A literature survey of benchmark functions for global optimisation problems

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Abstract: Test functions are important to validate and compare the performance of optimisation algorithms. There have been many test or benchmark functions reported in the literature; however, there is no standard list or set of benchmark functions. Ideally, test functions should have diverse properties to be truly useful to test new algorithms in an unbiased way. For this purpose, we have reviewed and compiled a rich set of 175 benchmark functions for unconstrained optimisation problems with diverse properties in terms of modality, separability, and valley landscape. This is by far the most complete set of functions so far in the literature, and it can be expected that this complete set of functions can be used for validation of new optimisation in the future.

Keywords: global optimisation; test functions; unimodal; multimodal; separable; non-separable.

Reference to this paper should be made as follows: Jamil, M. and Yang, X-S. (2013) 'A literature survey of benchmark functions for global optimisation problems', *Int. J. Mathematical Modelling and Numerical Optimisation*, Vol. 4, No. 2, pp.150–194.

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1 Introduction

The test of reliability, efficiency and validation of optimisation algorithms is frequently carried out by using a chosen set of common standard benchmarks or test functions from the literature. The number of test functions in most papers varied from a few to about two dozens. Ideally, the test functions used should be diverse and unbiased, however, there is no agreed set of test functions in the literature. Therefore, the major aim of this paper is to review and compile the most complete set of test functions that we can find from all the available literature so that they can be used for future validation and comparison of optimisation algorithms.

For any new optimisation, it is essential to validate its performance and compare with other existing algorithms over a good set of test functions. A common practice followed by many researches is to compare different algorithms on a large test set, especially when the test involves function optimisation (Gordon and Whitley, 1993; Whitley et al., 1996). However, it must be noted that effectiveness of one algorithm against others simply cannot be measured by the problems that it solves if the the set of problems are too specialised and without diverse properties. Therefore, in order to evaluate an algorithm, one must identify the kind of problems where it performs better compared to others. This helps in characterising the type of problems for which an algorithm is suitable. This is only possible if the test suite is large enough to include a wide variety of problems, such as unimodal, multimodal, regular, irregular, separable, non-separable and multi-dimensional problems.

Many test functions may be scattered in different textbooks, in individual research articles or at different websites. Therefore, searching for a single source of test function with a wide variety of characteristics is a cumbersome and tedious task. The most notable attempts to assemble global optimisation (GO) test problems can be found in Ali et al. (2005), Averick et al. (1991, 1992), Branin (1972), Chung and Reynolds (1998), Dixon and Szegó (1978), Dixon and Price (1989), Fletcher and Powell (1963), Flouda et al. (1999), Moré et al. (1981), Powell (1962, 1964), Price et al. (2005), Salomon (1996), Schwefel (1981, 1995), Suganthan et al. (2005), Tang et al. (2008, 2010) and Whitley et al. (1996). Online collections of test problems also exist, such as the GLOBAL library at the cross-entropy toolbox (The Cross-Entropy Toolbox, http://www.maths.uq.edu.au/CEToolBox/), GAMS World (2000) CUTE (Gould et al., 2001), GO test problems collection by Hedar (n.d.), collection of test functions (Andrei, 2008; GEATbx, http://www.geatbx.com/; Test Problems for Global Optimization, http://www2.imm.dtu.dk/ kajm/Test ex forms/test ex.html; Mishra, 2006a, 2006b, 2006c, 2006d, 2006e, 2006f, 2006g), a collection of continuous GO test problems COCONUT (Neumaier, 2003) and a subset of commonly used test functions (Yang, 2010a). This motivates us to carry out a thorough analysis and compile a comprehensive collection of unconstrained optimisation test problems.

In general, unconstrained problems can be classified into two categories: test functions and real-world problems. Test functions are artificial problems, and can be used to evaluate the behaviour of an algorithm in sometimes diverse and difficult situations. Artificial problems may include single global minimum, single or multiple global minima in the presence of many local minima, long narrow valleys, null-space effects, and flat surfaces. These problems can be easily manipulated and modified to test the algorithms in diverse scenarios. On the other hand, real-world problems originate from different fields such as physics, chemistry, engineering, mathematics, etc. These problems are hard to manipulate and may contain complicated algebraic or differential expressions and may require a significant amount of data to compile. A collection of real-world unconstrained optimisation problems can be found in Averick et al. (1991, 1992).

In this present work, we will focus on the test function benchmarks and their diverse properties such as modality and separability. A function with more than one local optimum is called multimodal. These functions are used to test the ability of an algorithm to escape from any local minimum. If the exploration process of an algorithm is poorly designed, then it cannot search the function landscape effectively. This, in turn, leads to an algorithm getting stuck at a local minimum. Multi-modal functions with many local minima are among the most difficult class of problems for many algorithms. Functions with flat surfaces pose a difficulty for the algorithms, since the flatness of the function does not give the algorithm any information to direct the search process towards the minima (Stepint, Matyas, PowerSum).

Another group of test problems is formulated by separable and non-separable functions. According to Boyer et al. (2005), the dimensionality of the search space is an important issue with the problem. In some functions, the area that contains that global minima are very small, when compared to the whole search space, such as Easom, Michalewicz (m=10) and Powell. For problems such as Perm, Kowalik and Schaffer, the global minimum is located very close to the local minima. If the algorithm cannot keep up the direction changes in the functions with a narrow curved valley, in case of functions like Beale, Colville, or cannot explore the search space effectively, in case of function like Pen Holder, Testtube-Holder having multiple global minima, the algoritm will fail in these kinds of problems. Another problem that the algorithms may suffer is the scaling problem with many orders of magnitude differences between the domain and the function hyper-surface (Junior et al., 2004), such as Goldstein-Price and Trid.

2 Characteristics of test functions

The goal of any GO is to find the best possible solutions $\mathbf{x}^*$ from a set $\mathbb{X}$ according to a set of criteria $F = \{f_1, f_2, \cdots f_n\}$. These criteria are called objective functions expressed in the form of mathematical functions. An objective function is a mathematical function $f : D \subset \Re^n \to \Re$ subject to additional constraints. The set D is referred to as the set of feasible points in a search space. In the case of optimising a single criterion f, an optimum is either its maximum or minimum. The GO problems are often defined as minimisation problems, however, these problems can be easily converted to maximisation problems by negating f. A general global optimum problem can be defined as follows:

$$\underset{\mathbf{x}}{\text{minimise}} f(\mathbf{x}) \tag{1}$$

The true optimal solution of an optimisation problem may be a set of $\mathbf{x}^* \in D$ of all optimal points in D, rather than a single minimum or maximum value in some cases. There could be multiple, even an infinite number of optimal solutions, depending on the domain of the search space. The tasks of any good GO algorithm is to find globally optimal or at least sub-optimal solutions. The objective functions could be characterised as continuous, discontinuous, linear, non-linear, convex, non-conxex, unimodal, multimodal, separable[1] and non-separable.

According to Chung and Reynolds (1998), it is important to ask the following two questions before solving an optimisation problem;

1. What aspects of the function landscape make the optimisation process difficult?
2. What type of *a priori* knowledge is most effective for searching particular types of function landscape?

In order to answer these questions, benchmark functions can be classified in terms of features like modality, basins, valleys, separability and dimensionality (Winston, 1992).

2.1 Modality

The number of ambiguous peaks in the function landscape corresponds to the modality of a function. If algorithms encounters these peaks during a search process, there is a tendency that the algorithm may be trapped in one of the peaks. This will have a negative impact on the search process, as this can direct the search away from the true optimal solutions.

2.2 Basins

A relatively steep decline surrounding a large area is called a basin. Optimisation algorithms can be easily attracted to such regions. Once in these regions, the search process of an algorithm is severely hampered. This is due to lack of information to direct the search process towards the minimum. According to Chung and Reynolds (1998), a basin corresponds to the plateau for a maximisation problem, and a problem can have multiple plateaus.

2.3 Valleys

A valley occurs when a narrow area of little change is surrounded by regions of steep descent (Chung and Reynolds, 1998). As with the basins, minimisers are initially attracted to this region. The progress of a search process of an algorithm may be slowed down considerably on the floor of the valley.

2.4 Separability

The separability is a measure of difficulty of different benchmark functions. In general, separable functions are relatively easy to solve, when compared with their inseperable counterpart, because each variable of a function is independent of the other variables. If all the parameters or variables are independent, then a sequence of n independent optimisation processes can be performed. As a result, each design variable or parameter can be optimised independently. According to Salomon (1996), the general condition of separability to see if the function is easy to optimise or not is given as

$$\frac{\partial f(\overline{x})}{\partial x_i} = g(x_i)h(\overline{x}) \tag{2}$$

where $g(\overline{x_i})$ means any function of x_i only and $h(\overline{x})$ any function of any $\overline{x}$. If this condition is satisfied, the function is called partially separable and easy to optimise, because solutions for each x_i can be obtained independently of all the other parameters. This separability condition can be illustrated by the following two examples.

For example, function (f_{105}) is not separable, because it does not satisfy the condition (2)

$$\frac{\partial f_{105}(x_1, x_2)}{\partial x_1} = 400(x_1^2 - x_2)x_1 - 2x_1 - 2$$
$$\frac{\partial f_{105}(x_1, x_2)}{\partial x_2} = -200(x_1^2 - x_2)$$

On the other hand, the sphere function (f_{137}) with two variables can indeed satisfy the above condition (2) as shown below.

$$\frac{\partial f_{137}(x_1, x_2)}{\partial x_1} = 2x_1 \qquad \frac{\partial f_{137}(x_1, x_2)}{\partial x_2} = 2x_2$$

where $h(x)$ is regarded as 1.

In Boyer et al. (2005), the formal definition of separability is given as

$$\underset{x_1,\ldots,x_p}{\text{arg minimise}} f(x_1, \ldots, x_p) = \big(\underset{x_1}{\text{arg minimise}} f(x_1, \ldots), \ldots, \underset{x_p}{\text{arg minimise}} f(\ldots, x_p)\big) \tag{3}$$

In other words, a function of p variables is called separable, if it can written as a sum of p functions of just one variable (Boyer et al., 2005). On the other hand, a function is called non-separable, if its variables show inter-relation among themselves or are not independent. If the objective function variables are independent of each other, then the objective functions can be decomposed into sub-objective functions. Then, each of these sub-objectives involves only one decision variable, while treating all the others as constant and can be expressed as

$$f(x_1, x_2, \cdots, x_p) = \sum_{i=1}^{p} f_i(x_i) \tag{4}$$

2.5 Dimensionality

The difficulty of a problem generally increases with its dimensionality. According to Winston (1992) and Yao and Liu (1996), as the number of parameters or dimension increases, the search space also increases exponentially. For highly non-linear problems, this dimensionality may be a significant barrier for almost all optimisation algorithms.

3 Benchmark test functions for GO

Now, we present a collection of 175 unconstrained optimisation test problems which can be used to validate the performance of optimisation algorithms. The dimensions, problem domain size and optimal solution are denoted by D, $Lb \leq \mathbf{x}_i \leq Ub$ and $f(\mathbf{x}^*) = f(x_1, ...x_n)$, respectively. The symbols Lb and Ub represent lower, upper bound of the variables, respectively. It is worth noting that in several cases, the optimal solution vectors and their corresponding solutions are known only as numerical approximations.

