The impossibility of measuring productivity in small samples

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Tokens & types

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

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our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

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type-frequency list

w	f_w
recently	1
very	5
not	3
otherwise	1
much	2
merely	2
now	1

Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

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Zipf ranking

w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1

Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

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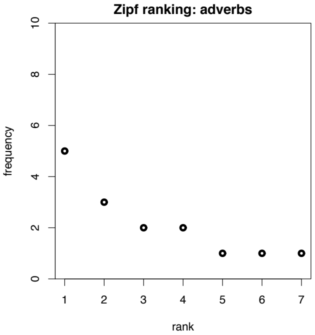

A realistic Zipf ranking: the Brown corpus

top frequencies			bottom frequencies		
r	**f**	**word**	**rank range**	**f**	**randomly selected examples**
1	69836	the	7731 – 8271	10	schedules, polynomials, bleak
2	36365	of	8272 – 8922	9	tolerance, shaved, hymn
3	28826	and	8923 – 9703	8	decreased, abolish, irresistible
4	26126	to	9704 – 10783	7	immunity, cruising, titan
5	23157	a	10784 – 11985	6	geographic, lauro, portrayed
6	21314	in	11986 – 13690	5	grigori, slashing, developer
7	10777	that	13691 – 15991	4	sheath, gaulle, ellipsoids
8	10182	is	15992 – 19627	3	mc, initials, abstracted
9	9968	was	19628 – 26085	2	thar, slackening, deluxe
10	9801	he	26086 – 45215	1	beck, encompasses, second-place

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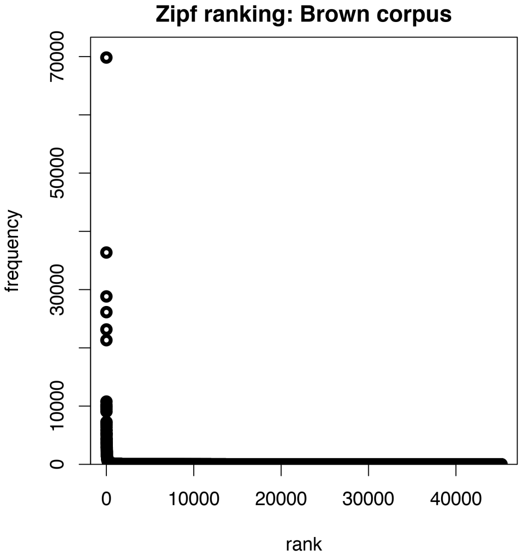

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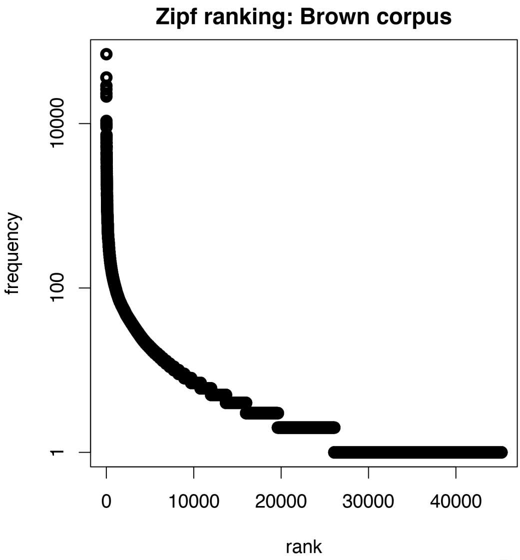

Frequency spectrum

- pool types with $f = 1$ (hapax legomena), types with $f = 2$ (dis legomena), ..., $f = m$, ...
- $V_1 = 3$: number of hapax legomena (*now, otherwise, recently*)
- $V_2 = 2$: number of dis legomena (*merely, much*)
- general definition: $V_m = |\{w \mid f_w = m\}|$

Zipf ranking

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frequency spectrum

m	V_m
1	3
2	2
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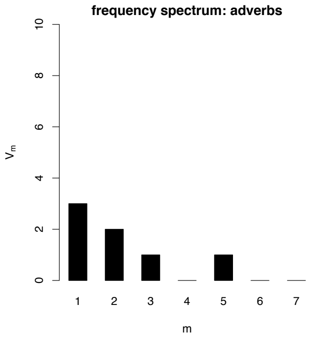

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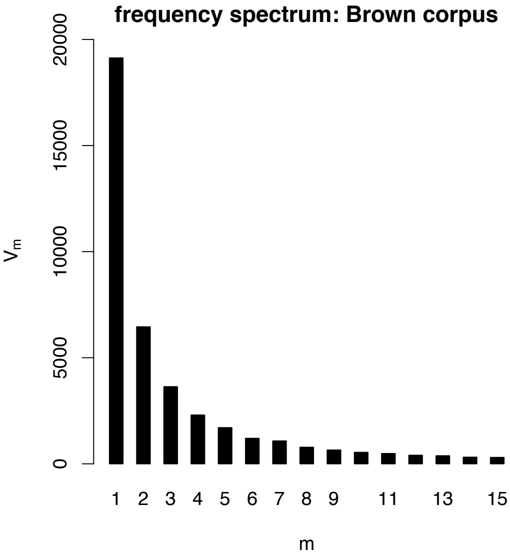

Vocabulary growth curve

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 1,\ V(N) = 1,\ V_1(N) = 1$

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- $N = 12,\ V(N) = 7,\ V_1(N) = 4$
- $N = 15,\ V(N) = 7,\ V_1(N) = 3$

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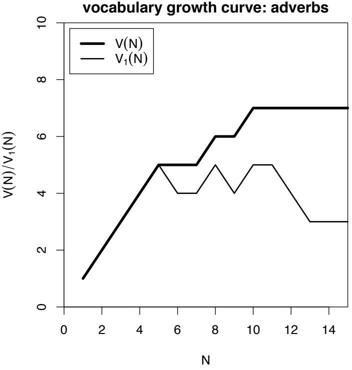

