Tutorial: Working with categorical data with R and the vcd package

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Abstract

This tutorial describes the creation of frequency and contingency tables from categorical variables, along with tests of independence, measures of association, and methods for graphically displaying results. The framework is provided by the R package **vcd**, but other packages are used to help with various tasks.

Keywords: contingency tables, mosaic plots, sieve plots, categorical data, independence, conditional independence, R.

1. Introduction

This tutorial, part of the **vcdExtra** package, describes the creation of frequency and contingency tables from categorical variables, along with tests of independence, measures of association, and methods for graphically displaying results. It borrows structure and some ideas from from Robert Kabakoff's *Quick-R* web page, http://www.statmethods.net/stats/frequencies.html.

There is much more to the analysis of categorical data than is described here, where the emphasis is on cross-tabulated tables of frequencies ("contingency tables"), statistical tests, associated loglinear models, and visualization of *how* variables are related.

A more general treatment of graphical methods for categorical data is contained in my book, *Visualizing Categorical Data* (Friendly 2000), for which **vcd** is a partial R companion, covering topics not otherwise available in R. On the other hand, the implementation of graphical methods in **vcd** is more general in many respects than what I provided in SAS.

A more complete theoretical description of these statistical methods is provided in Agresti's (2002) Categorical Data Analysis. For this, see the Splus/R companion by Laura Thompson, https://home.comcast.net/~lthompson221/Splusdiscrete2.pdf.

2. Creating and manipulating frequency tables

R provides many methods for creating frequency and contingency tables. Several are described below. In the examples below, we use some real examples and some anonymous ones, where the variables A, B, and C represent categorical variables, and X represents an arbitrary R data object.

The first thing you need to know is that categorical data can be represented in three different forms in R, and it is sometimes necessary to convert from one form to another, for carrying out statistical tests, fitting models or visualizing the results. Once a data object exists in R, you can examine its complete structure with the str() function, or view the names of its components with the names() function.

case form a data frame containing individual observations, with one or more factors, used as the classifying variables. In case form, there may also be numeric covariates. The total number of observations is nrow(X), and the number of variables is ncol(X).

Example: The Arthritis data is available in case form in the vcd package. There are two explanatory factors: Treatment and Sex. Age is a covariate, and Improved is the response—an ordered factor, with levels None < Some < Marked. Excluding Age, we would have a $2 \times 2 \times 3$ contingency table for Treatment, Sex and Improved.

```
> names(Arthritis)
                        # show the variables
Γ1] "ID"
                "Treatment" "Sex"
                                         "Age"
                                                     "Improved"
> str(Arthritis)
                        # show the structure
                     84 obs. of 5 variables:
'data.frame':
            : int 57 46 77 17 36 23 75 39 33 55 ...
 $ Treatment: Factor w/ 2 levels "Placebo", "Treated": 2 2 2 2 2 2 2 2 2 ...
 $ Sex
            : Factor w/ 2 levels "Female", "Male": 2 2 2 2 2 2 2 2 2 ...
            : int 27 29 30 32 46 58 59 59 63 63 ...
 $ Improved : Ord.factor w/ 3 levels "None"<"Some"<..: 2 1 1 3 3 3 1 3 1 1 ...</pre>
> head(Arthritis,5)
                        # first 5 observations, same as Arthritis[1:5,]
  ID Treatment Sex Age Improved
1 57
       Treated Male
                     27
                            Some
2 46
       Treated Male
                     29
                            None
3 77
      Treated Male
                     30
                            None
4 17
      Treated Male
                     32
                          Marked
5 36
       Treated Male
                          Marked
```

frequency form a data frame containing one or more factors, and a frequency variable, often called Freq or count. The total number of observations is sum(X\$Freq), sum(X[,"Freq"]) or some equivalent form. The number of cells in the table is nrow(X).

Example: For small frequency tables, it is often convenient to enter them in frequency form using expand.grid() for the factors and c() to list the counts in a vector. The example below, from Agresti (2002) gives results for the 1991 General Social Survey, with respondents classified by sex and party identification.

```
> # Agresti (2002), table 3.11, p. 106
> GSS <- data.frame(
    expand.grid(sex=c("female", "male"),
                party=c("dem", "indep", "rep")),
    count=c(279,165,73,47,225,191))
> GSS
     sex party count
1 female
           dem
                 279
    male
           dem
                 165
                  73
3 female indep
    male indep
                  47
5 female
           rep
                 225
    male
           rep
                 191
```

table form a matrix, array or table object, whose elements are the frequencies in an n-way table. The variable names (factors) and their levels are given by dimnames(X). The total number of observations is sum(X). The number of dimensions of the table is length(dimnames(X)), and the table sizes are given by sapply(dimnames(X), length).

Example: The HairEyeColor is stored in table form in vcd.

Example: Enter frequencies in a matrix, and assign dimnames, giving the variable names and category labels. Note that, by default, matrix() uses the elements supplied by columns in the result, unless you specify byrow=TRUE.

satisfaction

income	VeryD	${\tt LittleD}$	${\tt ModerateS}$	VeryS
< 15k	1	3	10	6
15-25k	2	3	10	7
25-40k	1	6	14	12
> 40k	0	1	9	11

JobSat is a matrix, not an object of class("table"), and some functions are happier with tables than matrices. You can coerce it to a table with as.table(),

2.1. Ordered factors and reordered tables

In table form, the values of the table factors are ordered by their position in the table. Thus in the JobSat data, both income and satisfaction represent ordered factors, and the *positions* of the values in the rows and columns reflects their ordered nature.

Yet, for analysis, there are time when you need *numeric* values for the levels of ordered factors in a table, e.g., to treat a factor as a quantitative variable. In such cases, you can simply re-assign the dimnames attribute of the table variables. For example, here, we assign numeric values to income as the middle of their ranges, and treat satisfaction as equally spaced with integer scores.

```
> dimnames(JobSat)$income<-c(7.5,20,32.5,60)
> dimnames(JobSat)$satisfaction<-1:4</pre>
```

For the HairEyeColor data, hair color and eye color are ordered arbitrarily. For visualizing the data using mosaic plots and other methods described below, it turns out to be more useful to assure that both hair color and eye color are ordered from dark to light. Hair colors are actually ordered this way already, and it is easiest to re-order eye colors by indexing. Again str() is your friend.

```
> HairEyeColor <- HairEyeColor[, c(1,3,4,2), ]
> str(HairEyeColor)

num [1:4, 1:4, 1:2] 32 53 10 3 10 25 7 5 3 15 ...
- attr(*, "dimnames")=List of 3
    ..$ Hair: chr [1:4] "Black" "Brown" "Red" "Blond"
    ..$ Eye : chr [1:4] "Brown" "Hazel" "Green" "Blue"
    ..$ Sex : chr [1:2] "Male" "Female"
```

This is also the order for both hair color and eye color shown in the result of a correspondence analysis (Figure 5) below.

