Statistics for Linguists with R – a SIGIL course

Unit 2: Corpus Frequency Data & Statistical Inference

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Frequency estimates & comparison

- How often is *kick the bucket* really used?
- What are the characteristics of "translationese"?
- Do Americans use more split infinitives than Britons? What about British teenagers?
- What are the typical collocates of *cat*?
- Can the next word in a sentence be predicted?
- Do native speakers prefer constructions that are grammatical according to some linguistic theory?

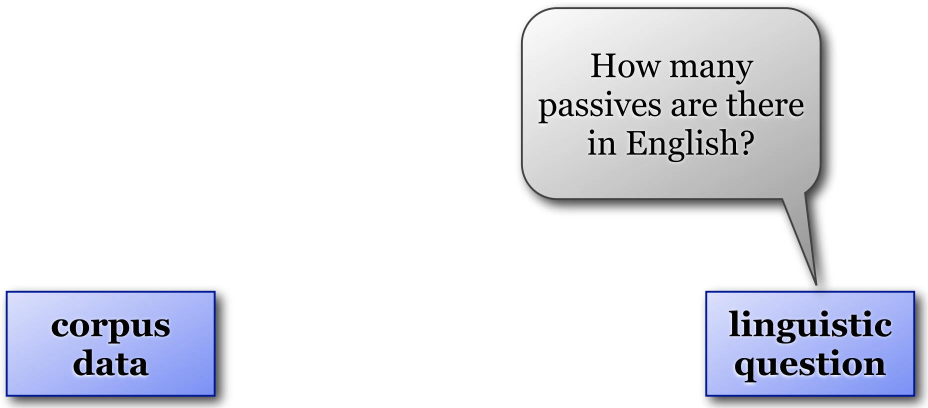
➡ evidence from frequency comparisons / estimates

A simple toy problem

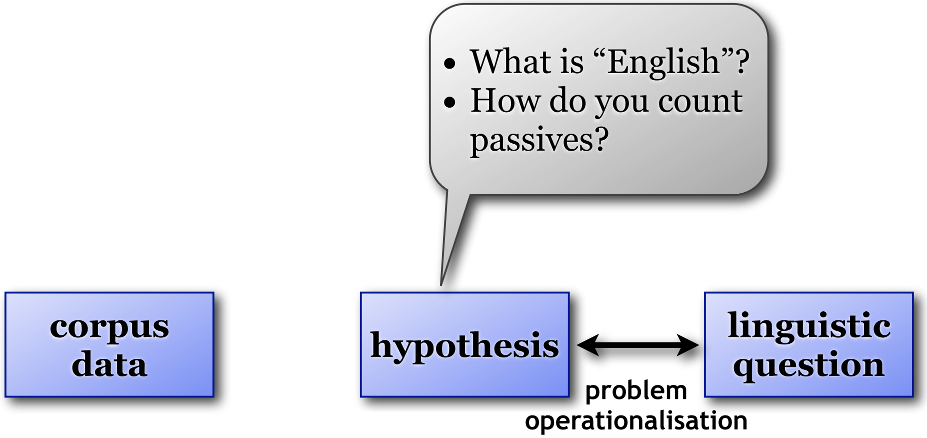
How many passives are there in English?

- American English style guide claims that
  - "*In an average English text, no more than 15% of the sentences are in passive voice. So use the passive sparingly, prefer sentences in active voice.*"
  - http://www.ego4u.com/en/business-english/grammar/passive actually states that only 10% of English sentences are passives (as of January 2009)!
- We have doubts and want to verify this claim

From research question to statistical analysis



From research question to statistical analysis



What is English?

- Sensible definition: group of speakers
  - e.g. American English as language spoken by native speakers raised and living in the U.S.
  - may be restricted to certain communicative situation
- Also applies to definition of sublanguage
  - dialect (Bostonian, Cockney), social group (teenagers), genre (advertising), domain (statistics), …
- Here: professional writing by native speakers of AmE (➪ target audience of style guide)

How do you count passives?

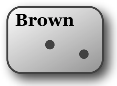
- Types vs. tokens
  - **type** count: How many *different* passives are there?
  - **token** count: How many *instances* are there?
- How many passive tokens are there in English?
  - infinitely many, of course!

∞

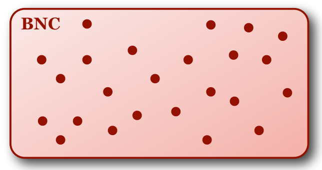
- **Absolute frequency** is not meaningful here

Against "absolute" frequency

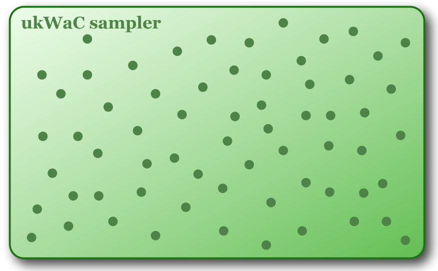
- Are there 20,000 passives?



  - Brown (1M words)



- Or 1 million?
  - BNC (90M words)



- Or 5.1 million?
  - ukWaC sampler (450M words)

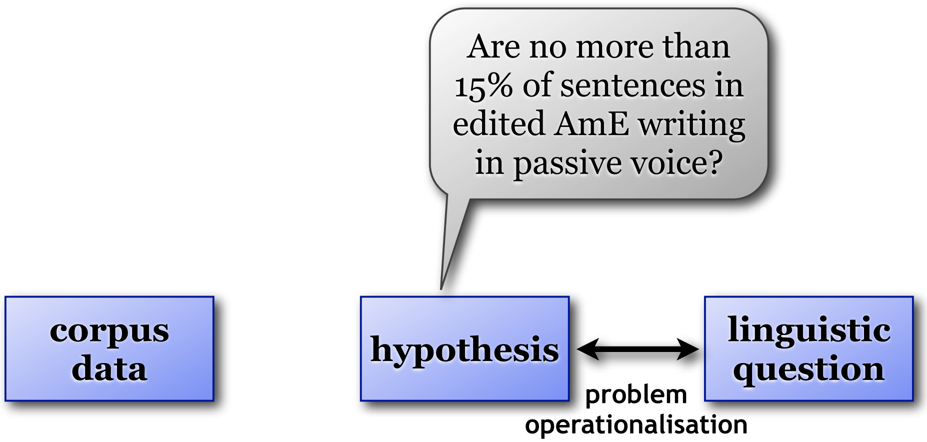
How do you count passives?

