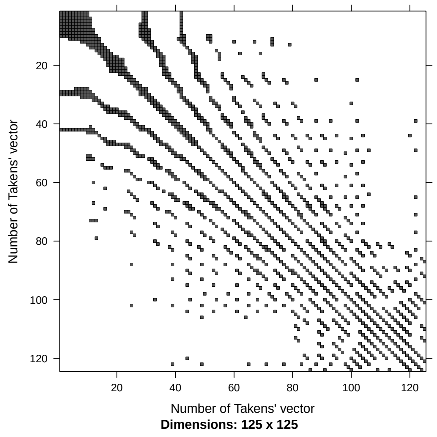
STATISTICAL DATA ANALYSIS: RECURRENCE PLOT

GÜNTHER SAWITZKI



Contents

1. Setup

Input

Date: 2013-11.
Key words and phrases. data analysis, distribution diagnostics, recurrence plot.
This waste book is a companion to "G. Sawitzki: Statistical Data Analysis"
Typeset, with minor revisions: February 6, 2014 from cvs *Revision* : 1.2
`gs@statlab.uni-heidelberg.de` .

```
save.RNGseed <- 87149 #.Random.seed
save.RNGkind <- RNGkind()
# save.RNGseed
save.RNGkind
```

Output

```
[1] "Mersenne-Twister" "Inversion"
```

Input

```
set.seed(save.RNGseed, save.RNGkind[1])
```

Input

```
laptime <- function(){
return(round(structure(proc.time() - chunk.time.start, class = "proc_time")[3],3))
chunk.time.start <<- proc.time()
}
```

Input

```
# install.packages("sintro",repos="http://r-forge.r-project.org",type="source")
library(sintro)
```

We use

Input

```
library(nonlinearTseries)
```

To display state space, we us a variant of pairs().

Input

```
statepairs <- function(states, rank=FALSE){
        main <- paste("Takens states:",deparse(substitute(states)))
        if (rank) {states <- apply(uniftakens,2,rank,ties.method="random")
        main <- paste(main," ranked")}
        pairs(states,
        main=main,
        col=rgb(0,0,0,0.2))
}
```

2. Test Cases

We set up a small series of test signals.

For convenience, some source code from other libraries is included to make this self-contained.

As a global constant, we set up the length of the series to be used.

Input

```
nsignal <- 128
system.time.start <- proc.time()
```

For representation, we use a common layout.

Input
```
plotsignal <- function (signal) {
par(mfrow=c(1,2))
plot(signal, col="blue", pch=20, xlab="t" )

plot(signal, type="l",
        main=deparse(substitute(signal)), xlab="t")
points(signal, col="blue", pch=20 )
}
```

Input
```
sin10 <- function(n=nsignal) {sin( (1:n)/n* 2*pi*10)}
plotsignal(sin10())
```

See Figure 1,

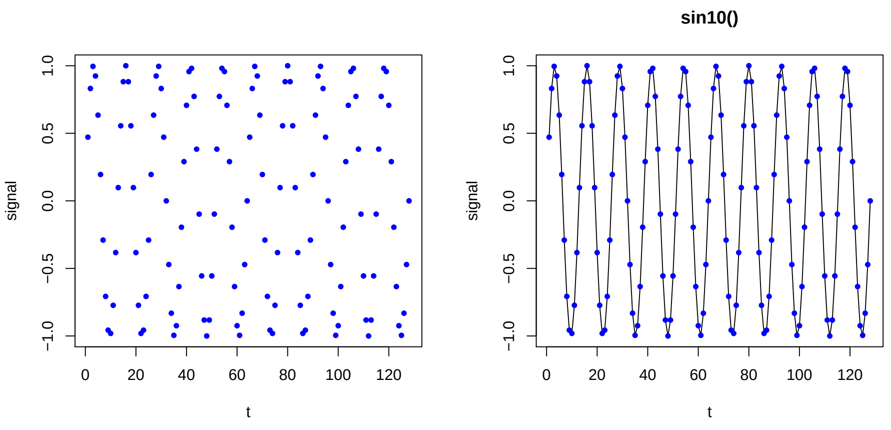


Figure 1. Test case: sin10. Signal and linear interpolation.

Input
```
unif <- function(n=nsignal) {runif(n)}
xunif<-unif()
plotsignal(xunif)
```

See Figure 2 on the next page,

Input
```
chirp <- function(n=nsignal) {
# this is copied from library(signal)
signal.chirp <- function(t, f0 = 0, t1 = 1, f1 = 100,
                  form = c("linear", "quadratic", "logarithmic"), phase = 0){

  form <- match.arg(form)
  phase <- 2*pi*phase/360

  switch(form,
    "linear" = {
```

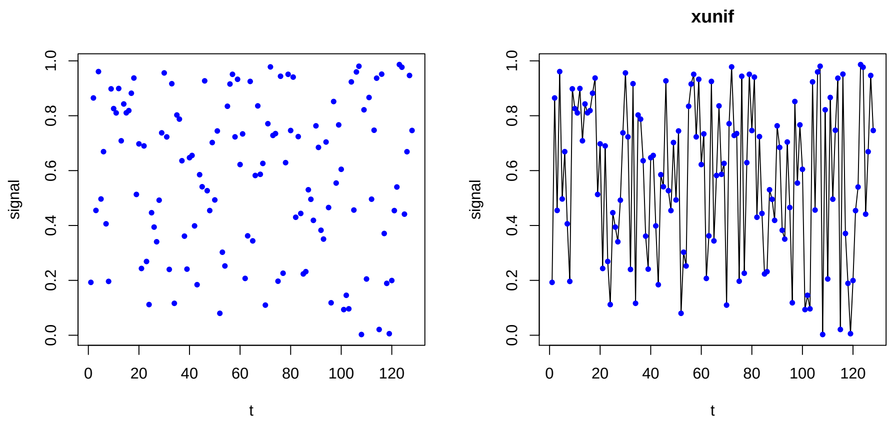


FIGURE 2. Test case: unif - uniform random numbers. Signal and linear interpolation.

```
        a <- pi*(f1 - f0)/t1
        b <- 2*pi*f0
        cos(a*t^2 + b*t + phase)
    },
    "quadratic" = {
        a <- (2/3*pi*(f1-f0)/t1/t1)
        b <- 2*pi*f0
        cos(a*t^3 + b*t + phase)
    },
    "logarithmic" = {
        a <- 2*pi * t1 / log(f1 - f0)
        b <- 2*pi * f0
        x <- (f1-f0)^(1/t1)
        cos(a*x^t + b*t + phase)
    })
}

signal.chirp(seq(0, 0.6, len=nsignal))
}
plotsignal(chirp())
```

ToDo: include doppler waveslim

See Figure 3 on the facing page,

3. Recurrence States

Recurrence plots have been introduced in an attempt to understand near periodic in hydrodynamics. On the one hand, and etended theory on dynamical systems was available, covering deterministic models. A fundamental concept is that at a certain time a system is in some state, and developing from this. Defining the proper state space is a critical step in modelling.

The other toolkit ist that of stochastics processes, in particular Markov models. Classical time series assumes stationarity, and this is obviously not the way to go. A fundamental idea for Markov models is that the system state is seen in a temporal context: you have a Markov process, if you can define a (non-anticpating) state that has sufficient information for prediction: given this state, the future is independent from the past.

Recurrence, coming back to some state, is often a key to understand a near periodic system.

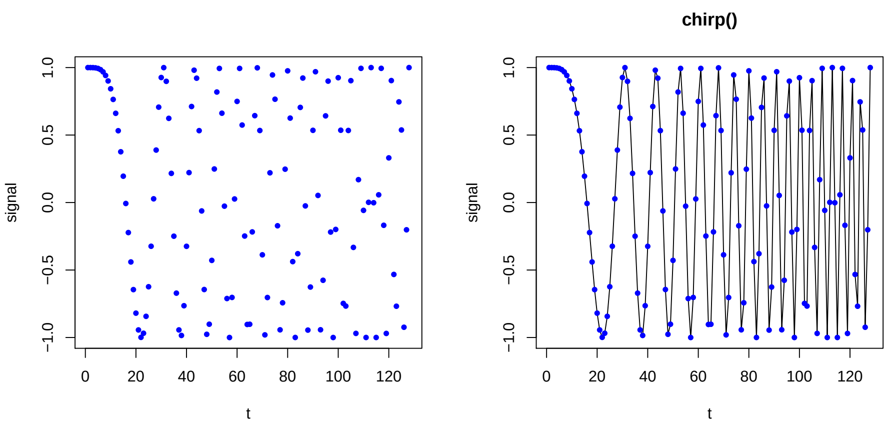


Figure 3. Test case: chirp signal. Signal and linear interpolation.

