Econometric Computing with HC and HAC Covariance Matrix Estimators

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Abstract

sandwich

Keywords: covariance matrix estimator, heteroskedasticity, autocorrelation, estimating functions, econometric computing, R.

1. Introduction

R Development Core Team (2004) Cribari-Neto and Zarkos (2003)

stress econometric computing

reusable components

covariance matrices not only as options to certain test but as stand-alone functions which can be plugged into various inference procedures

2. The linear regression model

To fix notations,

3. Estimating covariance matrices

3.1. Dealing with heteroskedasticity

White (1980) MacKinnon and White (1985) Long and Ervin (2000) Cribari-Neto (2004)

3.2. Dealing with autocorrelation

Newey and West (1987) Andrews (1991) Andrews and Monahan (1992) Lumley and Heagerty (1999)

4. Applications and illustrations

4.1. Testing coefficients in cross-sectional data

Greene (1993) Cribari-Neto (2004) Zeileis and Hothorn (2002) Fox (2002)

```
R> data(PublicSchools)
R> ps <- na.omit(PublicSchools)
R> ps$Income <- ps$Income * 1e-04
```

```
R> fm.ps <- lm(Expenditure ~ Income + I(Income^2), data = ps)

R> coeftest(fm.ps, df = Inf, vcov = vcovHC(fm.ps, type = "HC0"))

z test of coefficients of "lm" object 'fm.ps':

             Estimate Std. Error z value Pr(>|z|)
(Intercept)    832.91     460.89  1.8072  0.07073 .
Income       -1834.20    1243.04 -1.4756  0.14006
I(Income^2)   1587.04     829.99  1.9121  0.05586 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R> coeftest(fm.ps, df = Inf, vcov = vcovHC(fm.ps, type = "HC4"))

z test of coefficients of "lm" object 'fm.ps':

             Estimate Std. Error z value Pr(>|z|)
(Intercept)    832.91    3008.01  0.2769   0.7819
Income       -1834.20    8183.19 -0.2241   0.8226
I(Income^2)   1587.04    5488.93  0.2891   0.7725

vcovHC(fm.ps, type = "HC0")
```

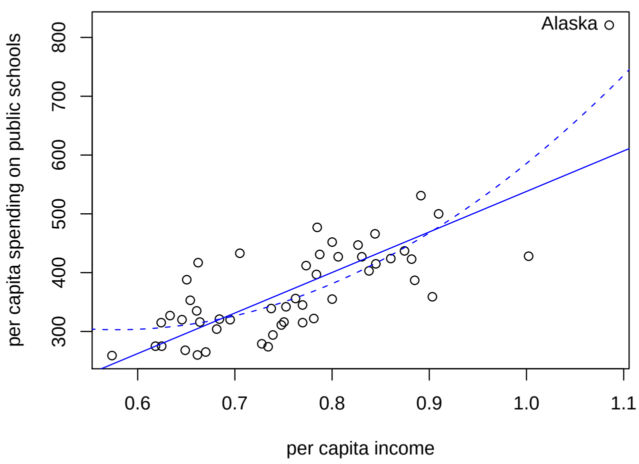

Figure 1: Expenditure on public schools and income

4.2. Testing coefficients in time-series data

Greene (1993)

```
R> data(Investment)

R> fm.inv <- lm(RealInv ~ RealInt + RealGNP, data = Investment)
```

```
R> coeftest(fm.inv, df = Inf, vcov = NeweyWest(fm.inv, lag = 4))

z test of coefficients of "lm" object 'fm.inv':

              Estimate Std. Error z value Pr(>|z|)
(Intercept) -12.533601  18.958298 -0.6611   0.5085
RealInt      -1.001438   3.342375 -0.2996   0.7645
RealGNP       0.169136   0.016751 10.0972   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

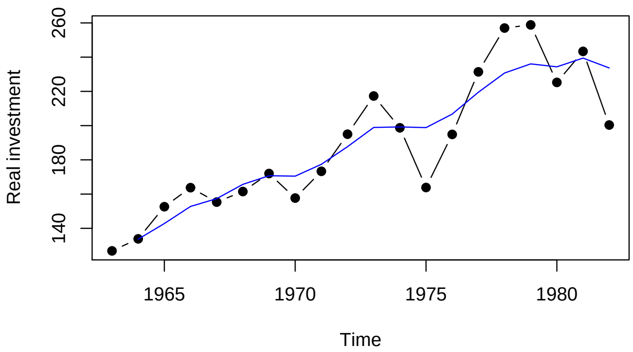

Figure 2: Investment equation data

4.3. Testing and dating structural changes in the presence of heteroskedasticity and autocorrelation

Bai and Perron (2003) Andrews (1993) Ploberger and Krämer (1992)
Zeileis, Leisch, Hornik, and Kleiber (2002) Zeileis (2004)

