Statistics for Linguists with R – a SIGIL course

Unit 2: Corpus Frequency Data & Statistical Inference

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http://SIGIL.R-Forge.R-Project.org/

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Frequency estimates & comparison

- How often is *kick the bucket* really used?

- What are the characteristics of "translationese"?

- Do Americans use more split infinitives than Britons? What about British teenagers?

- What are the typical collocates of *cat*?

- Can the next word in a sentence be predicted?

- Do native speakers prefer constructions that are grammatical according to some linguistic theory?
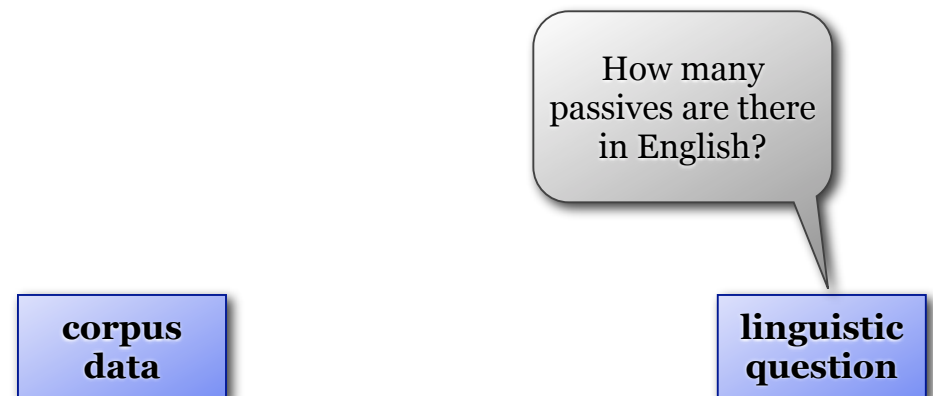
➡ evidence from frequency comparisons / estimates
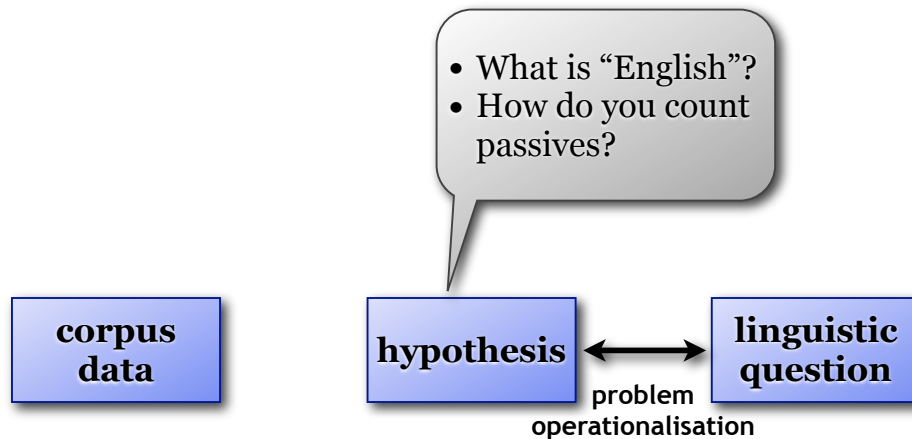
A simple toy problem

How many passives are there in English?

- American English style guide claims that
  - "*In an average English text, no more than 15% of the sentences are in passive voice. So use the passive sparingly, prefer sentences in active voice.*"
  - http://www.ego4u.com/en/business-english/grammar/passive actually states that only 10% of English sentences are passives (as of January 2009)!

- We have doubts and want to verify this claim

From research question to statistical analysis

How many passives are there in English?

corpus data

linguistic question

From research question to statistical analysis

- What is "English"?
- How do you count passives?

corpus data

hypothesis ↔ linguistic question

problem operationalisation

What is English?

◆ Sensible definition: group of speakers

- e.g. American English as language spoken by native speakers raised and living in the U.S.
- may be restricted to certain communicative situation

◆ Also applies to definition of sublanguage

- dialect (Bostonian, Cockney), social group (teenagers), genre (advertising), domain (statistics), …

◆ Here: professional writing by native speakers of AmE (⇨ target audience of style guide)

How do you count passives?

◆ Types vs. tokens

- **type** count: How many *different* passives are there?
- **token** count: How many *instances* are there?

◆ How many passive tokens are there in English?

- infinitely many, of course!

∞

◆ **Absolute frequency** is not meaningful here

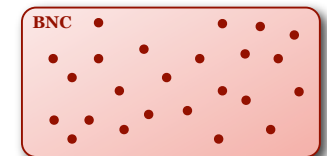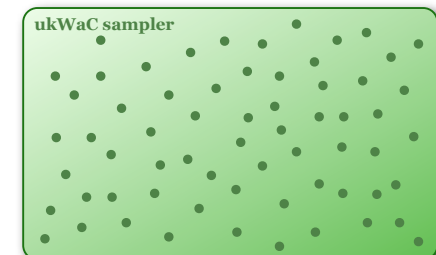# Against "absolute" frequency

◆ Are there **20,000** passives?   Brown

- Brown (1M words)

BNC

◆ Or **1 million**?

- BNC (90M words)

ukWaC sampler

◆ Or **5.1 million**?

- ukWaC sampler (450M words)

How do you count passives?

- Only **relative frequency** can be meaningful!
- What is the relative frequency of passives?
    - … **20,300** per **million words**?
    - … **390** per **thousand sentences**?
    - … **28** per **hour** of recorded speech?
    - … **4,000** per **book**?
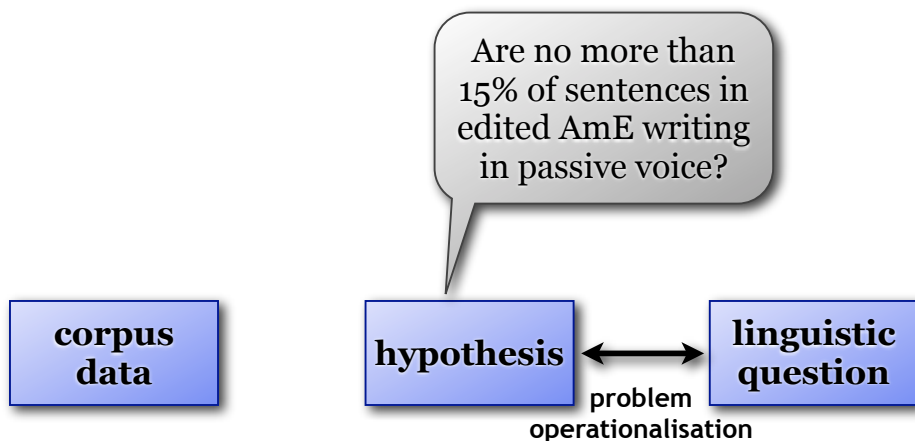
- What is a sensible unit of measurement?