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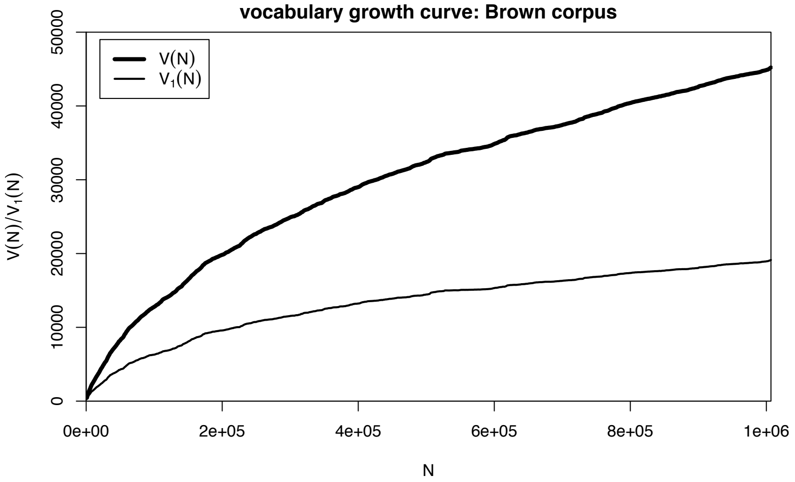
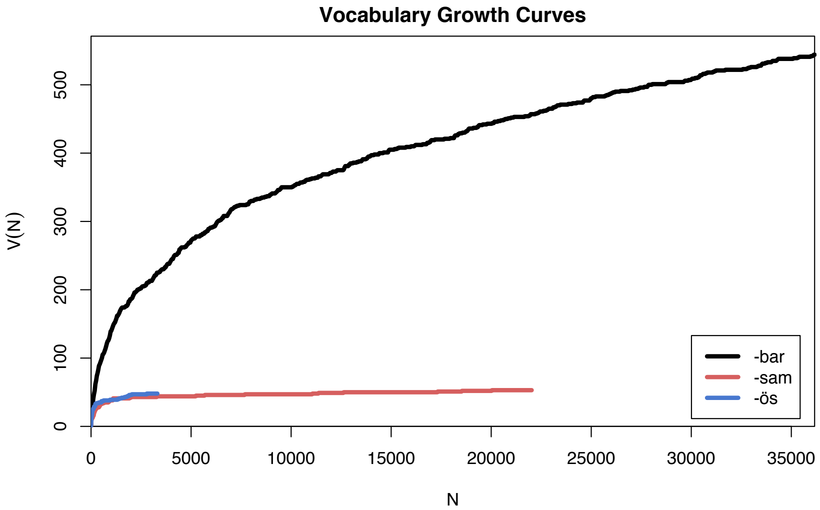


Measuring morphological productivity

example from Evert and Lüdeling (2001)



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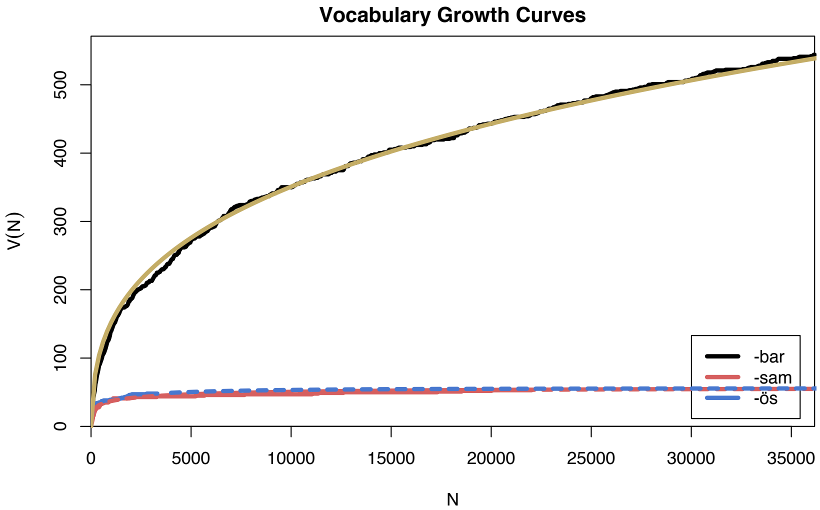

Quantitative measures of productivity

(Tweedie and Baayen 1998; Baayen 2001)

- Baayen's (1991) productivity index $\mathcal{P}$
  (slope of vocabulary growth curve)

$$\mathcal{P} = \frac{V_1}{N}$$

- TTR = type-token ratio

$$\mathrm{TTR} = \frac{V}{N}$$

- Population size

$$S = \lim_{N \to \infty} V(N)$$

- Herdan's law (1964)

$$C = \frac{\log V}{\log N}$$

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$$C = \frac{\log V}{\log N}$$

- Yule (1944) / Simpson (1949)

$$K = 10\,000 \cdot \frac{\sum_m m^2 V_m - N}{N^2}$$

- Guiraud (1954)

$$R = \frac{V}{\sqrt{N}}$$

- Sichel (1975)

$$S = \frac{V_2}{V}$$

- Honoré (1979)

$$H = \frac{\log N}{1 - \frac{V_1}{V}}$$

Productivity measures for bare singulars in the BNC

	spoken	written
V	2,039	12,876
N	6,766	85,750
K	86.84	28.57
R	24.79	43.97
S	0.13	0.15
C	0.86	0.83
$\mathcal{P}$	0.21	0.08
TTR	0.301	0.150
pop. S	15,958	36,874

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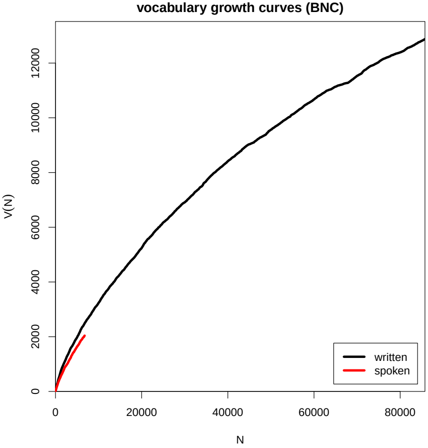
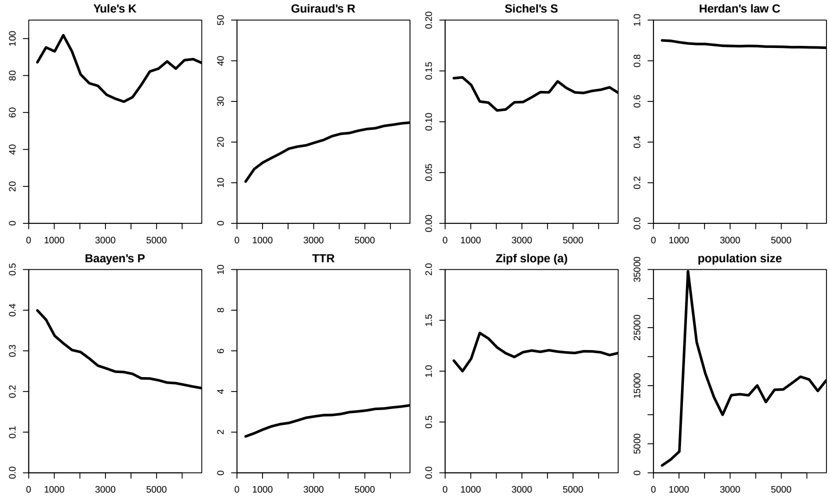
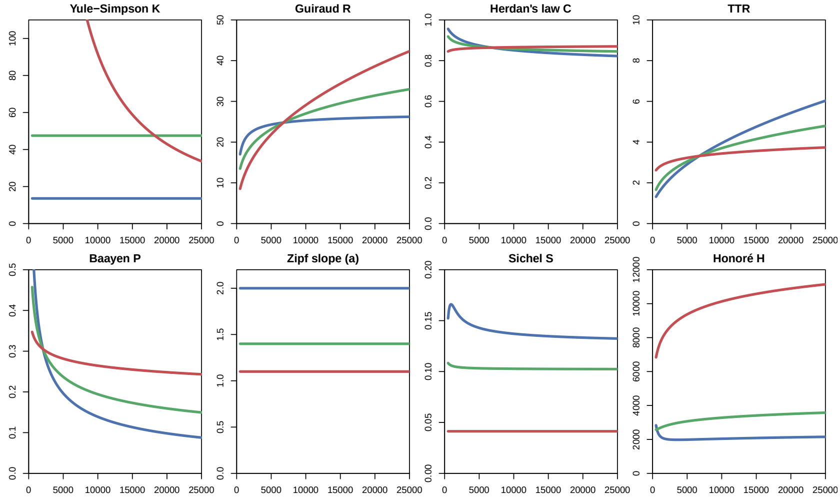