- Only **relative frequency** can be meaningful!
- What is the relative frequency of passives?
  - … **20,300** per **million words**?
  - … **390** per **thousand sentences**?
  - … **28** per **hour** of recorded speech?
  - … **4,000** per **book**?
- What is a sensible unit of measurement?

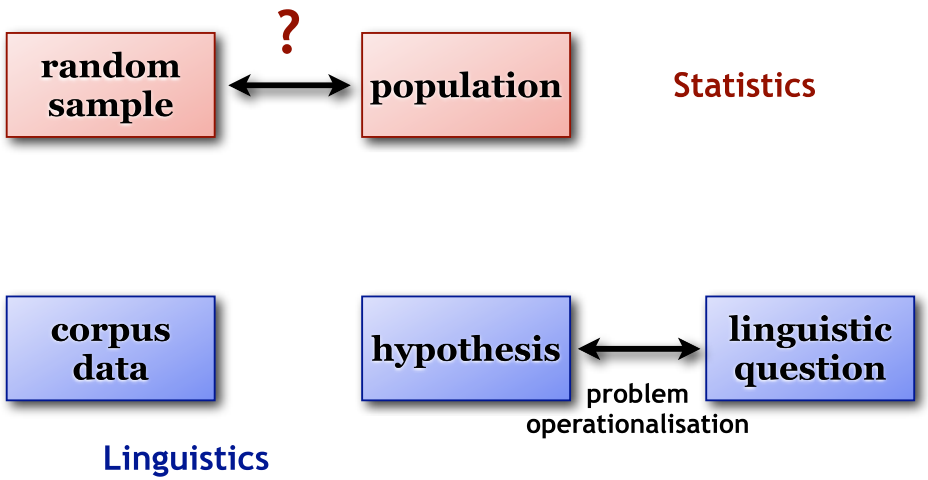
How do you count passives?

- How many passives could there be at most?
  - every VP can be in active or passive voice
  - frequency of passives only has a meaningful interpretation by comparison with frequency of potential passives
- What proportion of VPs are in passive voice?
  - easier: proportion of sentences that contain a passive
  - in general, proportion wrt. some **unit of measurement**
- **Relative frequency** = **proportion** π

From research question to statistical analysis



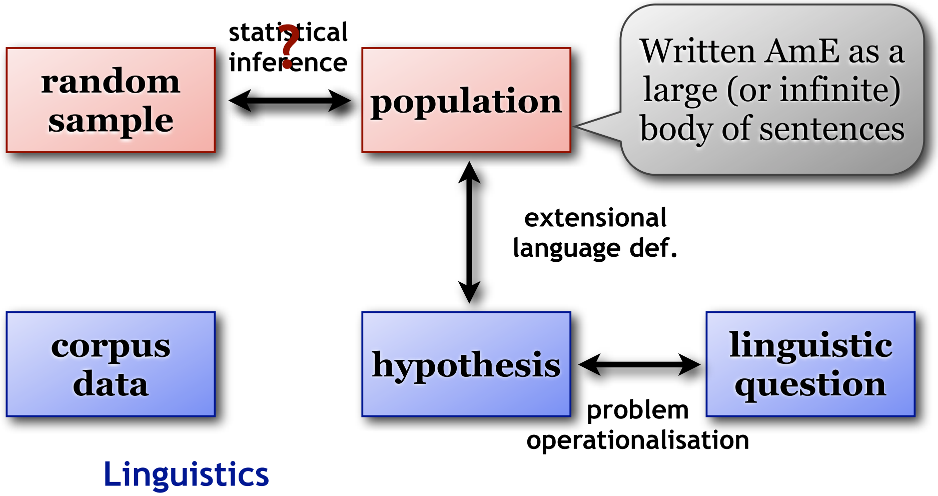
From research question to statistical analysis



How do you count tokens in an infinite language?

- Statistics deals with similar problems:
  - goal: determine properties of **large population** (human populace, objects produced in factory, …)
  - method: take (completely) **random sample** of objects, then extrapolate from sample to population
  - this works only because of **random** sampling!
- Many statistical methods are readily available

From research question to statistical analysis

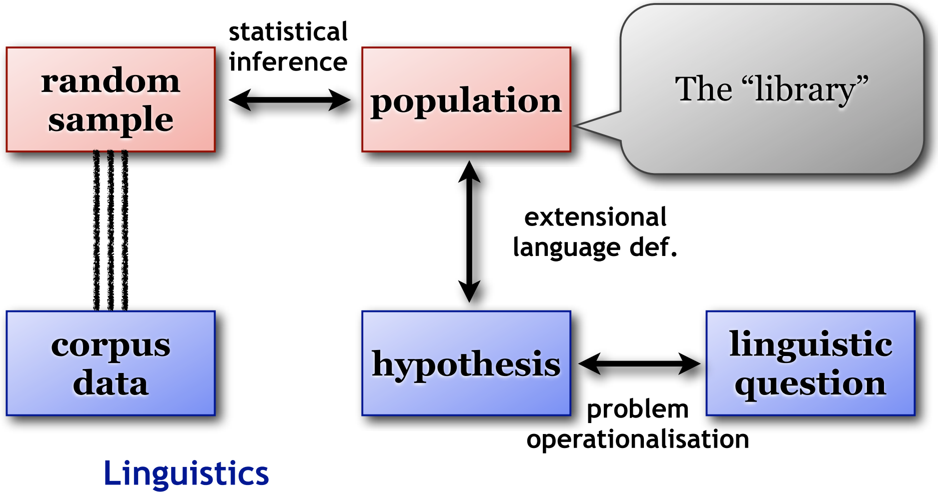


The library metaphor

- Extensional definition of a language:
  "All utterances made by speakers of the language under appropriate conditions, plus all utterances they *could* have made"
- Imagine a huge library with all the books written in a language, as well as all the hypothetical books that have never been written

