A Tutorial on fitting Cumulative Link Models with the ordinal Package

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Abstract

It is shown by example how a cumulative link mixed model is fitted with the `clm` function in package ordinal. Model interpretation and inference is briefly discussed.

We will consider the data on the bitterness of wine from Randall (1989) presented in Table 1 and available as the object `wine` in package ordinal. The data were also analyzed with mixed effects models by Tutz and Hennevogl (1996). The following gives an impression of the wine data object:

```
> data(wine)
> head(wine)

  response rating temp contact bottle judge
1       36      2 cold      no      1     1
2       48      3 cold      no      2     1
3       47      3 cold     yes      3     1
4       67      4 cold     yes      4     1
5       77      4 warm      no      5     1
6       60      4 warm      no      6     1

> str(wine)

'data.frame':        72 obs. of  6 variables:
 $ response: num  36 48 47 67 77 60 83 90 17 22 ...
 $ rating  : Ord.factor w/ 5 levels "1"<"2"<"3"<"4"<..: 2 3 3 4 4 4 5 5 1 2 ...
 $ temp    : Factor w/ 2 levels "cold","warm": 1 1 1 1 2 2 2 2 1 1 ...
 $ contact : Factor w/ 2 levels "no","yes": 1 1 2 2 1 1 2 2 1 1 ...
 $ bottle  : Factor w/ 8 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 1 2 ...
 $ judge   : Factor w/ 9 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 2 2 ...
```

The data represent a factorial experiment on factors determining the bitterness of wine with 1 = "least bitter" and 5 = "most bitter". Two treatment factors (temperature and contact) each have two levels. Temperature and contact between juice and skins can be controlled when crushing grapes during wine production. Nine judges each assessed wine from two bottles from each of the four treatment conditions, hence there are 72 observations in all. For more information see the manual entry for the wine data: `help(wine)`.

We will fit the following cumulative link mixed model to the wine data:

$$\begin{gathered}\operatorname{logit}(P(Y_i \leq j)) = \theta_j - \beta_1(\texttt{temp}_i) - \beta_2(\texttt{contact}_i) \\ i = 1, \ldots, n, \quad j = 1, \ldots, J-1\end{gathered} \tag{1}$$

Table 1: Ratings of the bitterness of some white wines. Data are adopted from Randall (1989).

			Judge								
Temperature	Contact	Bottle	1	2	3	4	5	6	7	8	9
cold	no	1	2	1	2	3	2	3	1	2	1
cold	no	2	3	2	3	2	3	2	1	2	2
cold	yes	3	3	1	3	3	4	3	2	2	3
cold	yes	4	4	3	2	2	3	2	2	3	2
warm	no	5	4	2	5	3	3	2	2	3	3
warm	no	6	4	3	5	2	3	4	3	3	2
warm	yes	7	5	5	4	5	3	5	2	3	4
warm	yes	8	5	4	4	3	3	4	3	4	4

This is a model for the cumulative probability of the ith rating falling in the jth category or below, where i index all observations and $j = 1, \ldots, J$ index the response categories ($J = 5$). $\{\theta_j\}$ are known as threshold parameters or cut-points.

We fit this cumulative link model by maximum likelihood with the `clm` function in package ordinal. Here we save the fitted `clm` model in the object `fm1` (short for `f`itted `m`odel `1`) and `print` the model by simply typing its name:

```
> fm1 <- clm(rating ~ temp + contact, data = wine)
> fm1

Call:
clm(location = rating ~ temp + contact, data = wine)

Location coefficients:
  tempwarm contactyes
  2.503102   1.527798

No Scale coefficients

Threshold coefficients:
      1|2       2|3       3|4       4|5
-1.344383  1.250809  3.466887  5.006405

log-likelihood: -86.49192
AIC: 184.9838
```

Additional information is provided with the `summary` method:

```
> summary(fm1)

Call:
clm(location = rating ~ temp + contact, data = wine)

Location coefficients:
           Estimate Std. Error z value Pr(>|z|)
tempwarm    2.5031   0.5287     4.7346 2.1946e-06
contactyes  1.5278   0.4766     3.2055 0.0013484

No scale coefficients
```

```
Threshold coefficients:
    Estimate Std. Error z value
1|2 -1.3444   0.5171    -2.5998
2|3  1.2508   0.4379     2.8565
3|4  3.4669   0.5978     5.7998
4|5  5.0064   0.7309     6.8496

log-likelihood: -86.49192
AIC: 184.9838
Condition number of Hessian: 26.59441
```