With data in case form or frequency form, when you have ordered factors represented with character values, you must ensure that they are treated as ordered in R.¹

Imagine that the Arthritis data was read from a text file. By default the Improved will be ordered alphabetically: Marked, None, Some—not what we want. In this case, the function ordered() (and others) can be useful.

¹In SAS, many procedures offer the option order = data | internal | formatted to allow character values to be ordered according to (a) their order in the data set, (b) sorted internal value, or (c) sorted formatted representation provided by a SAS format.

- > Arthritis <- read.csv("arthritis.txt",header=TRUE)
- > Arthritis\$Improved <- ordered(Arthritis\$Improved, levels=c("None", "Some", "Marked"))

With this order of Improved, the response in this data, a mosaic display of Treatment and Improved (Figure 1)shows a clearly interpretable pattern.

Arthritis: [Treatment] [Improved]

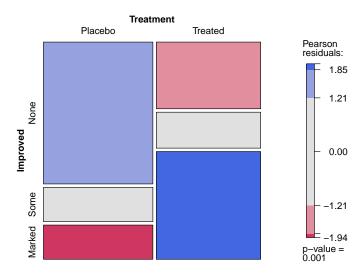


Figure 1: Mosaic plot for the Arthritis data, showing the marginal model of independence for Treatment and Improved. Age, a covariate, and Sex are ignored here.

Finally, there are situations where, particularly for display purposes, you want to re-order the *dimensions* of an *n*-way table, or change the labels for the variables or levels. This is easy when the data are in table form: aperm() permutes the dimensions, and assigning to names and dimnames changes variable names and level labels respectively. We will use the following version of UCBAdmissions in Section 3.4 below.²

```
> UCB <- aperm(UCBAdmissions, c(2, 1, 3))
> dimnames(UCB)[[2]] <- c("Yes", "No")
> names(dimnames(UCB)) <- c("Sex", "Admit?", "Department")
> ftable(UCB)
```

		Department	Α	В	C	D	E	F
Sex	Admit?							
Male	Yes		512	353	120	138	53	22
	No		313	207	205	279	138	351
Female	Yes		89	17	202	131	94	24
	No		19	8	391	244	299	317

² Changing Admit to Admit? might be useful for display purposes, but is dangerous— because it is then difficult to use that variable name in a model formula.

2.2. structable()

For 3-way and larger tables the **structable()** function in **vcd** provides a convenient and flexible tabular display. The variables assigned to the rows and columns of a two-way display can be specified by a model formula.

> structable(HairEyeColor)

show the table: default

		Eye	Brown	Hazel	Green	Blue
Hair	Sex					
Black	Male		32	10	3	11
	${\tt Female}$		36	5	2	9
${\tt Brown}$	Male		53	25	15	50
	${\tt Female}$		66	29	14	34
Red	Male		10	7	7	10
	${\tt Female}$		16	7	7	7
Blond	Male		3	5	8	30
	Female		4	5	8	64

> structable(Hair+Sex ~ Eye, HairEyeColor) # specify col ~ row variables

	Hair Sex	Black	Female	Brown	Female	Red	Fomalo	Blond	Female
	pex	Mare	Гешате	Mare	Гешате	Mare	гешате	Hare	remare
Eye									
Brown		32	36	53	66	10	16	3	4
Hazel		10	5	25	29	7	7	5	5
Green		3	2	15	14	7	7	8	8
Blue		11	9	50	34	10	7	30	64

It also returns an object of class "structable" which may be plotted with mosaic() (not shown here).

```
> HSE < - structable(Hair+Sex ~ Eye, HairEyeColor) # save structable object
> mosaic(HSE) # plot it
```

2.3. table() and friends

You can generate frequency tables from factor variables using the table() function, tables of proportions using the prop.table() function, and marginal frequencies using margin.table().

```
> n=500
> A <- factor(sample(c("a1","a2"), n, rep=TRUE))
> B <- factor(sample(c("b1","b2"), n, rep=TRUE))
> C <- factor(sample(c("c1","c2"), n, rep=TRUE))
> mydata <- data.frame(A,B,C)

> # 2-Way Frequency Table
> attach(mydata)
> mytable <- table(A,B)  # A will be rows, B will be columns
> mytable  # print table
> margin.table(mytable, 1) # A frequencies (summed over B)
```

```
> margin.table(mytable, 2) # B frequencies (summed over A)
> prop.table(mytable) # cell percentages
> prop.table(mytable, 1) # row percentages
> prop.table(mytable, 2) # column percentages
```

table() can also generate multidimensional tables based on 3 or more categorical variables. In this case, use the ftable() function to print the results more attractively.

```
> # 3-Way Frequency Table
> mytable <- table(A, B, C)
> ftable(mytable)
```

Table ignores missing values. To include NA as a category in counts, include the table option exclude=NULL if the variable is a vector. If the variable is a factor you have to create a new factor using newfactor <- factor(oldfactor, exclude=NULL).

2.4. xtabs

The xtabs() function allows you to create crosstabulations using formula style input.

```
> # 3-Way Frequency Table
> mytable <- xtabs(~A+B+C, data=mydata)
> ftable(mytable) # print table
> summary(mytable) # chi-square test of indepedence
```

If a variable is included on the left side of the formula, it is assumed to be a vector of frequencies (useful if the data have already been tabulated).

2.5. Converting among frequency tables and data frames

As we've seen, a given contingency table can be represented equivalently in different forms, but some R functions were designed for one particular representation. Table 1 shows some handy tools for converting from one form to another.

A contingency table in table form (an object of class(table)) can be converted to a data.frame with as.data.frame().³ The resulting data.frame contains columns representing the classifying

³ Because R is object-oriented, this is actually a short-hand for the function as.data.frame.table().