How do you count passives?

- How many passives could there be at most?
    - every VP can be in active or passive voice
    - frequency of passives only has a meaningful interpretation by comparison with frequency of potential passives

- What proportion of VPs are in passive voice?
    - easier: proportion of sentences that contain a passive
    - in general, proportion wrt. some **unit of measurement**

- **Relative frequency** = **proportion** π

From research question to statistical analysis

Are no more than 15% of sentences in edited AmE writing in passive voice?

corpus data

hypothesis ←→ **linguistic question**

problem operationalisation

From research question to statistical analysis

?

random sample ←→ **population**   **Statistics**

corpus data

hypothesis ←→ **linguistic question**

problem operationalisation
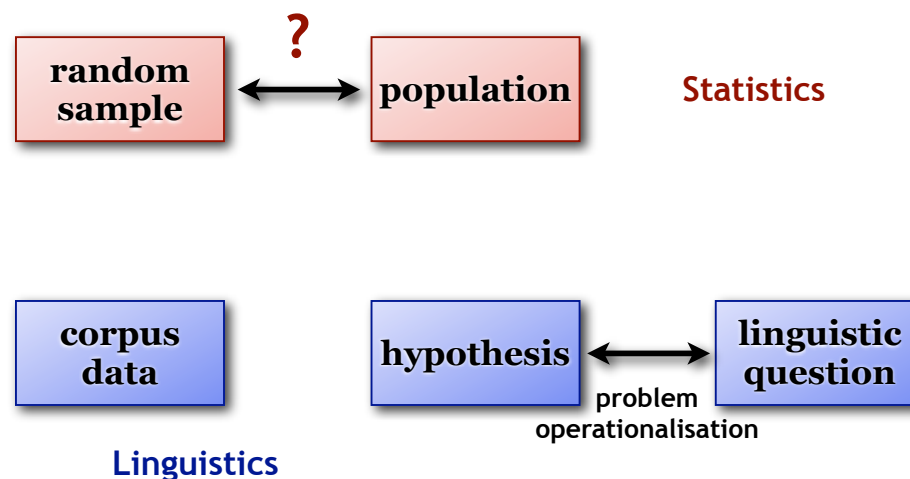
Linguistics

How do you count tokens in an infinite language?

- ◆ Statistics deals with similar problems:
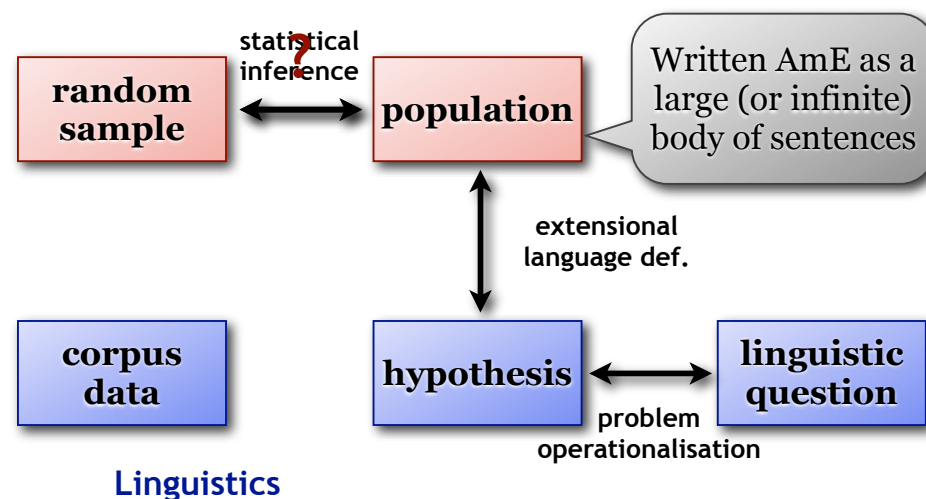 - goal: determine properties of **large population** (human populace, objects produced in factory, …)
  - method: take (completely) **random sample** of objects, then extrapolate from sample to population
  - this works only because of **random** sampling!
- ◆ Many statistical methods are readily available
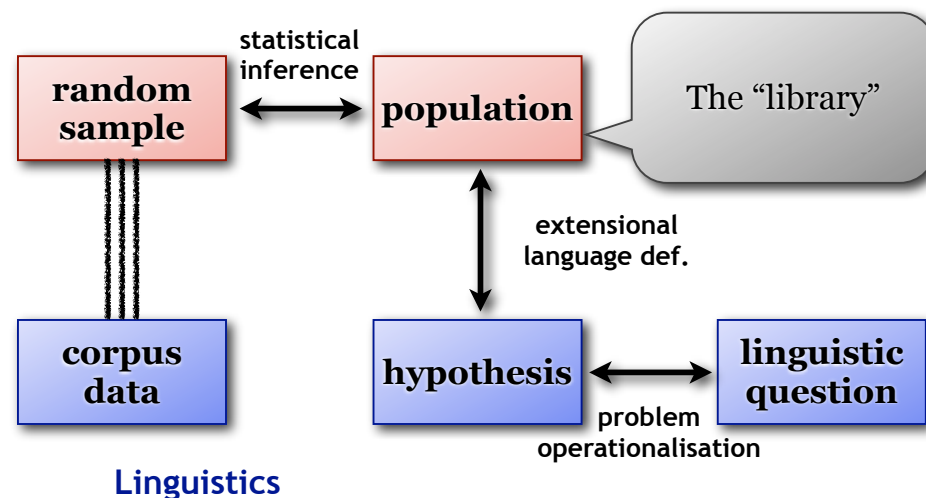
From research question to statistical analysis

random sample — statistical inference **?** → **population** — Written AmE as a large (or infinite) body of sentences

population — extensional language def. — **hypothesis**

corpus data

hypothesis ↔ **linguistic question** — problem operationalisation

Linguistics

The library metaphor

- ◆ Extensional definition of a language: "All utterances made by speakers of the language under appropriate conditions, plus all utterances they *could* have made"