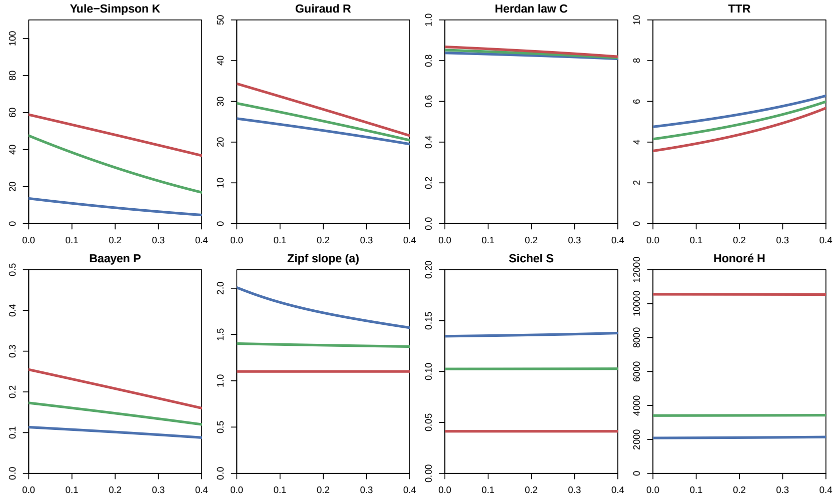
Are these "lexical constants" really constant?



Simulation: sample size



Simulation: frequent lexicalized types



LNRE models

- State-of-the-art approach to measuring productivity: **LNRE models** (Baayen 2001)
  - LNRE = Large Number of Rare Events
  - Baayen (2001) has 887 citations on Google Scholar
- Standard implementation: **zipfR** (Evert and Baroni 2007)
  - 76 citations on Google Scholar
  - only a few search results for Baayen's `lexstats` software
- LNRE uses various approximations and simplifications to obtain a tractable and elegant model
  - LNRE model usually minor component of complex procedure
  - often applied to very large samples ($N > 1$ M tokens)

The LNRE population

- Population: set of S types w_i with occurrence **probabilities** π_i
- $S =$ **population diversity** can be finite or infinite ($S = \infty$)
- Not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$

- **parametric** statistical **model** to describe full population (esp. for $S = \infty$), i.e. a function $i \mapsto \pi_i$
  - type probabilities π_i cannot be estimated reliably from a sample, but parameters of this function can
  - NB: population index $i \neq$ Zipf rank r

Zipf-Mandelbrot law as a population model

- Zipf-Mandelbrot law for type probabilities:

$$\pi_i := \frac{C}{(i+b)^a}$$

- Two free parameters: $a > 1$ and $b \geq 0$
  - ☞ C is not a parameter but a normalization constant, needed to ensure that $\sum_i \pi_i = 1$
- Third parameter: $S > 0$ or $S = \infty$

- This is the **Zipf-Mandelbrot** population model (Evert 2004)

Samples: type frequency list & spectrum

rank r	f_r	type i
1	37	6
2	36	1
3	33	3
4	31	7
5	31	10
6	30	5
7	28	12
8	27	2
9	24	4
10	24	16
11	23	8
12	22	14
⋮	⋮	⋮

m	V_m
1	83
2	22
3	20
4	12
5	10
6	5
7	5
8	3
9	3
10	3
⋮	⋮

sample #1

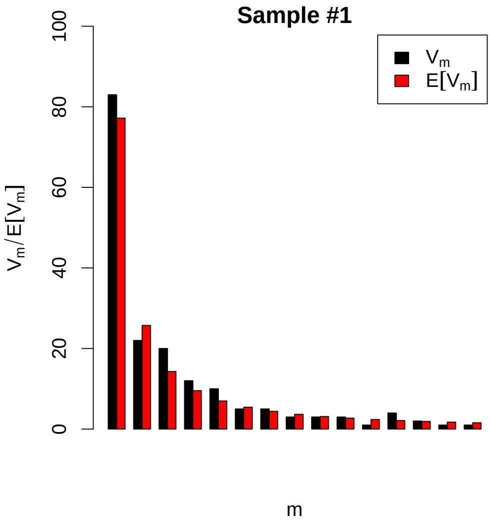
Samples: type frequency list & spectrum

rank r	f_r	type i
1	39	2
2	34	3
3	30	5
4	29	10
5	28	8
6	26	1
7	25	13
8	24	7
9	23	6
10	23	11
11	20	4
12	19	17
$\vdots$	$\vdots$	$\vdots$

m	V_m
1	76
2	27
3	17
4	10
5	6
6	5
7	7
8	3
10	4
11	2
$\vdots$	$\vdots$

sample #2

Expectation: frequency spectrum



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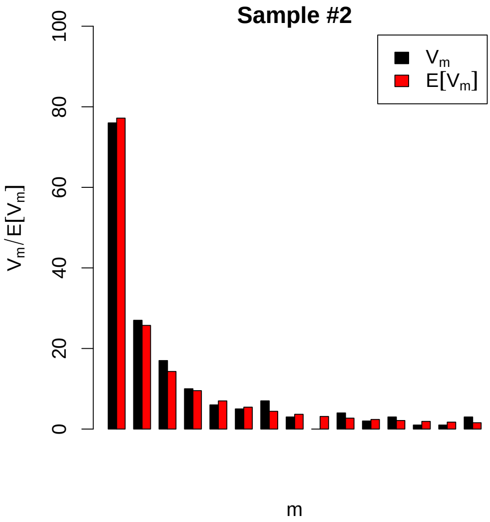

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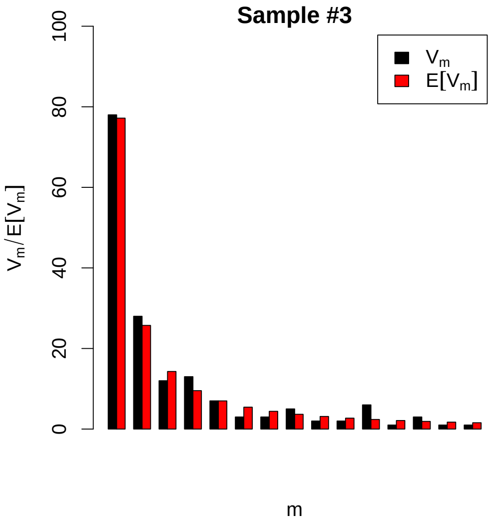