Table 1: Tools for converting among different forms for categorical data

		To this	
From this	Case form	Frequency form	Table form
Case form	noop	xtabs(~A+B)	table(A,B)
Frequency form	<pre>expand.dft(X)</pre>	noop	<pre>xtabs(count~A+B)</pre>
Table form	<pre>expand.dft(X)</pre>	as.data.frame(X)	noop

factors and the table entries (as a column named by the responseName argument, defaulting to Freq. This is the inverse of xtabs().

Example: Convert the GSStab in table form to a data frame in frequency form.

> as.data.frame(GSStab)

```
sex party Freq
1 female
           dem
                279
   male
           dem
               165
3 female indep
                 73
   male indep
                 47
5 female
           rep
                225
   male
           rep
                191
```

Example: Convert the Arthritis data in case form to a 3-way table of Treatment \times Sex \times Improved.⁴

> ftable(Art.tab)

Improved None Some Marked

${\tt Treatment}$	Sex			
Placebo	Female	19	7	6
	Male	10	0	1
Treated	Female	6	5	16
	Male	7	2	5

There may also be times that you wil need an equivalent case form data.frame with factors representing the table variables rather than the frequency table. For example, the mca() function in

⁴ table() does not allow a data argument to provide an environment in which the table variables are to be found. In the examples in Section 2.3 I used attach(mydata) for this purpose, but attach() leaves the variables in the global environment, while with() just evaluates the table() expression in a temporary environment of the data.

package MASS only operates on data in this format. Marc Schwartz provided code for expand.dft() on the Rhelp mailing list for converting a table back into a case form data.frame. This function is included in vcdExtra.

Example: Convert the Arthritis data in table form (Art.tab) back to a data.frame in case form, with factors Treatment, × Sex and Improved.

```
> Art.df <- expand.dft(Art.tab)
> str(Art.df)

'data.frame': 84 obs. of 3 variables:
$ Treatment: Factor w/ 2 levels "Placebo", "Treated": 1 1 1 1 1 1 1 1 1 1 1 ...
$ Sex : Factor w/ 2 levels "Female", "Male": 1 1 1 1 1 1 1 1 1 1 ...
$ Improved : Factor w/ 3 levels "Marked", "None",...: 2 2 2 2 2 2 2 2 2 2 ...
```

2.6. A complex example

If you've followed so far, you're ready for a more complicated example. The data file, tv.dat represents a 4-way table of size $5 \times 11 \times 5 \times 3$ where the table variables (unnamed in the file) are read as V1 - V4, and the cell frequency is read as V5. The file, stored in the data/ directory of vcdExtra, can be read as follows:

```
> tv.data<-read.table(system.file("data","tv.dat",package="vcdExtra"))
> head(tv.data,5)

V1 V2 V3 V4 V5
1  1  1  1  1  6
2  2  1  1  1  18
3  3  1  1  1  6
4  4  1  1  1  2
5  5  1  1  1  11
```

For a local file, just use read.table() in this form:

```
> tv.data<-read.table("C:/R/data/tv.dat")</pre>
```

The data tv.dat came from the initial implementation of mosaic displays in R by Jay Emerson. In turn, they came from the initial development of mosaic displays (Hartigan and Kleiner 1984) that illustrated the method with data on a large sample of TV viewers whose behavior had been recorded for the Neilson ratings. This data set contains sample television audience data from Neilsen Media Research for the week starting November 6, 1995.

The table variables are:

```
V1- values 1:5 correspond to the days Monday-Friday;
```

- V2- values 1:11 correspond to the quarter hour times 8:00PM through 10:30PM;
- V3- values 1:5 correspond to ABC, CBS, NBC, Fox, and non-network choices;
- V4— values 1:3 correspond to transition states: turn the television Off, Switch channels, or Persist in viewing the current channel.

We are interested just the cell frequencies, and rely on the facts that the (a) the table is complete—there are no missing cells, so nrow(tv.data)=825; (b) the observations are ordered so that V1 varies most rapidly and V4 most slowly. From this, we can just extract the frequency column and reshape it into an array.

But this table is too large and awkward to work with. Among the networks, Fox and Other occur infrequently. We can also cut it down to a 3-way table by considering only viewers who persist with the current station.

```
> TV <- TV[,,1:3,]
                          # keep only ABC, CBS, NBC
> TV <- TV[,,,3]
                          # keep only Persist -- now a 3 way table
> structable(TV)
                    Time 8:00 8:15 8:30 8:45 9:00 9:15 9:30 9:45 10:00 10:15 10:30
Day
           Network
Monday
           ABC
                           146
                                151
                                      156
                                             83
                                                  325
                                                       350
                                                             386
                                                                   340
                                                                         352
                                                                                280
                                                                                       278
                                                                                       272
           CBS
                           337
                                293
                                      304
                                            233
                                                 311
                                                       251
                                                             241
                                                                   164
                                                                         252
                                                                                265
           NBC
                           263
                                219
                                      236
                                            140
                                                 226
                                                       235
                                                             239
                                                                   246
                                                                         279
                                                                                263
                                                                                       283
                                181
Tuesday
           ABC
                           244
                                      231
                                            205
                                                 385
                                                       283
                                                             345
                                                                   192
                                                                         329
                                                                                351
                                                                                       364
           CBS
                           173
                                180
                                      184
                                            109
                                                 218
                                                       235
                                                             256
                                                                   250
                                                                         274
                                                                                263
                                                                                       261
           NBC
                           315
                                254
                                      280
                                            241
                                                 370
                                                       214
                                                             195
                                                                   111
                                                                         188
                                                                                190
                                                                                       210
                                      194
                                                 339
                                                       264
                                                             279
Wednesday ABC
                           233
                                161
                                            156
                                                                   140
                                                                         237
                                                                                228
                                                                                       203
                                126
                                      207
                                             59
                                                   98
                                                       103
                                                             122
                                                                    86
                                                                         109
                                                                                105
           CBS
                           158
                                                                                       110
           NBC
                           134
                                146
                                      166
                                             66
                                                 194
                                                       230
                                                             264
                                                                   143
                                                                         274
                                                                                289
                                                                                       306
                                                 187
                                                                                122
Thursday
           ABC
                           174
                                183
                                      197
                                            181
                                                       198
                                                             211
                                                                    86
                                                                         110
                                                                                       117
           CBS
                           196
                                185
                                      195
                                            104
                                                 106
                                                       116
                                                             116
                                                                    47
                                                                         102
                                                                                 84
                                                                                        84
           NBC
                                            477
                                                  590
                                                                                705
                           515
                                463
                                      472
                                                       473
                                                             446
                                                                   349
                                                                         649
                                                                                       747
Friday
           ABC
                           294
                                281
                                      305
                                            239
                                                 278
                                                       246
                                                             245
                                                                   138
                                                                         246
                                                                                232
                                                                                       233
           CBS
                           130
                                144
                                      154
                                             81
                                                  129
                                                       153
                                                             136
                                                                   126
                                                                         138
                                                                                136
                                                                                       152
           NBC
                           195
                                220
                                      248
                                            160
                                                 172
                                                       164
                                                             169
                                                                    85
                                                                         183
                                                                                198
                                                                                       204
```