- ◆ Imagine a huge library with all the books written in a language, as well as all the hypothetical books that have never been written
  - → **library metaphor** (Evert 2006)

From research question to statistical analysis

random sample — statistical inference → **population** — The "library"

population — extensional language def. — **hypothesis**

corpus data

hypothesis ↔ **linguistic question** — problem operationalisation

Linguistics

A random sample of a language

◆ Apply statistical procedure to linguistic problem
⇨ need random sample of objects from population

◆ Quiz: *What are the objects in our population?*

• words? sentences? texts? …

◆ Objects = whatever **unit of measurement** the proportions of interest are based on

• we need to take a random sample of such units

The library metaphor

◆ Random sampling in the library metaphor

• in order to take a sample of sentences:

• walk to a random shelf …
… pick a random book …
… open a random page …
… and choose a random sentence from the page

• this gives us 1 item for our sample

• repeat *n* times for **sample size *n***

Types, tokens and proportions

◆ Proportions and relative sample frequencies are defined formally in terms of types & tokens

◆ Relative frequency of type v in sample $\{t_1, …, t_n\}$ = proportion of tokens t_i that belong to this type

$$p = \frac{f(v)}{n}$$

frequency of type

sample size

◆ Compare relative sample frequency p against (hypothesised) population proportion π

Types, tokens and proportions

◆ Example: word frequencies

• word type = dictionary entry (distinct word)

• word token = instance of a word in library texts

◆ Example: passive VPs

• relevant VP types = active or passive (→ abstraction)

• VP token = instance of VP in library texts

◆ Example: verb sucategorisation

• relevant types = itr., tr., ditr., PP-comp., X-comp, …

• verb token = occurrence of selected verb in text

Inference from a sample

- ◆ Principle of inferential statistics
  - if a sample is picked at random, proportions should be roughly the same in sample and population

- ◆ Take a sample of 100 sentences
  - observe 19 passives → p = 19% = .19
  - style guide → population proportion π = 15%
  - $p > \pi$ → reject claim of style guide?

- ◆ Take another sample, just to be sure
  - observe 13 passives → p = 13% = .13
 - $p < \pi$ → claim of style guide confirmed?

Sampling variation

- ◆ Random choice of sample ensures proportions are the same on average in sample & population

- ◆ But it also means that for every sample we will get a different value because of chance effects → **sampling variation**
  - problem: erroneous rejection of style guide's claim results in publication of a false result

- ◆ The main purpose of statistical methods is to estimate & correct for sampling variation
  - that's all there is to inferential statistics, really

Reminder: The role of statistics
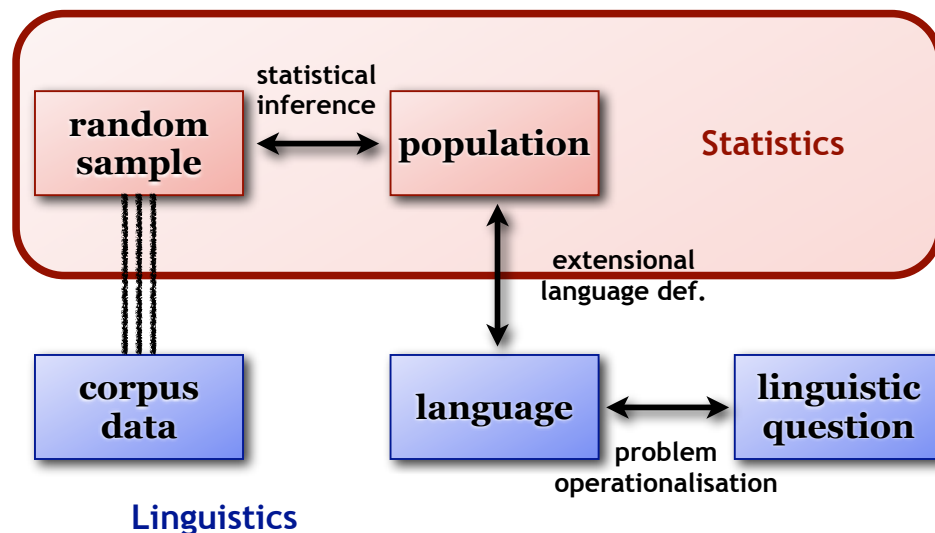
The null hypothesis

- ◆ Our "goal" is to refute the style guide's claim, which we call the **null hypothesis** H_0

$$H_0 : \pi = .15$$

  - we also refer to π_0 = .15 as the **null proportion**

- ◆ Erroneous rejection of H_0 is problematic
  - leads to embarrassing publication of false result
  - known as a **type I error** in statistics

- ◆ Need to control risk of a type I error

Estimating sampling variation

- ◆ Assume that style guide's claim H_0 is correct
  - i.e. rejection of H_0 is always a type I error
- ◆ Many corpus linguists set out to test H_0
  - each one draws a random sample of size $n = 100$
  - how many of the samples have the expected $k = 15$ passives, how many have $k = 19$, etc.?
  - if we are willing to reject H_0 for $k = 19$ passives in a sample, all corpus linguists with such a sample will publish a false result
  - risk of type I error = percentage of such cases

