STATISTICAL DATA ANALYSIS: RECURRENCE PLOT

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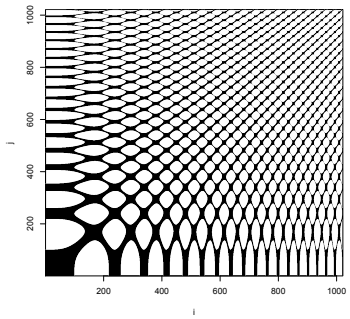

Contents

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1. Background

1.0.1. *Takens' Recurrence States.* Recurrence plots have been introduced in an attempt to understand near periodic in hydrodynamics. On the one hand, and extended theory on dynamical systems was available, covering deterministic models. A fundamental concept is that at a certain time a system is in some state, and developing from this. Defining the proper state space is a critical step in modelling.

The other toolkit is that of stochastics processes, in particular Markov models. Classical time series assumes stationarity, and this is obviously not the way to go. A fundamental idea for Markov models is that the system state is seen in a temporal context: you have a Markov process, if you can define a (non-anticipating) state that has sufficient information for prediction: given this state, the future is independent from the past.

Recurrence, coming back to some state, is often a key to understand a near periodic system.

Hydrodynamics is a challenging problem. Understanding planetary motion is a historical challenge, and may be useful as an illustration.

As a simple illustration, let $x = (x_i)$ be a sequence, maybe near periodic. For now, think of i as a time index.

Recurrence plots have two steps. The first was a bold step by Floris Takens. If you do not know the state space of a system, for a choice of "dimension" d, take the sequence of d tuples taken from your data to define the states.

$$u_i = (x_i, \ldots, x_{i+d})$$

This is Takens' delay embedding state (re)construction Takens [1981].

As a mere technical refinement: you may know that your data are a flattened representation of t dimensional data. So you take

$$u_i = (x_i, \ldots, x_{i+d*m}).$$

ToDo: add support for higher dimensional signals

This may be a relict of FORTRAN times, where it was common to flatten two-dimensional structures by case. We ignore this detail here and take $m = 1$.

Conceptually, you define states by observed histories. For classical Markov setup, the state is defined by the previous information x_{i-1}, but for more complex situations you may have to step back in the past. Finding the appropriate d is the challenge. So it may be appropriate to view the Takens states as a family, indexed by the time scope d. The rest is structural information how to arrange items.

Of course it is possible to compress information here, sorting states and removing duplicates. Keeping the original definition as the advantage that we have the index i, so that u_i is the state at index position i.

But the states may have an inherent structure, which we may take into account or ignore. Since for this example, we are just in 4-dimensional space, marginal scatterplots may give enough information.

Takens states are vectors in M dimensions. There are standard statistical techniques to visiulaize aspects of M dimensional vectors, at least for not too high dimension. One is the draftsman's plot, a scatterplot matrix by marginals. For Takens states, this is implemented as `statepairs()`. The other is coplots, a variant of scatterplot matrix by

marginals, contitionend by one or two additional variables. For Takens states, this is implemented as *statecolplots()*

To display the Takens state space, we us a variant of pairs().

ToDo: the takens state plot may be critically affected by outliers. Find a good rescaling.

By convention, the states are defined using overlapping sliding windows. This imposes considerable dependence between the states: one state is the shifted previous states, with only the end sub-state replaced. As an option, the states can be subsampled, using only non-overlapping ranges.

The Takens states may be stationary, that is asymptotically, that is the states starting at i do not depend on i. In this case, the first row (or column) contains all information, and pairs plot form an inclusion sequence by. In general, we will use state plots in 8 dimensions, a limit suggested by the print area.

ToDo: consider dimension-adjusted radius

1.1. **Recurrence Plots.** The next step, taken in Eckmann *et al.* [1987] was to use a two dimensional display. Take a scatterplot with the Taken's states a marginal. Take a sliding window of your process data, and for each i, find the "distance" of u_i from and to any of the collected states. If the distance is below some chosen threshold, mark the point (i, j) for which u(j) is in the ball of radius r(i) centred at u(i).

The original publication Eckmann *et al.* [1987] actually used a nearest neighbourhood environment to cover about 10 data points.

The construction has considerable arbitrary choices. The critical radius may depend on the point i. In practical applications, using a constant radius is a common first step. Using a dichotomous marking was what presumably was necessary when the idea was introduced. With todays technology, we can allow a markup on a finer scale, as has been seen in Orion-1.

ToDo: support distance instead of 0/1 indicators

We can gain additional freedom by using a correlation view: instead of looking from one axis, we can walk along the diagonal, using two reference axis.

Helpful hints how to interpret recurrence plots are in "Recurrence Plots At A Glance" `<http://www.recurrence-plot.tk/glance.php>`.

1.2. **Recurrence Quantification Analysis.** While visual inspection is the prime way to assess recurrence plots, quantification of some aspects revealed of the plot may be helpful. A collection of indices is provided by a recurrence quantification analysis (RQA) Zbilut and Webber [2006], Webber Jr and Zbilut [2005].

See Table 1 on the following page.

2. R Setup

Input

```
save.RNGseed <- 87149 #.Random.seed
save.RNGkind <- RNGkind()
# save.RNGseed
save.RNGkind
```

Output

```
[1] "Mersenne-Twister" "Inversion"
```

TABLE 1. Recurrence Quantification Analysis (RQA)

REC	Recurrence. Percentage of recurrence points in a recurrence Plot.
DET	Determinism. Percentage of recurrence points that form diagonal lines.
LAM	Percentage of recurrent points that form vertical lines.
RATIO	Ratio between *DET* and *RR*.
Lmax	Length of the longest diagonal line.
Lmean	Mean length of the diagonal lines. The main diagonal is not taken into account.
DIV	Inverse of *Lmax*.
Vmax	Longest vertical line.
Vmean	Average length of the vertical lines. This parameter is also referred to as the Trapping time.
ENTR	Shannon entropy of the diagonal line lengths distribution
TREND	Trend of the number of recurrent points depending on the distance to the main diagonal
diagonalHistogram	Histogram of the length of the diagonals.
recurrenceRate	Number of recurrent points depending on the distance to the main diagonal.

Input
```
set.seed(save.RNGseed, save.RNGkind[1])
```

Input
```
laptime <- function(){
return(round(structure(proc.time() - chunk.time.start, class = "proc_time")[3],3))
chunk.time.start <<- proc.time()
}
```

Input
```
# install.packages("sintro",repos="http://r-forge.r-project.org",type="source")
library(sintro)
```

We use

Input
```
library(nonlinearTseries)
```

ToDo: improve choice of alpha

2.0.1. *Takens states.*

`statepairs` *Show marginal scatterplots of Takens states*

Usage.

```
statepairs(states, main, rank = FALSE, nooverlap = FALSE, alpha)
```

Arguments.

`states`	A matrix: Takens states by dimension, one state per row.
`main`	Optional: the main header.
`rank`	An experimental variant. If `rank`, the values are rank transformed.
`nooverlap`	An experimental variant. If `nooverlap`, the cases are subsampled by dimension.
`alpha`	The alpha value used for plotting. The current choice is $sqrtfrac1nrstates$ as a default.

ToDo: colour by time

Input

```
statepairs <- function(states, main, rank=FALSE, nooverlap= FALSE, alpha){
        n <- dim(states)[1]; dim <-  dim(states)[2]
        if (missing(main)) {
        main <- paste("Takens states:",deparse(substitute(states)), "\n",
                "n=", n," dim=", dim) }

        if (nooverlap) {states <- states[ seq(1,n, by=dim),]

        main <- paste(main," no overlap")}

        if (missing(alpha)) {alpha <- 1/sqrt(dim(states)[1])}
        #cat("alpha=",alpha)

        if (rank) {states <- apply(states, 2, rank, ties.method="random")
        main <- paste(main," ranked")}
        pairs(states, main=main,
        col=rgb(0,0,0, alpha), pch=19)
        #title(main=main, outer=TRUE, line=-2, cex.main=0.8)
}
```

Assuming the index is time, the `stateplot()` is layed out to show the highest index as response.

ToDo: extend for low dimensions, extend parameters

Input

```
statecoplot <- function(states, main, rank=FALSE, nooverlap= FALSE, alpha){
        n <- dim(states)[1]; dim <-  dim(states)[2]
        if (missing(main)) {
        main <- paste("Takens states:",deparse(substitute(states)), "\n",
                "n=", n," dim=", dim) }

        if (nooverlap) {states <- states[ seq(1,n, by=dim),]

        main <- paste(main," no overlap")}

        if (missing(alpha)) {alpha <- 1/sqrt(dim(states)[1])}
        #cat("alpha=",alpha)

        if (rank) {states <- apply(states, 2, rank, ties.method="random")
        main <- paste(main," ranked")}

        coplot((states[,4]~states[,3]|states[,1]+ states[,2]), main=main,
        col=rgb(0,0,0, alpha), pch=19)
        #title(main=main, outer=TRUE, line=-2, cex.main=0.8)
}
```

2.1. **Local Bottleneck.** To allow experimental implementations, functions from *nonlinearTseries* are aliased here.

Input

```
local.buildTakens <- function (time.series,
        embedding.dim, time.lag=1,
        id=deparse(substitute(time.series)))
{
        takens <- nonlinearTseries:::buildTakens(time.series, embedding.dim, time.lag)
        attr(takens,"time.lag") <- time.lag
        attr(takens,"id") <- id
        return(takens)
}
```

Input

```
local.findAllNeighbours <- nonlinearTseries:::findAllNeighbours
```

minor cosmetics added to recurrencePlotAux

ToDo: propagate parameters from *buildTakens* and *findAllNeighbours* in a slot of the result, instead of using explicit parameters in recurrencePlotAux.

Input

```
#non-sparse variant
#local.recurrencePlotAux <- nonlinearTseries:::recurrencePlotAux
local.recurrencePlotAux = function(neighs, dim=NULL, lag=NULL, radius=NULL){

        # just for reference. This function is inlined
        neighbourListNeighbourMatrix  = function(){
          #neighs.matrix = Diagonal(ntakens)
          for (i in 1:ntakens){
            if (length(neighs[[i]])>0){
              for (j in neighs[[i]]){
                neighs.matrix[i,j] = 1
              }
            }
          }
          return (neighs.matrix)
        }

  ntakens=length(neighs)
  neighs.matrix <- matrix(nrow=ntakens,ncol=ntakens)
  #neighbourListNeighbourMatrix()
    #neighs.matrix = Diagonal(ntakens)
          for (i in 1:ntakens){
                  neighs.matrix[i,i] = 1 # do we want the diagonal fixed to 1
            if (length(neighs[[i]])>0){
              for (j in neighs[[i]]){
                neighs.matrix[i,j] = 1
              }
            }
          }

        main <- paste("Recurrence Plot: ",
                      deparse(substitute(neighs))
                      )
        more <- NULL

        #use compones of neights if available
        if (!is.null(dim)) more <- paste(more," dim:",dim)
        if (!is.null(lag)) more <- paste(more," lag:",lag)
        if (!is.null(radius)) more <- paste(more," radius:",radius)
```

```
        if (!is.null(more)) main <- paste(main,"\n",more)

  # need no print because it is not a trellis object!!
  #print(
          image(x=1:ntakens, y=1:ntakens,
                 z=neighs.matrix,xlab="i", ylab="j",
                col="black",
                #xlim=c(1,ntakens), ylim=c(1,ntakens),
                useRaster=TRUE,  #? is this safe??
                main=main
                )
 #        )

}
```

ToDo: improve feedback for data structures in *nonlinearTseries*

ToDo: improve to a full *show* method

2.2. **Recurrence Plots.** This is a hack to report RQA information. $dim = NULL$ is added to align calling with other functions.

Input

```
showrqa <- function(takens, dim=NULL, radius,
                digits=3,
                do.hist = TRUE, log = "" )
{
        xxrqa <- rqa(takens=takens,radius=radius)
        cat(paste(deparse(substitute(takens)), " n=", dim(takens)[1],
                " Dim:", dim(takens)[2], "\n"))
        cat(paste("Radius:", radius,
                " Recurrence coverage REC:", round(xxrqa[1]$REC, digits),
                " log(REC)/log(R):", round(log(xxrqa[1]$REC)/log(radius), digits), "\n"))
        cat(paste("Determinism:", round(xxrqa$DET, digits),
                " Laminarity:",round(xxrqa$LAM, digits), "\n"))
        cat(paste("DIV:", round(xxrqa$DIV, digits), "\n"))
        cat(paste("Trend:", round(xxrqa$TREND, digits),
                " Entropy:",round(xxrqa$ENTR, digits), "\n"))
        cat(paste("Diagonal lines max:", round(xxrqa$Lmax, digits),
                 " Mean:",round(xxrqa$Lmean, digits),
                " Mean off main:",round(xxrqa$LmeanWithoutMain, digits), "\n"))
        cat(paste("Vertical lines max:", round(xxrqa$Vmax, digits),
                " Mean:",round(xxrqa$Vmean, digits), "\n"))
                # str(xxrqa[4:12])

        if (do.hist){
        if (log==TRUE) log<-"y"
        oldpar <- par(mfrow=c(2,1))
        xxrqa$diagonalHistogram[xxrqa$diagonalHistogram==0] <- NA # hack for zero counts
                barplot(xxrqa$diagonalHistogram,
                main=paste(deparse(substitute(takens)), "Diagonal",
                        "\n n=", dim(takens)[1], " Dim:",
                        dim(takens)[2], " Radius: ",radius),
                        xlab="length of diagonals",
                        log = log)

                        xxrqa$diagonalHistogram[xxrqa$diagonalHistogram==0] <- NA # hack for zero counts
        barplot(xxrqa$recurrenceRate,
                main=paste(deparse(substitute(takens)),"Recurrence Rate",
                        "\n n=", dim(takens)[1], " Dim:",
                        dim(takens)[2], " Radius: ",radius),
                        xlab="distance to diagonal",
```

```
                        log = log)
        par(oldpar)
                }
        invisible(xxrqa)
}
```

3. Test Cases

We set up a small series of test signals. Some synthetic test signals are introduced here. More test cases used later are the Geyser data set (Section 6 on page 22) and two examples of heart rate data sets (Section 7 on page 43 and Section 8 on page 57) from *library(rhrv)*.

For convenience, some source code from other libraries is included to make this self-contained.

As a global constant, we set up the length of the series to be used for test signals.

Input

```
#nsignal <- 256
nsignal <- 1024
#nsignal <- 4096
system.time.start <- proc.time()
```

For signal representation, we use a common layout.

Input

```
plotsignal <- function(signal, main, ylab) {
        #! alpha level should depend on expected number of overlaps

        if (missing(ylab)) { ylab <- deparse(substitute(signal)) }

        par(mfrow = c(1,
                2))
        plot(signal,
                main = "", xlab = "index",  ylab = ylab,
                col = rgb(0, 0, 1, 0.3), pch = 20)

        plot(signal, type = "l",
                main = "",  xlab = "index", ylab = ylab,
                col = rgb(0, 0, 0, 0.4))
        points(signal,
                col = rgb(0, 0, 1, 0.3), pch = 20)
        if (missing(main)) { main = deparse(substitute(signal)) }
        title(main = main,
                outer = TRUE, line = -2, cex.main = 1.2)
}
```

Input

```
sin10 <- function(n=nsignal) {sin( (1:n)/n* 2*pi*10)}
plotsignal(sin10())
```

See Figure 1 on the following page.

Input

```
unif <- function(n=nsignal) {runif(n)}
xunif<-unif()
plotsignal(xunif)
```

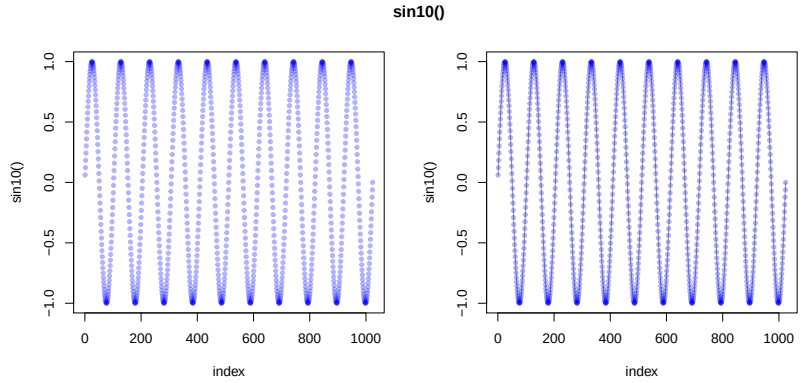

FIGURE 1. Test case: sin10. Signal and linear interpolation.

See Figure 2,

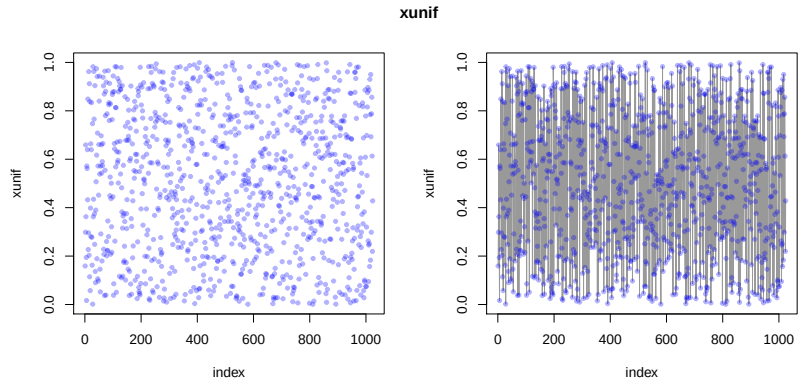

FIGURE 2. Test case: unif - uniform random numbers. Signal and linear interpolation.

Input

```
chirp <- function(n=nsignal) {
# this is copied from library(signal)
signal.chirp <- function(t, f0 = 0, t1 = 1, f1 = 100,
                  form = c("linear", "quadratic", "logarithmic"),
                  phase = 0){

  form <- match.arg(form)
  phase <- 2*pi*phase/360

  switch(form,
    "linear" = {
        a <- pi*(f1 - f0)/t1
        b <- 2*pi*f0
        cos(a*t^2 + b*t + phase)
    },
    "quadratic" = {
        a <- (2/3*pi*(f1-f0)/t1/t1)
        b <- 2*pi*f0
        cos(a*t^3 + b*t + phase)
    },
```

```
    "logarithmic" = {
        a <- 2*pi * t1 / log(f1 - f0)
        b <- 2*pi * f0
        x <- (f1-f0)^(1/t1)
        cos(a*x^t + b*t + phase)
    })
}

signal.chirp(seq(0, 0.6, len=nsignal))
}
plotsignal(chirp())
```

See Figure 3,

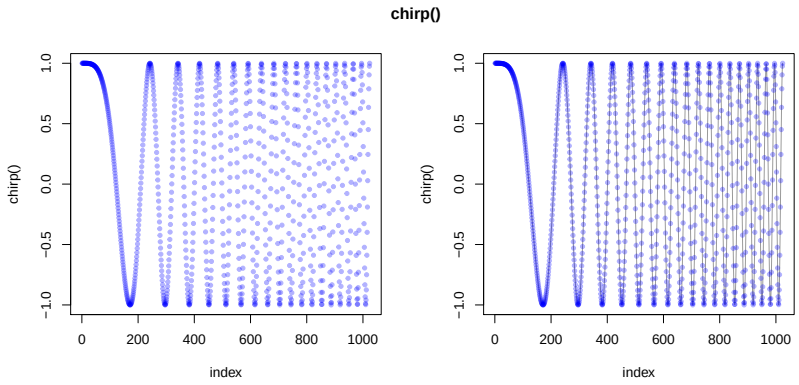

FIGURE 3. Test case: chirp signal. Signal and linear interpolation.

ToDo: include doppler waveslim

4. Takens' Recurrence States

Input
```
sintakens <- local.buildTakens(time.series=sin10(),
        embedding.dim=4, time.lag=1)
statepairs(sintakens) #4
```

See Figure 4.

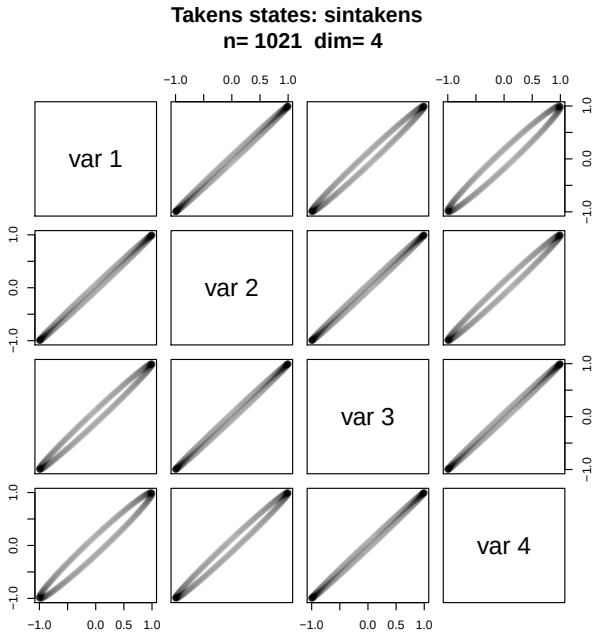

Figure 4. Takens states. Test case: sinus. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.654 sec.

Input
```
statepairs(sintakens, nooverlap=TRUE) #dim=4
```

See Figure 5 on the facing page.

Input
```
statecoplot(sintakens) #4
```

See Figure 6 on page 14.

Input
```
uniftakens <- local.buildTakens( time.series=xunif,
        embedding.dim=4, time.lag=1)
statepairs(uniftakens) #dim=4
```

See Figure 7 on page 14.

