nf= 55
## Iteration  55 :nf= 56
## Iteration  56 :nf= 57
## Iteration  57 :nf= 58
## Iteration  58 :nf= 59
## Iteration  59 :nf= 60
## Iteration  60 :nf= 61
## Iteration  61 :nf= 62
## Iteration  62 :nf= 63
## Iteration  63 :nf= 64
## Iteration  64 :nf= 65
## Iteration  65 :nf= 66
## Iteration  66 :nf= 67
## Iteration  67 :nf= 68
## Iteration  68 :nf= 69
## Iteration  69 :nf= 70
## Iteration  70 :nf= 71
## Iteration  71 :nf= 72
## Iteration  72 :nf= 73
## Iteration  73 :nf= 74
## Iteration  74 :nf= 75
## Iteration  75 :nf= 76
## Iteration  76 :nf= 77
## Iteration  77 :nf= 78
## Iteration  78 :nf= 79
## Iteration  79 :nf= 80
## Iteration  80 :nf= 81
## Iteration  81 :nf= 82
## Iteration  82 :nf= 83
## Iteration  83 :nf= 84
## Iteration  84 :nf= 85
## Iteration  85 :nf= 86
## Iteration  86 :nf= 87
## Iteration  87 :nf= 88
## Iteration  88 :nf= 89
## Iteration  89 :nf= 90
## Iteration  90 :nf= 91
## Iteration  91 :nf= 92
## Iteration  92 :nf= 93
## Iteration  93 :nf= 94
## Iteration  94 :nf= 95
## Iteration  95 :nf= 96
## Iteration  96 :nf= 97
## Iteration  97 :nf= 98
## Iteration  98 :nf= 99
## Iteration  99 :nf= 100
## Iteration  100 :nf= 101
## Iteration  101 :nf= 102
## Iteration  102 :nf= 103
## Iteration  103 :nf= 104
## Iteration  104 :nf= 105
## Iteration  105 :nf= 106
## Iteration  106 :nf= 107
## Iteration  107 :nf= 108
```

```
## Iteration  108 :nf= 109
## Iteration  109 :nf= 110
## Iteration  110 :nf= 111
## Iteration  111 :nf= 112
## Iteration  112 :nf= 113
## Iteration  113 :nf= 114
## Iteration  114 :nf= 115
## Iteration  115 :nf= 116
## Iteration  116 :nf= 117
## Iteration  117 :nf= 118
## Iteration  118 :nf= 119
## Iteration  119 :nf= 120
## Iteration  120 :nf= 121
## Iteration  121 :nf= 122
## Iteration  122 :nf= 123
## Iteration  123 :nf= 124
## Iteration  124 :nf= 125
## Iteration  125 :nf= 126
## Iteration  126 :nf= 127
## Iteration  127 :nf= 128
## Iteration  128 :nf= 129
## Iteration  129 :nf= 130
## Iteration  130 :nf= 131
## Iteration  131 :nf= 132
## Iteration  132 :nf= 133
## Iteration  133 :nf= 134
## Iteration  134 :nf= 135
## Iteration  135 :nf= 136
## Iteration  136 :nf= 137
## Iteration  137 :nf= 138
## Iteration  138 :nf= 139
## Iteration  139 :nf= 140
## Iteration  140 :nf= 141
## Iteration  141 :nf= 142
## Iteration  142 :nf= 143
## Iteration  143 :nf= 144
## Iteration  144 :nf= 145
## Iteration  145 :No progress before linesearch!
## No progress in linesearch!
```

```
print(sol50pi)
```

```
## $par
##  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [36] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
##
## $value
## [1] 6.108742e-29
##
## $grad
##  [1]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
##  [6]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [11]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [16]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [21]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
```

```
## [26]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [31]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [36]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [41]  0.000000e+00  0.000000e+00 -8.881784e-15  4.884981e-15  9.769963e-15
## [46] -1.199041e-14  7.549517e-15 -1.154632e-14  1.332268e-14  4.440892e-15
##
## $Hess
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
##  [1,]   80  -40    0    0    0    0    0    0    0     0     0     0     0
##  [2,]  -40  102  -40    0    0    0    0    0    0     0     0     0     0
##  [3,]    0  -40  102  -40    0    0    0    0    0     0     0     0     0
##  [4,]    0    0  -40  102  -40    0    0    0    0     0     0     0     0
##  [5,]    0    0    0  -40  102  -40    0    0    0     0     0     0     0
##  [6,]    0    0    0    0  -40  102  -40    0    0     0     0     0     0
##  [7,]    0    0    0    0    0  -40  102  -40    0     0     0     0     0
##  [8,]    0    0    0    0    0    0  -40  102  -40     0     0     0     0
##  [9,]    0    0    0    0    0    0    0  -40  102   -40     0     0     0
## [10,]    0    0    0    0    0    0    0    0  -40   102   -40     0     0
## [11,]    0    0    0    0    0    0    0    0    0   -40   102   -40     0
## [12,]    0    0    0    0    0    0    0    0    0     0   -40   102   -40
## [13,]    0    0    0    0    0    0    0    0    0     0     0   -40   102
## [14,]    0    0    0    0    0    0    0    0    0     0     0     0   -40
## [15,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [16,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [17,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [18,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [19,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [20,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [21,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [22,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [23,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [24,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [25,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [26,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [27,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [28,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [29,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [30,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [31,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [32,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [33,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [34,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [35,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [36,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [37,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [38,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [39,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [40,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [41,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [42,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [43,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [44,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [45,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [46,]    0    0    0    0    0    0    0    0    0     0     0     0     0
```

```
## [47,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [48,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [49,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [50,]    0    0    0    0    0    0    0    0    0     0     0     0     0
##       [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24]
##  [1,]     0     0     0     0     0     0     0     0     0     0     0
##  [2,]     0     0     0     0     0     0     0     0     0     0     0
##  [3,]     0     0     0     0     0     0     0     0     0     0     0
##  [4,]     0     0     0     0     0     0     0     0     0     0     0
##  [5,]     0     0     0     0     0     0     0     0     0     0     0
##  [6,]     0     0     0     0     0     0     0     0     0     0     0
##  [7,]     0     0     0     0     0     0     0     0     0     0     0
##  [8,]     0     0     0     0     0     0     0     0     0     0     0
##  [9,]     0     0     0     0     0     0     0     0     0     0     0
## [10,]     0     0     0     0     0     0     0     0     0     0     0
## [11,]     0     0     0     0     0     0     0     0     0     0     0
## [12,]     0     0     0     0     0     0     0     0     0     0     0
## [13,]   -40     0     0     0     0     0     0     0     0     0     0
## [14,]   102   -40     0     0     0     0     0     0     0     0     0
## [15,]   -40   102   -40     0     0     0     0     0     0     0     0
## [16,]     0   -40   102   -40     0     0     0     0     0     0     0
## [17,]     0     0   -40   102   -40     0     0     0     0     0     0
## [18,]     0     0     0   -40   102   -40     0     0     0     0     0
## [19,]     0     0     0     0   -40   102   -40     0     0     0     0
## [20,]     0     0     0     0     0   -40   102   -40     0     0     0
## [21,]     0     0     0     0     0     0   -40   102   -40     0     0
## [22,]     0     0     0     0     0     0     0   -40   102   -40     0
## [23,]     0     0     0     0     0     0     0     0   -40   102   -40
## [24,]     0     0     0     0     0     0     0     0     0   -40   102
## [25,]     0     0     0     0     0     0     0     0     0     0   -40
## [26,]     0     0     0     0     0     0     0     0     0     0     0
## [27,]     0     0     0     0     0     0     0     0     0     0     0
## [28,]     0     0     0     0     0     0     0     0     0     0     0
## [29,]     0     0     0     0     0     0     0     0     0     0     0
## [30,]     0     0     0     0     0     0     0     0     0     0     0
## [31,]     0     0     0     0     0     0     0     0     0     0     0
## [32,]     0     0     0     0     0     0     0     0     0     0     0
## [33,]     0     0     0     0     0     0     0     0     0     0     0
## [34,]     0     0     0     0     0     0     0     0     0     0     0
## [35,]     0     0     0     0     0     0     0     0     0     0     0
## [36,]     0     0     0     0     0     0     0     0     0     0     0
## [37,]     0     0     0     0     0     0     0     0     0     0     0
## [38,]     0     0     0     0     0     0     0     0     0     0     0
## [39,]     0     0     0     0     0     0     0     0     0     0     0
## [40,]     0     0     0     0     0     0     0     0     0     0     0
## [41,]     0     0     0     0     0     0     0     0     0     0     0
## [42,]     0     0     0     0     0     0     0     0     0     0     0
## [43,]     0     0     0     0     0     0     0     0     0     0     0
## [44,]     0     0     0     0     0     0     0     0     0     0     0
## [45,]     0     0     0     0     0     0     0     0     0     0     0
## [46,]     0     0     0     0     0     0     0     0     0     0     0
## [47,]     0     0     0     0     0     0     0     0     0     0     0
## [48,]     0     0     0     0     0     0     0     0     0     0     0
## [49,]     0     0     0     0     0     0     0     0     0     0     0
```