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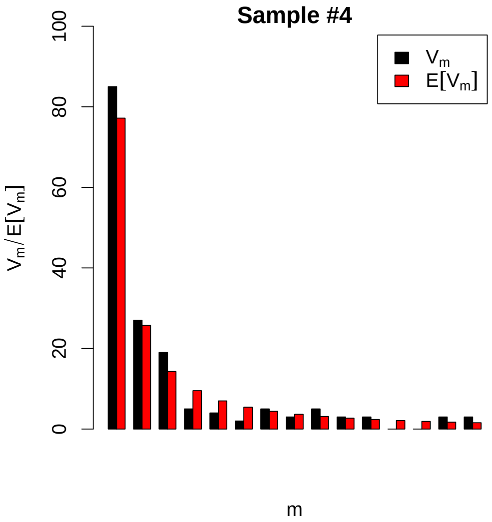
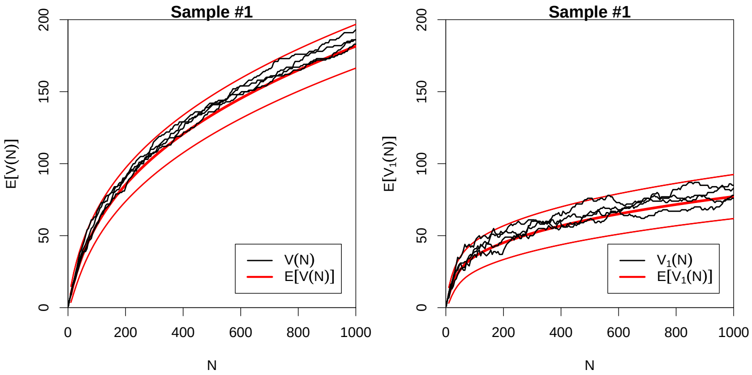


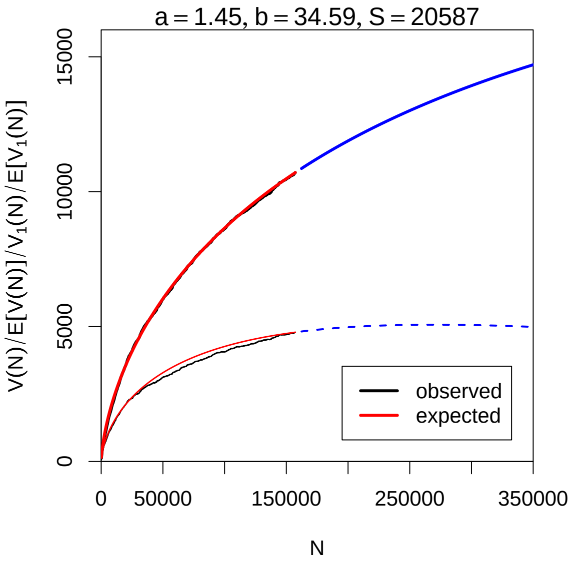
Expectation: vocabulary growth curve



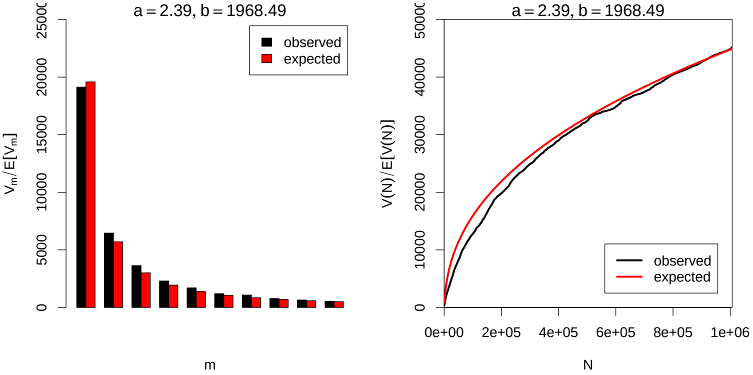
"Confidence intervals" indicate predicted sampling distribution:

☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines

Extrapolating vocabulary growth



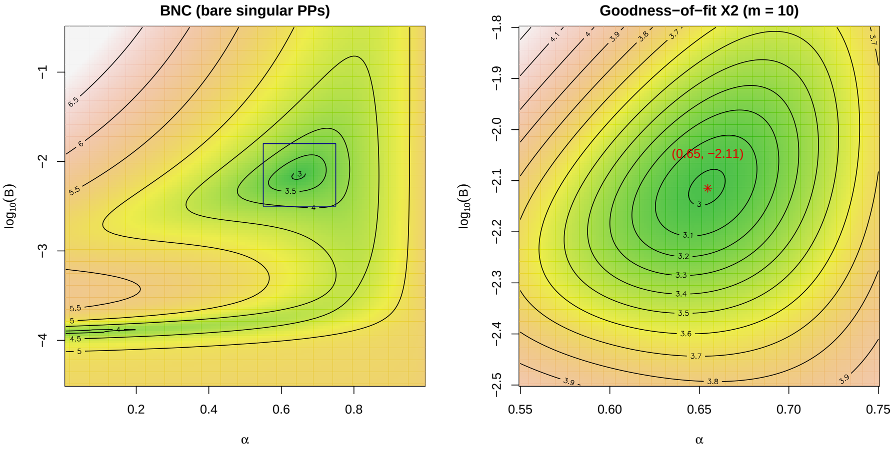
Parameter estimation



- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure based on minimisation of suitable cost function: $a = 2.39$ and $b = 1968$

Parameter estimation

ZM model with free parameters $0 < \alpha < 1$ and $B > 0$



Problems of LNRE models

- Assumption: corpus data = random sample
  - ☞ holds reasonably well for morphological productivity
  - ☞ simple but effective ECHO correction (Baroni and Evert 2007)

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➥ Wide-spread irresponsible application of LNRE models to small samples (e.g. Lüdeling and Evert 2005)

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   (i.e. training data differ from population distribution)
   - ☞ model fitted to training data, may not reflect true population
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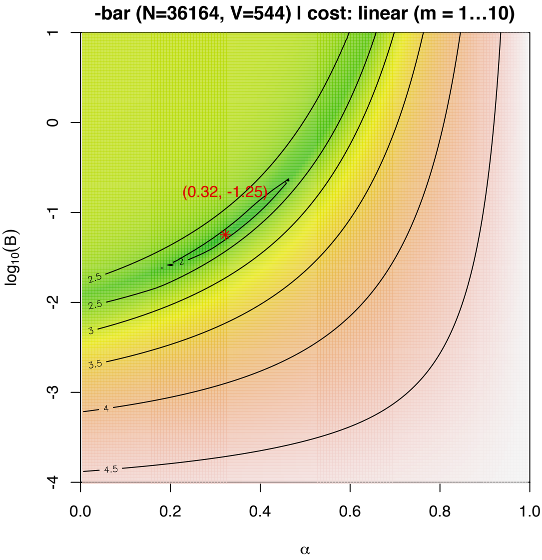
Cost functions for German word-formation affixes