# Estimating sampling variation

- ◆ We don't need an infinite number of monkeys (or corpus linguists) to answer these questions
  - randomly picking sentences from our metaphorical library is like drawing balls from an infinite urn
  - red ball = passive sent. / white ball = active sent.
  - H_0: assume proportion of red balls in urn is 15%
- ◆ This leads to a **binomial distribution**

$$\Pr(k) = \binom{n}{k} (\pi_0)^k (1 - \pi_0)^{n-k}$$

percentage of samples = **probability**

Binomial sampling distribution

→ risk of false rejection = **p-value** = 26.2%

tail probability = 9.9%

tail probability = 16.3%

percentage of samples with X=k

value k of observed frequency X
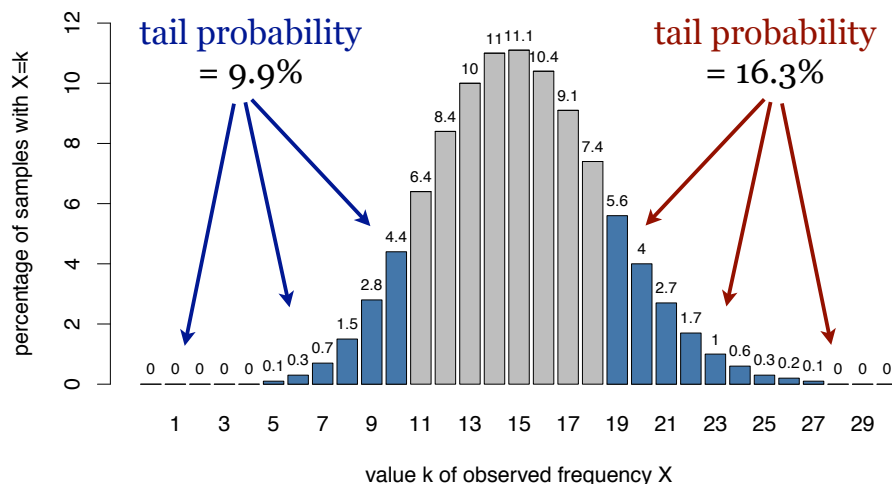
Statistical hypothesis testing

- ◆ Statistical **hypothesis tests**
  - define a **rejection criterion** for refuting H_0
  - control the risk of false rejection (**type I error**) to a "socially acceptable level" (**significance level** α)
  - **p-value** = risk of type I error given observation, interpreted as amount of evidence against H_0
- ◆ Two-sided vs. one-sided tests
  - in general, two-sided tests are recommended (safer)
  - one-sided test is plausible in our example

Hypothesis tests in practice

One sample: frequency estimate (confidence interval)

back to top

Frequency count	Sample size
19	100

(Clear fields)

95% ⬍ confidence interval
in automatic ⬍ format

☐ extrapolate to _____ items   (Calculate)

Two samples: frequency comparison

back to top

	Frequency count	Sample size
Sample 1	19	100
Sample 2	25	200

- http://sigil.collocations.de/wizard.html
- http://vassarstats.net/
- SPSS, SAS, Excel, ...
- We want to do it in Ⓡ, of course

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Binomial hypothesis test in R

◆ Relevant R function: `binom.test()`

◆ We need to specify

- **observed data**: **19** passives out of **100** sentences
- **null hypothesis**: H_0: $\pi = \mathbf{15\%}$

◆ Using the `binom.test()` function:

```
> binom.test(19, 100, p=.15) # two-sided
```

```
> binom.test(19, 100, p=.15, # one-sided
    alternative="greater")
```

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Binomial hypothesis test in R

```
> binom.test(19, 100, p=.15)

    Exact binomial test

data:  19 and 100

number of successes = 19, number of
trials = 100, p-value = 0.2623

alternative hypothesis: true probability of
success is not equal to 0.15

95 percent confidence interval:
 0.1184432 0.2806980

sample estimates:
probability of success
               0.19
```

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Rejection criterion & significance level

```
> binom.test(19, 100, p=.15)$p.value
[1] 0.2622728
```
$p > .05$ n.s.

```
> binom.test(23, 100, p=.15)$p.value
[1] 0.03430725
```
$p < .05 = \alpha$ *

```
> binom.test(25, 100, p=.15)$p.value
[1] 0.007633061
```
$p < .01 = \alpha$ **

```
> binom.test(29, 100, p=.15)$p.value
[1] 0.0003529264
```
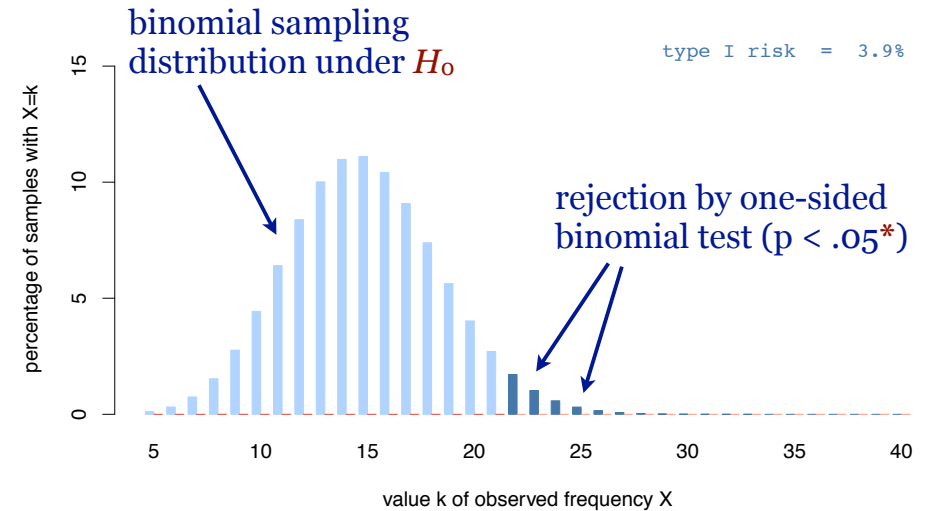$p < .001 = \alpha$ ***

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Type II errors

◆ Rejection criterion controls risk of type I error

- only for situation in which H_o is true

◆ Type II error = failure to reject incorrect H_o

- for situation in which H_o is not true
  → rejection correct, non-rejection is an error

◆ What is the risk of a type II error?