```
## [50,]     0     0     0     0     0     0     0     0     0     0     0
##       [,25] [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35]
##  [1,]     0     0     0     0     0     0     0     0     0     0     0
##  [2,]     0     0     0     0     0     0     0     0     0     0     0
##  [3,]     0     0     0     0     0     0     0     0     0     0     0
##  [4,]     0     0     0     0     0     0     0     0     0     0     0
##  [5,]     0     0     0     0     0     0     0     0     0     0     0
##  [6,]     0     0     0     0     0     0     0     0     0     0     0
##  [7,]     0     0     0     0     0     0     0     0     0     0     0
##  [8,]     0     0     0     0     0     0     0     0     0     0     0
##  [9,]     0     0     0     0     0     0     0     0     0     0     0
## [10,]     0     0     0     0     0     0     0     0     0     0     0
## [11,]     0     0     0     0     0     0     0     0     0     0     0
## [12,]     0     0     0     0     0     0     0     0     0     0     0
## [13,]     0     0     0     0     0     0     0     0     0     0     0
## [14,]     0     0     0     0     0     0     0     0     0     0     0
## [15,]     0     0     0     0     0     0     0     0     0     0     0
## [16,]     0     0     0     0     0     0     0     0     0     0     0
## [17,]     0     0     0     0     0     0     0     0     0     0     0
## [18,]     0     0     0     0     0     0     0     0     0     0     0
## [19,]     0     0     0     0     0     0     0     0     0     0     0
## [20,]     0     0     0     0     0     0     0     0     0     0     0
## [21,]     0     0     0     0     0     0     0     0     0     0     0
## [22,]     0     0     0     0     0     0     0     0     0     0     0
## [23,]     0     0     0     0     0     0     0     0     0     0     0
## [24,]   -40     0     0     0     0     0     0     0     0     0     0
## [25,]   102   -40     0     0     0     0     0     0     0     0     0
## [26,]   -40   102   -40     0     0     0     0     0     0     0     0
## [27,]     0   -40   102   -40     0     0     0     0     0     0     0
## [28,]     0     0   -40   102   -40     0     0     0     0     0     0
## [29,]     0     0     0   -40   102   -40     0     0     0     0     0
## [30,]     0     0     0     0   -40   102   -40     0     0     0     0
## [31,]     0     0     0     0     0   -40   102   -40     0     0     0
## [32,]     0     0     0     0     0     0   -40   102   -40     0     0
## [33,]     0     0     0     0     0     0     0   -40   102   -40     0
## [34,]     0     0     0     0     0     0     0     0   -40   102   -40
## [35,]     0     0     0     0     0     0     0     0     0   -40   102
## [36,]     0     0     0     0     0     0     0     0     0     0   -40
## [37,]     0     0     0     0     0     0     0     0     0     0     0
## [38,]     0     0     0     0     0     0     0     0     0     0     0
## [39,]     0     0     0     0     0     0     0     0     0     0     0
## [40,]     0     0     0     0     0     0     0     0     0     0     0
## [41,]     0     0     0     0     0     0     0     0     0     0     0
## [42,]     0     0     0     0     0     0     0     0     0     0     0
## [43,]     0     0     0     0     0     0     0     0     0     0     0
## [44,]     0     0     0     0     0     0     0     0     0     0     0
## [45,]     0     0     0     0     0     0     0     0     0     0     0
## [46,]     0     0     0     0     0     0     0     0     0     0     0
## [47,]     0     0     0     0     0     0     0     0     0     0     0
## [48,]     0     0     0     0     0     0     0     0     0     0     0
## [49,]     0     0     0     0     0     0     0     0     0     0     0
## [50,]     0     0     0     0     0     0     0     0     0     0     0
##       [,36] [,37] [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46]
##  [1,]     0     0     0     0     0     0     0     0     0     0     0
```

```
##  [2,]    0    0    0    0    0    0    0    0    0    0    0
##  [3,]    0    0    0    0    0    0    0    0    0    0    0
##  [4,]    0    0    0    0    0    0    0    0    0    0    0
##  [5,]    0    0    0    0    0    0    0    0    0    0    0
##  [6,]    0    0    0    0    0    0    0    0    0    0    0
##  [7,]    0    0    0    0    0    0    0    0    0    0    0
##  [8,]    0    0    0    0    0    0    0    0    0    0    0
##  [9,]    0    0    0    0    0    0    0    0    0    0    0
## [10,]    0    0    0    0    0    0    0    0    0    0    0
## [11,]    0    0    0    0    0    0    0    0    0    0    0
## [12,]    0    0    0    0    0    0    0    0    0    0    0
## [13,]    0    0    0    0    0    0    0    0    0    0    0
## [14,]    0    0    0    0    0    0    0    0    0    0    0
## [15,]    0    0    0    0    0    0    0    0    0    0    0
## [16,]    0    0    0    0    0    0    0    0    0    0    0
## [17,]    0    0    0    0    0    0    0    0    0    0    0
## [18,]    0    0    0    0    0    0    0    0    0    0    0
## [19,]    0    0    0    0    0    0    0    0    0    0    0
## [20,]    0    0    0    0    0    0    0    0    0    0    0
## [21,]    0    0    0    0    0    0    0    0    0    0    0
## [22,]    0    0    0    0    0    0    0    0    0    0    0
## [23,]    0    0    0    0    0    0    0    0    0    0    0
## [24,]    0    0    0    0    0    0    0    0    0    0    0
## [25,]    0    0    0    0    0    0    0    0    0    0    0
## [26,]    0    0    0    0    0    0    0    0    0    0    0
## [27,]    0    0    0    0    0    0    0    0    0    0    0
## [28,]    0    0    0    0    0    0    0    0    0    0    0
## [29,]    0    0    0    0    0    0    0    0    0    0    0
## [30,]    0    0    0    0    0    0    0    0    0    0    0
## [31,]    0    0    0    0    0    0    0    0    0    0    0
## [32,]    0    0    0    0    0    0    0    0    0    0    0
## [33,]    0    0    0    0    0    0    0    0    0    0    0
## [34,]    0    0    0    0    0    0    0    0    0    0    0
## [35,]  -40    0    0    0    0    0    0    0    0    0    0
## [36,]  102  -40    0    0    0    0    0    0    0    0    0
## [37,]  -40  102  -40    0    0    0    0    0    0    0    0
## [38,]    0  -40  102  -40    0    0    0    0    0    0    0
## [39,]    0    0  -40  102  -40    0    0    0    0    0    0
## [40,]    0    0    0  -40  102  -40    0    0    0    0    0
## [41,]    0    0    0    0  -40  102  -40    0    0    0    0
## [42,]    0    0    0    0    0  -40  102  -40    0    0    0
## [43,]    0    0    0    0    0    0  -40  102  -40    0    0
## [44,]    0    0    0    0    0    0    0  -40  102  -40    0
## [45,]    0    0    0    0    0    0    0    0  -40  102  -40
## [46,]    0    0    0    0    0    0    0    0    0  -40  102
## [47,]    0    0    0    0    0    0    0    0    0    0  -40
## [48,]    0    0    0    0    0    0    0    0    0    0    0
## [49,]    0    0    0    0    0    0    0    0    0    0    0
## [50,]    0    0    0    0    0    0    0    0    0    0    0
##       [,47] [,48] [,49] [,50]
##  [1,]    0    0    0    0
##  [2,]    0    0    0    0
##  [3,]    0    0    0    0
##  [4,]    0    0    0    0
```

```
##  [5,]     0     0     0     0
##  [6,]     0     0     0     0
##  [7,]     0     0     0     0
##  [8,]     0     0     0     0
##  [9,]     0     0     0     0
## [10,]     0     0     0     0
## [11,]     0     0     0     0
## [12,]     0     0     0     0
## [13,]     0     0     0     0
## [14,]     0     0     0     0
## [15,]     0     0     0     0
## [16,]     0     0     0     0
## [17,]     0     0     0     0
## [18,]     0     0     0     0
## [19,]     0     0     0     0
## [20,]     0     0     0     0
## [21,]     0     0     0     0
## [22,]     0     0     0     0
## [23,]     0     0     0     0
## [24,]     0     0     0     0
## [25,]     0     0     0     0
## [26,]     0     0     0     0
## [27,]     0     0     0     0
## [28,]     0     0     0     0
## [29,]     0     0     0     0
## [30,]     0     0     0     0
## [31,]     0     0     0     0
## [32,]     0     0     0     0
## [33,]     0     0     0     0
## [34,]     0     0     0     0
## [35,]     0     0     0     0
## [36,]     0     0     0     0
## [37,]     0     0     0     0
## [38,]     0     0     0     0
## [39,]     0     0     0     0
## [40,]     0     0     0     0
## [41,]     0     0     0     0
## [42,]     0     0     0     0
## [43,]     0     0     0     0
## [44,]     0     0     0     0
## [45,]     0     0     0     0
## [46,]   -40     0     0     0
## [47,]   102   -40     0     0
## [48,]   -40   102   -40     0
## [49,]     0   -40   102   -40
## [50,]     0     0   -40    22
##
## $counts
## $counts$niter
## [1] 145
##
## $counts$nfn
## [1] 146
##
```