1 *Ackley Function 1* (Báck and Schwefel, 1993) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_1(x) = -20e^{-0.02\sqrt{D^{-1}\sum_{i=1}^{D} x_i^2}} - e^{D^{-1}\sum_{i=1}^{D}\cos(2\pi x_i)} + 20 + e$$

subject to $-35 \leq x_i \leq 35$. The global minima is located at origin $\mathbf{x}^* = (0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

2 *Ackley Function 2* (Ackley, 1987) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_2(x) = -200e^{-0.02\sqrt{x_1^2 + x_2^2}}$$

subject to $-32 \leq x_i \leq 32$. The global minimum is located at origin $\mathbf{x}^* = (0, 0)$, $f(\mathbf{x}^*) = -200$.

3 *Ackley Function 3* (Ackley, 1987) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_3(x) = 200e^{-0.02\sqrt{x_1^2 + x_2^2}} + 5e^{\cos(3x_1) + \sin(3x_2)}$$

subject to $-32 \leq x_i \leq 32$. The global minimum is located at $\mathbf{x}^* = (0, \approx -0.4)$, $f(\mathbf{x}^*) \approx -219.1418$.

4 *Ackley Function 4 or Modified Ackley Function* (Rónkkónen, 2009) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_4(\mathbf{x}) = \sum_{i=1}^{D} \left(e^{-0.2} \sqrt{x_i^2 + x_{i+1}^2} + 3 \left(\cos(2x_i) + \sin(2x_{i+1}) \right) \right)$$

subject to $-35 \leq x_i \leq 35$. It is highly multimodal function with two global minimum close to origin
$\mathbf{x} = f(\{-1.479252, -0.739807\}, \{1.479252, -0.739807\})$, $f(\mathbf{x}^*) = -3.917275$.

5 *Adjiman Function* (Adjiman et al., 1998) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_5(x) = \cos(x_1)\sin(x_2) - \frac{x_1}{(x_2^2 + 1)}$$

subject to $-1 \leq x_1 \leq 2$, $-1 \leq x_2 \leq 1$. The global minimum is located at $\mathbf{x}^* = (2, 0.10578)$, $f(\mathbf{x}^*) = -2.02181$.

6 *Alpine Function 1* (Rahnamyan et al., 2007a) (continuous, non-differentiable, separable, non-scalable, multimodal)

$$f_6(\mathbf{x}) = \sum_{i=1}^{D} \left| x_i \sin(x_i) + 0.1 x_i \right|$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at origin $\mathbf{x}^* = (0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

7 *Alpine Function 2* (Clerc, 1999) (continuous, differentiable, separable, scalable, multimodal)

$$f_7(\mathbf{x}) = \prod_{i=1}^{D} \sqrt{x_i} \sin(x_i)$$

subject to $0 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = (7.917 \cdots 7.917)$, $f(\mathbf{x}^*) = 2.808^D$.

8 *Brad Function* (Brad, 1970) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_8(\mathbf{x}) = \sum_{i=1}^{15} \left[\frac{y_i - x_1 - u_i}{v_i x_2 + w_i x_3} \right]^2$$

where $u_i = i$, $v_i = 16 - i$, $w_i = \min(u_i, v_i)$ and $\underline{\mathbf{y}} = y_i = [0.14, 0.18, 0.22, 0.25, 0.29, 0.32, 0.35, 0.39, 0.37, 0.58, 0.73, 0.96, 1.34, 2.10, 4.39]^T$. It is subject to $-0.25 \leq x_1 \leq 0.25$, $0.01 \leq x_2, x_3 \leq 2.5$. The global minimum is located at $\mathbf{x}^* = (0.0824, 1.133, 2.3437)$, $f(\mathbf{x}^*) = 0.00821487$.

9 *Bartels Conn Function* (continuous, non-differentiable, non-separable, non-scalable, multimodal)

$$f_9(\mathbf{x}) = \left|x_1^2 + x_2^2 + x_1x_2\right| + \left|\sin(x_1)\right| + \left|\cos(x_2)\right|$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = (0, 0)$, $f(\mathbf{x}^*) = 1$.

10 *Beale Function* (Hedar, n.d.) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{10}(\mathbf{x}) = (1.5 - x_1 + x_1x_2)^2 + (2.25 - x_1 + x_1x_2^2)^2 \\ +(2.625 - x_1 + x_1x_2^3)^2$$

subject to $-4.5 \leq x_i \leq 4.5$. The global minimum is located at $\mathbf{x}^* = (3, 0.5)$, $f(\mathbf{x}^*) = 0$.

11 *Biggs EXP Function 2* (Biggs, 1971) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{11}(\mathbf{x}) = \sum_{i=1}^{10} \left(e^{-t_ix_1} - 5e^{-t_ix_2} - y_i\right)^2$$

where $t_i = 0.1i$, $y_i = e^{-t_i} - 5e^{10t_i}$. It is subject to $0 \leq x_i \leq 20$. The global minimum is located at $\mathbf{x}^* = (1, 10)$, $f(\mathbf{x}^*) = 0$.

12 *Biggs EXP Function 3* (Biggs, 1971) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{12}(\mathbf{x}) = \sum_{i=1}^{10} \left(e^{-t_ix_1} - x_3e^{-t_ix_2} - y_i\right)^2$$

where $t_i = 0.1i$, $y_i = e^{-t_i} - 5e^{10t_i}$. It is subject to $0 \leq x_i \leq 20$. The global minimum is located at $\mathbf{x}^* = (1, 10, 5)$, $f(\mathbf{x}^*) = 0$.

13 *Biggs EXP Function 4* (Biggs, 1971) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{13}(\mathbf{x}) = \sum_{i=1}^{10} \left(x_3e^{-t_ix_1} - x_4e^{-t_ix_2} - y_i\right)^2$$

where $t_i = 0.1i$, $y_i = e^{-t_i} - 5e^{10t_i}$. It is subject to $0 \leq x_i \leq 20$. The global minimum is located at $\mathbf{x}^* = (1, 10, 1, 5)$, $f(\mathbf{x}^*) = 0$.

14 *Biggs EXP Function 5* (Biggs, 1971) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{14}(\mathbf{x}) = \sum_{i=1}^{11} \left(x_3e^{-t_ix_1} - x_4e^{-t_ix_2} + 3e^{-t_ix_5} - y_i\right)^2$$

where $t_i = 0.1i$, $y_i = e^{-t_i} - 5e^{10t_i} + 3e^{-4t_i}$. It is subject to $0 \leq x_i \leq 20$. The global minimum is located at $\mathbf{x}^* = (1, 10, 1, 5, 4)$, $f(\mathbf{x}^*) = 0$.

15 *Biggs EXP Function 6* (Biggs, 1971) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{15}(\mathbf{x}) = \sum_{i=1}^{13} \left(x_3 e^{-t_i x_1} - x_4 e^{-t_i x_2} + x_6 e^{-t_i x_5} - y_i\right)^2$$

where $t_i = 0.1i$, $y_i = e^{-t_i} - 5e^{10t_i} + 3e^{-4t_i}$. It is subject to $-20 \leq x_i \leq 20$. The global minimum is located at $\mathbf{x}^* = (1, 10, 1, 5, 4, 3)$, $f(\mathbf{x}^*) = 0$.

16 *Bird Function* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{16}(\mathbf{x}) = \sin(x_1)e^{(1-\cos(x_2))^2} + \cos(x_2)e^{(1-\sin(x_1))^2} + (x_1 - x_2)^2$$

subject to $-2\pi \leq x_i \leq 2\pi$. The global minimum is located at $\mathbf{x}^* = (4.70104,$ $3.15294)$,$(-1.58214,$ $-3.13024)$, $f(\mathbf{x}^*) = -106.764537$.

17 *Bohachevsky Function 1* (Bohachevsky et al., 1986) (continuous, differentiable, separable, non-scalable, multimodal)

$$\begin{aligned} f_{17}(\mathbf{x}) = {} & x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) \\ & -0.4\cos(4\pi x_2) + 0.7 \end{aligned}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

18 *Bohachevsky Function 2* (Bohachevsky et al., 1986) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{18}(\mathbf{x}) = {} & x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) \cdot 0.4\cos(4\pi x_2) \\ & +0.3 \end{aligned}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

19 *Bohachevsky Function 3* (Bohachevsky et al., 1986) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{19}(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1 + 4\pi x_2) + 0.3$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

20 *Booth Function* (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{20}(\mathbf{x}) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(1, 3)$, $f(\mathbf{x}^*) = 0$.

21 *Box-Betts Quadratic Sum Function* (Ali et al., 2005) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{21}(\mathbf{x}) = \sum_{i=0}^{D-1} g(x_i)^2$$

where

$$g(x) = e^{-0.1(i+1)x_1} - e^{-0.1(i+1)x_2} - e^{[(-0.1(i+1))-e^{-(i+1)}]x_3}$$

subject to $0.9 \le x_1 \le 1.2$, $9 \le x_2 \le 11.2$, $0.9 \le x_2 \le 1.2$. The global minimum is located at $\mathbf{x}^* = f(1, 10, 1)$ $f(\mathbf{x}^*) = 0$.

22 *Branin RCOS Function* (Branin, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{22}(\mathbf{x}) = \left(x_2 - \frac{5.1x_1^2}{4\pi^2} + \frac{5x_1}{\pi} - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$$

with domain $-5 \le x_1 \le 10$, $0 \le x_1 \le 15$. It has three global minima at $\mathbf{x}^* = f(\{-\pi, 12.275\}, \{\pi, 2.275\}, \{3\pi, 2.425\})$, $f(\mathbf{x}^*) = 0.3978873$.

23 *Branin RCOS Function 2* (Muntenau and Lazarescu, 1998) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{23}(\mathbf{x}) = \left(x_2 - \frac{5.1x_1^2}{4\pi^2} + \frac{5x_1}{\pi} - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1)\cos(x_2)\ln(x_1^2 + x_2^2 + 1) + 10$$

with domain $-5 \le x_i \le 15$. The global minimum is located at $\mathbf{x}^* = f(-3.2, 12.53)$, $f(\mathbf{x}^*) = 5.559037$.

24 *Brent Function* (Branin, 1972) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{24}(\mathbf{x}) = (x_1 + 10)^2 + (x_2 + 10)^2 + e^{-x_1^2 - x_2^2}$$

with domain $-10 \le x_i \le 10$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

25 *Brown Function* (Begambre and Laier, 2009) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{25}(\mathbf{x}) = \sum_{i=1}^{n-1} (x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)}$$

subject to $-1 \le x_i \le 4$. The global minimum is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

Bukin functions (Silagadze, 2007) are almost fractal (with fine seesaw edges) in the surroundings of their minimal points. Due to this property, they are extremely difficult to optimise by any global or local optimisation methods.

26 *Bukin Function 2* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{26}(\mathbf{x}) = 100(x_2 - 0.01x_1^2 + 1) + 0.01(x_1 + 10)^2$$

subject to $-15 \le x_1 \le -5$ and $-3 \le x_2 \le -3$. The global minimum is located at $\mathbf{x}^* = f(-10, 0)$, $f(\mathbf{x}^*) = 0$.