Hydrodynamics is a challenging problem. Understanding planetary motion is a historical challenge, and may be useful as an illustration.

As a simple illustration, let $x = (x_i)$ be a sequence, maybe near periodic. For now, think of i as a time index.

Recurrency plots have two steps. The first was a bold step by Floris Takens. If you do not know the state space of a system, for a choice of "dimension" d, take the sequence of d tuples taken from your data to define the states.

$$u_i = (x_i, \ldots, x_{i+d})$$

As a mere technical refinement: you may know that your data are a flattenened representation of t dimensional data. So you take

$$u_i = (x_i, \ldots, x_{i+d*m}).$$

We ignore this detail here and take $m = 1$.

Conceptually, you define states by observed histories. For classical Markov setup, the state is defined by the previous information x_{i-1}, but for more comles situations you may have to step back in the past. Finding the appropriate d is the challenge. So it may be appropriate to view the Takens staes as a family, indexed by the time scope d. The rest is structural information how to arrange items.

Of course it is possible to compress information here, sorting states and removing duplicates. Keeping the original definition as the advantage that we have the index i, so that u_i is the state at index position i.

But the states may have an inherent structure, which we may take into account or ignore. Since for this example, we are just in 4-dimensional space, marginal scatterplots may give enough information.

Input
```
sintakens <- buildTakens( time.series=sin10(),embedding.dim=4, time.lag=1)
statepairs(sintakens)
```

See Figure 4 on the next page.

Input

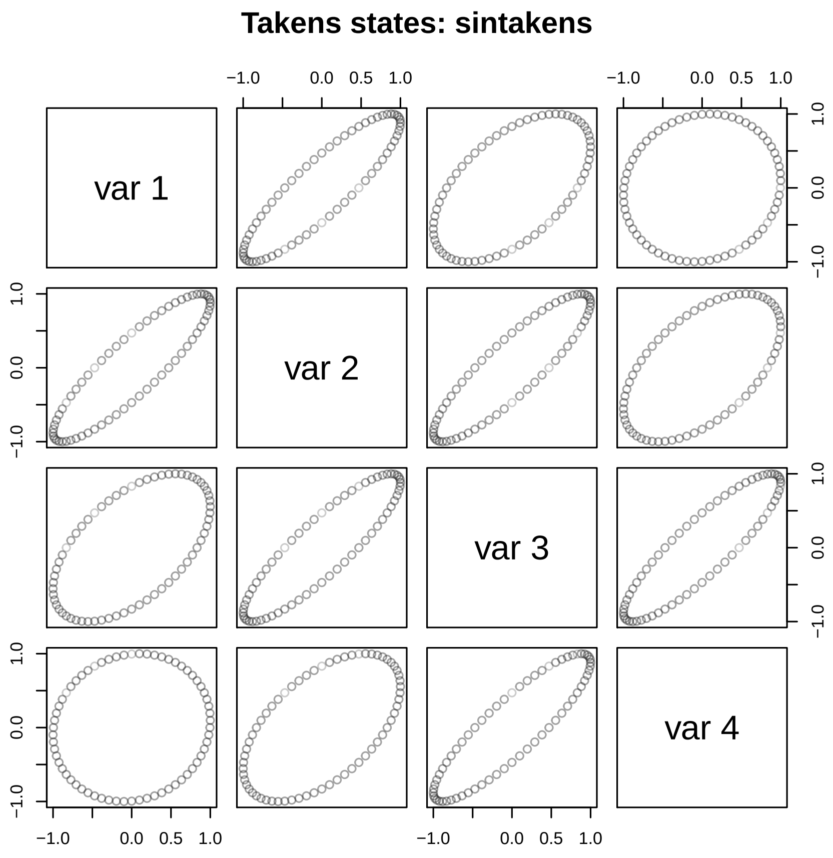


FIGURE 4. Test case: sinus. Note that marginal views of 1-dimensional circles in d space may appear as ellipses. Time used: 0.186 sec.

```
uniftakens <- buildTakens( time.series=xunif,embedding.dim=4,time.lag=1)
statepairs(uniftakens)
```

See Figure 5 on the facing page.

```
Input
statepairs(chirptakens)
```

See Figure 6 on page 8

4. Recurrence Plots

The next step, taken in Eckmann *et al.* [1987] was to use a two dimensional display. Take a scatterplot with the Taken's staes a marginal. Take a sliding window of your process data, and for each i, find the "distance" of u_i from and to any of the collected states. If the distance is below some chosen threshold, mark the point (i, j) for which u(j) is in the ball of radius r(i) centred at u(i).

The original publication Eckmann *et al.* [1987] actually used a nearest neighbourhood evnironment to cover about 10 data points.

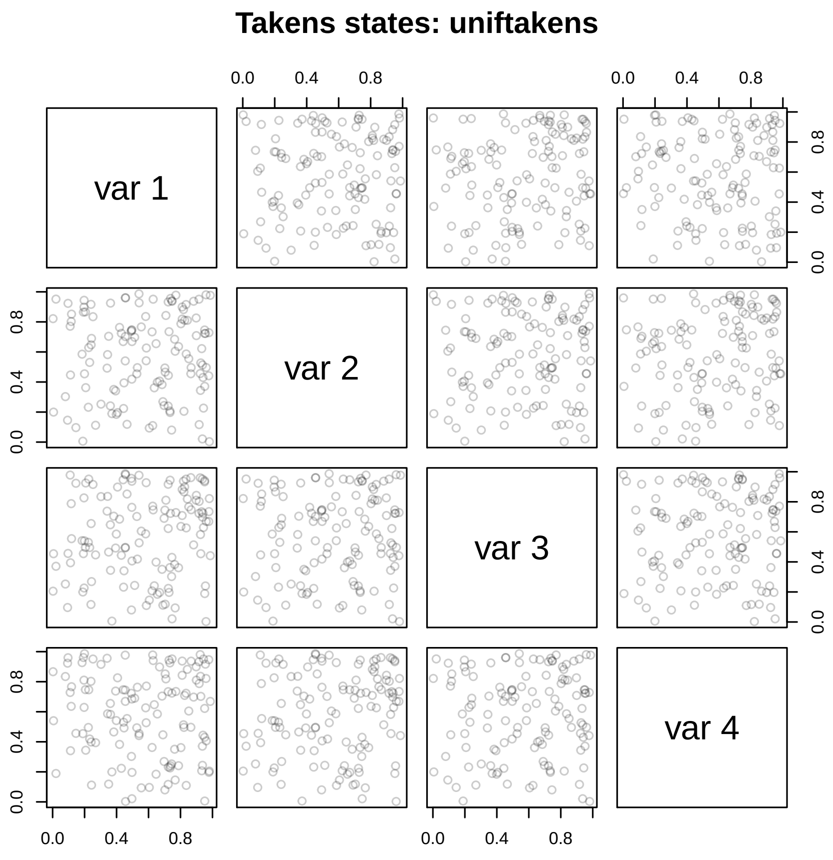


FIGURE 5. Test case: uniform random numbers. Time used: 0.21 sec.

The construction has considerable arbitrary choices. The critical radius may depend on the point i. In practical applications, using a constant radius is a common first step. Using a dichotomous marking was what presumably was necessary when the idea was introduced. With todays technology, we can allow a markup on a finer scale, as has been seen in Orion-1.

We can gain additional freedom by using a correlation view: instead of looking from one axis, we can walk along the diagonal, using two reference axis.