```
## $counts$ngr
## [1] 145
##
## $counts$nhess
## [1] 145
##
##
## $convcode
## [1] 93
```

```
print(eigen(sol50pi$Hess)$values)
```

```
##  [1] 181.84200 181.36863 180.58176 179.48449 178.08116 176.37730 174.37964
##  [8] 172.09607 169.53560 166.70834 163.62545 160.29911 156.74243 152.96948
## [15] 148.99513 144.83509 140.50578 136.02429 131.40832 126.67610 121.84632
## [22] 116.93804 111.97066 106.96381 101.93725  96.91085  91.90447  86.93791
## [29]  82.03080  77.20253  72.47223  67.85859  63.37989  59.05387  54.89766
## [36]  50.92776  47.15992  43.60907  40.28933  37.21385  34.39481  31.84332
## [43]  29.56937  27.58175  25.88797  24.49427  23.40556  22.62547  22.15648
## [50]   2.00000
```

```
sol50pim <- snewtonm(x0, genrosa.f, genrosa.g, genrosa.h, gs=10)
```

```
## trace = 1
##  Start snewtonm   f0= 22405.14   at   [1] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.14
##  [8] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [15] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [22] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [29] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [36] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [43] 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593 3.141593
## [50] 3.141593
## Iteration  1  fbest= 22405.14
##  lambda = 0.0001220703   fval= 4133.339
## Iteration  2  fbest= 4133.339
##  lambda = 1.831055e-05   fval= 731.8323
## Iteration  3  fbest= 731.8323
##  lambda = 2.746582e-06   fval= 115.5295
## Iteration  4  fbest= 115.5295
##  lambda = 4.119873e-07   fval= 16.41807
## Iteration  5  fbest= 16.41807
##  lambda = 6.17981e-08   fval= 4.728525
## Iteration  6  fbest= 4.728525
##  lambda = 9.269714e-09   fval= 3.909583
## Iteration  7  fbest= 3.909583
##  lambda = 1.390457e-09   fval= 3.643291
## Iteration  8  fbest= 3.643291
##  lambda = 2.085686e-10   fval= 3.385992
## Iteration  9  fbest= 3.385992
##  lambda = 3.128529e-11   fval= 3.134871
## Iteration  10  fbest= 3.134871
##  lambda = 4.692793e-12   fval= 2.89029
## Iteration  11  fbest= 2.89029
##  lambda = 7.039189e-13   fval= 2.652637
## Iteration  12  fbest= 2.652637
```

```
##  lambda = 1.055878e-13   fval= 2.42233
## Iteration  13  fbest= 2.42233
##  lambda = 1.583818e-14   fval= 2.199815
## Iteration  14  fbest= 2.199815
##  lambda = 2.375726e-15   fval= 1.98557
## Iteration  15  fbest= 1.98557
##  lambda = 3.56359e-16   fval= 1.780102
## Iteration  16  fbest= 1.780102
##  lambda = 5.345384e-17   fval= 1.583946
## Iteration  17  fbest= 1.583946
##  lambda = 8.018077e-18   fval= 1.397661
## Iteration  18  fbest= 1.397661
##  lambda = 1.202711e-18   fval= 1.221824
## Iteration  19  fbest= 1.221824
##  lambda = 1.804067e-19   fval= 1.057021
## Iteration  20  fbest= 1.057021
##  lambda = 2.706101e-20   fval= 0.9038294
## Iteration  21  fbest= 0.9038294
##  lambda = 4.059151e-21   fval= 0.7627985
## Iteration  22  fbest= 0.7627985
##  lambda = 6.088727e-22   fval= 0.6344206
## Iteration  23  fbest= 0.6344206
##  lambda = 9.13309e-23   fval= 0.5190938
## Iteration  24  fbest= 0.5190938
##  lambda = 1.369964e-23   fval= 0.4170763
## Iteration  25  fbest= 0.4170763
##  lambda = 2.054945e-24   fval= 0.3284352
## Iteration  26  fbest= 0.3284352
##  lambda = 3.082418e-25   fval= 0.2529935
## Iteration  27  fbest= 0.2529935
##  lambda = 4.623627e-26   fval= 0.1902851
## Iteration  28  fbest= 0.1902851
##  lambda = 6.935441e-27   fval= 0.1395286
## Iteration  29  fbest= 0.1395286
##  lambda = 1.040316e-27   fval= 0.09963431
## Iteration  30  fbest= 0.09963431
##  lambda = 1.560474e-28   fval= 0.06925367
## Iteration  31  fbest= 0.06925367
##  lambda = 2.340711e-29   fval= 0.0468708
## Iteration  32  fbest= 0.0468708
##  lambda = 3.511067e-30   fval= 0.03092309
## Iteration  33  fbest= 0.03092309
##  lambda = 5.2666e-31   fval= 0.01992538
## Iteration  34  fbest= 0.01992538
##  lambda = 7.8999e-32   fval= 0.01257024
## Iteration  35  fbest= 0.01257024
##  lambda = 1.184985e-32   fval= 0.007785817
## Iteration  36  fbest= 0.007785817
##  lambda = 1.777478e-33   fval= 0.004748192
## Iteration  37  fbest= 0.004748192
##  lambda = 2.666216e-34   fval= 0.002858877
## Iteration  38  fbest= 0.002858877
##  lambda = 3.999324e-35   fval= 0.001703596
## Iteration  39  fbest= 0.001703596
```

```
##  lambda = 5.998987e-36   fval= 0.001006829
## Iteration  40  fbest= 0.001006829
##  lambda = 8.99848e-37   fval= 0.0005911864
## Iteration  41  fbest= 0.0005911864
##  lambda = 1.349772e-37   fval= 0.0003453769
## Iteration  42  fbest= 0.0003453769
##  lambda = 2.024658e-38   fval= 0.0002009824
## Iteration  43  fbest= 0.0002009824
##  lambda = 3.036987e-39   fval= 0.0001166031
## Iteration  44  fbest= 0.0001166031
##  lambda = 4.555481e-40   fval= 6.749228e-05
## Iteration  45  fbest= 6.749228e-05
##  lambda = 6.833221e-41   fval= 3.899665e-05
## Iteration  46  fbest= 3.899665e-05
##  lambda = 1.024983e-41   fval= 2.250153e-05
## Iteration  47  fbest= 2.250153e-05
##  lambda = 1.537475e-42   fval= 1.297025e-05
## Iteration  48  fbest= 1.297025e-05
##  lambda = 2.306212e-43   fval= 7.470387e-06
## Iteration  49  fbest= 7.470387e-06
##  lambda = 3.459318e-44   fval= 4.300099e-06
## Iteration  50  fbest= 4.300099e-06
##  lambda = 5.188977e-45   fval= 2.474097e-06
## Iteration  51  fbest= 2.474097e-06
##  lambda = 7.783466e-46   fval= 1.423001e-06
## Iteration  52  fbest= 1.423001e-06
##  lambda = 1.16752e-46   fval= 8.182387e-07
## Iteration  53  fbest= 8.182387e-07
##  lambda = 1.75128e-47   fval= 4.704014e-07
## Iteration  54  fbest= 4.704014e-07
##  lambda = 2.62692e-48   fval= 2.703908e-07
## Iteration  55  fbest= 2.703908e-07
##  lambda = 3.940379e-49   fval= 1.554052e-07
## Iteration  56  fbest= 1.554052e-07
##  lambda = 5.910569e-50   fval= 8.93103e-08
## Iteration  57  fbest= 8.93103e-08
##  lambda = 8.865854e-51   fval= 5.132265e-08
## Iteration  58  fbest= 5.132265e-08
##  lambda = 1.329878e-51   fval= 2.949137e-08
## Iteration  59  fbest= 2.949137e-08
##  lambda = 1.994817e-52   fval= 1.694589e-08
## Iteration  60  fbest= 1.694589e-08
##  lambda = 2.992226e-53   fval= 9.736917e-09
## Iteration  61  fbest= 9.736917e-09
##  lambda = 4.488338e-54   fval= 5.594601e-09
## Iteration  62  fbest= 5.594601e-09
##  lambda = 6.732508e-55   fval= 3.214472e-09
## Iteration  63  fbest= 3.214472e-09
##  lambda = 1.009876e-55   fval= 1.846906e-09
## Iteration  64  fbest= 1.846906e-09
##  lambda = 1.514814e-56   fval= 1.061147e-09
## Iteration  65  fbest= 1.061147e-09
##  lambda = 2.272221e-57   fval= 6.096822e-10
## Iteration  66  fbest= 6.096822e-10
```

