# Unit 5: Word Frequency Distributions with the zipfR package

## Statistics for Linguists with R – A SIGIL Course

Designed by Marco Baroni[1] and Stefan Evert[2]

[1]Center for Mind/Brain Sciences (CIMeC)
University of Trento, Italy

[2]Corpus Linguistics Group
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

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# Outline







# Lexical statistics

Zipf (1949, 1965); Baayen (2001); Baroni (2008)

- Statistical study of the frequency distribution of **types** (words or other linguistic units) in texts
  - remember the distinction between **types** and **tokens**?
- Different from other categorical data because of the extreme richness of types
  - people often speak of **Zipf's law** in this context





# Outline

## Basic terminology

- $N$: sample / corpus size, number of **tokens** in the sample
- $V$: **vocabulary** size, number of distinct **types** in the sample
- $V_m$: **spectrum element** $m$, number of types in the sample with frequency $m$ (i.e. exactly $m$ occurrences)
- $V_1$: number of **hapax legomena**, types that occur only once in the sample (for hapaxes, #types = #tokens)

- A sample: `a b b c a a b a`
- $N = 8$, $V = 3$, $V_1 = 1$





## Rank / frequency profile

- The sample: `c a a b c c a c d`
- Frequency list ordered by decreasing frequency

| $t$ | $f$ |
|---|---|
| c | 4 |
| a | 3 |
| b | 1 |
| d | 1 |

- Rank / frequency profile: ranks instead of type labels

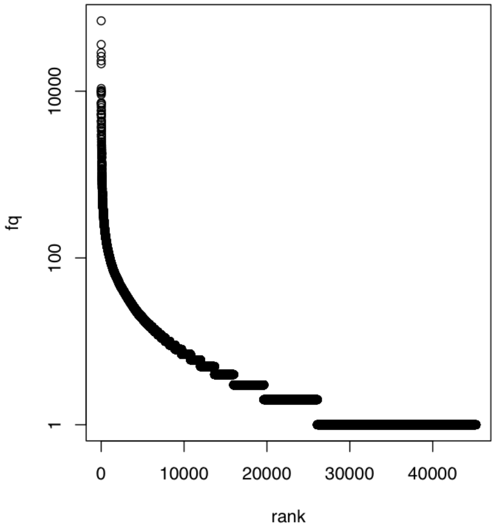
| $r$ | $f$ |
|---|---|
| 1 | 4 |
| 2 | 3 |
| 3 | 1 |
| 4 | 1 |

- Expresses type frequency $f_r$ as function of rank of a type





## Rank/frequency profile of Brown corpus








## Top and bottom ranks in the Brown corpus

| top frequencies | | | bottom frequencies | | |
|---|---|---|---|---|---|
| **$r$** | **$f$** | **word** | **rank range** | **$f$** | **randomly selected examples** |
| 1 | 69836 | the | 7731 – 8271 | 10 | schedules, polynomials, bleak |
| 2 | 36365 | of | 8272 – 8922 | 9 | tolerance, shaved, hymn |
| 3 | 28826 | and | 8923 – 9703 | 8 | decreased, abolish, irresistible |
| 4 | 26126 | to | 9704 – 10783 | 7 | immunity, cruising, titan |
| 5 | 23157 | a | 10784 – 11985 | 6 | geographic, lauro, portrayed |
| 6 | 21314 | in | 11986 – 13690 | 5 | grigori, slashing, developer |
| 7 | 10777 | that | 13691 – 15991 | 4 | sheath, gaulle, ellipsoids |
| 8 | 10182 | is | 15992 – 19627 | 3 | mc, initials, abstracted |
| 9 | 9968 | was | 19628 – 26085 | 2 | thar, slackening, deluxe |
| 10 | 9801 | he | 26086 – 45215 | 1 | beck, encompasses, second-place |

## Frequency spectrum

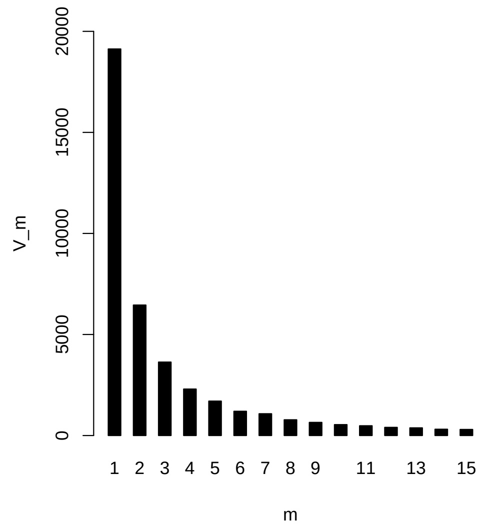
- The sample: `c a a b c c a c d`
- Frequency classes: 1 (`b`, `d`), 3 (`a`), 4 (`c`)
- Frequency spectrum:

| $m$ | $V_m$ |
|---|---|
| 1 | 2 |
| 3 | 1 |
| 4 | 1 |





## Frequency spectrum of Brown corpus








## Vocabulary growth curve

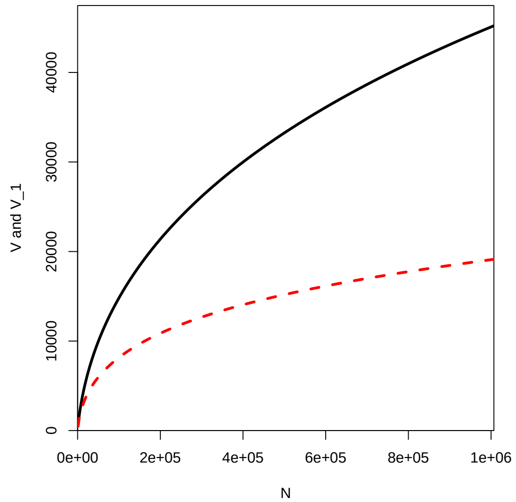
- The sample: `a b b c a a b a`
- $N = 1,\ V = 1,\ V_1 = 1 \quad (V_2 = 0, \ldots)$
- $N = 3,\ V = 2,\ V_1 = 1 \quad (V_2 = 1,\ V_3 = 0, \ldots)$
- $N = 5,\ V = 3,\ V_1 = 1 \quad (V_2 = 2,\ V_3 = 0, \ldots)$
- $N = 8,\ V = 3,\ V_1 = 1 \quad (V_2 = 0,\ V_3 = 1,\ V_4 = 1, \ldots)$