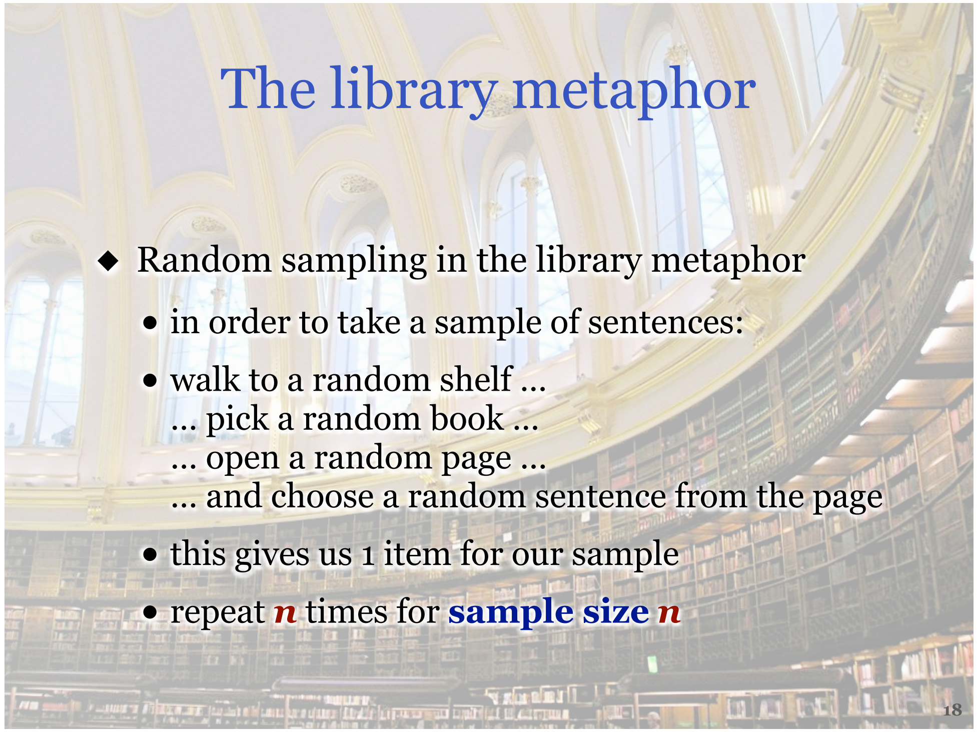
→ **library metaphor** (Evert 2006)

From research question to statistical analysis



A random sample of a language

- Apply statistical procedure to linguistic problem
  ➪ need random sample of objects from population
- Quiz: *What are the objects in our population?*
  - words? sentences? texts? …
- Objects = whatever **unit of measurement** the proportions of interest are based on
  - we need to take a random sample of such units



The library metaphor

- Random sampling in the library metaphor
  - in order to take a sample of sentences:
  - walk to a random shelf …
    … pick a random book …
    … open a random page …
    … and choose a random sentence from the page
  - this gives us 1 item for our sample
  - repeat ***n*** times for **sample size *n***

Types, tokens and proportions

- Proportions and relative sample frequencies are defined formally in terms of types & tokens
- Relative frequency of type v in sample $\{t_1, \ldots, t_n\}$ = proportion of tokens t_i that belong to this type

$$p = \frac{f(v)}{n}$$

frequency of type

sample size

- Compare relative sample frequency $\boldsymbol{p}$ against (hypothesised) population proportion $\boldsymbol{\pi}$

Types, tokens and proportions

- Example: word frequencies
  - word type = dictionary entry (distinct word)
  - word token = instance of a word in library texts
- Example: passive VPs
  - relevant VP types = active or passive (➞ abstraction)
  - VP token = instance of VP in library texts
- Example: verb sucategorisation
  - relevant types = itr., tr., ditr., PP-comp., X-comp, …
  - verb token = occurrence of selected verb in text

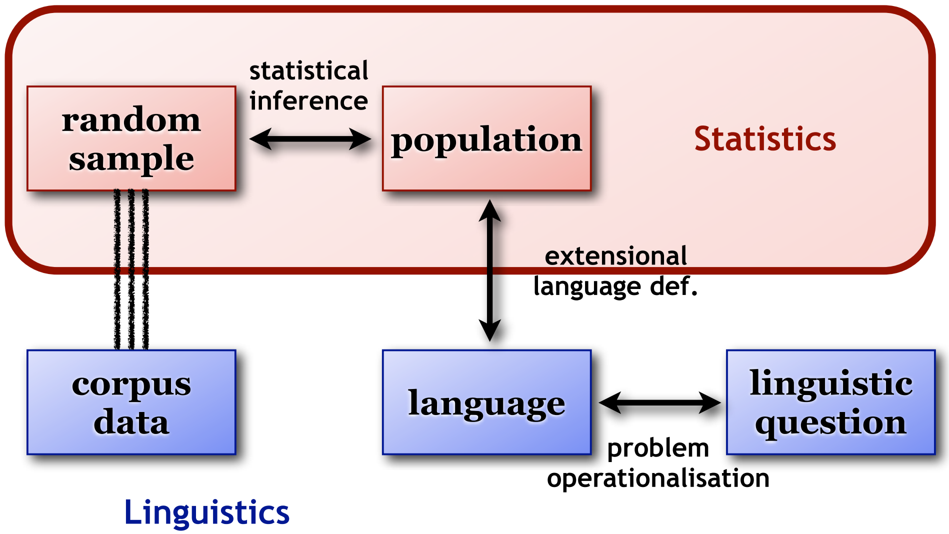
Inference from a sample

- Principle of inferential statistics
  - if a sample is picked at random, proportions should be roughly the same in sample and population
- Take a sample of 100 sentences
  - observe 19 passives → $p = 19\% = .19$
  - style guide → population proportion $\pi = 15\%$
  - $p > \pi$ → reject claim of style guide?
- Take another sample, just to be sure
  - observe 13 passives → $p = 13\% = .13$
  - $p < \pi$ → claim of style guide confirmed?

