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Generalized Measurement Invariance Tests for Factor Analysis

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Measurement Invariance

- Measurement invariance: Sets of tests/items consistently assigning scores across diverse groups of individuals.
- Notable violations of measurement invariance:
 - SAT for different ethnic groups (Atkinson, 2001)
 - Intelligence tests & the Flynn effect (Wicherts et al., 2004)

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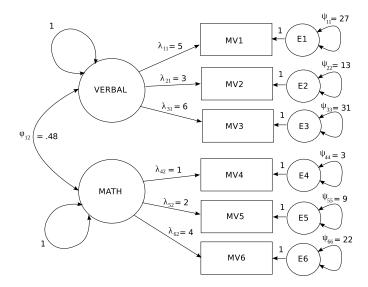
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Example (Age ≤ 16)

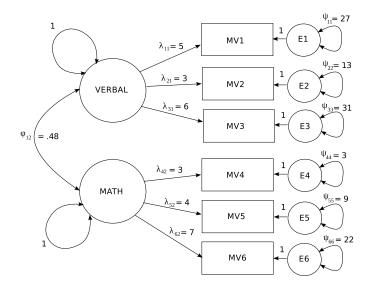


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Example (Age > 16)



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• Hypothesis of "full" measurement invariance:

$$H_0: \theta_i = \theta_0, i = 1, \dots, n$$

 $H_1: \text{Not all the } \theta_i = \theta_0$

where $\theta_i = (\lambda_{i,1,1}, \dots, \psi_{i,1,1}, \dots, \varphi_{i,1,2})^{\top}$ is the full p-dimensional parameter vector for individual i.

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- H_0 from the previous slide is difficult to fully assess due to all the ways by which individuals may differ.
- We typically place people into groups based on a meaningful auxiliary variable, then study measurement invariance across those groups (via Likelihood Ratio tests, Lagrange multiplier tests, Wald tests).
- If we did not know the groups in advance, we could conduct a LR or LM test for each possible grouping, then take the maximum. Requires different critical values! (Can be obtained from proposed tests.)

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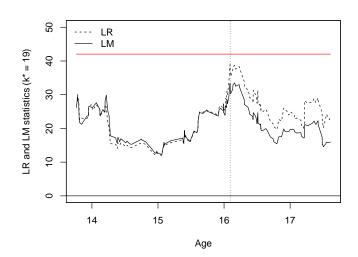
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Lack of Grouping



Conclusion

Proposed Tests

- In contrast to existing tests of measurement invariance, the proposed tests offer the abilities to:
 - Test for measurement invariance when groups are ill-defined (e.g., when the grouping variable is continuous).
 - Test for measurement invariance in any subset of model parameters.
 - Interpret the nature of measurement invariance violations.

Conclusions

Proposed Tests

- The proposed family of tests rely on first derivatives of the model's log-likelihood function.
- We consider individual terms (scores) of the gradient.
 These scores tell us how well a particular parameter describes a particular individual.

$$\sum_{i=1}^n s(\hat{\boldsymbol{\theta}}; \mathbf{x}_i) = \mathbf{0}$$
, where

$$s(\hat{\theta}; \mathbf{x}_i) = \frac{\partial}{\partial \boldsymbol{\theta}} \log \mathsf{L}(\mathbf{x}_i, \boldsymbol{\theta}) \big|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}}$$

Conclusions

Proposed Tests

- Under measurement invariance, parameter estimates should roughly describe everyone equally well. So people's scores should fluctuate around zero.
- If measurement invariance is violated, the scores should stray from zero.

Conclusions

Aggregating Scores

- We need a way to aggregate scores across people so that we can draw some general conclusions.
 - Order individuals by an auxiliary variable.
 - Define $t \in (1/n, n)$. The empirical cumulative score process is defined by:

$$\mathbf{B}(\hat{\theta};t) = \frac{1}{\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} s(\hat{\theta};\mathbf{x}_i).$$

where |nt| is the integer part of nt.