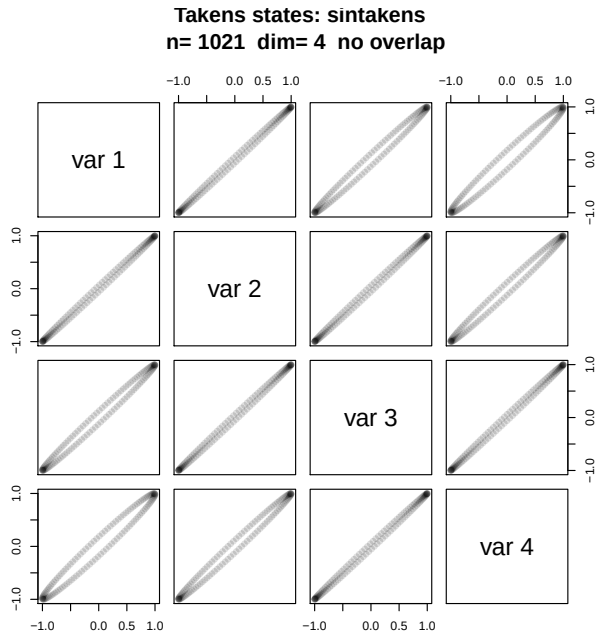

FIGURE 5. Takens states. Test case: sinus, no overlap. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.925 sec.

Input

```
statecoplot(uniftakens) #dim=4
```

See Figure 8 on page 15.

Input

```
statepairs(uniftakens, nooverlap=TRUE) #dim=4
```

See Figure 9 on page 15.

Input

```
chirptakens <- local.buildTakens( time.series=chirp(),
        embedding.dim=4,time.lag=1)
statepairs(chirptakens) #dim=4
```

See Figure 10 on page 16.

Input

```
statecoplot(chirptakens) #dim=4
```

See Figure 11 on page 16.

Input

```
statepairs(chirptakens, nooverlap=TRUE) #dim=4
```

See Figure 12 on page 17.

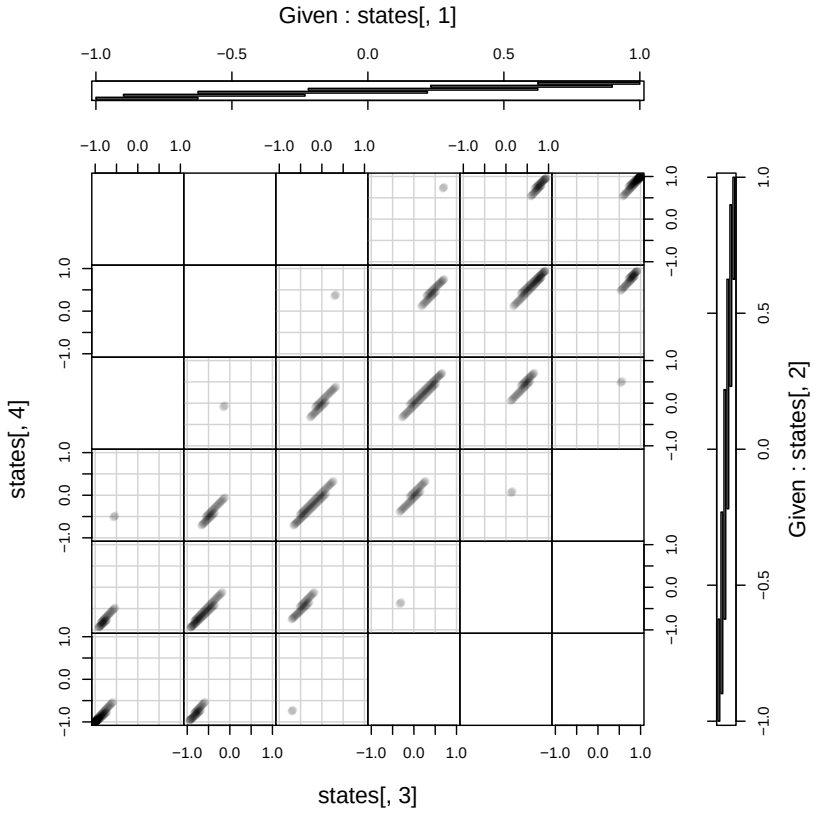

FIGURE 6. State coplot. Test case: sinus. Note that $2d$ marginal views of 1-dimensional circles in d space generally appear as ellipses. Time used: 0.287 sec.

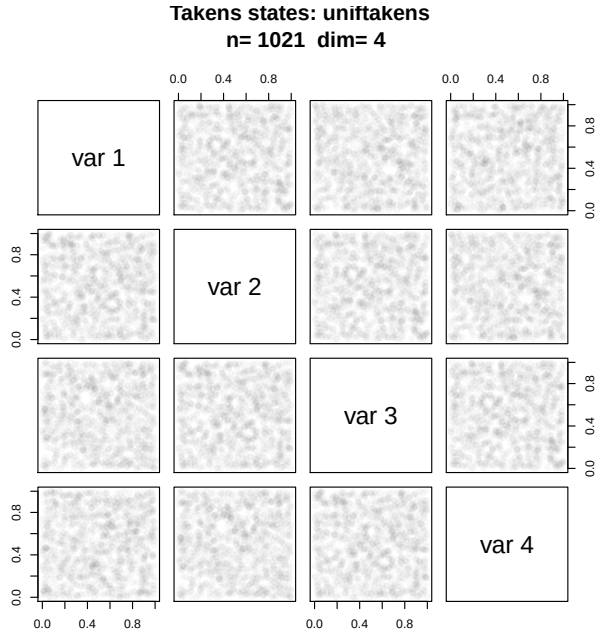

FIGURE 7. Takens states. Test case: uniform random numbers. Time used: 0.728 sec.

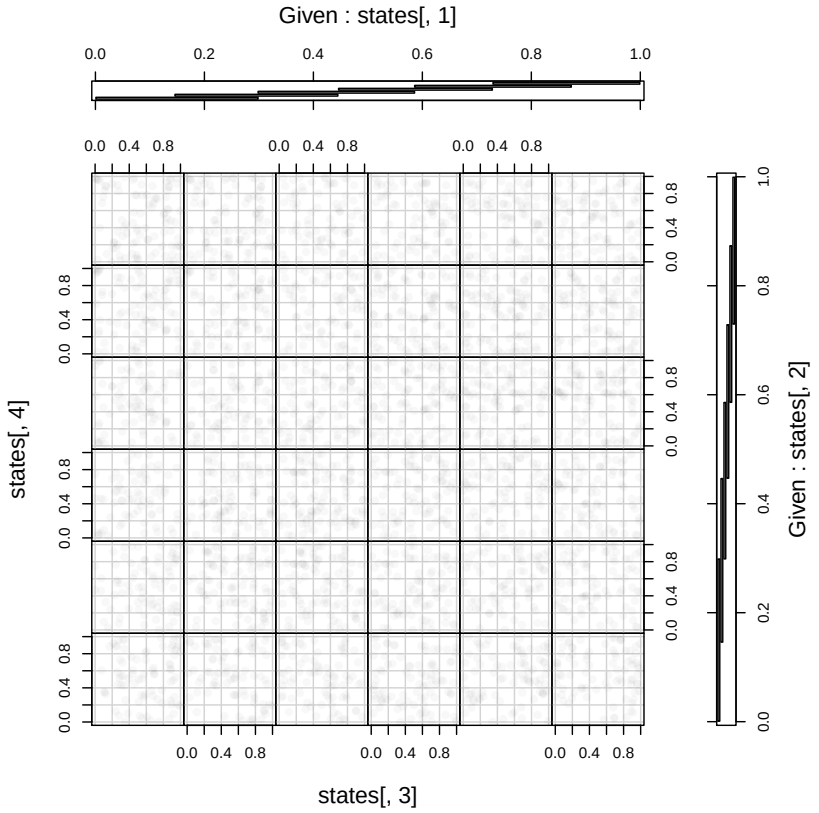

FIGURE 8. State coplot. Test case: uniform random numbers. Time used: 1.077 sec.

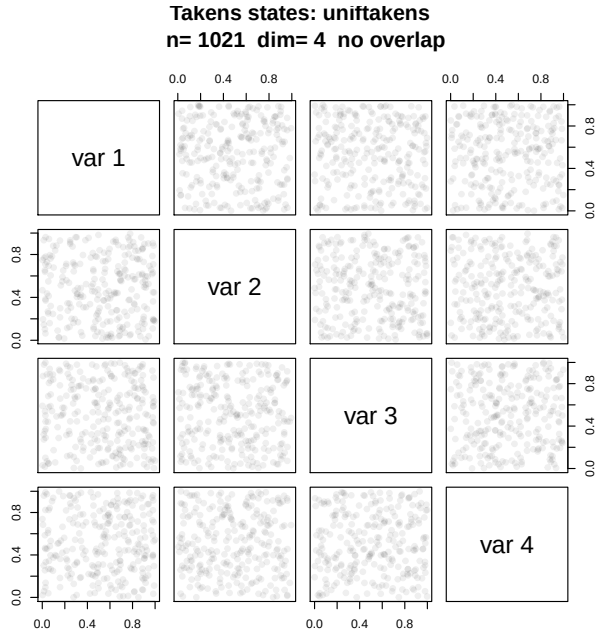

FIGURE 9. Takens states. Test case: uniform random numbers. Time used: 1.361 sec.

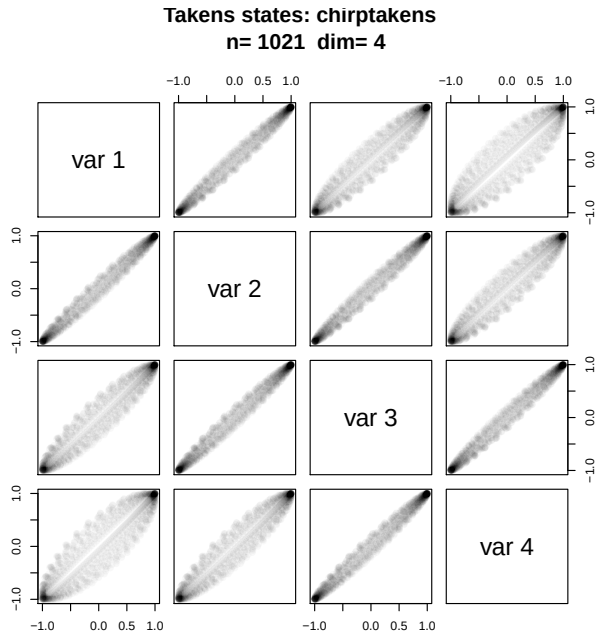

FIGURE 10. Takens states. Test case: chirp signal. Time used: 0.713 sec.

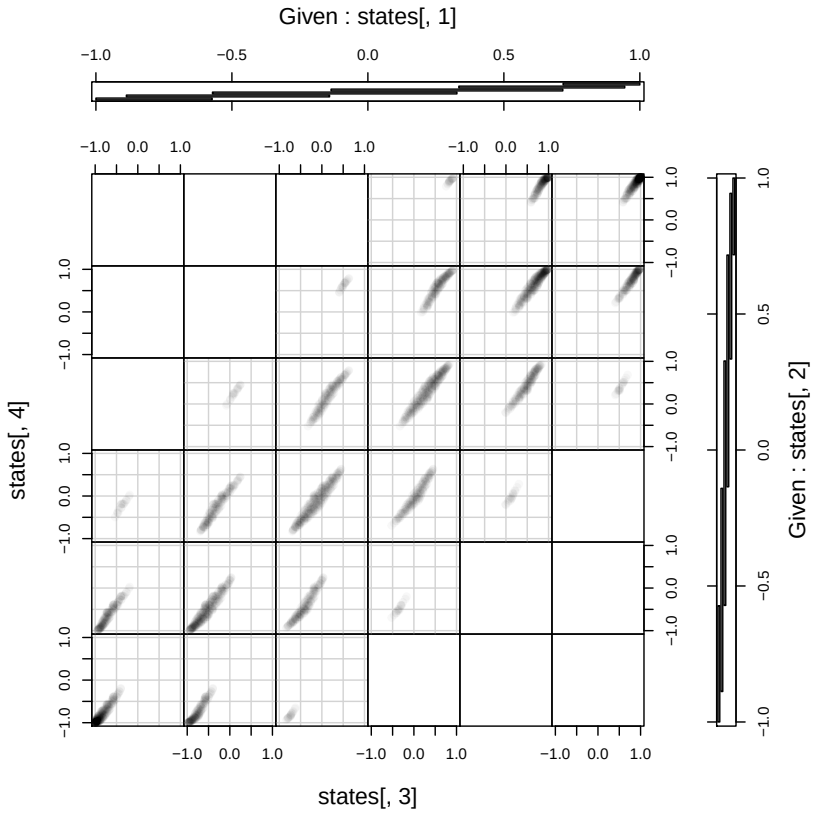

FIGURE 11. State coplot. Test case: chirp signal. Time used: 1.047 sec.

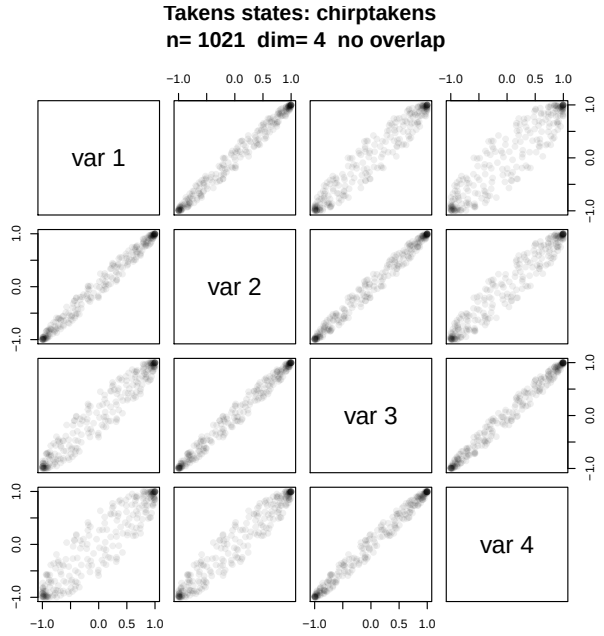

FIGURE 12. Takens states. Test case: chirp signal. Time used: 1.311 sec-

5. Applied Recurrence Plots

5.1. **Sinus.**

Input
```
sin10neighs<-local.findAllNeighbours(sintakens, radius=0.2)
save(sin10neighs, file="sin10neighs.Rdata")
```

Input
```
load(file="sin10neighs.RData")
local.recurrencePlotAux(sin10neighs, dim=2, radius=0.2)
```

See Figure 13.

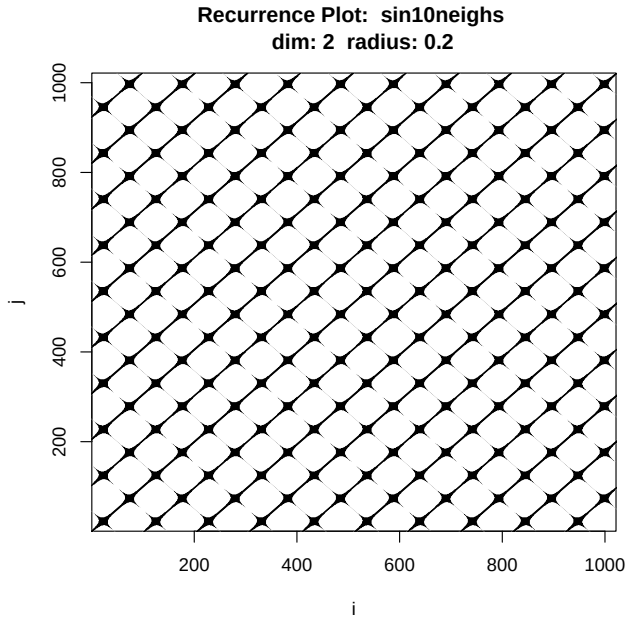

Figure 13. Recurrence plot. Recurrence Plot. Test case: sinus curves. Time used: 1.391 sec.

Input
```
showrqa(sintakens, radius=0.2)
```

Output
```
sintakens  n= 1021  Dim: 4
Radius: 0.2  Recurrence coverage REC: 0.136  log(REC)/log(R): 1.241
Determinism: 0.96  Laminarity: 0.982
DIV: 0.001
Trend: 0  Entropy: 2.74
Diagonal lines max: 1020  Mean: 14.263  Mean off main: 14.157
Vertical lines max: 25  Mean: 10.251
```

See Figure 14 on the next page.

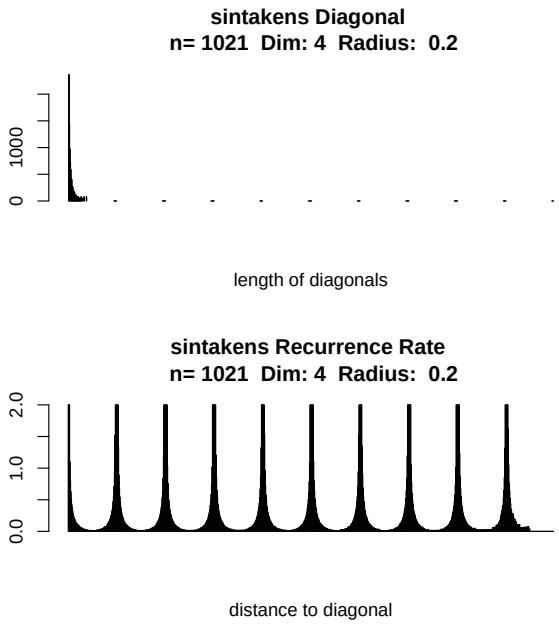

FIGURE 14. Recurrence plot. RQA. Test case: sinus curves. Time used: 0.5 sec.

5.2. **Uniform random.**

Input

```
load(file="unifneighs.RData")
local.recurrencePlotAux(unifneighs, radius=0.2)
```

See Figure 15 on the following page.

5.3. **Chirp Signal.**

Input

```
chirpneighs<-local.findAllNeighbours(chirptakens,radius=0.6)#0.4
save(chirpneighs, file="chirpneighs.RData")
```

Input

```
load(file="chirpneighs.RData")
local.recurrencePlotAux(chirpneighs)
```

See Figure 16 on the next page.

Input

```
showrqa(chirptakens, radius=0.6)
```

Output

```
chirptakens  n= 1021  Dim: 4
Radius: 0.6  Recurrence coverage REC: 0.341  log(REC)/log(R): 2.108
Determinism: 0.988  Laminarity: 0.998
```

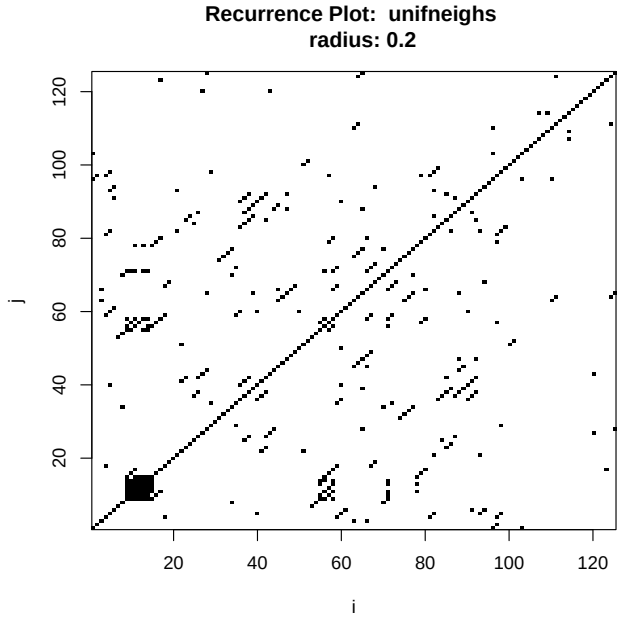

FIGURE 15. Recurrence plot. Recurrence Plot. Test case: uniform random numbers. Time used: 0.095 sec.

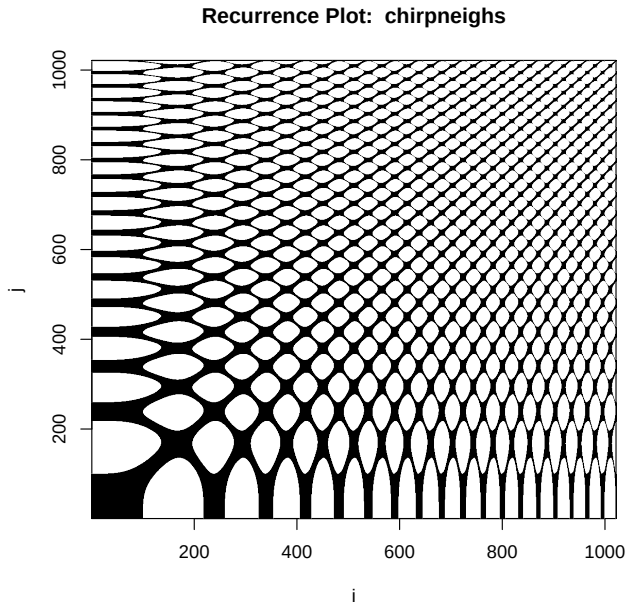

FIGURE 16. Recurrence plot. Recurrence Plot. Test case: chirp signal. Time used: 2.209 sec.

```
DIV: 0.001
Trend: 0  Entropy: 3.254
Diagonal lines max: 1020  Mean: 12.496  Mean off main: 12.46
Vertical lines max: 125  Mean: 14.721
```

See Figure 17.

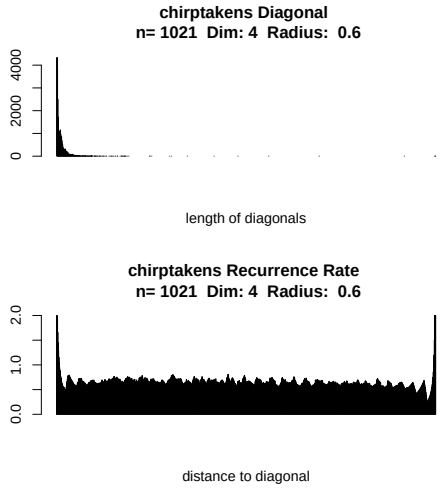

FIGURE 17. Recurrence Plot RQA. Test case: chirp signal. Time used: 2.862 sec.

