# The impossibility of measuring productivity in small samples

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## Tokens & types

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

**type-frequency list**

| $w$ | $f_w$ |
|---|---|
| *recently* | 1 |
| *very* | 5 |
| *not* | 3 |
| *otherwise* | 1 |
| *much* | 2 |
| *merely* | 2 |
| *now* | 1 |





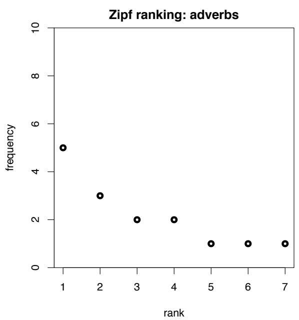
## Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

**Zipf ranking**

| $w$ | $r$ | $f_r$ |
|---|---|---|
| *very* | 1 | 5 |
| *not* | 2 | 3 |
| *merely* | 3 | 2 |
| *much* | 4 | 2 |
| *now* | 5 | 1 |
| *otherwise* | 6 | 1 |
| *recently* | 7 | 1 |



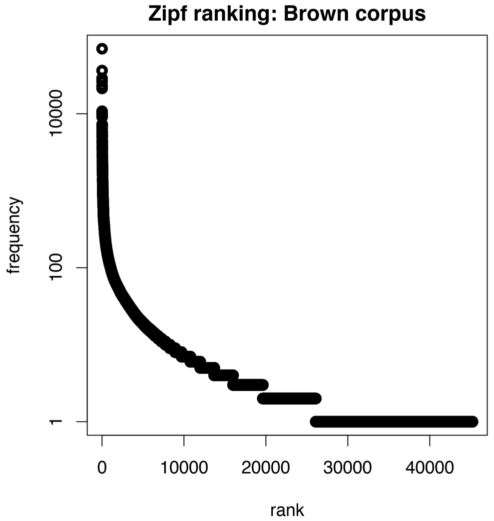





## A realistic Zipf ranking: the Brown corpus

| top frequencies | | | bottom frequencies | | |
|---|---|---|---|---|---|
| **r** | **f** | **word** | **rank range** | **f** | **randomly selected examples** |
| 1 | 69836 | the | 7731 – 8271 | 10 | schedules, polynomials, bleak |
| 2 | 36365 | of | 8272 – 8922 | 9 | tolerance, shaved, hymn |
| 3 | 28826 | and | 8923 – 9703 | 8 | decreased, abolish, irresistible |
| 4 | 26126 | to | 9704 – 10783 | 7 | immunity, cruising, titan |
| 5 | 23157 | a | 10784 – 11985 | 6 | geographic, lauro, portrayed |
| 6 | 21314 | in | 11986 – 13690 | 5 | grigori, slashing, developer |
| 7 | 10777 | that | 13691 – 15991 | 4 | sheath, gaulle, ellipsoids |
| 8 | 10182 | is | 15992 – 19627 | 3 | mc, initials, abstracted |
| 9 | 9968 | was | 19628 – 26085 | 2 | thar, slackening, deluxe |
| 10 | 9801 | he | 26086 – 45215 | 1 | beck, encompasses, second-place |

## A realistic Zipf ranking: the Brown corpus








## Frequency spectrum

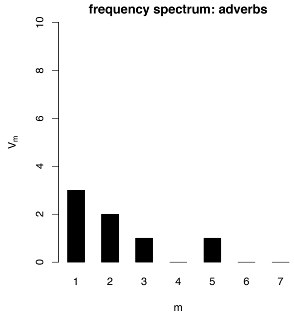
- pool types with $f = 1$ (hapax legomena), types with $f = 2$ (dis legomena), $\ldots$, $f = m$, $\ldots$
- $V_1 = 3$: number of hapax legomena (*now, otherwise, recently*)
- $V_2 = 2$: number of dis legomena (*merely, much*)
- general definition: $V_m = |\{w \mid f_w = m\}|$

**Zipf ranking**

| $w$ | $r$ | $f_r$ |
|---|---|---|
| *very* | 1 | 5 |
| *not* | 2 | 3 |
| *merely* | 3 | 2 |
| *much* | 4 | 2 |
| *now* | 5 | 1 |
| *otherwise* | 6 | 1 |
| *recently* | 7 | 1 |