```
##  lambda = 3.408332e-58   fval= 3.502911e-10
## Iteration  67  fbest= 3.502911e-10
##  lambda = 5.112498e-59   fval= 2.012578e-10
## Iteration  68  fbest= 2.012578e-10
##  lambda = 7.668747e-60   fval= 1.156312e-10
## Iteration  69  fbest= 1.156312e-10
##  lambda = 1.150312e-60   fval= 6.643493e-11
## Iteration  70  fbest= 6.643493e-11
##  lambda = 1.725468e-61   fval= 3.816955e-11
## Iteration  71  fbest= 3.816955e-11
##  lambda = 2.588202e-62   fval= 2.192992e-11
## Iteration  72  fbest= 2.192992e-11
##  lambda = 3.882303e-63   fval= 1.259959e-11
## Iteration  73  fbest= 1.259959e-11
##  lambda = 5.823455e-64   fval= 7.238953e-12
## Iteration  74  fbest= 7.238953e-12
##  lambda = 8.735182e-65   fval= 4.159055e-12
## Iteration  75  fbest= 4.159055e-12
##  lambda = 1.310277e-65   fval= 2.389535e-12
## Iteration  76  fbest= 2.389535e-12
##  lambda = 1.965416e-66   fval= 1.372878e-12
## Iteration  77  fbest= 1.372878e-12
##  lambda = 2.948124e-67   fval= 7.887702e-13
## Iteration  78  fbest= 7.887702e-13
##  lambda = 4.422186e-68   fval= 4.531781e-13
## Iteration  79  fbest= 4.531781e-13
##  lambda = 6.633279e-69   fval= 2.603678e-13
## Iteration  80  fbest= 2.603678e-13
##  lambda = 9.949918e-70   fval= 1.49591e-13
## Iteration  81  fbest= 1.49591e-13
##  lambda = 1.492488e-70   fval= 8.594562e-14
## Iteration  82  fbest= 8.594562e-14
##  lambda = 2.238732e-71   fval= 4.937896e-14
## Iteration  83  fbest= 4.937896e-14
##  lambda = 3.358097e-72   fval= 2.837005e-14
## Iteration  84  fbest= 2.837005e-14
##  lambda = 5.037146e-73   fval= 1.629965e-14
## Iteration  85  fbest= 1.629965e-14
##  lambda = 7.555719e-74   fval= 9.364757e-15
## Iteration  86  fbest= 9.364757e-15
##  lambda = 1.133358e-74   fval= 5.380401e-15
## Iteration  87  fbest= 5.380401e-15
##  lambda = 1.700037e-75   fval= 3.091241e-15
## Iteration  88  fbest= 3.091241e-15
##  lambda = 2.550055e-76   fval= 1.776033e-15
## Iteration  89  fbest= 1.776033e-15
##  lambda = 3.825083e-77   fval= 1.020397e-15
## Iteration  90  fbest= 1.020397e-15
##  lambda = 5.737624e-78   fval= 5.862559e-16
## Iteration  91  fbest= 5.862559e-16
##  lambda = 8.606437e-79   fval= 3.368258e-16
## Iteration  92  fbest= 3.368258e-16
##  lambda = 1.290965e-79   fval= 1.93519e-16
## Iteration  93  fbest= 1.93519e-16
```

```
##  lambda = 1.936448e-80   fval= 1.111838e-16
## Iteration  94  fbest= 1.111838e-16
##  lambda = 2.904672e-81   fval= 6.387925e-17
## Iteration  95  fbest= 6.387925e-17
##  lambda = 4.357009e-82   fval= 3.6701e-17
## Iteration  96  fbest= 3.6701e-17
##  lambda = 6.535513e-83   fval= 2.108609e-17
## Iteration  97  fbest= 2.108609e-17
##  lambda = 9.803269e-84   fval= 1.211474e-17
## Iteration  98  fbest= 1.211474e-17
##  lambda = 1.47049e-84   fval= 6.96037e-18
## Iteration  99  fbest= 6.96037e-18
##  lambda = 2.205736e-85   fval= 3.998991e-18
## Iteration  100  fbest= 3.998991e-18
##  lambda = 3.308603e-86   fval= 2.297569e-18
## Iteration  101  fbest= 2.297569e-18
##  lambda = 4.962905e-87   fval= 1.320039e-18
## Iteration  102  fbest= 1.320039e-18
##  lambda = 7.444357e-88   fval= 7.584114e-19
## Iteration  103  fbest= 7.584114e-19
##  lambda = 1.116654e-88   fval= 4.357355e-19
## Iteration  104  fbest= 4.357355e-19
##  lambda = 1.67498e-89   fval= 2.503463e-19
## Iteration  105  fbest= 2.503463e-19
##  lambda = 2.512471e-90   fval= 1.438332e-19
## Iteration  106  fbest= 1.438332e-19
##  lambda = 3.768706e-91   fval= 8.263755e-20
## Iteration  107  fbest= 8.263755e-20
##  lambda = 5.653059e-92   fval= 4.747835e-20
## Iteration  108  fbest= 4.747835e-20
##  lambda = 8.479588e-93   fval= 2.727809e-20
## Iteration  109  fbest= 2.727809e-20
##  lambda = 1.271938e-93   fval= 1.567227e-20
## Iteration  110  fbest= 1.567227e-20
##  lambda = 1.907907e-94   fval= 9.004297e-21
## Iteration  111  fbest= 9.004297e-21
##  lambda = 2.861861e-95   fval= 5.173295e-21
## Iteration  112  fbest= 5.173295e-21
##  lambda = 4.292792e-96   fval= 2.972255e-21
## Iteration  113  fbest= 2.972255e-21
##  lambda = 6.439187e-97   fval= 1.707674e-21
## Iteration  114  fbest= 1.707674e-21
##  lambda = 9.658781e-98   fval= 9.811258e-22
## Iteration  115  fbest= 9.811258e-22
##  lambda = 1.448817e-98   fval= 5.636946e-22
## Iteration  116  fbest= 5.636946e-22
##  lambda = 2.173226e-99   fval= 3.238625e-22
## Iteration  117  fbest= 3.238625e-22
##  lambda = 3.259839e-100   fval= 1.860713e-22
## Iteration  118  fbest= 1.860713e-22
##  lambda = 4.889758e-101   fval= 1.069047e-22
## Iteration  119  fbest= 1.069047e-22
##  lambda = 7.334637e-102   fval= 6.142062e-23
## Iteration  120  fbest= 6.142062e-23
```

```
##  lambda = 1.100196e-102   fval= 3.528866e-23
## Iteration  121  fbest= 3.528866e-23
##  lambda = 1.650293e-103   fval= 2.027416e-23
## Iteration  122  fbest= 2.027416e-23
##  lambda = 2.47544e-104   fval= 1.164851e-23
## Iteration  123  fbest= 1.164851e-23
##  lambda = 3.71316e-105   fval= 6.69274e-24
## Iteration  124  fbest= 6.69274e-24
##  lambda = 5.56974e-106   fval= 3.84531e-24
## Iteration  125  fbest= 3.84531e-24
##  lambda = 8.35461e-107   fval= 2.20946e-24
## Iteration  126  fbest= 2.20946e-24
##  lambda = 1.253191e-107   fval= 1.26947e-24
## Iteration  127  fbest= 1.26947e-24
##  lambda = 1.879787e-108   fval= 7.294277e-25
## Iteration  128  fbest= 7.294277e-25
##  lambda = 2.819681e-109   fval= 4.191511e-25
## Iteration  129  fbest= 4.191511e-25
##  lambda = 4.229521e-110   fval= 2.407963e-25
## Iteration  130  fbest= 2.407963e-25
##  lambda = 6.344282e-111   fval= 1.383034e-25
## Iteration  131  fbest= 1.383034e-25
##  lambda = 9.516423e-112   fval= 7.948656e-26
## Iteration  132  fbest= 7.948656e-26
##  lambda = 1.427463e-112   fval= 4.565858e-26
## Iteration  133  fbest= 4.565858e-26
##  lambda = 2.141195e-113   fval= 2.623978e-26
## Iteration  134  fbest= 2.623978e-26
##  lambda = 3.211793e-114   fval= 1.509071e-26
## Iteration  135  fbest= 1.509071e-26
##  lambda = 4.817689e-115   fval= 8.669335e-27
## Iteration  136  fbest= 8.669335e-27
##  lambda = 7.226534e-116   fval= 4.973768e-27
## Iteration  137  fbest= 4.973768e-27
##  lambda = 1.08398e-116   fval= 2.859917e-27
## Iteration  138  fbest= 2.859917e-27
##  lambda = 1.62597e-117   fval= 1.644134e-27
## Iteration  139  fbest= 1.644134e-27
##  lambda = 2.438955e-118   fval= 9.493941e-28
## Iteration  140  fbest= 9.493941e-28
##  lambda = 3.658433e-119   fval= 5.466806e-28
## Iteration  141  fbest= 5.466806e-28
##  lambda = 5.487649e-120   fval= 3.137694e-28
## Iteration  142  fbest= 3.137694e-28
##  lambda = 8.231473e-121   fval= 1.81931e-28
## Iteration  143  fbest= 1.81931e-28
##  lambda = 1.234721e-121   fval= 1.047706e-28
## Iteration  144  fbest= 1.047706e-28
##  lambda = 1.852082e-122   fval= 6.108742e-29
## Iteration  145  fbest= 6.108742e-29
##  lambda = 2.778122e-123   fval= 3.604108e-29
## Iteration  146  fbest= 3.604108e-29
##  lambda = 4.167183e-124   fval= 1.991874e-29
## Iteration  147  fbest= 1.991874e-29
```