Finally, for some purposes, we might want to collapse the 11 times into a smaller number. Here, we use as.data.frame.table() to convert the table back to a data frame, levels() to re-assign the values of Time, and finally, xtabs() to give a new, collapsed frequency table.

```
> TV.df <- as.data.frame.table(TV)</pre>
> levels(TV.df$Time) <- c(rep("8:00-8:59",4),rep("9:00-9:59",4), rep("10:00-10:44",3))
> TV2 <- xtabs(Freq ~ Day + Time + Network, TV.df)
> structable(Day ~ Time+Network, TV2)
                     Day Monday Tuesday Wednesday Thursday Friday
Time
            Network
8:00-8:59
            ABC
                             536
                                     861
                                                744
                                                          735
                                                                1119
            CBS
                            1167
                                     646
                                                550
                                                          680
                                                                 509
            NBC
                                                         1927
                                                                 823
                             858
                                    1090
                                                512
9:00-9:59
            ABC
                            1401
                                    1205
                                               1022
                                                          682
                                                                 907
```

CBS

NBC

10:00-10:44 ABC

CBS	789	798	324	270	426
NBC	825	588	869	2101	585

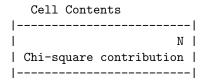
Whew!

3. Tests of Independence

3.1. CrossTable

OK, now we're ready to do some analyses. For tabular displays, the <code>CrossTable()</code> function in the <code>gmodels</code> package produces cross-tabulations modeled after PROC FREQ in SAS or CROSSTABS in SPSS. It has a wealth of options for the quantities that can be shown in each cell.

- > # 2-Way Cross Tabulation
- > library(gmodels)
- > CrossTable(GSStab,prop.t=FALSE,prop.r=FALSE,prop.c=FALSE)



Total Observations in Table: 980

	party			
sex	dem	indep	rep	Row Total
female	279 1.183			
male	165 1.693			403
Column Total	 444 	120	 416 	 980

There are options to report percentages (row, column, cell), specify decimal places, produce Chisquare, Fisher, and McNemar tests of independence, report expected and residual values (pearson, standardized, adjusted standardized), include missing values as valid, annotate with row and column titles, and format as SAS or SPSS style output! See help(CrossTable) for details.

3.2. Chi-square test

For 2-way tables you can use chisq.test() to test independence of the row and column variable. By default, the *p*-value is calculated from the asymptotic chi-squared distribution of the test statistic. Optionally, the *p*-value can be derived via Monte Carlo simulation.

> (HairEye <- margin.table(HairEyeColor, c(1, 2)))</pre>

Eye Brown Hazel Green Blue Hair Black 68 15 5 20 Brown 119 54 29 84 Red 26 14 14 17 Blond 10 16 94

> chisq.test(HairEye)

Pearson's Chi-squared test

data: HairEye

X-squared = 138.2898, df = 9, p-value < 2.2e-16

3.3. Fisher Exact Test

fisher.test(X) provides an exact test of independence. X must be a two-way contingency table in table form. Another form, fisher.test(X, Y) takes two categorical vectors of the same length. For tables larger than 2×2 the method can be computationally intensive (or can fail) if the frequencies are not small.

> fisher.test(GSStab)

Fisher's Exact Test for Count Data

data: GSStab
p-value = 0.03115

alternative hypothesis: two.sided

But this does not work because HairEye data has n=592 total frequency. An exact test is unnecessary in this case.

> fisher.test(HairEye)

Error in fisher.test(HairEye) : FEXACT error 6. LDKEY is too small for this problem. Try increasing the size of the workspace.

3.4. Mantel-Haenszel test and conditional association

Use the mantelhaen.test(X) function to perform a Cochran-Mantel-Haenszel χ^2 chi test of the null hypothesis that two nominal variables are *conditionally independent*, $A \perp B \mid C$, in each stratum, assuming that there is no three-way interaction. X is a 3 dimensional contingency table, where the last dimension refers to the strata.

The UCBAdmissions serves as an example of a $2 \times 2 \times 6$ table, with Dept as the stratifying variable.

```
> ## UC Berkeley Student Admissions
```

> mantelhaen.test(UCBAdmissions)

Mantel-Haenszel chi-squared test with continuity correction

The results show no evidence for association between admission and gender when adjusted for department. However, we can easily see that the assumption of equal association across the strata (no 3-way association) is probably violated. For $2 \times 2 \times k$ tables, this can be examined from the odds ratios for each 2×2 table (oddsratio()), and tested by using woolf_test() in vcd.

> oddsratio(UCBAdmissions, log=FALSE)

A B C D E F
0.3492120 0.8025007 1.1330596 0.9212838 1.2216312 0.8278727
> lor <- oddsratio(UCBAdmissions) # capture log odds ratios
> summary(lor)
Log Odds Ratio Std. Error z value Pr(>|z|)

```
0.259993 -4.0466 2.599e-05 ***
       -1.052076
Α
В
       -0.220023
                   0.427128 -0.5151
                                        0.3032
C
        0.124922
                   0.143727 0.8692
                                        0.1924
D
       -0.081987
                   0.149975 - 0.5467
                                        0.2923
Ε
        0.200187
                   0.199581 1.0030
                                        0.1579
F
       -0.188896
                   0.302085 -0.6253
                                        0.2659
```

Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1

> woolf_test(UCBAdmissions)

Woolf-test on Homogeneity of Odds Ratios (no 3-Way assoc.)