- depends on true population proportion π

- intuitively, risk of type II error will be low if the difference $\delta = \pi - \pi_o$ (the **effect size**) is large enough

Type II errors

binomial sampling distribution under H_o

`type I risk = 3.9%`

rejection by one-sided binomial test (p < .05*)

percentage of samples with X=k

value k of observed frequency X

Type II errors

type II risk for k ≤ 21 = 21.1%

binomial sampling distribution under H_o

`type I risk  =  3.9%`
`type II risk = 21.1%`

sampling distribution for hypothetical π=.25

percentage of samples with X=k

value k of observed frequency X
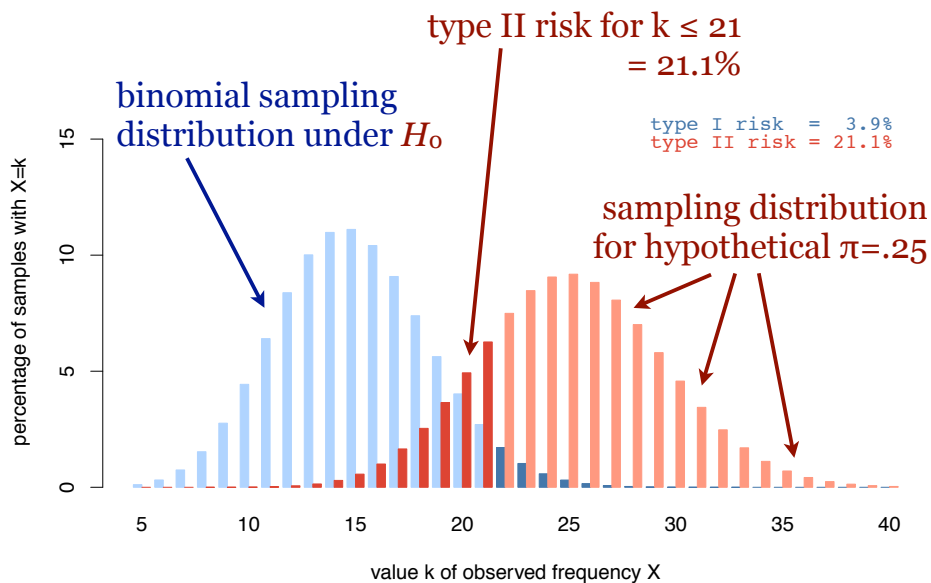
Type II errors
& effect size

effect size δ

$\delta = 0.15$

`type I risk  =  3.9%`
`type II risk =  2.9%`

percentage of samples with X=k

value k of observed frequency X
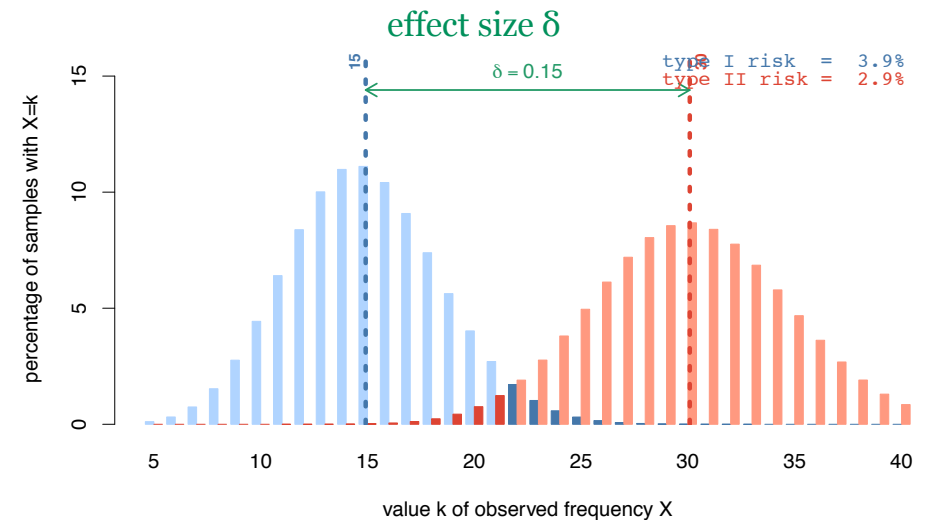
Type II errors
& sample size