6. Case Study: Geyser data

ToDo: double check: *MASS:::geyser* should be used, not *faithful*

ToDo: Geyser: extended to two-dimensional data in *geyserlin*. Experimental only. Check.

This is a classical data set with a two dimensional structure, *duration* and *waiting*.

The data structure asks for a variant of the recurence plot, adapted to a two dimensional series.

Input

```
library(MASS)
data(geyser)
```

6.1. **Geyser Eruptions.**

Input

```
plotsignal(geyser$duration)
```

See Figure 18,

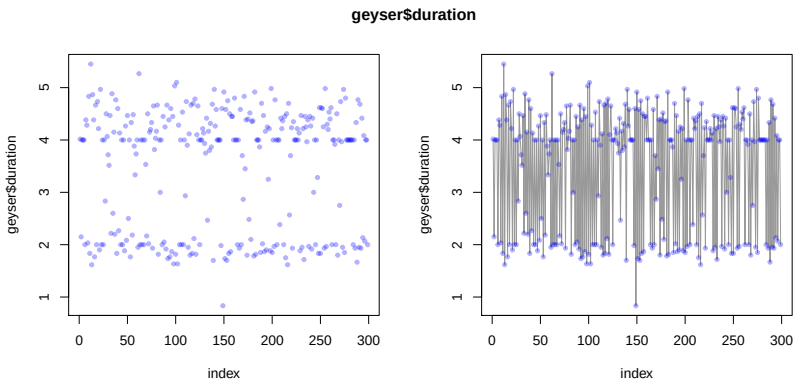

Figure 18. Example case: Old Faithful Geyser eruptions. Signal and linear interpolation.

Input

```
eruptionstakens4 <-
        local.buildTakens( time.series=geyser$duration,
                embedding.dim=4, time.lag=1)
        statepairs(eruptionstakens4) #dim=4
```

See Figure 28 on page 28.

Input

```
        statecoplot(eruptionstakens4) #dim=4
```

See Figure 20 on the next page.

Input

```
        statepairs(eruptionstakens4, nooverlap=TRUE) #dim=4
```

See Figure 29 on page 29.

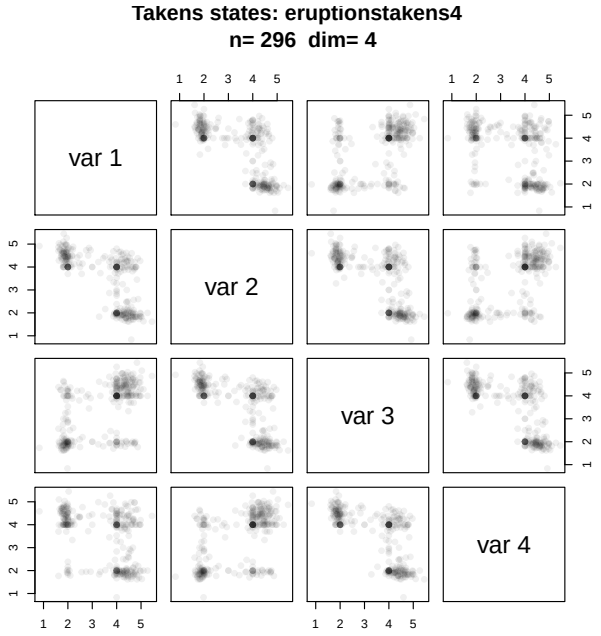

FIGURE 19. Recurrence plot. Example case: Old Faithful Geyser eruptions. Time used: 0.29 sec.

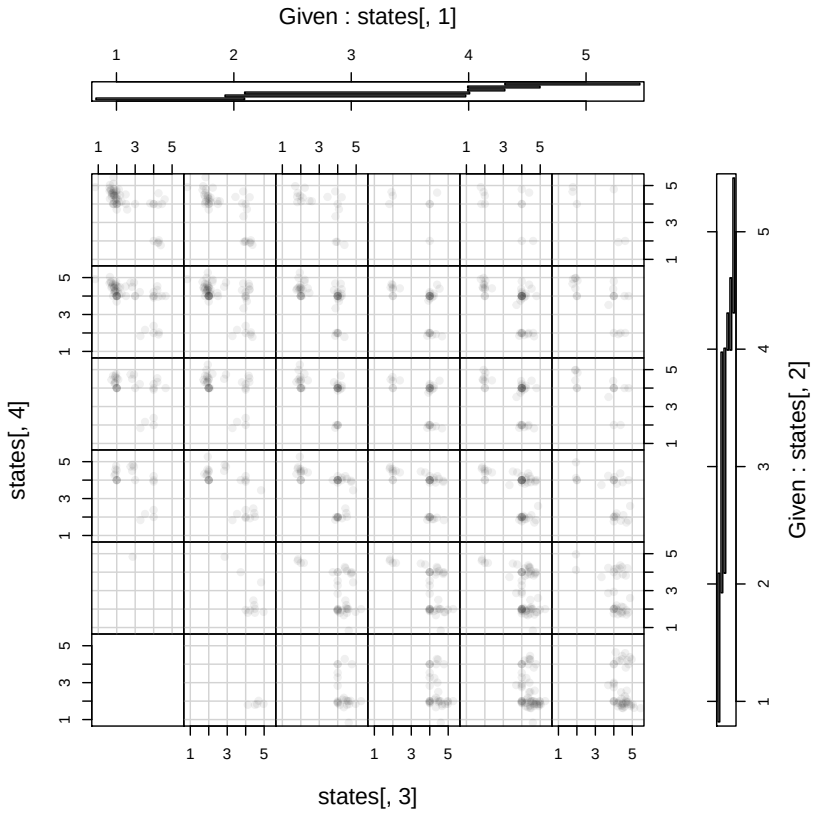

FIGURE 20. State coplot. Example case: Old Faithful Geyser eruptions. Time used: 0.526 sec.

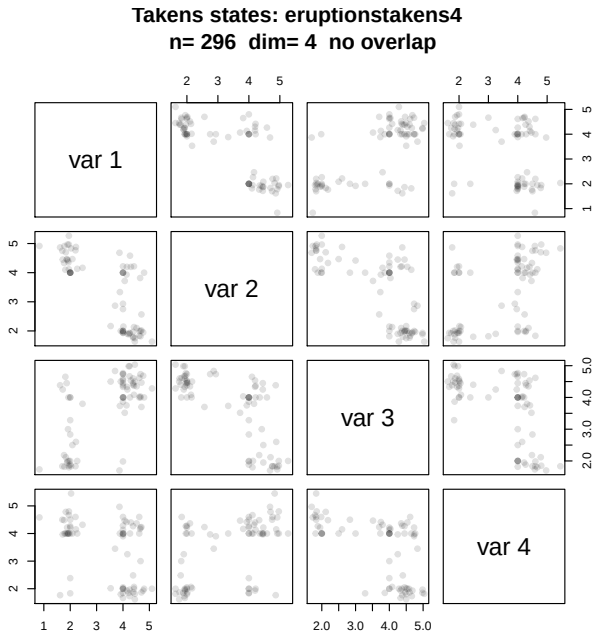

FIGURE 21. Recurrence plot. Example case: Old Faithful Geyser eruptions. Time used: 0.703 sec.

Input

```
eruptionsneighs4<-local.findAllNeighbours(eruptionstakens4, radius=0.8)
save(eruptionsneighs4, file="eruptionsneighs4.RData")
```

Input

```
load(file="eruptionsneighs4.RData")
local.recurrencePlotAux(eruptionsneighs4)
```

See Figure 30 on page 30.

Input

```
showrqa(eruptionstakens4, radius=0.8)
```

Output

```
eruptionstakens4  n= 296  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.112  log(REC)/log(R): 9.822
Determinism: 0.903  Laminarity: 0.076
DIV: 0.05
Trend: 0  Entropy: 1.779
Diagonal lines max: 20  Mean: 3.919  Mean off main: 3.79
Vertical lines max: 5  Mean: 3.041
```

See Figure 31 on page 30.

6.1.1. *Geyser eruptions. Dim=2.*

Input

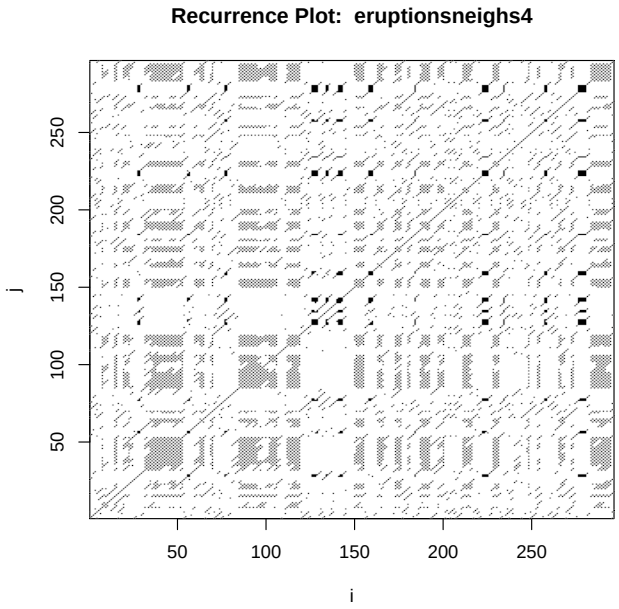

FIGURE 22. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.185 sec.

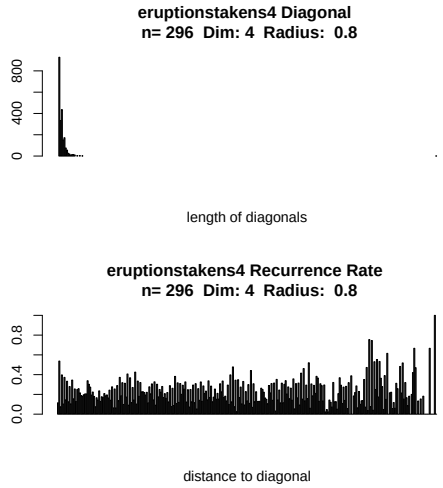

FIGURE 23. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.388 sec.

```
eruptionstakens2 <-
        local.buildTakens(time.series=geyser$duration,
                embedding.dim=2, time.lag=1)
statepairs(eruptionstakens2) #dim=2
```

See Figure 24 on the next page.

Input

```
statepairs(eruptionstakens2, nooverlap=TRUE) #dim=2
```

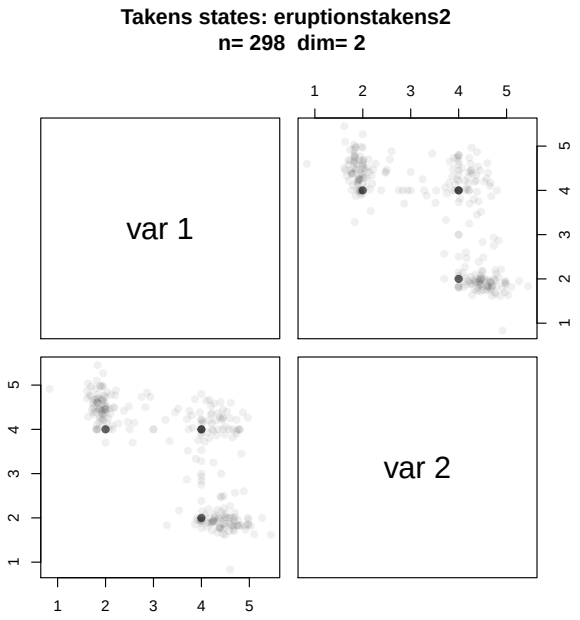

FIGURE 24. Recurrence plot. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.108 sec.

See Figure 25 on the facing page

Input

```
eruptionsneighs2<-local.findAllNeighbours(eruptionstakens2,
        radius=0.8)
save(eruptionsneighs2, file="eruptionsneighs2.RData")
```

Input

```
load(file="eruptionsneighs2.RData")
local.recurrencePlotAux(eruptionsneighs2)
```

See Figure 26 on the next page.

Input

```
showrqa(eruptionstakens2, radius=0.8)
```

Output

```
eruptionstakens2  n= 298  Dim: 2
Radius: 0.8  Recurrence coverage REC: 0.274  log(REC)/log(R): 5.806
Determinism: 0.892  Laminarity: 0.205
DIV: 0.045
Trend: 0  Entropy: 1.691
Diagonal lines max: 22  Mean: 3.644  Mean off main: 3.595
Vertical lines max: 7  Mean: 3.457
```

See Figure 27 on page 28.

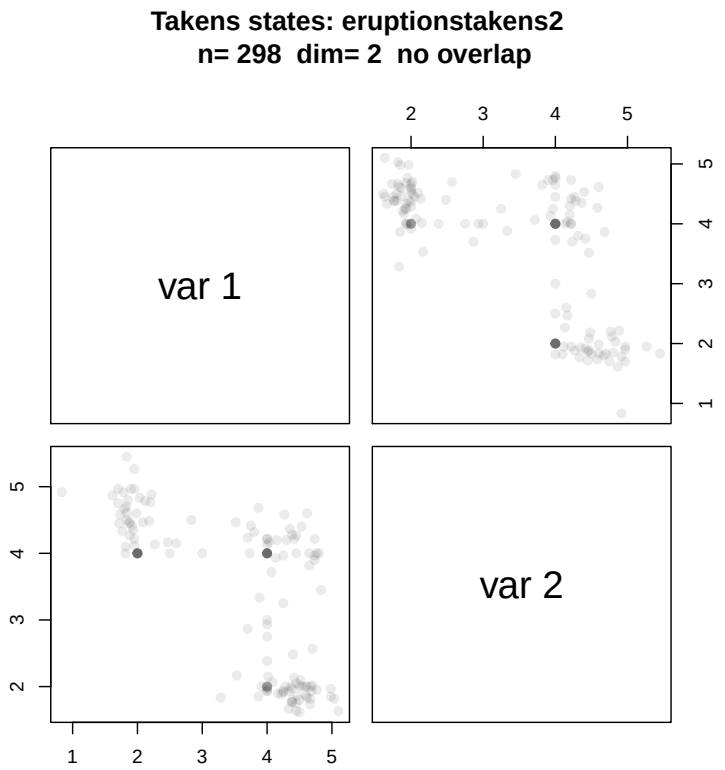

FIGURE 25. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.203 sec.

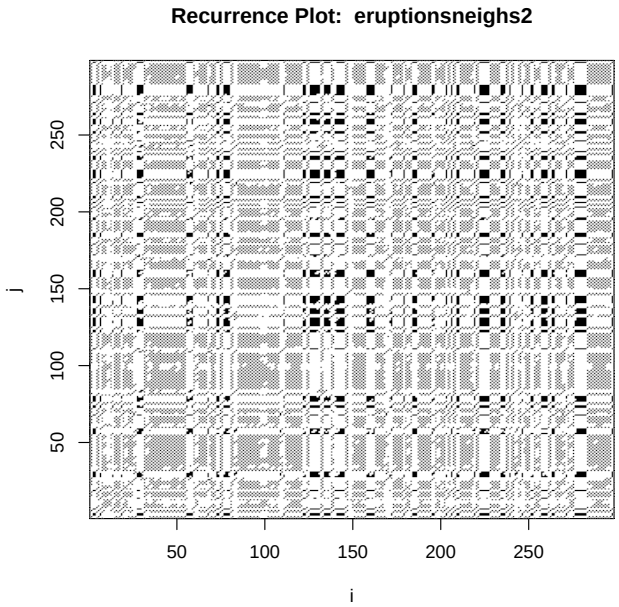

FIGURE 26. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser Geyser eruptions. Dim=2. Time used: 0.252 sec.

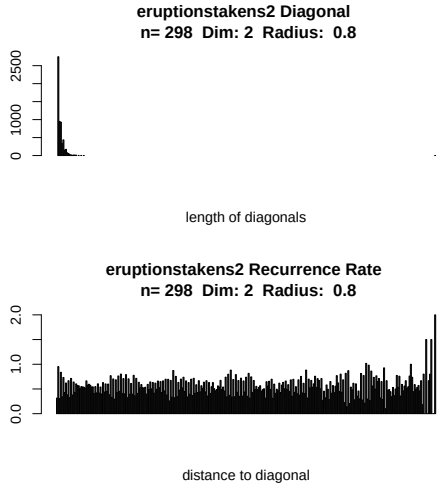

FIGURE 27. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.459 sec.

6.1.2. *Geyser eruptions. Dim=4.*

Input

```
eruptionstakens4 <- local.buildTakens( time.series=geyser$duration,
        embedding.dim=4, time.lag=1)
statepairs(eruptionstakens4) #dim=4
```

See Figure 28.

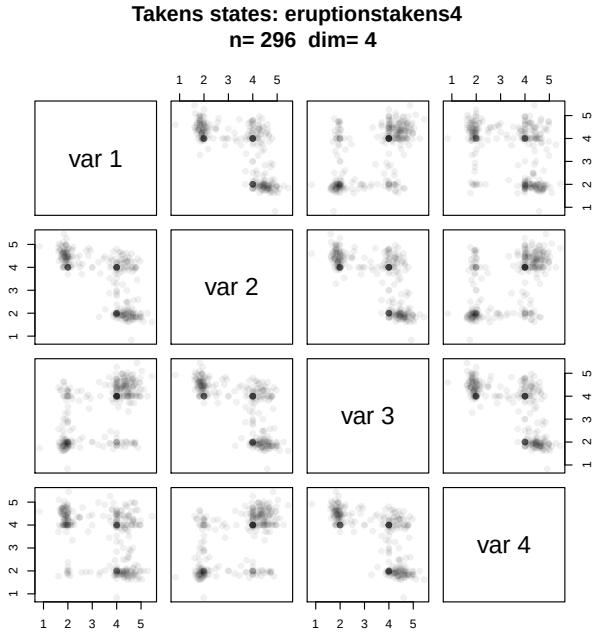

FIGURE 28. Recurrence plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.279 sec.

Input
```
statepairs(eruptionstakens4, nooverlap=TRUE) #dim=4
```

See Figure 29.

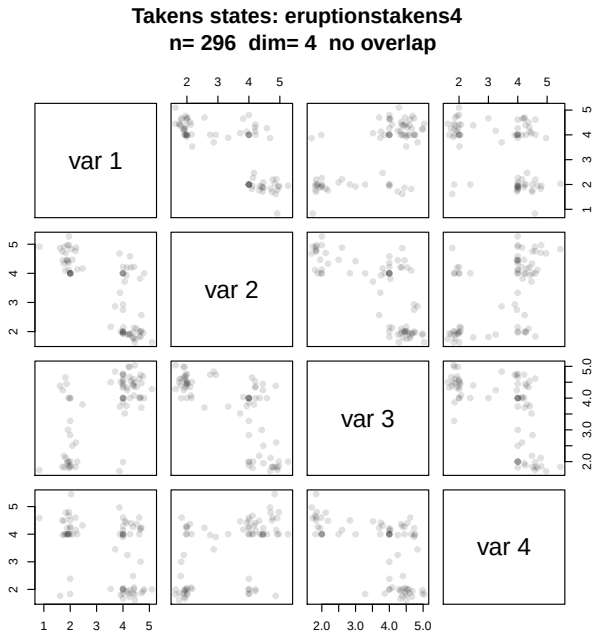

FIGURE 29. Recurrence plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.435 sec.

Input
```
eruptionsneighs4<-local.findAllNeighbours(eruptionstakens4,
        radius=0.8)
save(eruptionsneighs4, file="eruptionsneighs4.RData")
```

Input
```
load(file="eruptionsneighs4.RData")
local.recurrencePlotAux(eruptionsneighs4)
```

See Figure 30 on the next page.

Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.189 sec.

Input
```
showrqa(eruptionstakens4, radius=0.8)
```

Output
```
eruptionstakens4  n= 296  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.112  log(REC)/log(R): 9.822
Determinism: 0.903  Laminarity: 0.076
DIV: 0.05
Trend: 0  Entropy: 1.779
Diagonal lines max: 20  Mean: 3.919  Mean off main: 3.79
Vertical lines max: 5  Mean: 3.041
```

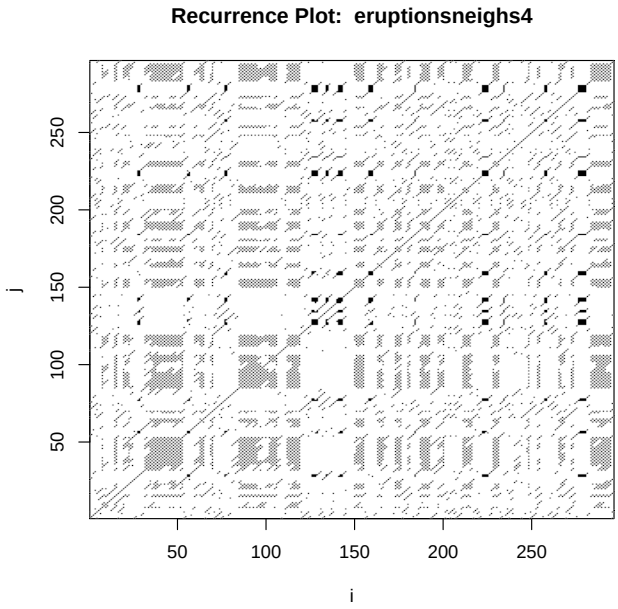

FIGURE 30. Recurrence plot. .

See Figure 31.

