STATISTICAL DATA ANALYSIS: RECURRENCE PLOT

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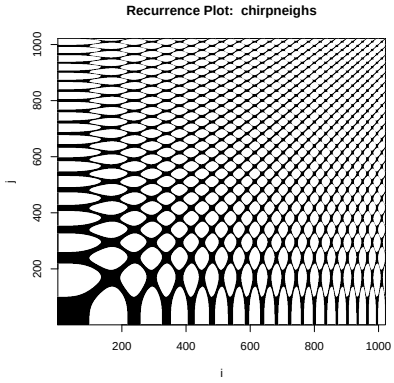

Contents

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1. Setup

Input

```
save.RNGseed <- 87149 #.Random.seed
save.RNGkind <- RNGkind()
# save.RNGseed
save.RNGkind
```

Output

```
[1] "Mersenne-Twister" "Inversion"
```

Input

```
set.seed(save.RNGseed, save.RNGkind[1])
```

Input

```
laptime <- function(){
return(round(structure(proc.time() - chunk.time.start, class = "proc_time")[3],3))
chunk.time.start <<- proc.time()
}
```

Input

```
# install.packages("sintro",repos="http://r-forge.r-project.org",type="source")
library(sintro)
```

We use

Input

```
library(nonlinearTseries)
```

ToDo: the takens state plot may be critically affected by outliers. Find a good rescaling.

1.0.1. *Taken states.* To display the Takens state space, we us a variant of pairs().

By convention, the states are defined using overlapping sliding windows. This imposes considerable dependence between the states: one state is the shifted previous states, with only the end sub-state replaced. As an option, the states can be subsampled, using only non-overlapping ranges.

The Takens states may be stationary, that is asymptotically, that is the states starting at i do not depend on i. In this case, the first row (or column) contains all information, and pairs plot form an inclusion sequence by. In general, we will use state plots in 8 dimensions, a limit suggested by the print area.

ToDo: improve choice of alpha

`statepairs` *Show marginal scatterplots of Takens states*

Usage.

```
statepairs(states, main, rank = FALSE, nooverlap = FALSE, alpha)
```

Arguments.

states	A matrix: Takens states by dimension, one state per row.
main	Optional: the main header.
rank	An experimental variant. If *rank*, the values are rank transformed.
nooverlap	An experimental variant. If *nooverlap*, the cases are subsampled by dimension.
alpha	The alpha value used for plotting. The current choice is $sqrtfrac1nrstates$ as a default.

Input

```
statepairs <- function(states, main, rank=FALSE, nooverlap= FALSE, alpha){
        n <- dim(states)[1]; dim <-  dim(states)[2]
        if (missing(main)) {
        main <- paste("Takens states:",deparse(substitute(states)), "\n",
                "n=", n," dim=", dim) }

        if (nooverlap) {states <- states[ seq(1,n, by=dim),]

        main <- paste(main," no overlap")}

        if (missing(alpha)) {alpha <- 1/sqrt(dim(states)[1])}
        #cat("alpha=",alpha)

        if (rank) {states <- apply(states, 2, rank, ties.method="random")
        main <- paste(main," ranked")}
        pairs(states, main=main,
        col=rgb(0,0,0, alpha), pch=19)
        #title(main=main, outer=TRUE, line=-2, cex.main=0.8)
}
```

1.1. **Local Bottleneck.** To allow experimental implementations, functions from *nonlinearTseries* are aliased here.

Input

```
local.buildTakens <- buildTakens
```

Input

```
local.findAllNeighbours <- nonlinearTseries:::findAllNeighbours
```

minor cosmetics added to recurrencePlotAux

ToDo: propagate parameters from *buildTakens* and *findAllNeighbours* in a slot of the result, instead of using explicit parameters in recurrencePlotAux.

Input

```
#non-sparse variant
#local.recurrencePlotAux <- nonlinearTseries:::recurrencePlotAux
local.recurrencePlotAux = function(neighs, dim=NULL, lag=NULL, radius=NULL){

        # just for reference. This function is inlined
        neighbourListNeighbourMatrix  = function(){
          #neighs.matrix = Diagonal(ntakens)
          for (i in 1:ntakens){
            if (length(neighs[[i]])>0){
              for (j in neighs[[i]]){
                neighs.matrix[i,j] = 1
              }
            }
          }
```

```
          return (neighs.matrix)
        }

  ntakens=length(neighs)
  neighs.matrix <- matrix(nrow=ntakens,ncol=ntakens)
  #neighbourListNeighbourMatrix()
    #neighs.matrix = Diagonal(ntakens)
        for (i in 1:ntakens){
                neighs.matrix[i,i] = 1 # do we want the diagonal fixed to 1
          if (length(neighs[[i]])>0){
            for (j in neighs[[i]]){
              neighs.matrix[i,j] = 1
            }
          }
        }

       main <- paste("Recurrence Plot: ",
                     deparse(substitute(neighs))
                     )
       more <- NULL

       #use compones of neights if available
       if (!is.null(dim)) more <- paste(more," dim:",dim)
       if (!is.null(lag)) more <- paste(more," lag:",lag)
       if (!is.null(radius)) more <- paste(more," radius:",radius)

       if (!is.null(more)) main <- paste(main,"\n",more)

  # need no print because it is not a trellis object!!
  #print(
        image(x=1:ntakens, y=1:ntakens,
              z=neighs.matrix,xlab="i", ylab="j",
              col="black",
              #xlim=c(1,ntakens), ylim=c(1,ntakens),
              useRaster=TRUE,  #? is this safe??
              main=main
              )
#       )

}
```

ToDo: improve feedback for data structures in *nonlinearTseries*

2. Test Cases

We set up a small series of test signals.

For convenience, some source code from other libraries is included to make this self-contained.

As a global constant, we set up the length of the series to be used for test signals.

Input

```
#nsignal <- 256
nsignal <- 1024
#nsignal <- 4096
system.time.start <- proc.time()
```

For signal representation, we use a common layout.

Input

```
plotsignal <- function(signal, main, ylab) {
        #! alpha level should depend on expected number of overlaps

        if (missing(ylab)) { ylab <- deparse(substitute(signal)) }

        par(mfrow = c(1,
                2))
        plot(signal,
                main = "", xlab = "index",  ylab = ylab,
                col = rgb(0, 0, 1, 0.3), pch = 20)

        plot(signal, type = "l",
                main = "",  xlab = "index", ylab = ylab,
                col = rgb(0, 0, 0, 0.4))
        points(signal,
                col = rgb(0, 0, 1, 0.3), pch = 20)
        if (missing(main)) { main = deparse(substitute(signal)) }
        title(main = main,
                outer = TRUE, line = -2, cex.main = 1.2)
}
```

Input

```
sin10 <- function(n=nsignal) {sin( (1:n)/n* 2*pi*10)}
plotsignal(sin10())
```

See figure 1.

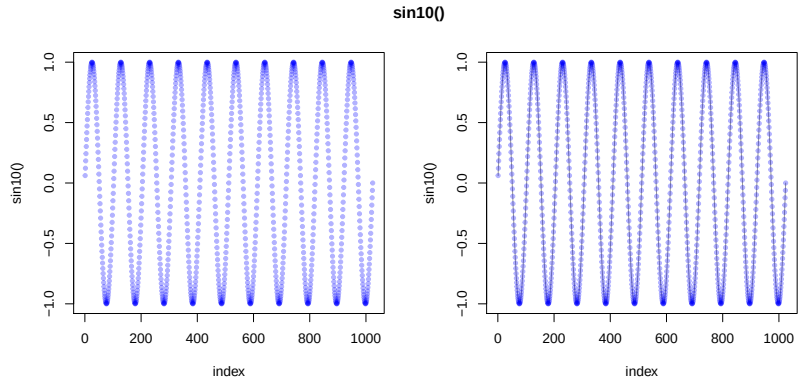

FIGURE 1. Test case: sin10. Signal and linear interpolation.

Input

```
unif <- function(n=nsignal) {runif(n)}
xunif<-unif()
plotsignal(xunif)
```

See figure 2 on the next page,

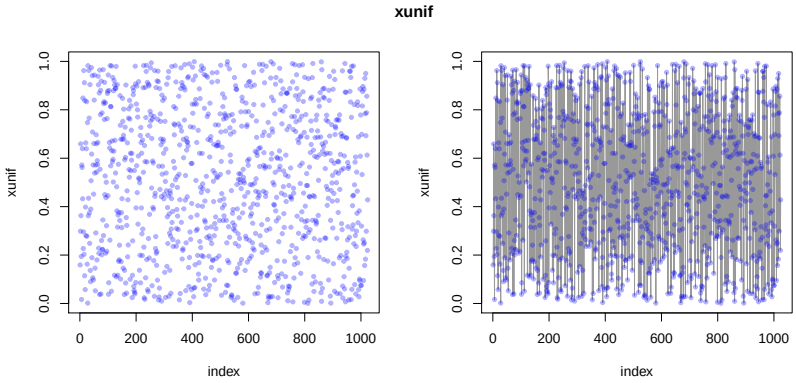

FIGURE 2. Test case: unif - uniform random numbers. Signal and linear interpolation.

Input

```
chirp <- function(n=nsignal) {
# this is copied from library(signal)
signal.chirp <- function(t, f0 = 0, t1 = 1, f1 = 100,
                  form = c("linear", "quadratic", "logarithmic"),
                  phase = 0){

  form <- match.arg(form)
  phase <- 2*pi*phase/360

  switch(form,
    "linear" = {
        a <- pi*(f1 - f0)/t1
        b <- 2*pi*f0
        cos(a*t^2 + b*t + phase)
    },
    "quadratic" = {
        a <- (2/3*pi*(f1-f0)/t1/t1)
        b <- 2*pi*f0
        cos(a*t^3 + b*t + phase)
    },
    "logarithmic" = {
        a <- 2*pi * t1 / log(f1 - f0)
        b <- 2*pi * f0
        x <- (f1-f0)^(1/t1)
        cos(a*x^t + b*t + phase)
    })
}

signal.chirp(seq(0, 0.6, len=nsignal))
}
plotsignal(chirp())
```

ToDo: include doppler waveslim

See figure 3 on the facing page,

3. TAKENS' RECURRENCE STATES

Recurrence plots have been introduced in an attempt to understand near periodic in hydrodynamics. On the one hand, and extended theory on dynamical systems was available,

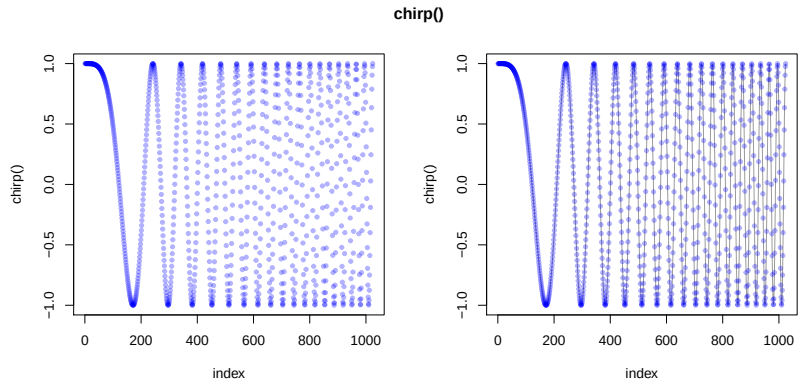

FIGURE 3. Test case: chirp signal. Signal and linear interpolation.

covering deterministic models. A fundamental concept is that at a certain time a system is in some state, and developing from this. Defining the proper state space is a critical step in modelling.

The other toolkit is that of stochastics processes, in particular Markov models. Classical time series assumes stationarity, and this is obviously not the way to go. A fundamental idea for Markov models is that the system state is seen in a temporal context: you have a Markov process, if you can define a (non-anticipating) state that has sufficient information for prediction: given this state, the future is independent from the past.

Recurrence, coming back to some state, is often a key to understand a near periodic system.

Hydrodynamics is a challenging problem. Understanding planetary motion is a historical challenge, and may be useful as an illustration.

As a simple illustration, let $x = (x_i)$ be a sequence, maybe near periodic. For now, think of i as a time index.

Recurrence plots have two steps. The first was a bold step by Floris Takens. If you do not know the state space of a system, for a choice of "dimension" d, take the sequence of d tuples taken from your data to define the states.

$$u_i = (x_i, \ldots, x_{i+d})$$

This is Takens' delay embedding state (re)construction Takens [1981].

As a mere technical refinement: you may know that your data are a flattened representation of t dimensional data. So you take

$$u_i = (x_i, \ldots, x_{i+d*m}).$$

This may be a relict of FORTRAN times, where it was common to flatten two-dimensional structures by case. We ignore this detail here and take $m = 1$.

ToDo: add support for higher dimensional signals

Conceptually, you define states by observed histories. For classical Markov setup, the state is defined by the previous information x_{i-1}, but for more complex situations you may have to step back in the past. Finding the appropriate d is the challenge. So it may be appropriate to view the Takens states as a family, indexed by the time scope d. The rest is structural information how to arrange items.

Of course it is possible to compress information here, sorting states and removing duplicates. Keeping the original definition as the advantage that we have the index i, so that u_i is the state at index position i.

But the states may have an inherent structure, which we may take into account or ignore. Since for this example, we are just in 4-dimensional space, marginal scatterplots may give enough information.

Input

```
sintakens <- local.buildTakens(time.series=sin10(),
        embedding.dim=4, time.lag=1)
statepairs(sintakens) #4
```

See figure 4.

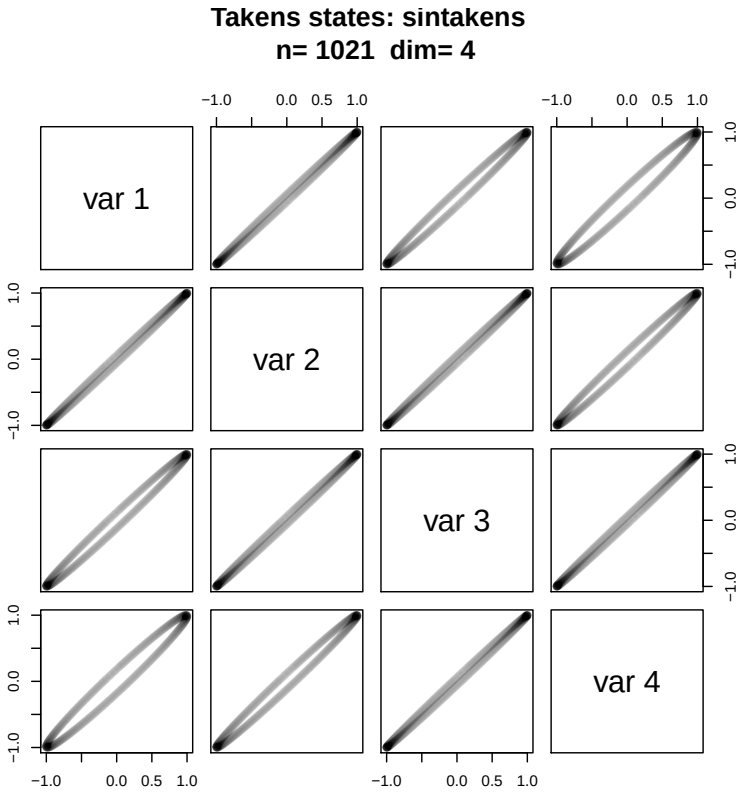

FIGURE 4. Test case: sinus. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.641 sec.

Input

```
statepairs(sintakens, nooverlap=TRUE) #dim=4
```

See figure 5 on the facing page.

Input

```
uniftakens <- local.buildTakens( time.series=xunif,
        embedding.dim=4, time.lag=1)
statepairs(uniftakens) #dim=4
```

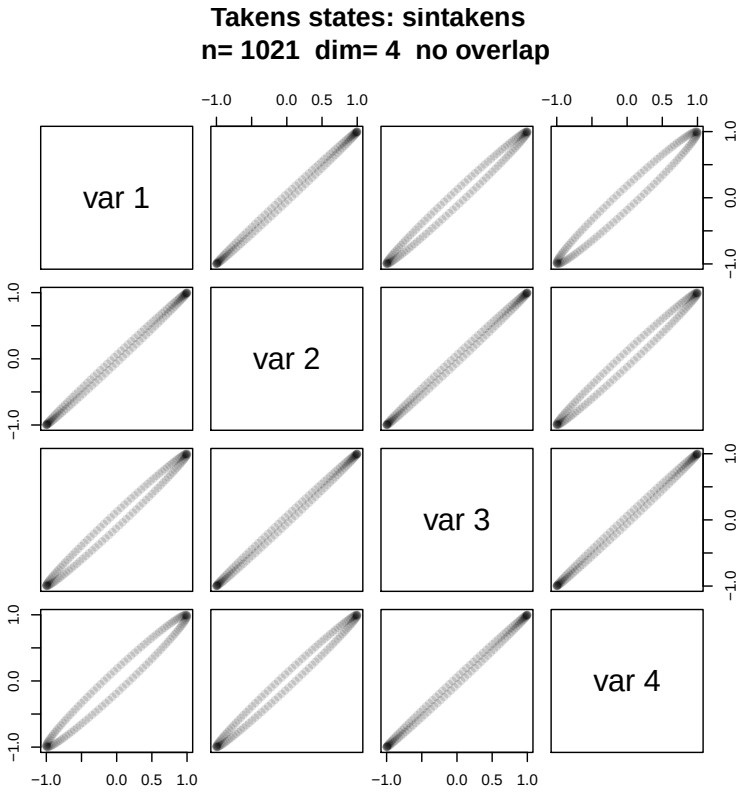

FIGURE 5. Test case: sinus, no overlap. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.888 sec.

See figure 6 on the next page.

Input
```
statepairs(uniftakens, nooverlap=TRUE) #dim=4
```

See figure 7 on page 11.

Input
```
chirptakens <- local.buildTakens( time.series=chirp(),
        embedding.dim=4,time.lag=1)
statepairs(chirptakens) #dim=4
```

See figure 8 on page 12

Input
```
statepairs(chirptakens, nooverlap=TRUE) #dim=4
```

See figure 9 on page 13

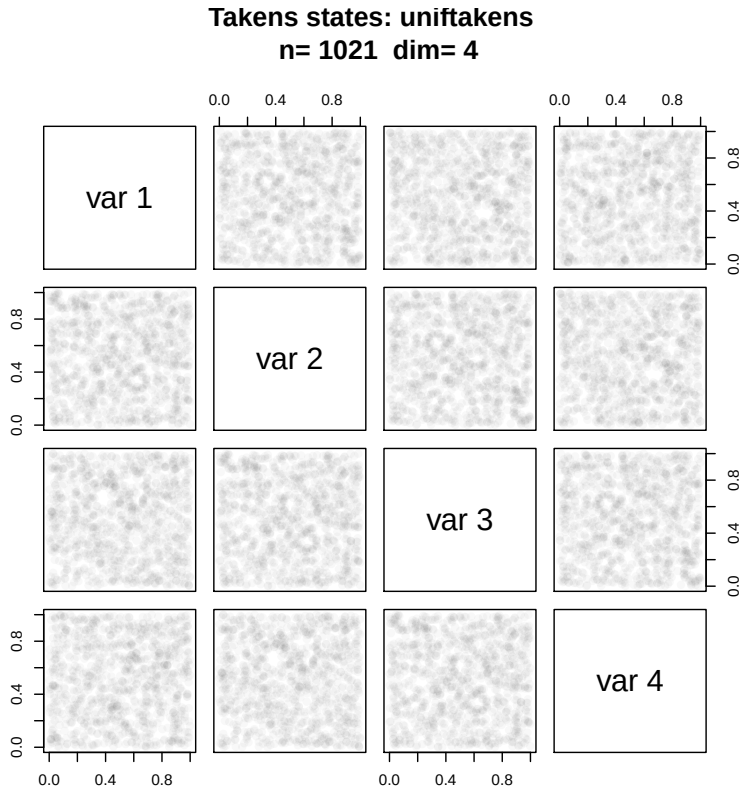

FIGURE 6. Test case: uniform random numbers. Time used: 0.712 sec.