Sampling variation

- Random choice of sample ensures proportions are the same on average in sample & population
- But it also means that for every sample we will get a different value because of chance effects → **sampling variation**
  - problem: erroneous rejection of style guide's claim results in publication of a false result
- The main purpose of statistical methods is to estimate & correct for sampling variation
  - that's all there is to inferential statistics, really

Reminder: The role of statistics



The null hypothesis

- Our "goal" is to refute the style guide's claim, which we call the **null hypothesis** H_0

$$H_0 : \pi = .15$$

  - we also refer to $\pi_0 = .15$ as the **null proportion**
- Erroneous rejection of H_0 is problematic
  - leads to embarrassing publication of false result
  - known as a **type I error** in statistics
- Need to control risk of a type I error

Estimating sampling variation

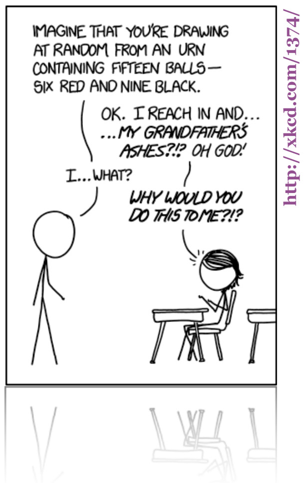
- Assume that style guide's claim H_0 is correct
  - i.e. rejection of H_0 is always a type I error
- Many corpus linguists set out to test H_0
  - each one draws a random sample of size n = 100
  - how many of the samples have the expected k = 15 passives, how many have k = 19, etc.?
  - if we are willing to reject H_0 for k = 19 passives in a sample, all corpus linguists with such a sample will publish a false result
  - risk of type I error = percentage of such cases

Estimating sampling variation

- We don’t need an infinite number of monkeys (or corpus linguists) to answer these questions
  - randomly picking sentences from our metaphorical library is like drawing balls from an infinite urn
  - red ball = passive sent. / white ball = active sent.
  - H_0: assume proportion of red balls in urn is 15%
- This leads to a **binomial distribution**

$$\Pr(k) = \binom{n}{k} (\pi_0)^k (1 - \pi_0)^{n-k}$$

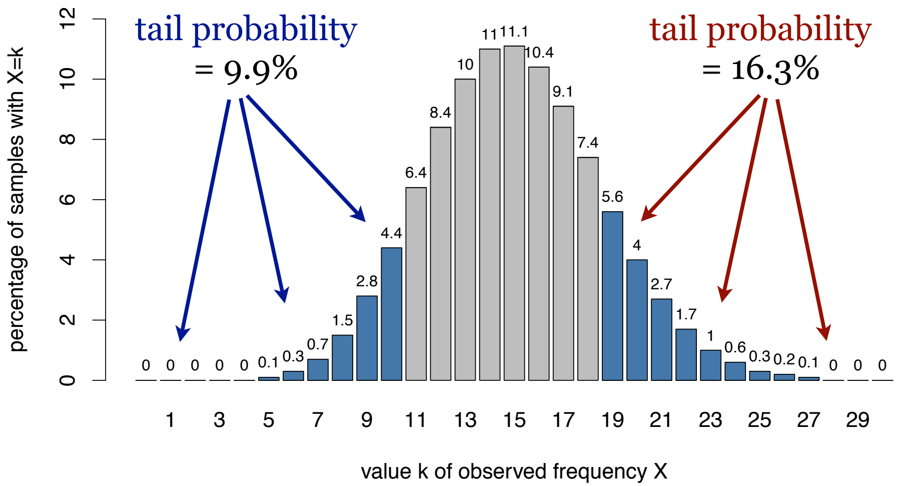
percentage of samples = **probability**



Comic relief

Binomial sampling distribution

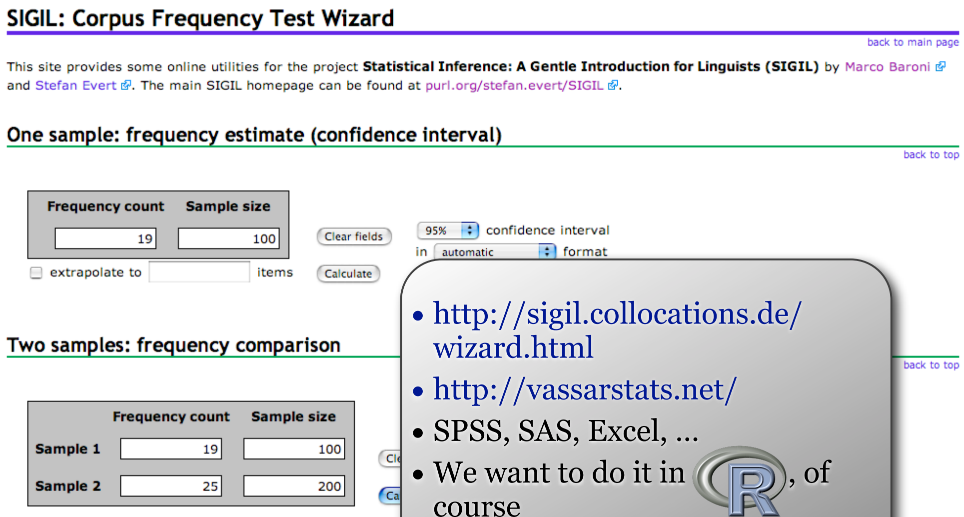
→ risk of false rejection = **p-value** = 26.2%



Statistical hypothesis testing

- Statistical **hypothesis tests**
  - define a **rejection criterion** for refuting H_0
  - control the risk of false rejection (**type I error**) to a "socially acceptable level" (**significance level** α)
  - **p-value** = risk of type I error given observation, interpreted as amount of evidence against H_0
- Two-sided vs. one-sided tests
  - in general, two-sided tests are recommended (safer)
  - one-sided test is plausible in our example

Hypothesis tests in practice



- http://sigil.collocations.de/wizard.html
- http://vassarstats.net/
- SPSS, SAS, Excel, …
- We want to do it in R, of course

Binomial hypothesis test in R

- Relevant R function: `binom.test()`
- We need to specify
  - **observed data**: **19** passives out of **100** sentences
  - **null hypothesis**: H_0: π = **15%**
- Using the `binom.test()` function:

```
> binom.test(19, 100, p=.15)  # two-sided
> binom.test(19, 100, p=.15,  # one-sided
     alternative="greater")
```


Binomial hypothesis test in R

```
> binom.test(19, 100, p=.15)

   Exact binomial test

data:  19 and 100

number of successes = 19, number of
trials = 100, p-value = 0.2623

alternative hypothesis: true probability of
success is not equal to 0.15

95 percent confidence interval:
 0.1184432 0.2806980

sample estimates:
probability of success
                  0.19
```