```
##  lambda = 6.250775e-125   fval= 1.311481e-29
## Iteration  148  fbest= 1.311481e-29
##  lambda = 9.376163e-126   fval= 7.937913e-30
## Iteration  149  fbest= 7.937913e-30
##  lambda = 1.406424e-126   fval= 4.930381e-30
## Iteration  150  fbest= 4.930381e-30
##  lambda = 2.109637e-127   fval= 4.289431e-30
## Iteration  151  fbest= 4.289431e-30
##  lambda = 3.164455e-128   fval= 1.528418e-30
## Iteration  152  fbest= 1.528418e-30
##  lambda = 4.746682e-129   fval= 2.021456e-30
##  lambda = 0.001220703   fval= 2.021456e-30
##  lambda = 0.01220703   fval= 2.021456e-30
##  lambda = 0.1220703   fval= 2.021456e-30
##  lambda = 1.220703   fval= 2.169367e-30
## Null step
## Finished
```

```
print(sol50pim)
```

```
## $xs
##  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [36] 1 1 1 1 1 1 1 1 1 1 1 1 1 1
##
## $fv
## [1] 1.528418e-30
##
## $grd
##  [1]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
##  [6]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [11]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [16]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [21]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [26]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [31]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [36]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [41]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## [46] -8.881784e-15  4.884981e-15  8.881784e-16  1.776357e-15  0.000000e+00
##
## $Hess
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
##  [1,]   80  -40    0    0    0    0    0    0    0     0     0     0     0
##  [2,]  -40  102  -40    0    0    0    0    0    0     0     0     0     0
##  [3,]    0  -40  102  -40    0    0    0    0    0     0     0     0     0
##  [4,]    0    0  -40  102  -40    0    0    0    0     0     0     0     0
##  [5,]    0    0    0  -40  102  -40    0    0    0     0     0     0     0
##  [6,]    0    0    0    0  -40  102  -40    0    0     0     0     0     0
##  [7,]    0    0    0    0    0  -40  102  -40    0     0     0     0     0
##  [8,]    0    0    0    0    0    0  -40  102  -40     0     0     0     0
##  [9,]    0    0    0    0    0    0    0  -40  102   -40     0     0     0
## [10,]    0    0    0    0    0    0    0    0  -40   102   -40     0     0
## [11,]    0    0    0    0    0    0    0    0    0   -40   102   -40     0
## [12,]    0    0    0    0    0    0    0    0    0     0   -40   102   -40
## [13,]    0    0    0    0    0    0    0    0    0     0     0   -40   102
## [14,]    0    0    0    0    0    0    0    0    0     0     0     0   -40
```

```
## [15,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [16,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [17,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [18,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [19,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [20,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [21,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [22,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [23,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [24,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [25,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [26,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [27,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [28,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [29,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [30,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [31,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [32,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [33,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [34,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [35,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [36,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [37,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [38,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [39,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [40,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [41,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [42,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [43,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [44,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [45,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [46,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [47,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [48,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [49,]    0    0    0    0    0    0    0    0    0     0     0     0     0
## [50,]    0    0    0    0    0    0    0    0    0     0     0     0     0
##       [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24]
##  [1,]     0     0     0     0     0     0     0     0     0     0     0
##  [2,]     0     0     0     0     0     0     0     0     0     0     0
##  [3,]     0     0     0     0     0     0     0     0     0     0     0
##  [4,]     0     0     0     0     0     0     0     0     0     0     0
##  [5,]     0     0     0     0     0     0     0     0     0     0     0
##  [6,]     0     0     0     0     0     0     0     0     0     0     0
##  [7,]     0     0     0     0     0     0     0     0     0     0     0
##  [8,]     0     0     0     0     0     0     0     0     0     0     0
##  [9,]     0     0     0     0     0     0     0     0     0     0     0
## [10,]     0     0     0     0     0     0     0     0     0     0     0
## [11,]     0     0     0     0     0     0     0     0     0     0     0
## [12,]     0     0     0     0     0     0     0     0     0     0     0
## [13,]   -40     0     0     0     0     0     0     0     0     0     0
## [14,]   102   -40     0     0     0     0     0     0     0     0     0
## [15,]   -40   102   -40     0     0     0     0     0     0     0     0
## [16,]     0   -40   102   -40     0     0     0     0     0     0     0
## [17,]     0     0   -40   102   -40     0     0     0     0     0     0
```

```
## [18,]    0    0    0  -40  102  -40    0    0    0    0    0
## [19,]    0    0    0    0  -40  102  -40    0    0    0    0
## [20,]    0    0    0    0    0  -40  102  -40    0    0    0
## [21,]    0    0    0    0    0    0  -40  102  -40    0    0
## [22,]    0    0    0    0    0    0    0  -40  102  -40    0
## [23,]    0    0    0    0    0    0    0    0  -40  102  -40
## [24,]    0    0    0    0    0    0    0    0    0  -40  102
## [25,]    0    0    0    0    0    0    0    0    0    0  -40
## [26,]    0    0    0    0    0    0    0    0    0    0    0
## [27,]    0    0    0    0    0    0    0    0    0    0    0
## [28,]    0    0    0    0    0    0    0    0    0    0    0
## [29,]    0    0    0    0    0    0    0    0    0    0    0
## [30,]    0    0    0    0    0    0    0    0    0    0    0
## [31,]    0    0    0    0    0    0    0    0    0    0    0
## [32,]    0    0    0    0    0    0    0    0    0    0    0
## [33,]    0    0    0    0    0    0    0    0    0    0    0
## [34,]    0    0    0    0    0    0    0    0    0    0    0
## [35,]    0    0    0    0    0    0    0    0    0    0    0
## [36,]    0    0    0    0    0    0    0    0    0    0    0
## [37,]    0    0    0    0    0    0    0    0    0    0    0
## [38,]    0    0    0    0    0    0    0    0    0    0    0
## [39,]    0    0    0    0    0    0    0    0    0    0    0
## [40,]    0    0    0    0    0    0    0    0    0    0    0
## [41,]    0    0    0    0    0    0    0    0    0    0    0
## [42,]    0    0    0    0    0    0    0    0    0    0    0
## [43,]    0    0    0    0    0    0    0    0    0    0    0
## [44,]    0    0    0    0    0    0    0    0    0    0    0
## [45,]    0    0    0    0    0    0    0    0    0    0    0
## [46,]    0    0    0    0    0    0    0    0    0    0    0
## [47,]    0    0    0    0    0    0    0    0    0    0    0
## [48,]    0    0    0    0    0    0    0    0    0    0    0
## [49,]    0    0    0    0    0    0    0    0    0    0    0
## [50,]    0    0    0    0    0    0    0    0    0    0    0
##       [,25] [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35]
##  [1,]    0    0    0    0    0    0    0    0    0    0    0
##  [2,]    0    0    0    0    0    0    0    0    0    0    0
##  [3,]    0    0    0    0    0    0    0    0    0    0    0
##  [4,]    0    0    0    0    0    0    0    0    0    0    0
##  [5,]    0    0    0    0    0    0    0    0    0    0    0
##  [6,]    0    0    0    0    0    0    0    0    0    0    0
##  [7,]    0    0    0    0    0    0    0    0    0    0    0
##  [8,]    0    0    0    0    0    0    0    0    0    0    0
##  [9,]    0    0    0    0    0    0    0    0    0    0    0
## [10,]    0    0    0    0    0    0    0    0    0    0    0
## [11,]    0    0    0    0    0    0    0    0    0    0    0
## [12,]    0    0    0    0    0    0    0    0    0    0    0
## [13,]    0    0    0    0    0    0    0    0    0    0    0
## [14,]    0    0    0    0    0    0    0    0    0    0    0
## [15,]    0    0    0    0    0    0    0    0    0    0    0
## [16,]    0    0    0    0    0    0    0    0    0    0    0
## [17,]    0    0    0    0    0    0    0    0    0    0    0
## [18,]    0    0    0    0    0    0    0    0    0    0    0
## [19,]    0    0    0    0    0    0    0    0    0    0    0
## [20,]    0    0    0    0    0    0    0    0    0    0    0
```