## Vocabulary growth curve of Brown corpus

With $V_1$ growth in red (curve smoothed with binomial interpolation)



# Outline

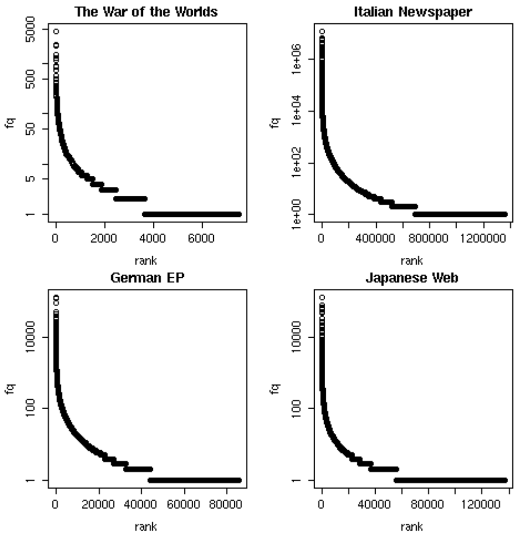






## Typical frequency patterns

### Across text types & languages

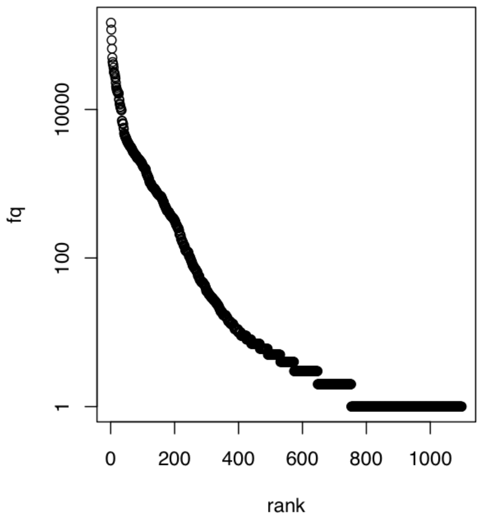







## Typical frequency patterns

### The Italian prefix *ri-* in the *la Repubblica* corpus

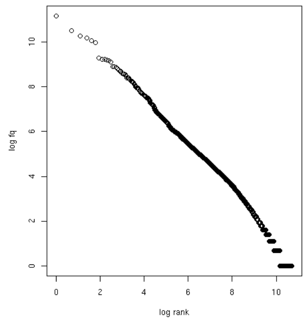







## Is there a general law?

- Language after language, corpus after corpus, linguistic type after linguistic type, . . . we observe the same "few giants, many dwarves" pattern
- Similarity of plots suggests that relation between rank and frequency could be captured by a general law
- Nature of this relation becomes clearer if we plot $\log f$ as a function of $\log r$



# Outline







# Zipf's law

- Straight line in double-logarithmic space corresponds to **power law** for original variables
- This leads to Zipf's (1949; 1965) famous law:

$$f(w) = \frac{C}{r(w)^a}$$

- With $a = 1$ and $C$ =60,000, Zipf's law predicts that:
  - most frequent word occurs 60,000 times
  - second most frequent word occurs 30,000 times
  - third most frequent word occurs 20,000 times
  - and there is a long tail of 80,000 words with frequencies between 1.5 and 0.5 occurrences(!)





# Zipf's law

## Logarithmic version

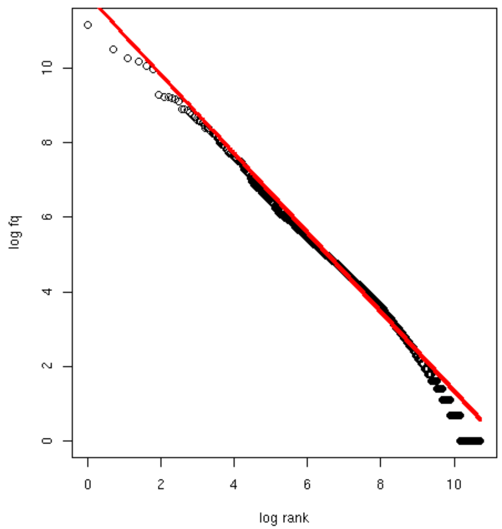
- Zipf's power law:
$$f(w) = \frac{C}{r(w)^a}$$
- If we take logarithm of both sides, we obtain:
$$\log f(w) = \log C - a \log r(w)$$
- Zipf's law predicts that rank / frequency profiles are straight lines in double logarithmic space
- Best fit $a$ and $C$ can be found with least-squares method
- Provides intuitive interpretation of $a$ and $C$:
  - $a$ is **slope** determining how fast log frequency decreases
  - $\log C$ is **intercept**, i.e., predicted log frequency of word with rank 1 (log rank 0) = most frequent word





# Zipf's law

## Fitting the Brown rank/frequency profile



## Zipf-Mandelbrot law

Mandelbrot (1953, 1962)