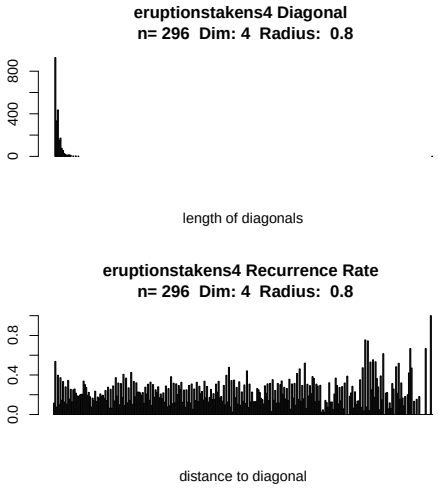

FIGURE 31. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.381 sec.

6.1.3. *Geyser eruptions. Dim=8.*

Input

```
eruptionstakens8 <- local.buildTakens( time.series=geyser$duration,
        embedding.dim=8,time.lag=1)
statepairs(eruptionstakens8) #dim=8
```

See Figure 32 on the next page.

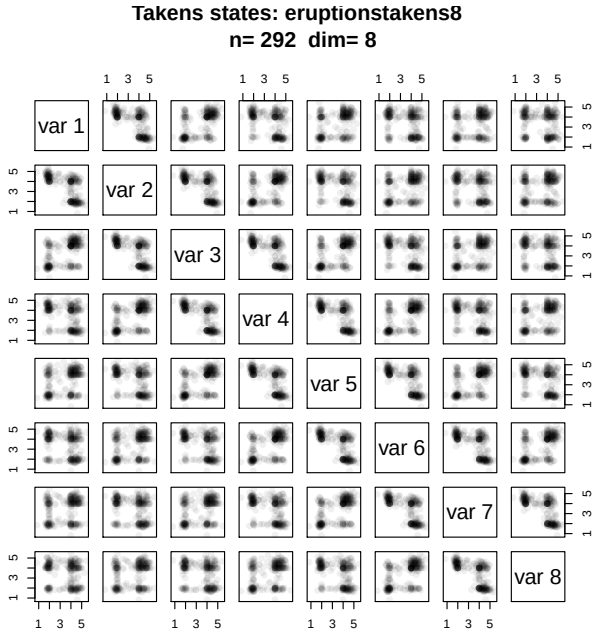

FIGURE 32. Recurrence plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.931 sec.

Input
```
statepairs(eruptionstakens8, nooverlap=TRUE) #dim=8
```

See Figure 33 on the following page.

Input
```
eruptionsneighs8<-local.findAllNeighbours(eruptionstakens8,
        radius=0.8)
save(eruptionsneighs8, file="eruptionsneighs8.RData")
```

Input
```
load(file="eruptionsneighs8.RData")
local.recurrencePlotAux(eruptionsneighs8)
```

See Figure 34 on page 33.

Input
```
showrqa(eruptionstakens8, radius=0.8)
```

Output
```
eruptionstakens8  n= 292  Dim: 8
Radius: 0.8  Recurrence coverage REC: 0.024  log(REC)/log(R): 16.799
Determinism: 0.924  Laminarity: 0
DIV: 0.062
Trend: 0  Entropy: 1.8
Diagonal lines max: 16  Mean: 4.583  Mean off main: 3.871
Vertical lines max: 0  Mean: 0
```

See Figure 35 on page 33.

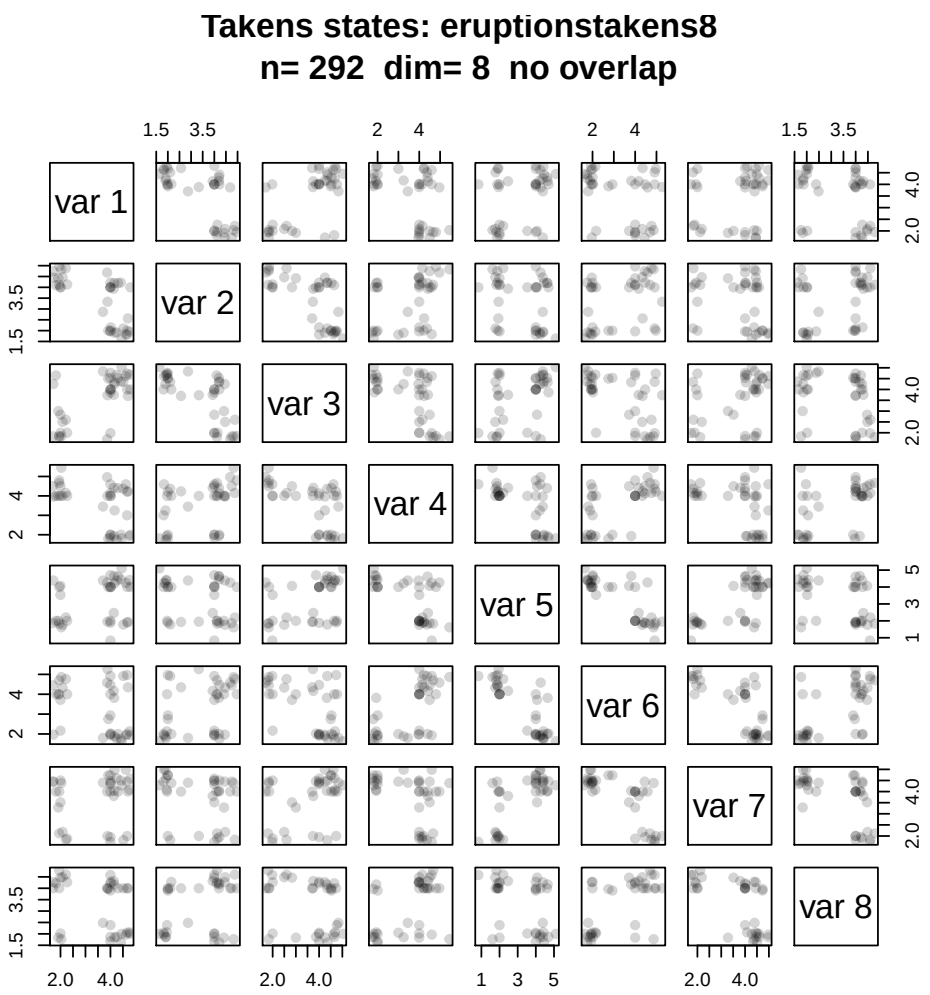

FIGURE 33. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 1.268 sec.

6.2. **Geyser Eruptions: Comparison by Dimension.** For comparison, recurrence plots for the Geyser data with varying dimension are in Figure 36 on page 34

6.3. **Geyser Waiting.**

Input

```
plotsignal(geyser$waiting)
```

See Figure 37 on page 34.

Input

```
waitingtakens <-
        local.buildTakens( time.series=geyser$waiting,
                embedding.dim=4, time.lag=4)
statepairs(waitingtakens) #dim=4
```

See Figure 38 on page 35.

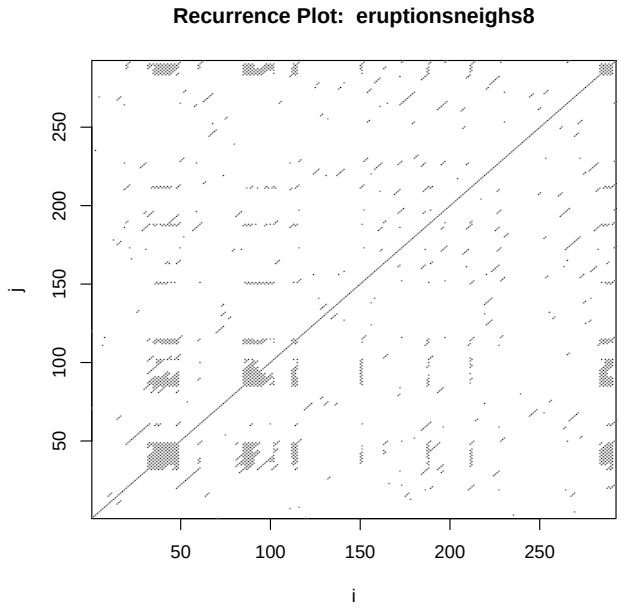

FIGURE 34. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.112 sec.

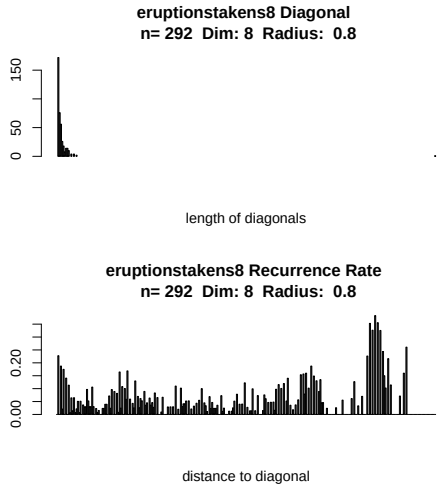

FIGURE 35. Recurrence Plot RQA. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.274 sec.

Input

```
waitingneighs<-local.findAllNeighbours(waitingtakens, radius=16)
save(waitingneighs, file="waitingneighs.Rdata")
```

Input

```
showrqa(waitingtakens, radius=16)
```

Output

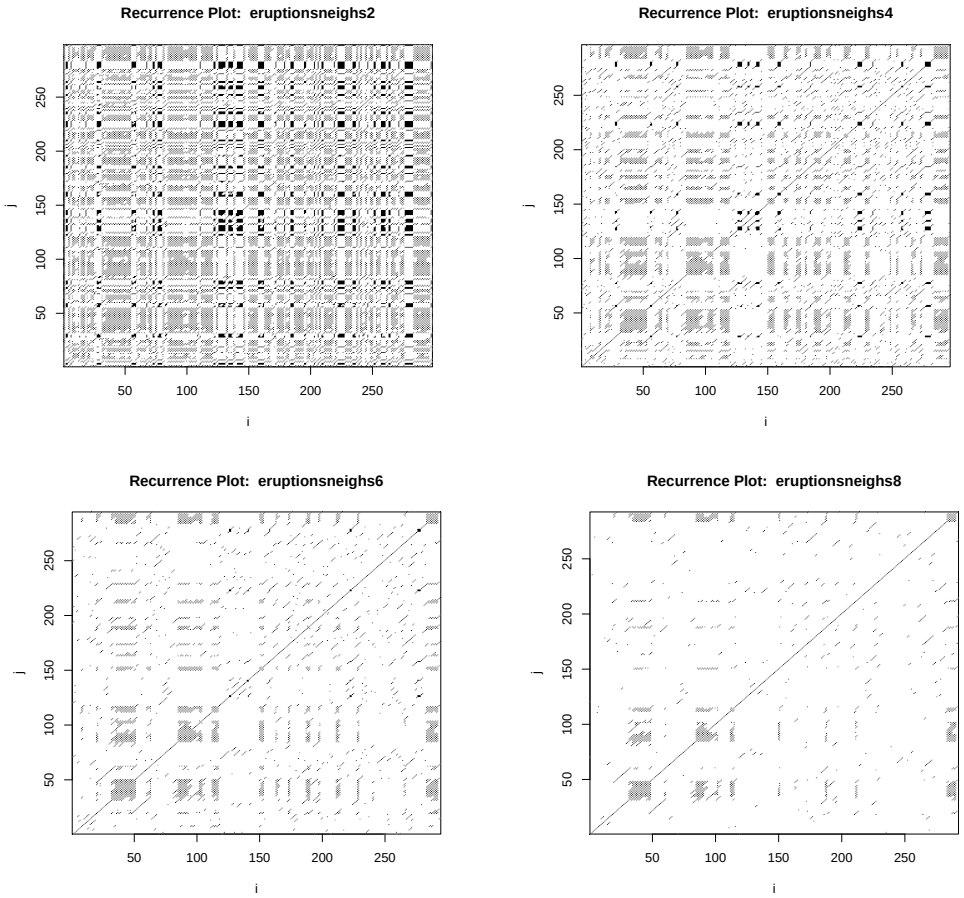

FIGURE 36. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=2, 4, 6, 8.

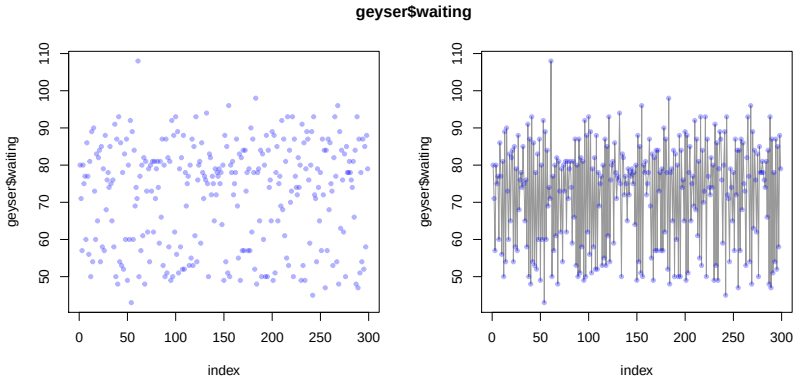

FIGURE 37. Example case: Old Faithful Geyser waiting. Signal and linear interpolation. Time used: 0.491 sec.

```
waitingtakens  n= 287  Dim: 4
Radius: 16  Recurrence coverage REC: 0.137  log(REC)/log(R): -0.718
Determinism: 0.382  Laminarity: 0.053
DIV: 0.053
```

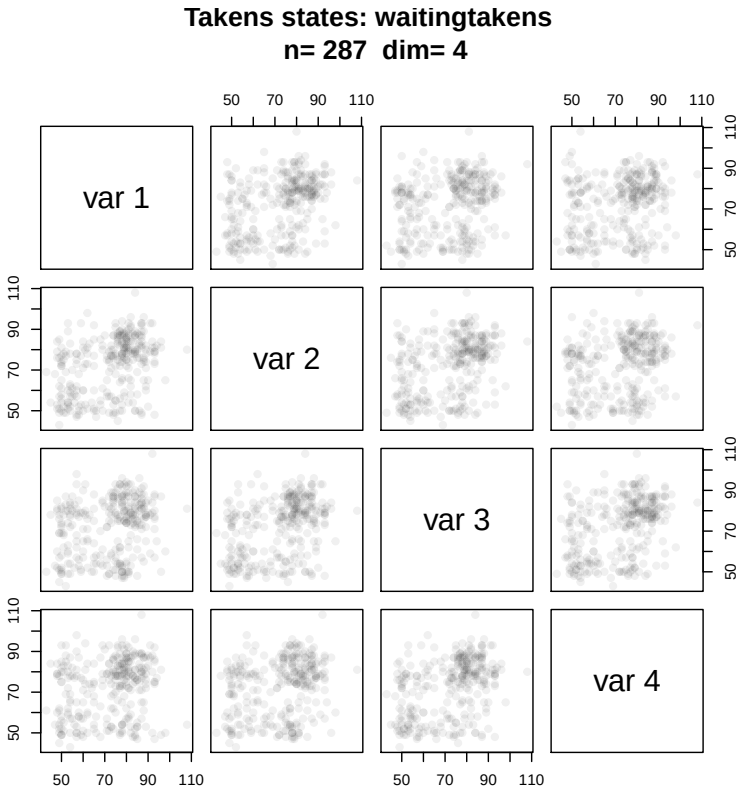

FIGURE 38. Example case: Old Faithful Geyser waiting. Time used: 0.298 sec.

```
Trend: 0  Entropy: 1.002
Diagonal lines max: 19  Mean: 2.878  Mean off main: 2.688
Vertical lines max: 3  Mean: 2.315
```

See Figure 39 on the following page.

Input

```
load(file="waitingneighs.RData")
local.recurrencePlotAux(waitingneighs)
```

See Figure 40 on the next page.

6.4. **Geyser - linearized.** So far, *nonlinearTseries* only handles multivariate data by FORTRAN conventions, using a lag parameter.

As a hack, we transform the data to FORTRAN conventions.

Input

```
geyserlin <- t(geyser)
dim(geyserlin)<-NULL
dimnames(geyserlin)<-NULL
```

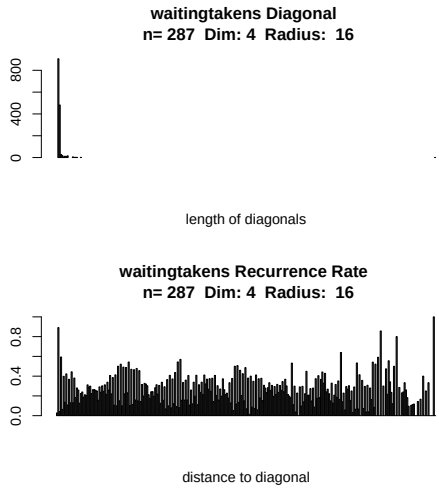

FIGURE 39. Recurrence Plot RQA. Example case: Old Faithful Geyser waiting. Time used: 0.175 sec.

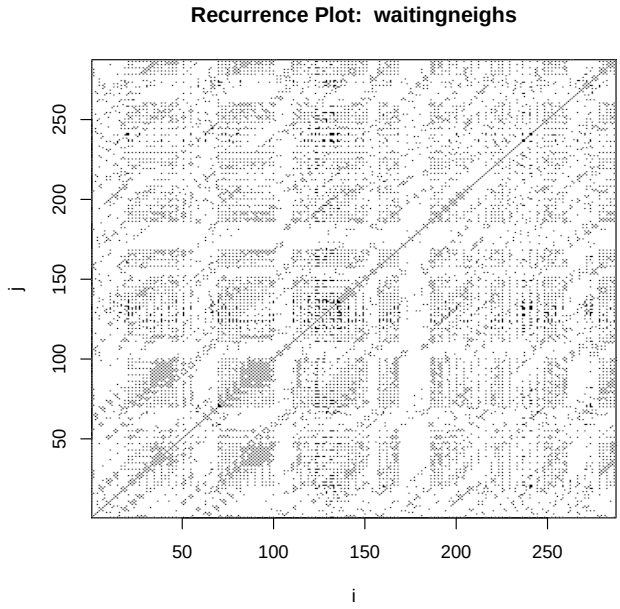

FIGURE 40. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser waiting. Time used: 0.401 sec.

Now duration and waiting are mixed. A $lag = 2$ separates the dimension again. The Taken states iterate over the index, giving alternating a duration and waiting state.

6.5. **Geyser Eruptions linearized.**

Input

```
plotsignal(geyserlin)
```

See Figure 41.

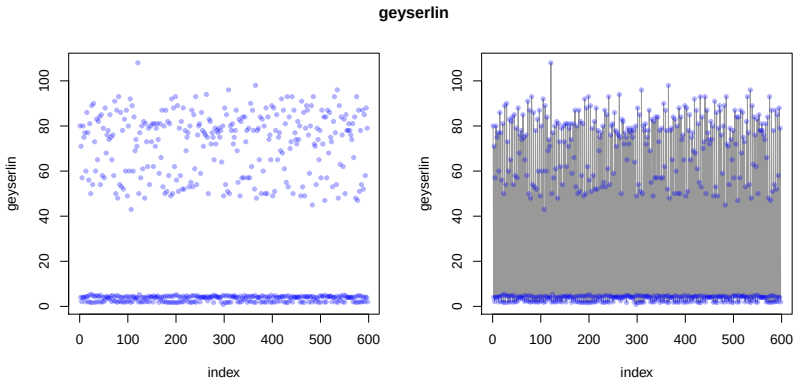

FIGURE 41. Example case: Old Faithful Geyser eruptions. Signal and linear interpolation.

Input

```
glineruptionstakens4 <-
        local.buildTakens( time.series=geyserlin,
                embedding.dim=4, time.lag=2)
statepairs(glineruptionstakens4) #dim=4
```

See Figure 42 on the next page.

Input

```
glineruptionsneighs4<-local.findAllNeighbours(glineruptionstakens4, radius=0.8)
save(glineruptionsneighs4, file="glineruptionsneighs4.RData")
```

Input

```
showrqa(glineruptionstakens4, radius=0.8)
```

Output

```
glineruptionstakens4  n= 592  Dim: 4
Radius: 0.8  Recurrence coverage REC: 0.029  log(REC)/log(R): 15.901
Determinism: 0.059  Laminarity: 0
DIV: Inf
Trend: 0  Entropy: 0
Diagonal lines max: 0  Mean: 592  Mean off main: NaN
Vertical lines max: 0  Mean: 0
```

See Figure 43 on the following page.

Input

```
load(file="glineruptionsneighs4.RData")
local.recurrencePlotAux(glineruptionsneighs4)
```

See Figure 44 on page 39.

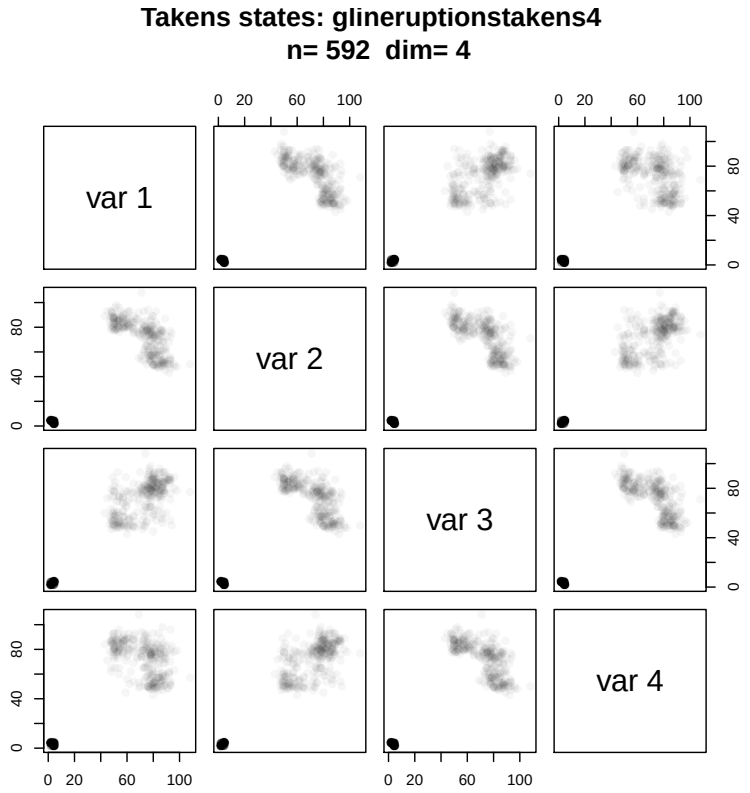

FIGURE 42. Example case: Old Faithful Geyser eruptions. Time used: 0.434 sec.