**frequency spectrum**

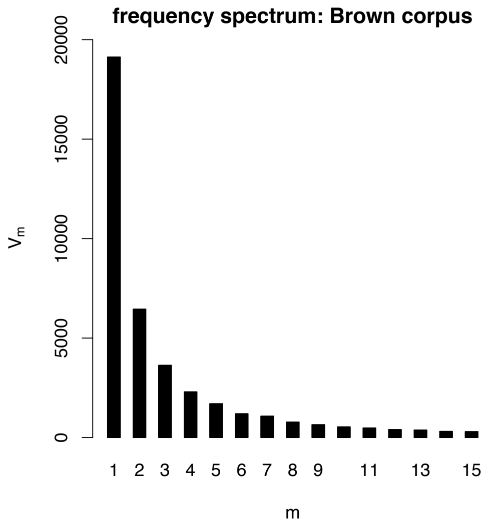
| $m$ | $V_m$ |
|---|---|
| 1 | 3 |
| 2 | 2 |
| 3 | 1 |
| 5 | 1 |








## A realistic frequency spectrum: the Brown corpus



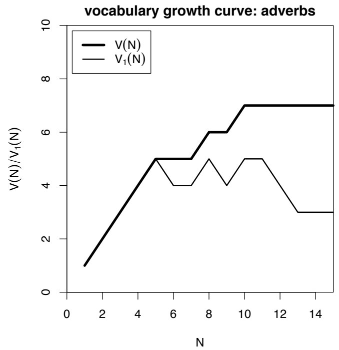




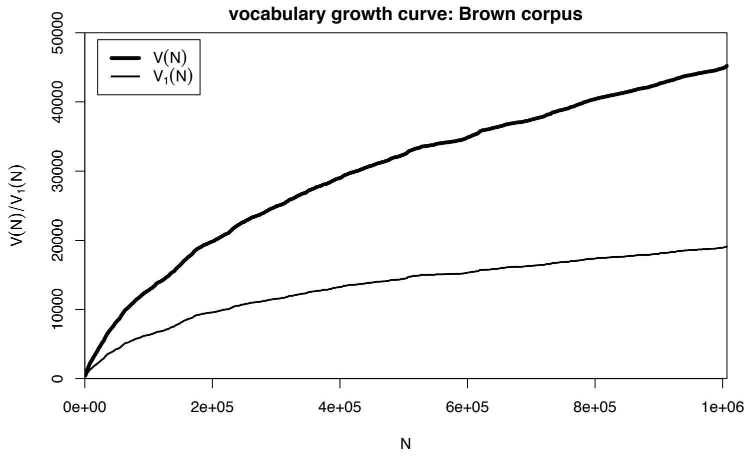

## Vocabulary growth curve

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 1$, $V(N) = 1$, $V_1(N) = 1$
- $N = 3$, $V(N) = 3$, $V_1(N) = 3$
- $N = 7$, $V(N) = 5$, $V_1(N) = 4$
- $N = 12$, $V(N) = 7$, $V_1(N) = 4$
- $N = 15$, $V(N) = 7$, $V_1(N) = 3$



# A realistic vocabulary growth curve: the Brown corpus

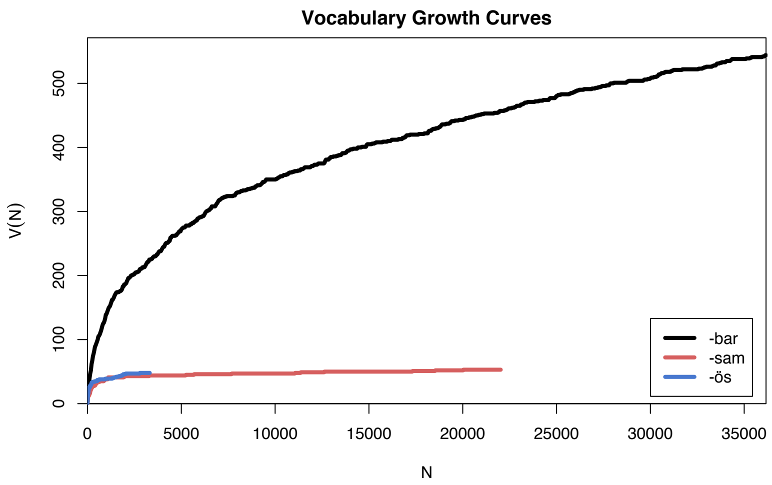







# Measuring morphological productivity

example from Evert and Lüdeling (2001)








# Quantitative measures of productivity

(Tweedie and Baayen 1998; Baayen 2001)

- Baayen's (1991) productivity index $\mathcal{P}$ (slope of vocabulary growth curve)

$$\mathcal{P} = \frac{V_1}{N}$$

- TTR = type-token ratio

$$\mathrm{TTR} = \frac{V}{N}$$

- Population size

$$S = \lim_{N \to \infty} V(N)$$

- Herdan's law (1964)

$$C = \frac{\log V}{\log N}$$

- Yule (1944) / Simpson (1949)

$$K = 10\,000 \cdot \frac{\sum_m m^2 V_m - N}{N^2}$$

- Guiraud (1954)

$$R = \frac{V}{\sqrt{N}}$$

- Sichel (1975)

$$S = \frac{V_2}{V}$$

- Honoré (1979)