4.1. **Sinus.**

Input

```
load(file="sin10neighs.RData")
nonlinearTseries:::recurrencePlotAux(sin10neighs)
```

4.2. **Uniform random.**

Input

```
load(file="unifneighs.RData")
nonlinearTseries:::recurrencePlotAux(unifneighs)
```

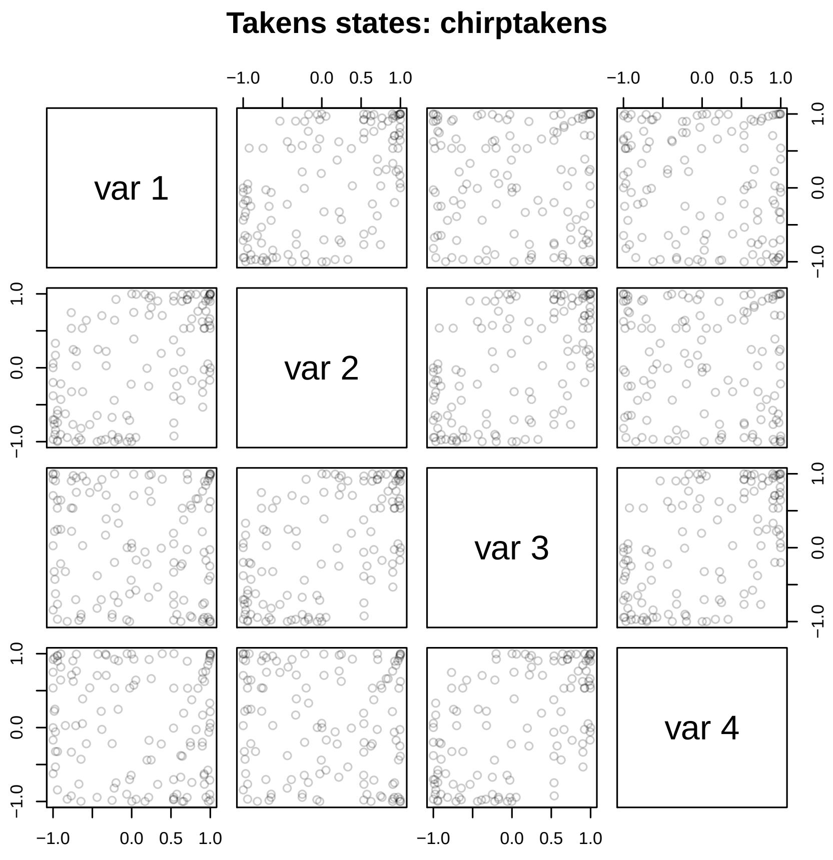


FIGURE 6. Test case: chirp signal. Time used: 0.191 sec.

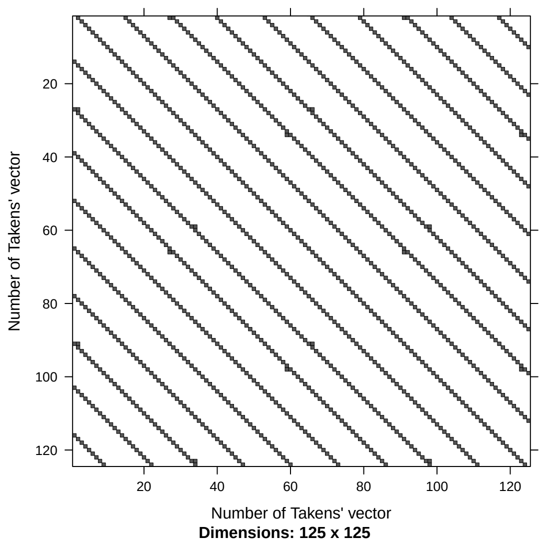


FIGURE 7. Recurrence Plot. Test case: sinus curves. Time used: 4.463 sec.

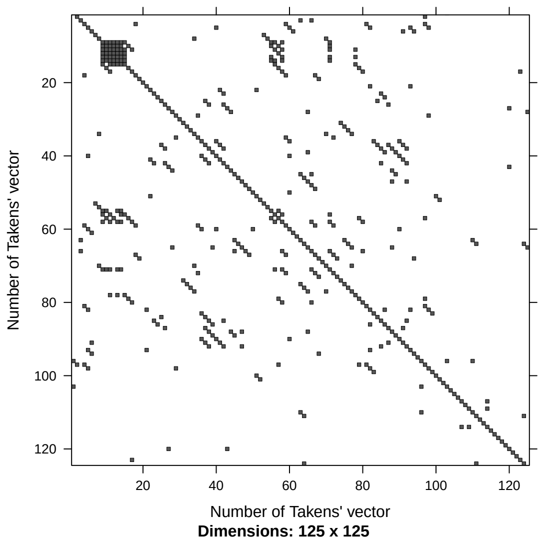


Figure 8. Recurrence Plot. Test case: uniform random numbers. Time used: 1.495 sec.

4.3. **Chirp Signal.**

Input
```
chirpneighs<-nonlinearTseries:::findAllNeighbours(chirptakens,radius=0.6)#0.4
save(chirpneighs, file="chirpneighs.RData")
```

Input
```
load(file="chirpneighs.RData")
nonlinearTseries:::recurrencePlotAux(chirpneighs)
```

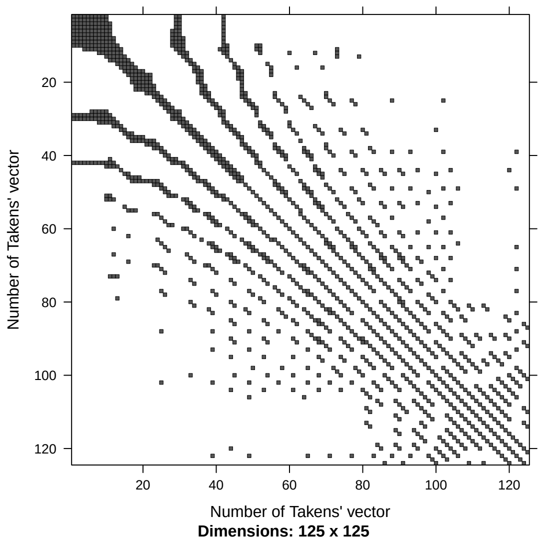


Figure 9. Recurrence Plot. Test case: chirp signal. Time used: 5.201 sec.

5. Case Study: Geyser data

ToDo: extend to two-dimensional data

This is a classical data set with a two dimensional structure, *waiting* and *waiting*.

Input
```
library(MASS)
data(faithful)
```

5.1. Geyser Eruptions.

Input
```
plotsignal(faithful$eruptions)
```

See Figure 10,

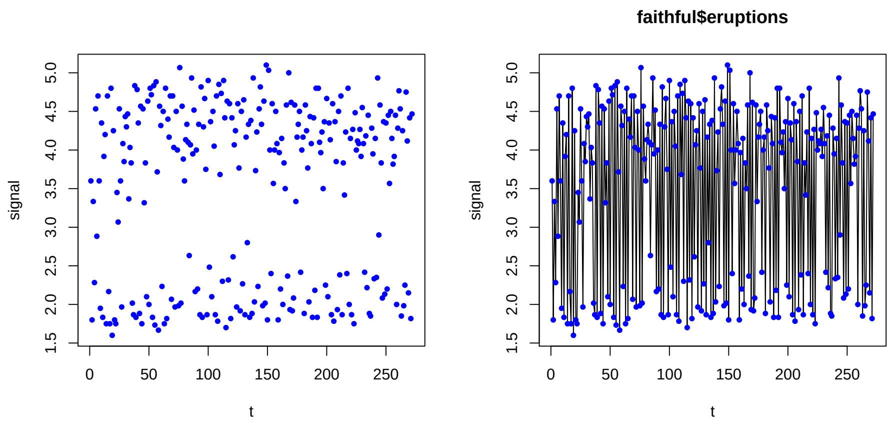


Figure 10. Example case: Old Faithful Geyser eruptions. Signal and linear interpolation.