```
R> data(RealInt)
```

```
R> ocus <- gefp(RealInt ~ 1, fit = lm, vcov = kernHAC)
```

plot(ocus), sctest(ocus)

```
R> fs <- Fstats(RealInt ~ 1, vcov = kernHAC)
```

sctest(fs), plot(fs)

```
R> bp <- breakpoints(RealInt ~ 1)
R> confint(bp, vcov = kernHAC)

         Confidence intervals for breakpoints
         of optimal 3-segment partition:

Call:
```

```
confint.breakpointsfull(object = bp, vcov = kernHAC)

Breakpoints at observation number:
  2.5 % breakpoints 97.5 %
1    37          47     48
2    77          79     81

Corresponding to breakdates:
  2.5 %   breakpoints 97.5 %
1 1970(1) 1972(3)     1972(4)
2 1980(1) 1980(3)     1981(1)
```

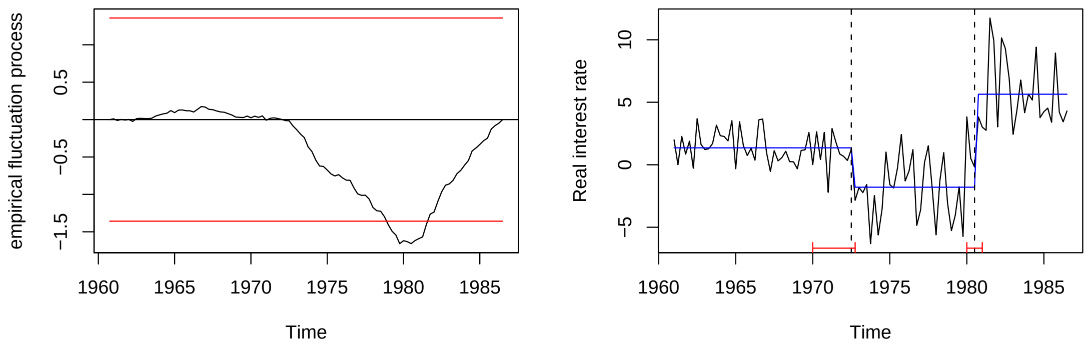

Figure 3: OLS-based CUSUM test (left) and fitted model (right) for real interest data

5. Summary

A. R code

A.1. Testing coefficients in cross-sectional data

Load public schools data, omit `NA` in Wisconsin and scale income:

```
data(PublicSchools)
ps <- na.omit(PublicSchools)
ps$Income <- ps$Income * 0.0001
```

Fit quadratic regression model:

```
fm.ps <- lm(Expenditure ~ Income + I(Income^2), data = ps)
```

Compare standard errors:

```
sqrt(diag(vcov(fm.ps)))
sqrt(diag(vcovHC(fm.ps, type = "const")))
sqrt(diag(vcovHC(fm.ps, type = "HC0")))
sqrt(diag(vcovHC(fm.ps, type = "HC3")))
sqrt(diag(vcovHC(fm.ps, type = "HC4")))
```

Test coefficient of quadratic term:

```
coeftest(fm.ps, df = Inf, vcov = vcovHC(fm.ps, type = "HC0"))
coeftest(fm.ps, df = Inf, vcov = vcovHC(fm.ps, type = "HC4"))
```

Visualization:

```
plot(Expenditure ~ Income, data = ps,
  xlab = "per capita income",
  ylab = "per capita spending on public schools")
inc <- seq(0.5, 1.2, by = 0.001)
lines(inc, predict(fm.ps, data.frame(Income = inc)), col = 4, lty = 2)
fm.ps2 <- lm(Expenditure ~ Income, data = ps)
abline(fm.ps2, col = 4)
text(ps[2,2], ps[2,1], rownames(ps)[2], pos = 2)
```

A.2. Testing coefficients in time-series data

Load investment equation data:

```
data(Investment)
```

Fit regression model:

```
fm.inv <- lm(RealInv ~ RealInt + RealGNP, data = Investment)
```

Test coefficients using Newey-West HAC estimator:

```
coeftest(fm.inv, df = Inf, vcov = NeweyWest(fm.inv, lag = 4))
```

Visualization:

```
plot(Investment[, "RealInv"], type = "b", pch = 19, ylab = "Real investment")
lines(ts(fitted(fm.inv), start = 1964), col = 4)
```

A.3. Testing and dating structural changes in the presence of heteroskedasticity and autocorrelation

Load real interest series:

```
data(RealInt)
```

OLS-based CUSUM test with quadratic spectral kernel HAC estimate:

```
ocus <- gefp(RealInt ~ 1, fit = lm, vcov = kernHAC)
plot(ocus, aggregate = FALSE)
sctest(ocus)
```

sup$F$ test with quadratic spectral kernel HAC estimate:

```
fs <- Fstats(RealInt ~ 1, vcov = kernHAC)
plot(fs)
sctest(fs)
```

Breakpoint estimation and confidence intervals with quadratic spectral kernel HAC estimate:

```
bp <- breakpoints(RealInt ~ 1)
confint(bp, vcov = kernHAC)
```

Visualization:

```
plot(RealInt, ylab = "Real interest rate")
lines(ts(fitted(bp), start = start(RealInt), freq = 4), col = 4)
lines(confint(bp, vcov = kernHAC))
```

References

Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, **59**, 817–858.

Andrews DWK (1993). "Tests for Parameter Instability and Structural Change With Unknown Change Point." *Econometrica*, **61**, 821–856.

Andrews DWK, Monahan JC (1992). "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator." *Econometrica*, **60**(4), 953–966.

Bai J, Perron P (2003). "Computation and Analysis of Multiple Structural Change Models." *Journal of Applied Econometrics*, **18**, 1–22.

Cribari-Neto F (2004). "Asymptotic Inference Under Heteroskedasticity of Unknown Form." *Computational Statistics & Data Analysis*, **45**, 215–233.

Cribari-Neto F, Zarkos SG (2003). "Econometric and Statistical Computing Using Ox." *Computational Economics*, **21**, 277–295.

Fox J (2002). *An R and S-PLUS Companion to Applied Regression*. Sage Publications, Thousand Oaks, CA.

Greene WH (1993). *Econometric Analysis*. Macmillan Publishing Company, New York, 2nd edition.

Long JS, Ervin LH (2000). "Using Heteroscedasticity Consistent Standard Errors in the Linear Regression Model." *The American Statistician*, **54**, 217–224.

Lumley T, Heagerty P (1999). "Weighted Empirical Adaptive Variance Estimators for Correlated Data Regression." *Journal of the Royal Statistical Society B*, **61**, 459–477.

MacKinnon JG, White H (1985). "Some Heteroskedasticity-consistent Covariance Matrix Estimators with Improved Finite Sample Properties." *Journal of Econometrics*, **29**, 305–325.

Newey WK, West KD (1987). "A Simple, Positive-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, **55**, 703–708.

Ploberger W, Krämer W (1992). "The CUSUM Test With OLS Residuals." *Econometrica*, **60**(2), 271–285.

R Development Core Team (2004). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-00-3, URL http://www.R-project.org/.

White H (1980). "A Heteroskedasticity-consistent Covariance Matrix and a Direct Test for Heteroskedasticity." *Econometrica*, **48**, 817–838.

Zeileis A (2004). "Implementing a Class of Structural Change Tests: An Econometric Computing Approach." *Report 7*, Department of Statistics and Mathematics, Wirtschaftsuniversität Wien, Research Report Series. URL http://epub.wu-wien.ac.at/.

Zeileis A, Hothorn T (2002). "Diagnostic Checking in Regression Relationships." *R News*, **2**(3), 7–10. URL http://cran.R-project.org/doc/Rnews/.

Zeileis A, Leisch F, Hornik K, Kleiber C (2002). "**strucchange**: An R Package for Testing for Structural Change in Linear Regression Models." *Journal of Statistical Software*, **7**(2), 1–38. URL http://www.jstatsoft.org/v07/i02/.