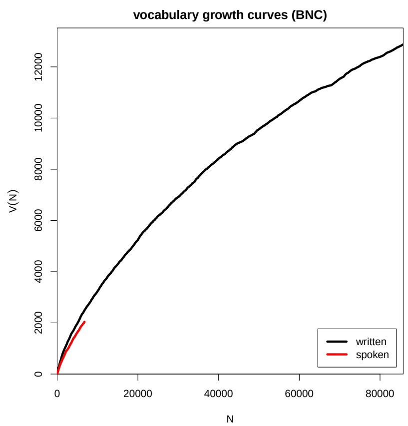
$$H = \frac{\log N}{1 - \frac{V_1}{V}}$$



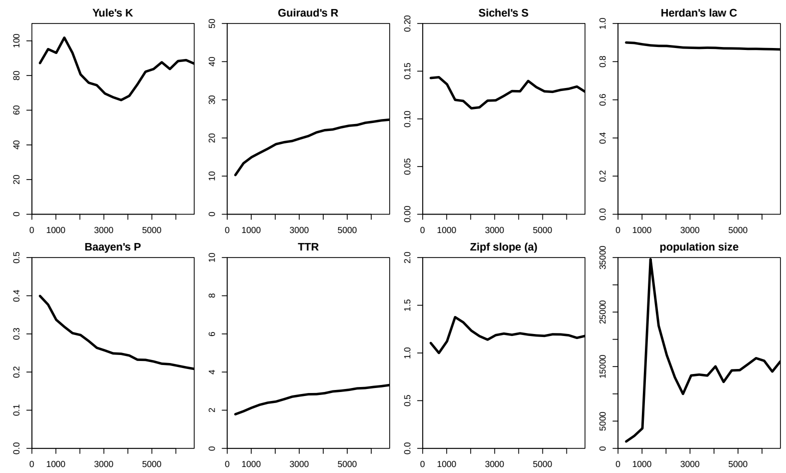


# Productivity measures for bare singulars in the BNC

| | spoken | written |
|---|---|---|
| $V$ | 2,039 | 12,876 |
| $N$ | 6,766 | 85,750 |
| $K$ | 86.84 | 28.57 |
| $R$ | 24.79 | 43.97 |
| $S$ | 0.13 | 0.15 |
| $C$ | 0.86 | 0.83 |
| $\mathcal{P}$ | 0.21 | 0.08 |
| TTR | 0.301 | 0.150 |
| pop. $S$ | 15,958 | 36,874 |



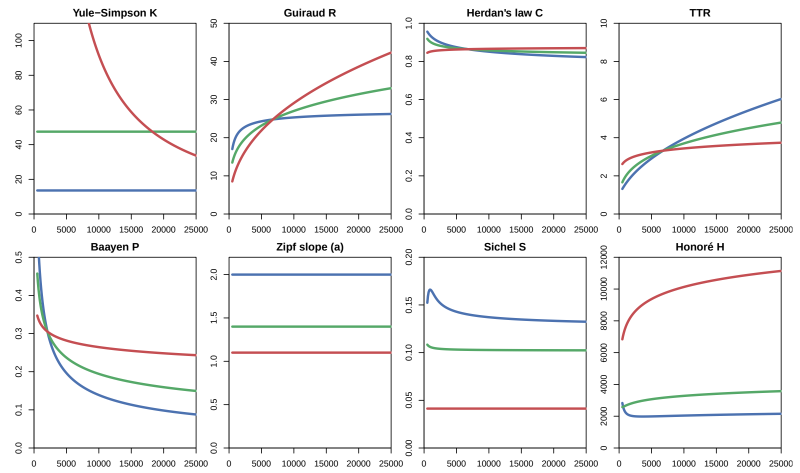
## Are these "lexical constants" really constant?