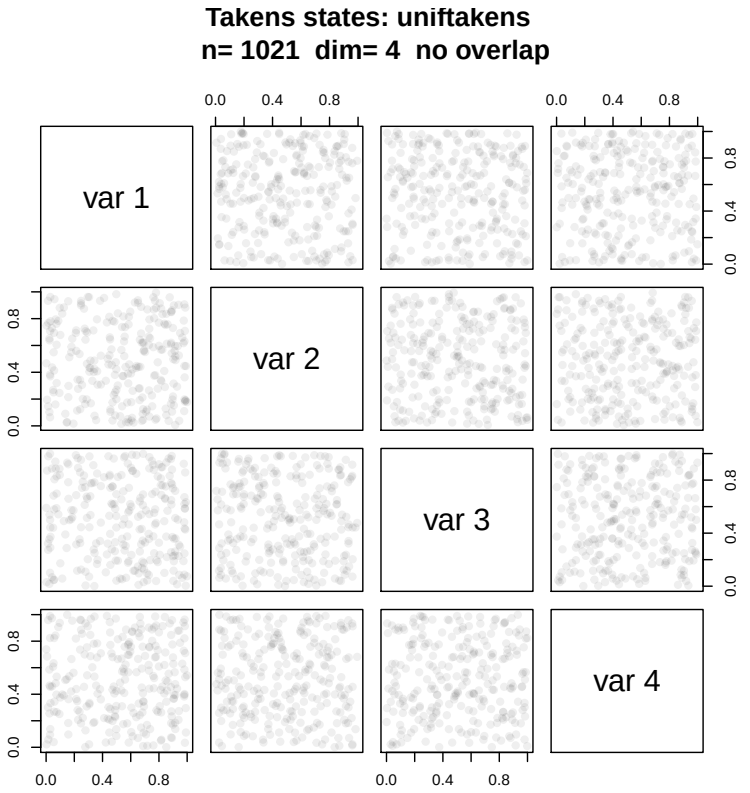

FIGURE 7. Test case: uniform random numbers. Time used: 1.009 sec.

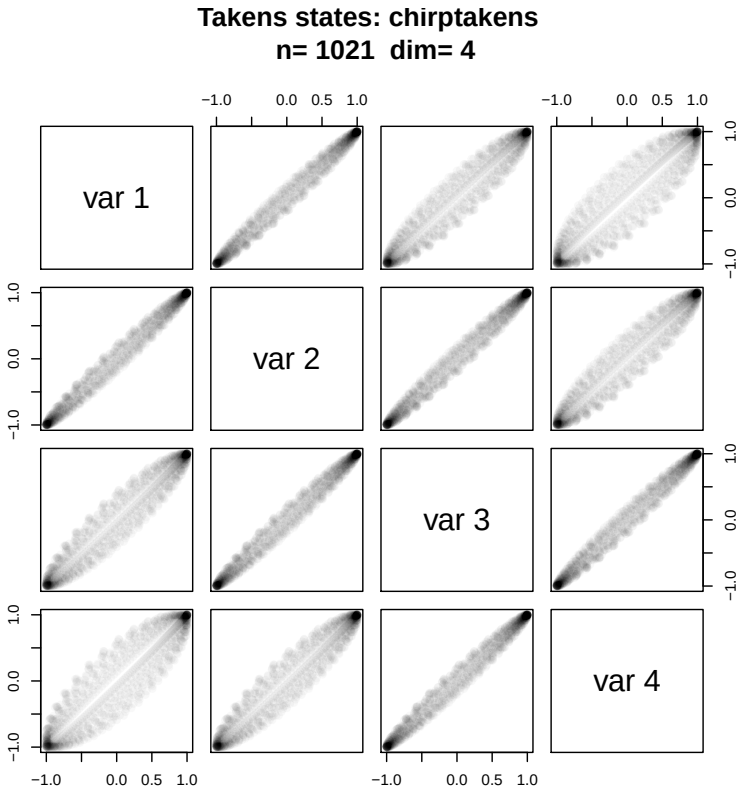

FIGURE 8. Test case: chirp signal. Time used: 0.713 sec.

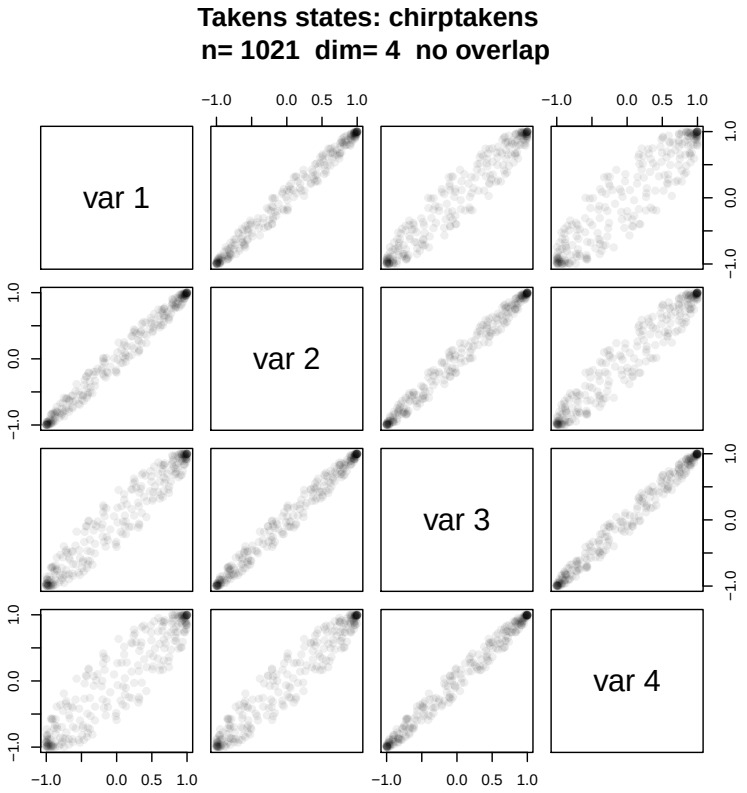

FIGURE 9. Test case: chirp signal. Time used: 1.001 sec.

4. Recurrence Plots

ToDo: consider dimension-adjusted radius

The next step, taken in Eckmann *et al.* [1987] was to use a two dimensional display. Take a scatterplot with the Taken's states a marginal. Take a sliding window of your process data, and for each i, find the "distance" of u_i from and to any of the collected states. If the distance is below some chosen threshold, mark the point (i, j) for which u(j) is in the ball of radius r(i) centred at u(i).

The original publication Eckmann *et al.* [1987] actually used a nearest neighbourhood environment to cover about 10 data points.

The construction has considerable arbitrary choices. The critical radius may depend on the point i. In practical applications, using a constant radius is a common first step. Using a dichotomous marking was what presumably was necessary when the idea was introduced. With todays technology, we can allow a markup on a finer scale, as has been seen in Orion-1.

ToDo: support distance instead of 0/1 indicators

We can gain additional freedom by using a correlation view: instead of looking from one axis, we can walk along the diagonal, using two reference axis.

Helpful hints how to interpret recurrence plots are in "Recurrence Plots At A Glance" `<http://www.recurrence-plot.tk/glance.php>`.

5. Recurrence Quantification Analysis

While visual inspection is the prime way to assess recurrence plots, quantification of some aspects revealed of the plot may be helpful. A collection of indices is provided by a recurrence quantification analysis (RQA) Zbilut and Webber [2006], Webber Jr and Zbilut [2005].

See Table table 1 on the next page.

ToDo: improve to a full *show* method

This is a hack to report RQA information. $dim = NULL$ is added to align calling with other functions.

Input

```
showrqa <- function(takens, dim=NULL, radius, do.hist = TRUE)
{
        xxrqa <- rqa(takens=takens,radius=radius)
        cat(paste(deparse(substitute(takens)), " n=", dim(takens)[1],
                " Dim:", dim(takens)[2], "\n"))
        cat(paste("Radius:", radius,
                " Recurrence coverage REC:", round(xxrqa[1]$REC, 3),
                " log(REC)/log(R):", round(log(xxrqa[1]$REC)/log(radius),3), "\n"))
        cat(paste("Determinism:", round(xxrqa$DET,3),
                " Laminarity:",round(xxrqa$LAM,3), "\n"))
        cat(paste("DIV:", round(xxrqa$DIV,3), "\n"))
        cat(paste("Trend:", round(xxrqa$TREND,3),
                " Entropy:",round(xxrqa$ENTR,3), "\n"))
        cat(paste("Diagonal lines max:", round(xxrqa$Lmax,3),
                 " Mean:",round(xxrqa$Lmean,3),
                " Mean off main:",round(xxrqa$LmeanWithoutMain,3), "\n"))
        cat(paste("Vertical lines max:", round(xxrqa$Vmax,3),
                " Mean:",round(xxrqa$Vmean,3), "\n"))
                # str(xxrqa[4:12])
```

TABLE 1. Recurrence Quantification Analysis (RQA)

REC	Recurrence. Percentage of recurrence points in a recurrence Plot.
DET	Determinism. Percentage of recurrence points that form diagonal lines.
LAM	Percentage of recurrent points that form vertical lines.
RATIO	Ratio between *DET* and *RR*.
Lmax	Length of the longest diagonal line.
Lmean	Mean length of the diagonal lines. The main diagonal is not taken into account.
DIV	Inverse of *Lmax*.
Vmax	Longest vertical line.
Vmean	Average length of the vertical lines. This parameter is also referred to as the Trapping time.
ENTR	Shannon entropy of the diagonal line lengths distribution
TREND	Trend of the number of recurrent points depending on the distance to the main diagonal
diagonalHistogram	Histogram of the length of the diagonals.
recurrenceRate	Number of recurrent points depending on the distance to the main diagonal.

```
    oldpar <- par(mfrow=c(2,1))
    if (do.hist){
    barplot(xxrqa$diagonalHistogram,
            main=paste(deparse(substitute(takens)), "Diagonal",
                    "\n n=", dim(takens)[1], " Dim:",
                    dim(takens)[2], " Radius: ",radius))
    barplot(xxrqa$recurrenceRate,
            main=paste(deparse(substitute(takens)),"Recurrence Rate",
                    "\n n=", dim(takens)[1], " Dim:",
                    dim(takens)[2], " Radius: ",radius))
            }
    par(oldpar)
    invisible(xxrqa)
}
```

6. Applied Recurrence Plots

6.1. **Sinus.**

Input

```
sin10neighs<-local.findAllNeighbours(sintakens, radius=0.2)
save(sin10neighs, file="sin10neighs.Rdata")
```

Input

```
load(file="sin10neighs.RData")
local.recurrencePlotAux(sin10neighs, dim=2, radius=0.2)
```

See figure 10 on the following page.

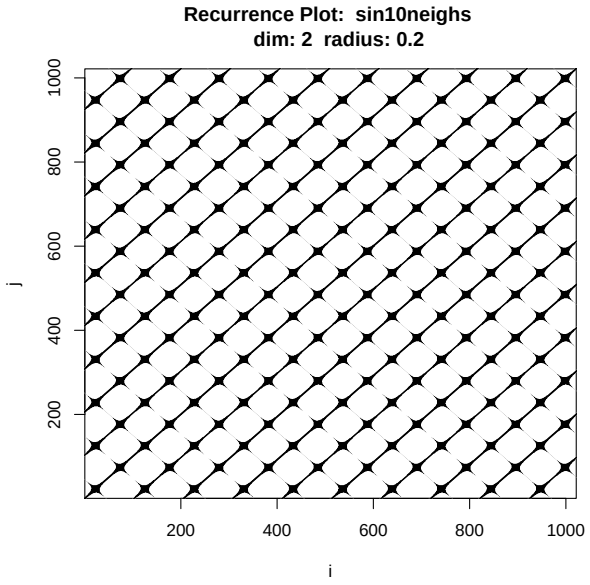

FIGURE 10. Recurrence Plot. Test case: sinus curves. Time used: 1.199 sec.

Input

```
showrqa(sintakens, radius=0.2)
```

Output

```
sintakens  n= 1021  Dim: 4
Radius: 0.2  Recurrence coverage REC: 0.136  log(REC)/log(R): 1.241
Determinism: 0.96  Laminarity: 0.982
DIV: 0.001
Trend: 0  Entropy: 2.74
Diagonal lines max: 1020  Mean: 14.263  Mean off main: 14.157
Vertical lines max: 25  Mean: 10.251
```

See figure 11 on the next page.

6.2. **Uniform random.**

Input

```
load(file="unifneighs.RData")
local.recurrencePlotAux(unifneighs, radius=0.2)
```

See figure 12 on the facing page.

6.3. **Chirp Signal.**

Input

```
chirpneighs<-local.findAllNeighbours(chirptakens,radius=0.6)#0.4
save(chirpneighs, file="chirpneighs.RData")
```

Input

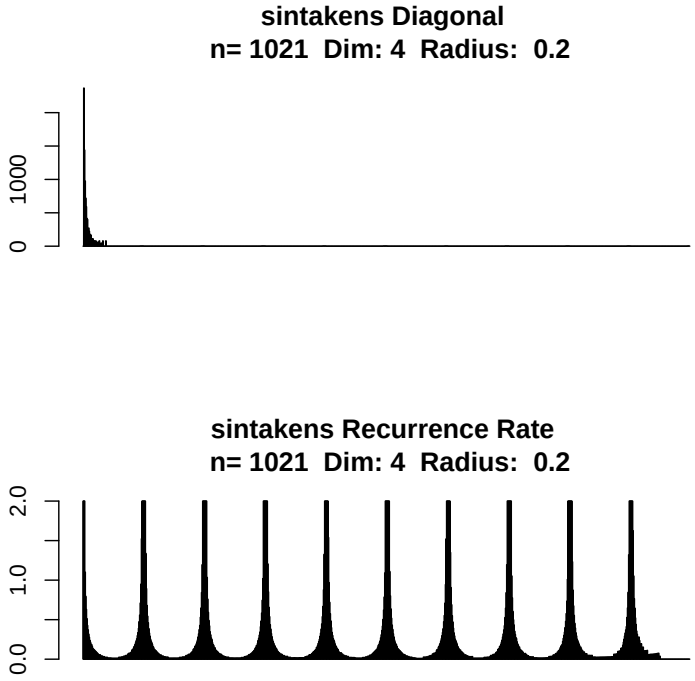

FIGURE 11. Recurrence Plot RQA. Test case: sinus curves. Time used: 0.488 sec.

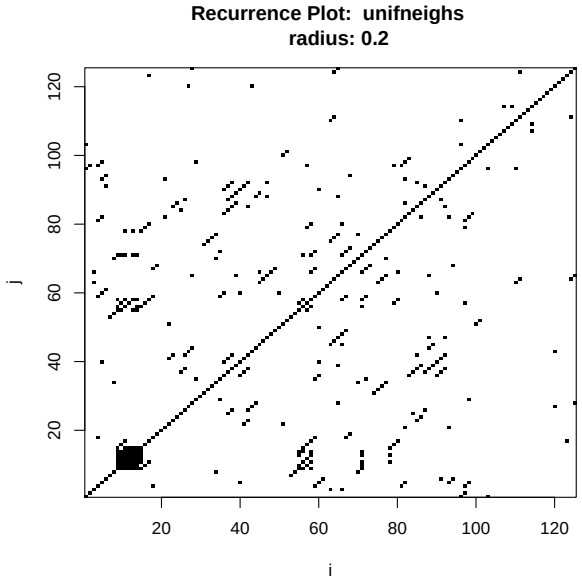

FIGURE 12. Recurrence Plot. Test case: uniform random numbers. Time used: 0.092 sec.

```
load(file="chirpneighs.RData")
local.recurrencePlotAux(chirpneighs)
```

See figure 13.

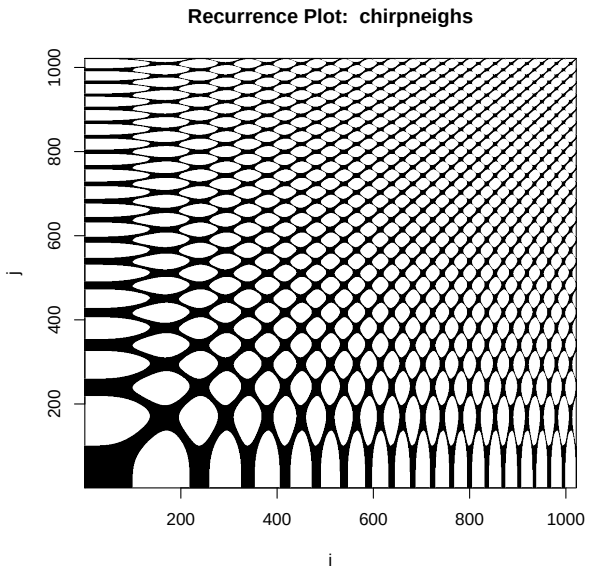

FIGURE 13. Recurrence Plot. Test case: chirp signal. Time used: 2.256 sec.

Input

```
showrqa(chirptakens, radius=0.6)
```

Output

```
chirptakens  n= 1021  Dim: 4
Radius: 0.6  Recurrence coverage REC: 0.341  log(REC)/log(R): 2.108
Determinism: 0.988  Laminarity: 0.998
DIV: 0.001
Trend: 0  Entropy: 3.254
Diagonal lines max: 1020  Mean: 12.496  Mean off main: 12.46
Vertical lines max: 125  Mean: 14.721
```

See figure 14 on the facing page.

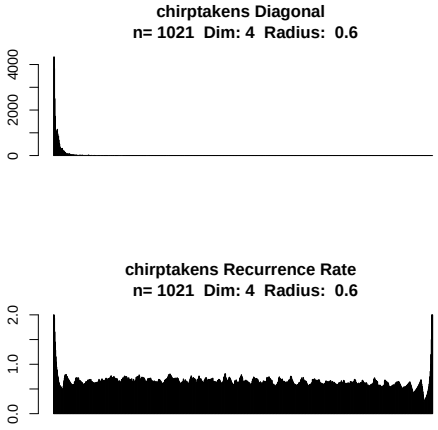

FIGURE 14. Recurrence Plot RQA. Test case: chirp signal. Time used: 2.889 sec.

7. Case Study: Geyser data

ToDo: double check: *MASS:::geyser* should be used, not *faithful*

ToDo: Geyser: extended to two-dimensional data in *geyserlin*. Check.

This is a classical data set with a two dimensional structure, *duration* and *waiting*.

Input

```
library(MASS)
data(geyser)
```

7.1. Geyser Eruptions.

Input

```
plotsignal(geyser$duration)
```

See figure 15,

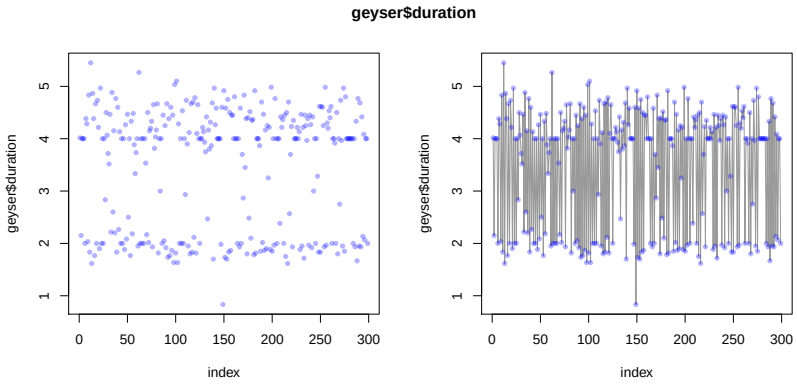

Figure 15. Example case: Old Faithful Geyser eruptions. Signal and linear interpolation.