- Mandelbrot's extra parameter:

$$f(w) = \frac{C}{(r(w) + b)^a}$$

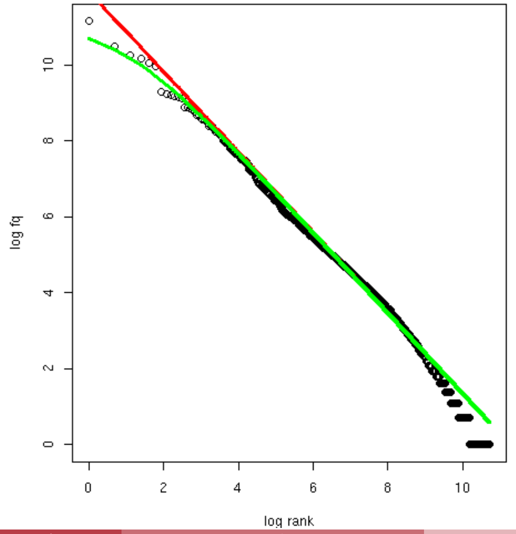
- Zipf's law is special case with $b = 0$
- Assuming $a = 1$, $C$ =60,000, $b = 1$:
  - For word with rank 1, Zipf's law predicts frequency of 60,000; Mandelbrot's variation predicts frequency of 30,000
  - For word with rank 1,000, Zipf's law predicts frequency of 60; Mandelbrot's variation predicts frequency of 59.94
- Zipf-Mandelbrot law forms basis of statistical LNRE models
  - ZM law derived mathematically as limiting distribution of vocabulary generated by a character-level Markov process





## Zipf-Mandelbrot vs. Zipf's law

Fitting the Brown rank/frequency profile








## Outline







## Applications of word frequency distributions

- Most important application: **extrapolation** of vocabulary size and frequency spectrum to larger sample sizes
  - morphological productivity (e.g. Lüdeling and Evert 2005)
  - lexical richness in stylometry (Efron and Thisted 1976), language acquisition, clinical linguistics (Garrard *et al.* 2005)
  - language technology (estimate proportion of OOV words, unseen grammra rules, typos, ...)

☞ need method for predicting vocab. growth on unseen data

- Direct applications of Zipf's law
  - population model for Good-Turing smoothing (Good 1953; Gale and Sampson 1995)
  - realistic prior for Bayesian language modelling

☞ need model of type probability distribution in the population

## Vocabulary growth: Pronouns vs. *ri-* in Italian

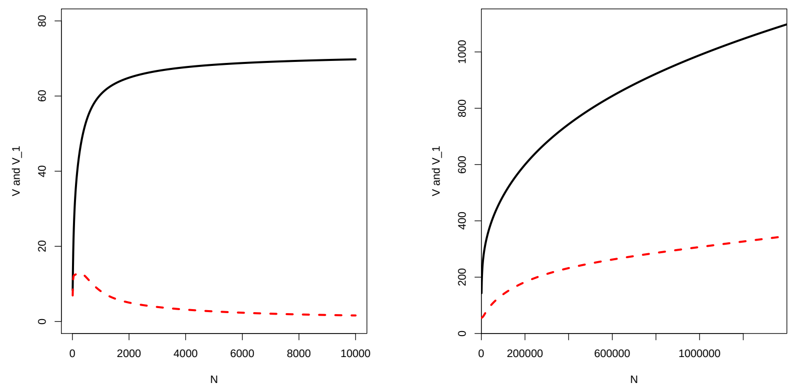
| $N$ | $V$ (pron.) | $V$ (*ri-*) |
|---|---|---|
| 5000 | 67 | 224 |
| 10000 | 69 | 271 |
| 15000 | 69 | 288 |
| 20000 | 70 | 300 |
| 25000 | 70 | 322 |
| 30000 | 71 | 347 |
| 35000 | 71 | 364 |
| 40000 | 71 | 377 |
| 45000 | 71 | 386 |
| 50000 | 71 | 400 |
| ... | ... | ... |





## Vocabulary growth: Pronouns vs. *ri-* in Italian

### Vocabulary growth curves







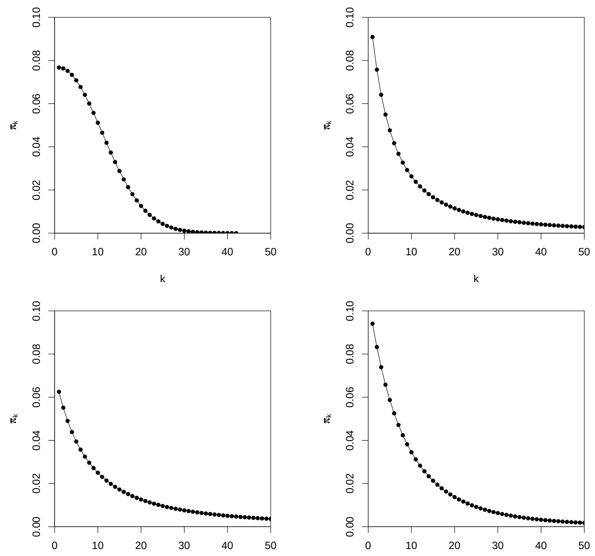

## LNRE models for word frequency distributions

- LNRE = large number of rare events (cf. Baayen 2001)
- Statistics: corpus = random sample from **population**
  - population characterised by vocabulary of **types** $w_k$ with occurrence **probabilities** $\pi_k$
  - not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - NB: not necessarily identical to Zipf ranking in sample!
- **LNRE model** = population model for type probabilities, i.e. a function $k \mapsto \pi_k$ (with small number of parameters)
  - type probabilities $\pi_k$ cannot be estimated reliably from a corpus, but parameters of LNRE model can





## Examples of population models



## The Zipf-Mandelbrot law as a population model

What is the right family of models for lexical frequency distributions?