Input
```
eruptionstakens4 <- buildTakens( time.series=faithful$eruptions, embedding.dim=4, time.lag=1)
statepairs(eruptionstakens4)
```

See Figure 11 on the next page

Input
```
eruptionsneighs4<-nonlinearTseries:::findAllNeighbours(eruptionstakens4, radius=0.8)
save(eruptionsneighs, file="eruptionsneighs4.RData")
```

Input
```
load(file="eruptionsneighs4.RData")
nonlinearTseries:::recurrencePlotAux(eruptionsneighs4)
```

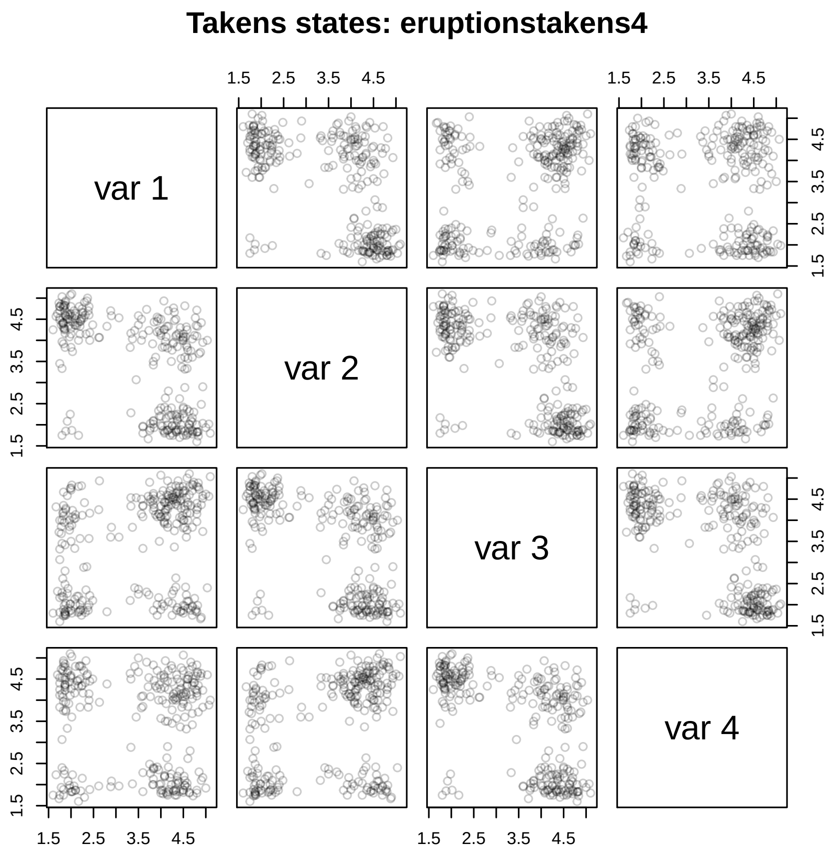


FIGURE 11. Example case: Old Faithful Geyser eruptions. Time used: 0.306 sec.

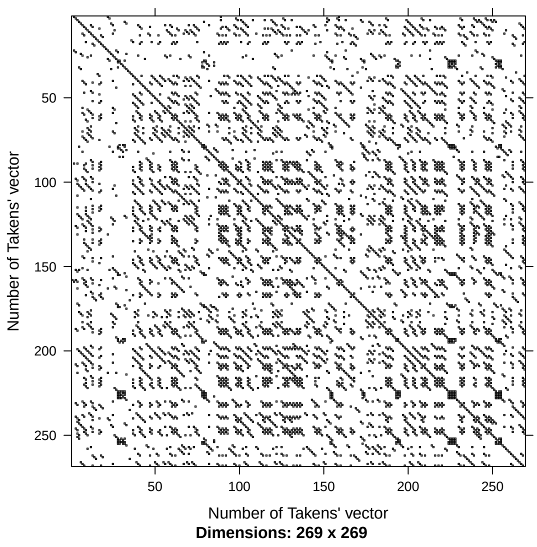


FIGURE 12. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=4. Time used: 23.381 sec.

5.1.1. *Dim=2.*

```
                              Input
eruptionstakens2 <- buildTakens(time.series=faithful$eruptions, embedding.dim=2, time.lag=1)
statepairs(eruptionstakens2)
```

See Figure 13

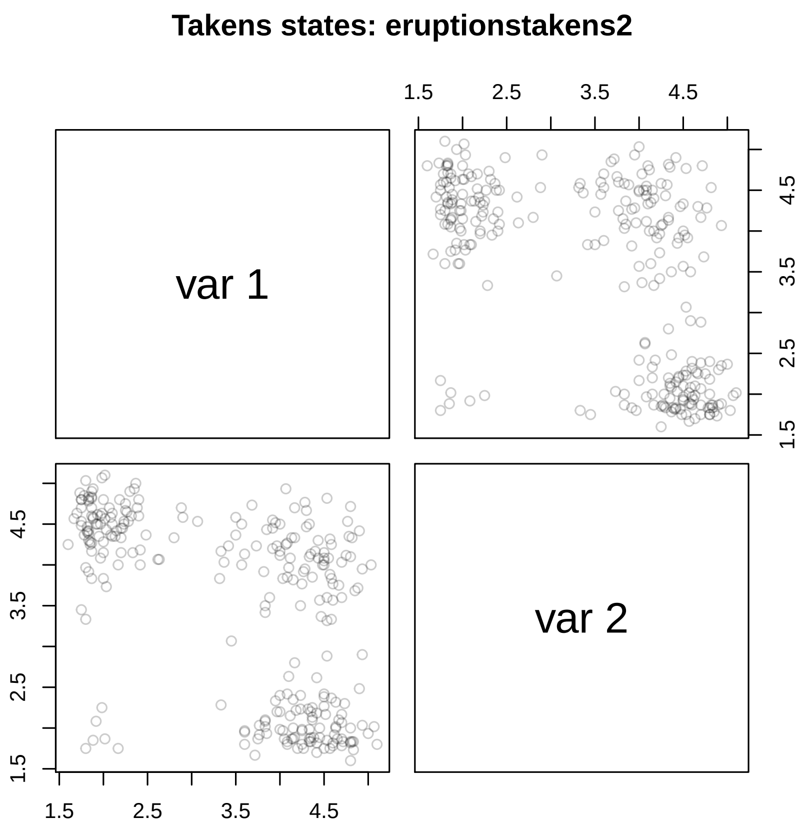


Figure 13. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 0.118 sec.

```
                              Input
eruptionsneighs2<-nonlinearTseries:::findAllNeighbours(eruptionstakens2, radius=0.8)
save(eruptionsneighs2, file="eruptionsneighs2.RData")
```

```
                              Input
load(file="eruptionsneighs2.RData")
nonlinearTseries:::recurrencePlotAux(eruptionsneighs2)
```

5.1.2. *Dim=6.*

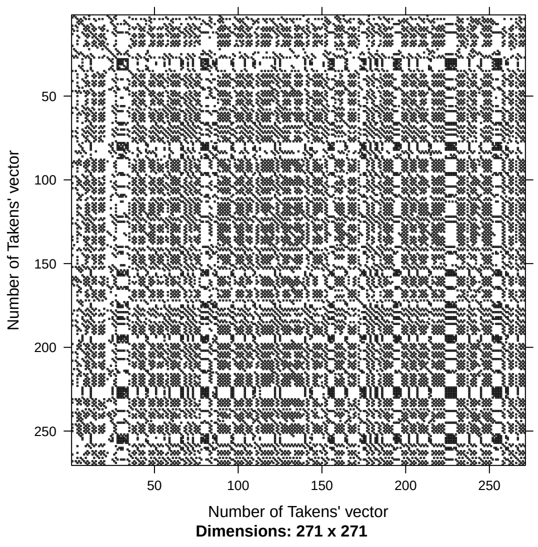


FIGURE 14. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=2. Time used: 74.734 sec.