Input

```
eruptionstakens4 <-
        local.buildTakens( time.series=geyser$duration,
                embedding.dim=4, time.lag=1)
        statepairs(eruptionstakens4) #dim=4
```

See figure 24 on page 26

Input

```
        statepairs(eruptionstakens4, nooverlap=TRUE) #dim=4
```

See figure 25 on page 27

Input

```
eruptionsneighs4<-local.findAllNeighbours(eruptionstakens4, radius=0.8)
save(eruptionsneighs4, file="eruptionsneighs4.RData")
```

Input

```
load(file="eruptionsneighs4.RData")
local.recurrencePlotAux(eruptionsneighs4)
```

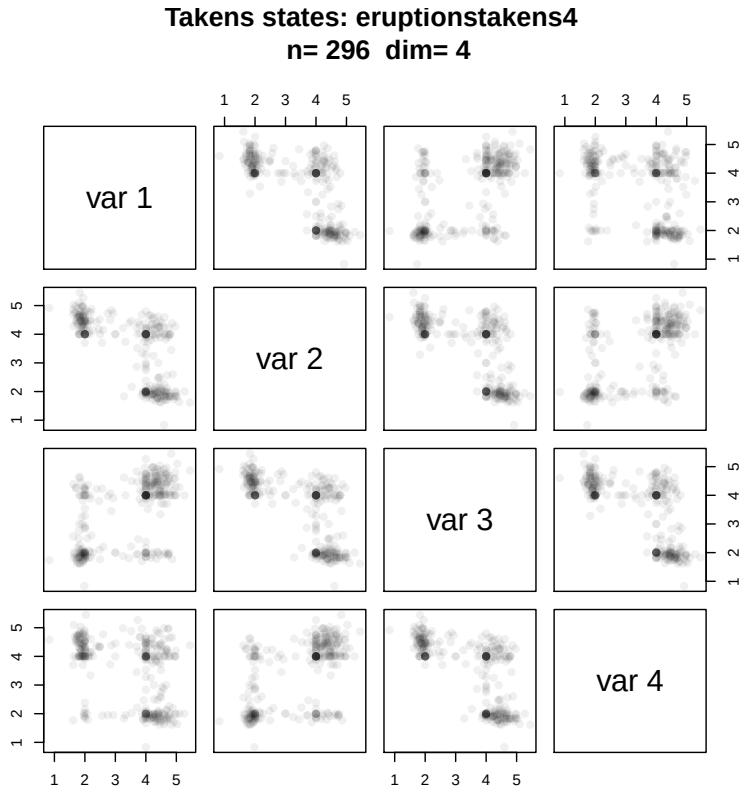

FIGURE 16. Example case: Old Faithful Geyser eruptions. Time used: 0.301 sec.

See figure 26 on page 27.

Input

```
showrqa(eruptionstakens4, radius=0.8)
```

Output

```
eruptionstakens4  n= 296  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.112  log(REC)/log(R): 9.822
Determinism: 0.903  Laminarity: 0.076
DIV: 0.05
Trend: 0  Entropy: 1.779
Diagonal lines max: 20  Mean: 3.919  Mean off main: 3.79
Vertical lines max: 5  Mean: 3.041
```

See figure 27 on page 28.

7.1.1. *Geyser eruptions. Dim=2.*

Input

```
eruptionstakens2 <-
        local.buildTakens(time.series=geyser$duration,
                embedding.dim=2, time.lag=1)
statepairs(eruptionstakens2) #dim=2
```

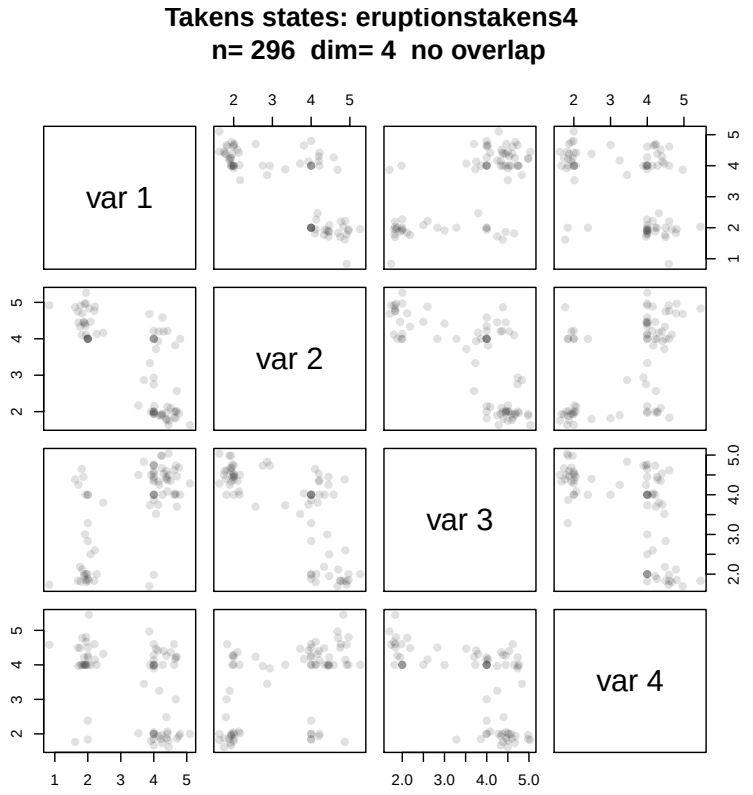

FIGURE 17. Example case: Old Faithful Geyser eruptions. Time used: 0.495 sec.

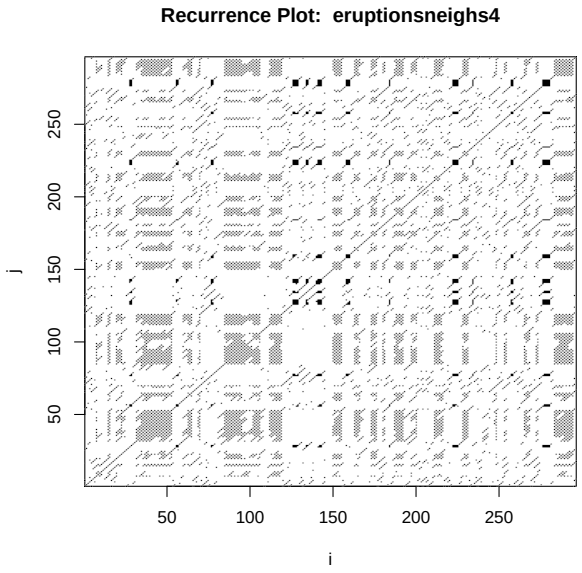

FIGURE 18. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.168 sec.

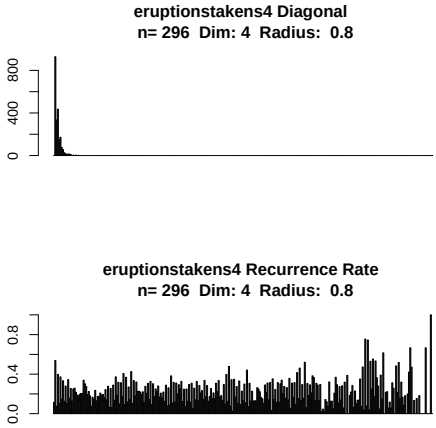

FIGURE 19. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.371 sec.

See figure 20

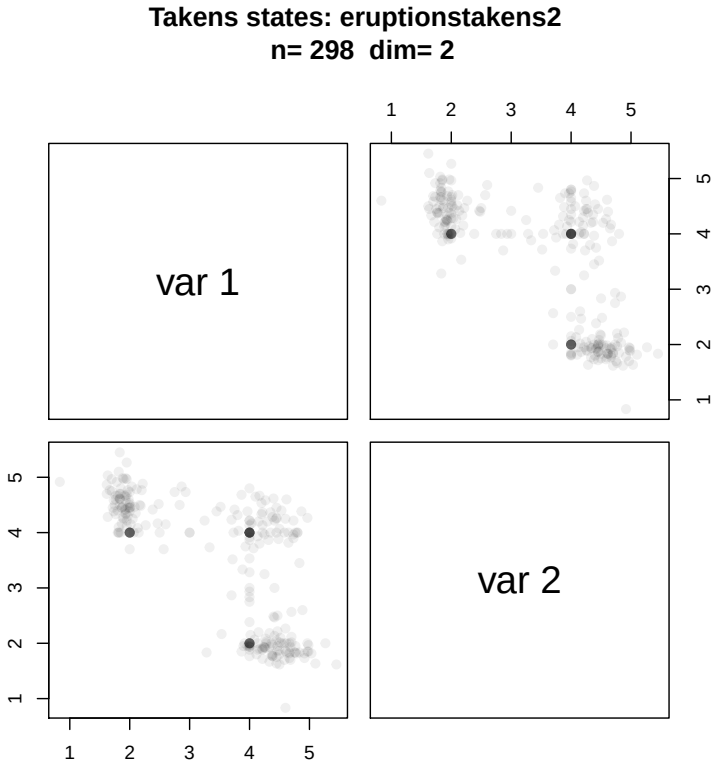

FIGURE 20. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.133 sec.

Input

```
statepairs(eruptionstakens2, nooverlap=TRUE) #dim=2
```

See figure 21

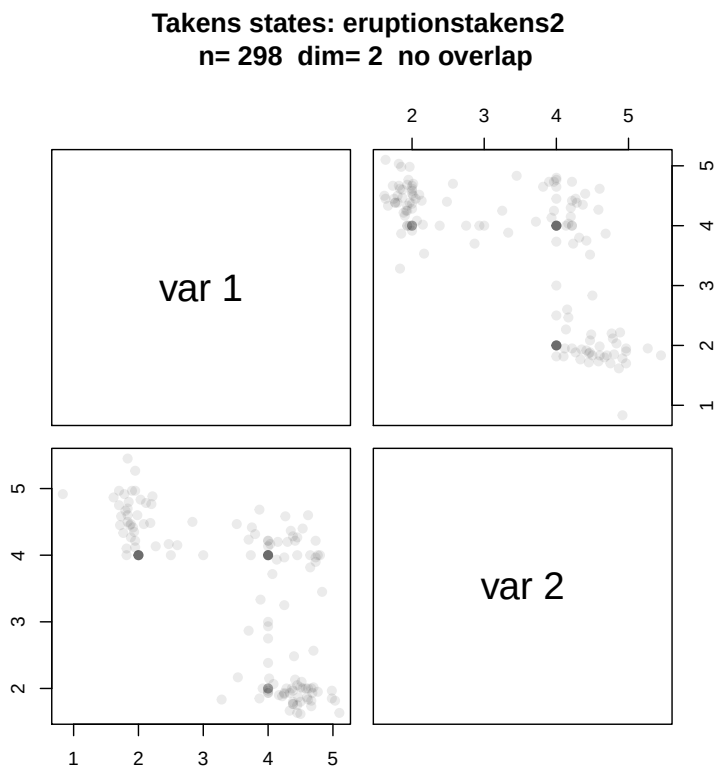

FIGURE 21. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.241 sec.

Input

```
eruptionsneighs2<-local.findAllNeighbours(eruptionstakens2,
        radius=0.8)
save(eruptionsneighs2, file="eruptionsneighs2.RData")
```

Input

```
load(file="eruptionsneighs2.RData")
local.recurrencePlotAux(eruptionsneighs2)
```

See figure 22 on the facing page.

Input

```
showrqa(eruptionstakens2, radius=0.8)
```

Output

```
eruptionstakens2  n= 298  Dim: 2
Radius: 0.8  Recurrence coverage REC: 0.274  log(REC)/log(R): 5.806
Determinism: 0.892  Laminarity: 0.205
DIV: 0.045
Trend: 0  Entropy: 1.691
Diagonal lines max: 22  Mean: 3.644  Mean off main: 3.595
Vertical lines max: 7  Mean: 3.457
```

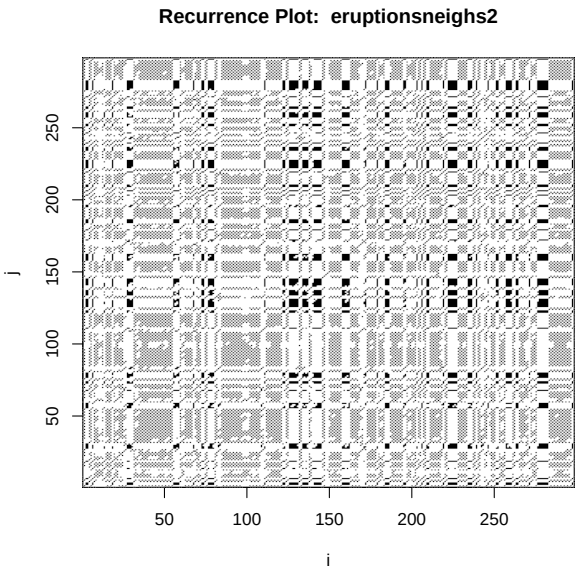

FIGURE 22. Recurrence Plot. Example case: Old Faithful Geyser Geyser eruptions. Dim=2. Time used: 0.246 sec.

See figure 23.

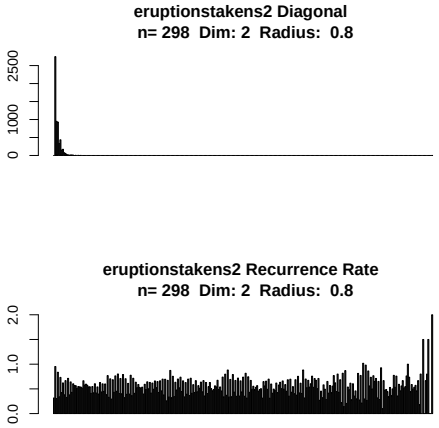

FIGURE 23. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.403 sec.

7.1.2. *Geyser eruptions. Dim=4.*

Input

```
eruptionstakens4 <- local.buildTakens( time.series=geyser$duration,
        embedding.dim=4, time.lag=1)
statepairs(eruptionstakens4) #dim=4
```

See figure 24 on the following page.

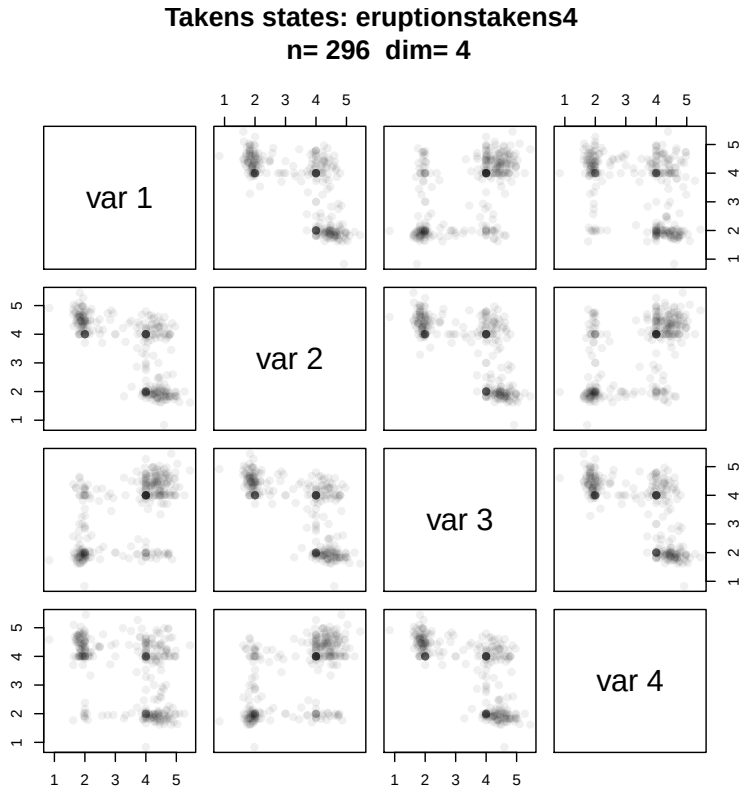

FIGURE 24. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.293 sec.

Input
```
statepairs(eruptionstakens4, nooverlap=TRUE) #dim=4
```

See figure 25 on the next page.

Input
```
eruptionsneighs4<-local.findAllNeighbours(eruptionstakens4,
        radius=0.8)
save(eruptionsneighs4, file="eruptionsneighs4.RData")
```

Input
```
load(file="eruptionsneighs4.RData")
local.recurrencePlotAux(eruptionsneighs4)
```

See figure 26 on the facing page.

Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.17 sec.

Input
```
showrqa(eruptionstakens4, radius=0.8)
```

Output

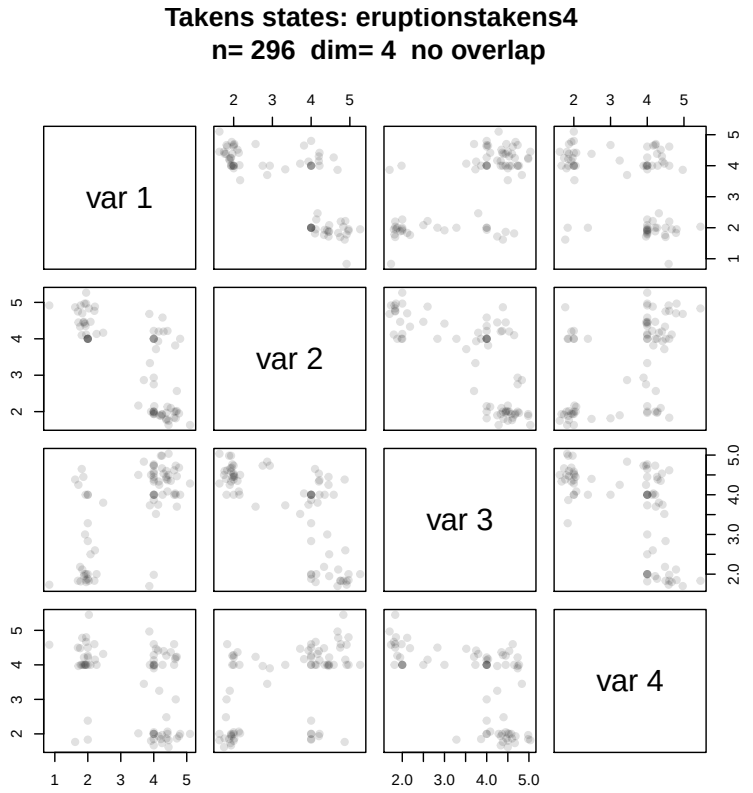

FIGURE 25. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.471 sec.

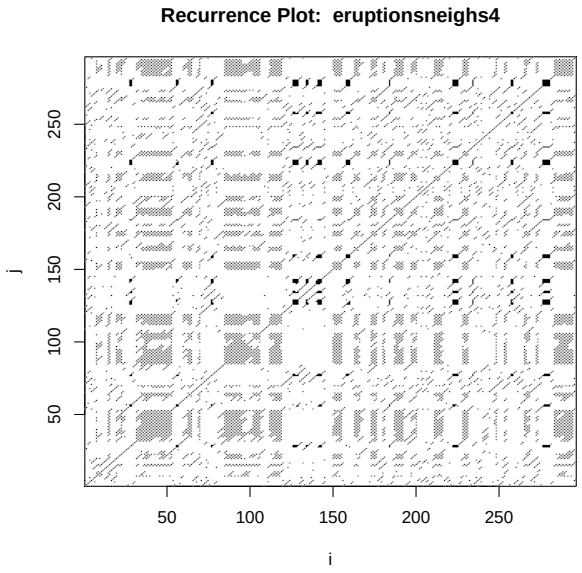

FIGURE 26. .

```
eruptionstakens4  n= 296  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.112  log(REC)/log(R): 9.822
Determinism: 0.903  Laminarity: 0.076
DIV: 0.05
Trend: 0  Entropy: 1.779
Diagonal lines max: 20  Mean: 3.919  Mean off main: 3.79
Vertical lines max: 5  Mean: 3.041
```

See figure 27.

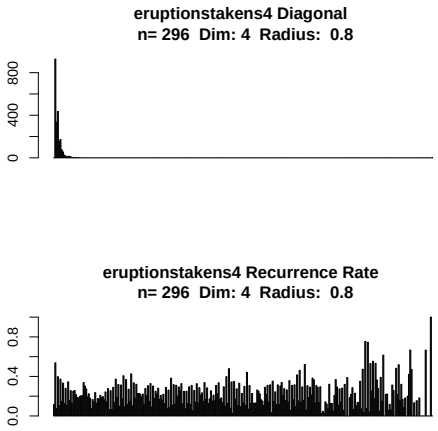

FIGURE 27. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.37 sec.

7.1.3. *Geyser eruptions. Dim=8.*

Input

```
eruptionstakens8 <- local.buildTakens( time.series=geyser$duration,
        embedding.dim=8,time.lag=1)
statepairs(eruptionstakens8) #dim=8
```

See figure 28 on the facing page.

Input

```
statepairs(eruptionstakens8, nooverlap=TRUE) #dim=8
```

See figure 29 on page 30.