- We have already seen that the Zipf-Mandelbrot law captures the distribution of observed frequencies very well
- Re-phrase the law for type probabilities:

$$\pi_k := \frac{C}{(k+b)^a}$$

- Two free parameters: $a > 1$ and $b \geq 0$
- $C$ is not a parameter but a normalization constant, needed to ensure that $\sum_k \pi_k = 1$
- this is the **Zipf-Mandelbrot** population model





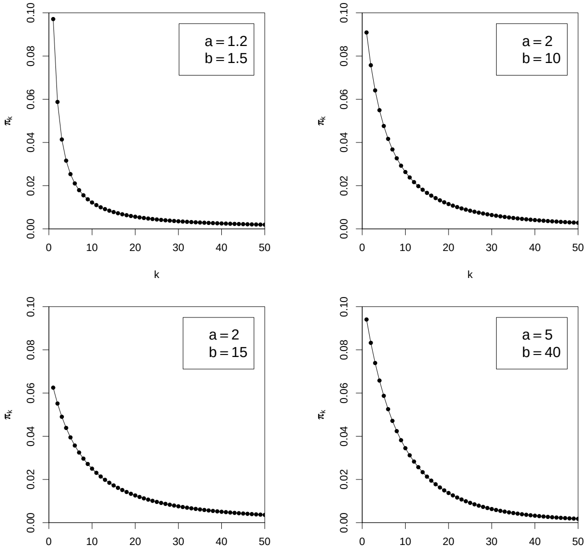
## Outline







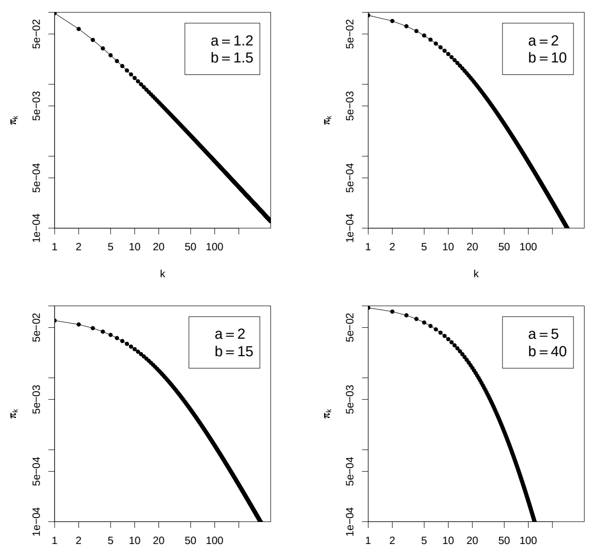
## The parameters of the Zipf-Mandelbrot model








## The parameters of the Zipf-Mandelbrot model



## The finite Zipf-Mandelbrot model

- Zipf-Mandelbrot population model characterizes an *infinite* type population: there is no upper bound on $k$, and the type probabilities $\pi_k$ can become arbitrarily small
- $\pi = 10^{-6}$ (once every million words), $\pi = 10^{-9}$ (once every billion words), $\pi = 10^{-12}$ (once on the entire Internet), $\pi = 10^{-100}$ (once in the universe?)
- Alternative: finite (but often very large) number of types in the population
- We call this the **population vocabulary size** $S$ (and write $S = \infty$ for an infinite type population)





## The finite Zipf-Mandelbrot model
Evert (2004)

- The **finite Zipf-Mandelbrot** model simply stops after the first $S$ types ($w_1, \ldots, w_S$)
- $S$ becomes a new parameter of the model
  $\rightarrow$ the finite Zipf-Mandelbrot model has 3 parameters

Abbreviations:

- **ZM** for Zipf-Mandelbrot model
- **fZM** for finite Zipf-Mandelbrot model





## Outline

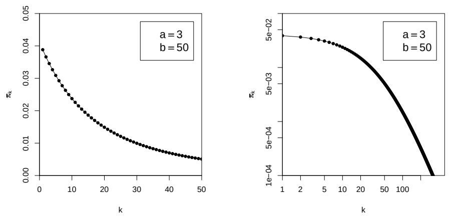






## Sampling from a population model

Assume we believe that the population we are interested in can be described by a Zipf-Mandelbrot model:




Use computer simulation to sample from this model:

- Draw $N$ tokens from the population such that in each step, type $w_k$ has probability $\pi_k$ to be picked
- This allows us to make predictions for samples (= corpora) of arbitrary size $N$ ➜ extrapolation

## Sampling from a population model

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **#1:** | 1 | 42 | 34 | 23 | 108 | 18 | 48 | 18 | 1 | … |
| | time | order | room | school | town | course | area | course | time | … |
| **#2:** | 286 | 28 | 23 | 36 | 3 | 4 | 7 | 4 | 8 | … |
| **#3:** | 2 | 11 | 105 | 21 | 11 | 17 | 17 | 1 | 16 | … |
| **#4:** | 44 | 3 | 110 | 34 | 223 | 2 | 25 | 20 | 28 | … |
| **#5:** | 24 | 81 | 54 | 11 | 8 | 61 | 1 | 31 | 35 | … |
| **#6:** | 3 | 65 | 9 | 165 | 5 | 42 | 16 | 20 | 7 | … |
| **#7:** | 10 | 21 | 11 | 60 | 164 | 54 | 18 | 16 | 203 | … |
| **#8:** | 11 | 7 | 147 | 5 | 24 | 19 | 15 | 85 | 37 | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |





## Samples: type frequency list & spectrum

| rank $r$ | $f_r$ | type $k$ |
|---|---|---|
| 1 | 37 | 6 |
| 2 | 36 | 1 |
| 3 | 33 | 3 |
| 4 | 31 | 7 |
| 5 | 31 | 10 |
| 6 | 30 | 5 |
| 7 | 28 | 12 |
| 8 | 27 | 2 |
| 9 | 24 | 4 |
| 10 | 24 | 16 |
| 11 | 23 | 8 |
| 12 | 22 | 14 |
| ⋮ | ⋮ | ⋮ |

| $m$ | $V_m$ |
|---|---|
| 1 | 83 |
| 2 | 22 |
| 3 | 20 |
| 4 | 12 |
| 5 | 10 |
| 6 | 5 |
| 7 | 5 |
| 8 | 3 |
| 9 | 3 |
| 10 | 3 |
| ⋮ | ⋮ |