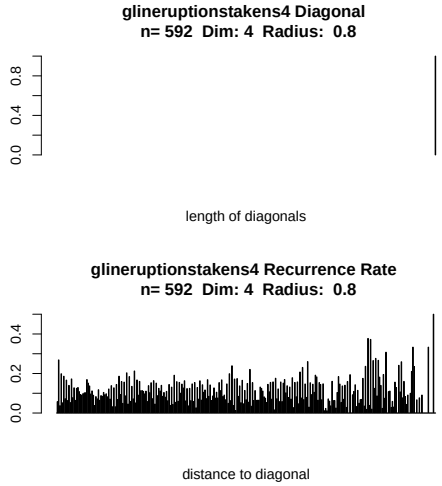

FIGURE 43. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.284 sec.

6.5.1. *Geyser eruptions - linearized. Dim=2.*

Input

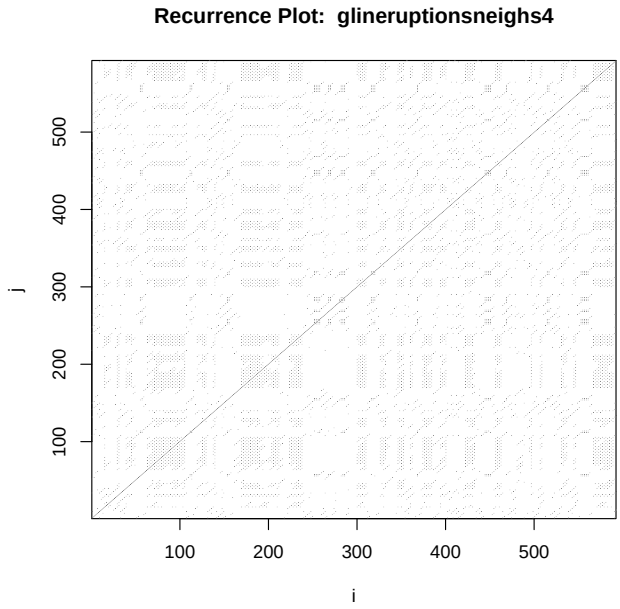

FIGURE 44. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 0.601 sec.

```
glineruptionstakens2 <-
        local.buildTakens(time.series=geyserlin,
                embedding.dim=2, time.lag=2)
statepairs(glineruptionstakens2) #dim=2
```

See Figure 45 on the following page.

Input

```
glineruptionsneighs2<-local.findAllNeighbours(glineruptionstakens2, radius=0.8)
save(glineruptionsneighs2, file="glineruptionsneighs2.RData")
```

Input

```
load(file="glineruptionsneighs2.RData")
local.recurrencePlotAux(glineruptionsneighs2)
```

See Figure 46 on the next page.

6.5.2. *Geyser eruptions - linearized. Dim=8.*

Input

```
glineruptionstakens8 <- local.buildTakens( time.series=geyserlin,
        embedding.dim=8,time.lag=2)
statepairs(glineruptionstakens8) #dim=8
```

See Figure 47 on page 41

Input

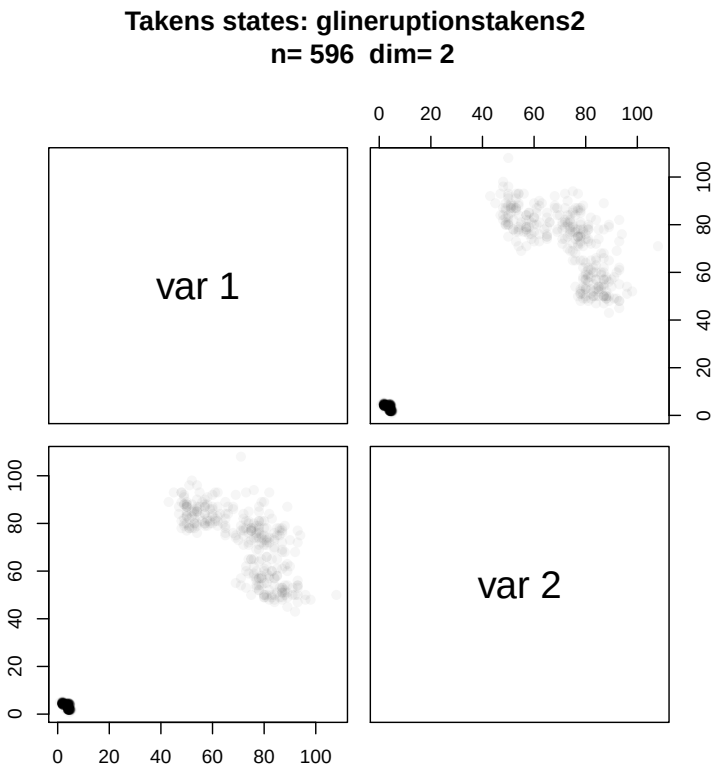

FIGURE 45. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.149 sec.

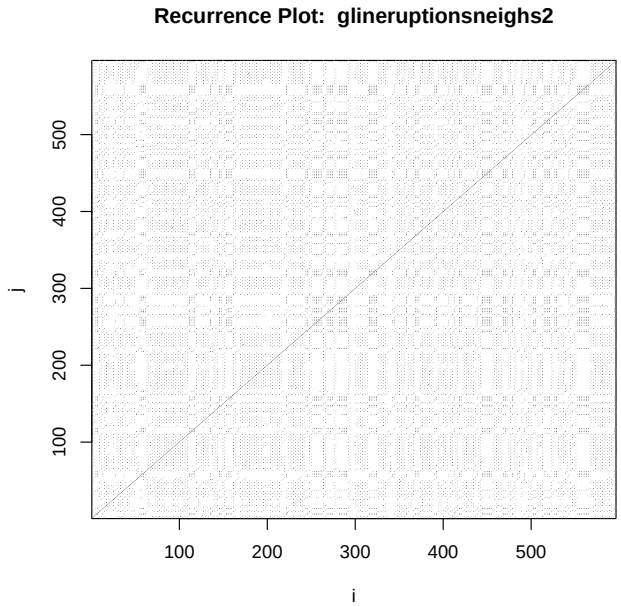

FIGURE 46. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser eruptions linearized. Dim=2. Time used: 0.376 sec.

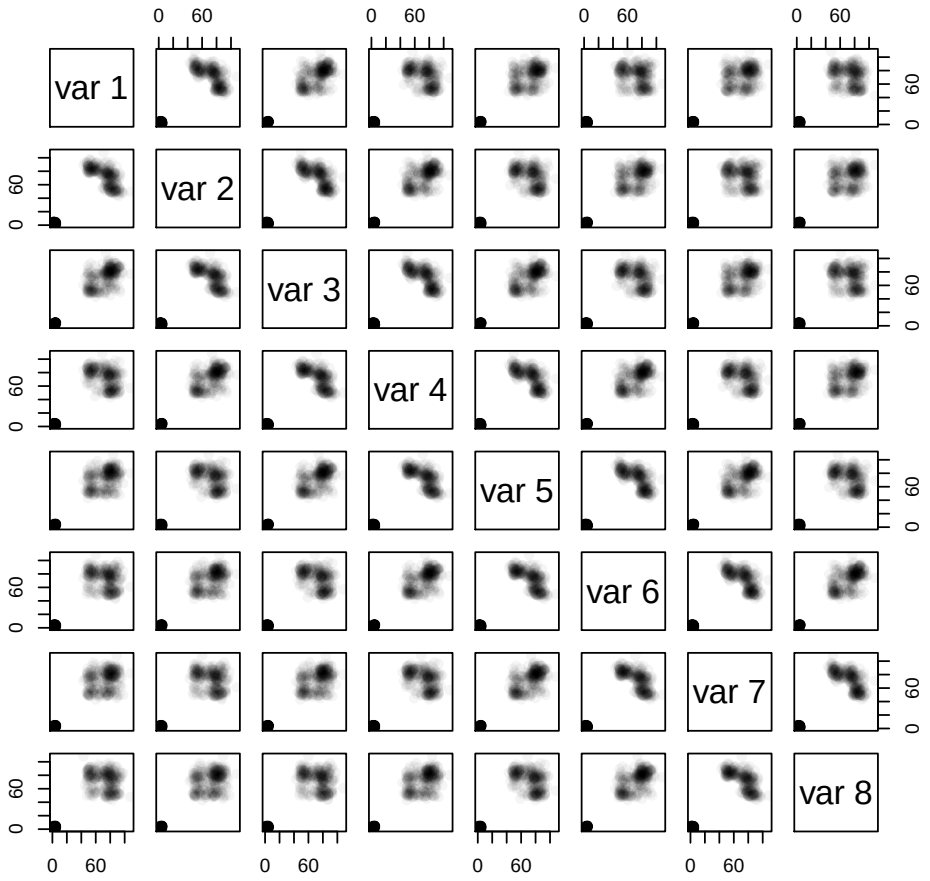

FIGURE 47. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 1.512 sec.

```
glineruptionsneighs8<-local.findAllNeighbours(glineruptionstakens8,
        radius=0.8)
save(glineruptionsneighs8, file="glineruptionsneighs8.RData")
```

Input

```
load(file="glineruptionsneighs8.RData")
local.recurrencePlotAux(glineruptionsneighs8)
```

See Figure 48 on the next page.

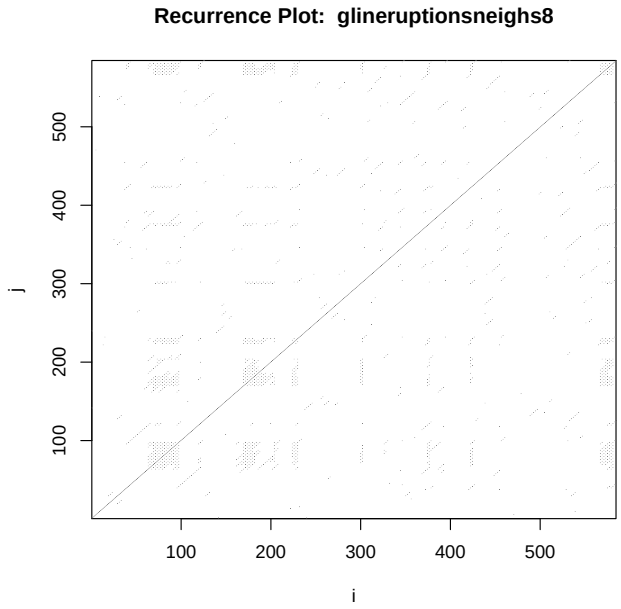

FIGURE 48. Recurrence plot. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 0.238 sec.

7. Case Study: HRV data example.beats

Only 1024 data points used in this section

Input

```
library(RHRV)
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVData.rda")
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVProcessedData.rda")
#####################################################
### code chunk number 1: creation
#####################################################
hrv.data  = CreateHRVData()
hrv.data = SetVerbose(hrv.data, TRUE )
#####################################################
### code chunk number 3: loading
#####################################################
hrv.data = LoadBeatAscii(hrv.data, "example.beats",
       RecordPath = "/users/gs/projects/rforge/rhrv/tutorial/beatsFolder")
```

Output

```
** Loading beats positions for record: example.beats **
   Path: /users/gs/projects/rforge/rhrv/tutorial/beatsFolder
   Scale: 1
   Date: 01/01/1900
   Time: 00:00:00
   Number of beats: 17360
```

Input

```
#       RecordPath = "beatsFolder")


#####################################################
### code chunk number 4: derivating
#####################################################
hrv.data = BuildNIHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 17360
```

Input

Input

```
plotsignal(hrv.data$Beat$RR)
```

See Figure 49 on the following page.

ToDo: We have outliers at approximately 2*RR. Could this be an artefact of preprocessing, filtering out too many impulses?

Input

```
hrvRRtakens4 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=4,time.lag=1)
statepairs(hrvRRtakens4) #dim=4
```

See Figure 53 on page 46.

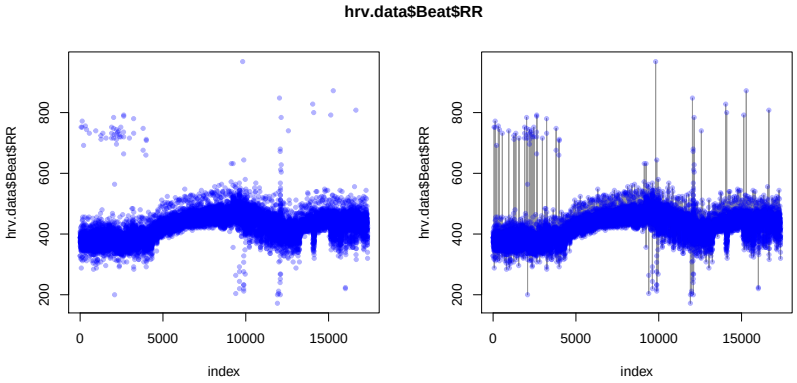

FIGURE 49. RHRV tutorial example.beats. Signal and linear interpolation.

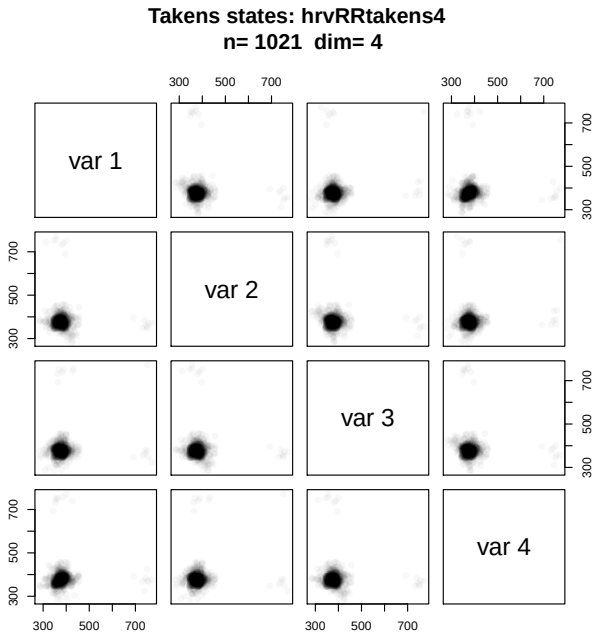

FIGURE 50. Recurrence plot. RHRV tutorial example.beats. Time used: 0.576 sec.

Input

```
statepairs(hrvRRtakens4, rank=TRUE) #dim=4
```

See Figure 51 on the facing page.

Input

```
statecoplot(hrvRRtakens4) #dim=4
```

See Figure 52 on page 46.

See Figure 53 on page 46.

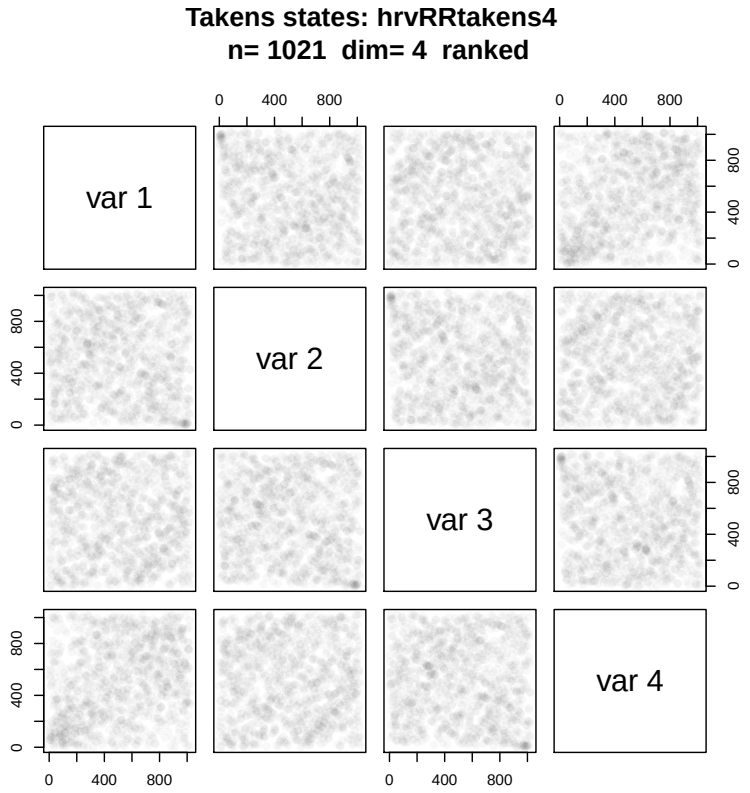

FIGURE 51. RHRV tutorial example.beats. Ranked data. Time used: 1.241 sec.

Input

```
hrvRRneighs4 <-local.findAllNeighbours(hrvRRtakens4, radius=16)
save(hrvRRneighs4, file="hrvRRneighs4.Rdata")
```

Time used: 0.143 sec.

Input

```
load(file="hrvRRneighs4.RData")
local.recurrencePlotAux(hrvRRneighs4)
```

See Figure 54 on page 47.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

7.1. RHRV: example.beats - Comparison by Dimension.

Input

```
hrvRRtakens2 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrvRRneighs2 <-local.findAllNeighbours(hrvRRtakens2, radius=16)
save(hrvRRneighs2, file="hrvRRneighs2.Rdata")
```

Time used: 0.151 sec.

Input

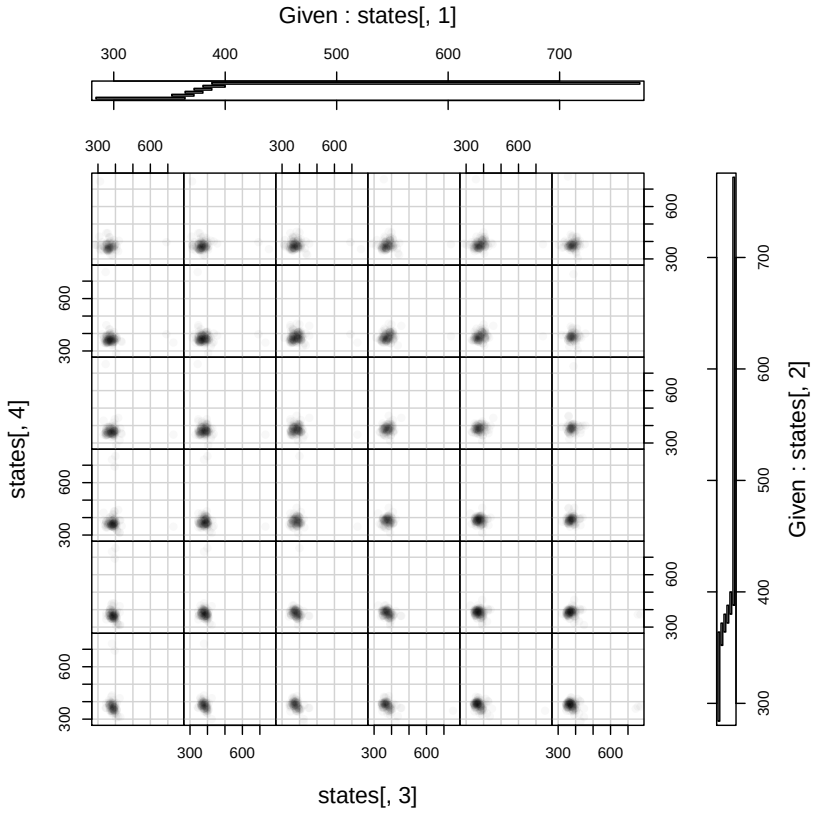

FIGURE 52. State coplot. RHRV tutorial example.beats. Time used: 0.283 sec.

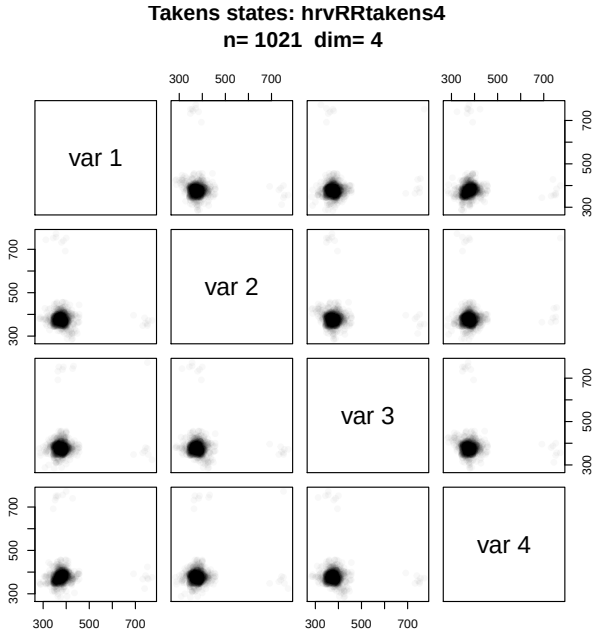

FIGURE 53. Recurrence plot. RHRV tutorial example.beats. Time used: 0.001 sec.