Input

```
eruptionsneighs8<-local.findAllNeighbours(eruptionstakens8,
        radius=0.8)
save(eruptionsneighs8, file="eruptionsneighs8.RData")
```

Input

```
load(file="eruptionsneighs8.RData")
local.recurrencePlotAux(eruptionsneighs8)
```

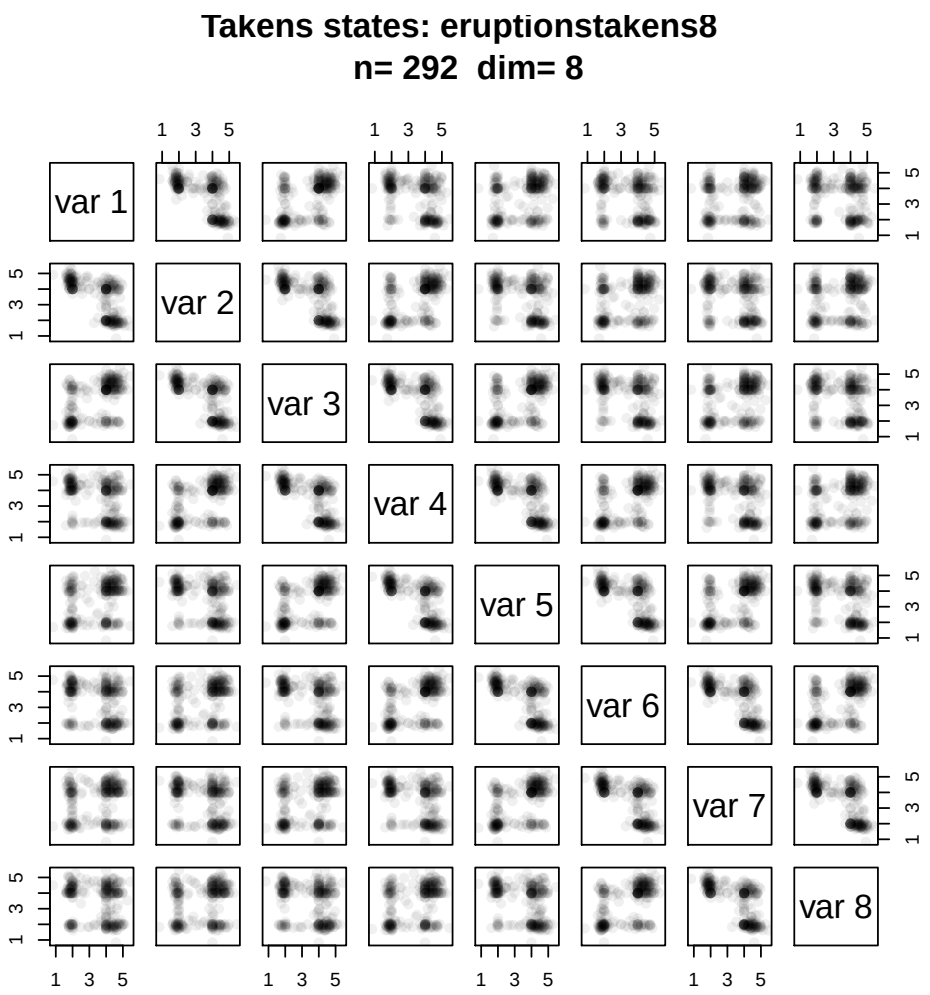

FIGURE 28. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.903 sec.

See figure 30 on page 31.

Input

```
showrqa(eruptionstakens8, radius=0.8)
```

Output

```
eruptionstakens8  n= 292  Dim: 8
Radius: 0.8  Recurrence coverage REC: 0.024  log(REC)/log(R): 16.799
Determinism: 0.924  Laminarity: 0
DIV: 0.062
Trend: 0  Entropy: 1.8
Diagonal lines max: 16  Mean: 4.583  Mean off main: 3.871
Vertical lines max: 0  Mean: 0
```

See figure 31 on page 31.

7.2. **Geyser Eruptions: Comparison by Dimension.** For comparison, recurrence plots for the Geyser data with varying dimension are in figure 32 on page 32

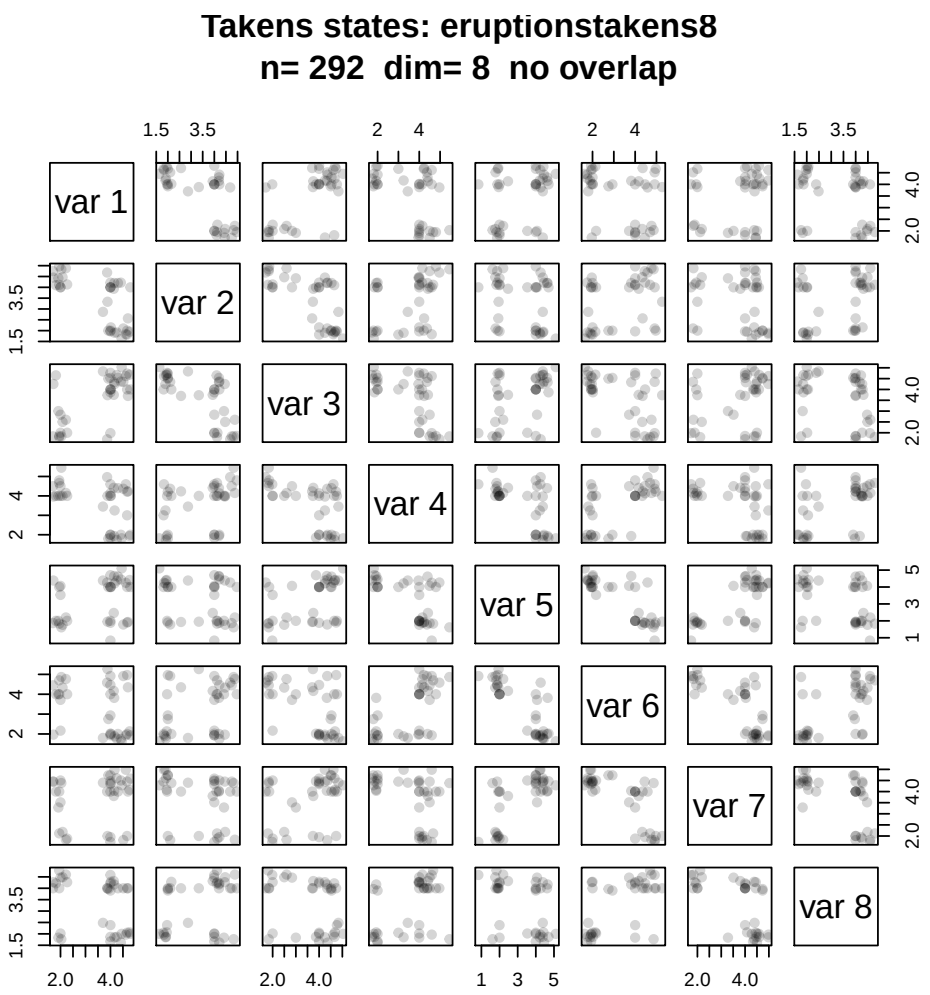

FIGURE 29. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 1.258 sec.

7.3. **Geyser Waiting.**

Input

```
plotsignal(geyser$waiting)
```

See figure 33 on page 32.

Input

```
waitingtakens <-
        local.buildTakens( time.series=geyser$waiting,
                embedding.dim=4, time.lag=4)
statepairs(waitingtakens) #dim=4
```

See figure 34 on page 33.

Input

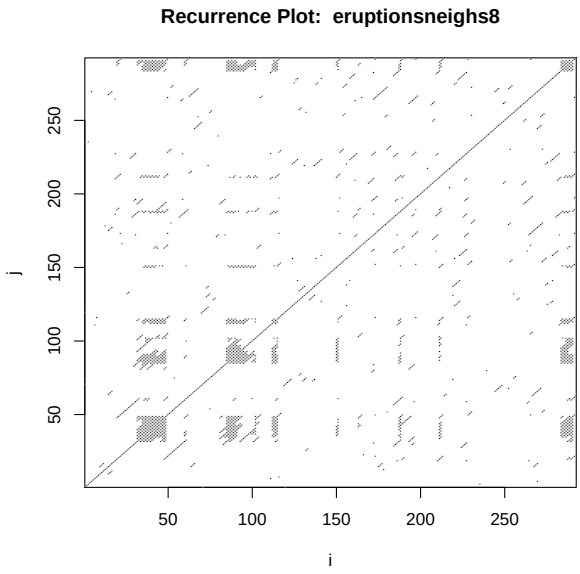

FIGURE 30. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.106 sec.

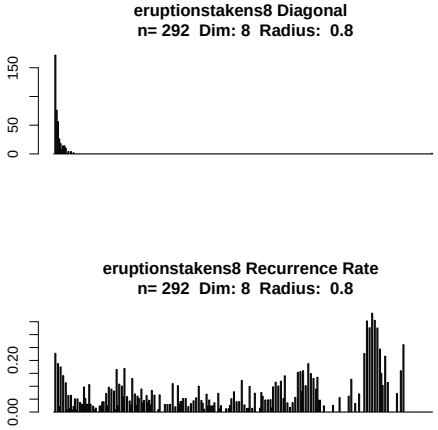

FIGURE 31. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.281 sec.

```
waitingneighs<-local.findAllNeighbours(waitingtakens, radius=16)
save(waitingneighs, file="waitingneighs.Rdata")
```

Input

```
showrqa(waitingtakens, radius=16)
```

Output

```
waitingtakens  n= 287  Dim: 4
Radius: 16  Recurrence coverage REC: 0.137  log(REC)/log(R): -0.718
Determinism: 0.382  Laminarity: 0.053
```

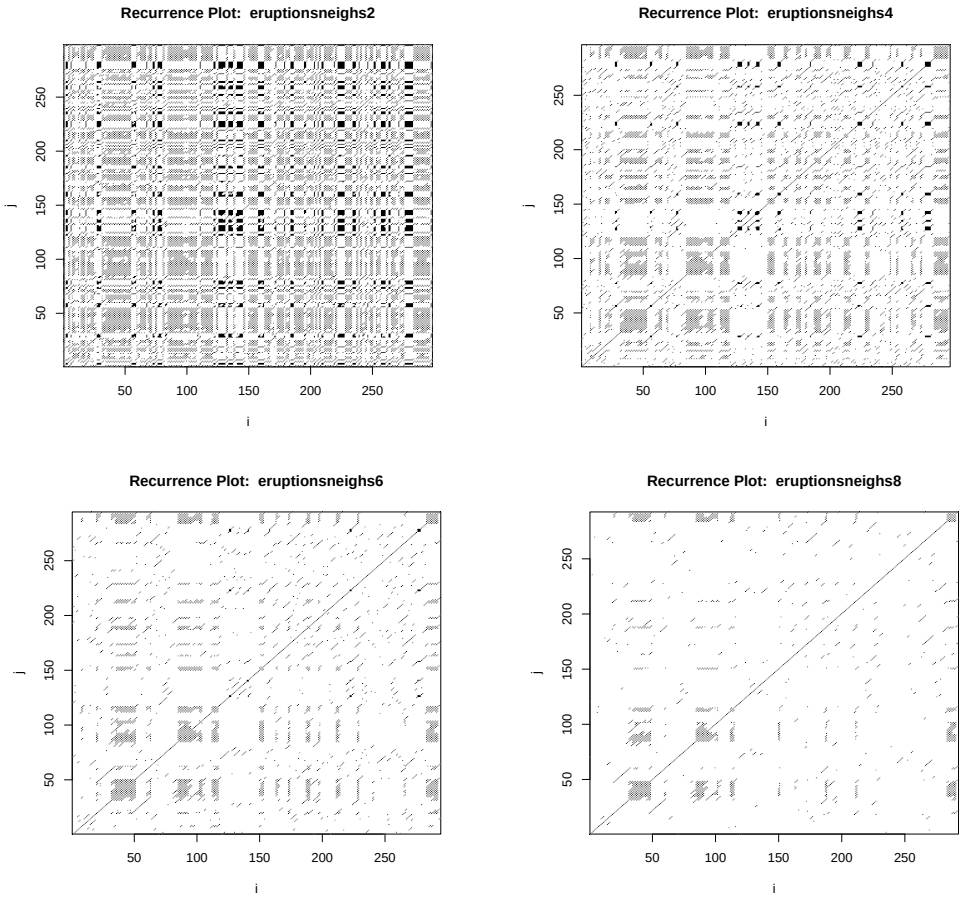

FIGURE 32. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=2, 4, 6, 8.

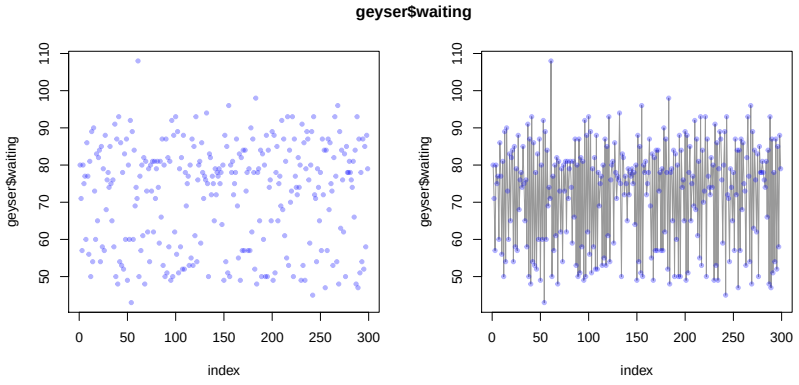

FIGURE 33. Example case: Old Faithful Geyser waiting. Signal and linear interpolation. Time used: 0.533 sec.

```
DIV: 0.053
Trend: 0  Entropy: 1.002
Diagonal lines max: 19  Mean: 2.878  Mean off main: 2.688
Vertical lines max: 3  Mean: 2.315
```

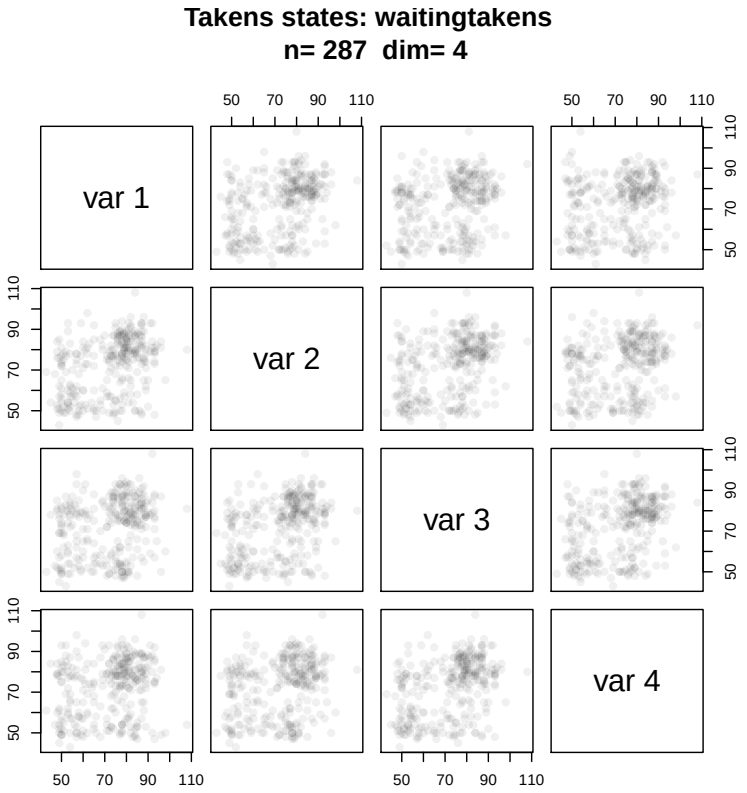

FIGURE 34. Example case: Old Faithful Geyser waiting. Time used: 0.29 sec.

See figure 35.

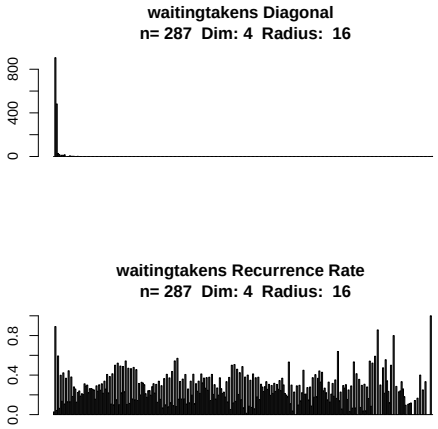

FIGURE 35. Recurrence Plot RQA. Example case: Old Faithful Geyser waiting. Time used: 0.183 sec.

Input

```
load(file="waitingneighs.RData")
local.recurrencePlotAux(waitingneighs)
```

See figure 36.

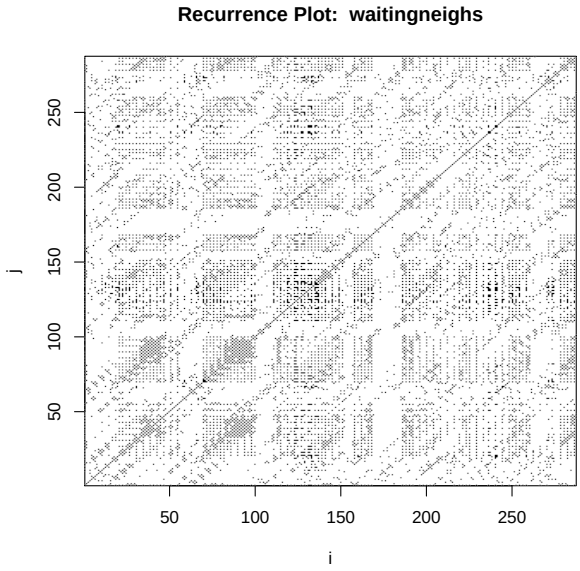

FIGURE 36. Recurrence Plot. Example case: Old Faithful Geyser waiting. Time used: 0.4 sec.

7.4. **Geyser - linearized.** So far, *nonlinearTseries* only handles multivariate data by FORTRAN conventions, using a lag parameter.

As a hack, we transform the data to FORTRAN conventions.

Input

```
geyserlin <- t(geyser)
dim(geyserlin)<-NULL
dimnames(geyserlin)<-NULL
```

Now duration and waiting are mixed. A $lag = 2$ separates the dimension again. The Taken states iterate over the index, giving alternating a duration and waiting state.

7.5. **Geyser Eruptions linearized.**

Input

```
plotsignal(geyserlin)
```

See figure 37 on the next page.

Input

```
glineruptionstakens4 <-
        local.buildTakens( time.series=geyserlin,
                embedding.dim=4, time.lag=2)
statepairs(glineruptionstakens4) #dim=4
```

See figure 38 on the facing page.

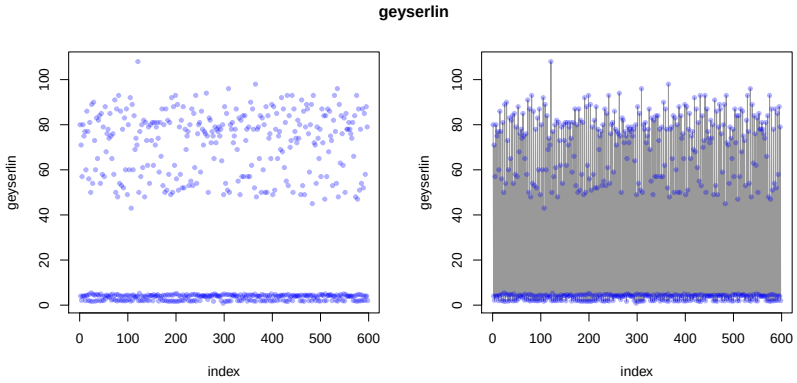

FIGURE 37. Example case: Old Faithful Geyser eruptions. Signal and linear interpolation.