Conclusion

 Under the hypothesis of measurement invariance, a functional central limit theorem holds:

$$\mathbf{I}(\widehat{\boldsymbol{\theta}})^{-1/2}\mathbf{B}(\widehat{\boldsymbol{\theta}};\cdot)\stackrel{d}{\to}\mathbf{B}^0(\cdot),$$

where $\mathbf{I}(\widehat{\theta})$ is the observed information matrix and $\mathbf{B}^0(\cdot)$ is a p-dimensional Brownian bridge.

- Testing procedure: Compute an aggregated statistic of the empirical score process and compare with corresponding quantile of aggregated Brownian motion.
- Test statistics: Special cases include double maximum (DM), Cramér-von Mises (CvM), maximum of LM statistics.

Conclusions

Simulation

- Simulation: What is the power of the proposed tests?
 - Two-factor model, with three indicators each.
 - Measurement invariance violation in three factor loading parameters, with magnitude from 0–4 standard errors.
 - Sample size in {100, 200, 500}.
 - Model parameters tested in {3,19}.
 - Three test statistics.

Measurement Invariance

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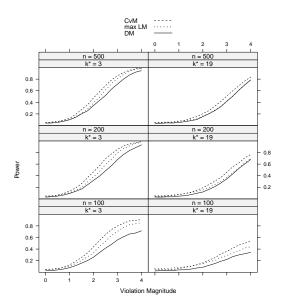
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Simulation



Conclusion

Example

- Example: Studying stereotype threat via factor analysis (Wicherts et al., 2005)
 - Stereotype threat: Knowledge of stereotypes about one's social group might cause one to fulfill the stereotypes.
 - Wicherts et al. study: 295 students were administered three intelligence tests. Stereotypes were primed for half of the students.
 - Groups defined by: Ethnicity (majority/minority) and whether or not stereotypes were primed.

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Illustration

Conclusion

- To study the data, Wicherts et al. employed a series of four-group, one-factor models.
 - General finding: Minorities with stereotype primes have different measurement parameters than other groups.
 - Current example: Is measurement further impacted by academic performance (as measured by student GPA)?

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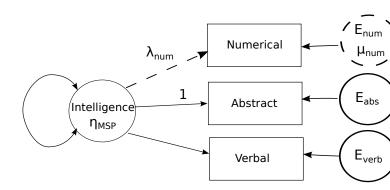
- We utilize a model employed by Wicherts et al., where four model parameters are specific to the "minority, stereotype prime" group.
 - Test for measurement invariance in these parameters wrt the student GPA variable (either all four together or individually).
 - Violations of measurement invariance imply that stereotype threat is more problematic for students of low or high GPA.

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Model

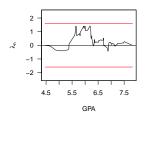


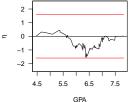
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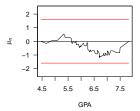
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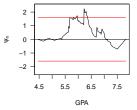
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Results for Single Parameters









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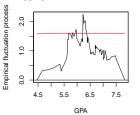
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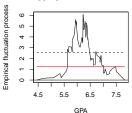
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Aggregated Results

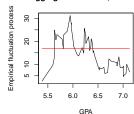




Aggregated Process, CvM



Aggregated Process, max LM



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Conclusions

- Measurement invariance tests utilizing stochastic processes have important advantages over existing tests:
 - Isolating specific parameters that violate measurement invariance, allowing the researcher to define specific types of measurement invariance "post hoc" instead of "a priori".
 - Isolating groups of individuals whose parameter values differ.
 - Studying the impact of continuous variables on model estimates, without "ruining" the rest of the model.
- Power is reasonable, with specific tests being better in specific circumstances.

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Conclusions

- To carry out the tests, we utilize
 - lavaan for model estimation.
 - estfun() for score extraction, which is currently a combination of our own code and lavaan code.
 - strucchange for carrying out the proposed tests with the scores.
 - Required input: Fitted model, function for score extraction, and information matrix (optional).
 - gefp() constructs the process.
 - sctest() and plot() calculate and visualize test statistics.

Conclusions

Current Work

- Continued test implementation via strucchange and lavaan (and possibly OpenMx).
- Detailed examination of test properties.
- Extension to related psychometric issues.
- Working paper: http://econpapers.repec.org/RePEc:inn:wpaper: 2011 00
 - 2011-09

Measurement Invariance

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• Questions?