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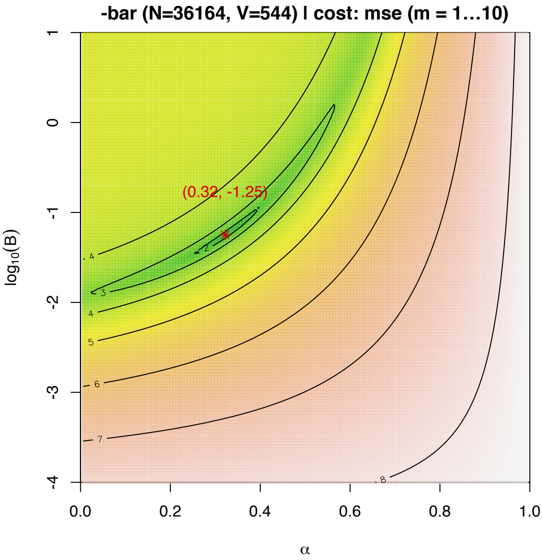

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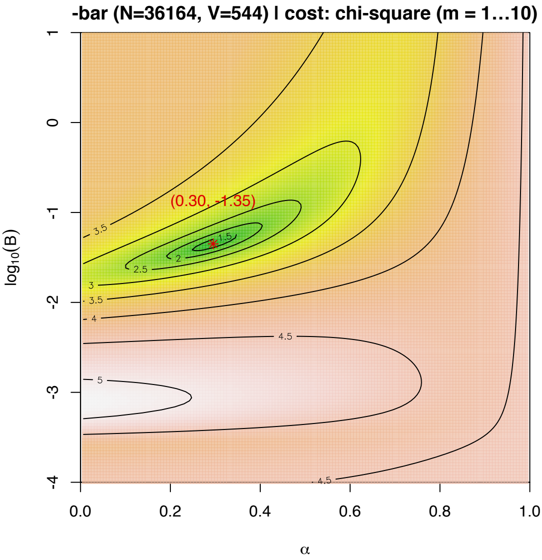

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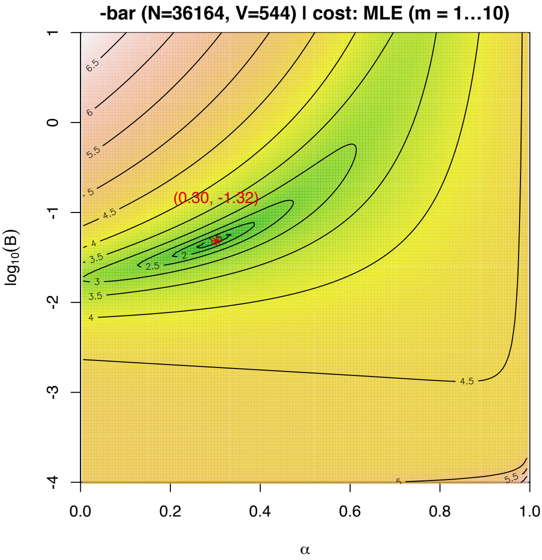

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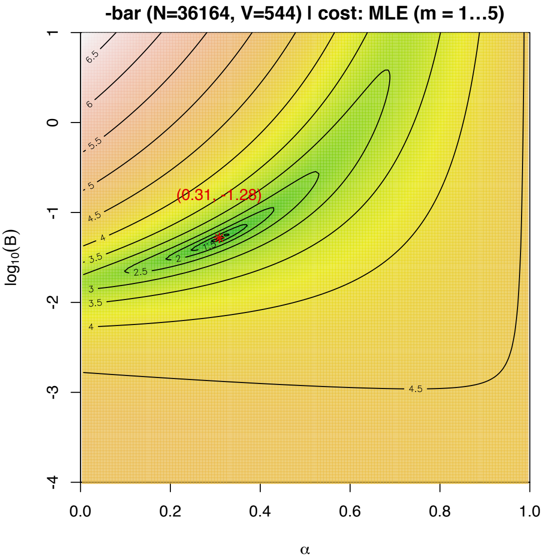

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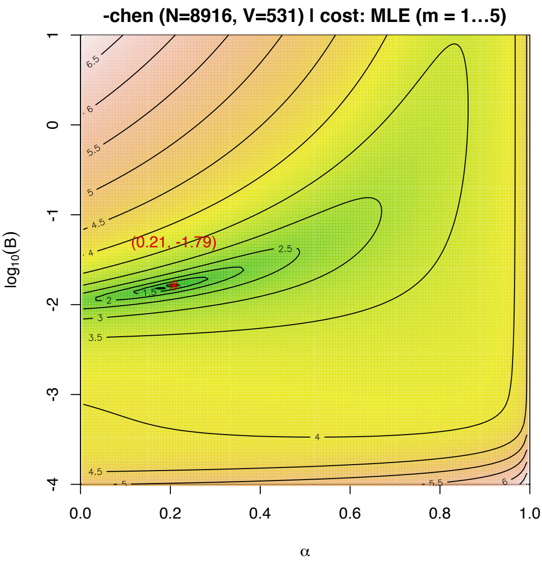

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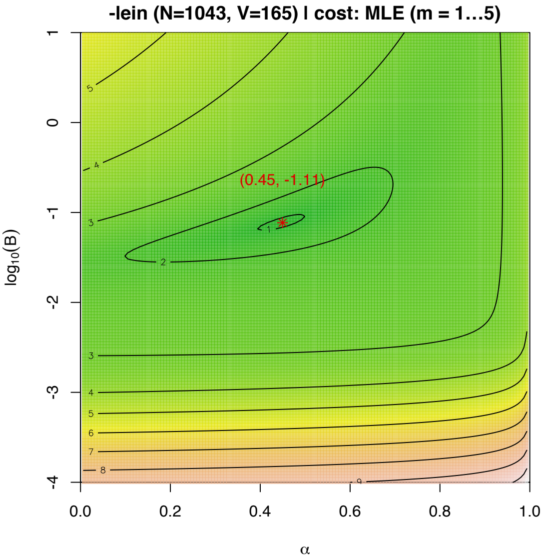
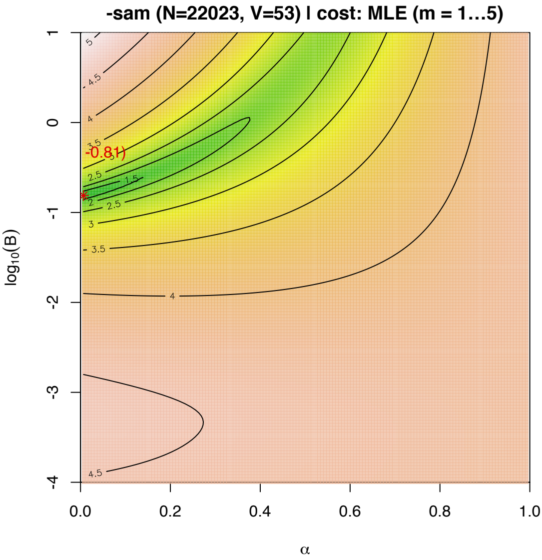