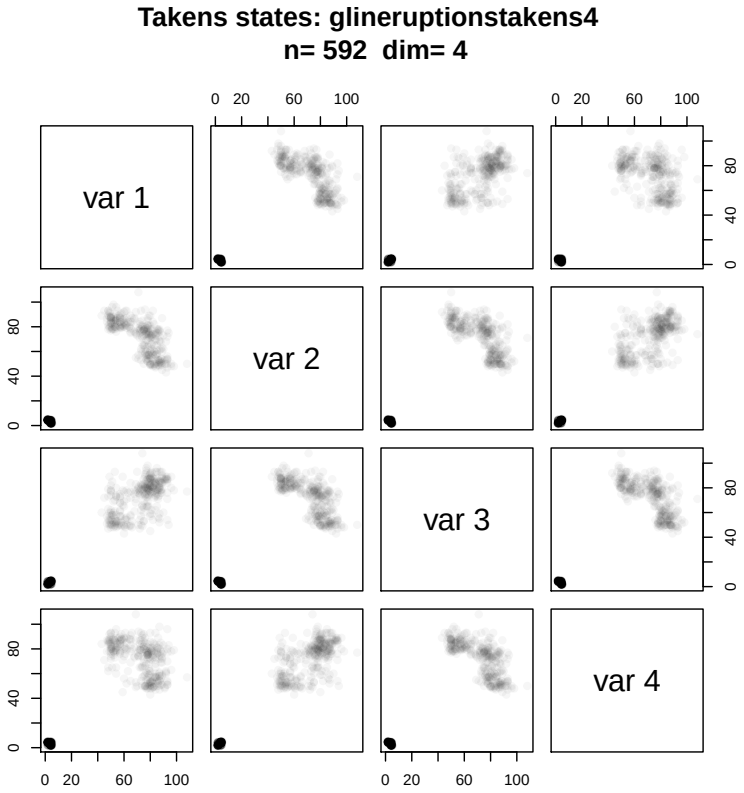

FIGURE 38. Example case: Old Faithful Geyser eruptions. Time used: 0.476 sec.

Input

```
glineruptionsneighs4<-local.findAllNeighbours(glineruptionstakens4, radius=0.8)
save(glineruptionsneighs4, file="glineruptionsneighs4.RData")
```

Input

```
showrqa(glineruptionstakens4, radius=0.8)
```

Output

```
glineruptionstakens4  n= 592  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.029  log(REC)/log(R): 15.901
Determinism: 0.059  Laminarity: 0
DIV: Inf
Trend: 0  Entropy: 0
Diagonal lines max: 0  Mean: 592  Mean off main: NaN
Vertical lines max: 0  Mean: 0
```

See figure 39.

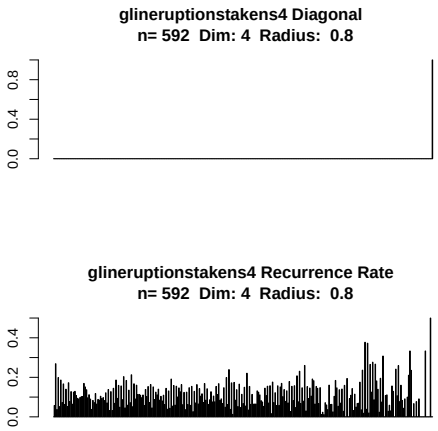

FIGURE 39. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.335 sec.

Input

```
load(file="glineruptionsneighs4.RData")
local.recurrencePlotAux(glineruptionsneighs4)
```

See figure 40 on the facing page.

7.5.1. *Geyser eruptions - linearized. Dim=2.*

Input

```
glineruptionstakens2 <-
        local.buildTakens(time.series=geyserlin,
                embedding.dim=2, time.lag=2)
statepairs(glineruptionstakens2) #dim=2
```

See figure 41 on the next page.

Input

```
glineruptionsneighs2<-local.findAllNeighbours(glineruptionstakens2, radius=0.8)
save(glineruptionsneighs2, file="glineruptionsneighs2.RData")
```

Input

```
load(file="glineruptionsneighs2.RData")
local.recurrencePlotAux(glineruptionsneighs2)
```

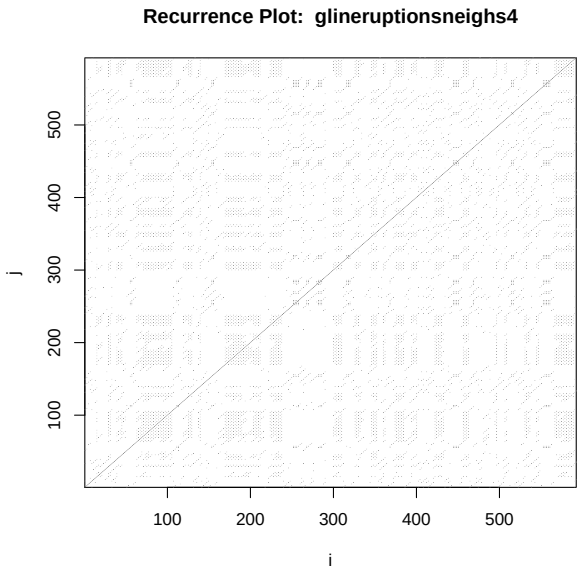

FIGURE 40. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.644 sec.

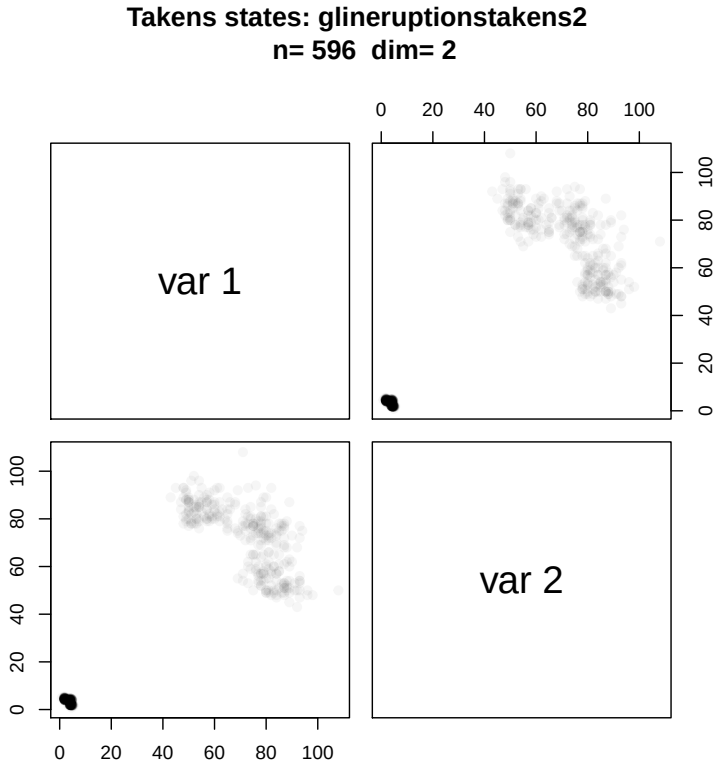

FIGURE 41. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.157 sec.

See figure 42.

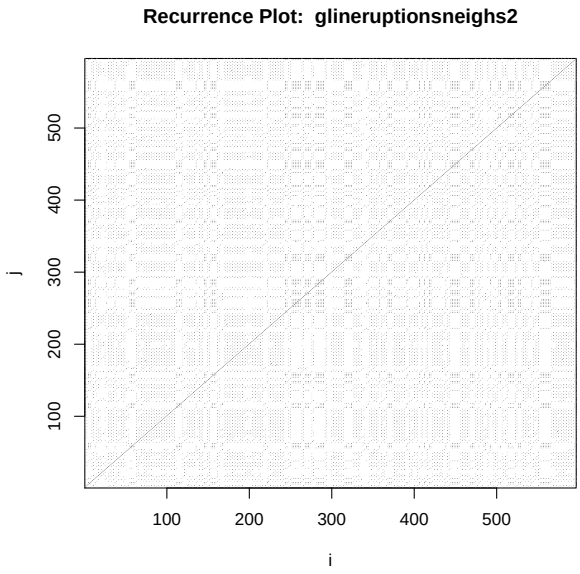

FIGURE 42. Recurrence Plot. Example case: Old Faithful Geyser eruptions linearized. Dim=2. Time used: 0.378 sec.

7.5.2. *Geyser eruptions - linearized. Dim=8.*

Input

```
glineruptionstakens8 <- local.buildTakens( time.series=geyserlin,
        embedding.dim=8,time.lag=2)
statepairs(glineruptionstakens8) #dim=8
```

See figure 43 on the next page

Input

```
glineruptionsneighs8<-local.findAllNeighbours(glineruptionstakens8,
        radius=0.8)
save(glineruptionsneighs8, file="glineruptionsneighs8.RData")
```

Input

```
load(file="glineruptionsneighs8.RData")
local.recurrencePlotAux(glineruptionsneighs8)
```

See figure 44 on page 40.

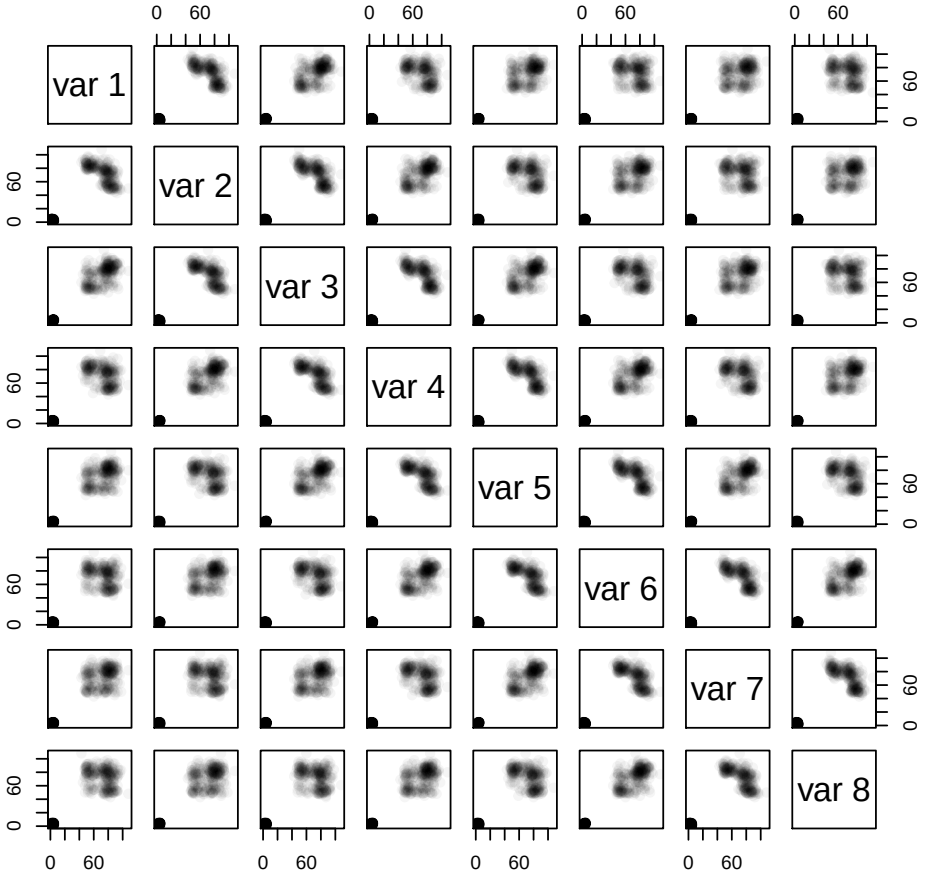

FIGURE 43. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 1.541 sec.

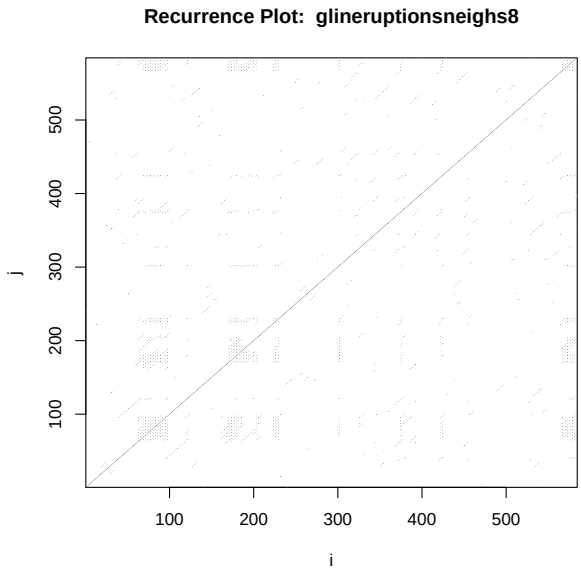

FIGURE 44. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.253 sec.

8. Case Study: HRV data example.beats

Only 1024 data points used in this section

Input

```
library(RHRV)
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVData.rda")
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVProcessedData.rda")
#####################################################
### code chunk number 1: creation
#####################################################
hrv.data  = CreateHRVData()
hrv.data = SetVerbose(hrv.data, TRUE )
#####################################################
### code chunk number 3: loading
#####################################################
hrv.data = LoadBeatAscii(hrv.data, "example.beats",
       RecordPath = "/users/gs/projects/rforge/rhrv/tutorial/beatsFolder")
```

Output

```
** Loading beats positions for record: example.beats **
   Path: /users/gs/projects/rforge/rhrv/tutorial/beatsFolder
   Scale: 1
   Date: 01/01/1900
   Time: 00:00:00
   Number of beats: 17360
```

Input

```
#       RecordPath = "beatsFolder")


#####################################################
### code chunk number 4: derivating
#####################################################
hrv.data = BuildNIHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 17360
```

Input

Input

```
plotsignal(hrv.data$Beat$RR)
```

See figure 45 on the next page.

ToDo: We have outliers at approximately 2*RR. Could this be an artefact of preprocessing, filtering out too many impulses?

Input

```
hrvRRtakens4 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=4,time.lag=1)
statepairs(hrvRRtakens4) #dim=4
```

See figure 46 on the following page.

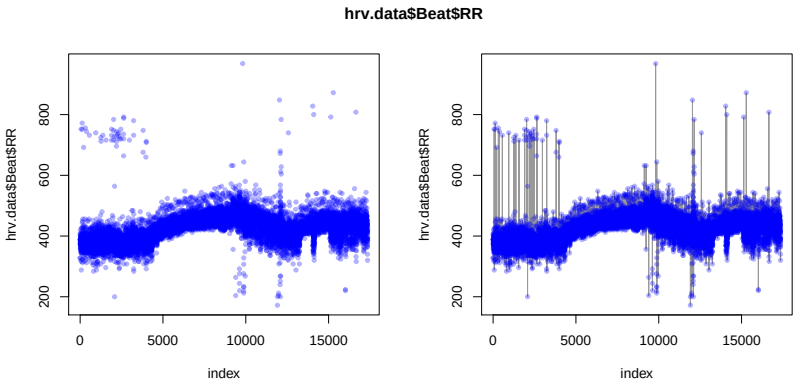

FIGURE 45. RHRV tutorial example.beats. Signal and linear interpolation.

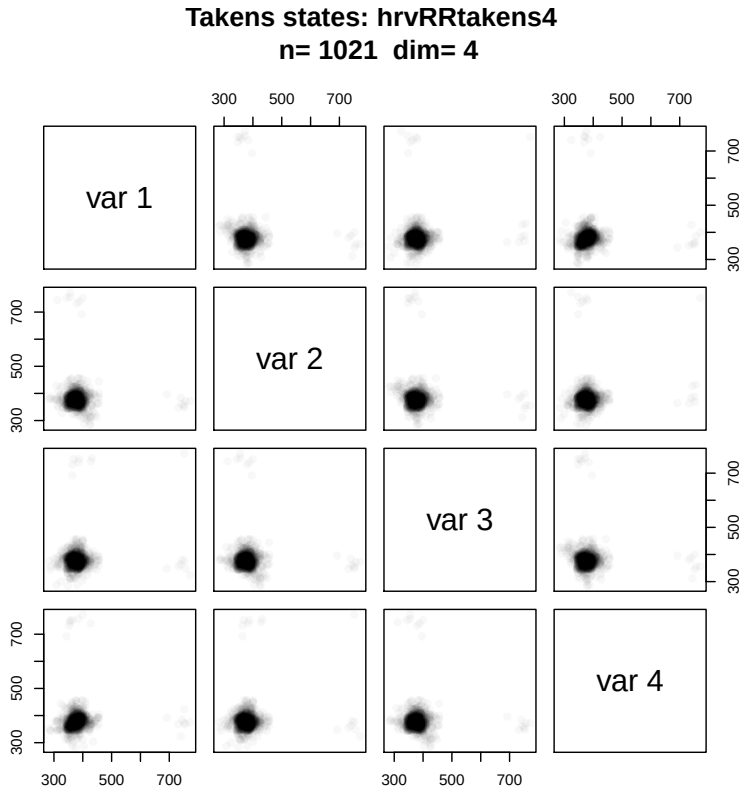

FIGURE 46. RHRV tutorial example.beats. Time used: 0.623 sec.

Input
```
statepairs(hrvRRtakens4, rank=TRUE) #dim=4
```

See figure 47 on the next page.

Input
```
hrvRRneighs4 <-local.findAllNeighbours(hrvRRtakens4, radius=16)
save(hrvRRneighs4, file="hrvRRneighs4.Rdata")
```

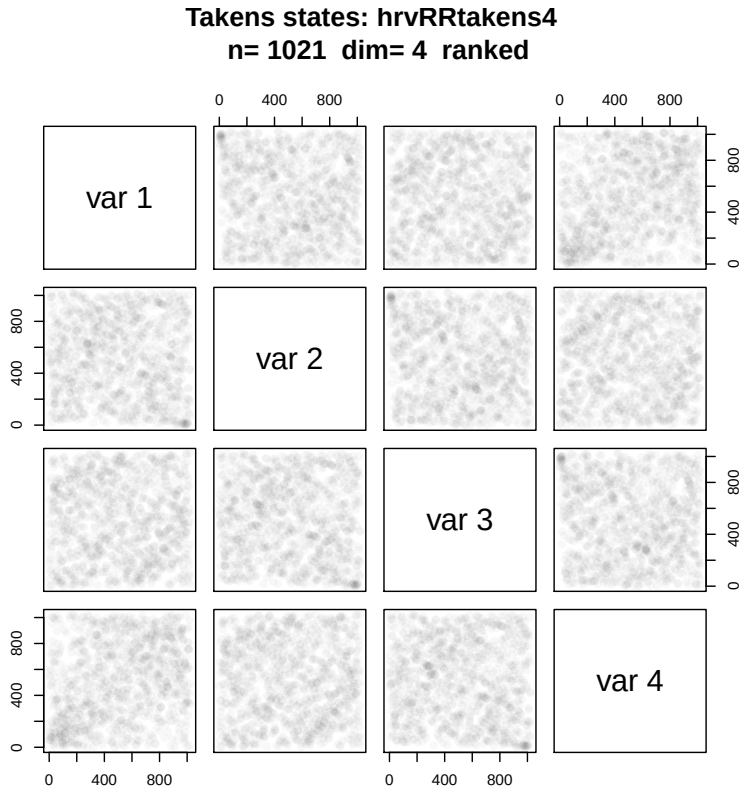

FIGURE 47. RHRV tutorial example.beats. Ranked data. Time used: 1.343 sec.

Time used: 0.13 sec.

Input

```
load(file="hrvRRneighs4.RData")
local.recurrencePlotAux(hrvRRneighs4)
```

See figure 48 on the following page.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

8.1. **RHRV: Comparison by Dimension.**

Input

```
hrvRRtakens2 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrvRRneighs2 <-local.findAllNeighbours(hrvRRtakens2, radius=16)
save(hrvRRneighs2, file="hrvRRneighs2.Rdata")
```

Time used: 0.164 sec.

Input

```
load(file="hrvRRneighs2.RData")
local.recurrencePlotAux(hrvRRneighs2)
```

See figure 49 on page 46. Time used: 1.572 sec.

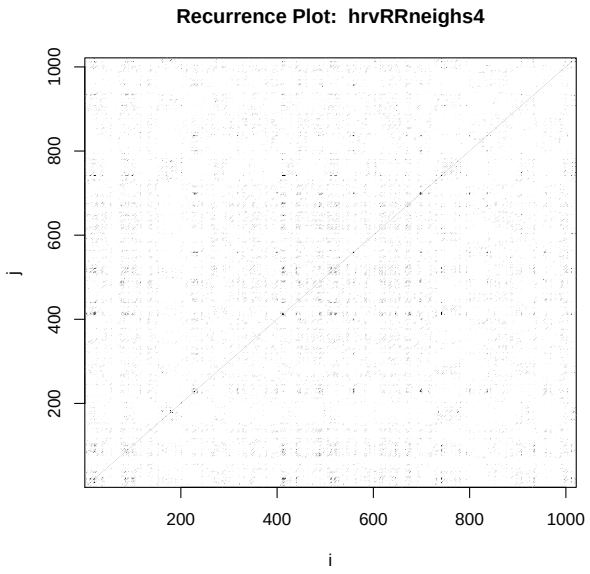

FIGURE 48. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=4. Time used: 0.762 sec.