```
## [21,]    0    0    0    0    0    0    0    0    0    0    0
## [22,]    0    0    0    0    0    0    0    0    0    0    0
## [23,]    0    0    0    0    0    0    0    0    0    0    0
## [24,]  -40    0    0    0    0    0    0    0    0    0    0
## [25,]  102  -40    0    0    0    0    0    0    0    0    0
## [26,]  -40  102  -40    0    0    0    0    0    0    0    0
## [27,]    0  -40  102  -40    0    0    0    0    0    0    0
## [28,]    0    0  -40  102  -40    0    0    0    0    0    0
## [29,]    0    0    0  -40  102  -40    0    0    0    0    0
## [30,]    0    0    0    0  -40  102  -40    0    0    0    0
## [31,]    0    0    0    0    0  -40  102  -40    0    0    0
## [32,]    0    0    0    0    0    0  -40  102  -40    0    0
## [33,]    0    0    0    0    0    0    0  -40  102  -40    0
## [34,]    0    0    0    0    0    0    0    0  -40  102  -40
## [35,]    0    0    0    0    0    0    0    0    0  -40  102
## [36,]    0    0    0    0    0    0    0    0    0    0  -40
## [37,]    0    0    0    0    0    0    0    0    0    0    0
## [38,]    0    0    0    0    0    0    0    0    0    0    0
## [39,]    0    0    0    0    0    0    0    0    0    0    0
## [40,]    0    0    0    0    0    0    0    0    0    0    0
## [41,]    0    0    0    0    0    0    0    0    0    0    0
## [42,]    0    0    0    0    0    0    0    0    0    0    0
## [43,]    0    0    0    0    0    0    0    0    0    0    0
## [44,]    0    0    0    0    0    0    0    0    0    0    0
## [45,]    0    0    0    0    0    0    0    0    0    0    0
## [46,]    0    0    0    0    0    0    0    0    0    0    0
## [47,]    0    0    0    0    0    0    0    0    0    0    0
## [48,]    0    0    0    0    0    0    0    0    0    0    0
## [49,]    0    0    0    0    0    0    0    0    0    0    0
## [50,]    0    0    0    0    0    0    0    0    0    0    0
##       [,36] [,37] [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46]
##  [1,]    0    0    0    0    0    0    0    0    0    0    0
##  [2,]    0    0    0    0    0    0    0    0    0    0    0
##  [3,]    0    0    0    0    0    0    0    0    0    0    0
##  [4,]    0    0    0    0    0    0    0    0    0    0    0
##  [5,]    0    0    0    0    0    0    0    0    0    0    0
##  [6,]    0    0    0    0    0    0    0    0    0    0    0
##  [7,]    0    0    0    0    0    0    0    0    0    0    0
##  [8,]    0    0    0    0    0    0    0    0    0    0    0
##  [9,]    0    0    0    0    0    0    0    0    0    0    0
## [10,]    0    0    0    0    0    0    0    0    0    0    0
## [11,]    0    0    0    0    0    0    0    0    0    0    0
## [12,]    0    0    0    0    0    0    0    0    0    0    0
## [13,]    0    0    0    0    0    0    0    0    0    0    0
## [14,]    0    0    0    0    0    0    0    0    0    0    0
## [15,]    0    0    0    0    0    0    0    0    0    0    0
## [16,]    0    0    0    0    0    0    0    0    0    0    0
## [17,]    0    0    0    0    0    0    0    0    0    0    0
## [18,]    0    0    0    0    0    0    0    0    0    0    0
## [19,]    0    0    0    0    0    0    0    0    0    0    0
## [20,]    0    0    0    0    0    0    0    0    0    0    0
## [21,]    0    0    0    0    0    0    0    0    0    0    0
## [22,]    0    0    0    0    0    0    0    0    0    0    0
## [23,]    0    0    0    0    0    0    0    0    0    0    0
```

```
## [24,]     0     0     0     0     0     0     0     0     0     0     0
## [25,]     0     0     0     0     0     0     0     0     0     0     0
## [26,]     0     0     0     0     0     0     0     0     0     0     0
## [27,]     0     0     0     0     0     0     0     0     0     0     0
## [28,]     0     0     0     0     0     0     0     0     0     0     0
## [29,]     0     0     0     0     0     0     0     0     0     0     0
## [30,]     0     0     0     0     0     0     0     0     0     0     0
## [31,]     0     0     0     0     0     0     0     0     0     0     0
## [32,]     0     0     0     0     0     0     0     0     0     0     0
## [33,]     0     0     0     0     0     0     0     0     0     0     0
## [34,]     0     0     0     0     0     0     0     0     0     0     0
## [35,]   -40     0     0     0     0     0     0     0     0     0     0
## [36,]   102   -40     0     0     0     0     0     0     0     0     0
## [37,]   -40   102   -40     0     0     0     0     0     0     0     0
## [38,]     0   -40   102   -40     0     0     0     0     0     0     0
## [39,]     0     0   -40   102   -40     0     0     0     0     0     0
## [40,]     0     0     0   -40   102   -40     0     0     0     0     0
## [41,]     0     0     0     0   -40   102   -40     0     0     0     0
## [42,]     0     0     0     0     0   -40   102   -40     0     0     0
## [43,]     0     0     0     0     0     0   -40   102   -40     0     0
## [44,]     0     0     0     0     0     0     0   -40   102   -40     0
## [45,]     0     0     0     0     0     0     0     0   -40   102   -40
## [46,]     0     0     0     0     0     0     0     0     0   -40   102
## [47,]     0     0     0     0     0     0     0     0     0     0   -40
## [48,]     0     0     0     0     0     0     0     0     0     0     0
## [49,]     0     0     0     0     0     0     0     0     0     0     0
## [50,]     0     0     0     0     0     0     0     0     0     0     0
##       [,47] [,48] [,49] [,50]
##  [1,]     0     0     0     0
##  [2,]     0     0     0     0
##  [3,]     0     0     0     0
##  [4,]     0     0     0     0
##  [5,]     0     0     0     0
##  [6,]     0     0     0     0
##  [7,]     0     0     0     0
##  [8,]     0     0     0     0
##  [9,]     0     0     0     0
## [10,]     0     0     0     0
## [11,]     0     0     0     0
## [12,]     0     0     0     0
## [13,]     0     0     0     0
## [14,]     0     0     0     0
## [15,]     0     0     0     0
## [16,]     0     0     0     0
## [17,]     0     0     0     0
## [18,]     0     0     0     0
## [19,]     0     0     0     0
## [20,]     0     0     0     0
## [21,]     0     0     0     0
## [22,]     0     0     0     0
## [23,]     0     0     0     0
## [24,]     0     0     0     0
## [25,]     0     0     0     0
## [26,]     0     0     0     0
```

```
## [27,]     0     0     0     0
## [28,]     0     0     0     0
## [29,]     0     0     0     0
## [30,]     0     0     0     0
## [31,]     0     0     0     0
## [32,]     0     0     0     0
## [33,]     0     0     0     0
## [34,]     0     0     0     0
## [35,]     0     0     0     0
## [36,]     0     0     0     0
## [37,]     0     0     0     0
## [38,]     0     0     0     0
## [39,]     0     0     0     0
## [40,]     0     0     0     0
## [41,]     0     0     0     0
## [42,]     0     0     0     0
## [43,]     0     0     0     0
## [44,]     0     0     0     0
## [45,]     0     0     0     0
## [46,]   -40     0     0     0
## [47,]   102   -40     0     0
## [48,]   -40   102   -40     0
## [49,]     0   -40   102   -40
## [50,]     0     0   -40    22
##
## $counts
## $counts$niter
## [1] 158
##
## $counts$nfn
## [1] 157
##
## $counts$ngr
## [1] 152
##
## $counts$nhess
## [1] 152
```

```
print(eigen(sol50pim$Hess)$values)
```

```
##  [1] 181.84200 181.36863 180.58176 179.48449 178.08116 176.37730 174.37964
##  [8] 172.09607 169.53560 166.70834 163.62545 160.29911 156.74243 152.96948
## [15] 148.99513 144.83509 140.50578 136.02429 131.40832 126.67610 121.84632
## [22] 116.93804 111.97066 106.96381 101.93725  96.91085  91.90447  86.93791
## [29]  82.03080  77.20253  72.47223  67.85859  63.37989  59.05387  54.89766
## [36]  50.92776  47.15992  43.60907  40.28933  37.21385  34.39481  31.84332
## [43]  29.56937  27.58175  25.88797  24.49427  23.40556  22.62547  22.15648
## [50]   2.00000
```

```
# ?? do we want to try nlm, nlminb, and optimx versions??
```

The Hobbs weed infestation problem

??ref

```
## Optimization test function HOBBS
## ?? refs (put in .doc??)
## Nash and Walker-Smith (1987, 1989) ...


hobbs.f<- function(x){ # # Hobbs weeds problem -- function
    if (abs(12*x[3]) > 500) { # check computability
       fbad<-.Machine$double.xmax
       return(fbad)
    }
    res<-hobbs.res(x)
    f<-sum(res*res)
}


hobbs.res<-function(x){ # Hobbs weeds problem -- residual
# This variant uses looping
    if(length(x) != 3) stop("hobbs.res -- parameter vector n!=3")
    y<-c(5.308, 7.24, 9.638, 12.866, 17.069, 23.192, 31.443, 38.558, 50.156, 62.948,
         75.995, 91.972)
    t<-1:12
    if(abs(12*x[3])>50) {
       res<-rep(Inf,12)
    } else {
       res<-x[1]/(1+x[2]*exp(-x[3]*t)) - y
    }
}

hobbs.jac<-function(x){ # Jacobian of Hobbs weeds problem
   jj<-matrix(0.0, 12, 3)
   t<-1:12
    yy<-exp(-x[3]*t)
    zz<-1.0/(1+x[2]*yy)
     jj[t,1] <- zz
     jj[t,2] <- -x[1]*zz*zz*yy
     jj[t,3] <- x[1]*zz*zz*yy*x[2]*t
   return(jj)
}

hobbs.g<-function(x){ # gradient of Hobbs weeds problem
    # NOT EFFICIENT TO CALL AGAIN
    jj<-hobbs.jac(x)
    res<-hobbs.res(x)
    gg<-as.vector(2.*t(jj) %*% res)
    return(gg)
}


hobbs.rsd<-function(x) { # Jacobian second derivative
    rsd<-array(0.0, c(12,3,3))
    t<-1:12
    yy<-exp(-x[3]*t)
    zz<-1.0/(1+x[2]*yy)
```