Cost functions for German word-formation affixes



Goodness-of-fit

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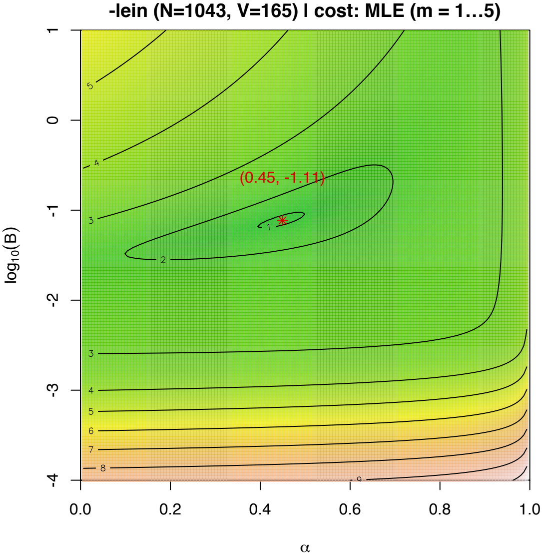
- Test statistic

$$X^2 = (\mathbf{V} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{V} - \boldsymbol{\mu}) \sim \chi^2_{k+1}$$

➥ Multivariate chi-squared test of **goodness-of-fit**

- ☞ significant rejection of the LNRE model for $p < .05$

Confidence sets based on goodness-of-fit test?



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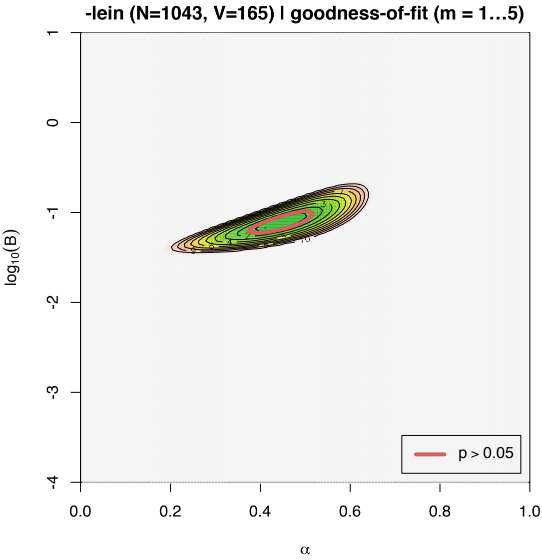

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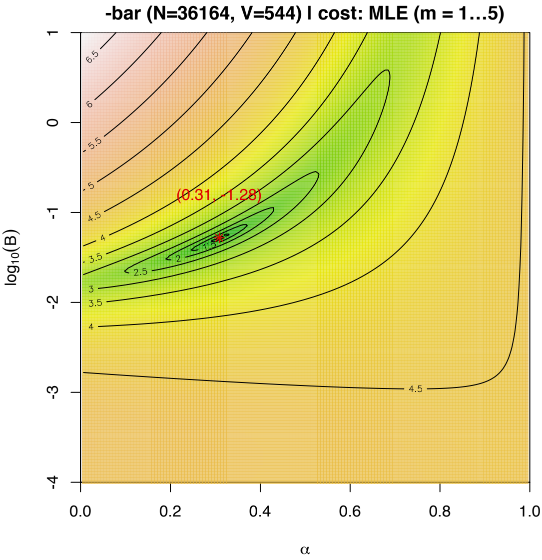

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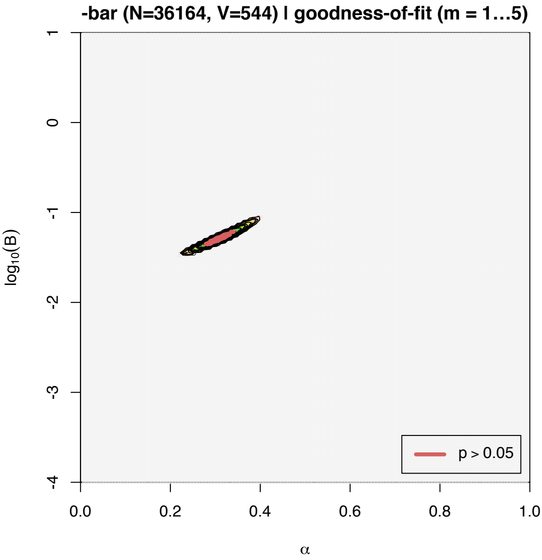

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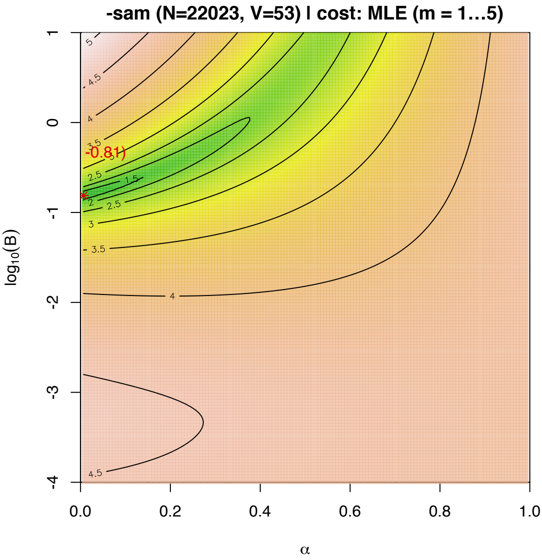

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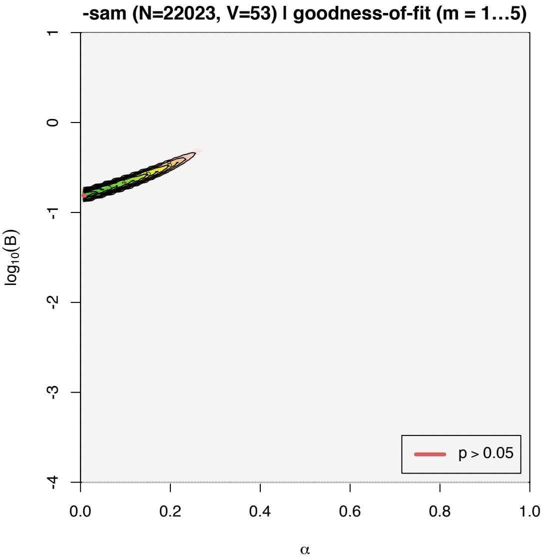
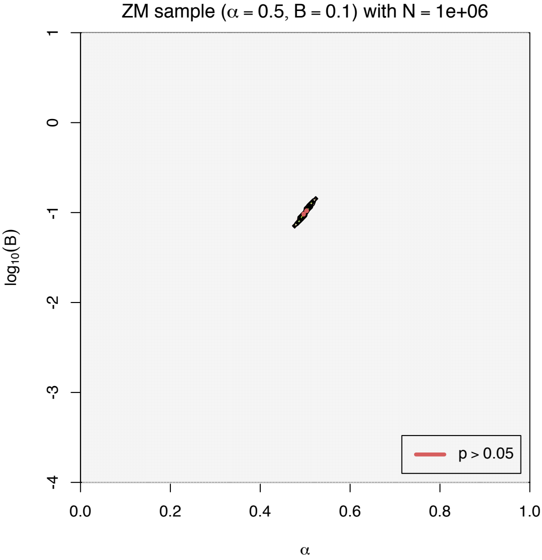