27 *Bukin Function 4* (continuous, non-differentiable, separable, non-scalable, multimodal)

$$f_{27}(\mathbf{x}) = 100x_2^2 + 0.01\|x_1 + 10\|$$

subject to $-15 \le x_1 \le -5$ and $-3 \le x_2 \le -3$. The global minimum is located at $\mathbf{x}^* = f(-10, 0)$, $f(\mathbf{x}^*) = 0$.

28 *Bukin Function 6* (continuous, non-differentiable, non-separable, non-scalable, multimodal)

$$f_{28}(\mathbf{x}) = 100\sqrt{\|x_2 - 0.01x_1^2\|} + 0.01\|x_1 + 10\|$$

subject to $-15 \le x_1 \le -5$ and $-3 \le x_2 \le -3$. The global minimum is located at $\mathbf{x}^* = f(-10, 1)$, $f(\mathbf{x}^*) = 0$.

29 *Camel Function – Three Hump* (Branin, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{29}(\mathbf{x}) = 2x_1^2 - 1.05x_1^4 + x_1^6/6 + x_1x_2 + x_2^2$$

subject to $-5 \le x_i \le 5$. The global minima is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

30 *Camel Function – Six Hump* (Branin, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{30}(\mathbf{x}) = {} & (4 - 2.1x_1^2 + \frac{x_1^4}{3})x_1^2 \\ & + x_1x_2 + (4x_2^2 - 4)x_2^2 \end{aligned}$$

subject to $-5 \le x_i \le 5$. The two global minima are located at $\mathbf{x}^* = f(\{-0.0898, 0.7126\}, \{0.0898, -0.7126, 0\})$, $f(\mathbf{x}^*) = -1.0316$.

31 *Chen Bird Function* (Chen, 2003) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{31}(\mathbf{x}) = {} & -\frac{0.001}{\lfloor (0.001)^2 + (x_1 - 0.4x_2 - 0.1)^2 \rfloor} - \\ & \frac{0.001}{\lfloor (0.001)^2 + (2x_1 + x_2 - 1.5)^2 \rfloor} \end{aligned}$$

subject to $-500 \leq x_i \leq 500$ The global minimum is located at $\mathbf{x}^* = f(-\frac{7}{18}, -\frac{13}{18})$, $f(\mathbf{x}^*) = -2000$.

32 *Chen V Function* (Chen, 2003) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{32}(\mathbf{x}) = -\frac{0.001}{\lfloor (0.001)^2 + (x_1^2 + x_2^2 - 1)^2 \rfloor} - \frac{0.001}{\lfloor (0.001)^2 + (x_1^2 + x_2^2 - 0.5)^2 \rfloor} - \frac{0.001}{\lfloor (0.001)^2 + (x_1^2 - x_2^2)^2 \rfloor}$$

subject to $-500 \leq x_i \leq 500$ The global minimum is located at $\mathbf{x}^* = f(-0.3888889, 0.7222222)$, $f(\mathbf{x}^*) = -2000$.

33 *Chichinadze Function* (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{33}(\mathbf{x}) = x_1^2 - 12x_1 + 11 + 10\cos(\pi x_1/2) + 8\sin(5\pi x_1/2) - (1/5)^{0.5} \exp(-0.5(x_2 - 0.5)^2)$$

subject to $-30 \leq x_i \leq 30$. The global minimum is located at $\mathbf{x}^* = f(5.90133, 0.5)$, $f(\mathbf{x}^*) = -43.3159$.

34 *Chung Reynolds Function* (Chung and Reynolds, 1998) (continuous, differentiable, partially-separable, scalable, unimodal)

$$f_{34}(\mathbf{x}) = \left(\sum_{i=1}^{D} x_i^2 \right)^2$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

35 *Cola Function* (Adorio and Dilman, 2005) (continuous, differentiable, non-separable, non-scalable, multimodal)

The 17-dimensional function computes indirectly the formula (D, u) by setting $x_0 = y_0, x_1 = u_0, x_i = u_{2(i-2)}, y_i = u_{2(i-2)+1}$

$$f_{35}(n, u) = h(x, y) = \sum_{j<i} (r_{i,j} - d_{i,j})^2$$

where $r_{i,j}$ is given by

$$r_{i,j} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$$

and d is a symmetric matrix given by

$$\mathbf{d} = [d_{ij}] = \begin{pmatrix} 1.27 \\ 1.69\,1.43 \\ 2.04\,2.35\,2.43 \\ 3.09\,3.18\,3.26\,2.85 \\ 3.20\,3.22\,3.27\,2.88\,1.55 \\ 2.86\,2.56\,2.58\,2.59\,3.12\,3.06 \\ 3.17\,3.18\,3.18\,3.12\,1.31\,1.64\,3.00 \\ 3.21\,3.18\,3.18\,3.17\,1.70\,1.36\,2.95\,1.32 \\ 2.38\,2.31\,2.42\,1.94\,2.85\,2.81\,2.56\,2.91\,2.97 \end{pmatrix}$$

This function has bounds $0 \le x_0 \le 4$ and $-4 \le x_i \le 4$ for $i = 1 \ldots D-1$. It has a global minimum of $f(\mathbf{x}^*) = 11.7464$.

36 *Colville Function* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{36}(\mathbf{x}) = {} & 100(x_1 - x_2^2)^2 + (1 - x_1)^2 + \\ & 90(x_4 - x_3^2)^2 + (1 - x_3)^2 + \\ & 10.1((x_2 - 1)^2 + (x_4 - 1)^2) + \\ & 19.8(x_2 - 1)(x_4 - 1) \end{aligned}$$

subject to $-10 \le x_i \le 10$. The global minima is located at $\mathbf{x}^* = f(1, \cdots, 1)$, $f(\mathbf{x}^*) = 0$.

37 *Corana Function* (Corana et al., 1987) (Discontinuous, non-differentiable, separable, scalable, multimodal)

$$f_{37}(\mathbf{x}) = \begin{cases} 0.15\left(z_i - 0.05\mathrm{sgn}(z_i)^2\right) d_i & \text{if} |v_i| < A \\ d_i x_i^2 & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} & v_i = |x_i - z_i|, \quad A = 0.05 \\ & z_i = 0.2 \left\lfloor \left| \frac{x_i}{0.2} \right| + 0.49999 \right\rfloor \mathrm{sgn}\,(x_i) \\ & d_i = (1, 1000, 10, 100) \end{aligned}$$

subject to $-500 \le x_i \le 500$. The global minimum is located at $\mathbf{x}^* = f(0, 0, 0, 0)$, $f(\mathbf{x}^*) = 0$.

38 *Cosine Mixture Function* (Ali et al., 2005) (Discontinuous, non-differentiable, separable, scalable, multimodal)

$$f_{38}(\mathbf{x}) = -0.1 \sum_{i=1}^{n} \cos(5\pi x_i) - \sum_{i=1}^{n} x_i^2$$

subject to $-1 \le x_i \le 1$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = (0.2\,\mathrm{or}\,0.4)$ for n $= 2\,\mathrm{and}\,4$ respectively.

39 *Cross-in-Tray Function* (Mishra, 2006f) (continuous, non-separable, non-scalable, multimodal)

$$f_{39}(\mathbf{x}) = -0.0001[|\sin(x_1)\sin(x_2) \\ e^{|100-[(x_1^2+x_2^2)]^{0.5}/\pi|}| + 1]^{0.1}$$

subject to $-10 \le x_i \le 10$.
The four global minima are located at $\mathbf{x}^* = f(\pm 1.349406685353340, \pm 1.349406608602084)$, $f(\mathbf{x}^*) = -2.06261218$.

40 *Csendes Function* (Csendes and Ratz, 1997) (continuous, differentiable, separable, scalable, multimodal)

$$f_{40}(\mathbf{x}) = \sum_{i=1}^{D} x_i^6 \left(2 + \sin \frac{1}{x_i}\right)$$

subject to $-1 \le x_i \le 1$. The global minimum is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

41 *Cube Function* (Lavi and Vogel, 1966) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{41}(\mathbf{x}) = 100\left(x_2 - x_1^3\right)^2 + (1 - x_1)^2$$

subject to $-10 \le x_i \le 10$. The global minimum is located at $\mathbf{x}^* = f(-1, 1)$, $f(\mathbf{x}^*) = 0$.

42 *Damavandi Function* (Damavandi and Safavi-Naeini, 2005) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{42}(\mathbf{x}) = \left[1 - \left|\frac{\sin[\pi(x_1-2)]\sin[\pi(x_2-2)]}{\pi^2(x_1-2)(x_2-2)}\right|^5\right] \\ \left[2 + (x_1-7)^2 + 2(x_2-7)^2\right]$$

subject to $0 \le x_i \le 14$. The global minimum is located at $\mathbf{x}^* = f(2, 2)$, $f(\mathbf{x}^*) = 0$.

43 *Deb Function 1* (Rónkkónen, 2009) (continuous, differentiable, separable, scalable, multimodal)

$$f_{43}(\mathbf{x}) = -\frac{1}{D}\sum_{i=1}^{D} \sin^6(5\pi x_i)$$

subject to $-1 \le x_i \le 1$. The number of global minima is 5^D that are evenly spaced in the function landscape, where D represents the dimension of the problem.

44 *Deb Function 3* (Rónkkónen, 2009) (continuous, differentiable, separable, scalable, multimodal)

$$f_{44}(\mathbf{x}) = -\frac{1}{D}\sum_{i=1}^{D}\sin^6(5\pi(x_i^{3/4} - 0.05))$$

subject to $-1 \leq x_i \leq 1$. The number of global minima is 5^D that are unevenly spaced in the function landscape, where D represents the dimension of the problem.

45 *Deckkers-Aarts Function* (Ali et al., 2005) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{45}(\mathbf{x}) = 10^5 x_1^2 + x_2^2 - (x_1^2 + x_2^2)^2 + 10^{-5}(x_1^2 + x_2^2)^4$$

subject to $-20 \leq x_i \leq 20$. The two global minima are located at $\mathbf{x}^* = f(0, \pm 15)$ $f(\mathbf{x}^*) = -24777$.

46 *deVilliers Glasser Function 1* (deVillers and Glasser, 1981)(continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{46}(\mathbf{x}) = \sum_{i=1}^{24}\left[x_1 x_2^{t_i}\sin(x_3 t_i + x_4) - y_i\right]^2$$

where $t_i = 0.1(i-1)$, $y_i = 60.137 \times 1.371^{t_i}\sin(3.112 t_i + 1.761)$. It is subject to $-500 \leq x_i \leq 500$. The global minimum is $f(\mathbf{x}^*) = 0$.

47 *deVilliers Glasser Function 2* (deVillers and Glasser, 1981) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{47}(\mathbf{x}) = \sum_{i=1}^{16}\left[x_1 x_2^{t_i}\tanh\left[x_3 t_i + \sin(x_4 t_i)\right]\cos(t_i e^{x_5}) - y_i\right]^2$$

where $t_i = 0.1(i-1)$,
$y_i = 53.81 \times 1.27^{t_i}\tanh(3.012 t_i + \sin(2.13 t_i))\cos(e^{0.507} t_i)$. It is subject to $-500 \leq x_i \leq 500$. The global minimum is $f(\mathbf{x}^*) = 0$.

48 *Dixon & Price Function* (Dixon and Price, 1989) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{48}(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^{D} i(2x_i^2 - x_{i-1})^2$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(2^{(\frac{2^i - 2}{2^i})})$, $f(\mathbf{x}^*) = 0$.