```
    rsd[t,1,1]<- 0.0
    rsd[t,2,1]<- -yy*zz*zz
    rsd[t,1,2]<- -yy*zz*zz
    rsd[t,2,2]<- 2.0*x[1]*yy*yy*zz*zz*zz
    rsd[t,3,1]<- t*x[2]*yy*zz*zz
    rsd[t,1,3]<- t*x[2]*yy*zz*zz
    rsd[t,3,2]<- t*x[1]*yy*zz*zz*(1-2*x[2]*yy*zz)
    rsd[t,2,3]<- t*x[1]*yy*zz*zz*(1-2*x[2]*yy*zz)
##    rsd[t,3,3]<- 2*t*t*x[1]*x[2]*x[2]*yy*yy*zz*zz*zz
    rsd[t,3,3]<- -t*t*x[1]*x[2]*yy*zz*zz*(1-2*yy*zz*x[2])
    return(rsd)
}


hobbs.h <- function(x) { ## compute Hessian
#   cat("Hessian not yet available\n")
#   return(NULL)
    H<-matrix(0,3,3)
    res<-hobbs.res(x)
    jj<-hobbs.jac(x)
    rsd<-hobbs.rsd(x)
##    H<-2.0*(t(res) %*% rsd + t(jj) %*% jj)
    for (j in 1:3) {
       for (k in 1:3) {
          for (i in 1:12) {
             H[j,k]<-H[j,k]+res[i]*rsd[i,j,k]
          }
       }
    }
    H<-2*(H + t(jj) %*% jj)
    return(H)
}


hobbsrsd.tst<-function(x) { # test rsd calculations
   hh<-1e-7 # use this for delta for derivatives
   Ja<-hobbs.jac(x)
   rsd<-hobbs.rsd(x)
   x1<-x+c(hh,0,0)
   x2<-x+c(0,hh,0)
   x3<-x+c(0,0,hh)
   Ja1<-hobbs.jac(x1)
   Ja2<-hobbs.jac(x2)
   Ja3<-hobbs.jac(x3)
   cat("w.r.t. x1 ")
   print(maxard((Ja1-Ja)/hh,rsd[,,1] ))
   cat("w.r.t. x2 ")
   print(maxard((Ja2-Ja)/hh,rsd[,,2] ))
   cat("w.r.t. x3 ")
   print(maxard((Ja3-Ja)/hh,rsd[,,3] ))
}
```

```
hobbs.doc <- function() { ## documentation for hobbs
   cat("One generalization of the Rosenbrock banana valley function (n parameters)\n")
   ## How should we do the documentation output?
}


hobbs.setup <- function(n=NULL, dotdat=NULL) {
  # if (is.null(gs) ) { gs<-100.0 } # set the scaling
  # if ( is.null(n) ) {
  #    n <- readline("Order of problem (n):")
  # }
   n<-3 # fixed for Hobbs, as is m=12
   x<-rep(2,n)
   lower<-rep(-100.0, n)
   upper<-rep(100.0, n)
   bdmsk<-rep(1,n)
   if (! is.null(dotdat) ) {
       fargs<-paste("gs=",gs,sep='') # ?? still need to do this nicely
   } else { fargs<-NULL }
   gsu<-list(x=x,lower=lower,upper=upper,bdmsk=bdmsk,fargs=fargs)
   return(gsu)

}

hobbs.fgh <- function(x) { # all 3 for trust method
         stopifnot(is.numeric(x))
         stopifnot(length(x) == 3)
         f<-hobbs.f(x)
         g<-hobbs.g(x)
         B<-hobbs.h(x)
         list(value = f, gradient = g, hessian = B)
}

require(snewton)
x0 <- c(200, 50, .3)
cat("Start for Hobbs:")
```

```
## Start for Hobbs:
```

```
print(x0)
```

```
## [1] 200.0  50.0   0.3
```

```
solx0 <- snewton(x0, hobbs.f, hobbs.g, hobbs.h)
```

```
## trace = 1
## Initial function value =  158.2324
## nf= 1
## Iteration  1 :nf= 2
## Iteration  2 :nf= 3
## Iteration  3 :nf= 4
## Iteration  4 :nf= 5
## Iteration  5 :nf= 6
## Iteration  6 :nf= 7
## Iteration  7 :nf= 8
```

```
## Iteration  8 :nf= 9
## Iteration  9 :nf= 10
## Iteration  10 :No progress before linesearch!
## No progress in linesearch!
```

```
print(solx0)
```

```
## $par
## [1] 196.1862618  49.0916395   0.3135697
##
## $value
## [1] 2.587277
##
## $grad
## [1] -4.440892e-16  2.109424e-15 -3.979039e-13
##
## $Hess
##             [,1]         [,2]        [,3]
## [1,]    1.265461    -3.256125    1602.105
## [2,]   -3.256125     8.627095   -4095.206
## [3,] 1602.105263 -4095.206388 2043434.033
##
## $counts
## $counts$niter
## [1] 10
##
## $counts$nfn
## [1] 11
##
## $counts$ngr
## [1] 10
##
## $counts$nhess
## [1] 10
##
##
## $convcode
## [1] 93
```

```
print(eigen(solx0$Hess)$values)
```

```
## [1] 2.043443e+06 4.249248e-01 4.413953e-03
```

```
cat("This test finds a saddle point\n")
```

```
## This test finds a saddle point
```

```
x1s <- c(100, 10, .1)
cat("Start for Hobbs:")
```

```
## Start for Hobbs:
```

```
print(x1s)
```

```
## [1] 100.0  10.0   0.1
```

```
solx1s <- snewton(x1s, hobbs.f, hobbs.g, hobbs.h, control=list(trace=2))
```

```
## trace = 2
## Initial function value =  10685.29
## [1] 100.0  10.0   0.1
## Termination test:nf= 1
## current gradient norm = 82341.59
## Iteration  1 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 22
## current gradient norm = 82341.59
## Iteration  2 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
```

```
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 43
## current gradient norm = 82341.59
## Iteration  3 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
```

```
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 65
## current gradient norm = 82341.59
## Iteration  4 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
```

```
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 87
## current gradient norm = 82341.59
## Iteration  5 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
```

```
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 109
## current gradient norm = 82341.59
## Iteration  6 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
```

```
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 131
## current gradient norm = 82341.59
## Iteration  7 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
```

```
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## Stepsize now = 4.194304e-16
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 154
## current gradient norm = 82341.59
## Iteration  8 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
```

```
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 176
## current gradient norm = 82341.59
## Iteration  9 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 197
```

```
## current gradient norm = 82341.59
## Iteration  10 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 218
## current gradient norm = 82341.59
## Iteration  11 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
```

```
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 239
## current gradient norm = 82341.59
## Iteration  12 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
```

```
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## Stepsize now = 4.194304e-16
## * f(xnew)= 10685.29
## end major loop
## Termination test:nf= 262
## current gradient norm = 82341.59
## Iteration  13 :Search vector:[1] -75.4852409   4.0706426  -0.4183675
## Gradient projection =  45255.98
## f(xnew)= 24228.5
## Stepsize now = 0.2
## * f(xnew)= 18092.97
## Stepsize now = 0.04
## * f(xnew)= 12450.3
## Stepsize now = 0.008
## * f(xnew)= 11045.81
## Stepsize now = 0.0016
## * f(xnew)= 10757.64
## Stepsize now = 0.00032
## * f(xnew)= 10699.77
## Stepsize now = 6.4e-05
## * f(xnew)= 10688.18
## Stepsize now = 1.28e-05
## * f(xnew)= 10685.87
## Stepsize now = 2.56e-06
## * f(xnew)= 10685.4
```

```
## Stepsize now = 5.12e-07
## * f(xnew)= 10685.31
## Stepsize now = 1.024e-07
## * f(xnew)= 10685.29
## Stepsize now = 2.048e-08
## * f(xnew)= 10685.29
## Stepsize now = 4.096e-09
## * f(xnew)= 10685.29
## Stepsize now = 8.192e-10
## * f(xnew)= 10685.29
## Stepsize now = 1.6384e-10
## * f(xnew)= 10685.29
## Stepsize now = 3.2768e-11
## * f(xnew)= 10685.29
## Stepsize now = 6.5536e-12
## * f(xnew)= 10685.29
## Stepsize now = 1.31072e-12
## * f(xnew)= 10685.29
## Stepsize now = 2.62144e-13
## * f(xnew)= 10685.29
## Stepsize now = 5.24288e-14
## * f(xnew)= 10685.29
## Stepsize now = 1.048576e-14
## * f(xnew)= 10685.29
## Stepsize now = 2.097152e-15
## * f(xnew)= 10685.29
## Stepsize now = 4.194304e-16
## * f(xnew)= 10685.29
## No progress in linesearch!
```