Confidence sets for idealized samples from ZM population

➜ X^2 tests model parameters rather than goodness-of-fit



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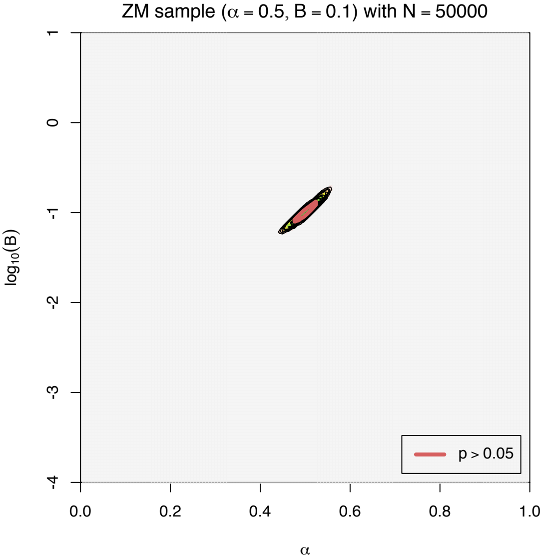

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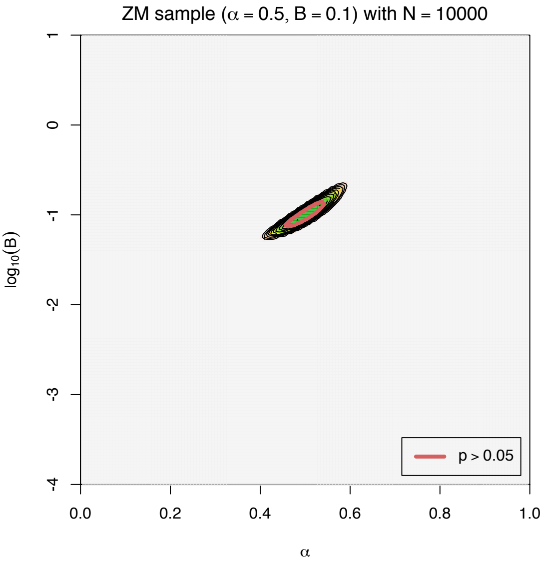

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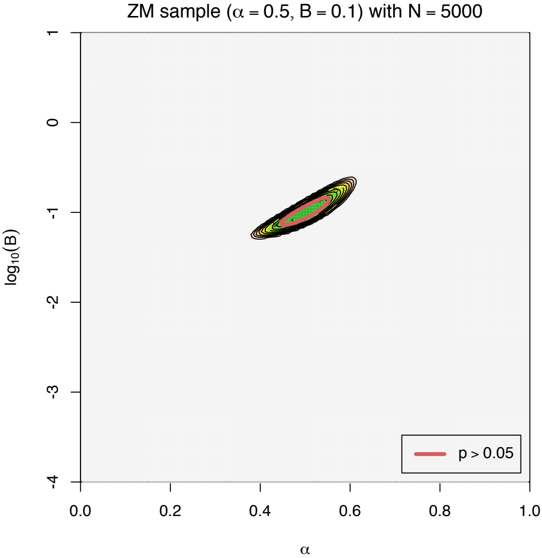

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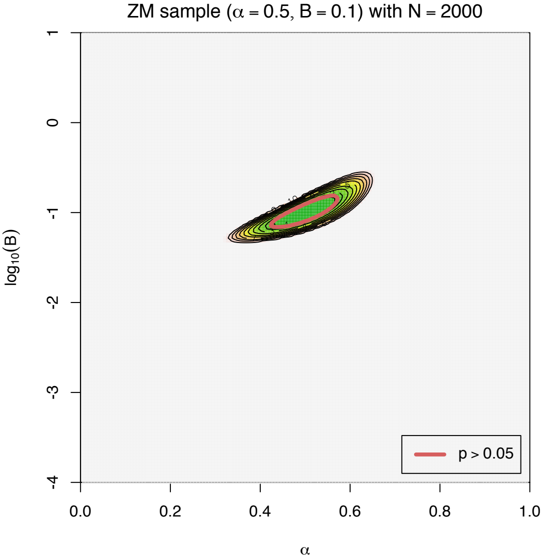

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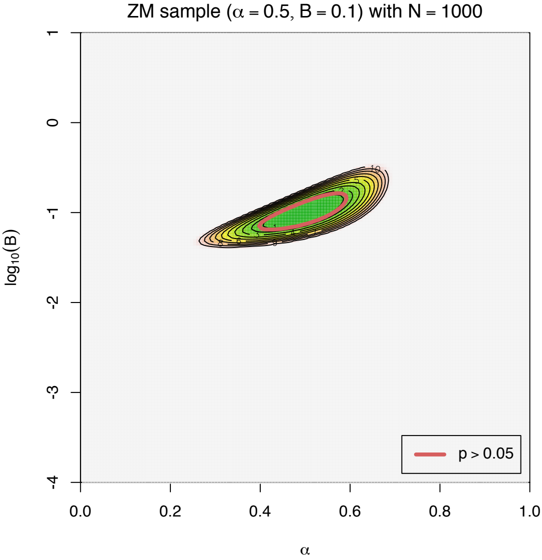

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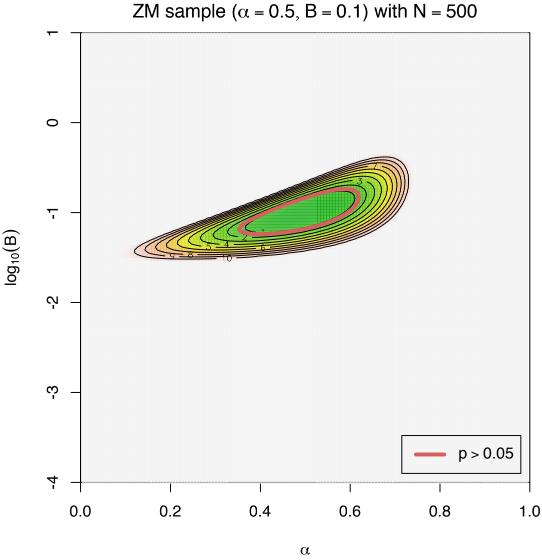

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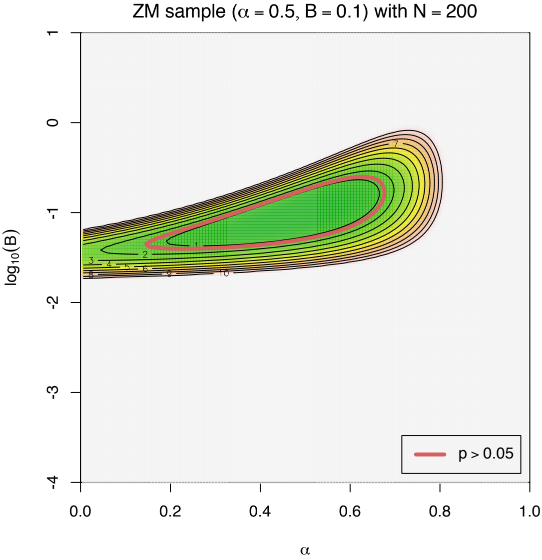

How reliable are the fitted models?