49 *Dolan Function* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{49}(\mathbf{x}) = (x_1 + 1.7x_2)\sin(x_1) - 1.5x_3 - 0.1x_4\cos(x_4 + x_5 - x_1) + \\ 0.2x_5^2 - x_2 - 1$$

subject to $-100 \leq x_i \leq 100$. The global minimum is $f(\mathbf{x}^*) = 0$.

50 *Easom Function* (Chung and Reynolds, 1998)(continuous, differentiable, separable, non-scalable, multimodal)

$$f_{50}(\mathbf{x}) = -\cos(x_1)\cos(x_2)\exp[-(x_1 - \pi)^2 \\ -(x_2 - \pi)^2]$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(\pi, \pi)$, $f(\mathbf{x}^*) = -1$.

51 *El-Attar-Vidyasagar-Dutta Function* (El-Attar et al., 1979) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{51}(\mathbf{x}) = (x_1^2 + x_2 - 10)^2 + (x_1 + x_2^2 - 7)^2 + \\ (x_1^2 + x_2^3 - 1)^2$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = f(2.842503, 1.920175)$, $f(\mathbf{x}^*) = 0.470427$.

52 *Egg Crate Function* (continuous, separable, non-scalable)

$$f_{52}(\mathbf{x}) = x_1^2 + x_2^2 + 25(\sin^2(x_1) + \sin^2(x_2))$$

subject to $-5 \leq x_i \leq 5$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

53 *Egg Holder Function* (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{53}(\mathbf{x}) = \sum_{i=1}^{m-1} [-(x_{i+1} + 47)\sin\sqrt{|x_{i+1} + x_i/2 + 47|} \\ -x_i\sin\sqrt{|x_i - (x_{i+1} + 47)|}]$$

subject to $-512 \leq x_i \leq 512$. The global minimum is located at $\mathbf{x}^* = f(512, 404.2319)$, $f(\mathbf{x}^*) \approx 959.64$.

54 *Exponential Function* (Rahnamyan et al., 2007b) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{54}(\mathbf{x}) = -\exp\left(-0.5\sum_{i=1}^{D} x_i^2\right)$$

subject to $-1 \leq x_i \leq 1$. The global minima is located at $\mathbf{x} = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 1$.

55 *EX Function 1* (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{55}(\mathbf{x}) = 0.1(1 - x_1^2) + 0.1\sin(10x_1) + (11 - x_2)^2 + \sin(10x_2)$$

subject to $x_i \in [0, 2] \bigcap [10, 12]$. The global minima is located at $\mathbf{x} = f(1.764, 11.150)$, $f(\mathbf{x}^*) \approx -1.28186$.

56 *Freudenstein Roth Function* (Rao, 2009) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{56}(\mathbf{x}) = &(x_1 - 13 + ((5 - x_2)x_2 - 2)x_2)^2 + \\ &(x_1 - 29 + ((x_2 + 1)x_2 - 14)x_2)^2 \end{aligned}$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(5, 4)$, $f(\mathbf{x}^*) = 0$.

57 *Giunta Function* (Mishra, 2006f) (continuous, differentiable, separable, scalable, multimodal)

$$\begin{aligned} f_{57}(\mathbf{x}) = 0.6 + &\sum_{i=1}^{2} [\sin(\frac{16}{15}x_i - 1) \\ &+ \sin^2(\frac{16}{15}x_i - 1) \\ &+ \frac{1}{50}\sin(4(\frac{16}{15}x_i - 1))] \end{aligned}$$

subject to $-1 \leq x_i \leq 1$. The global minimum is located at $\mathbf{x}^* = f(0.45834282, 0.45834282)$, $f(\mathbf{x}^*) = 0.060447$.

58 *Goldstein Price Function* (Goldstein and Price, 1971) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{58}(\mathbf{x}) = &[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 \\ &+ 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \\ &\times [30 + (2x_1 - 3x_2)^2 \\ &(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)] \end{aligned}$$

subject to $-2 \leq x_i \leq 2$. The global minimum is located at $\mathbf{x}^* = f(0, -1)$, $f(\mathbf{x}^*) = 3$.

59 *Griewank Function* (Griewank, 1981) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{59}(\mathbf{x}) = \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

60 *Gulf Research Problem* (Shanno, 1970) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{60}(\mathbf{x}) = \sum_{i=1}^{99} \left[\exp\left(-\frac{(u_i - x_2)^{x_3}}{x_i} \right) - 0.01i \right]^2$$

where $u_i = 25 + [-50 \ln(0.01i)]^{1/1.5}$ subject to $0.1 \leq x_1 \leq 100$, $0 \leq x_2 \leq 25.6$ and $0 \leq x_1 \leq 5$. The global minimum is located at $\mathbf{x}^* = f(50, 25, 1.5)$, $f(\mathbf{x}^*) = 0$.

61 *Hansen Function* (Fraley et al., 1989) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{61}(\mathbf{x}) = \sum_{i}^{4} (i+1)\cos(ix_1 + i + 1)$$
$$\sum_{j=0}^{4} (j+1)\cos((j+2)x_2 + j + 1)$$

subject to $-10 \leq x_i \leq 10$. The multiple global minima are located at

$$\mathbf{x}^* = f(\{-7.589893, -7.708314\}, \quad \{-7.589893, -1.425128\},$$

$$\{-7.589893, \quad 4.858057\}, \quad \{-1.306708, -7.708314\},$$

$$\{-1.306708, \quad 4.858057\}, \quad \{\ 4.976478, \quad 4.858057\},$$

$$\{\ 4.976478, -1.425128\}, \quad \{\ 4.976478, -7.708314\}),$$

62 *Hartman Function 3* (Hartman, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{62}(\mathbf{x}) = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2 \right]$$

subject to $0 \leq x_j \leq 1$, $j \in \{1, 2, 3\}$ with constants a_{ij}, p_{ij} and c_i are given as

$$\mathbf{A} = [A_{ij}] = \begin{pmatrix} 3 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3 & 10 & 30 \\ 0.1 & 10 & 35 \end{pmatrix}, \ \underline{\mathbf{c}} = c_i = \begin{bmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{bmatrix},$$

$$\underline{\mathbf{p}} = p_i = \begin{pmatrix} 0.3689 & 0.1170 & 0.2673 \\ 0.4699 & 0.4837 & 0.7470 \\ 0.1091 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{pmatrix}$$

The global minimum is located at $\mathbf{x}^* = f(0.1140, 0.556, 0.852)$, $f(\mathbf{x}^*) \approx -3.862782$.

63 *Hartman Function 6* (Hartman, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{63}(\mathbf{x}) = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right]$$

subject to $0 \le x_j \le 1$, $j \in \{1, \cdots, 6\}$ with constants a_{ij}, p_{ij} and c_i are given as

$$\mathbf{A} = [A_{ij}] = \begin{pmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{pmatrix}, \ \underline{\mathbf{c}} = c_i = \begin{bmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{bmatrix}$$

$$\underline{\mathbf{p}} = p_i = \begin{pmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5586 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1451 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{pmatrix}$$

The global minima is located at
$\mathbf{x} = f(0.201690, 0.150011, 0.476874, 0.275332, ...0.311652, 0.657301)$,
$f(\mathbf{x}^*) \approx -3.32236$.

64 *Helical Valley* (Fletcher and Powell, 1963) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{64}(\mathbf{x}) = 100\left[(x_2 - 10\theta)^2 + \left(\sqrt{x_1^2 + x_2^2} - 1\right)\right] \\ +x_3^2$$

where

$$\theta = \begin{cases} \frac{1}{2\pi}\tan^{-1}\left(\frac{x_1}{x_2}\right), & \text{if } x_1 \ge 0 \\ \frac{1}{2\pi}\tan^{-1}\left(\frac{x_1}{x_2} + 0.5\right) & \text{if } x_1 < 0 \end{cases}$$

subject to $-10 \le x_i \le 10$. The global minima is located at $\mathbf{x}^* = f(1, 0, 0)$, $f(\mathbf{x}^*) = 0$.

65 *Himmelblau Function* (Himmelblau, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{65}(\mathbf{x}) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

subject to $-5 \le x_i \le 5$. The global minimum is located at $\mathbf{x}^* = f(3, 2)$, $f(\mathbf{x}^*) = 0$.

66 *Hosaki Function* (Bekey and Ung, 1974) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{66}(\mathbf{x}) = (1 - 8x_1 + 7x_1^2 - 7/3x_1^3 + 1/4x_1^4)x_2^2e^{-x_2}$$

subject to $0 \le x_1 \le 5$ and $0 \le x_2 \le 6$. The global minimum is located at $\mathbf{x}^* = f(4, 2)$, $f(\mathbf{x}^*) \approx -2.3458$.

67 *Jennrich-Sampson Function* (Jennrich and Sampson, 1968) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{67}(\mathbf{x}) = \sum_{i=1}^{10} \left(2 + 2i - \left(\mathrm{e}^{ix_1} + e^{ix_2}\right)\right)^2$$

subject to $-1 \leq x_i \leq 1$. The global minimum is located at $\mathbf{x}^* = f(0.257825, 0.257825)$, $f(\mathbf{x}^*) = 124.3612$.

68 *Langerman Function 5* (Bersini et al., 1996) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{68}(\mathbf{x}) = -\sum_{i=1}^{m} c_i e^{-\frac{1}{\pi}\sum_{j=1}^{D}(x_j - a_{ij})^2} \cos\left(\pi \sum_{j=1}^{D}(x_j - a_{ij})^2\right)$$

subject to $0 \leq x_j \leq 10$, where $j \in [0, D-1]$ and $m = 5$. It has a global minimum value of $f(\mathbf{x}^*) = -1.4$. The matrix A and column vector c are given as The matrix A is given by

$$\mathbf{A} = [A_{ij}] = \begin{bmatrix} 9.681\;0.667\;4.783\;9.095\;3.517\;9.325\;6.544\;0.211\;5.122\;2.020 \\ 9.400\;2.041\;3.788\;7.931\;2.882\;2.672\;3.568\;1.284\;7.033\;7.374 \\ 8.025\;9.152\;5.114\;7.621\;4.564\;4.711\;2.996\;6.126\;0.734\;4.982 \\ 2.196\;0.415\;5.649\;6.979\;9.510\;9.166\;6.304\;6.054\;9.377\;1.426 \\ 8.074\;8.777\;3.467\;1.863\;6.708\;6.349\;4.534\;0.276\;7.633\;1.567 \end{bmatrix}$$

$$\underline{\mathbf{c}} = c_i = \begin{bmatrix} 0.806 \\ 0.517 \\ 1.5 \\ 0.908 \\ 0.965 \end{bmatrix}$$

69 *Keane Function* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{69}(\mathbf{x}) = \frac{\sin^2(x_1 - x_2)\sin^2(x_1 + x_2)}{\sqrt{x_1^2 + x_2^2}}$$

subject to $0 \leq x_i \leq 10$.
The multiple global minima are located at $\mathbf{x}^* = f(\{0, 1.39325\}, \{1.39325, 0\})$, $f(\mathbf{x}^*) = -0.673668$.

70 *Leon Function* (Lavi and Vogel, 1966) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{70}(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

subject to $-1.2 \leq x_i \leq 1.2$. A global minimum is located at $f(\mathbf{x}^*) = f(1, 1)$, $f(\mathbf{x}^*) = 0$.