data: UCBAdmissions
X-squared = 17.9017, df = 5, p-value = 0.003072

We can visualize the odds ratios of Admission for each department with fourfold displays using fourfold(). The cell frequencies n_{ij} of each 2×2 table are shown as a quarter circle whose radius is proportional to $\sqrt{n_{ij}}$, so that its area is proportional to the cell frequency. Confidence rings for the odds ratio allow a visual test of the null of no association; the rings for adjacent quadrants overlap iff the observed counts are consistent with the null hypothesis. In the extended version (the default), brighter colors are used where the odds ratio is significantly different from 1. The following lines produce Figure 2.⁵

```
> col <- c("#99CCFF", "#6699CC", "#F9AFAF", "#6666AO", "#FF0000", "#000080")
> fourfold(UCB,mfrow=c(2,3), color=col)
```

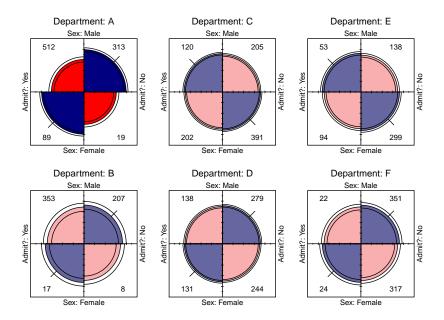


Figure 2: Fourfold display for the UCBAdmissions data. Where the odds ratio differs significantly from 1.0, the confidence bands do not overlap, and the circle quadrants are shaded more intensely.

Another vcd function, cotabplot(), provides a more general approach to visualizing conditional associations in contingency tables, similar to trellis-like plots produced by coplot() and lattice graphics. The panel argument supplies a function used to render each conditional subtable. The following gives a display (not shown) similar to Figure 2.

> cotabplot(UCB, panel = cotab_fourfold)

Finally, the there is a plot() method for oddsratio objects. By default, it shows the 95% confidence interval for the log odds ratio. Figure 3 is produced by:

> plot(lor, xlab="Department", ylab="Log Odds Ratio (Admit | Gender)")

3.5. Measures of Association

There are a variety of statistical measures of *strength* of association for contingency tables—similar in spirit to r or r^2 for continuous variables. With a large sample size, even a small degree of association can show a significant χ^2 , as in the example below for the GSS data.

The assocstats() function in vcd calculates the ϕ contingency coefficient, and Cramer's V for an $r \times c$ table. The input must be in table form, a two-way $r \times c$ table. It won't work with GSS in frequency form, but by now you should know how to convert.

> assocstats(GSStab)

⁵The color values co1[3:4] were modified from their default values to show a greater contrast between significant and insignificant associations here.

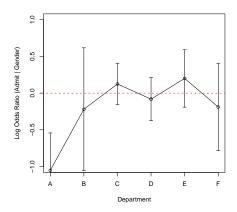


Figure 3: Log odds ratio plot for the UCBAdmissions data.

```
X^2 df P(> X^2)
Likelihood Ratio 7.0026 2 0.030158
Pearson 7.0095 2 0.030054
```

Phi-Coefficient : 0.085 Contingency Coeff.: 0.084 Cramer's V : 0.085

For tables with ordinal variables, like JobSat, some people prefer the Goodman-Kruskal γ statistic (?, §2.4.3) based on a comparison of concordant and discordant pairs of observations in the case-form equivalent of a two-way table.

> GKgamma(JobSat)

gamma : 0.221 std. error : 0.117

CI : -0.009 0.451

A web article by Richard Darlington, http://www.psych.cornell.edu/Darlington/crosstab/ TABLEO.HTM gives further description of these and other measures of association.

3.6. Measures of Agreement

The Kappa() function in the vcd package calculates Cohen's κ and weighted κ for a square two-way table with the same row and column categories (Cohen 1960).⁶ Normal-theory z-tests are obtained by dividing κ by its asymptotic standard error (ASE). A confint() method for Kappa objects provides confidence intervals.

> (K <- Kappa(SexualFun))</pre>

⁶ Don't confuse this with kappa() in base R that computes something entirely different (the condition number of a matrix).

```
value ASE z
Unweighted 0.1293303 0.06884553 1.878557
Weighted 0.2373806 0.12646539 1.877040

> confint(K)

Kappa lwr upr
Unweighted -0.00560450 0.2642650
Weighted -0.01048698 0.4852482
```

A visualization of agreement, both unweighted and weighted for degree of departure from exact agreement is provided by the agreementplot() function. Figure 4 shows the agreementplot for the SexualFun data, produced as shown below. The Bangdiwala measures represent the proportion of the shaded areas of the diagonal rectangles, using weights w_1 for exact agreement, and w_2 for partial agreement one step from the main diagonal.

```
> agree <- agreementplot(SexualFun, main="Is sex fun?")
> unlist(agree)
```



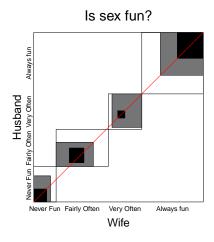


Figure 4: Agreement plot for the SexualFun data.

In other examples, the agreement plot can help to show *sources* of disagreement. For example, when the shaded boxes are above or below the diagonal (red) line, a lack of exact agreement can be attributed in part to different frequency of use of categories by the two raters—lack of *marginal homogeneity*.

3.7. Correspondence analysis

Use the **ca** package for correspondence analysis for visually exploring relationships between rows and columns in contingency tables. For an $r \times c$ table, the method provides a breakdown of the Pearson χ^2 for association in up to $M = \min(r-1, c-1)$ dimensions, and finds scores for the row (x_{im}) and column (y_{jm}) categories such that the observations have the maximum possible correlations.

Here, we carry out a simple correspondence analysis of the HairEye data. The printed results show that nearly 99% of the association between hair color and eye color can be accounted for in 2 dimensions.