Rejection criterion & significance level

```
> binom.test(19, 100, p=.15)$p.value
[1] 0.2622728
```

$p > .05$ n.s.

```
> binom.test(23, 100, p=.15)$p.value
[1] 0.03430725
```

$p < .05 = \alpha$ *

```
> binom.test(25, 100, p=.15)$p.value
[1] 0.007633061
```

$p < .01 = \alpha$ **

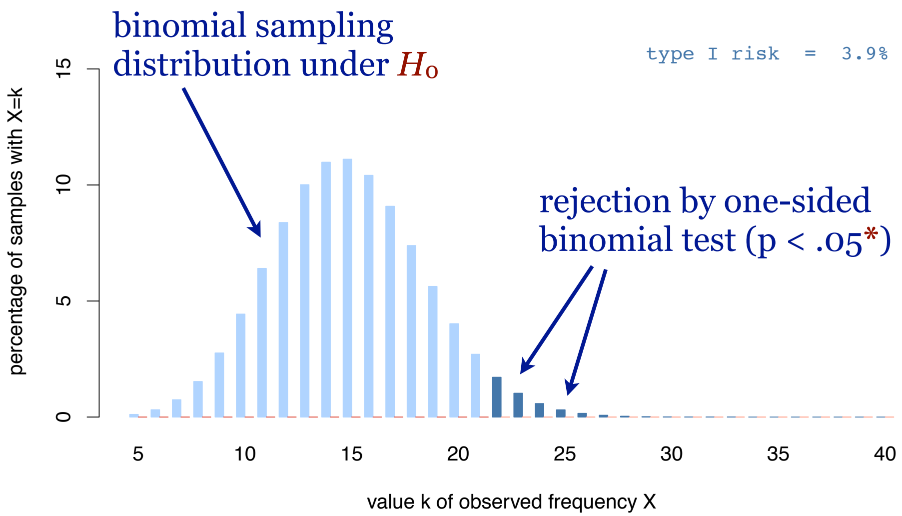
```
> binom.test(29, 100, p=.15)$p.value
[1] 0.0003529264
```

$p < .001 = \alpha$ ***

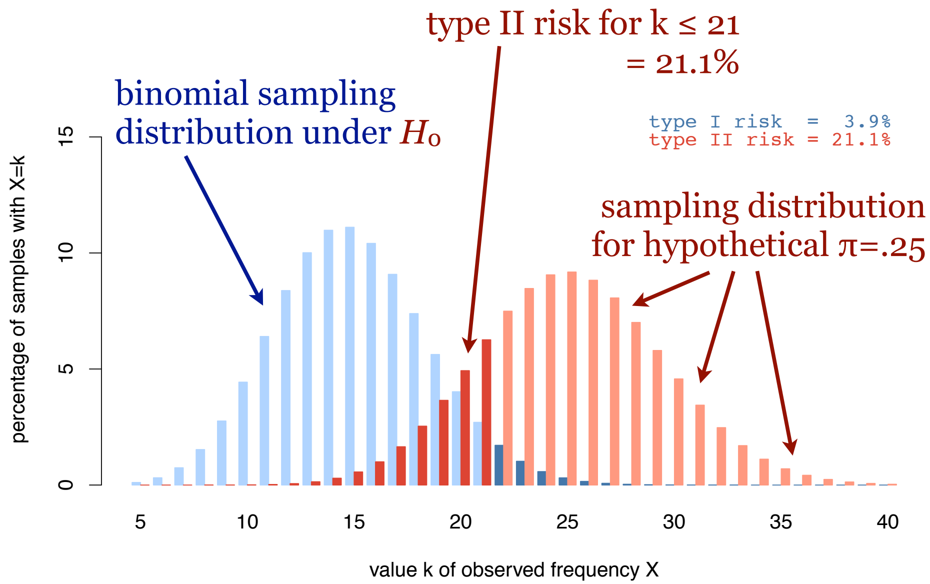
Type II errors

- Rejection criterion controls risk of type I error
  - only for situation in which H_0 is true
- Type II error = failure to reject incorrect H_0
  - for situation in which H_0 is not true
    → rejection correct, non-rejection is an error
- What is the risk of a type II error?
  - depends on true population proportion π
  - intuitively, risk of type II error will be low if the difference $\delta = \pi - \pi_0$ (the **effect size**) is large enough