Input

```
hrvRRtakens6 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=6,time.lag=1)
hrvRRneighs6 <-local.findAllNeighbours(hrvRRtakens6, radius=16)
save(hrvRRneighs6, file="hrvRRneighs6.Rdata")
```

Time used: 0.229 sec.

Input

```
load(file="hrvRRneighs6.RData")
local.recurrencePlotAux(hrvRRneighs6)
```

Dim=6. Time used: 0.774 sec.

Input

```
hrvRRtakens8 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=8,time.lag=1)
hrvRRneighs8 <-local.findAllNeighbours(hrvRRtakens8, radius=32)
save(hrvRRneighs8, file="hrvRRneighs8.Rdata")
```

Time used: 0.361 sec.

Input

```
load(file="hrvRRneighs8.RData")
local.recurrencePlotAux(hrvRRneighs8)
```

Dim=8. Time used: 0.937 sec.

Input

```
hrvRRtakens12 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrvRRneighs12 <-local.findAllNeighbours(hrvRRtakens12, radius=16)
save(hrvRRneighs12, file="hrvRRneighs12.Rdata")
```

Time used: 1.25 sec.

Input

```
load(file="hrvRRneighs12.RData")
local.recurrencePlotAux(hrvRRneighs12)
```

Time used: 1.481 sec.

Input

```
hrvRRtakens16 <- local.buildTakens(
        time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=16,time.lag=1)
hrvRRneighs16 <-local.findAllNeighbours(hrvRRtakens16, radius=32)
save(hrvRRneighs16, file="hrvRRneighs16.Rdata")
```

Time used: 1.881 sec.

Input

```
load(file="hrvRRneighs16.RData")
local.recurrencePlotAux(hrvRRneighs16)
```

Time used: 0.695 sec.

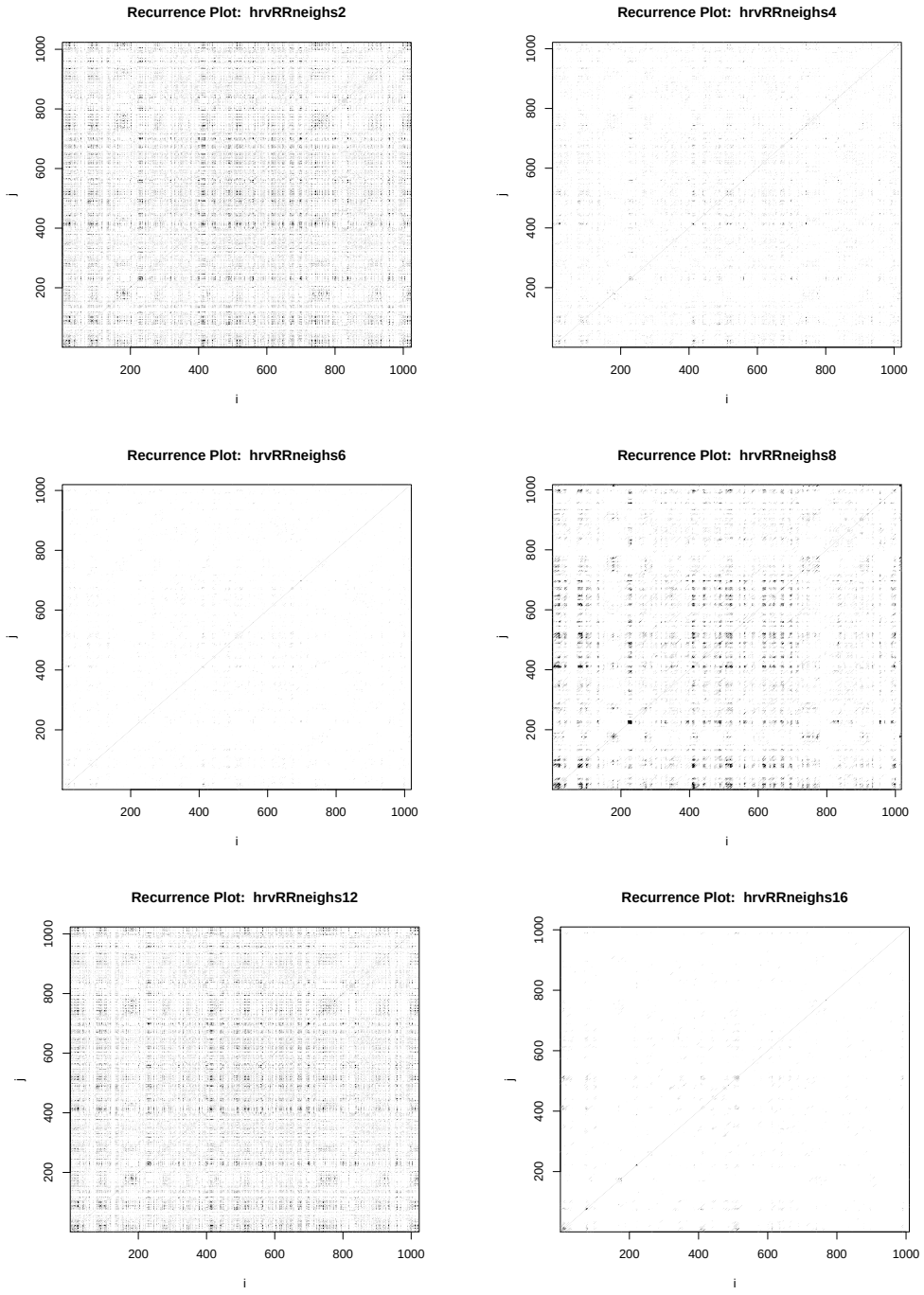

FIGURE 49. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 0.695 sec.

ToDo: Consider using differences

8.2. **Hart Rate Variation.** Since we are not interested in heart rate (or pulse), but in heart rate variation, a proposal is to use scaled differences.

Input

```
# source('/users/gs/projects/rforge/rhrv/pkg/R/BuildNIHR2.R', chdir = TRUE)
BuildNIDHR <-
function(HRVData, verbose=NULL) {
#------------------------------------------------------
# Obtains instantaneous heart rate variation from beats positions
# D for difference
#------------------------------------------------------
        if (!is.null(verbose)) {
                cat("  --- Warning: deprecated argument, using SetVerbose() instead ---\n ",
                        "   --- See help for more information!! ---\n")
                SetVerbose(HRVData,verbose)
        }

        if (HRVData$Verbose) {
                cat("** Calculating non-interpolated heart rate differences **\n")
        }

        if (is.null(HRVData$Beat$Time)) {
                cat("   --- ERROR: Beats positions not present... Impossible to calculate Heart Rate!! -
                return(HRVData)
        }

        NBeats=length(HRVData$Beat$Time)
        if (HRVData$Verbose) {
                cat("   Number of beats:",NBeats,"\n");
        }

  # addition gs
   #using NA, not constant extrapolation as else in RHRV
   #drr=c(NA,NA,1000.0*        diff(HRVData$Beat$Time, lag=1 , differences=2))
   HRVData$Beat$dRR=c(NA, NA,
           1000.0*diff(HRVData$Beat$Time, lag=1, differences=2))

   HRVData$Beat$avRR=(c(NA,HRVData$Beat$RR[-1])+HRVData$Beat$RR)/2

   HRVData$Beat$HRRV <- HRVData$Beat$dRR/HRVData$Beat$avRR

        return(HRVData)
}
```

differences for HRV

Input

```
hrv.data <- BuildNIDHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate differences **
   Number of beats: 17360
```

Input

```
HRRV <- hrv.data$Beat$HRRV
```

These are the displays of the Takens state space we used before, now for HRRV:

Input
```
plotsignal(HRRV)
```

See figure 50,

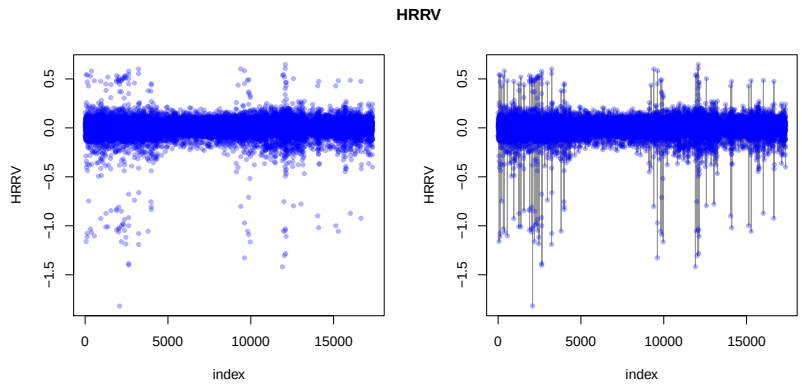

FIGURE 50. RHRV tutorial example.beats. HRRV Signal and linear interpolation.

Only 1024 data points used in this section

Input
```
hrvRRVtakens4 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=4,time.lag=1)
statepairs(hrvRRVtakens4) #dim=4
```

See figure 51 on the next page

Input
```
statepairs(hrvRRVtakens4, rank=TRUE) #dim=4
```

ToDo: findAllNeighbours does not handle NAs

See figure 52 on page 50

Input
```
#use hack: findAllNeighbours does not handle NAs
hrvRRVneighs4 <-local.findAllNeighbours(hrvRRVtakens4[-(1:2),], radius=0.125)
save(hrvRRVneighs4, file="hrvRRVneighs4.Rdata")
```

Time used: 0.257 sec.

Input
```
load(file="hrvRRVneighs4.RData")
local.recurrencePlotAux(hrvRRVneighs4,dim=4, radius=0.125)
```

ToDo: check. There seem to be strange artefacts.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

ToDo: fix default setting for radius.

8.3. RHRV Variation: Comparison by Dimension.

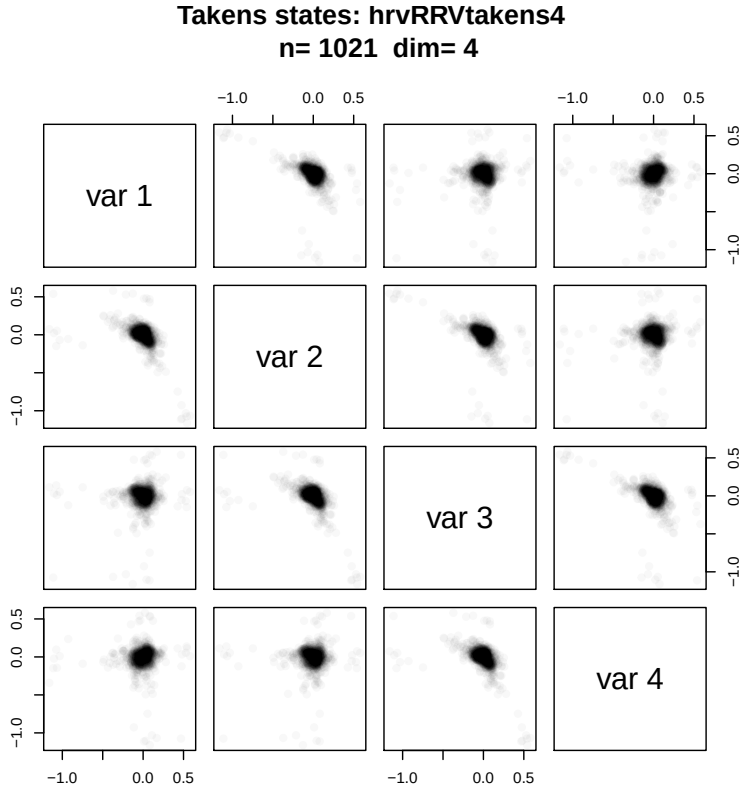

FIGURE 51. RHRV tutorial example.beats. HRRV Time used: 0.758 sec.

Input

```
hrvRRVtakens2 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=2, time.lag=1)
hrvRRVneighs2 <-local.findAllNeighbours(hrvRRVtakens2[-(1:2),],
        radius=0.125)
save(hrvRRVneighs2, file="hrvRRVneighs2.Rdata")
```

Time used: 0.531 sec.

Input

```
showrqa(hrvRRVtakens2[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens2[-(1:2), ]  n= 1021  Dim: 2
Radius: 0.125  Recurrence coverage REC: 0.515  log(REC)/log(R): 0.319
Determinism: 0.939  Laminarity: 0.81
DIV: 0.027
Trend: 0  Entropy: 2.346
Diagonal lines max: 37  Mean: 5.373  Mean off main: 5.362
Vertical lines max: 48  Mean: 3.998
```

Input

```
load(file="hrvRRVneighs2.RData")
local.recurrencePlotAux(hrvRRVneighs2, dim=2,  radius=0.125)
```

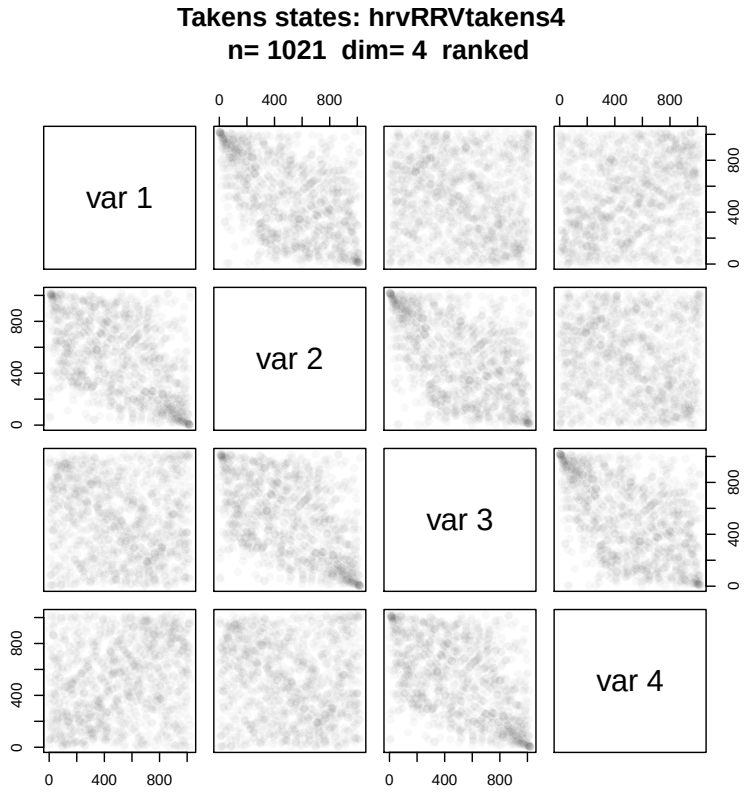

FIGURE 52. RHRV tutorial example.beats. Ranked HRRV data. Time used: 1.501 sec.

Time used: 3.642 sec.

Input

```
hrvRRVtakens6 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=6,time.lag=1)
hrvRRVneighs6 <-local.findAllNeighbours(hrvRRVtakens6[-(1:2),], radius=0.125)
save(hrvRRVneighs6, file="hrvRRVneighs6.Rdata")
```

Time used: 0.53 sec.

Input

```
showrqa(hrvRRVtakens6[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens6[-(1:2), ]  n= 1017  Dim: 6
Radius: 0.125  Recurrence coverage REC: 0.179  log(REC)/log(R): 0.827
Determinism: 0.943  Laminarity: 0.451
DIV: 0.03
Trend: 0  Entropy: 2.305
Diagonal lines max: 33  Mean: 5.243  Mean off main: 5.213
Vertical lines max: 22  Mean: 2.887
```

Input

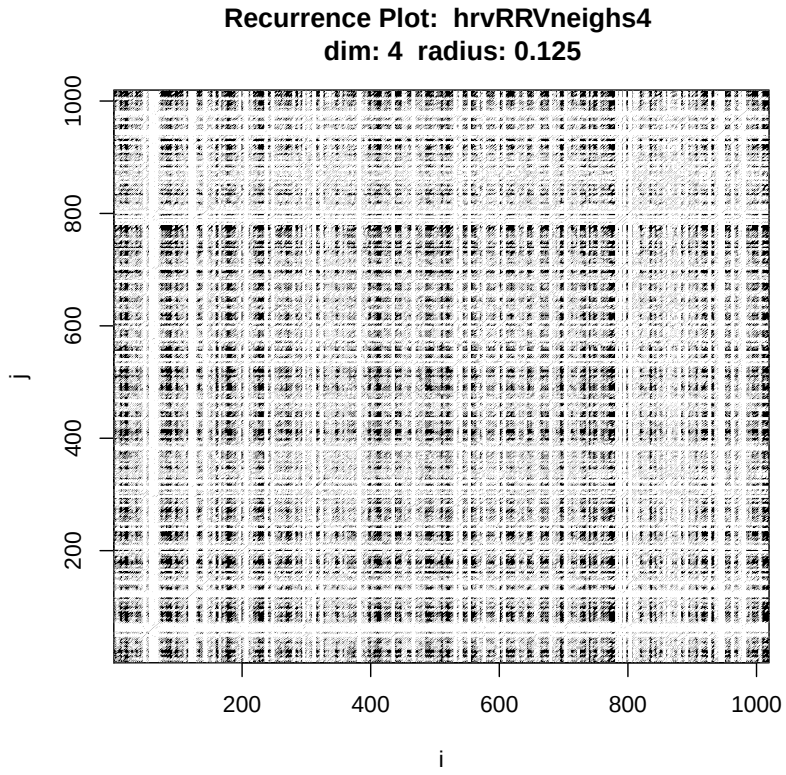

FIGURE 53. Recurrence Plot. Example case: RHRV tutorial example.beats. HRRV Dim=4. Time used: 2.293 sec.

```
load(file="hrvRRVneighs6.RData")
local.recurrencePlotAux(hrvRRVneighs6, dim=6, radius=0.125)
```

Dim=6. Time used: 1.632 sec.

Input

```
hrvRRVtakens8 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=8,time.lag=1)
hrvRRVneighs8 <-local.findAllNeighbours(hrvRRVtakens8[-(1:2),], radius=0.125)
save(hrvRRVneighs8, file="hrvRRVneighs8.Rdata")
```

Time used: 0.395 sec.

Input

```
showrqa(hrvRRVtakens8[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens8[-(1:2), ]  n= 1015  Dim: 8
Radius: 0.125  Recurrence coverage REC: 0.105  log(REC)/log(R): 1.084
Determinism: 0.943  Laminarity: 0.337
DIV: 0.032
Trend: 0  Entropy: 2.329
Diagonal lines max: 31  Mean: 5.361  Mean off main: 5.308
Vertical lines max: 17  Mean: 2.715
```

Input

```
load(file="hrvRRVneighs8.RData")
local.recurrencePlotAux(hrvRRVneighs8, dim=8, radius=0.125)
```

Dim=8. Time used: 1.242 sec.

Input

```
hrvRRVtakens12 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=12,time.lag=1)
hrvRRVneighs12 <-
        local.findAllNeighbours(hrvRRVtakens12[-(1:2),], radius=3/16)
save(hrvRRVneighs12, file="hrvRRVneighs12.Rdata")
```

Time used: 1.776 sec.

Input

```
showrqa(hrvRRVtakens12[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrvRRVtakens12[-(1:2), ]  n= 1011  Dim: 12
Radius: 0.1875  Recurrence coverage REC: 0.235  log(REC)/log(R): 0.865
Determinism: 0.988  Laminarity: 0.635
DIV: 0.015
Trend: 0  Entropy: 3.075
Diagonal lines max: 68  Mean: 9.775  Mean off main: 9.733
Vertical lines max: 52  Mean: 3.656
```

Input

```
load(file="hrvRRVneighs12.RData")
local.recurrencePlotAux(hrvRRVneighs12, dim=12,  radius=3/16)
```

Time used: 1.784 sec.

Input

```
hrvRRVtakens16 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=16,time.lag=1)
hrvRRVneighs16 <-local.findAllNeighbours(hrvRRVtakens16[-(1:2),], radius=3/16)
save(hrvRRVneighs16, file="hrvRRVneighs16.Rdata")
```

Time used: 2.282 sec.