Input
```
eruptionstakens6 <- buildTakens( time.series=faithful$eruptions,embedding.dim=6,time.lag=1)
statepairs(eruptionstakens6)
```

See Figure 15 on the next page

Input
```
eruptionsneighs6<-nonlinearTseries:::findAllNeighbours(eruptionstakens6, radius=0.8)
save(eruptionsneighs6, file="eruptionsneighs6.RData")
```

Input
```
load(file="eruptionsneighs6.RData")
nonlinearTseries:::recurrencePlotAux(eruptionsneighs6)
```

5.1.3. *Dim=8.*

Input
```
eruptionstakens8 <- buildTakens( time.series=faithful$eruptions,embedding.dim=8,time.lag=1)
statepairs(eruptionstakens8)
```

See Figure 17 on page 15

Input
```
eruptionsneighs8<-nonlinearTseries:::findAllNeighbours(eruptionstakens8, radius=0.8)
save(eruptionsneighs8, file="eruptionsneighs8.RData")
```

Input

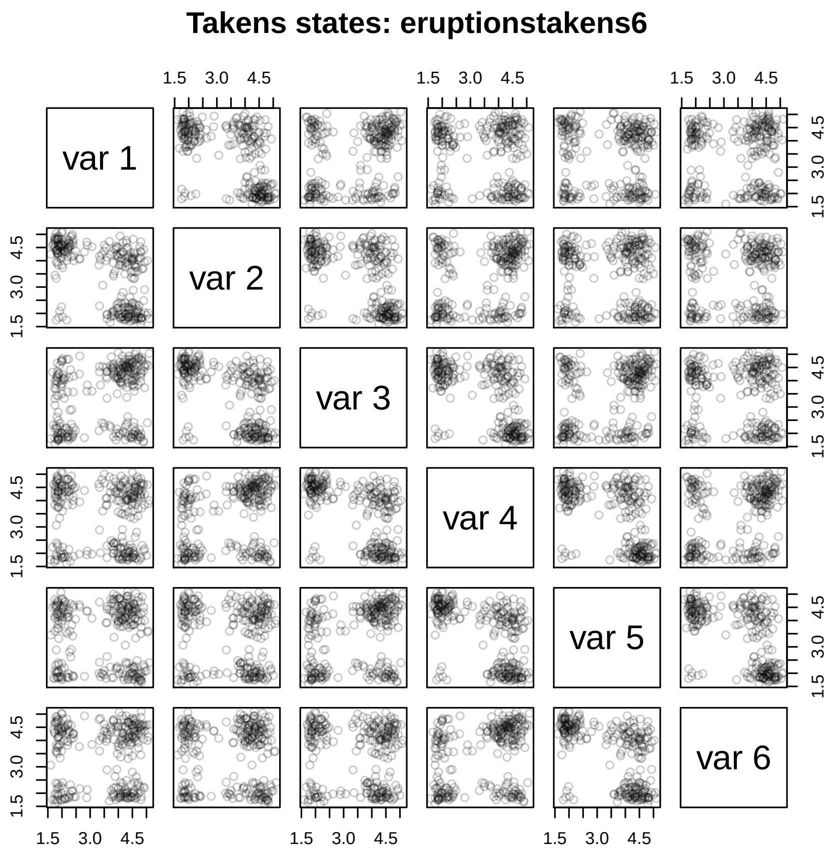


FIGURE 15. Example case: Old Faithful Geyser eruptions. Dim=6. Time used: 0.529 sec.

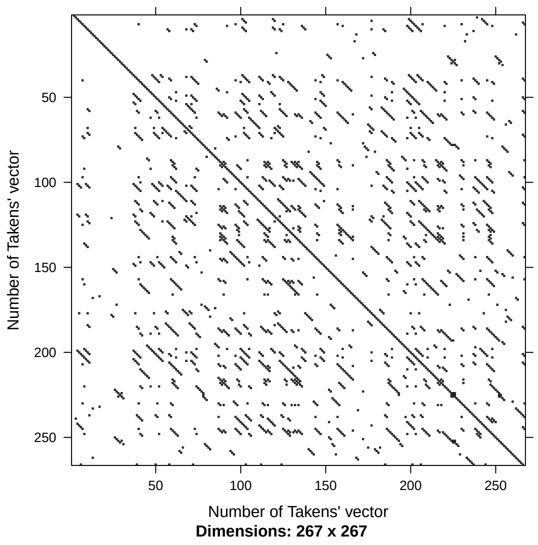


FIGURE 16. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=6. Time used: 7.491 sec.

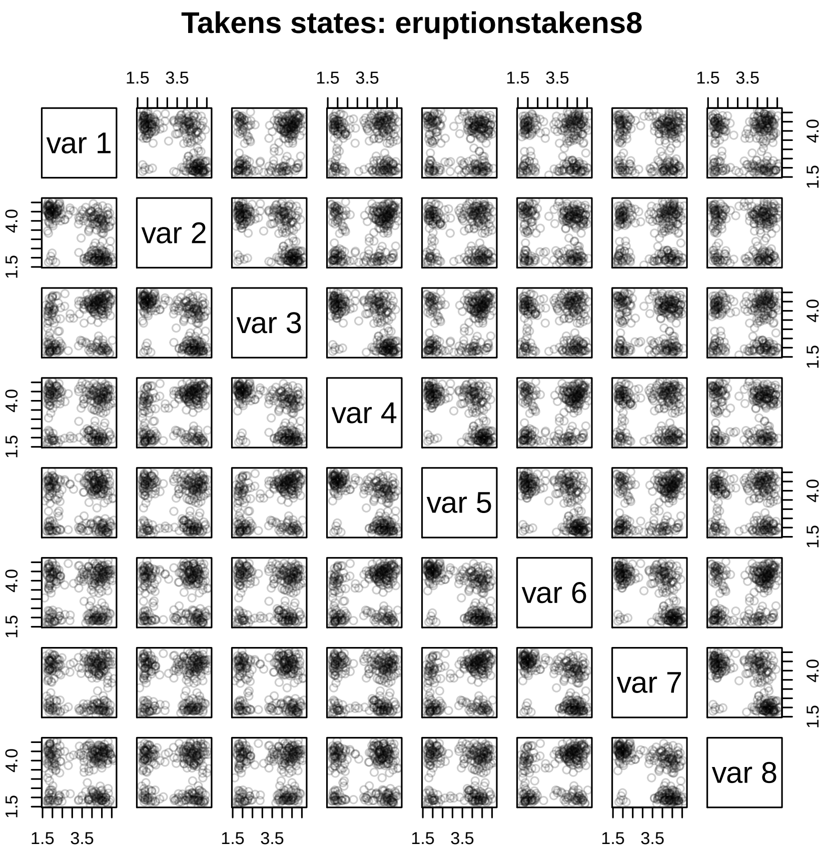


FIGURE 17. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 1.028 sec.

```
load(file="eruptionsneighs8.RData")
nonlinearTseries:::recurrencePlotAux(eruptionsneighs8)
```

5.2. **Geyser Eruptions: Comparison by Dimension.** For comparison, recurrence plots for the Geyser data with varying dimension are in Figure 19 on page 17

5.3. **Geyser Waiting.**

Input
```
plotsignal(faithful$waiting)
```

See Figure 20 on page 17,

Input
```
waitingtakens <- buildTakens( time.series=faithful$waiting,embedding.dim=4,time.lag=4)
statepairs(waitingtakens)
```

See Figure 21 on page 18

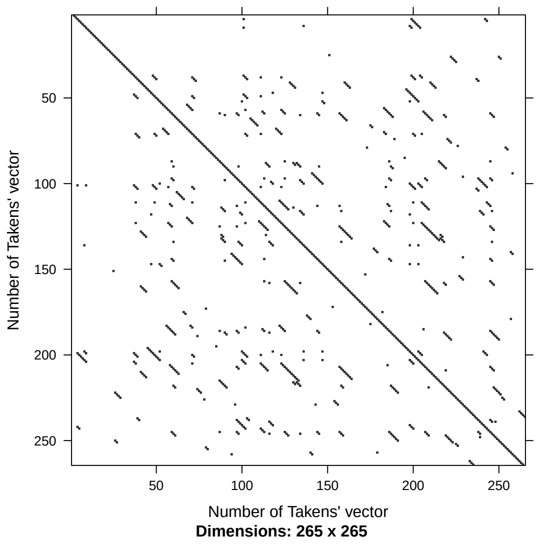


FIGURE 18. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=8. Time used: 2.759 sec.