Type II errors

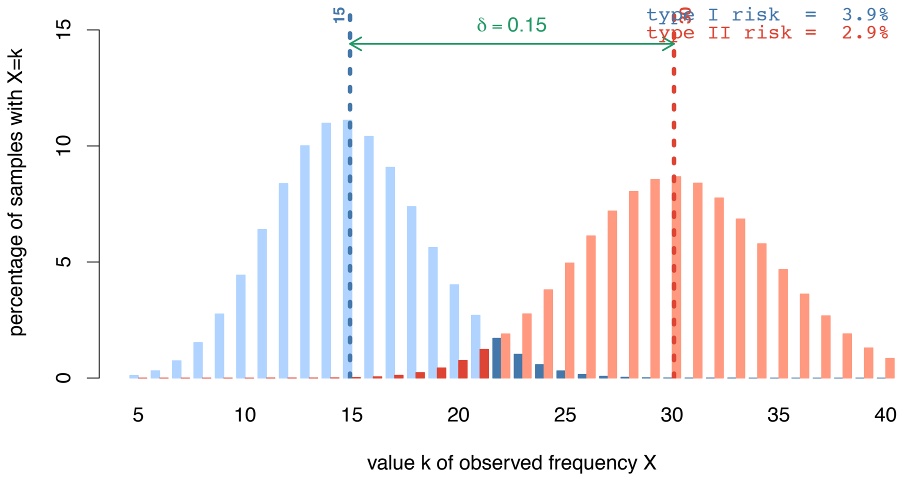


Type II errors



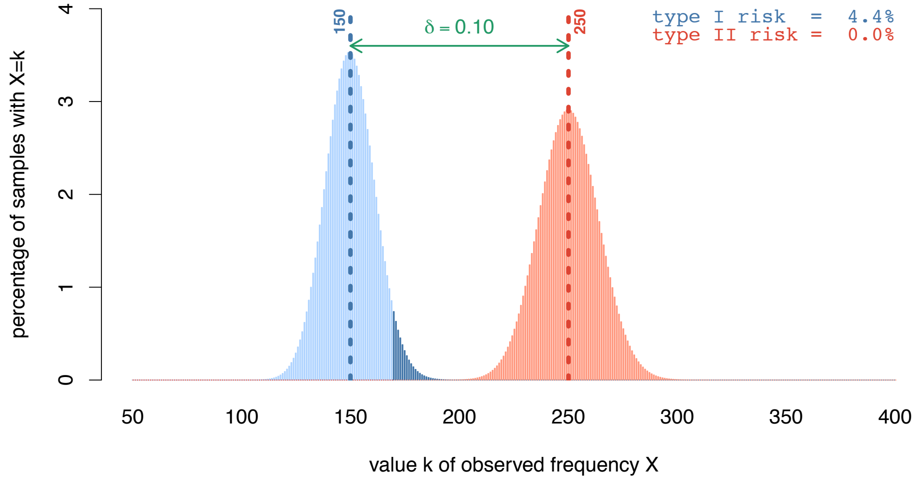
Type II errors
& effect size

effect size δ



Type II errors
& sample size

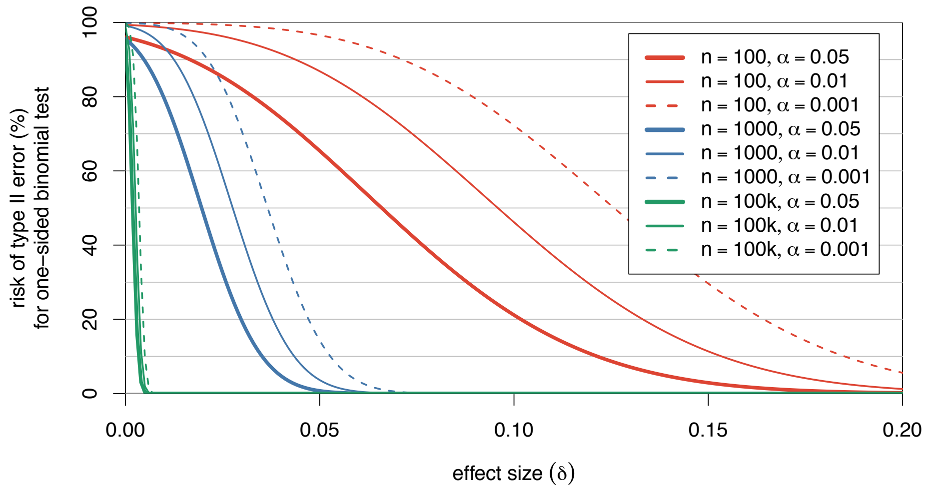
n = 1000



Power

- Type II error = failure to reject incorrect H_0
  - the larger the difference between H_0 and the true population proportion, the more likely it is that H_0 can be rejected based on a given sample
  - a **powerful** test has a low **type II error**
  - power analysis explores the relationship between effect size and risk of type II error
- Key insight: larger sample = more power
  - relative sampling variation becomes smaller
  - power also depends on significance level

Power analysis for binomial test



Power analysis for binomial test

- Key factors determining the power of a test
  - **sample size** → more evidence = greater power
  - **significance level** → trade-off btw. type I / II errors
- Influence of hypothesis test procedure
  - one-sided test more powerful than two-sided test
  - parametric tests more powerful than non-parametric
  - statisticians look for “uniformly most powerful” test
- Tests can become too powerful!
  - reject H_0 for 15.1% passives with n = 1,000,000

Parametric vs. non-parametric

- People often talk about parametric and non-parametric tests without precise definition
- Parametric tests make stronger assumptions
  - not just normality assuming (= Gaussian distribution)
  - binomial test: strong random sampling assumption → might be considered a parametric test in this sense!
- Parametric tests are usually more powerful
  - strong assumptions allow less conservative estimate of sampling variation → less evidence needed against H_0

Trade-offs in statistics

- Inferential statistics is a trade-off between type I errors and type II errors
  - i.e. between **significance** and **power**
- Significance level
  - determines trade-off point
  - low significance level α → low power
- Conservative tests
  - put more weight on avoiding type I errors → weaker
  - most non-parametric methods are conservative

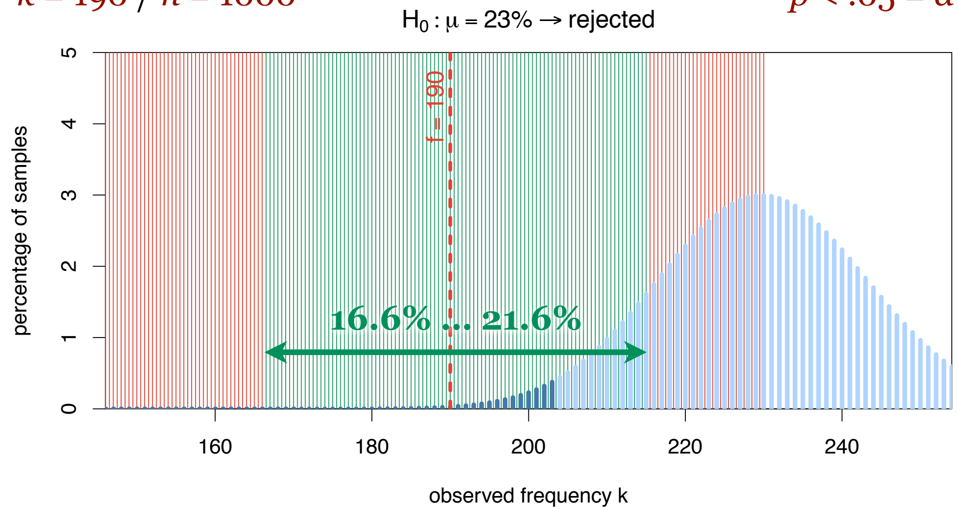
Confidence interval

- We now know how to test a null hypothesis H_0, rejecting it only if there is sufficient evidence
- But what if we do not have an obvious null hypothesis to start with?
  - this is typically the case in (computational) linguistics
- We can estimate the true population proportion from the sample data (relative frequency)
  - sampling variation → range of plausible values
  - such a **confidence interval** can be constructed by inverting hypothesis tests (e.g. binomial test)