The primary result is the coefficient table with parameter estimates, standard errors and Wald (or normal) based p-values for tests of the parameters being zero. The maximum likelihood estimates of the parameters are:

$$\hat{\beta}_1 = 2.50, \quad \hat{\beta}_2 = 1.53, \quad \{\hat{\theta}_j\} = [-1.34,\ 1.25,\ 3.47,\ 5.01]. \tag{2}$$

The condition number of the Hessian measures the empirical identifiability of the model. High numbers, say larger than 10^4 or 10^6 indicate that the model is ill defined. This would indicate that the model can be simplified, that possibly some parameters are not identifiable, and that optimization of the model can be difficult. In this case the condition number of the Hessian does not indicate a problem with the model.

The coefficients for `temp` and `contact` are positive indicating that higher temperature and more contact increase the bitterness of wine, i.e., rating in higher categories is more likely. The odds ratio of the event $Y \geq j$ is $\exp(\beta_{\text{treatment}})$, thus the odds ratio of bitterness being rated in category j or above at warm relative to cold temperatures is

```
> exp(coef(fm1)[5])

tempwarm
12.22034
```

The p-values for the location coefficients provided by the `summary` method are based on the so-called Wald statistic. More accurate test are provided by likelihood ratio tests. These can be obtained with the `anova` method, for example, the likelihood ratio test of `contact` is

```
> fm2 <- clm(rating ~ temp, data = wine)
> anova(fm2, fm1)

Likelihood ratio tests of cumulative link models

Response: rating
                 Model Resid. df -2logLik   Test    Df LR stat.      Pr(Chi)
1           temp |  |         67 184.0269
2 temp + contact |  |         66 172.9838 1 vs 2     1   11.043 0.0008902248
```

which in this case produce a slightly lower p-value. Equivalently we can use `dropterm` to obtain likelihood ratio tests of the explanatory variables while *controlling* for the remaining variables:

```
> dropterm(fm1, test = "Chi")

Single term deletions
```

```
Model:
location : rating ~ temp + contact
        Df    AIC    LRT   Pr(Chi)
<none>     184.98
temp     1 209.91 26.928 2.112e-07 ***
contact  1 194.03 11.043 0.0008902 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Likelihood ratio tests of the explanatory variables while *ignoring* the remaining variables are provided by the `addterm` method:

```
> fm0 <- clm(rating ~ 1, data = wine)
> addterm(fm0, scope = ~temp + contact, test = "Chi")

Single term additions

Model:
location : rating ~ 1
        Df    AIC     LRT   Pr(Chi)
<none>     215.44
temp     1 194.03 23.4113 1.308e-06 ***
contact  1 209.91  7.5263   0.00608 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this case these latter tests are not as strong as the tests controlling for the other variable.

Confidence intervals are provided by the `confint` method:

```
> confint(fm1)

                    2.5 %   97.5 %
loc.tempwarm   1.5097624 3.595212
loc.contactyes 0.6157917 2.492394
```

These are based on the profile likelihood function and generally fairly accurate. Less accurate, but simple and symmetric confidence intervals based on the standard errors of the parameters (so-called Wald confidence intervals) can be obtained with

```
> confint.default(fm1)

                2.5 %     97.5 %
1|2        -2.3578846 -0.3308819
2|3         0.3925795  2.1090383
3|4         2.2952980  4.6384757
4|5         3.5738543  6.4389547
tempwarm    1.4669080  3.5392958
contactyes  0.5936344  2.4619608
```

The cumulative link model in (1) assume that the thresholds, $\{\theta_j\}$ are constant for all values of the remaining explanatory variables, here `temp` and `contact`. This is generally referred to as the *proportional odds assumption* or *equal slopes assumption*, while both names are not entirely adequate since the assumption is also made for other links than the logit and for categorical variables as well as continuous ones. We can relax that assumption in two general ways: with nominal effects and scale effects which we will now discuss in turn:

Nominal effects:

References

Randall, J. (1989). The analysis of sensory data by generalised linear model. *Biometrical journal 7*, pp. 781–793.

Tutz, G. and W. Hennevogl (1996). Random effects in ordinal regression models. *Computational Statistics & Data Analysis 22*, pp. 537–557.