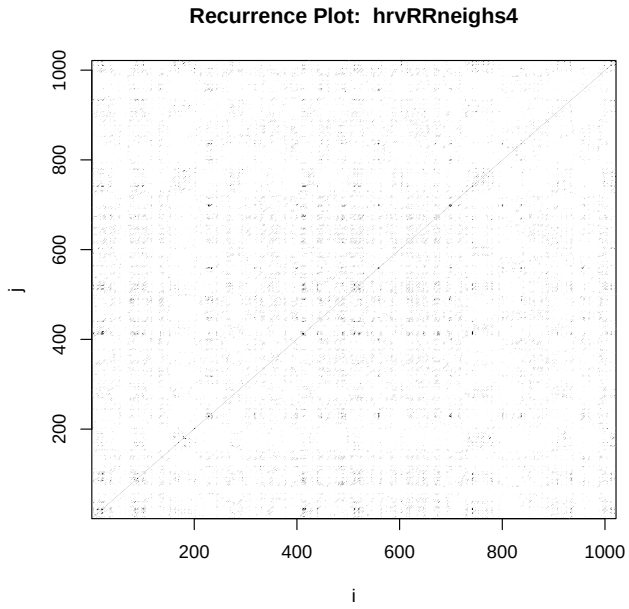

FIGURE 54. Recurrence plot. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=4. Time used: 0.794 sec.

```
load(file="hrvRRneighs2.RData")
local.recurrencePlotAux(hrvRRneighs2)
```

See Figure 55 on page 49. Time used: 1.606 sec.

Input

```
hrvRRtakens6 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=6,time.lag=1)
hrvRRneighs6 <-local.findAllNeighbours(hrvRRtakens6, radius=16)
save(hrvRRneighs6, file="hrvRRneighs6.Rdata")
```

Time used: 0.344 sec.

Input

```
load(file="hrvRRneighs6.RData")
local.recurrencePlotAux(hrvRRneighs6)
```

Dim=6. Time used: 0.614 sec.

Input

```
hrvRRtakens8 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=8,time.lag=1)
hrvRRneighs8 <-local.findAllNeighbours(hrvRRtakens8, radius=32)
save(hrvRRneighs8, file="hrvRRneighs8.Rdata")
```

Time used: 0.325 sec.

Input

```
load(file="hrvRRneighs8.RData")
local.recurrencePlotAux(hrvRRneighs8)
```

Dim=8. Time used: 0.937 sec.

Input

```
hrvRRtakens12 <- local.buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrvRRneighs12 <-local.findAllNeighbours(hrvRRtakens12, radius=16)
save(hrvRRneighs12, file="hrvRRneighs12.Rdata")
```

Time used: 1.212 sec.

Input

```
load(file="hrvRRneighs12.RData")
local.recurrencePlotAux(hrvRRneighs12)
```

Time used: 1.511 sec.

Input

```
hrvRRtakens16 <- local.buildTakens(
        time.series=hrv.data$Beat$RR[1:nsignal],
        embedding.dim=16,time.lag=1)
hrvRRneighs16 <-local.findAllNeighbours(hrvRRtakens16, radius=32)
save(hrvRRneighs16, file="hrvRRneighs16.Rdata")
```

Time used: 1.826 sec.

Input

```
load(file="hrvRRneighs16.RData")
local.recurrencePlotAux(hrvRRneighs16)
```

Time used: 0.558 sec.

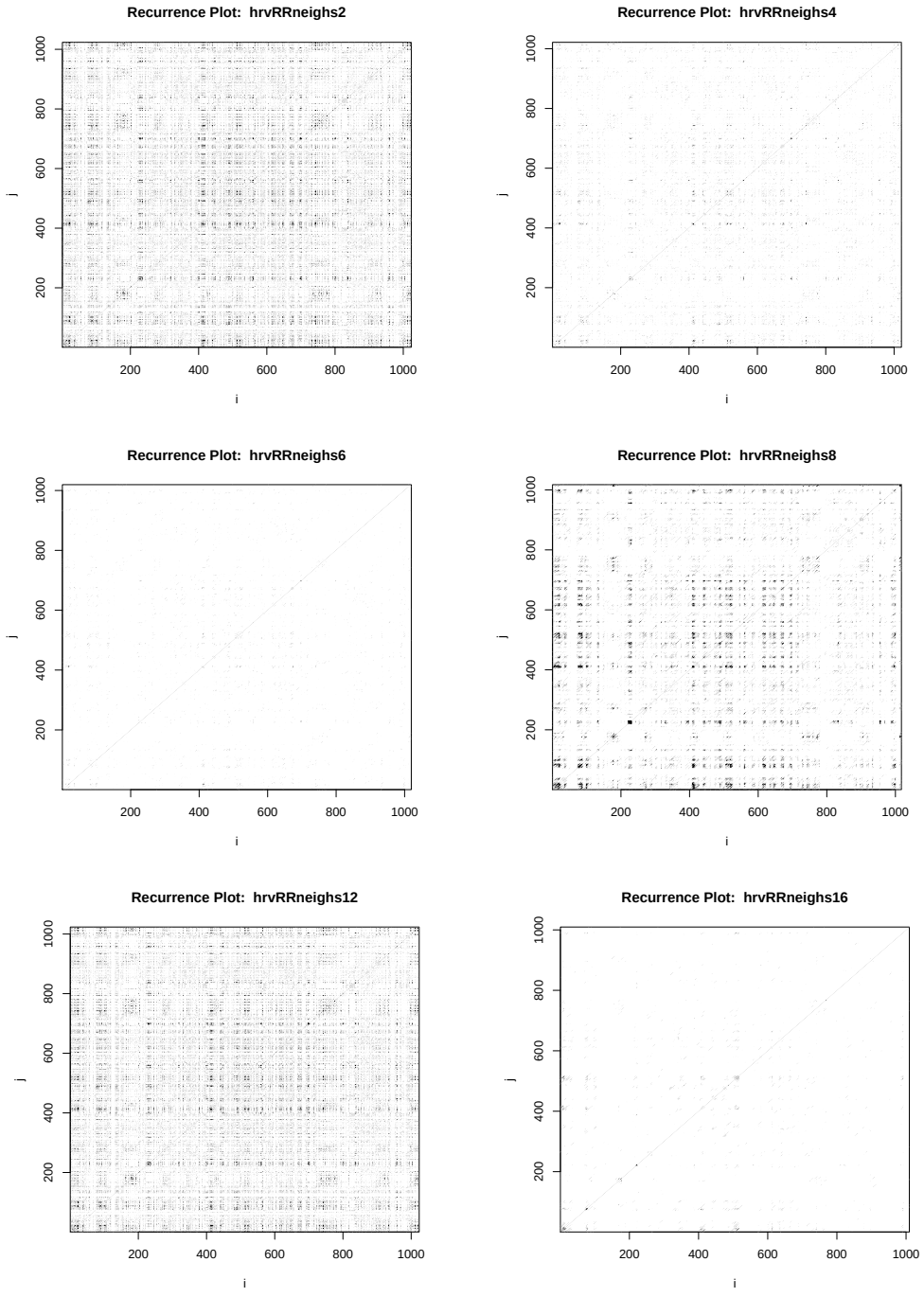

FIGURE 55. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 0.558 sec.

ToDo: This is an experimental proposal

7.2. **RHRV: example.beats - Hart Rate Variation.** Since we are not interested in heart rate (or pulse), but in heart rate variation, a proposal is to use scaled differences.

Input

```
# source('/users/gs/projects/rforge/rhrv/pkg/R/BuildNIHR2.R', chdir = TRUE)
BuildNIDHR <-
function(HRVData, verbose=NULL) {
#------------------------------------------------------
# Obtains instantaneous heart rate variation from beats positions
# D for difference
#------------------------------------------------------
        if (!is.null(verbose)) {
                cat("  --- Warning: deprecated argument, using SetVerbose() instead ---\n ",
                        "   --- See help for more information!! ---\n")
                SetVerbose(HRVData,verbose)
        }

        if (HRVData$Verbose) {
                cat("** Calculating non-interpolated heart rate differences **\n")
        }

        if (is.null(HRVData$Beat$Time)) {
                cat("   --- ERROR: Beats positions not present... Impossible to calculate Heart Rate!! -
                return(HRVData)
        }

        NBeats=length(HRVData$Beat$Time)
        if (HRVData$Verbose) {
                cat("   Number of beats:",NBeats,"\n");
        }

  # addition gs
   #using NA, not constant extrapolation as else in RHRV
   #drr=c(NA,NA,1000.0*        diff(HRVData$Beat$Time, lag=1 , differences=2))
   HRVData$Beat$dRR=c(NA, NA,
           1000.0*diff(HRVData$Beat$Time, lag=1, differences=2))

   HRVData$Beat$avRR=(c(NA,HRVData$Beat$RR[-1])+HRVData$Beat$RR)/2

   HRVData$Beat$HRRV <- HRVData$Beat$dRR/HRVData$Beat$avRR

        return(HRVData)
}
```

differences for HRV

Input

```
hrv.data <- BuildNIDHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate differences **
   Number of beats: 17360
```

Input

```
HRRV <- hrv.data$Beat$HRRV
```

These are the displays of the Takens state space we used before, now for HRRV:

Input
```
plotsignal(HRRV)
```

See Figure 56,

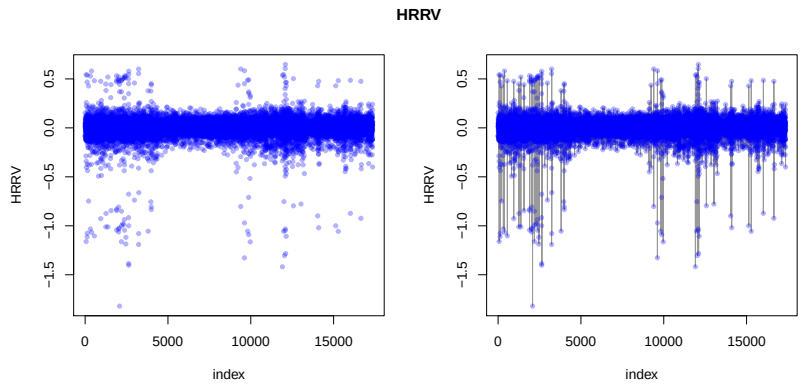

FIGURE 56. RHRV tutorial example.beats. HRRV Signal and linear interpolation.

Only 1024 data points used in this section

Input
```
hrvRRVtakens4 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=4,time.lag=1)
statepairs(hrvRRVtakens4) #dim=4
```

See Figure 57 on the following page

Input
```
statepairs(hrvRRVtakens4, rank=TRUE) #dim=4
```

See Figure 58 on page 53

ToDo: findAllNeighbours does not handle NAs

Input
```
#use hack: findAllNeighbours does not handle NAs
hrvRRVneighs4 <-local.findAllNeighbours(hrvRRVtakens4[-(1:2),], radius=0.125)
save(hrvRRVneighs4, file="hrvRRVneighs4.Rdata")
```

Time used: 0.25 sec.

Input
```
load(file="hrvRRVneighs4.RData")
local.recurrencePlotAux(hrvRRVneighs4,dim=4, radius=0.125)
```

ToDo: check. There seem to be strange artefacts.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

ToDo: fix default setting for radius.

7.3. RHRV: example.beats - Variation: Comparison by Dimension.

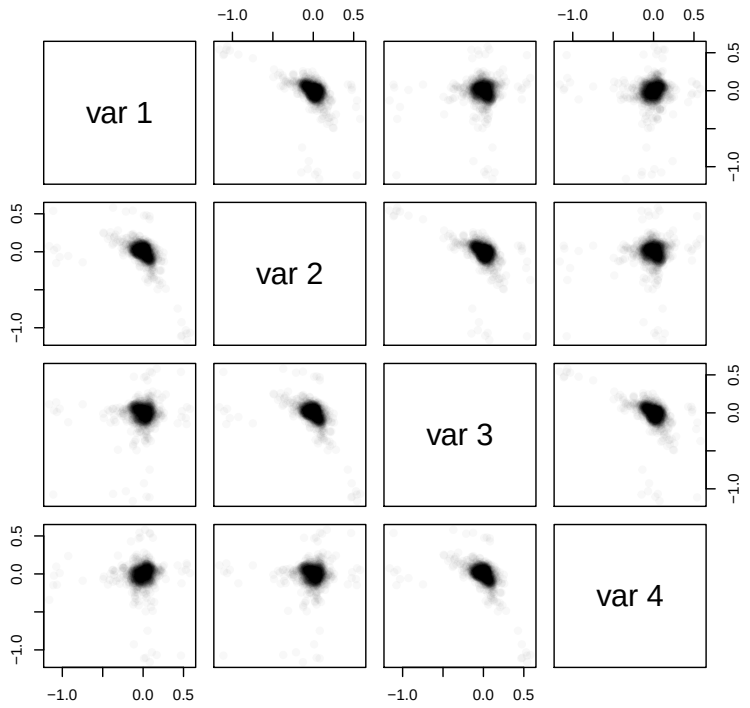

FIGURE 57. RHRV tutorial example.beats. HRRV Time used: 0.75 sec.

Input

```
hrvRRVtakens2 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=2, time.lag=1)
hrvRRVneighs2 <-local.findAllNeighbours(hrvRRVtakens2[-(1:2),],
        radius=0.125)
save(hrvRRVneighs2, file="hrvRRVneighs2.Rdata")
```

Time used: 0.474 sec.

Input

```
showrqa(hrvRRVtakens2[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens2[-(1:2), ]  n= 1021  Dim: 2
Radius: 0.125  Recurrence coverage REC: 0.515  log(REC)/log(R): 0.319
Determinism: 0.939  Laminarity: 0.81
DIV: 0.027
Trend: 0  Entropy: 2.346
Diagonal lines max: 37  Mean: 5.373  Mean off main: 5.362
Vertical lines max: 48  Mean: 3.998
```

Input

```
load(file="hrvRRVneighs2.RData")
local.recurrencePlotAux(hrvRRVneighs2, dim=2,  radius=0.125)
```

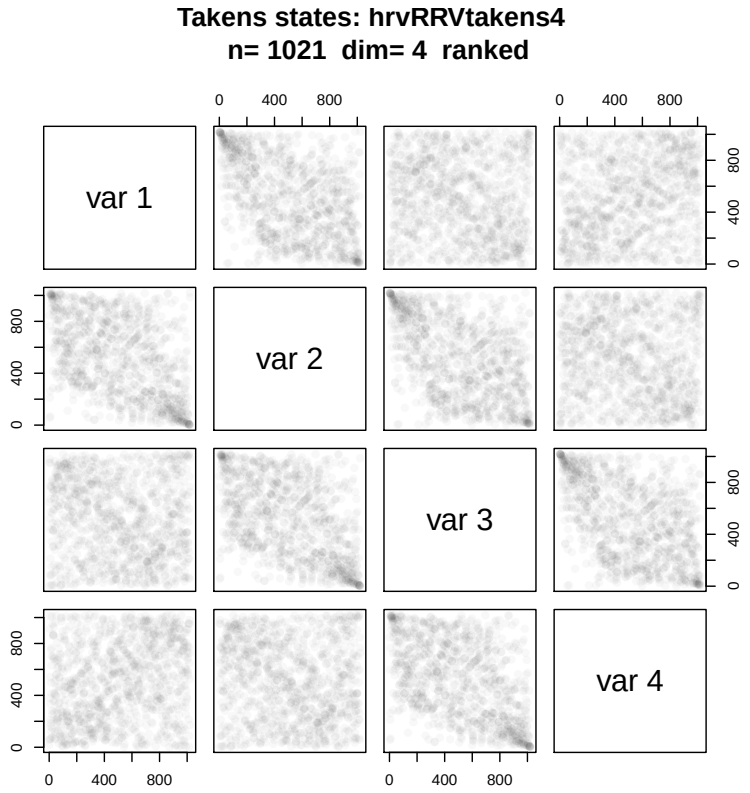

FIGURE 58. RHRV tutorial example.beats. Ranked HRRV data. Time used: 1.508 sec.

Time used: 3.776 sec.

Input

```
hrvRRVtakens6 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=6,time.lag=1)
hrvRRVneighs6 <-local.findAllNeighbours(hrvRRVtakens6[-(1:2),], radius=0.125)
save(hrvRRVneighs6, file="hrvRRVneighs6.Rdata")
```

Time used: 0.406 sec.

Input

```
showrqa(hrvRRVtakens6[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens6[-(1:2), ]  n= 1017  Dim: 6
Radius: 0.125  Recurrence coverage REC: 0.179  log(REC)/log(R): 0.827
Determinism: 0.943  Laminarity: 0.451
DIV: 0.03
Trend: 0  Entropy: 2.305
Diagonal lines max: 33  Mean: 5.243  Mean off main: 5.213
Vertical lines max: 22  Mean: 2.887
```

Input

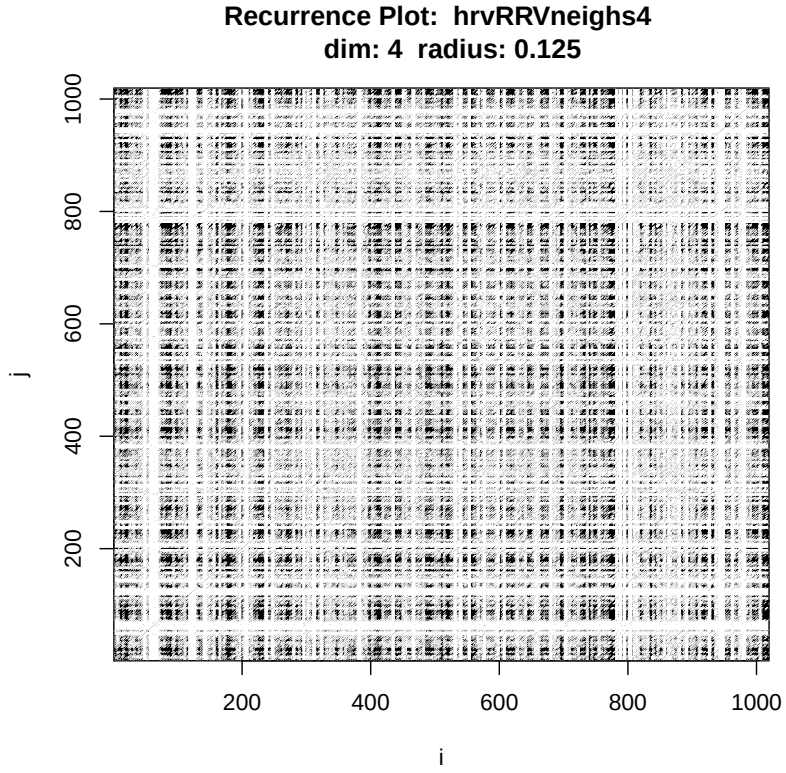

FIGURE 59. Recurrence Plot. Example case: RHRV tutorial example.beats. HRRV Dim=4. Time used: 2.479 sec.

```
load(file="hrvRRVneighs6.RData")
local.recurrencePlotAux(hrvRRVneighs6, dim=6, radius=0.125)
```

Dim=6. Time used: 1.826 sec.

Input

```
hrvRRVtakens8 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=8,time.lag=1)
hrvRRVneighs8 <-local.findAllNeighbours(hrvRRVtakens8[-(1:2),], radius=0.125)
save(hrvRRVneighs8, file="hrvRRVneighs8.Rdata")
```

Time used: 0.535 sec.

Input

```
showrqa(hrvRRVtakens8[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrvRRVtakens8[-(1:2), ]  n= 1015  Dim: 8
Radius: 0.125  Recurrence coverage REC: 0.105  log(REC)/log(R): 1.084
Determinism: 0.943  Laminarity: 0.337
DIV: 0.032
Trend: 0  Entropy: 2.329
Diagonal lines max: 31  Mean: 5.361  Mean off main: 5.308
Vertical lines max: 17  Mean: 2.715
```

Input

```
load(file="hrvRRVneighs8.RData")
local.recurrencePlotAux(hrvRRVneighs8, dim=8, radius=0.125)
```

Dim=8. Time used: 1.177 sec.

Input

```
hrvRRVtakens12 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=12,time.lag=1)
hrvRRVneighs12 <-
        local.findAllNeighbours(hrvRRVtakens12[-(1:2),], radius=3/16)
save(hrvRRVneighs12, file="hrvRRVneighs12.Rdata")
```

Time used: 1.905 sec.

Input

```
showrqa(hrvRRVtakens12[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrvRRVtakens12[-(1:2), ]  n= 1011  Dim: 12
Radius: 0.1875  Recurrence coverage REC: 0.235  log(REC)/log(R): 0.865
Determinism: 0.988  Laminarity: 0.635
DIV: 0.015
Trend: 0  Entropy: 3.075
Diagonal lines max: 68  Mean: 9.775  Mean off main: 9.733
Vertical lines max: 52  Mean: 3.656
```

Input

```
load(file="hrvRRVneighs12.RData")
local.recurrencePlotAux(hrvRRVneighs12, dim=12,  radius=3/16)
```

Time used: 1.97 sec.

Input

```
hrvRRVtakens16 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=16,time.lag=1)
hrvRRVneighs16 <-local.findAllNeighbours(hrvRRVtakens16[-(1:2),], radius=3/16)
save(hrvRRVneighs16, file="hrvRRVneighs16.Rdata")
```

Time used: 2.446 sec.

Input

```
showrqa(hrvRRVtakens16[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrvRRVtakens16[-(1:2), ]  n= 1007  Dim: 16
Radius: 0.1875  Recurrence coverage REC: 0.147  log(REC)/log(R): 1.144
Determinism: 0.99  Laminarity: 0.527
DIV: 0.016
Trend: 0  Entropy: 3.163
Diagonal lines max: 64  Mean: 10.594  Mean off main: 10.523
Vertical lines max: 40  Mean: 3.114
```

Input

```
load(file="hrvRRVneighs16.RData")
local.recurrencePlotAux(hrvRRVneighs16, dim=16, radius=3/16)
```

Time used: 1.318 sec.