$n = 1000$

type I risk = 4.4%
type II risk = 0.0%

$\delta = 0.10$

percentage of samples with X=k

value k of observed frequency X
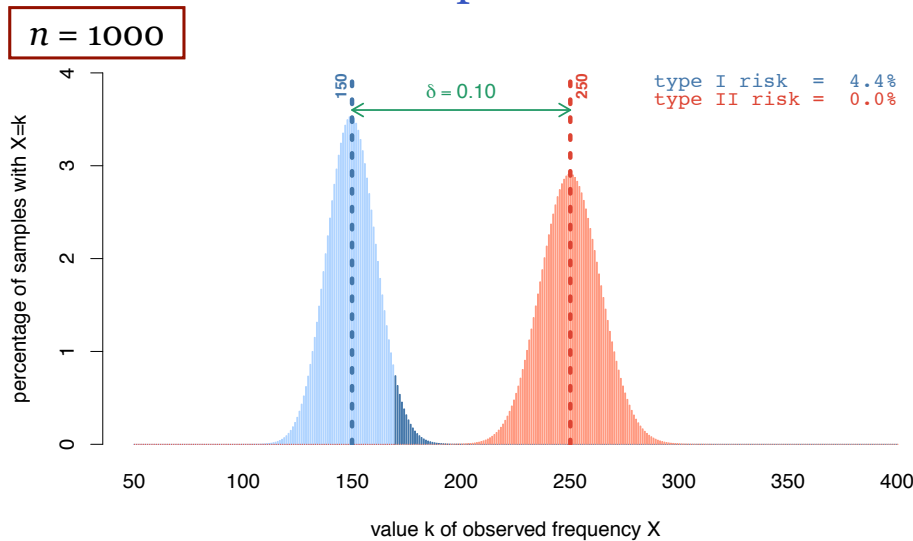
Power

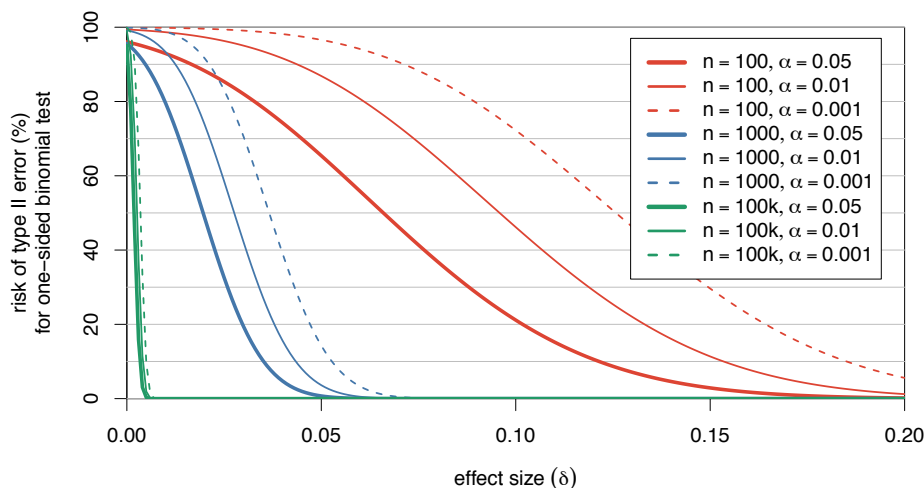◆ Type II error = failure to reject incorrect H_o

- the larger the difference between H_o and the true population proportion, the more likely it is that H_o can be rejected based on a given sample

- a **powerful** test has a low **type II error**

- power analysis explores the relationship between effect size and risk of type II error

◆ Key insight: larger sample = more power

- relative sampling variation becomes smaller

- power also depends on significance level

Power analysis for binomial test

risk of type II error (%)
for one–sided binomial test

—	n = 100, α = 0.05
—	n = 100, α = 0.01
- -	n = 100, α = 0.001
—	n = 1000, α = 0.05
—	n = 1000, α = 0.01
- -	n = 1000, α = 0.001
—	n = 100k, α = 0.05
—	n = 100k, α = 0.01
- -	n = 100k, α = 0.001

effect size (δ)

Power analysis for binomial test

◆ Key factors determining the power of a test

- **sample size** → more evidence = greater power
- **significance level** → trade-off btw. type I / II errors

◆ Influence of hypothesis test procedure

- one-sided test more powerful than two-sided test
- parametric tests more powerful than non-parametric
- statisticians look for "uniformly most powerful" test

◆ Tests can become too powerful!

- reject H_0 for 15.1% passives with $n = 1,000,000$

Parametric vs. non-parametric

◆ People often talk about parametric and non-parametric tests without precise definition

◆ Parametric tests make stronger assumptions

  • not just normality assuming (= Gaussian distribution)

  • binomial test: strong random sampling assumption → might be considered a parametric test in this sense!

◆ Parametric tests are usually more powerful

  • strong assumptions allow less conservative estimate of sampling variation → less evidence needed against H_0

Trade-offs in statistics

◆ Inferential statistics is a trade-off between type I errors and type II errors

  • i.e. between **significance** and **power**

◆ Significance level

  • determines trade-off point

  • low significance level α → low power

◆ Conservative tests

  • put more weight on avoiding type I errors → weaker

  • most non-parametric methods are conservative
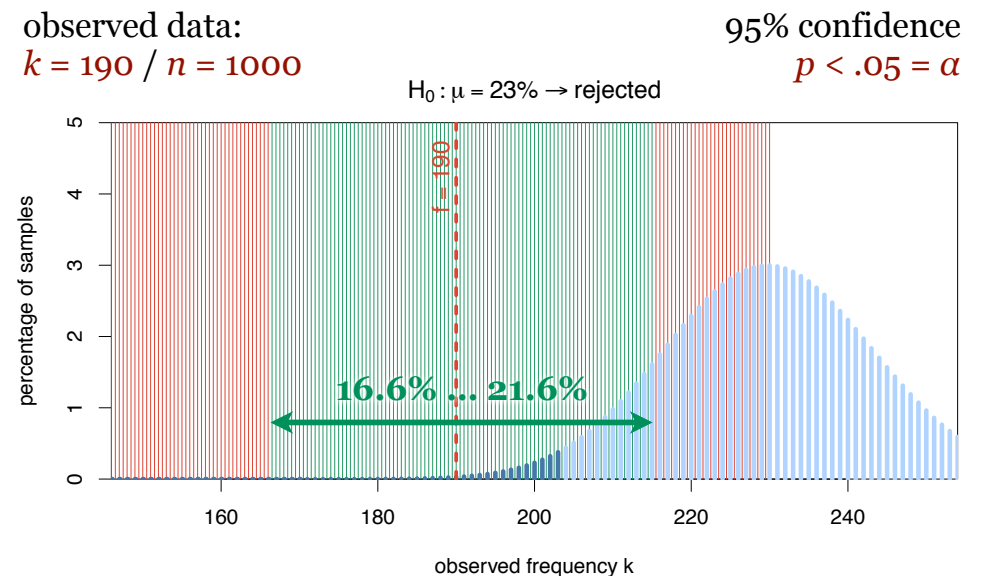
Confidence interval

◆ We now know how to test a null hypothesis H_0, rejecting it only if there is sufficient evidence

◆ But what if we do not have an obvious null hypothesis to start with?