71 *Matyas Function* (Hedar, n.d.) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{71}(\mathbf{x}) = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

72 *McCormick Function* (Lootsma, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{72}(\mathbf{x}) = \sin(x_1 + x_2) + (x_1 - x_2)^2 - (3/2)x_1 + (5/2)x_2 + 1$$

subject to $-1.5 \leq x_1 \leq 4$ and $-3 \leq x_2 \leq 3$. The global minimum is located at $\mathbf{x}^* = f(-0.547, -1.547)$, $f(\mathbf{x}^*) \approx -1.9133$.

73 *Miele Cantrell Function* (Cragg and Levy, 1969) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$\begin{aligned} f_{73}(\mathbf{x}) = {} & \left(e^{-x_1} - x_2\right)^4 + 100(x_2 - x_3)^6 \\ & + (\tan(x_3 - x_4))^4 + x_1^8 \end{aligned}$$

subject to $-1 \leq x_i \leq 1$. The global minimum is located at $\mathbf{x}^* = f(0, 1, 1, 1)$, $f(\mathbf{x}^*) = 0$.

74 *Mishra Function 1* (Mishra, 2006a) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{74}(\mathbf{x}) = \left(1 + D - \sum_{i=1}^{N-1} x_i\right)^{N - \sum_{i=1}^{N-1} x_i}$$

subject to $0 \leq x_i \leq 1$. The global minimum is $f(\mathbf{x}^*) = 2$.

75 *Mishra Function 2* (Mishra, 2006a) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{75}(\mathbf{x}) = \left(1 + D - \sum_{i=1}^{N-1} 0.5(x_i + x_{i+1})\right)^{N - \sum_{i=1}^{N-1} 0.5(x_i + x_{i+1})}$$

subject to $0 \leq x_i \leq 1$. The global minimum is $f(\mathbf{x}^*) = 2$.

76 *Mishra Function 3* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{76}(\mathbf{x}) = \sqrt{\left|\cos\sqrt{\left|x_1^2 + x_2^2\right|}\right|} + 0.01(x_1 + x_2)$$

The global minimum is located at $\mathbf{x}^* = f(-8.466, -10)$, $f(\mathbf{x}^*) = -0.18467$.

77 *Mishra Function 4* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{77}(\mathbf{x}) = \sqrt{\left|\sin\sqrt{|x_1^2+x_2^2|}\right|} + 0.01(x_1+x_2)$$

The global minimum is located at $\mathbf{x}^* = f(-9.94112, -10)$, $f(\mathbf{x}^*) = -0.199409$.

78 *Mishra Function 5* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{78}(\mathbf{x}) = \Big[\sin^2(\cos((x_1)+\cos(x_2)))^2 + \cos^2(\sin(x_1)+\sin(x_2)) + x_1\Big]^2$$
$$+0.01(x_1+x_2)$$

The global minimum is located at $\mathbf{x}^* = f(-1.98682, -10)$, $f(\mathbf{x}^*) = -1.01983$.

79 *Mishra Function 6* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{79}(\mathbf{x}) = -\ln\Big[\sin^2(\cos((x_1)+\cos(x_2)))^2 - \cos^2(\sin(x_1)+\sin(x_2)) + x_1\Big]^2$$
$$+0.01((x_1-1)^2+(x_2-1)^2)$$

The global minimum is located at $\mathbf{x}^* = f(2.88631, 1.82326)$, $f(\mathbf{x}^*) = -2.28395$.

80 *Mishra Function 7* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{80}(\mathbf{x}) = \Big[\prod_{i=1}^{D} x_i - N!\Big]^2$$

The global minimum is $f(\mathbf{x}^*) = 0$.

81 *Mishra Function 8* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{81}(\mathbf{x}) = 0.001\Big[\Big|x_1^{10} - 20x_1^9 + 180x_1^8 - 960x_1^7 + 3360x_1^6 - 8064x_1^5$$
$$1334x_1^4 - 15360x_1^3 + 11520x_1^2 - 5120x_1 + 2624\Big|$$
$$\Big|x_2^4 + 12x_2^3 + 54x_2^2 + 108x_2 + 81\Big|\Big]^2$$

The global minimum is located at $\mathbf{x}^* = f(2, -3)$, $f(\mathbf{x}^*) = 0$.

82 *Mishra Function 9* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{82}(\mathbf{x}) = \Big[ab^2c + abc^2 + b^2 + (x_1+x_2-x_3)^2\Big]^2$$

where $a = 2x_1^3 + 5x_1x_2 + 4x_3 - 2x_1^2x_3 - 18$, $b = x_1 + x_2^3 + x_1x_3^2 - 22$ $c = 8x_1^2 + 2x_2x_3 + 2x_2^2 + 3x_2^3 - 52$. The global minimum is located at $\mathbf{x}^* = f(1, 2, 3)$, $f(\mathbf{x}^*) = 0$.

83 *Mishra Function 10* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{83}(\mathbf{x}) = \Big[\lfloor x_1 \perp x_2 \rfloor - \lfloor x_1 \rfloor - \lfloor x_2 \rfloor\Big]^2$$

The global minimum is located at $\mathbf{x}^* = f\{(0,0),(2,2)\}$, $f(\mathbf{x}^*) = 0$.

84 *Mishra Function 11* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{84}(\mathbf{x}) = \Big[\frac{1}{D}\sum_{i=1}^{D}|x_i| - \big(\prod_{i=1}^{D}|x_i|\big)^{\frac{1}{N}}\Big]^2$$

The global minimum is $f(\mathbf{x}^*) = 0$.

85 *Parsopoulos Function* (continuous, differentiable, separable, scalable, multimodal)

$$f_{85}(\mathbf{x}) = \cos(x_1)^2 + \sin(x_2)^2$$

subject to $-5 \leq x_i \leq 5$, where $(x_1, x_2) \in \mathbb{R}^2$. This function has infinite number of global minima in $\mathbb{R}^2$, at points $(\kappa\frac{\pi}{2}, \lambda\pi)$, where $\kappa = \pm1, \pm3, ...$ and $\lambda = 0, \pm1, \pm2, ...$ In the given domain problem, function has 12 global minima all equal to zero.

86 *Pen Holder Function* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{86}(\mathbf{x}) = -\exp[|\cos(x_1)\cos(x_2)e^{|1-[(x_1^2+x_2^2)]^{0.5}/\pi|}|^{-1}]$$

subject to $-11 \leq x_i \leq 11$. The four global minima are located at $\mathbf{x}^* = f(\pm9.646168, \pm9.646168)$, $f(\mathbf{x}^*) = -0.96354$.

87 *Pathological Function* (Rahnamyan et al., 2007a) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{87}(\mathbf{x}) = \sum_{i=1}^{D-1}\left(0.5 + \frac{\sin^2\sqrt{100x_i^2 + x_{i+1}^2} - 0.5}{1 + 0.001(x_i^2 - 2x_i x_{i+1} + x_{i+1}^2)^2}\right)$$

subject to $-100 \leq x_i \leq 100$. The global minima is located $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

88 *Paviani Function* (Himmelblau, 1972) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{88}(\mathbf{x}) = \sum_{i=1}^{10}\Big[(\ln(x_i - 2))^2 + (\ln(10 - x_i))^2\Big] - \left(\prod_{i=1}^{10} x_i\right)^{0.2}$$

subject to $2.0001 \leq x_i \leq 10$, $i \in 1, 2, ..., 10$. The global minimum is located at $\mathbf{x}^* \approx f(9.351, ..., 9.351)$, $f(\mathbf{x}^*) \approx -45.778$.

89 *Pintér Function* (Pintér, 1996) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{89}(\mathbf{x}) = \sum_{i=1}^{D} ix_i^2 + \sum_{i=1}^{D} 20i\sin^2 A + \sum_{i=1}^{D} i\log_{10}\left(1 + iB^2\right)$$

where

$$A = (x_{i-1}\sin x_i + \sin x_{i+1})$$
$$B = \left(x_{i-1}^2 - 2x_i + 3x_{i+1} - \cos x_i + 1\right)$$

where $x_0 = x_D$ and $x_{D+1} = x_1$, subject to $-10 \leq x_i \leq 10$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

90 *Periodic Function* (Ali et al., 2005) (Separable)

$$f_{90}(\mathbf{x}) = 1 + \sin^2(x_1) + \sin^2(x_2) - 0.1e^{-(x_1^2+x_2^2)}$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0.9$.

91 *Powell Singular Function* (Powell, 1962) (continuous, differentiable, non-separable Scalable, unimodal)

$$\begin{aligned} f_{91}(\mathbf{x}) = \sum_{i=1}^{D/4} & (x_{4i-3} + 10x_{4i-2})^2 \\ & +5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - x_{4i-1})^4 \\ & +10(x_{4i-3} - x_{4i})^4 \end{aligned}$$

subject to $-4 \leq x_i \leq 5$. The global minima is located at $\mathbf{x}^* = f(3, -1, 0, 1, \cdots, 3, -1, 0, 1)$, $f(\mathbf{x}^*) = 0$.

92 *Powell Singular Function 2* (Fu et al., 2006) (continuous, differentiable, non-separable Scalable, unimodal)

$$\begin{aligned} f_{92}(\mathbf{x}) = \sum_{i=1}^{D-2} & (x_{i-1} + 10x_i)^2 \\ & +5(x_{i+1} - x_{i+2})^2 + (x_i - 2x_{i+1})^4 \\ & +10(x_{i-1} - x_{i+2})^4 \end{aligned}$$

subject to $-4 \leq x_i \leq 5$. The global minimum is $f(\mathbf{x}^*) = 0$.

93 *Powell Sum Function* (Rahnamyan et al., 2007a) (continuous, differentiable, Separable Scalable, unimodal)

$$f_{93}(\mathbf{x}) = \sum_{i=1}^{D} \left|x_i\right|^{i+1}$$

subject to $-1 \leq x_i \leq 1$. The global minimum is $f(\mathbf{x}^*) = 0$.

94 *Price Function 1* (Price, 1977) (continuous, non-differentiable, Separable Non-Scalable, multimodal)

$$f_{94}(\mathbf{x}) = (|x_1| - 5)^2 + (|x_2| - 5)^2$$

subject to $-500 \leq x_i \leq 500$. The global minimum are located at $\mathbf{x}^* = f(\{-5, -5\}, \{-5, 5\}, \{5, -5\}, \{5, 5\})$, $f(\mathbf{x}^*) = 0$.

95 *Price Function 2* (Price, 1977) (continuous, differentiable, non-separable Non-Scalable, multimodal)

$$f_{95}(\mathbf{x}) = 1 + \sin^2 x_1 + \sin^2 x_2 - 0.1e^{-x_1^2 - x_2^2}$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(0 \cdots 0)$, $f(\mathbf{x}^*) = 0.9$.

96 *Price Function 3* (Price, 1977) (continuous, differentiable, non-separable Non-Scalable, multimodal)

$$f_{96}(\mathbf{x}) = 100(x_2 - x_1^2)^2 + 6\Big[6.4(x_2 - 0.5)^2 - x_1 - 0.6\Big]^2$$

subject to $-500 \leq x_i \leq 500$. The global minimum are located at $\mathbf{x}^* = f(\{-5, -5\}, \{-5, 5\}, \{5, -5\}, \{5, 5\})$, $f(\mathbf{x}^*) = 0$.