**sample #1**





## Samples: type frequency list & spectrum

| rank $r$ | $f_r$ | type $k$ |
|---|---|---|
| 1 | 39 | 2 |
| 2 | 34 | 3 |
| 3 | 30 | 5 |
| 4 | 29 | 10 |
| 5 | 28 | 8 |
| 6 | 26 | 1 |
| 7 | 25 | 13 |
| 8 | 24 | 7 |
| 9 | 23 | 6 |
| 10 | 23 | 11 |
| 11 | 20 | 4 |
| 12 | 19 | 17 |
| ⋮ | ⋮ | ⋮ |

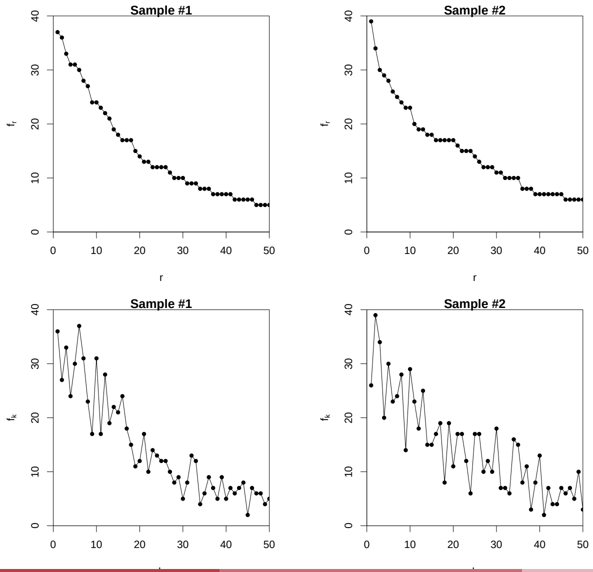
| $m$ | $V_m$ |
|---|---|
| 1 | 76 |
| 2 | 27 |
| 3 | 17 |
| 4 | 10 |
| 5 | 6 |
| 6 | 5 |
| 7 | 7 |
| 8 | 3 |
| 10 | 4 |
| 11 | 2 |
| ⋮ | ⋮ |

**sample #2**





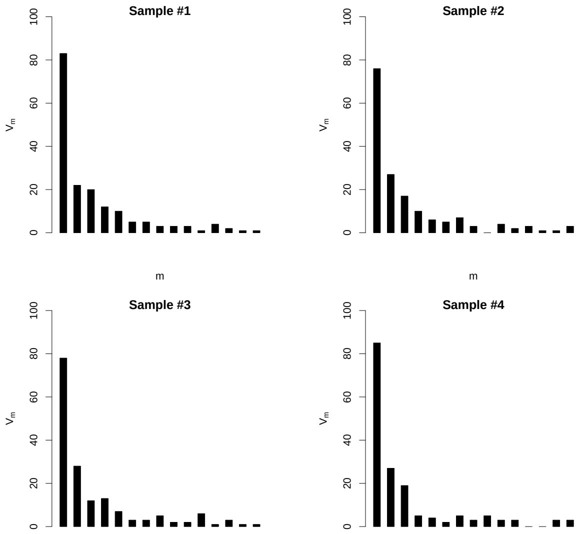
## Random variation in type-frequency lists




$r \leftrightarrow f_r$

$k \leftrightarrow f_k$

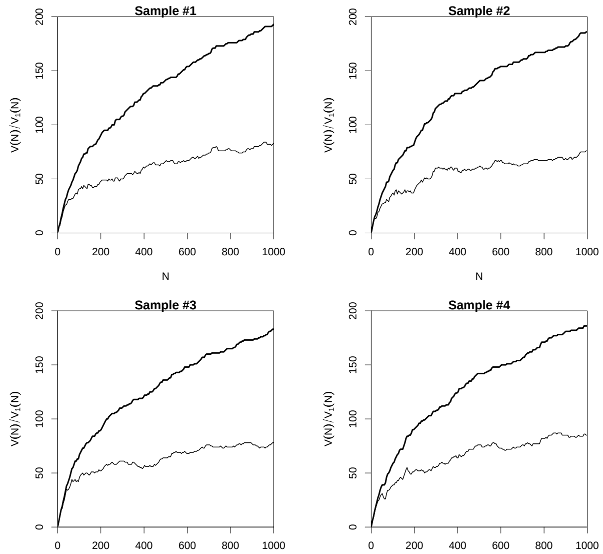
# Random variation: frequency spectrum








# Random variation: vocabulary growth curve








# Outline



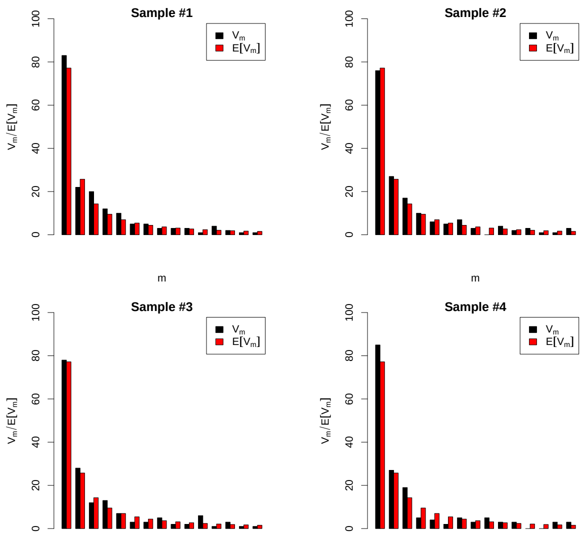




# Expected values

- There is no reason why we should choose a particular sample to make a prediction for the real data – each one is equally likely or unlikely
- Take the average over a large number of samples, called **expected value** or **expectation** in statistics
- Notation: $\mathrm{E}\big[V(N)\big]$ and $\mathrm{E}\big[V_m(N)\big]$
  - indicates that we are referring to expected values for a sample of size $N$
  - rather than to the specific values $V$ and $V_m$ observed in a particular sample or a real-world data set
- Expected values can be calculated efficiently *without* generating thousands of random samples