```
print(solx1s)
```

```
## $par
## [1] 100.0  10.0   0.1
##
## $value
## [1] 10685.29
##
## $grad
## [1]   -100.9131    783.5327 -82341.5897
##
## $Hess
##              [,1]       [,2]         [,3]
## [1,]    0.7158366   2.050058    -350.4172
## [2,]    2.0500582 -74.869818     774.4752
## [3,] -350.4171783 774.475203 -126055.8038
##
## $counts
## $counts$niter
## [1] 13
##
## $counts$nfn
## [1] 285
##
## $counts$ngr
```

```
## [1] 13
##
## $counts$nhess
## [1] 13
##
##
## $convcode
## [1] 93
```

```
print(eigen(solx1s$Hess)$values)
```

```
## [1]  1.690081e+00 -7.010902e+01 -1.260615e+05
```

```
cat("Following test fails with ERROR -- Why?\n")
```

```
## Following test fails with ERROR -- Why?
```

```
x1 <- c(1, 1, 1)
cat("Start for Hobbs:")
```

```
## Start for Hobbs:
```

```
print(x1)
```

```
## [1] 1 1 1
```

```
ftest <- try(solx1 <- snewton(x1, hobbs.f, hobbs.g, hobbs.h, control=list(trace=2)))
```

```
## trace = 2
## Initial function value =  23520.58
## [1] 1 1 1
## Termination test:nf= 1
## current gradient norm = 824.0421
## Iteration  1 :Search vector:[1] 41.42169 15.89584 16.90107
## Gradient projection =  -34243.81
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= 22180.36
## end major loop
## Termination test:nf= 4
## current gradient norm = 788.357
## Iteration  2 :Search vector:[1] 34.646760  2.720555  4.942733
## Gradient projection =  -27342.91
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= 17274.02
## end major loop
## Termination test:nf= 6
## current gradient norm = 626.2841
## Iteration  3 :Search vector:[1] 24.001827  2.813229 21.382377
## Gradient projection =  -14880.99
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= 16690.2
```

```
## end major loop
## Termination test:nf= 9
## current gradient norm = 601.6369
## Iteration  4 :Search vector:[1] 24.5128581  0.5681561 10.9076966
## Gradient projection =  -14683.69
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= 16114.55
## end major loop
## Termination test:nf= 12
## current gradient norm = 577.618
## Iteration  5 :Search vector:[1] 23.7542447  0.2788724  8.2638815
## Gradient projection =  -13675.8
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= 16005.58
## end major loop
## Termination test:nf= 16
## current gradient norm = 572.9978
## Iteration  6 :Search vector:[1] 23.5866837  0.2513985  7.9269186
## Gradient projection =  -13472.22
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= 15898.23
## end major loop
## Termination test:nf= 20
## current gradient norm = 568.4145
## Iteration  7 :Search vector:[1] 23.4174706  0.2277944  7.6221392
## Gradient projection =  -13269.91
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= 15792.5
## end major loop
## Termination test:nf= 24
## current gradient norm = 563.8677
## Iteration  8 :Search vector:[1] 23.2471543  0.2073461  7.3446873
## Gradient projection =  -13069.21
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
```

```
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= 15771.6
## end major loop
## Termination test:nf= 29
## current gradient norm = 562.9655
## Iteration  9 :Search vector:[1] 23.2130700  0.2036327  7.2924028
## Gradient projection =  -13029.39
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= 15767.43
## end major loop
## Termination test:nf= 35
## current gradient norm = 562.7854
## Iteration  10 :Search vector:[1] 23.2062534  0.2029037  7.2820653
## Gradient projection =  -13021.44
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= 15763.27
## end major loop
## Termination test:nf= 41
## current gradient norm = 562.6053
## Iteration  11 :Search vector:[1] 23.1994359  0.2021785  7.2717632
## Gradient projection =  -13013.49
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= 15759.1
```

```
## end major loop
## Termination test:nf= 47
## current gradient norm = 562.4253
## Iteration  12 :Search vector:[1] 23.192618  0.201457  7.261496
## Gradient projection =  -13005.54
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= 15758.27
## end major loop
## Termination test:nf= 54
## current gradient norm = 562.3893
## Iteration  13 :Search vector:[1] 23.1912540  0.2013133  7.2594478
## Gradient projection =  -13003.95
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= 15757.44
## end major loop
## Termination test:nf= 61
## current gradient norm = 562.3533
## Iteration  14 :Search vector:[1] 23.1898903  0.2011697  7.2574005
## Gradient projection =  -13002.36
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
```

```
## Stepsize now = 1.28e-05
## * f(xnew)= 15757.27
## end major loop
## Termination test:nf= 69
## current gradient norm = 562.3461
## Iteration  15 :Search vector:[1] 23.189618  0.201141  7.256991
## Gradient projection =  -13002.05
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= 15757.11
## end major loop
## Termination test:nf= 77
## current gradient norm = 562.3389
## Iteration  16 :Search vector:[1] 23.1893448  0.2011123  7.2565820
## Gradient projection =  -13001.73
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= 15756.94
## end major loop
## Termination test:nf= 85
## current gradient norm = 562.3317
## Iteration  17 :Search vector:[1] 23.1890721  0.2010836  7.2561729
## Gradient projection =  -13001.41
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
```

```
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= 15756.91
## end major loop
## Termination test:nf= 94
## current gradient norm = 562.3302
## Iteration  18 :Search vector:[1] 23.1890175  0.2010779  7.2560910
## Gradient projection =  -13001.35
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= 15756.91
## end major loop
## Termination test:nf= 105
## current gradient norm = 562.3302
## Iteration  19 :Search vector:[1] 23.1890154  0.2010776  7.2560878
## Gradient projection =  -13001.35
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
```

```
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 116
## current gradient norm = 562.3301
## Iteration  20 :Search vector:[1] 23.1890132  0.2010774  7.2560845
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 128
## current gradient norm = 562.3301
## Iteration  21 :Search vector:[1] 23.1890127  0.2010774  7.2560838
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
```

```
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 140
## current gradient norm = 562.3301
## Iteration  22 :Search vector:[1] 23.1890123  0.2010773  7.2560832
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 154
## current gradient norm = 562.3301
## Iteration  23 :Search vector:[1] 23.1890123  0.2010773  7.2560832
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
```

```
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 168
## current gradient norm = 562.3301
## Iteration  24 :Search vector:[1] 23.1890123  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 182
```

```
## current gradient norm = 562.3301
## Iteration  25 :Search vector:[1] 23.1890123  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 196
## current gradient norm = 562.3301
## Iteration  26 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
```

```
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 211
## current gradient norm = 562.3301
## Iteration  27 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 227
## current gradient norm = 562.3301
## Iteration  28 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
```

```
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 243
## current gradient norm = 562.3301
## Iteration  29 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
```

```
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 260
## current gradient norm = 562.3301
## Iteration  30 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 277
## current gradient norm = 562.3301
## Iteration  31 :Search vector:[1] 23.1890122  0.2010773  7.2560831
```

```
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 295
## current gradient norm = 562.3301
## Iteration  32 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
```

```
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 314
## current gradient norm = 562.3301
## Iteration  33 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
```

```
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 333
## current gradient norm = 562.3301
## Iteration  34 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
```

```
## end major loop
## Termination test:nf= 352
## current gradient norm = 562.3301
## Iteration  35 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= 15756.9
## end major loop
## Termination test:nf= 371
## current gradient norm = 562.3301
## Iteration  36 :Search vector:[1] 23.1890122  0.2010773  7.2560831
## Gradient projection =  -13001.34
## f(xnew)= Inf
## Stepsize now = 0.2
## * f(xnew)= Inf
## Stepsize now = 0.04
## * f(xnew)= Inf
## Stepsize now = 0.008
## * f(xnew)= Inf
```

```
## Stepsize now = 0.0016
## * f(xnew)= Inf
## Stepsize now = 0.00032
## * f(xnew)= Inf
## Stepsize now = 6.4e-05
## * f(xnew)= Inf
## Stepsize now = 1.28e-05
## * f(xnew)= Inf
## Stepsize now = 2.56e-06
## * f(xnew)= Inf
## Stepsize now = 5.12e-07
## * f(xnew)= Inf
## Stepsize now = 1.024e-07
## * f(xnew)= Inf
## Stepsize now = 2.048e-08
## * f(xnew)= Inf
## Stepsize now = 4.096e-09
## * f(xnew)= Inf
## Stepsize now = 8.192e-10
## * f(xnew)= Inf
## Stepsize now = 1.6384e-10
## * f(xnew)= Inf
## Stepsize now = 3.2768e-11
## * f(xnew)= Inf
## Stepsize now = 6.5536e-12
## * f(xnew)= Inf
## Stepsize now = 1.31072e-12
## * f(xnew)= Inf
## Stepsize now = 2.62144e-13
## * f(xnew)= Inf
## Stepsize now = 5.24288e-14
## * f(xnew)= Inf
## end major loop
## Termination test:nf= 391
## current gradient norm = Inf
## Iteration  37 :Search vector:[1] NaN NaN NaN
## Gradient projection =  NaN
```

```
if (class(ftest) != "try-error") {
   print(solx1)
   print(eigen(solx1$Hess)$values)
}

# we can also use nlm and nlminb
#??

# and call them from optimx (i.e., test this gives same results)
```