97 *Price Function 4* (Price, 1977) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{97}(\mathbf{x}) = (2x_1^3 x_2 - x_2^3)^2 + (6x_1 - x_2^2 + x_2)^2$$

subject to $-500 \leq x_i \leq 500$. The three global minima are located at $\mathbf{x}^* = f(\{0, 0\}, \{2, 4\}, \{1.464, -2.506\})$, $f(\mathbf{x}^*) = 0$.

98 *Qing Function* (Qing, 2006) (continuous, differentiable, Separable Scalable, multimodal)

$$f_{98}(\mathbf{x}) = \sum_{i=1}^{D} (x_i^2 - i)^2$$

subject to $-500 \leq x_i \leq 500$. The global minima are located at $\mathbf{x}^* = f(\pm\sqrt{i})$, $f(\mathbf{x}^*) = 0$.

99 *Quadratic Function* (continuous, differentiable, non-separable, non-scalable)

$$\begin{aligned} f_{99}(\mathbf{x}) = &-3803.84 - 138.08x_1 - 232.92x_2 \\ &+128.08x_1^2 + 203.64x_2^2 + 182.25x_1x_2 \end{aligned}$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(0.19388, 0.48513)$, $f(\mathbf{x}^*) = -3873.7243$.

100 *Quartic Function* (Storn and Price, 1996) (continuous, differentiable, separable, scalable)

$$f_{100}(\mathbf{x}) = \sum_{i=1}^{D} i x_i^4 + \text{random}[0, 1)$$

subject to $-1.28 \leq x_i \leq 1.28$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

101 *Quintic Function* (Mishra, 2006f) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{101}(\mathbf{x}) = \sum_{i=1}^{D} |x_i^5 - 3x_i^4 + 4x_i^3 + 2x_i^2 - 10x_i - 4|$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(-1 \text{ or } 2)$, $f(\mathbf{x}^*) = 0$.

102 *Rana Function* (Price et al., 2005) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{102}(\mathbf{x}) = \sum_{i=0}^{D-2} (x_{i+1} + 1)\cos(t_2)\sin(t_1) + x_i * \cos(t_1)\sin(t_2)$$

subject to $-500 \leq x_i \leq 500$, where $t_1 = \sqrt{\|x_{i+1} + x_i + 1\|}$ and $t_2 = \sqrt{\|x_{i+1} - x_i + 1\|}$.

103 *Ripple Function 1* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{103}(\mathbf{x}) = \sum_{i=1}^{2} -e^{-2 \ln 2 (\frac{x_i - 0.1}{0.8})^2} (\sin^6(5\pi x_i) + 0.1\cos^2(500\pi x_i))$$

subject to $0 \leq x_i \leq 1$. It has one global minimum and 252004 local minima. The global form of the function consists of 25 holes, which forms a 5×5 regular grid. Additionally, the whole function landscape is full of small ripples caused by high frequency cosine function which creates a large number of local minima.

104 *Ripple Function 25* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{104}(\mathbf{x}) = \sum_{i=1}^{2} -e^{-2 \ln 2 (\frac{x_i - 0.1}{0.8})^2} (\sin^6(5\pi x_i))$$

subject to $0 \leq x_i \leq 1$. It has one global form of the Ripple function 1 without any ripples due to absence of cosine term.

105 *Rosenbrock Function 1* (Rosenbrock, 1960) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{105}(\mathbf{x}) = \sum_{i=1}^{D-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$$

subject to $-30 \leq x_i \leq 30$. The global minima is located at $\mathbf{x}^* = f(1, \cdots, 1)$, $f(\mathbf{x}^*) = 0$.

106 *Rosenbrock Modified Function* (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{106}(\mathbf{x}) = 74 + 100(x_2 - x_1^2)^2 + (1 - x)^2 \\ -400e^{-\frac{(x_1+1)^2+(x_2+1)^2}{0.1}}$$

subject to $-2 \leq x_i \leq 2$. In this function, a Gaussian bump at $(-1, 1)$ is added, which causes a local minimum at $(1, 1)$ and global minimum is located at $\mathbf{x}^* = f(-1, -1)$, $f(\mathbf{x}^*) = 0$. This modification makes it a difficult to optimise because local minimum basin is larger than the global minimum basin.

107 *Rotated Ellipse Function* (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{107}(\mathbf{x}) = 7x_1^2 - 6\sqrt{3}x_1x_2 + 13x_2^2$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

108 *Rotated Ellipse Function 2* (Price et al., 2005) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{108}(\mathbf{x}) = x_1^2 - x_1x_2 + x_2^2$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.
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109 *Rump Function* (Moore, 1988) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{109}(\mathbf{x}) = (333.75 - x_1^2)x_2^6 + x_1^2(11x_1^2x_2^2 - 121x_2^4 - 2) + 5.5x_2^8 + \frac{x_1}{2x_2}$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

110 *Salomon Function* (Salomon, 1996) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{110}(\mathbf{x}) = = 1 - \cos\left(2\pi\sqrt{\sum_{i=1}^{D} x_i^2}\right) + 0.1\sqrt{\sum_{i=1}^{D} x_i^2}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0,0)$, $f(\mathbf{x}^*) = 0$.

111 *Sargan Function* (Dixon and Szegó, 1978) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{111}(\mathbf{x}) = = \sum_{i=1} D\Big(x_i^2 + 0.4\sum_{j\neq 1} x_i x_j\Big)$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0,\cdots,0)$, $f(\mathbf{x}^*) = 0$.

112 *Scahffer Function 1* (Mishra, 2006g) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{112}(\mathbf{x}) = 0.5 + \frac{\sin^2(x_1^2 + x_2^2)^2 - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0,0)$, $f(\mathbf{x}^*) = 0$.

113 *Scahffer Function 2* (Mishra, 2006g) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{113}(\mathbf{x}) = 0.5 + \frac{\sin^2(x_1^2 - x_2^2)^2 - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0,0)$, $f(\mathbf{x}^*) = 0$.

114 *Scahffer Function 3* (Mishra, 2006g) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{114}(\mathbf{x}) = 0.5 + \frac{\sin^2\Big(\cos\Big|x_1^2 - x_2^2\Big|\Big) - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, 1.253115)$, $f(\mathbf{x}^*) = 0.00156685$.

115 *Scahffer Function 4* (Mishra, 2006g) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{115}(\mathbf{x}) = 0.5 + \frac{\cos^2\Big(\sin(x_1^2 - x_2^2)\Big) - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, 1.253115)$, $f(\mathbf{x}^*) = 0.292579$.

116 *Schmidt Vetters Function* (Lootsma, 1972) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{116}(\mathbf{x}) = \frac{1}{1+(x_1-x_2)^2} + \sin\left(\frac{\pi x_2 + x_3}{2}\right) + e^{\left(\frac{x_1+x_2}{x_2}-2\right)^2}$$

The global minimum is located at $\mathbf{x}^* = f(0.78547, 0.78547, 0.78547)$, $f(\mathbf{x}^*) = 3$.

117 *Schumer Steiglitz Function* (Schumer and Steiglitz, 1968) (continuous, differentiable, separable, scalable, unimodal)

$$f_{117}(\mathbf{x}) = \sum_{i=1}^{D} x_i^4$$

The global minimum is located at $\mathbf{x}^* = f(0, \ldots, 0)$, $f(\mathbf{x}^*) = 0$.

118 *Schwefel Function* (Schwefel, 1981) (continuous, differentiable, partially-separable, scalable, unimodal)

$$f_{118}(\mathbf{x}) = \left(\sum_{i=1}^{D} x_i^2\right)^{\alpha}$$

where $\alpha \geq 0$, subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

119 *Schwefel Function 1.2* (Schwefel, 1981) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{119}(\mathbf{x}) = \sum_{i=1}^{D}\left(\sum_{j=1}^{i} x_j\right)^2$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

120 *Schwefel Function 2.4* (Schwefel, 1981) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{120}(\mathbf{x}) = \sum_{i=1}^{D}(x_i - 1)^2 + (x_1 - x_i^2)^2$$

subject to $0 \leq x_i \leq 10$. The global minima is located at $\mathbf{x}^* = f(1, \cdots, 1)$, $f(\mathbf{x}^*) = 0$.

121 *Schwefel Function 2.6* (Schwefel, 1981) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{121}(\mathbf{x}) = \max(|x_1 + 2x_2 - 7|, |2x_1 + x_2 - 5|)$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(1, 3)$, $f(\mathbf{x}^*) = 0$.

122 *Schwefel Function 2.20* (Schwefel, 1981) (continuous, non-differentiable, separable, scalable, unimodal)

$$f_{122}(\mathbf{x}) = -\sum_{i=1}^{n} |x_i|$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

123 *Schwefel Function 2.21* (Schwefel, 1981) (continuous, non-differentiable, separable, scalable, unimodal)

$$f_{123}(\mathbf{x}) = \max_{1 \leq i \leq D} |x_i|$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

124 *Schwefel Function 2.22* (Schwefel, 1981) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{124}(\mathbf{x}) = \sum_{i=1}^{D} |x_i| + \prod_{i=1}^{n} |x_i|$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

125 *Schwefel Function 2.23* (Schwefel, 1981) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{125}(\mathbf{x}) = \sum_{i=1}^{D} x_i^{10}$$

subject to $-10 \leq x_i \leq 10$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

126 *Schwefel Function 2.25* (Schwefel, 1981) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{126}(\mathbf{x}) = \sum_{i=2}^{D} (x_i - 1)^2 + (x_1 - x_i^2)^2$$

subject to $0 \leq x_i \leq 10$. The global minima is located at $\mathbf{x}^* = f(1, \cdots, 1)$, $f(\mathbf{x}^*) = 0$.

127 *Schwefel Function 2.26* (Schwefel, 1981) (continuous, differentiable, separable, scalable, multimodal)

$$f_{127}(\mathbf{x}) = -\frac{1}{D} \sum_{i=1}^{D} x_i \sin \sqrt{|x_i|}$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = \pm[\pi(0.5+k)]^2$, $f(\mathbf{x}^*) = -418.983$.

128 *Schwefel Function 2.36* (Schwefel, 1981) (continuous, differentiable, separable, scalable, multimodal)

$$f_{128}(\mathbf{x}) = -x_1 x_2(72 - 2x_1 - 2x_2)$$

subject to $0 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = f(12, \cdots, 12)$, $f(\mathbf{x}^*) = -3456$.

129 *Shekel Function 5* (Opačić, 1973) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{129}(\mathbf{x}) = -\sum_{i=1}^{5} \frac{1}{\sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i}$$

where $\mathbf{A} = [A_{ij}] = \begin{bmatrix} 4\,4\,4\,4 \\ 1\,1\,1\,1 \\ 8\,8\,8\,8 \\ 6\,6\,6\,6 \\ 3\,7\,3\,7 \end{bmatrix}$, $\underline{\mathbf{c}} = c_i = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$

subject to $0 \leq x_j \leq 10$. The global minima is located at $\mathbf{x}^* = f(4, 4, 4, 4)$, $f(\mathbf{x}^*) \approx -10.1499$.