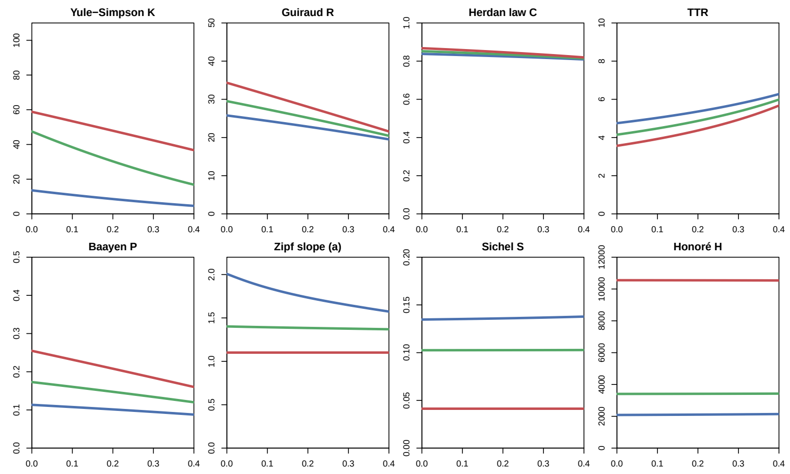

## Simulation: sample size








## Simulation: frequent lexicalized types








## LNRE models

- State-of-the-art approach to measuring productivity: **LNRE models** (Baayen 2001)
  - LNRE = Large Number of Rare Events
  - Baayen (2001) has 887 citations on Google Scholar
- Standard implementation: **zipfR** (Evert and Baroni 2007)
  - 76 citations on Google Scholar
  - only a few search results for Baayen's `lexstats` software
- LNRE uses various approximations and simplifications to obtain a tractable and elegant model
  - LNRE model usually minor component of complex procedure
  - often applied to very large samples ($N > 1$ M tokens)

## The LNRE population

- Population: set of $S$ types $w_i$ with occurrence **probabilities** $\pi_i$
- $S$ = **population diversity** can be finite or infinite ($S = \infty$)
- Not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$

- **parametric** statistical **model** to describe full population (esp. for $S = \infty$), i.e. a function $i \mapsto \pi_i$
  - type probabilities $\pi_i$ cannot be estimated reliably from a sample, but parameters of this function can
  - NB: population index $i \neq$ Zipf rank $r$





## Zipf-Mandelbrot law as a population model

- Zipf-Mandelbrot law for type probabilities:

$$\pi_i := \frac{C}{(i+b)^a}$$

- Two free parameters: $a > 1$ and $b \geq 0$
  - ☞ $C$ is not a parameter but a normalization constant, needed to ensure that $\sum_i \pi_i = 1$
- Third parameter: $S > 0$ or $S = \infty$

- This is the **Zipf-Mandelbrot** population model (Evert 2004)





## Samples: type frequency list & spectrum

| rank $r$ | $f_r$ | type $i$ |
|---|---|---|
| 1 | 37 | 6 |
| 2 | 36 | 1 |
| 3 | 33 | 3 |
| 4 | 31 | 7 |
| 5 | 31 | 10 |
| 6 | 30 | 5 |
| 7 | 28 | 12 |
| 8 | 27 | 2 |
| 9 | 24 | 4 |
| 10 | 24 | 16 |
| 11 | 23 | 8 |
| 12 | 22 | 14 |
| ⋮ | ⋮ | ⋮ |

| $m$ | $V_m$ |
|---|---|
| 1 | 83 |
| 2 | 22 |
| 3 | 20 |
| 4 | 12 |
| 5 | 10 |
| 6 | 5 |
| 7 | 5 |
| 8 | 3 |
| 9 | 3 |
| 10 | 3 |
| ⋮ | ⋮ |

**sample #1**





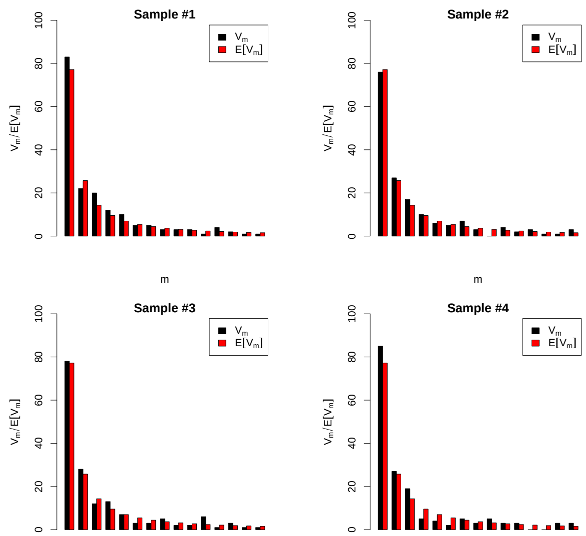
## Samples: type frequency list & spectrum

| rank $r$ | $f_r$ | type $i$ |
|---|---|---|
| 1 | 39 | 2 |
| 2 | 34 | 3 |
| 3 | 30 | 5 |
| 4 | 29 | 10 |
| 5 | 28 | 8 |
| 6 | 26 | 1 |
| 7 | 25 | 13 |
| 8 | 24 | 7 |
| 9 | 23 | 6 |
| 10 | 23 | 11 |
| 11 | 20 | 4 |
| 12 | 19 | 17 |
| ⋮ | ⋮ | ⋮ |

| $m$ | $V_m$ |
|---|---|
| 1 | 76 |
| 2 | 27 |
| 3 | 17 |
| 4 | 10 |
| 5 | 6 |
| 6 | 5 |
| 7 | 7 |
| 8 | 3 |
| 10 | 4 |
| 11 | 2 |
| ⋮ | ⋮ |