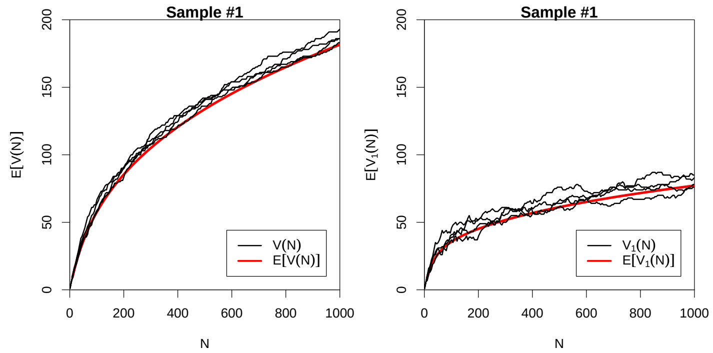
## The expected frequency spectrum







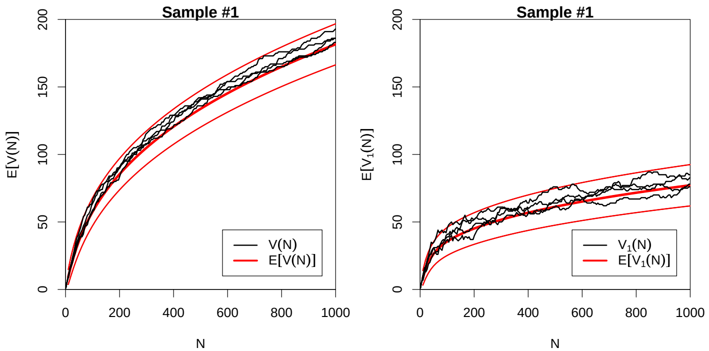

## The expected vocabulary growth curve








## Confidence intervals for the expected VGC




"Confidence intervals" indicate predicted sampling distribution:

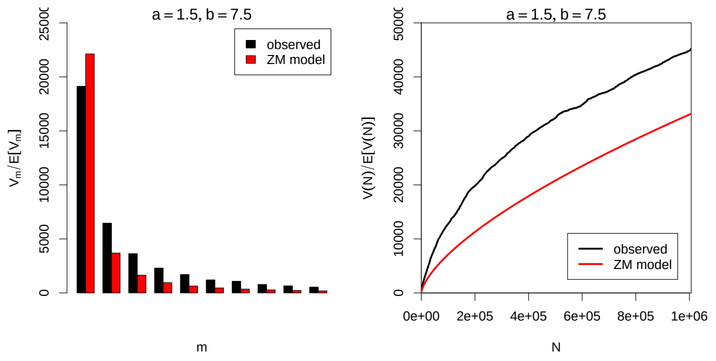
☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines





## Outline

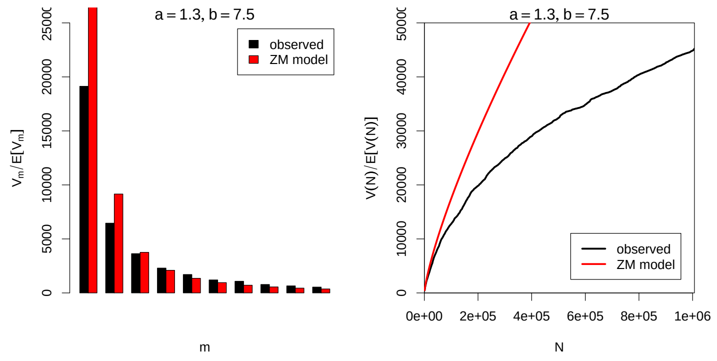
# Parameter estimation by trial & error







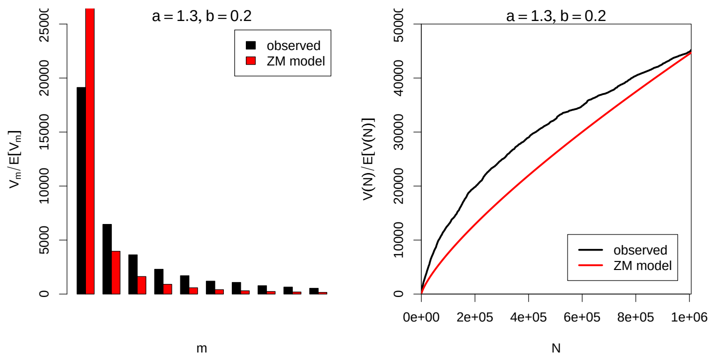

# Parameter estimation by trial & error







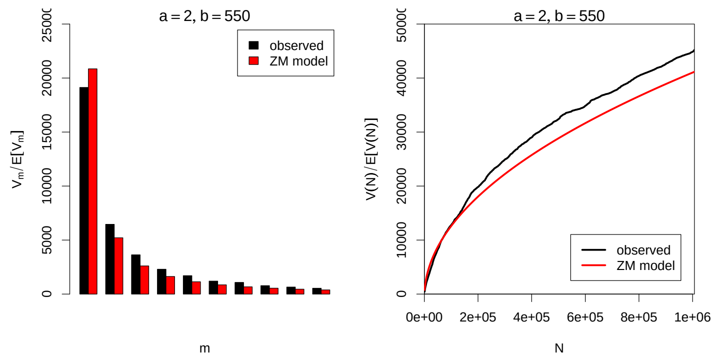

# Parameter estimation by trial & error



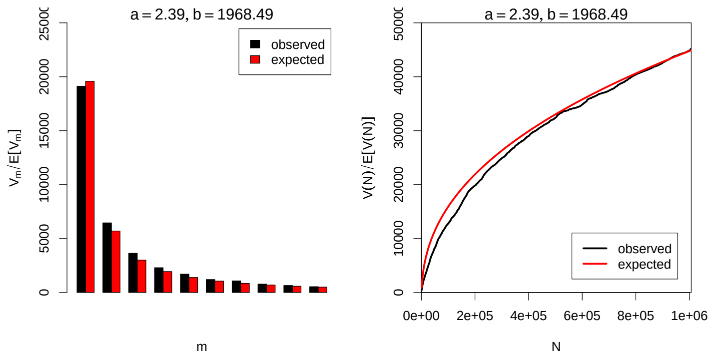





# Parameter estimation by trial & error



## Automatic parameter estimation




- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure: $a = 2.39$ and $b = 1968$





## Outline







## Goodness-of-fit

- Goodness-of-fit statistics measure how well the model has been fitted to the observed training data
- Compare observed vs. expected frequency distribution
  - frequency spectrum (➜ easier)
  - vocabulary growth curve
- Similarity measures
  - mean square error (➜ dominated by large $V$ / $V_m$)
  - multivariate chi-squared statistic $X^2$ takes sampling variaton (and covariance of spectrum elements) into account
- Multivariate chi-squared test for goodness-of-fit
  - $H_0$ : observed data = sample from LNRE model (i.e. fitted LNRE model describes the true population)
  - p-value derived from $X^2$ statistic ($X^2 \sim \chi_{\text{df}}$ under $H_0$)
  - in previous example: $p \approx 0$ :-(





## How reliable are the fitted models?

Three potential issues:

1. Model assumptions $\neq$ population
   (e.g. distribution does not follow a Zipf-Mandelbrot law)
   ☞ model cannot be adequate, regardless of parameter settings
2. Parameter estimation unsuccessful
   (i.e. suboptimal goodness-of-fit to training data)
   ☞ optimization algorithm trapped in local minimum
   ☞ can result in highly inaccurate model
3. Uncertainty due to sampling variation
   (i.e. observed training data differ from population distribution)
   ☞ model fitted to training data, may not reflect true population
   ☞ another training sample would have led to different parameters

# Bootstrapping

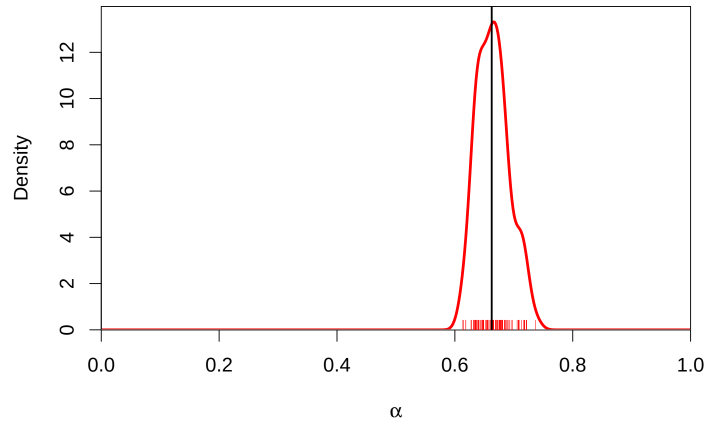
- An empirical approach to sampling variation:
  - take many random samples from the same population
  - estimate LNRE model from each sample
  - analyse distribution of model parameters, goodness-of-fit, etc. (mean, median, s.d., boxplot, histogram, ...)
  - problem: how to obtain the additional samples?
- Bootstrapping (Efron 1979)
  - resample from observed data *without replacement*
  - this approach is not suitable for type-token distributions (resamples underestimate vocabulary size $V$!)
- Parametric bootstrapping
  - use fitted model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known





# Bootstrapping

parametric bootstrapping with 100 replicates

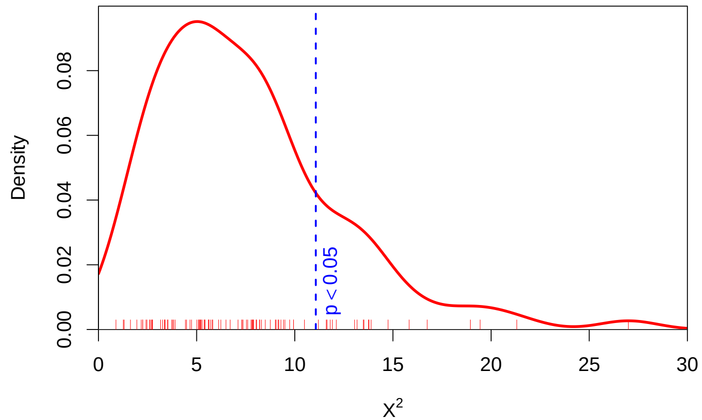







# Bootstrapping

parametric bootstrapping with 100 replicates

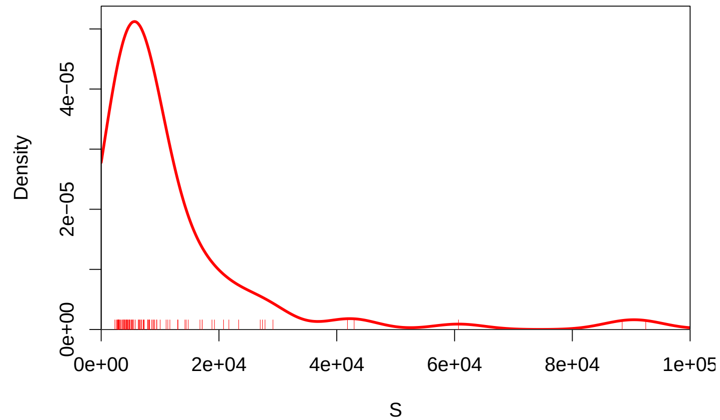







# Bootstrapping

parametric bootstrapping with 100 replicates



## Summary

LNRE modelling in a nutshell:

1. compile **observed** frequency spectrum (and vocabulary growth curves) for a given corpus or data set
2. estimate parameters of **LNRE model** by matching observed and expected frequency spectrum
3. evaluate **goodness-of-fit** on spectrum (Baayen 2001) or by testing extrapolation accuracy (Baroni and Evert 2007)
   - in principle, you should only go on if model gives a plausible explanation of the observed data!
4. use LNRE model to compute **expected** frequency spectrum for arbitrary sample sizes
   ➜ **extrapolation** of vocabulary growth curve
   - or use population model directly as Bayesian prior etc.





## zipfR

Evert and Baroni (2007)

- `http://zipfR.R-Forge.R-Project.org/`
- Conveniently available from CRAN repository
  - see Unit 1 for general package installation guides





## Loading

```
> library(zipfR)

> ?zipfR

> data(package="zipfR")

# package overview in HTML help leads to zipfR tutorial
> help.start()
```





## Importing data

```
> data(ItaRi.spc)        # not necessary in recent package versions
> data(ItaRi.emp.vgc)

# load your own data sets (see ?read.spc etc. for file format)
> my.spc <- read.spc("my.spc.txt")
> my.vgc <- read.vgc("my.vgc.txt")

> my.tfl <- read.tfl("my.tfl.txt")
> my.spc <- tfl2spc(my.tfl)  # compute spectrum from frequency list
```

## Looking at spectra

```
> summary(ItaRi.spc)
> ItaRi.spc

> N(ItaRi.spc)
> V(ItaRi.spc)
> Vm(ItaRi.spc, 1)
> Vm(ItaRi.spc, 1:5)

# Baayen's P = estimate for slope of VGC
> Vm(ItaRi.spc, 1) / N(ItaRi.spc)

> plot(ItaRi.spc)
> plot(ItaRi.spc, log="x")
```





## Looking at VGCs

```
> summary(ItaRi.emp.vgc)
> ItaRi.emp.vgc

> N(ItaRi.emp.vgc)

> plot(ItaRi.emp.vgc, add.m=1)
```





## Smoothing VGCs with binomial interpolation

(for details, see Baayen 2001, Sec. 2.6.1)

```
# interpolated VGC
> ItaRi.bin.vgc <-
  vgc.interp(ItaRi.spc, N(ItaRi.emp.vgc), m.max=1)

> summary(ItaRi.bin.vgc)

# comparison
> plot(ItaRi.emp.vgc, ItaRi.bin.vgc,
       legend=c("observed", "interpolated"))
```





## *ultra-*

- Load the spectrum and empirical VGC of the less common prefix *ultra-*
- Compute binomially interpolated VGC for *ultra-*
- Plot the binomially interpolated *ri-* and *ultra-* VGCs together

## Estimating LNRE models

```
# fit a fZM model
# (you can also try ZM and GIGP, and compare them with fZM)

> ItaUltra.fzm <- lnre("fzm", ItaUltra.spc)

> summary(ItaUltra.fzm)
```





## Observed/expected spectra at estimation size

```
# expected spectrum
> ItaUltra.fzm.spc <-
  lnre.spc(ItaUltra.fzm, N(ItaUltra.fzm))

# compare
> plot(ItaUltra.spc, ItaUltra.fzm.spc,
       legend=c("observed", "fzm"))

# plot first 10 elements only
> plot(ItaUltra.spc, ItaUltra.fzm.spc,
       legend=c("observed", "fzm"), m.max=10)
```





## Compare growth of two categories

```
# extrapolation of ultra- VGC to sample size of ri- data
> ItaUltra.ext.vgc <-
  lnre.vgc(ItaUltra.fzm, N(ItaRi.emp.vgc))

# compare
> plot(ItaUltra.ext.vgc, ItaRi.bin.vgc,
       N0=N(ItaUltra.fzm), legend=c("ultra-", "ri-"))

# zooming in
> plot(ItaUltra.ext.vgc, ItaRi.bin.vgc,
       N0=N(ItaUltra.fzm), legend=c("ultra-", "ri-"),
       xlim=c(0, 100e3))
```





## Model validation by parametric bootstrapping

```
# define function to extract relevant information from fitted model
> extract.stats <- function (m)
    data.frame(alpha=m$param$alpha, A=m$param$A,
               B=m$param$B, S=m$S, X2=m$gof$X2)

# run bootstrapping procedure (default = 100 replicates)
> runs <- lnre.bootstrap(ItaUltra.fzm, N(ItaUltra.fzm),
                         lnre, extract.stats, type="fzm")

# combine results into single data frame
> runs <- do.call(rbind, runs)

# NB: don't try this with large samples (N > 1M tokens)
```

## Model validation by parametric bootstrapping

```
# distribution of estimated model parameters
> hist(runs$alpha, freq=FALSE, xlim=c(0, 1))
> lines(density(runs$alpha), lwd=2, col="red")
> abline(v=ItaUltra.fzm$param$alpha, lwd=2, col="blue")

# try the other parameters for yourself!

# distribution of goodness-of-fit values
> hist(runs$X2, freq=FALSE)
> lines(density(runs$X2), lwd=2, col="red")

# estimated population vocabulary size
> hist(runs$S)  # what is wrong here?
```





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