Rows:

	Black	Brown	Red	Blond
Mass	0.182432	0.483108	0.119932	0.214527
${\tt ChiDist}$	0.551192	0.159461	0.354770	0.838397
Inertia	0.055425	0.012284	0.015095	0.150793
Dim. 1	-1.104277	-0.324463	-0.283473	1.828229
Dim. 2	1.440917	-0.219111	-2.144015	0.466706

Columns:

	Brown	Hazel	Green	Blue
Mass	0.371622	0.157095	0.108108	0.363176
${\tt ChiDist}$	0.500487	0.288654	0.385727	0.553684
${\tt Inertia}$	0.093086	0.013089	0.016085	0.111337
Dim. 1	-1.077128	-0.465286	0.354011	1.198061
Dim. 2	0.592420	-1.122783	-2.274122	0.556419

The resulting ca object can be plotted just by running the plot() method on the ca object, giving the result in Figure 5. plot.ca() does not allow labels for dimensions; these can be added with title(). It can be seen that most of the association is accounted for by the ordering of both hair color and eye color along Dimension 1, a dark to light dimension.

```
> plot(ca(HairEye), main="Hair Color and Eye Color")
> title(xlab="Dim 1 (89.4%)", ylab="Dim 2 (9.5%)")
```

4. Loglinear Models

You can use the loglm() function in the MASS package to fit log-linear models. Equivalent models can also be fit (from a different perspective) as generalized linear models with the glm() function using the family='poisson' argument, and the gnm package provides a wider range of generalized nonlinear models, particularly for testing structured associations. The visualization methods for these models are most advanced for models fit using loglm(), so this approach is emphasized here. Some extensions of these methods for models fit using glm() and gnm() are contained in the vcdExtra package.

Assume we have a 3-way contingency table based on variables A, B, and C. The possible different forms of loglinear models for a 3-way table are shown in Table 2. The **Model formula** column

-0.4

-0.2

#Blue Blond | Prown | Brown |

Hair Color and Eye Color

Figure 5: Correspondence analysis plot for the HairEye data.

0.2

Dim 1 (89.4%)

0.4

0.6

8.0

0.0

shows how to express each model for loglm() in R.⁷ In the Interpretation column, the symbol "\(\perp\)" is to be read as "is independent of," and "|" means "conditional on," or "adjusting for," or just "given".

Model	Model formula	Symbol	Interpretation
Mutual independence	~A + B + C	[A][B][C]	$A \perp B \perp C$
Joint independence	~A*B + C	[AB][C]	$(A B) \perp C$
Conditional independence	~(A+B)*C	[AC][BC]	$(A \perp B) \mid C$
All two-way associations	~A*B + A*C + B*C	[AB][AC][BC]	homogeneous association
Saturated model	~A*B*C	[ABC]	3-way association

Table 2: Log-linear Models for Three-Way Tables

For example, the formula $^{\sim}A$ + B + C specifies the model of mutual independence with no associations among the three factors. In standard notation for the expected frequencies m_{ijk} , this corresponds to

$$\log(m_{ijk}) = \mu + \lambda_i^A + \lambda_j^B + \lambda_k^C \equiv \mathbf{A} + \mathbf{B} + \mathbf{C}$$

The parameters λ_i^A, λ_j^B and λ_k^C pertain to the differences among the one-way marginal frequencies for the factors A, B and C.

⁷ For glm(), or gnm() with the data in the form of a frequency data.frame, the same model is specified in the form glm(Freq $\sim \ldots$, family="poisson").

Similarly, the model of joint independence allows an association between A and B, but specifies that C is independent of both of these and their combinations,

$$\log(m_{ijk}) = \mu + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_{ij}^{AB} \equiv \texttt{A} \, * \, \texttt{B} \, + \, \texttt{C}$$

In the literature or text books, you will often find these models expressed in shorthand symbolic notation, using brackets, [] to enclose the *high-order terms* in the model. Thus, the joint independence model can be denoted [AB] [C], as shown in the Symbol column in Table 2.

4.1. Fitting with loglm()

Pearson

For example, we can fit the model of mutual independence among hair color, eye color and sex in HairEyeColor as

164.9247 24

Similarly, the models of conditional independence and joint independence are specified as

0

```
> ## Conditional independence
> hec.2 <- loglm(~(Hair + Eye) * Sex, data=HairEyeColor)</pre>
> hec.2
loglm(formula = ~(Hair + Eye) * Sex, data = HairEyeColor)
Statistics:
                      X^2 df P(> X^2)
Likelihood Ratio 156.6779 18
                                     0
Pearson
                 147.9440 18
> ## Joint independence model.
> hec.3 <- loglm(~Hair*Eye + Sex, data=HairEyeColor)
> hec.3
loglm(formula = ~Hair * Eye + Sex, data = HairEyeColor)
Statistics:
                      X^2 df P(> X^2)
Likelihood Ratio 19.85656 15 0.1775045
                 19.56712 15 0.1891745
Pearson
```

Note that printing the model gives a brief summary of the goodness of fit. A set of models can be compared using the anova() function.

```
> anova(hec.1, hec.2, hec.3)
LR tests for hierarchical log-linear models
Model 1:
 "Hair + Eye + Sex
Model 2:
 ~(Hair + Eye) * Sex
Model 3:
 ~Hair * Eye + Sex
           Deviance df Delta(Dev) Delta(df) P(> Delta(Dev)
Model 1
          166.30014 24
Model 2
          156.67789 18
                           9.62225
                                            6
                                                     0.14149
           19.85656 15
Model 3
                                           3
                                                     0.00000
                         136.82133
Saturated
            0.00000 0
                          19.85656
                                           15
                                                     0.17750
```

4.2. Fitting with glm() and gnm()

The glm() approach, and extensions of this in the gnm package allows a much wider class of models for frequency data to be fit than can be handled by loglm(). Of particular importance are models for ordinal factors and for square tables, where we can test more structured hypotheses about the patterns of association than are provided in the tests of general association under loglm().