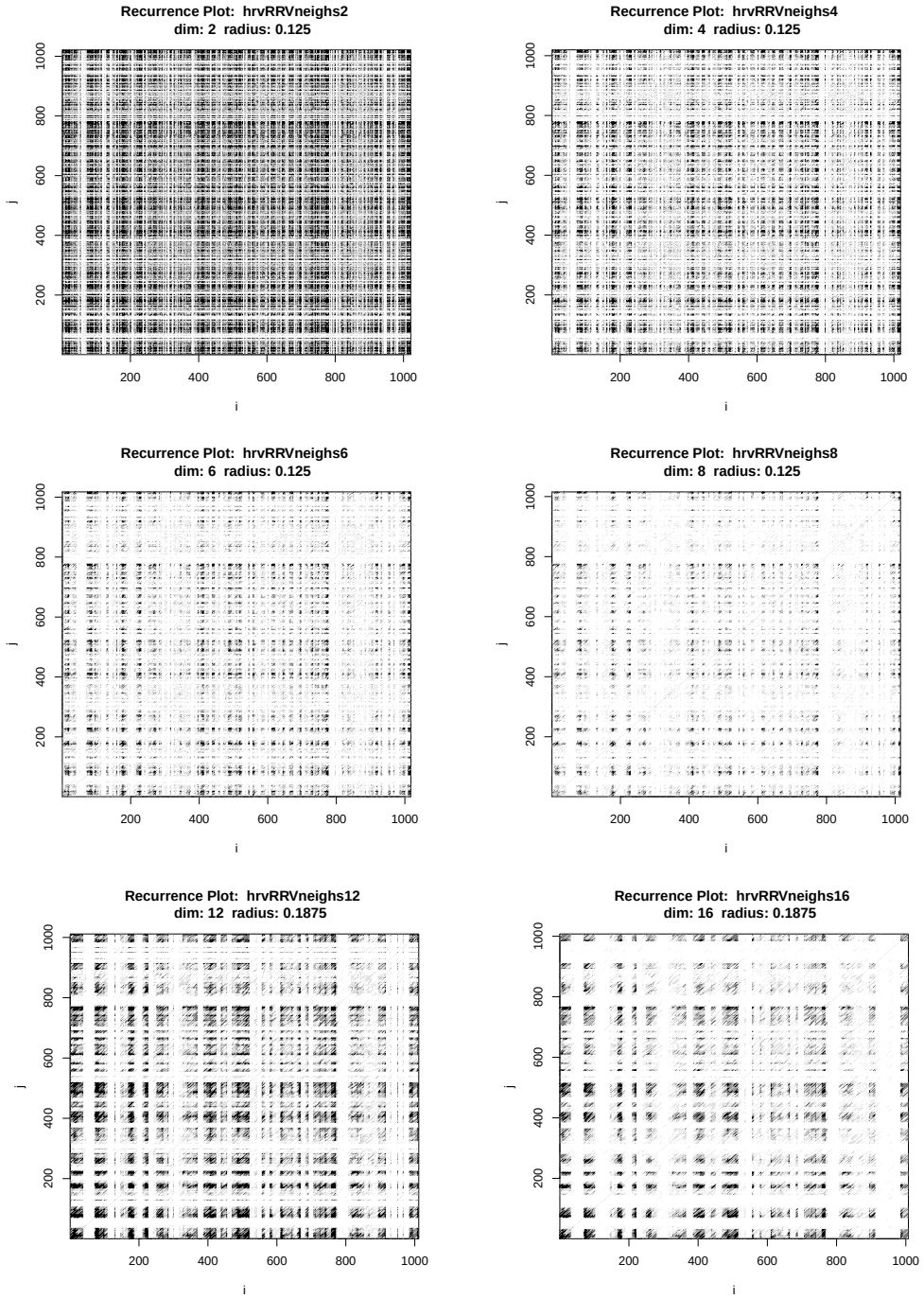

FIGURE 60. Recurrence Plot. Example case: RHRV tutorial example.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 1.318 sec.

8. Case Study: HRV data example2.beats

This is a copy of the previous section, now applied to HRV data example2.beats.

Input

```
library(RHRV)
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVData.rda")
load("/users/gs/projects/rforge/rhrv/pkg/data/HRVProcessedData.rda")
#####################################################
### code chunk number 1: creation
#####################################################
hrv2.data  = CreateHRVData()
hrv2.data = SetVerbose(hrv2.data, TRUE )
#####################################################
### code chunk number 3: loading
#####################################################
hrv2.data = LoadBeatAscii(hrv2.data, "example2.beats",
       RecordPath = "/users/gs/projects/rforge/rhrv/tutorial/beatsFolder")
```

Output

```
** Loading beats positions for record: example2.beats **
   Path: /users/gs/projects/rforge/rhrv/tutorial/beatsFolder
   Scale: 1
   Removed 2437 duplicated beats
   Date: 01/01/1900
   Time: 00:00:00
   Number of beats: 2437
```

Input

```
#       RecordPath = "beatsFolder")


#####################################################
### code chunk number 4: derivating
#####################################################
hrv2.data = BuildNIHR(hrv2.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 2437
```

Input

Input

```
plotsignal(hrv2.data$Beat$RR)
```

See Figure 61 on the following page.

ToDo: We have outliers at approximately 2*RR. Could this be an artefact of preprocessing, filtering out too many impulses?

Input

```
hrv2RRtakens4 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=4,time.lag=1)
statepairs(hrv2RRtakens4) #dim=4
```

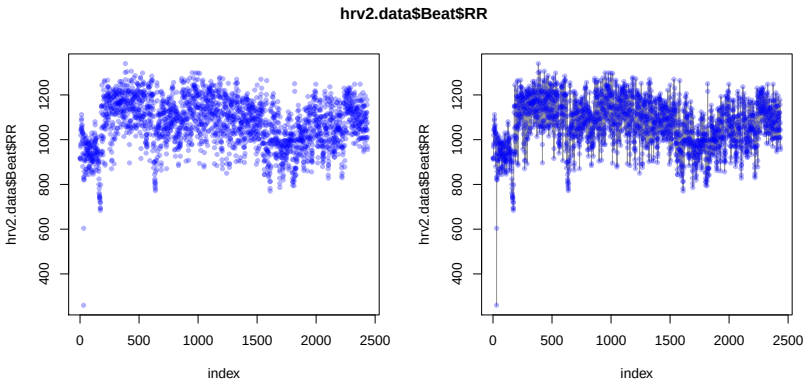

FIGURE 61. RHRV tutorial example2.beats. Signal and linear interpolation.

Only 1024 data points used in this plot

See Figure 62.

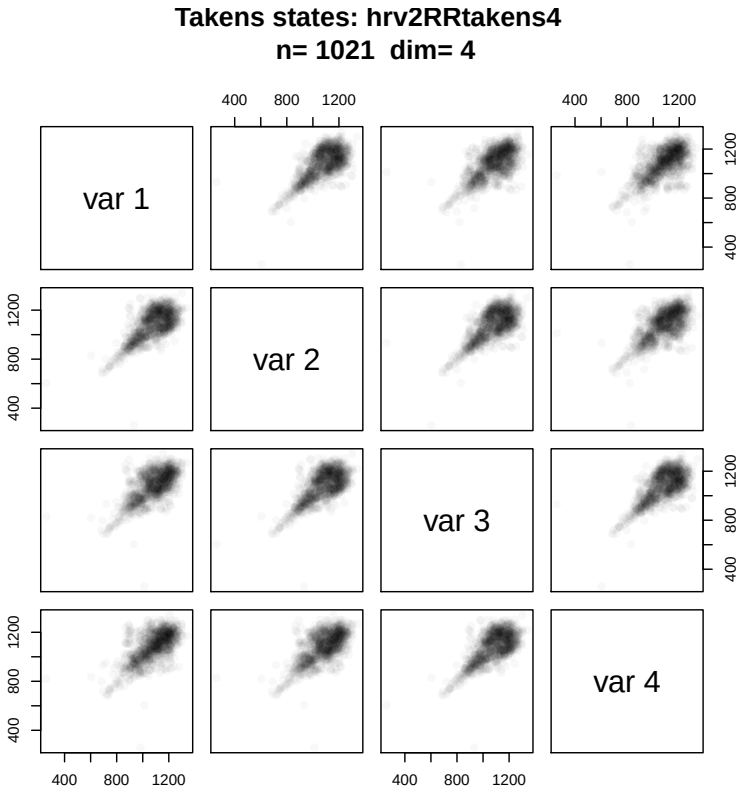

FIGURE 62. RHRV tutorial example2.beats. Time used: 0.684 sec.

Input

```
statepairs(hrv2RRtakens4, rank=TRUE) #dim=4
```

See Figure 63 on the facing page.

Input

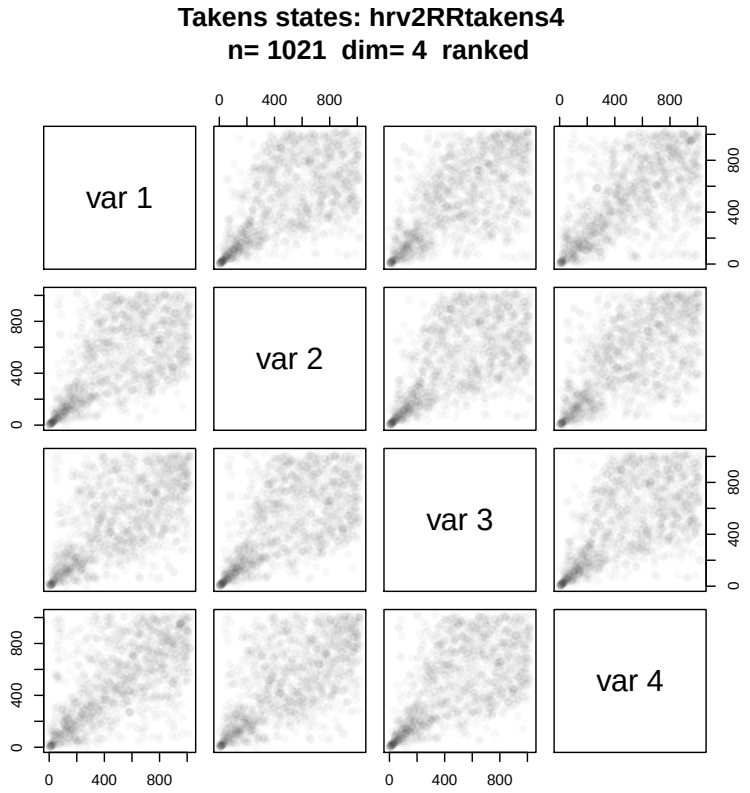

FIGURE 63. RHRV tutorial example2.beats. Ranked data. Time used: 1.413 sec.

```
statecoplot(hrv2RRtakens4) #dim=4
```

See Figure 64 on the next page.

Input

```
hrv2RRneighs4 <-local.findAllNeighbours(hrv2RRtakens4, radius = 48)
save(hrv2RRneighs4, file="hrv2RRneighs4.Rdata")
```

Time used: 0.39 sec.

Input

```
load(file="hrv2RRneighs4.RData")
local.recurrencePlotAux(hrv2RRneighs4)
```

See Figure 65 on page 61.

We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.

8.1. RHRV: example2.beats - Comparison by Dimension.

Input

```
hrv2RRtakens2 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrv2RRneighs2 <-local.findAllNeighbours(hrv2RRtakens2, radius=32)
save(hrv2RRneighs2, file="hrv2RRneighs2.Rdata")
```

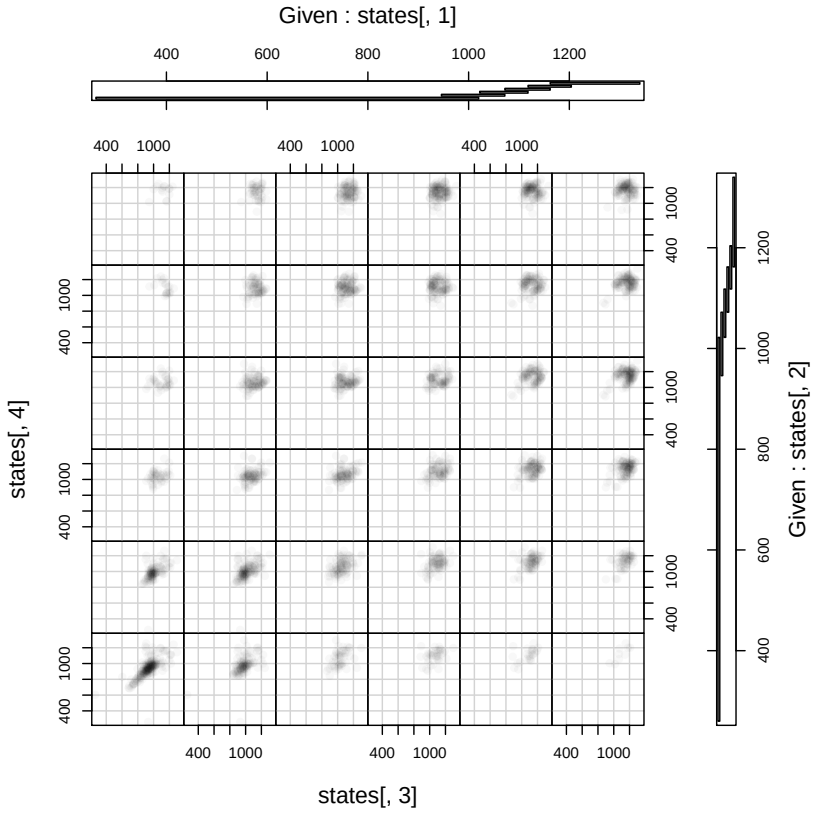

FIGURE 64. State coplot. RHRV tutorial example2.beats. Time used: 0.289 sec.

Time used: 0.246 sec.

Input

```
load(file="hrv2RRneighs2.RData")
local.recurrencePlotAux(hrv2RRneighs2)
```

See Figure 66 on page 63. Time used: 0.769 sec.

Input

```
hrv2RRtakens6 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=6,time.lag=1)
hrv2RRneighs6 <-local.findAllNeighbours(hrv2RRtakens6, radius=96)
save(hrv2RRneighs6, file="hrv2RRneighs6.Rdata")
```

Time used: 0.26 sec.

Input

```
load(file="hrv2RRneighs6.RData")
local.recurrencePlotAux(hrv2RRneighs6)
```

Dim=6. Time used: 0.997 sec.

Input

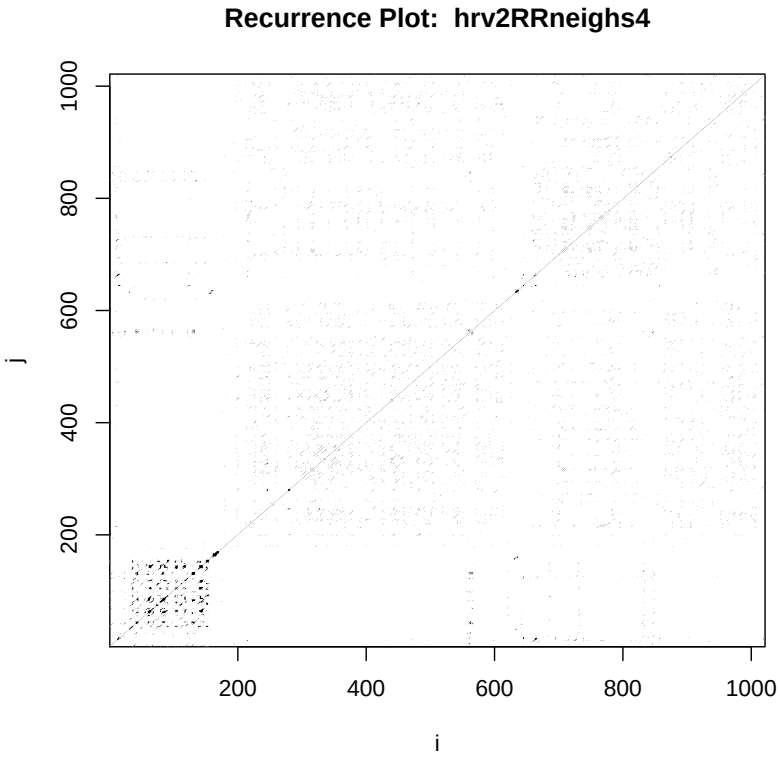

FIGURE 65. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=4. Time used: 0.742 sec.

```
hrv2RRtakens8 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=8,time.lag=1)
hrv2RRneighs8 <-local.findAllNeighbours(hrv2RRtakens8, radius=96)
save(hrv2RRneighs8, file="hrv2RRneighs8.Rdata")
```

Time used: 0.256 sec.

Input

```
load(file="hrv2RRneighs8.RData")
local.recurrencePlotAux(hrv2RRneighs8)
```

Dim=8. Time used: 0.853 sec.

Input

```
hrv2RRtakens12 <- local.buildTakens( time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=2,time.lag=1)
hrv2RRneighs12 <-local.findAllNeighbours(hrv2RRtakens12, radius=96)
save(hrv2RRneighs12, file="hrv2RRneighs12.Rdata")
```

Time used: 1.149 sec.

Input

```
load(file="hrv2RRneighs12.RData")
local.recurrencePlotAux(hrv2RRneighs12)
```

Time used: 2.178 sec.

Input

```
hrv2RRtakens16 <- local.buildTakens(
        time.series=hrv2.data$Beat$RR[1:nsignal],
        embedding.dim=16,time.lag=1)
hrv2RRneighs16 <-local.findAllNeighbours(hrv2RRtakens16, radius=192)
save(hrv2RRneighs16, file="hrv2RRneighs16.Rdata")
```

Time used: 2.626 sec.

Input

```
load(file="hrv2RRneighs16.RData")
local.recurrencePlotAux(hrv2RRneighs16)
```

Time used: 1.361 sec.

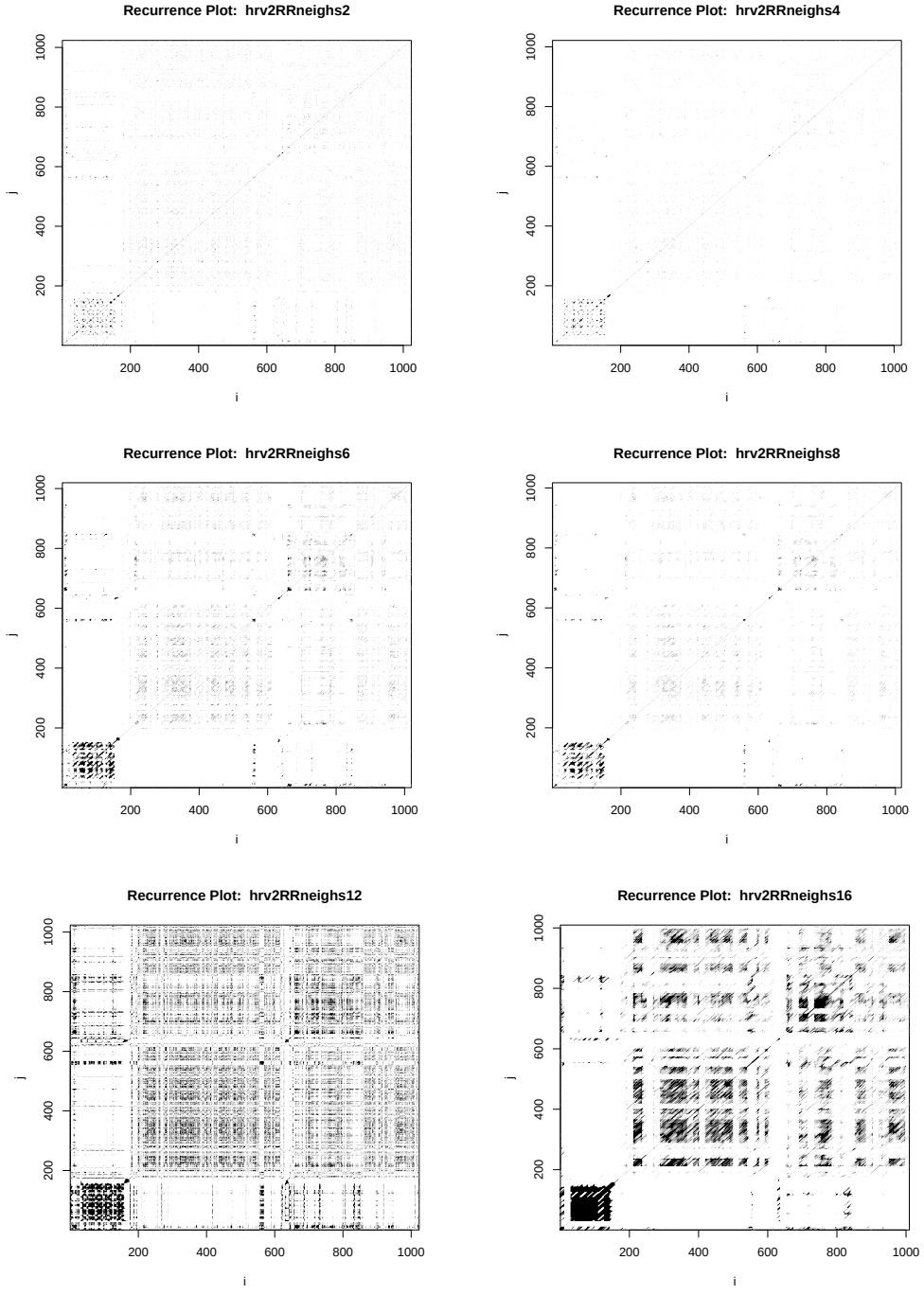

FIGURE 66. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 1.362 sec.

ToDo: Consider using differences

differences for HRV

8.2. **RHRV: example2.beats - Hart Rate Variation.** Since we are not interested in heart rate (or pulse), but in heart rate variation, a proposal is to use scaled differences.