**sample #2**

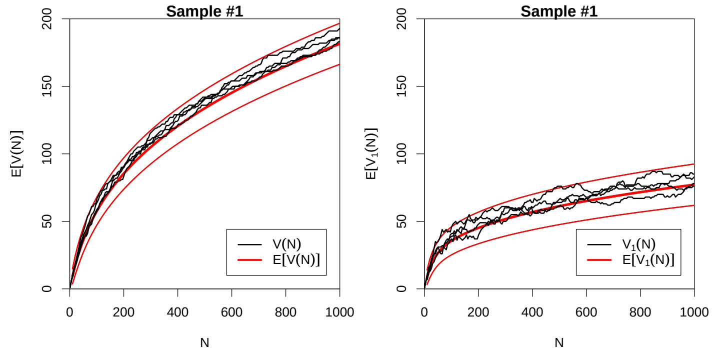
## Expectation: frequency spectrum








## Expectation: vocabulary growth curve



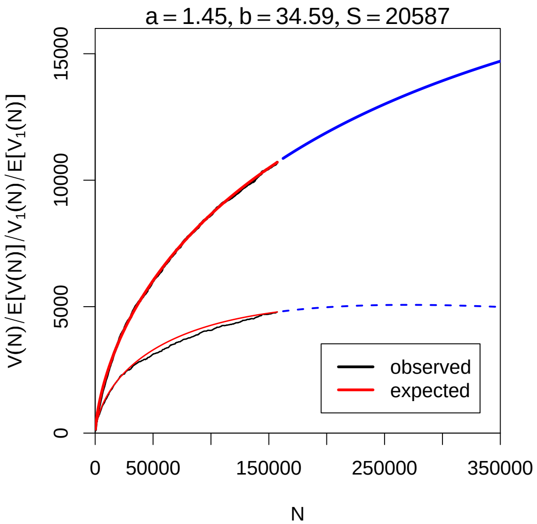

"Confidence intervals" indicate predicted sampling distribution:

☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines





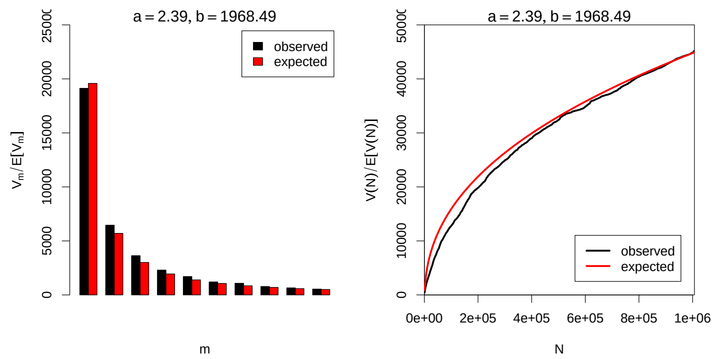
## Extrapolating vocabulary growth







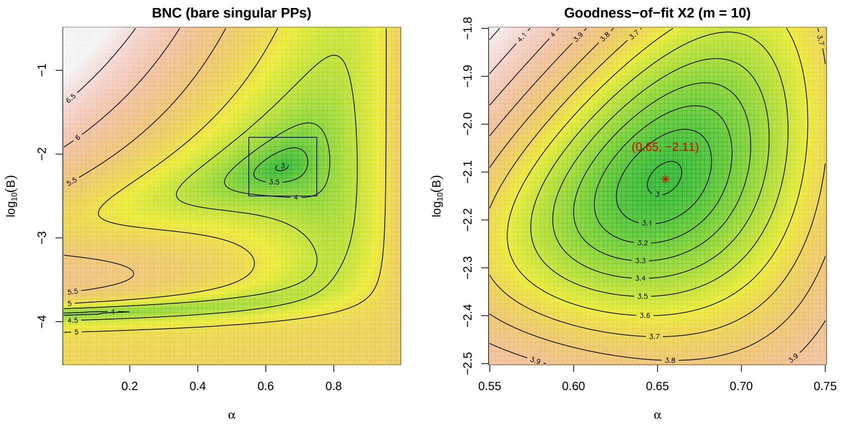

## Parameter estimation




- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure based on minimisation of suitable cost function: $a = 2.39$ and $b = 1968$