Example: The data Mental in the vcdExtra package gives a two-way table in frequency form classifying young people by their mental health status and parents' socioeconomic status (SES), where both of these variables are ordered factors.

```
> str(Mental)
'data.frame':
                      24 obs. of 3 variables:
         : Ord.factor w/ 6 levels "1"<"2"<"3"<"4"<...: 1 1 1 1 2 2 2 2 3 3 ...
 $ mental: Ord.factor w/ 4 levels "Well"<"Mild"<..: 1 2 3 4 1 2 3 4 1 2 ...</pre>
 $ Freq : int 64 94 58 46 57 94 54 40 57 105 ...
                                            # display the frequency table
> xtabs(Freq ~ mental+ses, data=Mental)
          ses
                                   6
                  2
                      3
                          4
                              5
mental
             1
                         72
  Well
            64
                57
                     57
                             36
                                  21
                94 105 141
 Mild
            94
                             97
                                 71
  Moderate
            58
                 54
                     65
                         77
                             54
                                 54
                             78
                                 71
  Impaired
            46
                40
                     60
                         94
```

Simple ways of handling ordinal variables involve assigning scores to the table categories, and the simplest cases are to use integer scores, either for the row variable ("column effects" model), the column variable ("row effects" model), or both ("uniform association" model).

```
> indep <- glm(Freq ~ mental + ses, family = poisson, data = Mental) # independence model
```

To fit more parsimonious models than general association, we can define numeric scores for the row and column categories

```
> # Use integer scores for rows/cols
> Cscore <- as.numeric(Mental$ses)
> Rscore <- as.numeric(Mental$mental)</pre>
```

Then, the row effects model, the column effects model, and the uniform association model can be fit as follows:

In model comparisons, we can carry out tests of *nested* models with anova() when those models are listed from smallest to largest. Here, there are two separate paths from the most restrictive (independence) model through the model of uniform association, to those that allow only one of row effects or column effects.

```
> anova(indep, linlin, coleff, test="Chisq")
```

Analysis of Deviance Table

```
Model 1: Freq ~ mental + ses
Model 2: Freq ~ mental + ses + Rscore:Cscore
Model 3: Freq ~ mental + ses + Rscore:ses
  Resid. Df Resid. Dev Df Deviance P(>|Chi|)
1
         15
                47.418
2
         14
                9.895 1
                            37.523 9.035e-10 ***
3
         10
                6.829 4
                            3.066
                                     0.5469
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
> anova(indep, linlin, roweff, test="Chisq")
Analysis of Deviance Table
Model 1: Freq ~ mental + ses
```

```
Model 2: Freq ~ mental + ses + Rscore:Cscore
Model 3: Freq ~ mental + ses + mental:Cscore
  Resid. Df Resid. Dev Df Deviance P(>|Chi|)
1
         15
                47.418
2
         14
                 9.895 1
                            37.523 9.035e-10 ***
                 6.281 2
3
         12
                             3.614
                                       0.1641
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The model of linear by linear association seems best on all accounts.

5. Mosaic plots

Mosaic plots provide an ideal method both for visualizing contingency tables and for visualizing the fit— or more importantly—lack of fit of a loglinear model. For a two-way table, mosaic() fits a model of independence, [A][B] or "A+B as an R formula. For n-way tables, mosaic() can fit any loglinear model, and can also be used to plot a model fit with loglm(). See Friendly (1994, 1999) for the statistical ideas behind these uses of mosaic displays in connection with loglinear models.

The essential idea is to recursively sub-divide a unit square into rectangular "tiles" for the cells of the table, such that the are area of each tile is proportional to the cell frequency. For a given loglinear model, the tiles can then be shaded in various ways to reflect the residuals (lack of fit) for a given model. The pattern of residuals can then be used to suggest a better model or understand where a given model fits or does not fit.

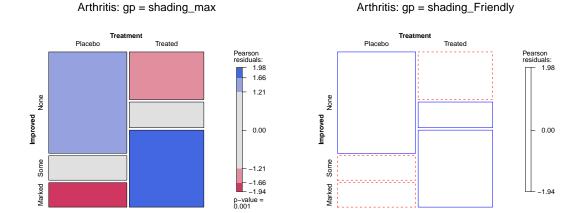
mosaic() provides a wide range of options for the directions of splitting, the specification of shading, labeling, spacing, legend and many other details. It is actually implemented as a special case of a more general class of displays for *n*-way tables called strucplot, including sieve diagrams, association plots, double-decker plots as well as mosaic plots. For details, see help(strucplot) and the "See also" links, and also Meyer, Zeileis, and Hornik (2006), which is available as an R vignette via vignette("strucplot", package="vcd").

Figure 1, showing the association between Treatment and Improved was produced with the following call to mosaic().

```
> mosaic(art, gp = shading_max, split_vertical = TRUE,
+ main="Arthritis: [Treatment] [Improved]")
```

Note that the residuals for the independence model were not large (as shown in the legend), yet the association between Treatment and Improved is highly significant.

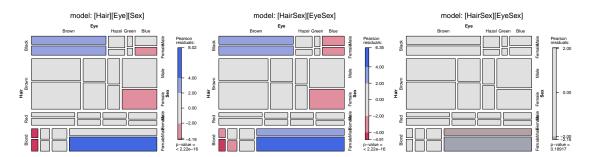
In contrast, one of the other shading schemes, from Friendly (1994) (use: gp = shading_Friendly), uses fixed cutoffs of $\pm 2, \pm 4$, to shade cells which are *individually* significant at approximately $\alpha = 0.05$ and $\alpha = 0.001$ levels, respectively. The right panel below uses gp = shading_Friendly.



5.1. Mosaics for loglinear models

When you have fit a loglinear model using loglm(), and saved the result (as a loglm object) the simplest way to display the results is to use the plot() method for the loglm object. Calling mosaic(loglm.object) has the same result. In Section 4.1 above, we fit several different models to the HairEyeColor data. We can produce mosaic displays of each just by plotting them:

```
> # mosaic plots, using plot.loglm() method
> plot(hec.1, main="model: [Hair][Eye][Sex]")
> plot(hec.2, main="model: [HairSex][EyeSex]")
> plot(hec.3, main="model: [HairEye][Sex]")
```



Alternatively, you can supply the model formula to mosaic() with the expected argument. This is passed to loglm(), which fits the model, and returns residuals used for shading in the plot. For example, here we examine the TV2 constructed in Section 2.6 above. The goal is to see how Network choice depends on (varies with) Day and Time. To do this:

- We fit a model of joint independence of Network on the combinations of Day and Time, with the model formula "Day:Time + Network.
- To make the display more easily read, we place Day and Time on the vertical axis and Network on the horizontal,
- The Time values overlap on the right vertical axis, so we use level() to abbreviate them. mosaic() also supports a more sophisticated set of labeling functions. Instead of changing the

data table, we could have used labeling_args = list(abbreviate = c(Time = 2)) for a similar effect.

The following call to mosaic() produces Figure 6