130 *Shekel Function 7* (Opačić, 1973) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{130}(\mathbf{x}) = -\sum_{i=1}^{7} \frac{1}{\sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i}$$

where $\mathbf{A} = [A_{ij}] = \begin{bmatrix} 4\,4\,4\,4 \\ 1\,1\,1\,1 \\ 8\,8\,8\,8 \\ 6\,6\,6\,6 \\ 3\,7\,3\,7 \\ 2\,9\,2\,9 \\ 5\,5\,3\,3 \end{bmatrix}$, $\underline{\mathbf{c}} = c_i = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \end{bmatrix}$

subject to $0 \leq x_j \leq 10$. The global minima is located at $\mathbf{x}^* = f(4, 4, 4, 4)$, $f(\mathbf{x}^*) \approx -10.3999$.

131 *Shekel Function 10* (Opačić, 1973) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{131}(\mathbf{x}) = -\sum_{i=1}^{10} \frac{1}{\sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i}$$

where $\mathbf{A} = [A_{ij}] = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 5 & 3 & 3 \\ 8 & 1 & 8 & 1 \\ 6 & 2 & 6 & 2 \\ 7 & 3.6 & 7 & 3.6 \end{bmatrix}$, $\underline{\mathbf{c}} = c_i = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \\ 0.5 \end{bmatrix}$

subject to $0 \leq x_j \leq 10$. The global minima is located at $\mathbf{x}^* = f(4, 4, 4, 4)$, $f(\mathbf{x}^*) \approx -10.5319$.

132 *Shubert Function* (Hennart, 1982) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{132}(\mathbf{x}) = \prod_{i=1}^{n} \left(\sum_{j=1}^{5} \cos((j+1)x_i + j) \right)$$

subject to $-10 \leq x_i \leq 10$, $i \in 1, 2, \cdots, n$. The 18 global minima are located at

$$\begin{aligned} \mathbf{x}^* = f(&\{-7.0835, \ 4.8580\}, \ \{-7.0835, -7.7083\}, \\ &\{-1.4251, -7.0835\}, \ \{\ 5.4828, \ 4.8580\}, \\ &\{-1.4251, -0.8003\}, \ \{\ 4.8580, \ 5.4828\}, \\ &\{-7.7083, -7.0835\}, \ \{-7.0835, -1.4251\}, \\ &\{-7.7083, -0.8003\}, \ \{-7.7083, \ 5.4828\}, \\ &\{-0.8003, -7.7083\}, \ \{-0.8003, -1.4251\}, \\ &\{-0.8003, \ 4.8580\}, \ \{-1.4251, \ 5.4828\}, \\ &\{\ 5.4828, -7.7083\}, \ \{\ 4.8580, -7.0835\}, \\ &\{\ 5.4828, -1.4251\}, \ \{\ 4.8580, -0.8003\}), \end{aligned}$$

$f(\mathbf{x}^*) \simeq -186.7309$.

133 *Shubert Function 3* (Adorio and Dilman, 2005) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{133}(\mathbf{x}) = \left(\sum_{i=1}^{D} \sum_{j=1}^{5} j\sin((j+1)x_i + j) \right)$$

subject to $-10 \leq x_i \leq 10$. The global minimum is $f(\mathbf{x}^*) \simeq -29.6733337$ with multiple solutions.

134 *Shubert Function 4* (Adorio and Dilman, 2005) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{134}(\mathbf{x}) = \left(\sum_{i=1}^{D} \sum_{j=1}^{5} j\cos((j+1)x_i + j) \right)$$

subject to $-10 \leq x_i \leq 10$. The global minimum is $f(\mathbf{x}^*) \simeq -25.740858$ with multiple solutions.

135 *Schaffer Function F6* (Schaffer et al., 1989) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{135}(\mathbf{x}) = \sum_{i=1}^{D} 0.5 + \frac{\sin^2 \sqrt{x_i^2 + x_{i+1}^2} - 0.5}{\left[1 + 0.001(x_i^2 + x_{i+1}^2)\right]^2}$$

subject to $-100 \leq x_i \leq 100$. The global minimum is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

136 *Sphere Function* (Schumer and Steiglitz, 1968) (continuous, differentiable, separable, scalable, multimodal)

$$f_{136}(\mathbf{x}) = \sum_{i=1}^{D} x_i^2$$

subject to $0 \leq x_i \leq 10$. The global minima is located $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

137 *Step Function* (discontinuous, non-differentiable, separable, scalable, unimodal)

$$f_{137}(\mathbf{x}) = \sum_{i=1}^{D} (\lfloor |x_i| \rfloor)$$

subject to $-100 \leq x_i \leq 100$. The global minima is located $\mathbf{x}^* = f(0, \cdots, 0) = 0$, $f(\mathbf{x}^*) = 0$.

138 *Step Function 2* (Báck and Schwefel, 1993) (discontinuous, non-differentiable, separable, scalable, unimodal)

$$f_{138}(\mathbf{x}) = \sum_{i=1}^{D} (\lfloor x_i + 0.5 \rfloor)^2$$

subject to $-100 \leq x_i \leq 100$. The global minima is located $\mathbf{x}^* = f(0.5, \cdots, 0.5) = 0$, $f(\mathbf{x}^*) = 0$.

139 *Step Function 3* (discontinuous, non-differentiable, separable, scalable, unimodal)

$$f_{139}(\mathbf{x}) = \sum_{i=1}^{D} \left(\lfloor x_i^2 \rfloor\right)$$

subject to $-100 \leq x_i \leq 100$. The global minima is located $\mathbf{x}^* = f(0, \cdots, 0) = 0$, $f(\mathbf{x}^*) = 0$.

140 *Stepint Function* (discontinuous, non-differentiable, separable, scalable, unimodal)

$$f_{140}(\mathbf{x}) = 25 + \sum_{i=1}^{D} (\lfloor x_i \rfloor)$$

subject to $-5.12 \leq x_i \leq 5.12$. The global minima is located $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

141 *Streched V Sine Wave Function* (Schaffer et al., 1989) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{141}(\mathbf{x}) = \sum_{i=1}^{D-1} (x_{i+1}^2 + x_i^2)^{0.25} \left[\sin^2\{50(x_{i+1}^2 + x_i^2)^{0.1}\} + 0.1 \right]$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = 0$.

142 *Sum Squares Function* (Hedar, n.d.) (continuous, differentiable, separable, scalable, unimodal)

$$f_{142}(\mathbf{x}) = \sum_{i=1}^{D} i x_i^2$$

subject to $-10 \leq x_i \leq 10$. The global minima is located $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

143 *Styblinski-Tang Function* (Silagadze, 2007) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{143}(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^{n} (x_i^4 - 16x_i^2 + 5x_i)$$

subject to $-5 \leq x_i \leq 5$. The global minimum is located $\mathbf{x}^* = f(-2.903534, -2.903534)$, $f(\mathbf{x}^*) = -78.332$.

144 *Holder Table Function 1* (Mishra, 2006f) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{144}(\mathbf{x}) = -|\cos(x_1)\cos(x_2)e^{|1-(x_1+x_2)^{0.5}/\pi|}|$$

subject to $-10 \leq x_i \leq 10$.
The four global minima are located at $\mathbf{x}^* = f(\pm 9.646168, \pm 9.646168)$, $f(\mathbf{x}^*) = -26.920336$.

145 *Holder Table Function 2* (Mishra, 2006f) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{145}(\mathbf{x}) = -|\sin(x_1)\cos(x_2)e^{|1-(x_1+x_2)^{0.5}/\pi|}|$$

subject to $-10 \leq x_i \leq 10$.

The four global minima are located at $\mathbf{x}^* = f(\pm 8.055023472141116, \pm 9.664590028909654)$, $f(\mathbf{x}^*) = -19.20850$.

146 *Carrom Table Function* (Mishra, 2006f) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{146}(\mathbf{x}) = -[(\cos(x_1)\cos(x_2) \\ \exp|1 - [(x_1^2 + x_2^2)^{0.5}]/\pi|)^2]/30$$

subject to $-10 \leq x_i \leq 10$.

The four global minima are located at $\mathbf{x}^* = f(\pm 9.646157266348881, \pm 9.646134286497169)$, $f(\mathbf{x}^*) = -24.1568155$.

147 *Testtube Holder Function* (Mishra, 2006f) (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{147}(\mathbf{x}) = -4\big[(\sin(x_1)\cos(x_2) \\ e^{|\cos[(x_1^2+x_2^2)/200]|})\big]$$

subject to $-10 \leq x_i \leq 10$. The two global minima are located at $\mathbf{x}^* = f(\pm\pi/2, 0)$, $f(\mathbf{x}^*) = -10.872300$.

148 *Trecanni Function* (Dixon and Szegó, 1978) (continuous, differentiable, separable, non-scalable, unimodal)

$$f_{148}(\mathbf{x}) = x_1^4 - 4x_1^3 + 4x_1 + x_2^2$$

subject to $-5 \leq x_i \leq 5$. The two global minima are located at $\mathbf{x}^* = f(\{0, 0\}, \{-2, 0\})$, $f(\mathbf{x}^*) = 0$.

149 *Trid Function 6* (Hedar, n.d.) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{149}(\mathbf{x}) = \sum_{i=1}^{D} (x_i - 1)^2 - \sum_{i=1}^{D} x_i x_{i-1}$$

subject to $-6^2 \leq x_i \leq 6^2$. The global minima is located at $f(\mathbf{x}^*) = -50$.

150 *Trid Function 10* (Hedar, n.d.) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{150}(\mathbf{x}) = \sum_{i=1}^{D} (x_i - 1)^2 - \sum_{i=1}^{D} x_i x_{i-1}$$

subject to $-100 \leq x_i \leq 100$. The global minima is located at $f(\mathbf{x}^*) = -200$.

151 *Trefethen Function* (Adorio and Dilman, 2005) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$
\begin{aligned}
f_{151}(\mathbf{x}) = & \, e^{\sin(50x_1)} + \sin(60e^{x_2}) \\
& + \sin(70\sin(x_1)) + \sin(\sin(80x_2)) \\
& - \sin(10(x_1 + x_2)) + \frac{1}{4}(x_1^2 + x_2^2)
\end{aligned}
$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = f(-0.024403, 0.210612)$, $f(\mathbf{x}^*) = -3.30686865$.

152 *Trigonometric Function 1* (Dixon and Szegó, 1978) (continuous, differentiable, non-separable, scalable, multimodal)

$$
\begin{aligned}
f_{152}(\mathbf{x}) = & \sum_{i=1}^{D} [D - \sum_{j=1}^{D} \cos x_j \\
& + i(1 - \cos(x_i) - \sin(x_i))]^2
\end{aligned}
$$

subject to $0 \leq x_i \leq pi$. The global minimum is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$

153 *Trigonometric Function 2* (Fu et al., 2006) (continuous, differentiable, non-separable, scalable, multimodal)

$$
f_{153}(\mathbf{x}) = 1 + \sum_{i=1}^{D} 8 \sin^2 \left[7(x_i - 0.9)^2\right] + 6 \sin^2 \left[14(x_1 - 0.9)^2\right] + (x_i - 0.9)^2
$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = f(0.9, \cdots, 0.9)$, $f(\mathbf{x}^*) = 1$

154 *Tripod Function* (Rahnamyan et al., 2007a) (discontinuous, non-differentiable, non-separable, non-scalable, multimodal)

$$
\begin{aligned}
f_{154}(\mathbf{x}) = & \, p(x_2)(1 + p(x_1)) \\
& + |x_1 + 50p(x_2)(1 - 2p(x_1))| \\
& + |x_2 + 50(1 - 2p(x_2))|
\end{aligned}
$$

subject to $-100 \leq x_i \leq 100$, where $p(x) = 1$ for $x \geq 0$. The global minimum is located at $\mathbf{x}^* = f(0, -50)$, $f(\mathbf{x}^*) = 0$.