Input
```
waitingneighs<-nonlinearTseries:::findAllNeighbours(waitingtakens, radius=16)
save(waitingneighs, file="waitingneighs.Rdata")
```

Input
```
load(file="waitingneighs.RData")
nonlinearTseries:::recurrencePlotAux(waitingneighs)
```

6. CASE STUDY: HRV DATA

Input
```
library(RHRV)
load("/data/pulse/rhrv/pkg/data/HRVData.rda")
load("/data/pulse/rhrv/pkg/data/HRVProcessedData.rda")
###################################################
### code chunk number 1: creation
###################################################
hrv.data  = CreateHRVData()
hrv.data = SetVerbose(hrv.data, TRUE )
###################################################
### code chunk number 3: loading
###################################################
hrv.data = LoadBeatAscii(hrv.data, "example.beats",
       RecordPath = "/data/pulse/rhrv/tutorial/beatsFolder")
```

Output
```
** Loading beats positions for record: example.beats **
   Path: /data/pulse/rhrv/tutorial/beatsFolder
   Scale: 1
   Date: 01/01/1900
```

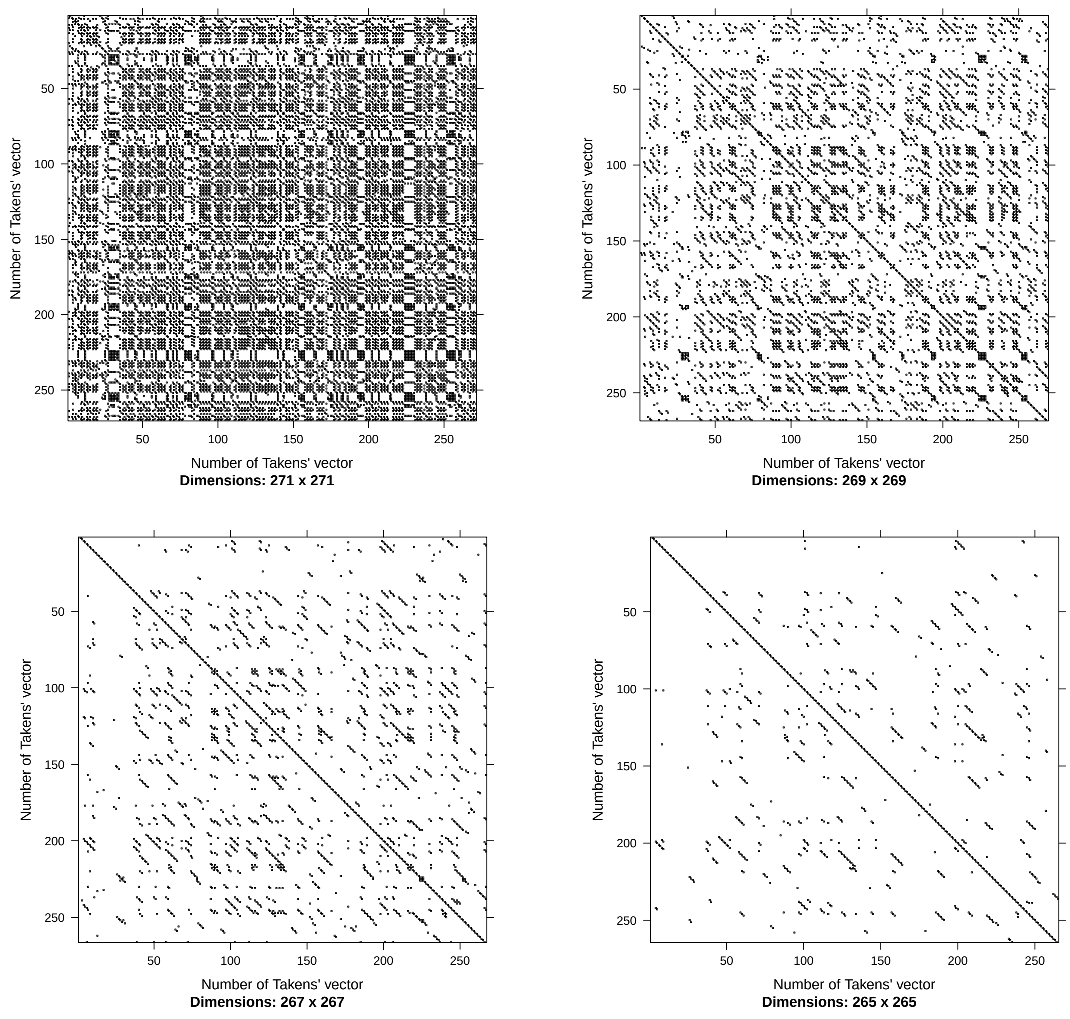


Figure 19. Recurrence Plot. Example case: Old Faithful Geyser eruptions. Dim=2, 4, 6, 8.

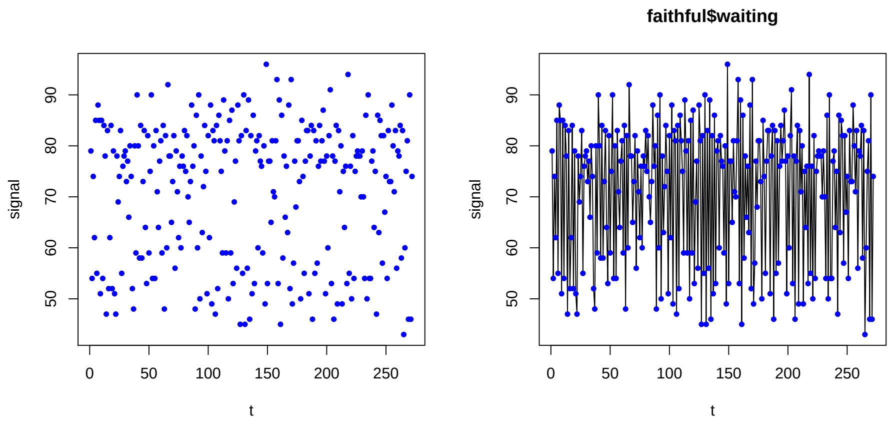


Figure 20. Example case: Old Faithful Geyser waiting. Signal and linear interpolation. Time used: 2.959 sec.

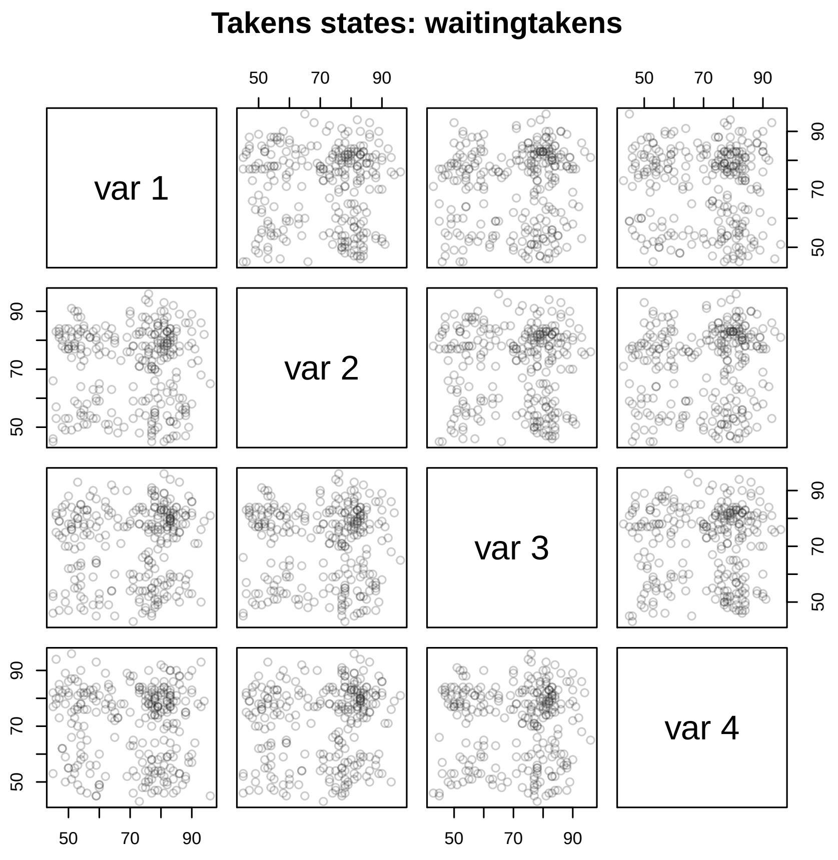


FIGURE 21. Example case: Old Faithful Geyser waiting. Time used: 0.258 sec.