## Parameter estimation

ZM model with free parameters $0 < \alpha < 1$ and $B > 0$








## Problems of LNRE models

- Assumption: corpus data = random sample
  - ☞ holds reasonably well for morphological productivity
  - ☞ simple but effective ECHO correction (Baroni and Evert 2007)
- Approximation: independent Poisson sampling
  - instead of correct multinomial sampling distribution
- Approximation: multivariate normal distribution of $V$ and $V_m$
  - true sampling distribution is completely intractable
- Approximation: continuous type density function $g(\pi)$
  - instead of discrete type probabilities of Z-M law

➥ Wide-spread irresponsible application of LNRE models to small samples (e.g. Lüdeling and Evert 2005)





## How reliable are the fitted models?

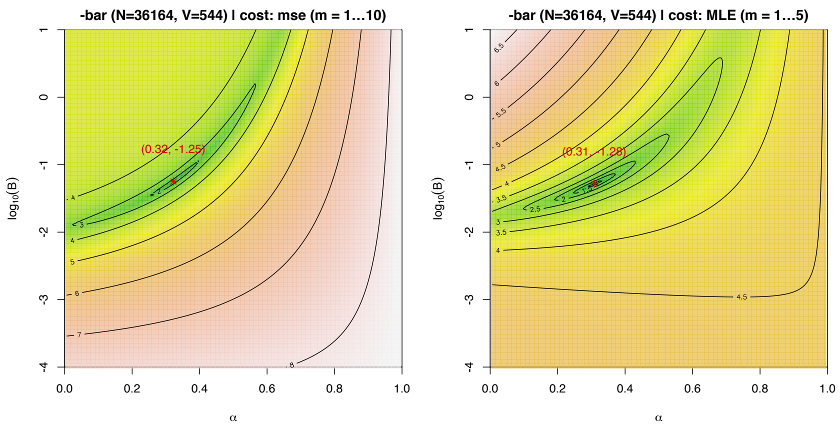
Three potential issues:

1. **Model assumptions $\neq$ population**
   (e.g. distribution does not follow a Zipf-Mandelbrot law)
   - ☞ model cannot be adequate, regardless of parameter settings
2. **Parameter estimation unsuccessful**
   (i.e. suboptimal goodness-of-fit to training data)
   - ☞ optimization algorithm trapped in local minimum
   - ☞ can result in highly inaccurate model
3. Uncertainty due to sampling variation
   (i.e. training data differ from population distribution)
   - ☞ model fitted to training data, may not reflect true population
   - ☞ another training sample would have led to different parameters
   - ☞ especially critical for small samples ($N < 10{,}000$)





## Cost functions for German word-formation affixes



## Goodness-of-fit

(Baayen 2001, Sec. 3.3)

- Statistics: confidence intervals for population coefficients by inverting hypothesis tests (all $H_0 : \mu = x$ with $p > .05$)
- Multivariate normal approximation for $\mathbf{V} = (V, V_1, \ldots, V_k)$:

$$\Pr(\mathbf{V} = \mathbf{v}) \sim \frac{e^{-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{v}-\boldsymbol{\mu})}}{\sqrt{(2\pi)^{k+1} \det \boldsymbol{\Sigma}}}$$

with $\boldsymbol{\mu} = (\mathrm{E}[V], \mathrm{E}[V_1], \mathrm{E}[V_2], \ldots)$ and $\boldsymbol{\Sigma}$ = covariance matrix

- Test statistic

$$X^2 = (\mathbf{V} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{V} - \boldsymbol{\mu}) \sim \chi^2_{k+1}$$

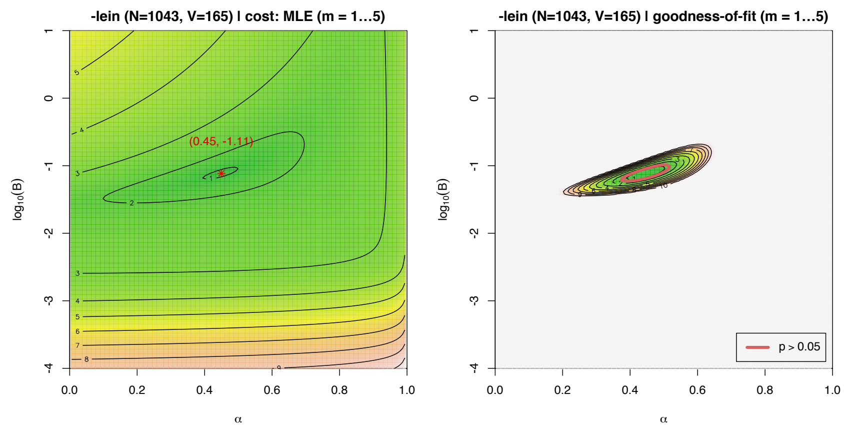
➥ Multivariate chi-squared test of **goodness-of-fit**