Confidence interval

observed data:
$k = 190$ / $n = 1000$

95% confidence
$p < .05 = \alpha$



Confidence intervals

I'm cheating here a tiny little bit (not always an interval)

- Confidence interval = range of plausible values for true population proportion
  - H_0 rejected by test iff π_0 is outside confidence interval
- Size of confidence interval depends on power of the test (i.e. sample size and significance level)

	$n = 100$ $k = 19$	$n = 1{,}000$ $k = 190$	$n = 10{,}000$ $k = 1{,}900$
$\alpha = .05$	11.8% . . . 28.1%	16.6% . . . 21.6%	18.2% . . . 19.8%
$\alpha = .01$	10.1% . . . 31.0%	15.9% . . . 22.4%	18.0% . . . 20.0%
$\alpha = .001$	8.3% . . . 34.5%	15.1% . . . 23.4%	17.7% . . . 20.3%

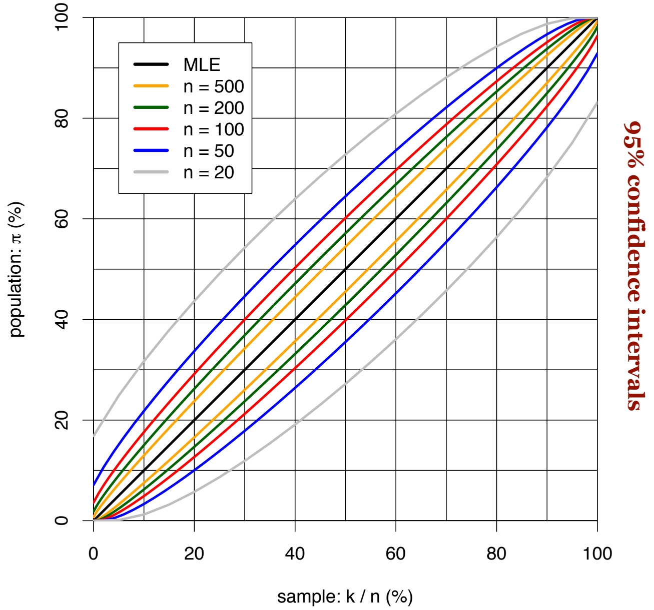
Confidence intervals in R

- Most hypothesis tests in R also compute a confidence interval (including `binom.test()`)
  - omit H_0 if only interested in confidence interval
- Significance level of underlying hypothesis test is controlled by `conf.level` parameter
  - expressed as confidence, e.g. `conf.level=.95` for significance level $\alpha = .05$, i.e. 95% confidence
- Can also compute one-sided confidence interval
  - controlled by `alternative` parameter
  - two-sided confidence intervals strongly recommended

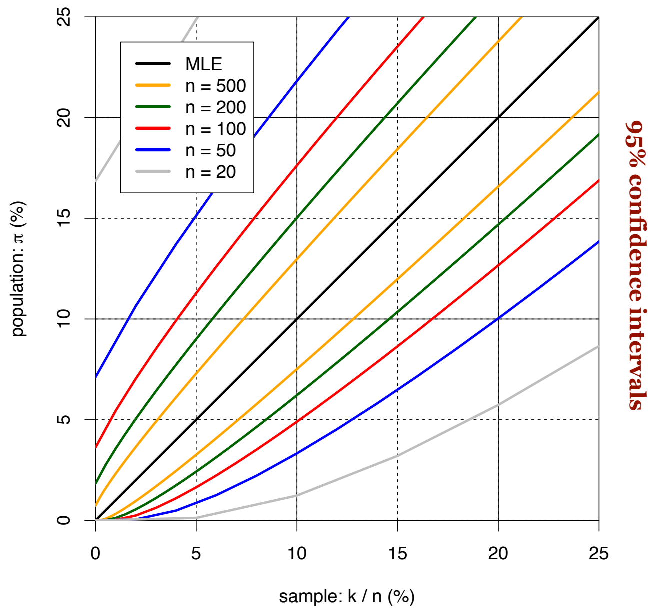
Confidence intervals in R

```
> binom.test(190, 1000, conf.level=.99)

   Exact binomial test

data:  190 and 1000

number of successes = 190, number of
trials = 1000, p-value < 2.2e-16

alternative hypothesis: true probability of
success is not equal to 0.5

99 percent confidence interval:
 0.1590920 0.2239133

sample estimates:
probability of success
                  0.19
```

Choosing sample size



Choosing sample size



Using R to choose sample size

- Call `binom.test()` with hypothetical values
- Plots on previous slides also created with R
  - requires calculation of large number of hypothetical confidence intervals
  - `binom.test()` is both inconvenient and inefficient
- The `corpora` package has a vectorised function

```
> library(corpora)
> prop.cint(190, 1000, conf.level=.99)
> ?prop.cint  # "conf. intervals for proportions"
```


Frequency comparison

- Many linguistic research questions can be operationalised as a frequency comparison
  - Are split infinitives more frequent in AmE than BrE?
  - Are there more definite articles in texts written by Chinese learners of English than native speakers?
  - Does *meow* occur more often in the vicinity of *cat* than elsewhere in the text?
  - Do speakers prefer *I couldn't agree more* over alternative realisations such as *I agree completely*?
- Compare observed frequencies in two samples

Frequency comparison

- Null hypothesis for frequency comparison

$$H_0 : \pi_1 = \pi_2$$

  - no assumptions about the precise value $\pi_1 = \pi_2 = \pi$
- Observed data
  - target count k_i and sample size n_i for each sample i
  - e.g. $k_1 = 19$ / $n_1 = 100$ passives vs. $k_2 = 25$ / $n_2 = 200$
- Effect size: difference of proportions
  - effect size $\delta = \pi_1 - \pi_2$ (and thus H_0: $\delta = 0$)

Frequency comparison in R

- Frequency comparison test: `prop.test()`
  - observed data: counts k_i and sample sizes n_i
  - also computes confidence interval for effect size
- E.g. for 19 passives out of 100 / 25 out of 200
  - parameters `conf.level` and `alternative` can be used in the familiar way

```
> prop.test(c(19,25), c(100,200))
```