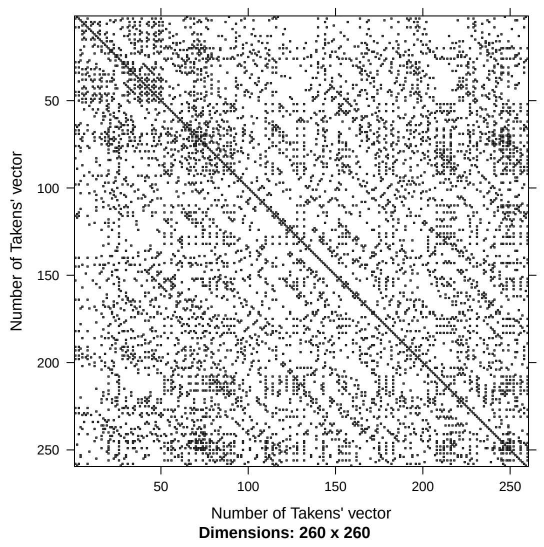


FIGURE 22. Recurrence Plot. Example case: Old Faithful Geyser waiting. Time used: 24.5 sec.

```
   Time: 00:00:00
   Number of beats: 17360
```

Input

```
 #       RecordPath = "beatsFolder")


 ###################################################
 ### code chunk number 4: derivating
 ###################################################
 hrv.data = BuildNIHR(hrv.data)
```

Output

```
** Calculating non-interpolated heart rate **
   Number of beats: 17360
```

Input

Input

```
 plotsignal(hrv.data$Beat$RR)
```

See Figure 23,

ToDo: We l
lies at appro
2*RR. Coul
an artefact
processing,
too many in

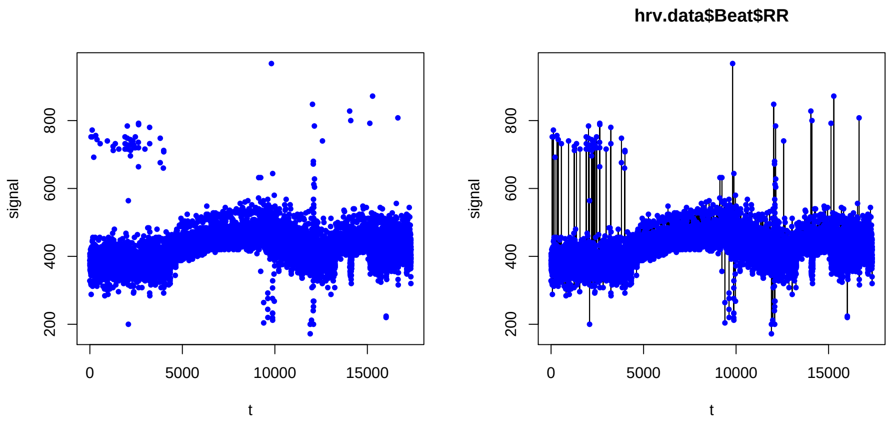


FIGURE 23. Example case: RHRV tutorial. Signal and linear interpolation.

Input

```
 hrvRRtakens4 <- buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],embedding.dim=4,time.lag=1)
 statepairs(hrvRRtakens4)
```

See Figure 24 on the next page

Input

```
 statepairs(hrvRRtakens4, rank=TRUE)
```

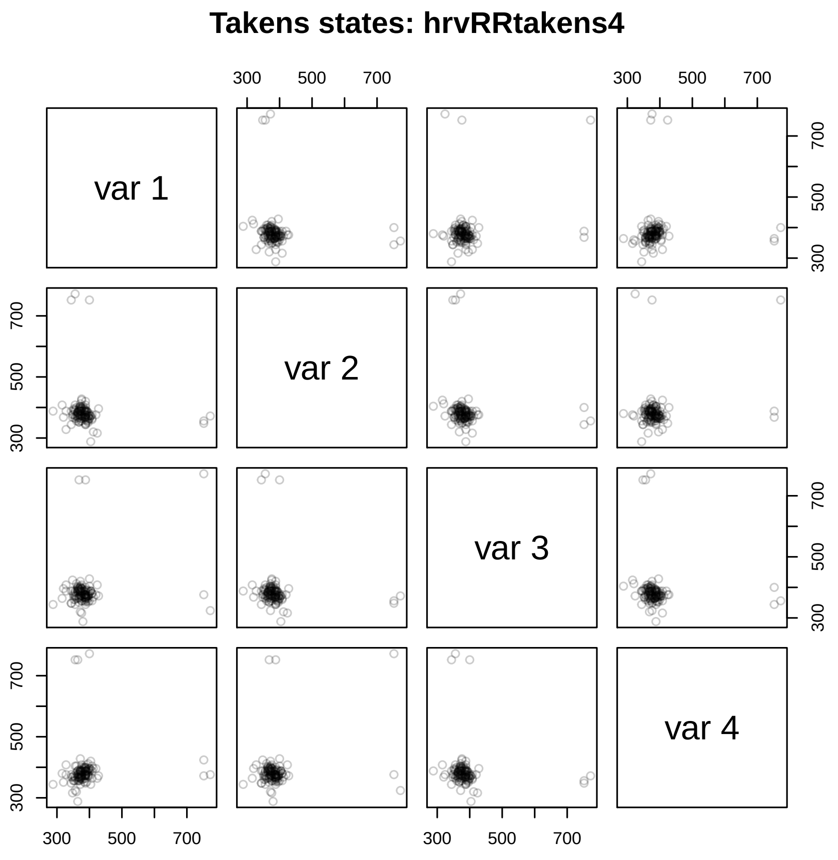


FIGURE 24. Example case: RHRV tutorial. Time used: 0.258 sec.

See Figure 25 on the facing page

```
Input
hrvRRneighs4 <-nonlinearTseries:::findAllNeighbours(hrvRRtakens, radius=16)
save(hrvRRneighs4, file="hrvRRneighs4.Rdata")
```

Time used: 0.033 sec.

```
Input
load(file="hrvRRneighs4.RData")
nonlinearTseries:::recurrencePlotAux(hrvRRneighs4)
```

6.1. **RHRV: Comparison by Dimension.**

```
Input
hrvRRtakens2 <- buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],embedding.dim=2,time.lag=1)
hrvRRneighs2 <-nonlinearTseries:::findAllNeighbours(hrvRRtakens2, radius=16)
save(hrvRRneighs2, file="hrvRRneighs2.Rdata")
```

Time used: 0.048 sec.

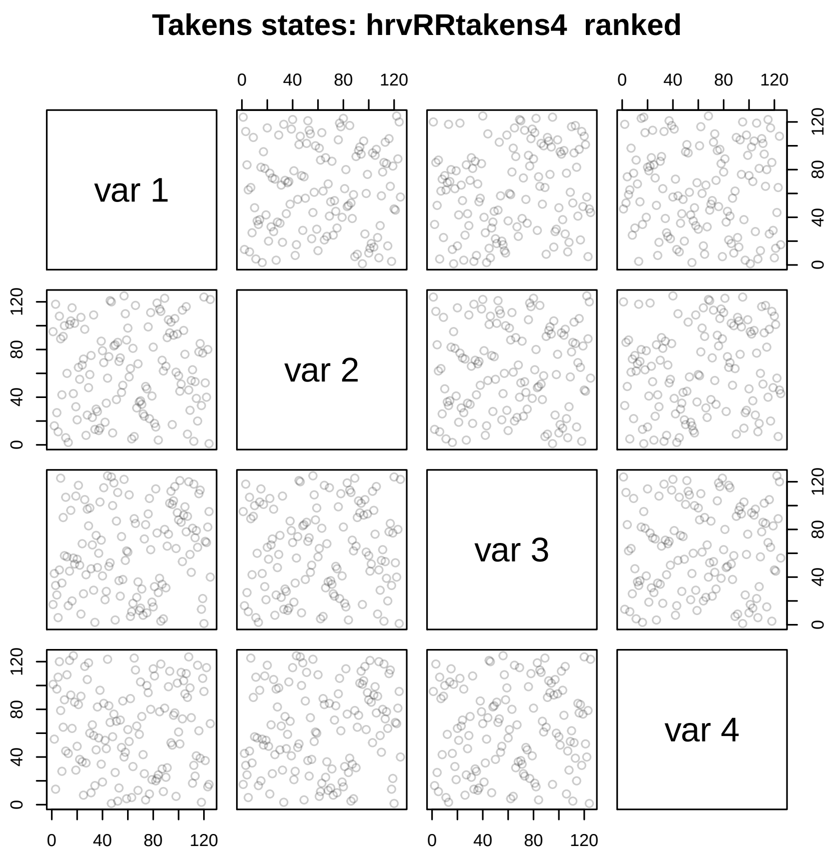


Figure 25. Example case: RHRV tutorial. Ranked data. Time used: 0.493 sec.