Input

```
showrqa(hrvRRVtakens16[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrvRRVtakens16[-(1:2), ]  n= 1007  Dim: 16
Radius: 0.1875  Recurrence coverage REC: 0.147  log(REC)/log(R): 1.144
Determinism: 0.99  Laminarity: 0.527
DIV: 0.016
Trend: 0  Entropy: 3.163
Diagonal lines max: 64  Mean: 10.594  Mean off main: 10.523
Vertical lines max: 40  Mean: 3.114
```

Input

```
load(file="hrvRRVneighs16.RData")
local.recurrencePlotAux(hrvRRVneighs16, dim=16, radius=3/16)
```

Time used: 1.451 sec.

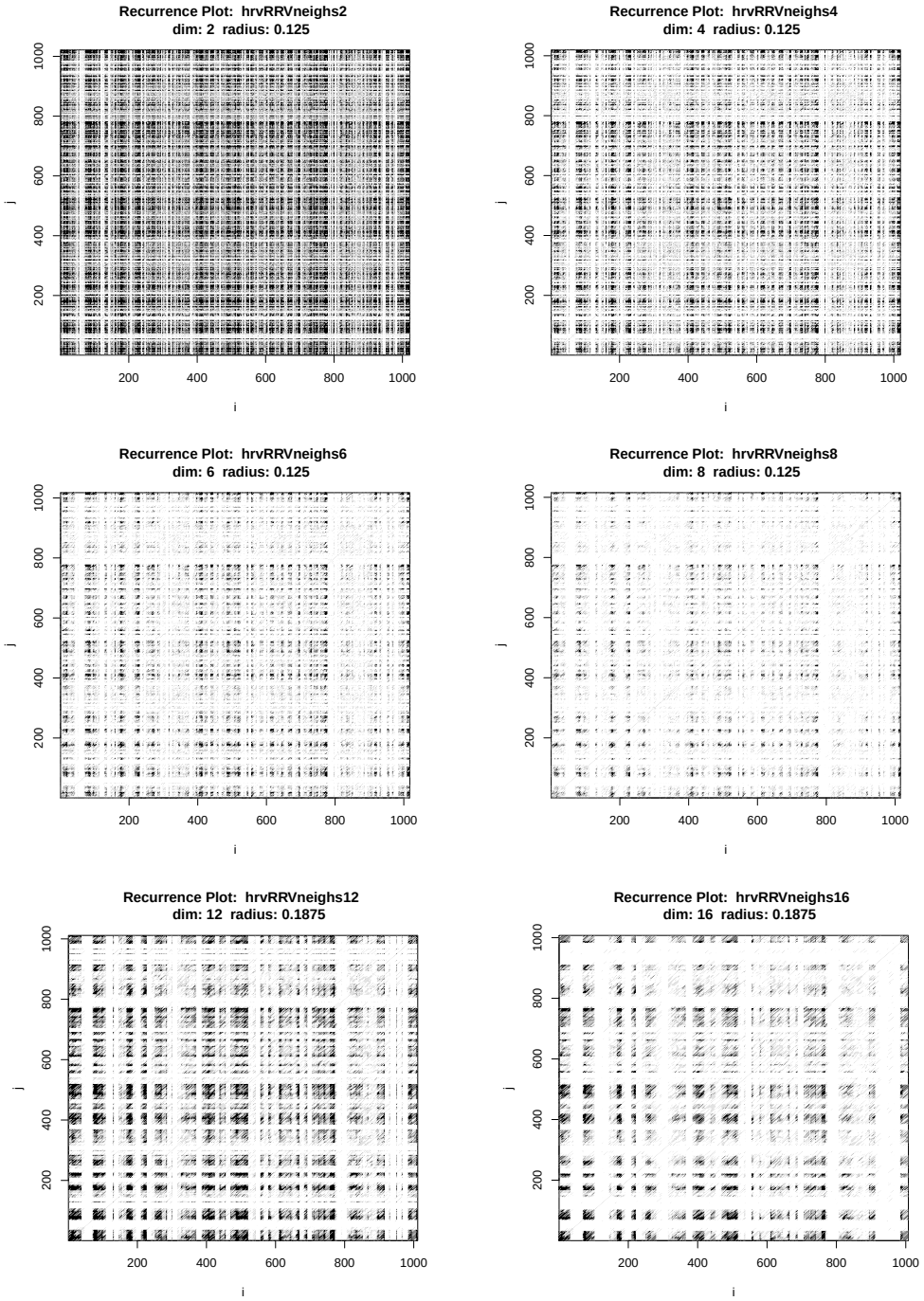

FIGURE 54. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 1.451 sec.

9. Case Study: HRV data example2.beats

Only 1024 data points used in this section

This is a copy of the previous section, now applied to HRV data example2.beats.

Input

```
library(RHRV)
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVData.rda")
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVProcessedData.rda")
#####################################################
### code chunk number 1: creation
#####################################################
hrv2.data  = CreateHRVData()
hrv2.data = SetVerbose(hrv2.data, TRUE )
#####################################################
### code chunk number 3: loading
#####################################################
hrv2.data = LoadBeatAscii(hrv2.data, "example2.beats",
       RecordPath = "/users/gs/projects/rforge/rhrv/tutorial/beatsFolder")
```

Output

```
** Loading beats positions for record: example2.beats **
   Path: /users/gs/projects/rforge/rhrv/tutorial/beatsFolder
   Scale: 1
   Removed 2437 duplicated beats
   Date: 01/01/1900
   Time: 00:00:00
   Number of beats: 2437
```

Input

```
#       RecordPath = "beatsFolder")


#####################################################
### code chunk number 4: derivating
#####################################################
hrv2.data = BuildNIHR(hrv2.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 2437
```

Input

Input

```
plotsignal(hrv2.data$Beat$RR)
```

ToDo: We have outliers at approximately 2*RR. Could this be an artefact of preprocessing, filtering out too many impulses?

See figure 55 on the next page.

Input

```
hrv2RRtakens4 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=4,time.lag=1)
statepairs(hrv2RRtakens4) #dim=4
```

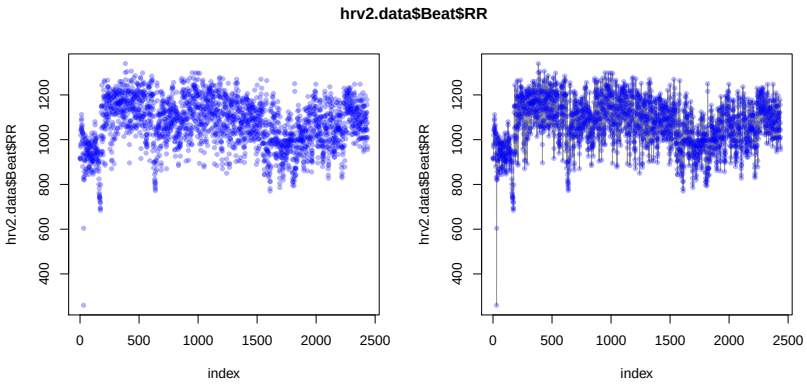

FIGURE 55. RHRV tutorial example2.beats. Signal and linear interpolation.

See figure 56.

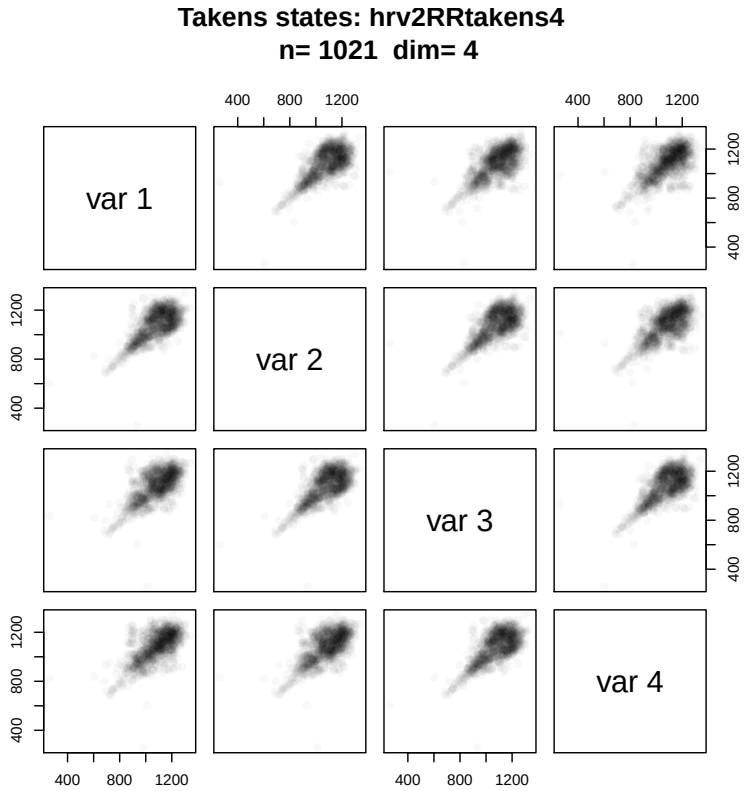

FIGURE 56. RHRV tutorial example2.beats. Time used: 0.692 sec.

Input

```
statepairs(hrv2RRtakens4, rank=TRUE) #dim=4
```

See figure 57 on the next page.

Input

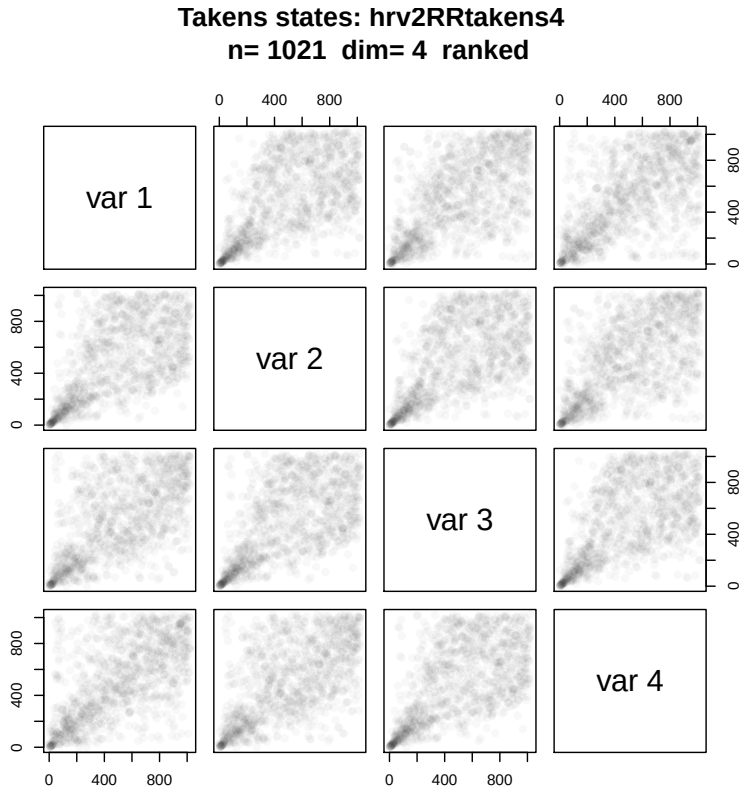

FIGURE 57. RHRV tutorial example2.beats. Ranked data. Time used: 1.413 sec.

```
hrv2RRneighs4 <-local.findAllNeighbours(hrv2RRtakens4, radius=16)
save(hrv2RRneighs4, file="hrv2RRneighs4.Rdata")
```

Time used: 0.076 sec.

Input

```
load(file="hrv2RRneighs4.RData")
local.recurrencePlotAux(hrv2RRneighs4)
```

See figure 58 on the facing page.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

9.1. **RHRV: Comparison by Dimension.**

Input

```
hrv2RRtakens2 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrv2RRneighs2 <-local.findAllNeighbours(hrv2RRtakens2, radius=16)
save(hrv2RRneighs2, file="hrv2RRneighs2.Rdata")
```

Time used: 0.128 sec.

Input

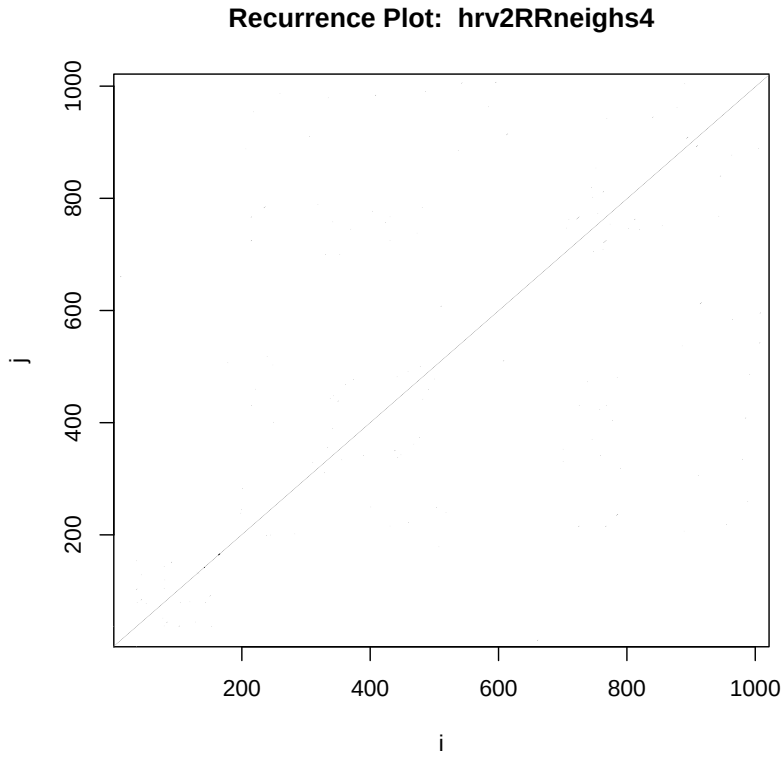

FIGURE 58. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=4. Time used: 0.55 sec.

```
load(file="hrv2RRneighs2.RData")
local.recurrencePlotAux(hrv2RRneighs2)
```

See figure 59 on page 59. Time used: 0.74 sec.

Input

```
hrv2RRtakens6 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=6,time.lag=1)
hrv2RRneighs6 <-local.findAllNeighbours(hrv2RRtakens6, radius=16)
save(hrv2RRneighs6, file="hrv2RRneighs6.Rdata")
```

Time used: 0.174 sec.

Input

```
load(file="hrv2RRneighs6.RData")
local.recurrencePlotAux(hrv2RRneighs6)
```

Dim=6. Time used: 0.79 sec.

Input

```
hrv2RRtakens8 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=8,time.lag=1)
hrv2RRneighs8 <-local.findAllNeighbours(hrv2RRtakens8, radius=32)
save(hrv2RRneighs8, file="hrv2RRneighs8.Rdata")
```

Time used: 0.172 sec.

Input

```
load(file="hrv2RRneighs8.RData")
local.recurrencePlotAux(hrv2RRneighs8)
```

Dim=8. Time used: 0.544 sec.

Input

```
hrv2RRtakens12 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrv2RRneighs12 <-local.findAllNeighbours(hrv2RRtakens12, radius=16)
save(hrv2RRneighs12, file="hrv2RRneighs12.Rdata")
```

Time used: 0.77 sec.

Input

```
load(file="hrv2RRneighs12.RData")
local.recurrencePlotAux(hrv2RRneighs12)
```

Time used: 0.755 sec.

Input

```
hrv2RRtakens16 <- local.buildTakens(
        time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=16,time.lag=1)
hrv2RRneighs16 <-local.findAllNeighbours(hrv2RRtakens16, radius=32)
save(hrv2RRneighs16, file="hrv2RRneighs16.Rdata")
```

Time used: 1.031 sec.

Input

```
load(file="hrv2RRneighs16.RData")
local.recurrencePlotAux(hrv2RRneighs16)
```

Time used: 0.647 sec.

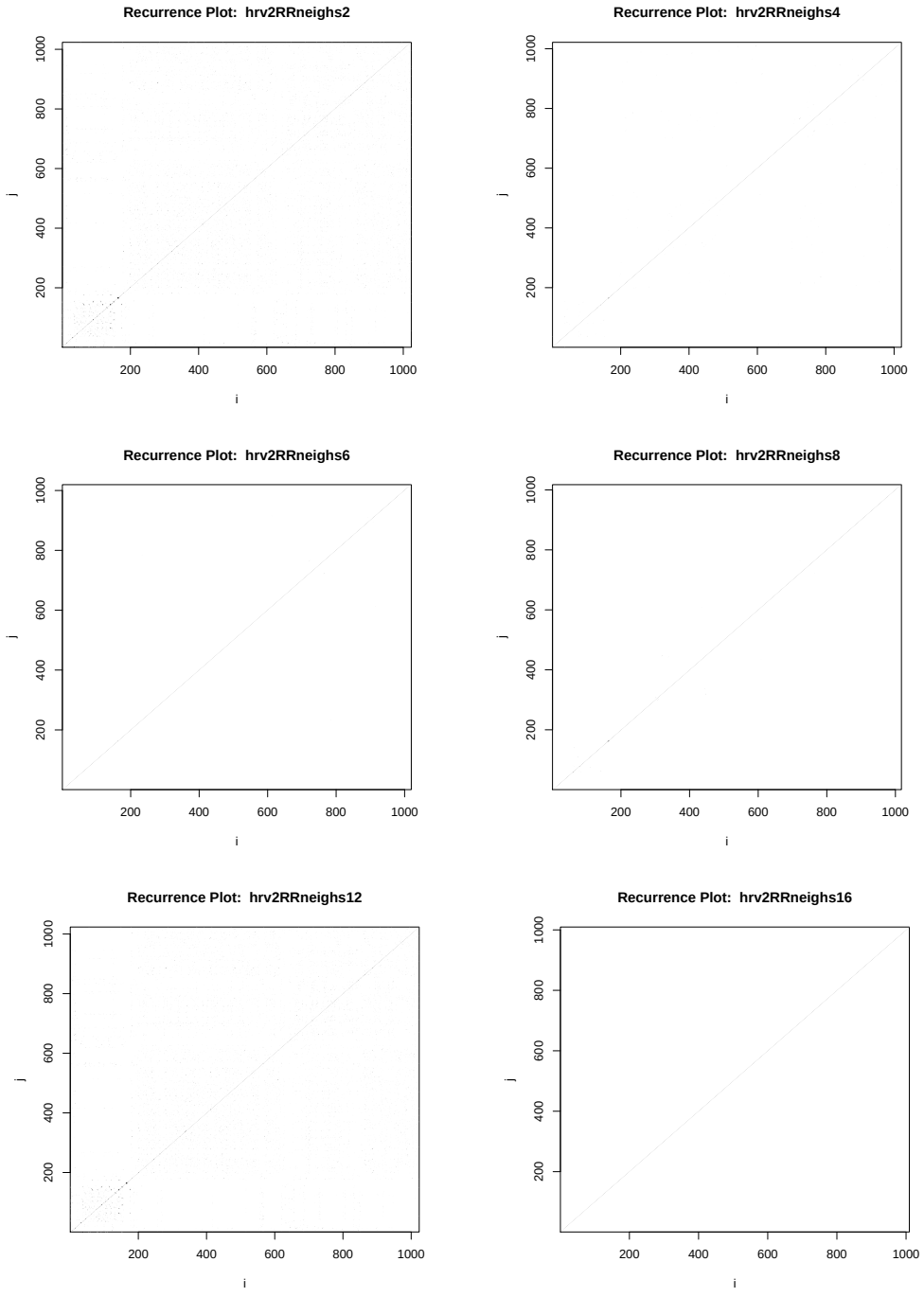

FIGURE 59. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 0.647 sec.

ToDo: Consider using differences

9.2. **Hart Rate Variation.** Since we are not interested in heart rate (or pulse), but in heart rate variation, a proposal is to use scaled differences.