Input

```
hrv2.data <- BuildNIDHR(hrv2.data)
```

Output

```
** Calculating non-interpolated heart rate differences **
   Number of beats: 2437
```

Input

```
HRRV <- hrv2.data$Beat$HRRV
```

These are the displays of the Takens state space we used before, now for HRRV:

Input

```
plotsignal(HRRV)
```

See Figure 67,

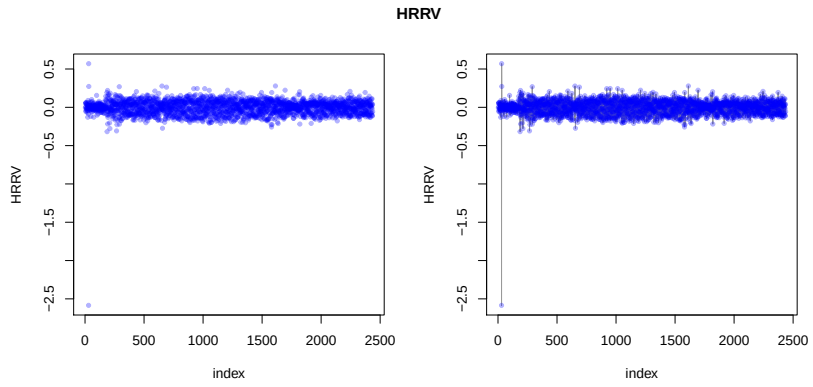

FIGURE 67. RHRV tutorial example2.beats. HRRV Signal and linear interpolation.

Only 1024 data points used in these plots

Input

```
hrv2RRVtakens4 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=4,time.lag=1)
statepairs(hrv2RRVtakens4) #dim=4
```

See Figure 68 on the next page.

Input

```
        statecoplot(hrv2RRVtakens4) #dim=4
```

Output

```
Missing rows: 1, 2
```

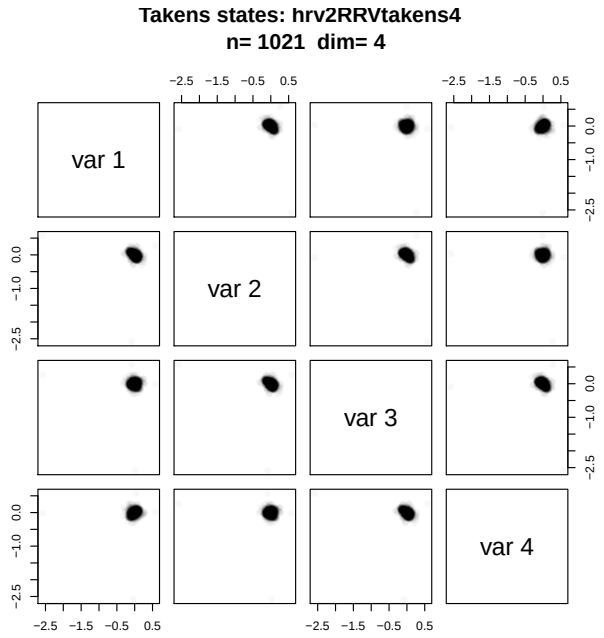

FIGURE 68. Recurrence plot. RHRV tutorial example2.beats. HRRV Time used: 0.761 sec.

See Figure 69 on the following page.

Input
```
statepairs(hrv2RRVtakens4, rank=TRUE) #dim=4
```

See Figure 70 on page 67

ToDo: findAllNeighbours does not handle NAs

Input
```
#use hack: findAllNeighbours does not handle NAs
hrv2RRVneighs4 <-local.findAllNeighbours(hrv2RRVtakens4[-(1:2),], radius=0.125)
save(hrv2RRVneighs4, file="hrv2RRVneighs4.Rdata")
```

Time used: 0.235 sec.

Input
```
load(file="hrv2RRVneighs4.RData")
local.recurrencePlotAux(hrv2RRVneighs4,dim=4, radius=0.125)
```

ToDo: check. There seem to be strange artefacts.
We should expect the breathing rhythm, so a time lag in the order of 10 is to be expected.
ToDo: fix default setting for radius. Eckmann uses nearest neighbours with NN=10

8.3. RHRV: example2.beats - Variation: Comparison by Dimension.

Input
```
hrv2RRVtakens2 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=2, time.lag=1)
hrv2RRVneighs2 <-local.findAllNeighbours(hrv2RRVtakens2[-(1:2),],
        radius=0.125)
save(hrv2RRVneighs2, file="hrv2RRVneighs2.Rdata")
```

Time used: 0.517 sec.

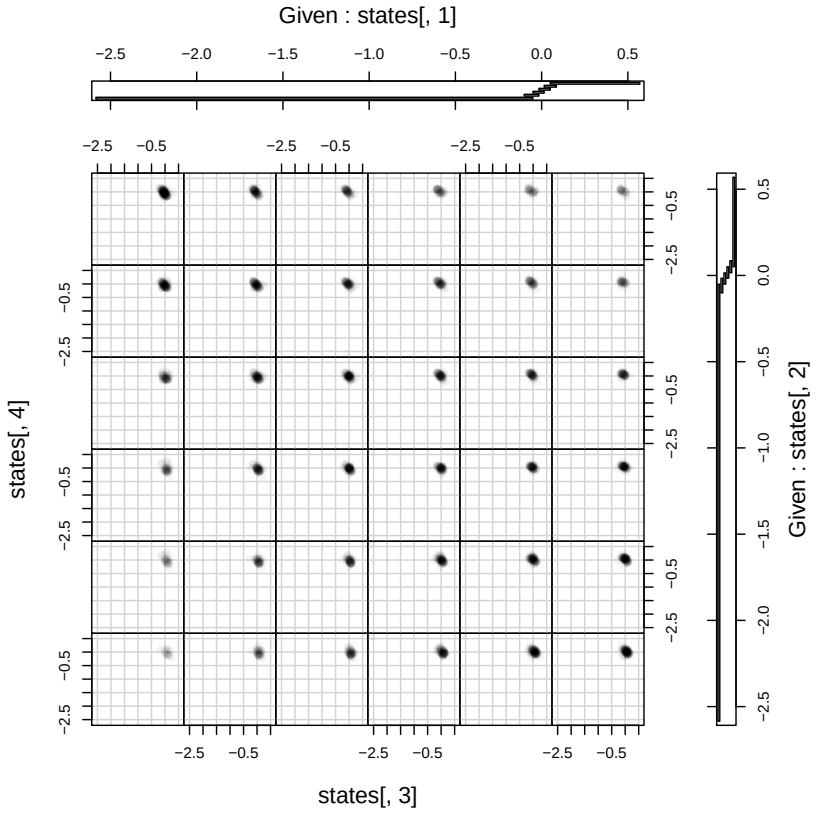

FIGURE 69. State coplot. RHRV tutorial example2.beats. HRRV. Time used: 1.319 sec.

Input

```
showrqa(hrv2RRVtakens2[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens2[-(1:2), ]  n= 1021  Dim: 2
Radius: 0.125  Recurrence coverage REC: 0.508  log(REC)/log(R): 0.326
Determinism: 0.913  Laminarity: 0.712
DIV: 0.009
Trend: 0  Entropy: 2.289
Diagonal lines max: 117  Mean: 5.305  Mean off main: 5.294
Vertical lines max: 86  Mean: 3.923
```

Input

```
load(file="hrv2RRVneighs2.RData")
local.recurrencePlotAux(hrv2RRVneighs2, dim=2,  radius=0.125)
```

Time used: 3.79 sec.

Input

```
hrv2RRVtakens6 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=6,time.lag=1)
hrv2RRVneighs6 <-local.findAllNeighbours(hrv2RRVtakens6[-(1:2),], radius=0.125)
save(hrv2RRVneighs6, file="hrv2RRVneighs6.Rdata")
```

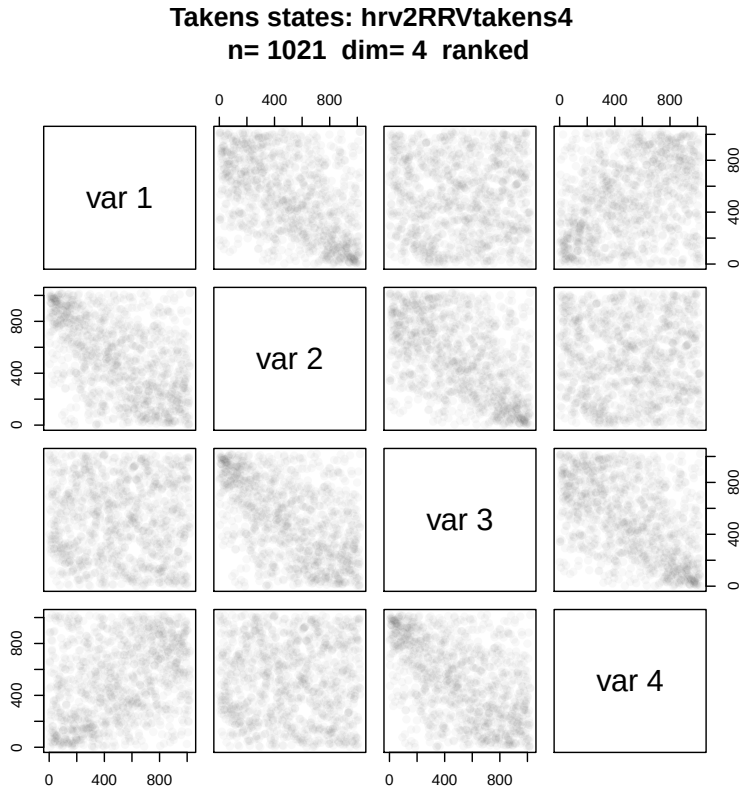

FIGURE 70. RHRV tutorial example2.beats. Ranked HRRV data. Time used: 2.05 sec.

Time used: 0.433 sec.

Input

```
showrqa(hrv2RRVtakens6[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens6[-(1:2), ]  n= 1017  Dim: 6
Radius: 0.125  Recurrence coverage REC: 0.187  log(REC)/log(R): 0.808
Determinism: 0.959  Laminarity: 0.368
DIV: 0.009
Trend: 0  Entropy: 2.528
Diagonal lines max: 113  Mean: 6.229  Mean off main: 6.195
Vertical lines max: 73  Mean: 3.978
```

Input

```
load(file="hrv2RRVneighs6.RData")
local.recurrencePlotAux(hrv2RRVneighs6, dim=6, radius=0.125)
```

Dim=6. Time used: 1.824 sec.

Input

```
hrv2RRVtakens8 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=8,time.lag=1)
```

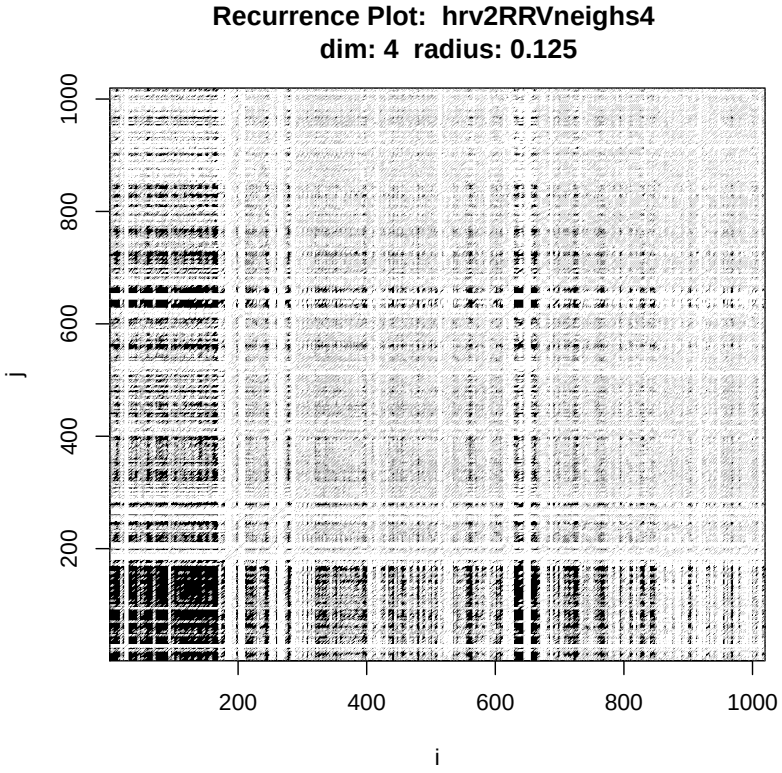

FIGURE 71. Recurrence Plot. Example case: RHRV tutorial example2.beats. HRRV Dim=4. Time used: 2.217 sec.

```
hrv2RRVneighs8 <-local.findAllNeighbours(hrv2RRVtakens8[-(1:2),], radius=0.125)
save(hrv2RRVneighs8, file="hrv2RRVneighs8.Rdata")
```

Time used: 0.417 sec.

Input

```
showrqa(hrv2RRVtakens8[-(1:2),], radius=0.125, do.hist=FALSE)
```

Output

```
hrv2RRVtakens8[-(1:2), ]  n= 1015  Dim: 8
Radius: 0.125  Recurrence coverage REC: 0.122  log(REC)/log(R): 1.012
Determinism: 0.96  Laminarity: 0.335
DIV: 0.009
Trend: 0  Entropy: 2.57
Diagonal lines max: 111  Mean: 6.482  Mean off main: 6.428
Vertical lines max: 71  Mean: 4.039
```

Input

```
load(file="hrv2RRVneighs8.RData")
local.recurrencePlotAux(hrv2RRVneighs8, dim=8, radius=0.125)
```

Dim=8. Time used: 1.38 sec.

Input

```
hrv2RRVtakens12 <-
        local.buildTakens( time.series=HRRV[1:nsignal],
                embedding.dim=12,time.lag=1)
hrv2RRVneighs12 <-
        local.findAllNeighbours(hrv2RRVtakens12[-(1:2),], radius=3/16)
save(hrv2RRVneighs12, file="hrv2RRVneighs12.Rdata")
```

Time used: 1.973 sec.

Input

```
showrqa(hrv2RRVtakens12[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens12[-(1:2), ]  n= 1011  Dim: 12
Radius: 0.1875  Recurrence coverage REC: 0.306  log(REC)/log(R): 0.708
Determinism: 0.993  Laminarity: 0.6
DIV: 0.007
Trend: 0  Entropy: 3.417
Diagonal lines max: 141  Mean: 13.443  Mean off main: 13.4
Vertical lines max: 143  Mean: 4.312
```

Input

```
load(file="hrv2RRVneighs12.RData")
local.recurrencePlotAux(hrv2RRVneighs12, dim=12,  radius=3/16)
```

Time used: 2.183 sec.

Input

```
hrv2RRVtakens16 <- local.buildTakens( time.series=HRRV[1:nsignal],
        embedding.dim=16,time.lag=1)
hrv2RRVneighs16 <-local.findAllNeighbours(hrv2RRVtakens16[-(1:2),], radius=3/16)
save(hrv2RRVneighs16, file="hrv2RRVneighs16.Rdata")
```

Time used: 2.766 sec.

Input

```
showrqa(hrv2RRVtakens16[-(1:2),], radius=3/16, do.hist=FALSE)
```

Output

```
hrv2RRVtakens16[-(1:2), ]  n= 1007  Dim: 16
Radius: 0.1875  Recurrence coverage REC: 0.221  log(REC)/log(R): 0.902
Determinism: 0.994  Laminarity: 0.555
DIV: 0.007
Trend: 0  Entropy: 3.419
Diagonal lines max: 137  Mean: 13.706  Mean off main: 13.644
Vertical lines max: 89  Mean: 4.104
```

Input

```
load(file="hrv2RRVneighs16.RData")
local.recurrencePlotAux(hrv2RRVneighs16, dim=16, radius=3/16)
```

Time used: 1.865 sec.

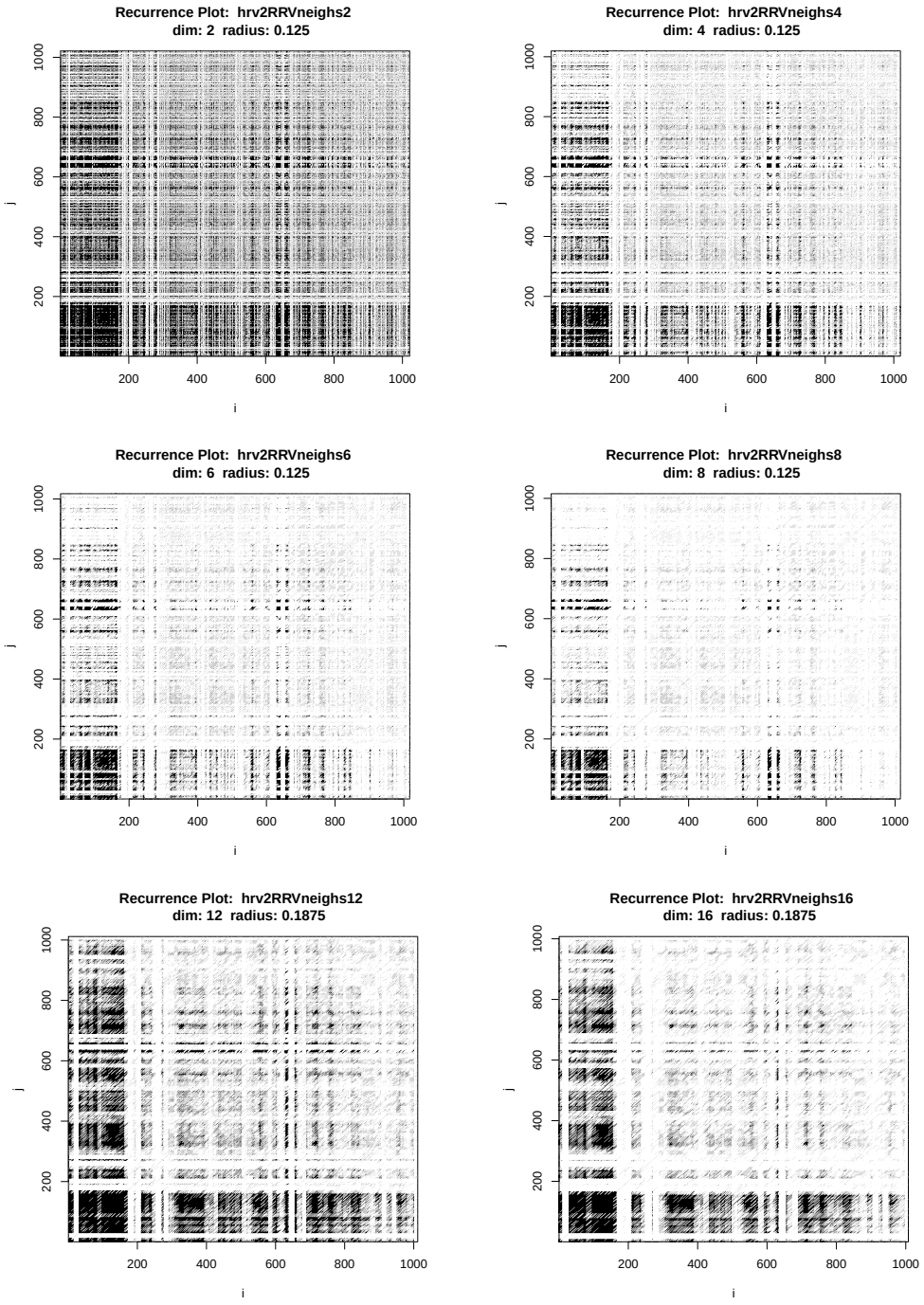

FIGURE 72. Recurrence Plot. Example case: RHRV tutorial example2.beats. Dim=2, 4, 6, 8, 12, 16. Time used: 1.865 sec.

References

Eckmann JP, Kamphorst SO, Ruelle D (1987). "Recurrence plots of dynamical systems." *Europhys. Lett*, **4**(9), 973–977.

Takens F (1981). "Detecting strange attractors in turbulence." In *Dynamical systems and turbulence, Warwick 1980*, pp. 366–381. Springer.

Webber Jr CL, Zbilut JP (2005). "Recurrence quantification analysis of nonlinear dynamical systems." *Tutorials in contemporary nonlinear methods for the behavioral sciences*, pp. 26–94.

Zbilut JP, Webber CL (2006). "Recurrence quantification analysis." *Wiley encyclopedia of biomedical engineering*. URL `http://onlinelibrary.wiley.com/doi/10.1002/9780471740360.ebs1355/full`.

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R session info:

Total Sweave time used: 82.166 sec. at Tue Aug 12 21:15:28 2014.

- R version 3.1.1 (2014-07-10), x86_64-apple-darwin13.1.0
- Locale: en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8
- Base packages: base, datasets, graphics, grDevices, methods, stats, tcltk, utils
- Other packages: leaps 2.9, locfit 1.5-9.1, MASS 7.3-33, Matrix 1.1-4, mgcv 1.8-1, nlme 3.1-117, nonlinearTseries 0.2.1, rgl 0.93.1098, RHRV 4.0, sintro 0.1-3, tkrplot 0.0-23, TSA 1.01, tseries 0.10-32, waveslim 1.7.3
- Loaded via a namespace (and not attached): grid 3.1.1, lattice 0.20-29, quadprog 1.5-5, tools 3.1.1, zoo 1.7-11

LaTeX information:

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textheight: 9.21922in

Bibliography style: jss

CVS/Svn repository information:

$Source: /u/math/j40/cvsroot/lectures/src/dataanalysis/Rnw/recurrence.Rnw,v $

copied to r-forge

HeadURL: svn+ssh://gsawitzki@scm.r-forge.r-project.org/svnroot/rhrv/gs/Rnw/recurrence.Rnw *

$Revision: 157 $

Date: 2014-08-12 21:13:26 +0200 (Tue, 12 Aug 2014) *

$Name: $

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