  • this is typically the case in (computational) linguistics

◆ We can estimate the true population proportion from the sample data (relative frequency)

  • sampling variation → range of plausible values

  • such a **confidence interval** can be constructed by inverting hypothesis tests (e.g. binomial test)

Confidence interval

observed data:
$k = 190 / n = 1000$

95% confidence
$p < .05 = \alpha$

$H_0 : \mu = 23\% \rightarrow$ rejected

16.6% ... 21.6%

percentage of samples

observed frequency k

Confidence intervals

I'm cheating here a tiny little bit (not always an interval)

◆ Confidence interval = range of plausible values for true population proportion

  • H_0 rejected by test iff π_0 is outside confidence interval

◆ Size of confidence interval depends on power of the test (i.e. sample size and significance level)

	$n = 100$ $k = 19$	$n = 1{,}000$ $k = 190$	$n = 10{,}000$ $k = 1{,}900$
$\alpha = .05$	11.8%…28.1%	16.6%…21.6%	18.2%…19.8%
$\alpha = .01$	10.1%…31.0%	15.9%…22.4%	18.0%…20.0%
$\alpha = .001$	8.3%…34.5%	15.1%…23.4%	17.7%…20.3%

Confidence intervals in R

◆ Most hypothesis tests in R also compute a confidence interval (including `binom.test()`)

  • omit H_0 if only interested in confidence interval

◆ Significance level of underlying hypothesis test is controlled by `conf.level` parameter

  • expressed as confidence, e.g. `conf.level=.95` for significance level $\alpha = .05$, i.e. 95% confidence

◆ Can also compute one-sided confidence interval

  • controlled by `alternative` parameter

  • two-sided confidence intervals strongly recommended

Confidence intervals in R

```
> binom.test(190, 1000, conf.level=.99)

    Exact binomial test

data:  190 and 1000

number of successes = 190, number of
trials = 1000, p-value < 2.2e-16

alternative hypothesis: true probability of
success is not equal to 0.5

99 percent confidence interval:
 0.1590920 0.2239133

sample estimates:
probability of success
              0.19
```
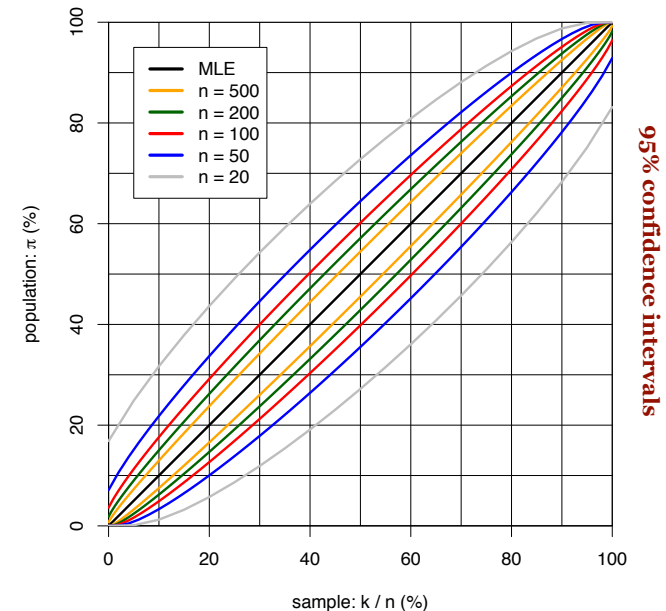
Choosing sample size

Choosing sample size

sample: k / n (%)

95% confidence intervals
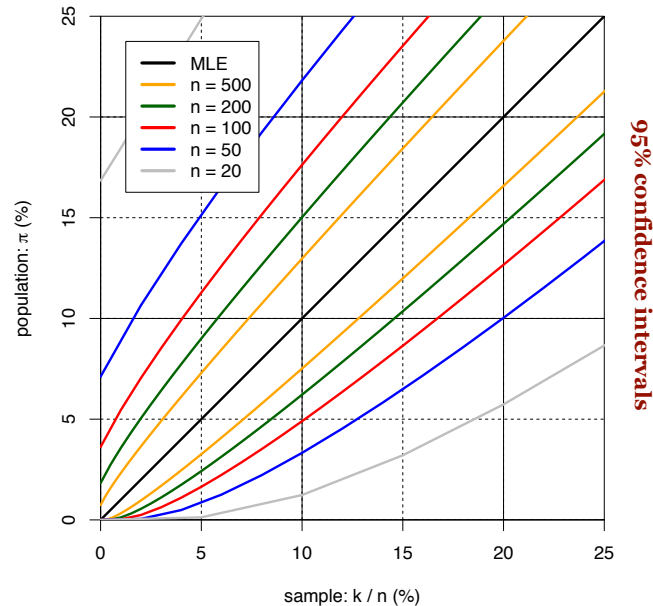
Using R to choose sample size

◆ Call `binom.test()` with hypothetical values

◆ Plots on previous slides also created with R

  • requires calculation of large number of hypothetical confidence intervals

  • `binom.test()` is both inconvenient and inefficient

◆ The `corpora` package has a vectorised function

```
> library(corpora)
> prop.cint(190, 1000, conf.level=.99)
> ?prop.cint  # "conf. intervals for proportions"
```

Frequency comparison

◆ Many linguistic research questions can be operationalised as a frequency comparison

  • Are split infinitives more frequent in AmE than BrE?

  • Are there more definite articles in texts written by Chinese learners of English than native speakers?

  • Does *meow* occur more often in the vicinity of *cat* than elsewhere in the text?

  • Do speakers prefer *I couldn't agree more* over alternative realisations such as *I agree completely*?

◆ Compare observed frequencies in two samples

Frequency comparison

◆ Null hypothesis for frequency comparison

$$H_0 : \pi_1 = \pi_2$$

  • no assumptions about the precise value $\pi_1 = \pi_2 = \pi$

◆ Observed data

  • target count k_i and sample size n_i for each sample i

  • e.g. $k_1 = 19$ / $n_1 = 100$ passives vs. $k_2 = 25$ / $n_2 = 200$

◆ Effect size: difference of proportions

  • effect size $\delta = \pi_1 - \pi_2$ (and thus H_0: $\delta = 0$)

Frequency comparison in R

- ◆ Frequency comparison test: `prop.test()`
  - observed data: counts k_i and sample sizes n_i
  - also computes confidence interval for effect size

- ◆ E.g. for 19 passives out of 100 / 25 out of 200
  - parameters `conf.level` and `alternative` can be used in the familiar way

  ```
  > prop.test(c(19,25), c(100,200))
  ```

Frequency comparison in R

```
> prop.test(c(19,25), c(100,200))

    2-sample test for equality of proportions with
continuity correction

data:  c(19, 25) out of c(100, 200)

X-squared = 1.7611, df = 1, p-value = 0.1845

alternative hypothesis: two.sided

95 percent confidence interval:
 -0.03201426  0.16201426

sample estimates:
prop 1 prop 2
 0.190  0.125
```