☞ significant rejection of the LNRE model for $p < .05$





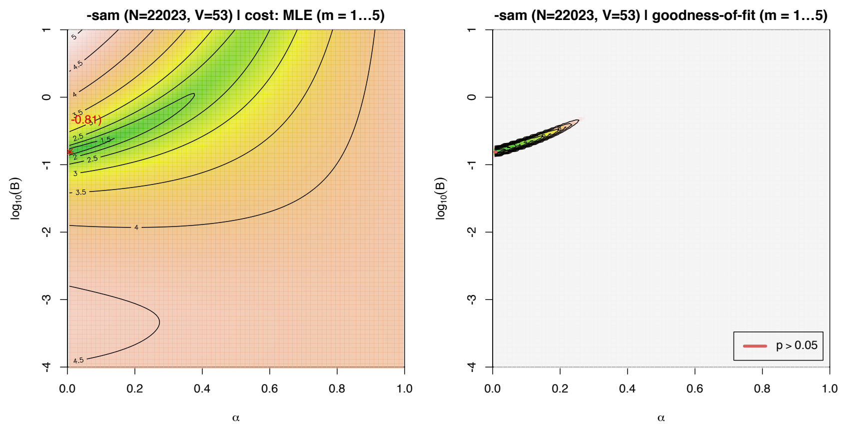
## Confidence sets based on goodness-of-fit test?








## Confidence sets based on goodness-of-fit test?

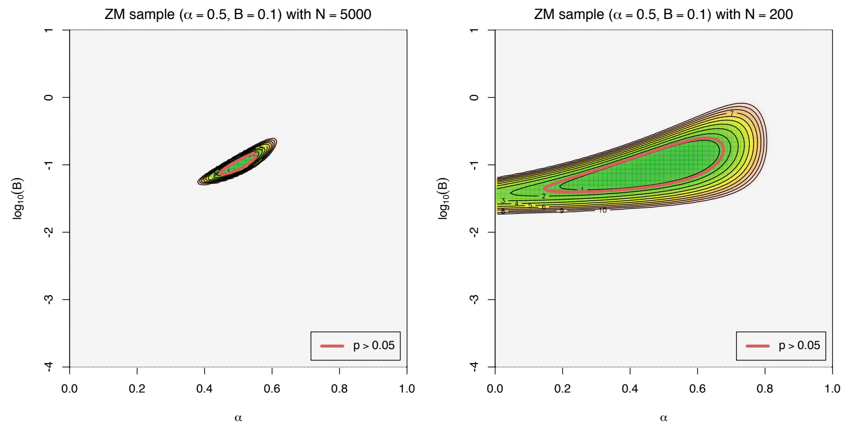







## Confidence sets for idealized samples from ZM population

➜ $X^2$ tests model parameters rather than goodness-of-fit



## How reliable are the fitted models?

Three potential issues:

1. Model assumptions $\neq$ population
   (e.g. distribution does not follow a Zipf-Mandelbrot law)
   - ☞ model cannot be adequate, regardless of parameter settings
2. Parameter estimation unsuccessful
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   - ☞ model fitted to training data, may not reflect true population
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## Bootstrapping

- An empirical approach to sampling variation:
  - take many random samples from the same population
  - estimate LNRE model from each sample
  - analyse distribution of model parameters, goodness-of-fit, etc. (mean, median, s.d., boxplot, histogram, ...)
  - problem: how to obtain the additional samples?
- Bootstrapping (Efron 1979)
  - resample from observed data *with replacement*
  - this approach is not suitable for type-token distributions (resamples underestimate vocabulary size $V$!)
- Parametric bootstrapping
  - use fitted model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known

