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. Kalman's revenge or

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evisited

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R–Package robKalman —

R. Kalman's revenge or

Robustness for Kalman Filtering Revisited

Peter Ruckdeschel[1] Bernhard Spangl[2]

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Rennes, July 9, 2009

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Euclidean State Space Models

Definitions and Assumptions:

— Time–Discrete, Euclidean Setup ideal model:

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\begin{aligned}
x_t &= F(x_{t-1}, t) + v_t, & v_t &\overset{\text{indep.}}{\sim} (0, Q_t), & &[p\text{–dim}],\\
y_t &= Z(x_t, t) + \varepsilon_t, & \varepsilon_t &\overset{\text{indep.}}{\sim} (0, V_t), & &[q\text{–dim}],\\
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$\{v_t\}, \{\varepsilon_t\}, x_0$ indep. as processes

functions F, Z smooth with known derivatives;
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extensible to:

- continuous time (SDE's)
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exogenous outliers affecting only singular observations

$$\text{AO} \quad :: \quad \varepsilon_t^{\text{re}} \sim (1 - r_{\text{AO}})\mathcal{L}(\varepsilon_t^{\text{id}}) + r_{\text{AO}}\mathcal{L}(\varepsilon_t^{\text{di}})$$

$$\text{SO} \quad :: \quad y_t^{\text{re}} \sim (1 - r_{\text{SO}})\mathcal{L}(y_t^{\text{id}}) + r_{\text{SO}}\mathcal{L}(y_t^{\text{di}})$$

endogenous outliers / structural changes

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Notation: $\cdot \mathrel{\hat{=}} \cdot^{\text{id}}$, $\hat{\cdot} \mathrel{\hat{=}} \cdot^{\text{re}}$, $\tilde{\cdot} \mathrel{\hat{=}} \cdot^{\text{di}}$

Different and competing goals

A/SO attenuation of "false alarms"

IO tracking: detect structural changes as fast as possible;
recovering: clean data from structural changes

A/SO & IO identification problem:
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Classical Method: Kalman–Filter

Filter Problem

$$\mathsf{E}\left|x_t - f_t(y_{1:t})\right|^2 = \min_{f_t} !,$$

with $y_{1:t} = (y_1, \ldots, y_t), \quad y_{1:0} := \emptyset$

General solution: $\mathsf{E}[x_t|y_{1:t}]$ —difficult to compute

Kalman–Filter assuming $F(x,t) = F_t x$, $Z(x,t) = Z_t x$

optimal solution among linear filters — Kalman[/Bucy] [60/61]:

Initialization: $x_{0|0} = a_0$

Prediction: $x_{t|t-1} = F_t x_{t-1|t-1}, \qquad [\Delta x_t = x_t - x_{t|t-1}]$

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Features of the Kalman–Filter

+ an easy, understandable structure:
  initialization, prediction, correction step
+ correction step is easily evaluable and interpretable: it is linear !
+ strict recursivity / Markovian structure:
  all information from the past useful for the future is captured in the value of $x_{t|t-1}$.
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- Kalman filter: filter, Kalman gain, covariances
- ACM-filter: filter, multivariate version, GM-estimator
- rLS-filter: filter, calibration of clipping height
  - AO/SO-robust version
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- extensible to further recursive filters:
  $\rightsquigarrow$ general interface `recursiveFilter`
  with arguments:
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Implementation concept

- Programming language
  - completely in S, perhaps some code in C later ($\rightsquigarrow$ FKF)
- Use existing infrastructure: zoo, timeSeries
  - for: graphics, diagnostics, management of date/time
- Code in different layers
  - internal functions: no S4-objects, no time stamps
    (helps bringing in code by "non-S4-people")
  - user interface: S4-objects, time stamps
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  - without using structured arguments:
    - too many arguments $\rightsquigarrow$ user looses track
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Implementation so far

Interfaces so far

- preliminary, "S4-free" interfaces
  - Kalman filter (in our context) KalmanFilter
  - rLS: rLSFilter (=rLS.AO.Filter),
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- availability: `robKalman` version `0.3` (incl. demos)

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Almost ready:

- S4 classes for SSMs, for output-classes, for method-classes,
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Work in process

Release Plans

- package `robKalman` should be on `CRAN` by UseR! 2009, but...
- at least: release on `CRAN` by end of August
- till then: refer to `r-forge`

Extensions

- robust smoothing (80% done)
- robust EM-Algorithm to estimate unknown hyper parameters (extending Shumway/Stoffer) (70% done)
- interpretation as random coefficient regression ⇝ robust regression-type approach (**rIC**, **mIC**) (30% done)
- connecttion to particle filters — theory and computer interface (10% done)
- speeding up things / bridging to fast Kalman filter of `FKF` by **David Luethi**, **Philipp Erb** (1% done)

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  very neat for collaborative R package development
  - version management (`svn`)
  - mail-forwarded `log`-files of committed code
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  - bug tracker, archived mailing lists, . . .
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  - consistency: coding & documentation conventions
- helpful: scheduling, reminders/deadlines for collaborators. . .
- summarizing:

  Collaborative programming is enjoyable and very exciting!

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