Contingency tables

	sample 1	sample 2
passive	k_1	k_2
active	$n_1{-}k_1$	$n_2{-}k_2$

19	25
81	175

- ◆ Data can also be given as a **contingency table**
  - e.g. $k_1 = 19$ / $n_1 = 100$ passives vs. $k_2 = 25$ / $n_2 = 200$
  - represents a cross-classification of $n = 300$ items
  - various tests for contingency tables can be applied
- ◆ Chi-squared X^2, likelihood ratio G^2, Fisher's test

Contingency tables

- ◆ Can easily carry out chi-squared (`chisq.test`) and Fisher's exact test (`fisher.test`) in R
  - likelihood ratio test not in R standard library
- ◆ Table for 19 / 100 vs. 25 / 200

  ```
  > ct <- cbind(c(19,81),
              c(25,175))
  > chisq.test(ct)
  > fisher.test(ct)
  ```

19	25
81	175

Contingency tables

- ◆ Tests are appropriate for different situations
  - Fisher: computationally expensive, small samples
  - X^2: for large samples and small balanced samples
  - G^2: suitable for highly skewed data
- ◆ Measures of effect size
  - **difference** $\delta = \pi_1 - \pi_2$
    → approx. confidence interval from proportions test
  - **relative risk** $r = \pi_1 / \pi_2$
  - **odds ratio** $\theta = \pi_1 (1 - \pi_2) / \pi_2 (1 - \pi_1)$
    → exact confidence interval from Fisher's test

Significance vs. relevance

- ◆ Much focus on significant p-value, but …
  - large differences may be non-significant if sample size is too small (e.g. $10/80 = 12.5\%$ vs. $20/80 = 25\%$)
  - increase sample size for more powerful/sensitive test
  - very large samples lead to highly significant p-values for minimal and irrelevant differences (e.g. 1M tokens with $150{,}000 = 15\%$ vs. $151{,}000 = 15.1\%$ occurrences)
- ◆ It is important to assess both **significance** and **relevance** (= effect size) of frequency data!
  - confidence intervals combine both aspects

A case study: passives

- ◆ As a case study, we will compare the frequency of passives in Brown (AmE) and LOB (BrE)
  - pooled data
  - separately for each genre category

- ◆ Data files provided in CSV format
  - **passives.brown.csv** & **passives.lob.csv**
  - `cat` = genre category, `passive` = number of passives, `n_w` = number of word, `n_s` = number of sentences, `name` = description of genre category

Preparing the data

```
> Brown <- read.csv("passives.brown.csv")
> LOB <- read.csv("passives.lob.csv")


> library(SIGIL)  # also included in SIGIL package
> Brown <- BrownPassives
> LOB <- LOBPassives
# now take a look at the two tables: what info do they provide?


# pooled data for entire corpus = column sums (col. 2 … 4)
> Brown.all <- colSums(Brown[, 2:4])
> LOB.all <- colSums(LOB[, 2:4])
```

Frequency tests for pooled data

```
# proportions test reports p-value is based on chi-squared test
# and approximate confidence interval for effect size δ
> prop.test(c(10123, 10934), c(49576, 49742))


> ct <- cbind(c(10123, 49576-10123),   # Brown
              c(10934, 49742-10934))   # LOB
> ct                 # contingency table for chi-squared / Fisher

> fisher.test(ct)  # exact confidence interval for odds ratio θ


# we could in principle do the same for all 15 genres …
```

Automation: user functions

```
# user function do.test() executes proportions test for samples
# k1/n1 and k2/n2, and summarizes relevant results in compact form
> do.test <- function (k1, n1, k2, n2) {

    # res contains results of proportions test (list = data structure)
    res <- prop.test(c(k1, k2), c(n1, n2))

    # data frames are a nice way to display summary tables
    fmt <- data.frame(p=res$p.value,
      lower=res$conf.int[1], upper=res$conf.int[2])

    fmt  # return value of function = last expression
}
> do.test(10123, 49576, 10934, 49742)  # pooled data
> do.test(146, 975, 134, 947)          # humour genre
```

A nicer user function

```
# nicer version of user function with genre category labels
> do.test <- function (k1, n1, k2, n2, cat="") {
    res <- prop.test(c(k1, k2), c(n1, n2))
    data.frame(
      p=res$p.value,
      lower=100*res$conf.int[1],   # scaled to % points
      upper=100*res$conf.int[2],
      row.names=cat   # add genre as row label
    ) # return data frame directly without local variable fmt
}

# extract relevant information directly from data frames
> do.test(Brown$passive[15], Brown$n_s[15],
          LOB$passive[15], LOB$n_s[15],
          cat=Brown$name[15])
```

Ad-hoc functions & loops

```
# ad-hoc convenience function to reduce typing/editing
# (works only if global Brown/LOB variables are set correctly!)
quick.test <- function (i) {
  do.test(k1=Brown$passive[i], n1=Brown$n_s[i],
          k2=LOB$passive[i], n2=LOB$n_s[i],
          cat=Brown$name[i])
}

quick.test(15)    # easy to repeat for different genres now
quick.test(9)


# loop over all 15 categories (more general: 1:nrow(Brown))
for (i in 1:15) {
  print( quick.test(i) )
}
```

R wizardry: working with lists

our code only works if rows of Brown/LOB are in the same order!
```
> all(Brown$cat == LOB$cat)
```

it would be nice to collect all these results in a single overview table
for this, we need a little bit of R wizardry …

apply function `quick.test()` to each number 1, …, 15
```
res.list <- lapply(1:15, quick.test)
```

pass `res.list` as individual arguments to `rbind()`
(think of this as an idiom you just have to remember …)
```
res <- do.call(rbind, res.list)
```

```
res                # data frame with one row for each genre
```

```
round(res, 3)    # rounded values are easier to read
```

It's your turn now …

◆ Questions:

- Which differences are significant?

- Are the effect sizes linguistically relevant?

◆ A different approach:

- You can construct a list of contingency tables with the `cont.table()` function from the `corpora` package

- Apply `fisher.test()` or `chisq.test()` directly to each table in the list using the `lapply()` function

- Try to extract relevant information with `sapply()`