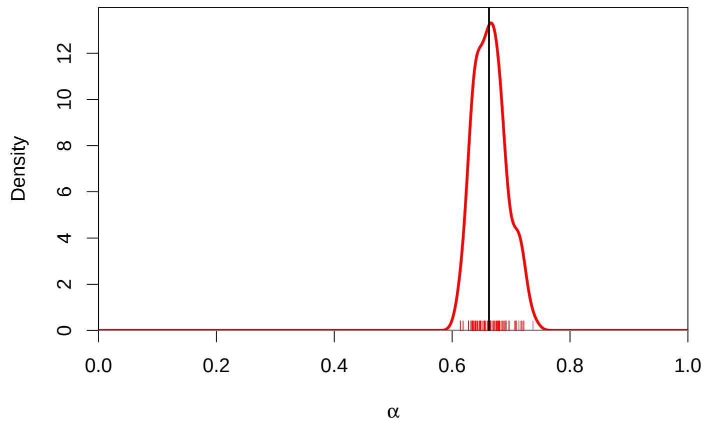




## Bootstrapping

parametric bootstrapping with 100 replicates, fZM samples for $N = 3467$

**Zipfian slope** $a = 1/\alpha$



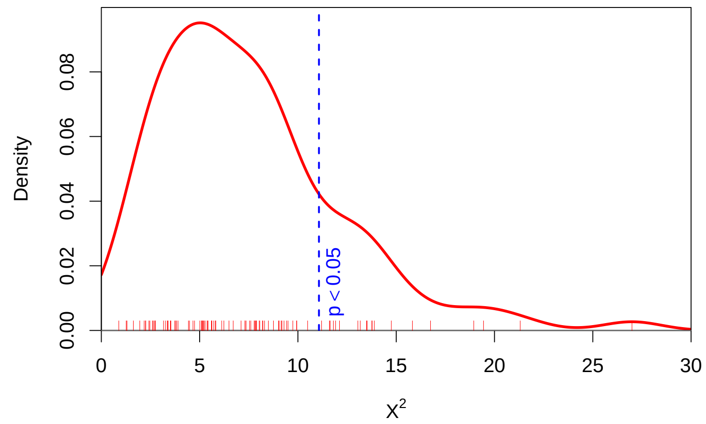





## Bootstrapping

parametric bootstrapping with 100 replicates, fZM samples for $N = 3467$

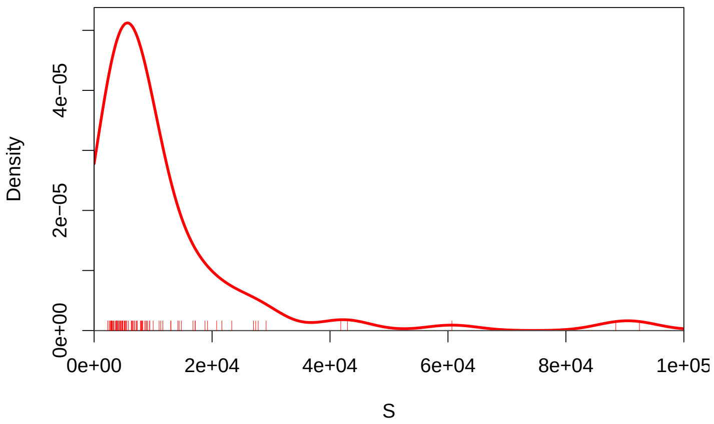
**Goodness-of-fit statistic** $X^2$ (model not plausible for $X^2 > 11$)



# Bootstrapping

parametric bootstrapping with 100 replicates, fZM samples for $N = 3467$

### Population diversity $S$

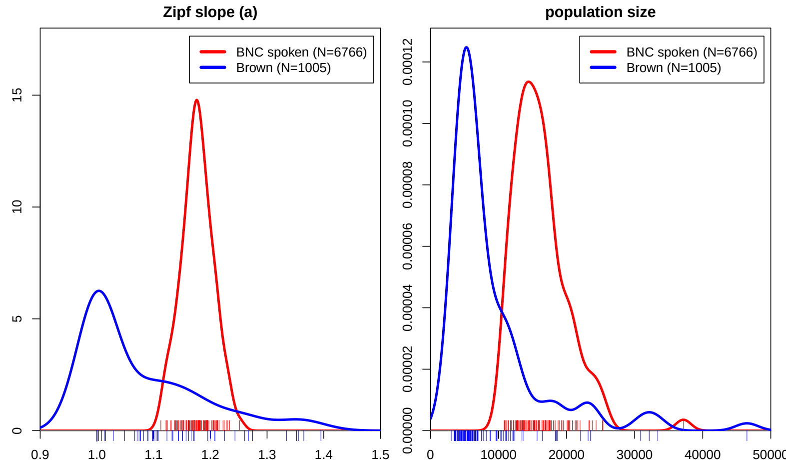







# Sample size matters!

Brown corpus is too small for reliable LNRE parameter estimation (bare singulars)








Thank you!





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