Frequency comparison in R

```
> prop.test(c(19,25), c(100,200))

  2-sample test for equality of proportions with
continuity correction

data:  c(19, 25) out of c(100, 200)

X-squared = 1.7611, df = 1, p-value = 0.1845

alternative hypothesis: two.sided

95 percent confidence interval:
 -0.03201426  0.16201426

sample estimates:
prop 1 prop 2
 0.190  0.125
```


Contingency tables

	sample 1	sample 2
passive	k_1	k_2
active	n_1-k_1	n_2-k_2

19	25
81	175

- Data can also be given as a **contingency table**
  - e.g. $k_1 = 19$ / $n_1 = 100$ passives vs. $k_2 = 25$ / $n_2 = 200$
  - represents a cross-classification of $n = 300$ items
  - various tests for contingency tables can be applied
- Chi-squared X^2, likelihood ratio G^2, Fisher's test

Contingency tables

- Can easily carry out chi-squared (`chisq.test`) and Fisher's exact test (`fisher.test`) in R
  - likelihood ratio test not in R standard library
- Table for 19 / 100 vs. 25 / 200

```
> ct <- cbind(c(19,81),
              c(25,175))
> chisq.test(ct)
> fisher.test(ct)
```

19	**25**
81	**175**

Contingency tables

- Tests are appropriate for different situations
  - Fisher: computationally expensive, small samples
  - X^2: for large samples and small balanced samples
  - G^2: suitable for highly skewed data
- Measures of effect size
  - **difference** $\delta = \pi_1 - \pi_2$
    → approx. confidence interval from proportions test
  - **relative risk** $r = \pi_1 / \pi_2$
  - **odds ratio** $\theta = \pi_1 (1 - \pi_2) / \pi_2 (1 - \pi_1)$
    → exact confidence interval from Fisher's test

Significance vs. relevance

- Much focus on significant p-value, but ...
  - large differences may be non-significant if sample size is too small (e.g. 10/80 = 12.5% vs. 20/80 = 25%)
  - increase sample size for more powerful/sensitive test
  - very large samples lead to highly significant p-values for minimal and irrelevant differences (e.g. 1M tokens with 150,000 = 15% vs. 151,000 = 15.1% occurrences)
- It is important to assess both **significance** and **relevance** (= effect size) of frequency data!
  - confidence intervals combine both aspects

A case study: passives

- As a case study, we will compare the frequency of passives in Brown (AmE) and LOB (BrE)
  - pooled data
  - separately for each genre category
- Data files provided in CSV format
  - **`passives.brown.csv`** & **`passives.lob.csv`**
  - `cat` = genre category, `passive` = number of passives, `n_w` = number of word, `n_s` = number of sentences, `name` = description of genre category

Preparing the data

```
> Brown <- read.csv("passives.brown.csv")
> LOB <- read.csv("passives.lob.csv")

> library(SIGIL)  # also included in SIGIL package
> Brown <- BrownPassives
> LOB <- LOBPassives
# now take a look at the two tables: what info do they provide?

# pooled data for entire corpus = column sums (col. 2 … 4)
> Brown.all <- colSums(Brown[, 2:4])
> LOB.all <- colSums(LOB[, 2:4])
```


Frequency tests for pooled data

```
# proportions test reports p-value is based on chi-squared test
# and approximate confidence interval for effect size δ
> prop.test(c(10123, 10934), c(49576, 49742))

> ct <- cbind(c(10123, 49576-10123), # Brown
              c(10934, 49742-10934)) # LOB
> ct               # contingency table for chi-squared / Fisher
> fisher.test(ct)  # exact confidence interval for odds ratio θ

# we could in principle do the same for all 15 genres …
```


Automation: user functions

```
# user function do.test() executes proportions test for samples
# k1/n1 and k2/n2, and summarizes relevant results in compact form
> do.test <- function (k1, n1, k2, n2) {
    # res contains results of proportions test (list = data structure)
    res <- prop.test(c(k1, k2), c(n1, n2))
    # data frames are a nice way to display summary tables
    fmt <- data.frame(p=res$p.value,
      lower=res$conf.int[1], upper=res$conf.int[2])
    fmt  # return value of function = last expression
}
> do.test(10123, 49576, 10934, 49742)  # pooled data
> do.test(146, 975, 134, 947)            # humour genre
```


A nicer user function

```
# nicer version of user function with genre category labels
> do.test <- function (k1, n1, k2, n2, cat="") {
    res <- prop.test(c(k1, k2), c(n1, n2))
    data.frame(
      p=res$p.value,
      lower=100*res$conf.int[1],  # scaled to % points
      upper=100*res$conf.int[2],
      row.names=cat  # add genre as row label
    ) # return data frame directly without local variable fmt
}

# extract relevant information directly from data frames
> do.test(Brown$passive[15], Brown$n_s[15],
          LOB$passive[15], LOB$n_s[15],
          cat=Brown$name[15])
```

Ad-hoc functions & loops

```
# ad-hoc convenience function to reduce typing/editing
# (works only if global Brown/LOB variables are set correctly!)
quick.test <- function (i) {
  do.test(k1=Brown$passive[i], n1=Brown$n_s[i],
          k2=LOB$passive[i], n2=LOB$n_s[i],
          cat=Brown$name[i])
}

quick.test(15)   # easy to repeat for different genres now
quick.test(9)

# loop over all 15 categories (more general: 1:nrow(Brown))
for (i in 1:15) {
  print( quick.test(i) )
}
```


R wizardry: working with lists

```
# our code only works if rows of Brown/LOB are in the same order!
> all(Brown$cat == LOB$cat)

# it would be nice to collect all these results in a single overview table
# for this, we need a little bit of R wizardry …

# apply function quick.test() to each number 1, …, 15
res.list <- lapply(1:15, quick.test)

# pass res.list as individual arguments to rbind()
# (think of this as an idiom you just have to remember …)
res <- do.call(rbind, res.list)

res             # data frame with one row for each genre

round(res, 3)   # rounded values are easier to read
```


It's your turn now …

- Questions:
  - Which differences are significant?
  - Are the effect sizes linguistically relevant?
- A different approach:
  - You can construct a list of contingency tables with the `cont.table()` function from the `corpora` package
  - Apply `fisher.test()` or `chisq.test()` directly to each table in the list using the `lapply()` function
  - Try to extract relevant information with `sapply()`