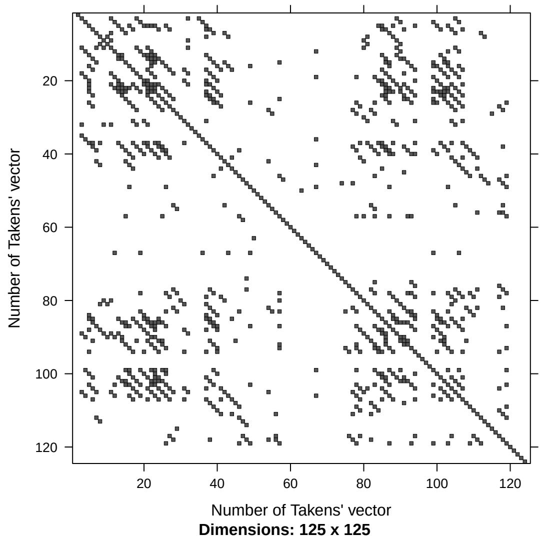


Figure 26. Recurrence Plot. Example case: RHRV tutorial. Dim=4. Time used: 3.594 sec.

Input
```
load(file="hrvRRneighs2.RData")
nonlinearTseries:::recurrencePlotAux(hrvRRneighs2)
```

Time used: 11.585 sec.

Input
```
hrvRRtakens6 <- buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],embedding.dim=6,time.lag=1)
hrvRRneighs6 <-nonlinearTseries:::findAllNeighbours(hrvRRtakens6, radius=16)
save(hrvRRneighs6, file="hrvRRneighs6.Rdata")
```

Time used: 0.041 sec.

Input
```
load(file="hrvRRneighs6.RData")
nonlinearTseries:::recurrencePlotAux(hrvRRneighs6)
```

Dim=6. Time used: 1.187 sec.

Input
```
hrvRRtakens8 <- buildTakens( time.series=hrv.data$Beat$RR[1:nsignal],embedding.dim=8,time.lag=1)
hrvRRneighs8 <-nonlinearTseries:::findAllNeighbours(hrvRRtakens8, radius=16)
save(hrvRRneighs8, file="hrvRRneighs8.Rdata")
```

Time used: 0.045 sec.

Input
```
load(file="hrvRRneighs8.RData")
nonlinearTseries:::recurrencePlotAux(hrvRRneighs8)
```

Dim=8. Time used: 0.572 sec.

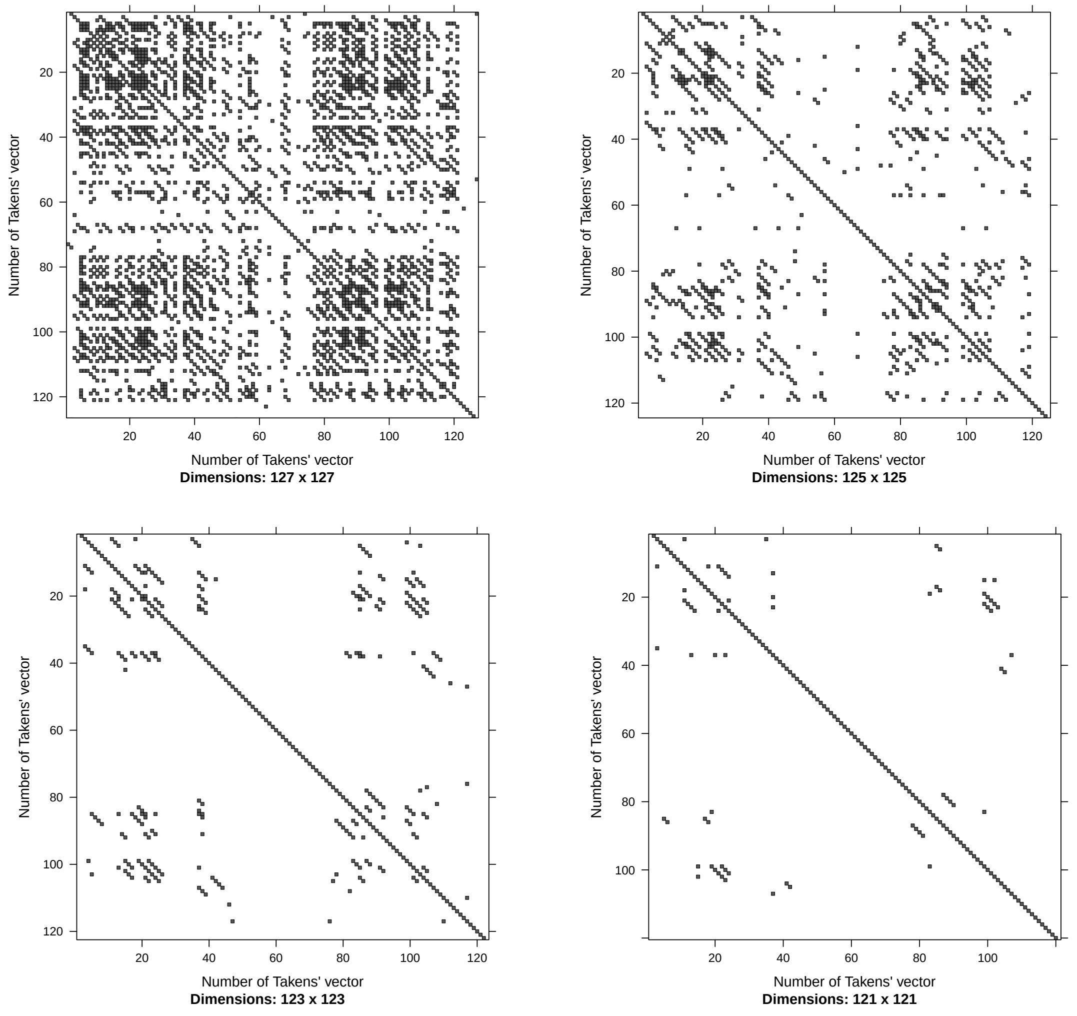


FIGURE 27. Recurrence Plot. Example case: RHRV tutorial. Dim=2, 4, 6, 8. Time used: 0.572 sec.

References

Eckmann, Jean-Pierre, Kamphorst, S Oliffson, & Ruelle, David. 1987. Recurrence plots of dynamical systems. *Europhys. Lett*, **4**(9), 973–977.

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R session info:

Total Sweave time used: 167.312 sec.

- R version 3.0.2 (2013-09-25), `x86_64-apple-darwin10.8.0`
- Locale: `en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8`
- Base packages: base, datasets, graphics, grDevices, methods, stats, tcltk, utils
- Other packages: leaps 2.9, locfit 1.5-9.1, MASS 7.3-29, Matrix 1.1-2, mgcv 1.7-27, nlme 3.1-113, nonlinearTseries 0.2, rgl 0.93.996, RHRV 4.0, sintro 0.1-3, tkrplot 0.0-23, TSA 1.01, tseries 0.10-32, waveslim 1.7.3
- Loaded via a namespace (and not attached): grid 3.0.2, lattice 0.20-25, quadprog 1.5-5, tools 3.0.2, zoo 1.7-11

LaTeX information:

textwidth: 6.00612in linewidth:6.00612in

textheight: 9.21922in

CVS/Svn repository information:

```
$Source: /u/math/j40/cvsroot/lectures/src/dataanalysis/Rnw/recurrence.Rnw,v $

$Revision: 1.2 $

$Date: 2014/02/05 20:05:07 $

$name:  $

$Author: j40 $
```

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