Input

```
# source('/users/gs/projects/rforge/rhrv/pkg/R/BuildNIHR2.R', chdir = TRUE)
BuildNIDHR <-
function(HRVData, verbose=NULL) {
#------------------------------------------------------
# Obtains instantaneous heart rate variation from beats positions
# D for difference
#------------------------------------------------------
        if (!is.null(verbose)) {
                cat("  --- Warning: deprecated argument, using SetVerbose() instead ---\n ",
                        "   --- See help for more information!! ---\n")
                SetVerbose(HRVData,verbose)
        }

        if (HRVData$Verbose) {
                cat("** Calculating non-interpolated heart rate differences **\n")
        }

        if (is.null(HRVData$Beat$Time)) {
                cat("   --- ERROR: Beats positions not present... Impossible to calculate Heart Rate!! -
                return(HRVData)
        }

        NBeats=length(HRVData$Beat$Time)
        if (HRVData$Verbose) {
                cat("   Number of beats:",NBeats,"\n");
        }

  # addition gs
   #using NA, not constant extrapolation as else in RHRV
   #drr=c(NA,NA,1000.0*        diff(HRVData$Beat$Time, lag=1 , differences=2))
   HRVData$Beat$dRR=c(NA, NA,
           1000.0*diff(HRVData$Beat$Time, lag=1, differences=2))

   HRVData$Beat$avRR=(c(NA,HRVData$Beat$RR[-1])+HRVData$Beat$RR)/2

   HRVData$Beat$HRRV <- HRVData$Beat$dRR/HRVData$Beat$avRR

        return(HRVData)
}
```

differences for HRV

Input

```
hrv2.data <- BuildNIDHR(hrv2.data)
```

Output

```
** Calculating non-interpolated heart rate differences **
   Number of beats: 2437
```

Input

```
HRRV <- hrv2.data$Beat$HRRV
```

These are the displays of the Takens state space we used before, now for HRRV:

Input
```
plotsignal(HRRV)
```

See figure 60,

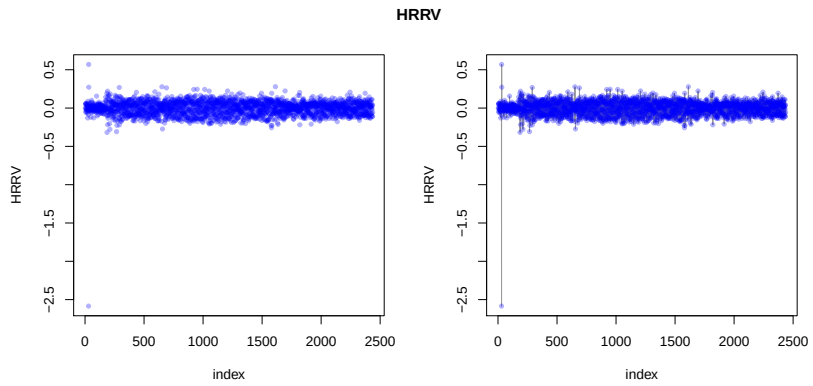

FIGURE 60. RHRV tutorial example2.beats. HRRV Signal and linear interpolation.

Only 1024 data points used in this section

Input
```
hrv2RRVtakens4 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=4,time.lag=1)
statepairs(hrv2RRVtakens4) #dim=4
```

See figure 61 on the next page

Input
```
statepairs(hrv2RRVtakens4, rank=TRUE) #dim=4
```

See figure 62 on page 63

ToDo: findAllNeighbours does not handle NAs

Input
```
#use hack: findAllNeighbours does not handle NAs
hrv2RRVneighs4 <-local.findAllNeighbours(hrv2RRVtakens4[-(1:2),], radius=0.125)
save(hrv2RRVneighs4, file="hrv2RRVneighs4.Rdata")
```

Time used: 0.258 sec.

Input
```
load(file="hrv2RRVneighs4.RData")
local.recurrencePlotAux(hrv2RRVneighs4,dim=4, radius=0.125)
```

ToDo: check. There seem to be strange artefacts.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

ToDo: fix default setting for radius.

9.3. RHRV Variation: Comparison by Dimension.

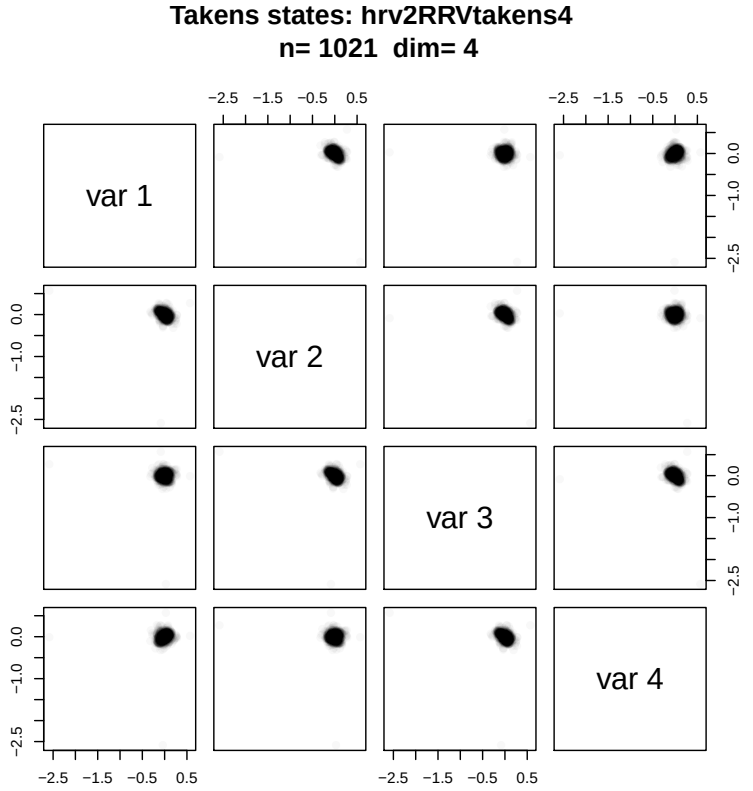

FIGURE 61. RHRV tutorial example2.beats. HRRV Time used: 0.74 sec.

Input

```
hrv2RRVtakens2 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=2, time.lag=1)
hrv2RRVneighs2 <-local.findAllNeighbours(hrv2RRVtakens2[-(1:2),],
        radius=0.125)
save(hrv2RRVneighs2, file="hrv2RRVneighs2.Rdata")
```

Time used: 0.549 sec.

Input

```
showrqa(hrv2RRVtakens2[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens2[-(1:2), ]  n= 1021  Dim: 2
Radius: 0.125  Recurrence coverage REC: 0.508  log(REC)/log(R): 0.326
Determinism: 0.913  Laminarity: 0.712
DIV: 0.009
Trend: 0  Entropy: 2.289
Diagonal lines max: 117  Mean: 5.305  Mean off main: 5.294
Vertical lines max: 86  Mean: 3.923
```

Input

```
load(file="hrv2RRVneighs2.RData")
local.recurrencePlotAux(hrv2RRVneighs2, dim=2,  radius=0.125)
```

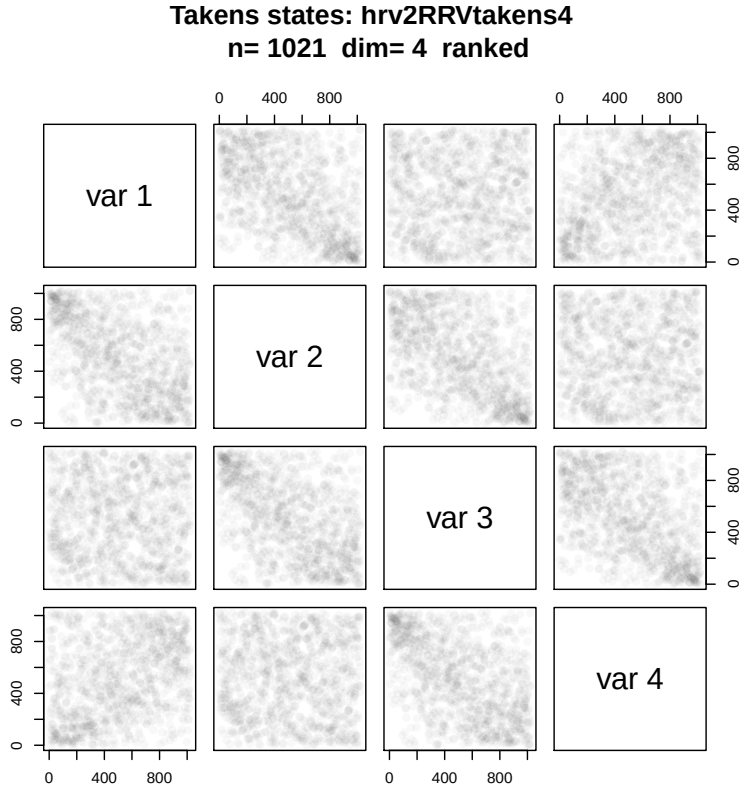

FIGURE 62. RHRV tutorial example2.beats. Ranked HRRV data. Time used: 1.51 sec.

Time used: 3.537 sec.

Input

```
hrv2RRVtakens6 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=6,time.lag=1)
hrv2RRVneighs6 <-local.findAllNeighbours(hrv2RRVtakens6[-(1:2),], radius=0.125)
save(hrv2RRVneighs6, file="hrv2RRVneighs6.Rdata")
```

Time used: 0.443 sec.

Input

```
showrqa(hrv2RRVtakens6[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens6[-(1:2), ]  n= 1017  Dim: 6
Radius: 0.125  Recurrence coverage REC: 0.187  log(REC)/log(R): 0.808
Determinism: 0.959  Laminarity: 0.368
DIV: 0.009
Trend: 0  Entropy: 2.528
Diagonal lines max: 113  Mean: 6.229  Mean off main: 6.195
Vertical lines max: 73  Mean: 3.978
```

Input

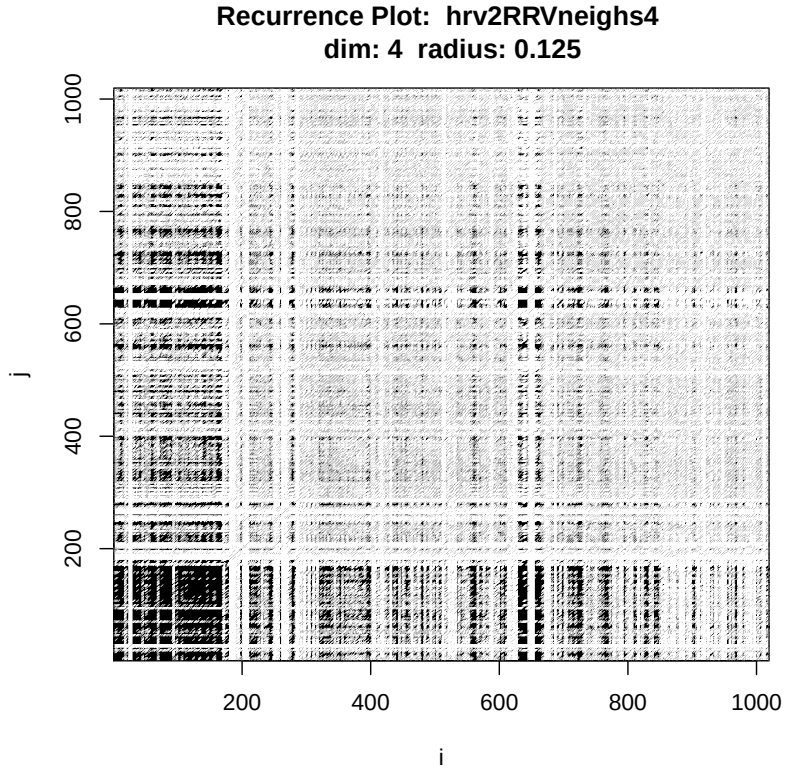

FIGURE 63. Recurrence Plot. Example case: RHRV tutorial example2.beats. HRRV Dim=4. Time used: 2.252 sec.

```
load(file="hrv2RRVneighs6.RData")
local.recurrencePlotAux(hrv2RRVneighs6, dim=6, radius=0.125)
```

Dim=6. Time used: 1.679 sec.

Input

```
hrv2RRVtakens8 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=8,time.lag=1)
hrv2RRVneighs8 <-local.findAllNeighbours(hrv2RRVtakens8[-(1:2),], radius=0.125)
save(hrv2RRVneighs8, file="hrv2RRVneighs8.Rdata")
```

Time used: 0.395 sec.

Input

```
showrqa(hrv2RRVtakens8[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens8[-(1:2), ]  n= 1015  Dim: 8
Radius: 0.125  Recurrence coverage REC: 0.122  log(REC)/log(R): 1.012
Determinism: 0.96  Laminarity: 0.335
DIV: 0.009
Trend: 0  Entropy: 2.57
Diagonal lines max: 111  Mean: 6.482  Mean off main: 6.428
Vertical lines max: 71  Mean: 4.039
```

Input

```
load(file="hrv2RRVneighs8.RData")
local.recurrencePlotAux(hrv2RRVneighs8, dim=8, radius=0.125)
```

Dim=8. Time used: 1.292 sec.

Input

```
hrv2RRVtakens12 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=12,time.lag=1)
hrv2RRVneighs12 <-
        local.findAllNeighbours(hrv2RRVtakens12[-(1:2),], radius=3/16)
save(hrv2RRVneighs12, file="hrv2RRVneighs12.Rdata")
```

Time used: 1.853 sec.

Input

```
showrqa(hrv2RRVtakens12[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens12[-(1:2), ]  n= 1011  Dim: 12
Radius: 0.1875  Recurrence coverage REC: 0.306  log(REC)/log(R): 0.708
Determinism: 0.993  Laminarity: 0.6
DIV: 0.007
Trend: 0  Entropy: 3.417
Diagonal lines max: 141  Mean: 13.443  Mean off main: 13.4
Vertical lines max: 143  Mean: 4.312
```

Input

```
load(file="hrv2RRVneighs12.RData")
local.recurrencePlotAux(hrv2RRVneighs12, dim=12,  radius=3/16)
```

Time used: 2.045 sec.

Input

```
hrv2RRVtakens16 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=16,time.lag=1)
hrv2RRVneighs16 <-local.findAllNeighbours(hrv2RRVtakens16[-(1:2),], radius=3/16)
save(hrv2RRVneighs16, file="hrv2RRVneighs16.Rdata")
```

Time used: 2.583 sec.

Input

```
showrqa(hrv2RRVtakens16[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens16[-(1:2), ]  n= 1007  Dim: 16
Radius: 0.1875  Recurrence coverage REC: 0.221  log(REC)/log(R): 0.902
Determinism: 0.994  Laminarity: 0.555
DIV: 0.007
Trend: 0  Entropy: 3.419
Diagonal lines max: 137  Mean: 13.706  Mean off main: 13.644
Vertical lines max: 89  Mean: 4.104
```

Input

```
load(file="hrv2RRVneighs16.RData")
local.recurrencePlotAux(hrv2RRVneighs16, dim=16, radius=3/16)
```

Time used: 1.588 sec.

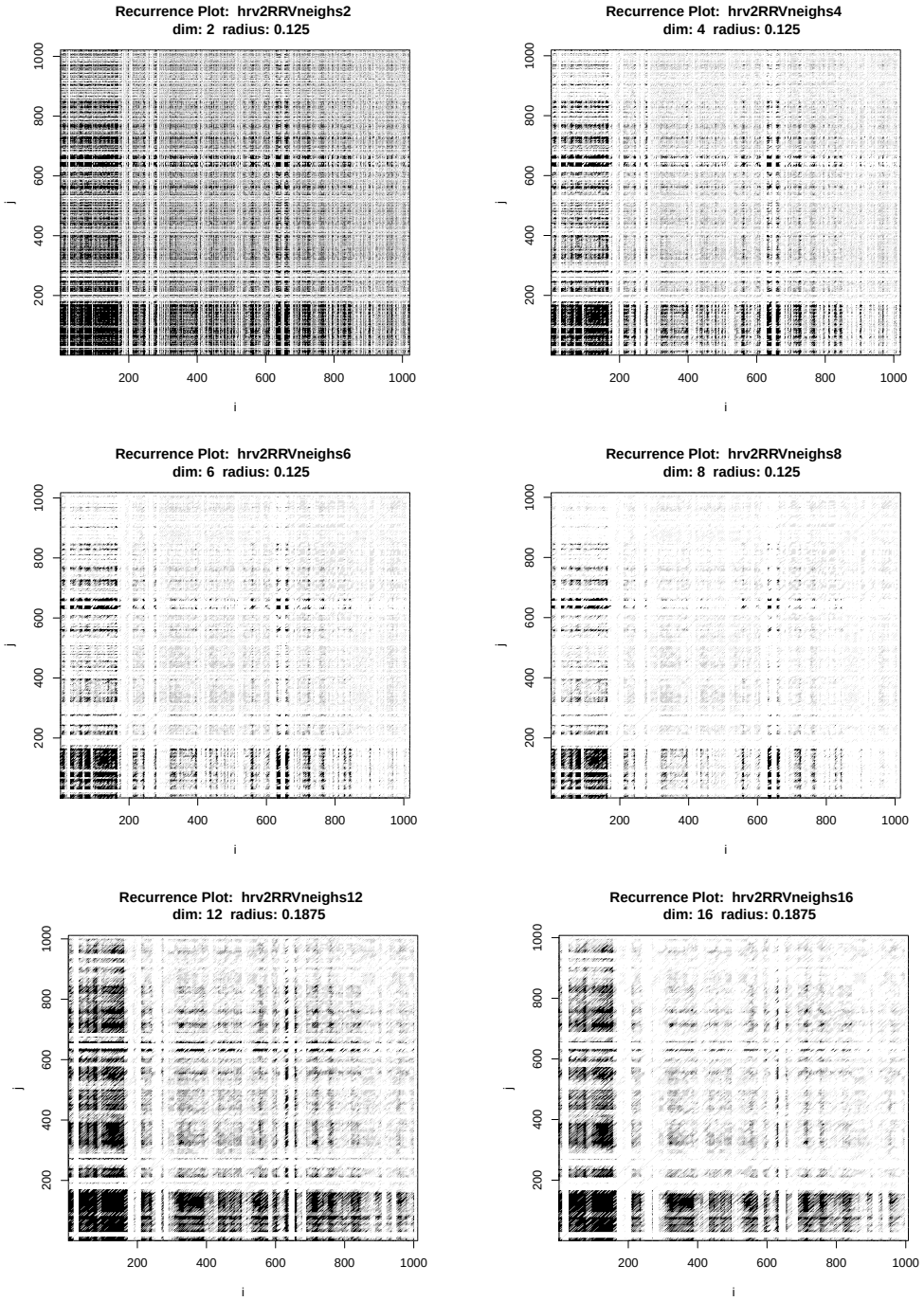

FIGURE 64. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 1.588 sec.

References

Eckmann JP, Kamphorst SO, Ruelle D (1987). "Recurrence plots of dynamical systems." *Europhys. Lett*, **4**(9), 973–977.

Takens F (1981). "Detecting strange attractors in turbulence." In *Dynamical systems and turbulence, Warwick 1980*, pp. 366–381. Springer.

Webber Jr CL, Zbilut JP (2005). "Recurrence quantification analysis of nonlinear dynamical systems." *Tutorials in contemporary nonlinear methods for the behavioral sciences*, pp. 26–94.

Zbilut JP, Webber CL (2006). "Recurrence quantification analysis." *Wiley encyclopedia of biomedical engineering*. URL `http://onlinelibrary.wiley.com/doi/10.1002/9780471740360.ebs1355/full`.

Index

R session info:

Total Sweave time used: 75.745 sec. at Sat Aug 9 19:46:16 2014.

- R version 3.1.1 (2014-07-10), x86_64-apple-darwin13.1.0
- Locale: en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8
- Base packages: base, datasets, graphics, grDevices, methods, stats, tcltk, utils
- Other packages: leaps 2.9, locfit 1.5-9.1, MASS 7.3-33, Matrix 1.1-4, mgcv 1.8-1, nlme 3.1-117, nonlinearTseries 0.2.1, rgl 0.93.1098, RHRV 4.0, sintro 0.1-3, tkrplot 0.0-23, TSA 1.01, tseries 0.10-32, waveslim 1.7.3
- Loaded via a namespace (and not attached): grid 3.1.1, lattice 0.20-29, quadprog 1.5-5, tools 3.1.1, zoo 1.7-11

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textheight: 9.21922in

Bibliography style: jss

CVS/Svn repository information:

$Source: /u/math/j40/cvsroot/lectures/src/dataanalysis/Rnw/recurrence.Rnw,v $

copied to r-forge

HeadURL: svn+ssh://gsawitzki@scm.r-forge.r-project.org/svnroot/rhrv/gs/Rnw/recurrence.Rnw *

$Revision: 153 $

Date: 2014-08-07 22:42:20 +0200 (Thu, 07 Aug 2014) *

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