```
> dimnames(TV2)$Time <- c("8", "9", "10") # re-level for mosaic display
> mosaic(~ Day + Network + Time, data=TV2, expected=~Day:Time + Network,
+ legend=FALSE, gp=shading_Friendly)
```

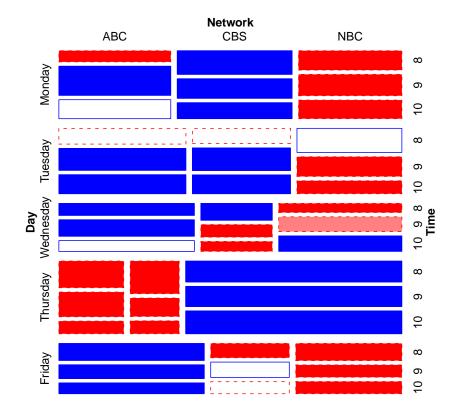


Figure 6: Mosaic plot for the TV data showing model of joint independence, Day: Time + Network.

From this, it is easy to read from the display how network choice varies with day and time. For example, CBS dominates in all time slots on Monday; ABC and NBC dominate on Tuesday, particularly in the later time slots; Thursday is an NBC day, while on Friday, ABC gets the greatest share.

In interpreting this mosaic and other plots, it is important to understand that associations included in the model—here, that between day and time—are *not* shown in the shading of the cells, because they have been fitted (taken into account) in the loglinear model.

For comparison, you might want to try fitting the model of homogeneous association. This allows all pairs of factors to be associated, but asserts that each pairwise association is the same across the levels of the remaining factor. The resulting plot displays the contributions to a 3-way association, but is not shown here.

```
> mosaic(~ Day + Network + Time, data=TV2,
+ expected=~Day:Time + Day:Network + Time:Network,
+ legend=FALSE, gp=shading_Friendly)
```

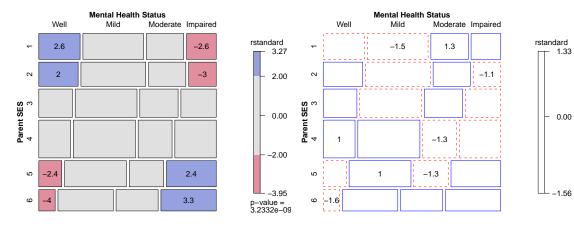
5.2. Mosaics for glm() and gnm() models

The **vcdExtra** provides some additional methods, mosaic.glm() and mosaic.gnm() for models fit with glm() and gnm(). These are not restricted to the Poisson family, but only apply to cases where the response variable is non-negative.

Here, we plot the independence and the linear-by-linear association model for the Mental health data from Section 4.2. These examples illustrate some of the options for labeling (variable names and residuals printed in cells).

Mental health data: Independence

Mental health data: Linear x Linear



6. Continuous predictors: spine and conditional density plots

When continuous predictors are available—and potentially important— in explaining a categorical outcome, models for that outcome include: logistic regression (binary response), the proportional odds model (ordered polytomous response), multinomial (generalized) logistic regression. Many of these are special cases of the generalized linear model using the "poisson" or "binomial" family and their relatives.

I don't go into fitting such models here, but I would be remiss not to illustrate some visualizations in **vcd** that are helpful here. The first of these is the spine plot or spinogram (Hummel 1996)

(produced with spine()). These are special cases of mosaic plots with specific spacing and shading to show how a categorical response varies with a continuous or categorical predictor.

They are also a generalization of stacked bar plots where not the heights but the widths of the bars corresponds to the relative frequencies of x. The heights of the bars then correspond to the conditional relative frequencies of y in every x group.

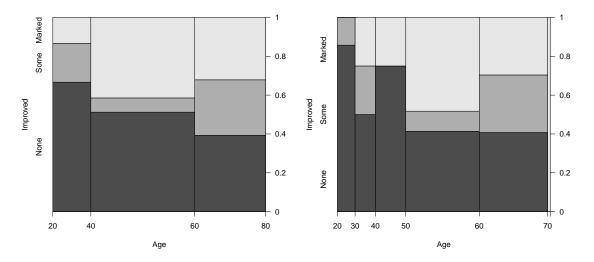
For the Arthritis data, we can see how Improved varies with Age as follows. spine() takes a formula of the form $y \sim x$ with a single dependent factor and a single explanatory variable x (a numeric variable or a factor). The range of a numeric variable x is divided into intervals based on the breaks argument, and stacked bars are drawn to show the distribution of y as x varies. As shown below, the discrete table that is visualized is returned by the function.

> (spine(Improved ~ Age, data = Arthritis, breaks = 3))

Improved Age None Some Marked [20,40] 10 3 2 (40,60] 21 3 17 (60,80] 11 8 9

> (spine(Improved ~ Age, data = Arthritis, breaks = "Scott"))

Improved					
Age	None	Some	Marked		
[20,30]	6	1	C		
(30,40]	4	2	2		
(40,50]	9	0	3		
(50,60]	12	3	14		
(60,70]	11	8	8		
(70,80]	0	0	1		



The conditional density plot (Hofmann and Theus 2005) is a further generalization. This visualization technique is similar to spinograms, but uses a smoothing approach rather than discretizing the explanatory variable. As well, it uses the original x axis and not a distorted one.

```
> cdplot(Improved ~ Age, data = Arthritis)
> with(Arthritis, rug(jitter(Age), col="white", quiet=TRUE))
```

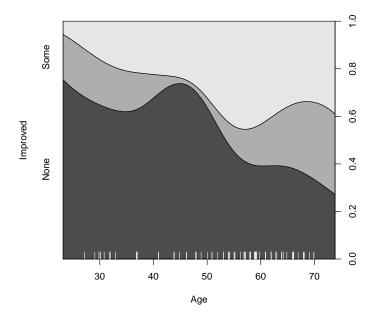


Figure 7: Conditional density plot for the Arthritis data showing the variation of Improved with Age.

In such plots, it is useful to also see the distribution of the observations across the horizontal axis, e.g., with a rug() plot. Figure 7 uses cdplot() from the graphics package rather than cd_plot() from vcd, and is produced with

```
> cdplot(Improved ~ Age, data = Arthritis)
> with(Arthritis, rug(jitter(Age), col="white", quiet=TRUE))
```

From Figure 7 it can be easily seen that the proportion of patients reporting Some or Marked improvement increases with Age, but there are some peculiar bumps in the distribution. These may be real or artifactual, but they would be hard to see with most other visualization methods. When we switch from non-parametric data exploration to parametric statistical models, such effects are easily missed.

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