155 *Ursem Function 1* (Rónkkónen, 2009) (continuous, differentiable, separable, non-scalable, multimodal)

$$
f_{155}(\mathbf{x}) = -\sin(2x_1 - 0.5\pi) - 3\cos(x_2) - 0.5x_1
$$

subject to $-2.5 \leq x_1 \leq 3$ and $-2 \leq x_2 \leq 2$, and has single global and local minima.

156 *Ursem Function 3* (Rónkkónen, 2009) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{156}(\mathbf{x}) = -\text{sin}(2.2\pi x_1 + 0.5\pi).\frac{2-|x_2|}{2}.\frac{3-|x_1|}{2}$$
$$-\text{sin}(0.5\pi x_2^2 + 0.5\pi).\frac{2-|x_2|}{2}.\frac{2-|x_1|}{2}$$

subject to $-2 \leq x_1 \leq 2$ and $-1.5 \leq x_2 \leq 1.5$, and has single global minimum and four regularly spaced local minima positioned in a direct line, such that global minimum is in the middle.

157 *Ursem Function 4* (Rónkkónen, 2009) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{157}(\mathbf{x}) = -3\text{sin}(0.5\pi x_1 + 0.5\pi).\frac{2-\sqrt{x_1^2+x_2^2}}{4}$$

subject to $-2 \leq x_i \leq 2$, and has single global minimum positioned at the middle and four local minima at the corners of the search space.

158 *Ursem Waves Function* (Rónkkónen, 2009) (continuous, differentiable, non-separable, non-scalable, multimodal)

$$f_{158}(\mathbf{x}) = -0.9x_1^2 + (x_2^2 - 4.5x_2^2)x_1x_2$$
$$+4.7\text{cos}(3x_1 - x_2^2(2+x_1))\text{sin}(2.5\pi x_1)$$

subject to $-0.9 \leq x_1 \leq 1.2$ and $-1.2 \leq x_2 \leq 1.2$, and has single global minimum and nine irregularly spaced local minima in the search space.

159 *Venter Sobiezcczanski-Sobieski Function* (Begambre and Laier, 2009) (continuous, differentiable, separable, non-scalable)

$$f_{159}(\mathbf{x}) = x_1^2 - 100\text{cos}(x_1)^2$$
$$-100\text{cos}(x_1^2/30) + x_2^2$$
$$-100\text{cos}(x_2)^2 - 100\text{cos}(x_2^2/30)$$

subject to $-50 \leq x_i \leq 50$. The global minimum is located at $\mathbf{x}^* = f(0, 0)$, $f(\mathbf{x}^*) = -400$.

160 *Watson Function* (Schwefel, 1981) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{160}(\mathbf{x}) = \sum_{i=0}^{29}\left[\sum_{j=0}^{4}((j-1)a_i^j x_{j+1}) - \left[\sum_{j=0}^{5} a_i^j x_{j+1}\right]^2 - 1\right]^2 + x_1^2$$

subject to $|x_i| \leq 10$, where the coefficient $a_i = i/29.0$. The global minimum is located at $\mathbf{x}^* = f(-0.0158, 1.012, -0.2329, 1.260, -1.513, 0.9928)$, $f(\mathbf{x}^*) = 0.002288$.

161 *Wayburn Seader Function 1* (Wayburn and Seader, 1987) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{161}(\mathbf{x}) = (x_1^6 + x_2^4 - 17)^2 + (2x_1 + x_2 - 4)^2$$

The global minimum is located at $\mathbf{x}^* = f\{(1,2),(1.597,0.806)\}$, $f(\mathbf{x}^*) = 0$.

162 *Wayburn Seader Function 2* (Wayburn and Seader, 1987) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{162}(\mathbf{x}) = \left[1.613 - 4(x_1 - 0.3125)^2 - 4(x_2 - 1.625)^2\right]^2 + (x_2 - 1)^2$$

subject to $-500 \leq 500$. The global minimum is located at $\mathbf{x}^* = f\{(0.2,1),(0.425,1)\}$, $f(\mathbf{x}^*) = 0$.

163 *Wayburn Seader Function 3* (Wayburn and Seader, 1987) (continuous, differentiable, non-separable, scalable, unimodal)

$$f_{163}(\mathbf{x}) = 2\frac{x_1^3}{3} - 8x_1^2 + 33x_1 - x_1x_2 + 5 + \left[(x_1 - 4)^2 + (x_2 - 5)^2 - 4\right]^2$$

subject to $-500 \leq 500$. The global minimum is located at $\mathbf{x}^* = f(5.611, 6.187)$, $f(\mathbf{x}^*) = 21.35$.

164 *W / Wavy Function* (Courrieu, 1997) (continuous, differentiable, separable, scalable, multimodal)

$$f_{164}(\mathbf{x}) = 1 - \frac{1}{D}\sum_{i=1}^{D} \cos(kx_i)e^{\frac{-x_i^2}{2}}$$

subject to $-\pi \leq x_i \leq \pi$. The global minimum is located at $\mathbf{x}^* = f(0,0)$, $f(\mathbf{x}^*) = 0$.
The number of local minima is kn and $(k+1)n$ for odd and even k respectively. For $D = 2$ and $k = 10$, there are 121 local minima.

165 *Weierstrass Function* (Suganthan et al., 2005)(continuous, differentiable, separable, scalable, multimodal)

$$f_{165}(\mathbf{x}) = \sum_{i=1}^{n}\left[\sum_{k=0}^{kmax} a^k \cos(2\pi b^k(x_i + 0.5)) - n\sum_{k=0}^{kmax} a^k \cos(\pi b^k)\right]$$

subject to $-0.5 \leq x_i \leq 0.5$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

166 *Whitley Function* (Whitley et al., 1996) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{166}(\mathbf{x}) = \sum_{i=1}^{D}\sum_{j=1}^{D}\Bigg[\frac{(100(x_i^2 - x_j)^2 + (1 - x_j)^2)^2}{4,000} \\ -\cos\left(100(x_i^2 - x_j)^2 + (1 - x_j)^2 + 1\right)\Bigg]$$

combines a very steep overall slope with a highly multimodal area around the global minimum located at $x_i = 1$, where $i = 1, ..., D$.

167 *Wolfe Function* (Schwefel, 1981) (continuous, differentiable, separable, scalable, multimodal)

$$f_{167}(\mathbf{x}) = \frac{4}{3}(x_1^2 + x_2^2 - x_1 x_2)^{0.75} + x_3$$

subject to $0 \le x_i \le 2$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

168 *Xin-She Yang Function 1* (discontinuous, differentiable, separable, scalable, multimodal)

This is a generic stochastic and non-smooth function proposed in Yang (2010a, 2010b).

$$f_{168}(\mathbf{x}) = \sum_{i=1}^{D} \epsilon_i |x_i|^i$$

subject to $-5 \le x_i \le 5$. The variable $\epsilon_i, (i = 1, 2, \cdots, D)$ is a random variable uniformly distributed in $[0, 1]$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

169 *Xin-She Yang Function 2* (Yang, 2010a,b) (discontinuous, non-differentiable, non-separable, scalable, multimodal)

$$f_{169}(\mathbf{x}) = \left(\sum_{i=1}^{D}|x_i|\right)\exp\left[-\sum_{i=1}^{D}\sin(x_i^2)\right]$$

subject to $-2\pi \le x_i \le 2\pi$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

170 *Xin-She Yang Function 3* (Yang, 2010a,b) (discontinuous, non-differentiable, non-separable, scalable, multimodal)

$$f_{170}(\mathbf{x}) = \left[e^{-\sum_{i=1}^{D}(x_i/\beta)^{2m}} - 2e^{-\sum_{i=1}^{D}(x_i)^2} . \prod_{i=1}^{D}\cos^2(x_i)\right]$$

subject to $-20 \le x_i \le 20$. The global minima for $m = 5$ and $\beta = 15$ is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = -1$.

171 *Xin-She Yang Function 4* (Yang, 2010a,b) (discontinuous, non-differentiable, non-separable, scalable, multimodal)

$$f_{171}(\mathbf{x}) = \left[e^{-\sum_{i=1}^{D}(x_i/\beta)^{2m}} - 2e^{-\sum_{i=1}^{D}(x_i-\pi)^2} . \prod_{i=1}^{D} \cos^2(x_i) \right]$$

subject to $-10 \leq x_i \leq 10$. The global minima for $m = 5$ and $\beta = 15$ is located at $\mathbf{x}^* = f(\pi, \cdots, \pi)$, $f(\mathbf{x}^*) = -1$.

172 *Zakharov Function* (Rahnamyan et al., 2007a) (continuous, differentiable, non-separable, scalable, multimodal)

$$f_{172}(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 + \left(\frac{1}{2} \sum_{i=1}^{n} i x_i \right)^2 + \left(\frac{1}{2} \sum_{i=1}^{n} i x_i \right)^4$$

subject to $-5 \leq x_i \leq 10$. The global minima is located at $\mathbf{x}^* = f(0, \cdots, 0)$, $f(\mathbf{x}^*) = 0$.

173 *Zettl Function* (Schwefel, 1995) (continuous, differentiable, non-separable, non-scalable, unimodal)

$$f_{173}(\mathbf{x}) = (x_1^2 + x_2^2 - 2x_1)^2 + 0.25x_1$$

subject to $-5 \leq x_i \leq 10$. The global minima is located at $\mathbf{x}^* = f(-0.0299, 0)$, $f(\mathbf{x}^*) = -0.003791$.

17 *Zirilli or Aluffi-Pentini's Function* (Ali et al., 2005) (continuous, differentiable, separable, non-scalable, unimodal)

$$f_{174}(\mathbf{x}) = 0.25x_1^4 - 0.5x_1^2 + 0.1x_1 + 0.5x_2^2$$

subject to $-10 \leq x_i \leq 10$. The global minimum is located at $\mathbf{x}^* = (-1.0465, 0)$, $f(\mathbf{x}^*) \approx -0.3523$.

175 *Zirilli Function 2* (continuous, differentiable, separable, non-scalable, multimodal)

$$f_{175}(\mathbf{x}) = 0.5x_1^2 + 0.5\big[1 - \cos(2x_1)\big] + x_2^2$$

subject to $-500 \leq x_i \leq 500$. The global minimum is located at $\mathbf{x}^* = (0, 0)$, $f(\mathbf{x}^*) = 0$.

4 Conclusions

Test functions are important to validate and compare optimisation algorithms, this is especially true for newly developed algorithms. Here, we attempted to provide the most comprehensive and concise list of known benchmarks or test functions. Any functions that is left out is just unintentional. The list is compiled on all the resources all the literature known to us by the time of writing. It can be expected that majority of these functions can be used for testing new optimisation algorithms so as to provide a more complete view on the performance of any algorithms of interest.

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