

R. Kalman's revenge or

Robustness for Kalman Filtering Revisited



R-Package robKalman —

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Euclidean State Space Models

Definitions and Assumptions:

— Time–Discrete, Euclidean Setup ideal model:

$$egin{aligned} x_t &= F(x_{t-1},t) + v_t, & v_t \stackrel{ ext{indep.}}{\sim} (0,Q_t), & [p- ext{dim}], \ y_t &= Z(x_t,t) + arepsilon_t, & arepsilon_t \stackrel{ ext{indep.}}{\sim} (0,V_t), & [q- ext{dim}], \ x_0 &\sim (a_0,Q_0), & [p- ext{dim}], \ \{v_t\}, \{arepsilon_t\}, x_0 &= 0. \end{aligned}$$

functions F, Z smooth with known derivatives, hyper–parameters Q_t, V_t, a_0 known

extensible to

- continuous time (SDE's)
- incorporate user-specified controls





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Types of Outliers

exogenous outliers affecting only singular observations

$$\begin{array}{lll} \mathsf{AO} & :: & \varepsilon_t^{\mathsf{re}} \sim (1 - r_{\mathsf{AO}}) \mathcal{L}(\varepsilon_t^{\mathsf{id}}) + r_{\mathsf{AO}} \mathcal{L}(\varepsilon_t^{\mathsf{di}}) \\ \mathsf{SO} & :: & y_t^{\mathsf{re}} \sim (1 - r_{\mathsf{SO}}) \mathcal{L}(y_t^{\mathsf{di}}) + r_{\mathsf{SO}} \mathcal{L}(y_t^{\mathsf{di}}) \end{array}$$

endogenous outliers / structural changes

$$egin{aligned} \mathsf{IO} &:: & oldsymbol{v}_t^{\mathsf{re}} \sim (\mathsf{1} - oldsymbol{r}_{\mathsf{lo}}) \mathcal{L}(oldsymbol{v}_t^{\mathsf{id}}) + oldsymbol{r}_{\mathsf{lo}} \mathcal{L}(oldsymbol{v}_t^{\mathsf{di}}) \end{aligned}$$

Notation: $\cdot \triangleq \cdot^{id}$, $\hat{\cdot} \triangleq \cdot^{re}$, $\tilde{\cdot} \triangleq \cdot^{di}$

Different and competing goals

A/SO attenuation of "false alarms"

IO tracking: detect structural changes as fast as possible; recovering: clean data from structural changes

A/SO & IO identification problem:

simultaneous treatment only possible with delay





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Classical Method: Kalman-Filter

Filter Problem

$$\mathsf{E} \left| x_t - f_t(y_{1:t}) \right|^2 = \min_{f_t} !,$$

with $y_{1:t} = (y_1, \dots, y_t), \quad y_{1:0} := \emptyset$

General solution: $E[x_t|y_{1:t}]$ —difficult to compute

Initialization: $x_{010} = a_0$

Prediction: $X_{t|t-1} = F_t X_{t-1|t-1}$, $[\Delta X_t = X_t - X_{t|t-1}]$

Correction: $X_{t|t} = X_{t|t-1} + M_t^0 \Delta y_t$, $[\Delta y_t = y_t - Z_t X_{t|t-1}]$





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Kalman–Filter assuming $F(x,t) = F_t x$, $Z(x,t) = Z_t x$ optimal solution among linear filters — Kalman[/Bucy] [60/61]:

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and corresponding recursions for the prediction/filtering error covariances $\Sigma_{t|t[-1]}$ and the Kalman gain M_t^0





Features of the Kalman-Filter

- + an easy, understandable structure: initialization, prediction, correction step
- + correction step is easily evaluable and interpretable: it is linear!
- + strict recursivity / Markovian structure: all information from the past useful for the future is captured in the value of $x_{t|t-1}$.
- the correction step is linear and thus not robust, as y enters unbounded;

Aim of robustification: try to retain all "+"'s, revise "-"





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- Kalman filter: filter, Kalman gain, covariances
- ACM-filter: filter, multivariate version, GM-estimator
- rLS-filter: filter, calibration of clipping height
 - AO/SO-robust version
 - IO-robust version
 - with a certain delay joint treatment of AO/SO's & IO's
- extensible to further recursive filters:
 - → general interface recursiveFilter with arguments:
 - data
 - state space model (hyper parameters)
 [will be: object of class SSM]
 - functions for the init./pred./corr.step
 [will be: object containing them]
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- Programming language
 - completely in S, perhaps some code in C later (→ FKF)
- Use existing infrastructure: zoo, timeSeries
 - for: graphics, diagnostics, management of date/time
- Code in different layers
 - internal functions: no S4-objects, no time stamps (helps bringing in code by "non-S4-people")
 - user interface: S4-objects, time stamps
- Use generating functions for encapsulation
 - without using structured arguments:
 - ★ too many arguments ~> user looses track
 - prone to name mis-matchings (positional, partial matching)
 - * bad alternative: fix defaults.
 - have generating functions to produce control objects
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Interfaces so far

- preliminary, "S4-free" interfaces
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 - rLS: rLSFilter (=rLS.AO.Filter),rLS.IO. Filter, rLS.IOAO.Filter
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http://r-forge.r-project.org/projects/robkalman/

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84 classes: for SSM's; for output-classes; for method-classes; for control-classes (reuse release code)

interfaces between 84-layer and 84-free layer

rome sam packages





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 to other SSM packages
 to robfilter (Roland Fried & K. Schettlinger)





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Work in process

Release Plans

- package robKalman should be on CRAN by UseR! 2009, but...
- at least: release on CRAN by end of August
- till then: refer to r-forge







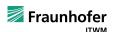
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Extensions

- robust smoothing (80% done)
- robust EM-Algorithm to estimate unknown hyper parameters (extending Shumway/Stoffer) (70% done)
- interpretation as random coefficient regression
 - → robust regression-type approach (rIC, mIC) (30% done)
- connecttion to particle filters theory and computer interface (10% done)
- speeding up things / bridging to fast Kalman filter of FKF by David Luethi, Philipp Erb
 (1% done)





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very neat for collaborative R package development

- version management (svn)
- mail-forwarded log-files of committed code
 → keep track of work of others
- bug tracker, archived mailing lists, ...
- see slides by **Stefan Theuss**l
- needs serious conceptional preparations

consistency: coding & documentation conventions

- helpful: scheduling, reminders/deadlines for collaborators.
- summarizing

Collaborative programming is enjoyable and very exciting:

Thanks for your attention





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References

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