Three potential issues:

1. Model assumptions $\neq$ population
   (e.g. distribution does not follow a Zipf-Mandelbrot law)
   - ☞ model cannot be adequate, regardless of parameter settings
2. Parameter estimation unsuccessful
   (i.e. suboptimal goodness-of-fit to training data)
   - ☞ optimization algorithm trapped in local minimum
   - ☞ can result in highly inaccurate model
3. **Uncertainty due to sampling variation**
   (i.e. training data differ from population distribution)
   - ☞ model fitted to training data, may not reflect true population
   - ☞ another training sample would have led to different parameters
   - ☞ especially critical for small samples ($N < 10{,}000$)

Bootstrapping

- An empirical approach to sampling variation:
  - take many random samples from the same population
  - estimate LNRE model from each sample
  - analyse distribution of model parameters, goodness-of-fit, etc. (mean, median, s.d., boxplot, histogram, ...)
  - problem: how to obtain the additional samples?

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  - resample from observed data *with replacement*
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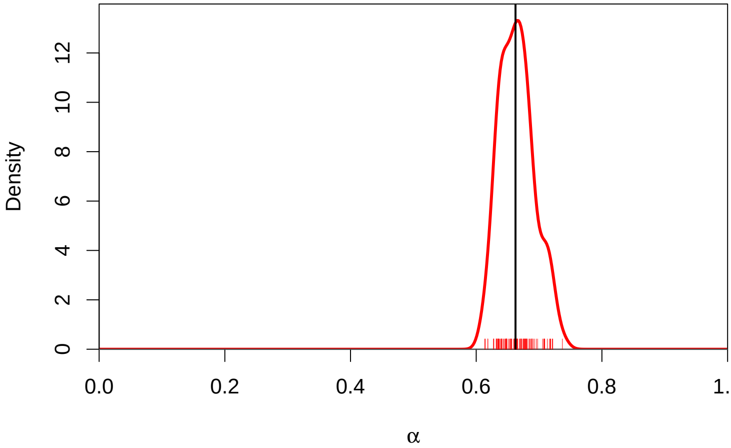
- Parametric bootstrapping
  - use fitted model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known

Bootstrapping

parametric bootstrapping with 100 replicates, fZM samples for $N = 3467$

Zipfian slope $a = 1/\alpha$



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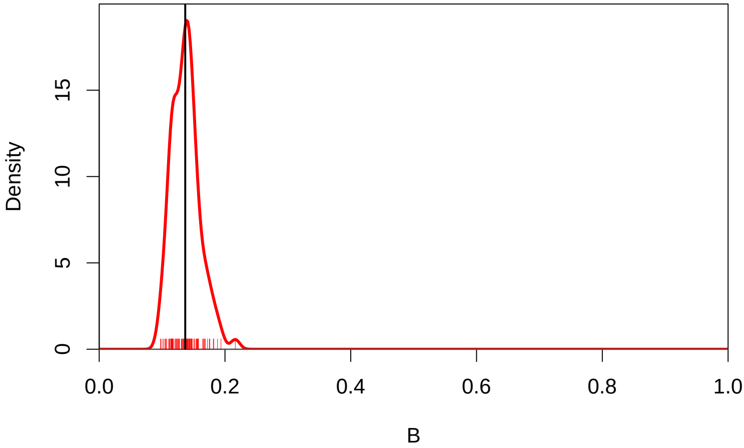
Offset $b = (1 - \alpha)/(B \cdot \alpha)$



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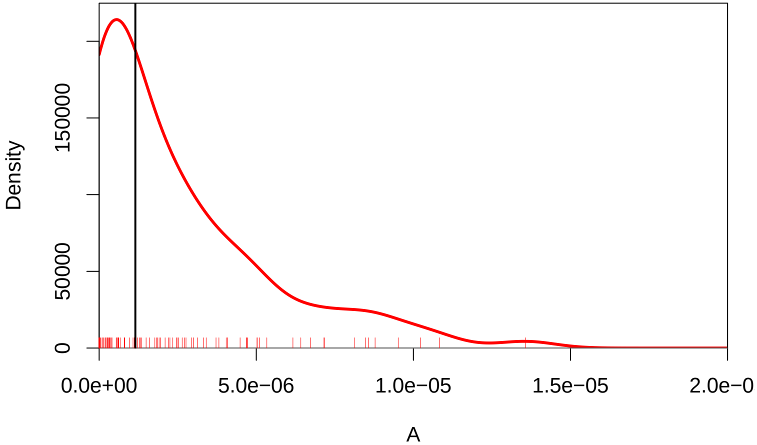
fZM probability cutoff $A = \pi_S$



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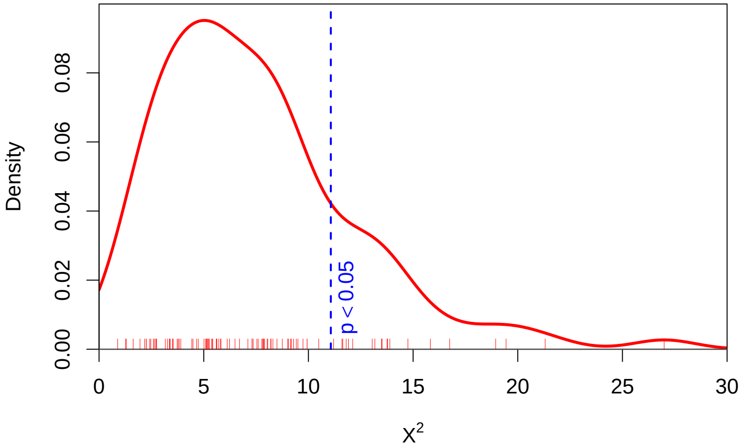
Goodness-of-fit statistic X^2 (model not plausible for $X^2 > 11$)



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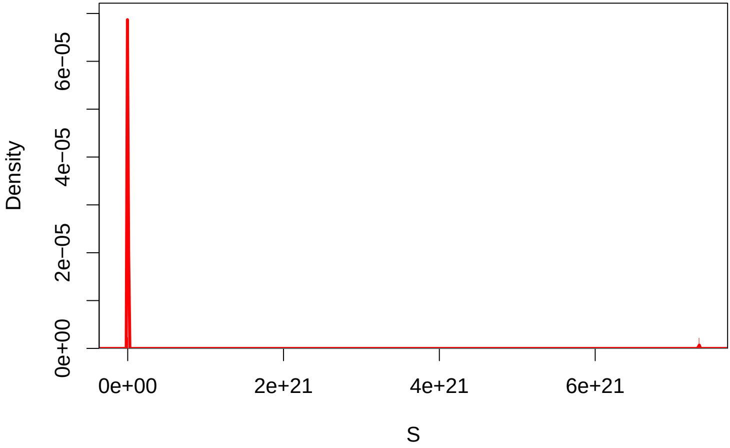
Population diversity S



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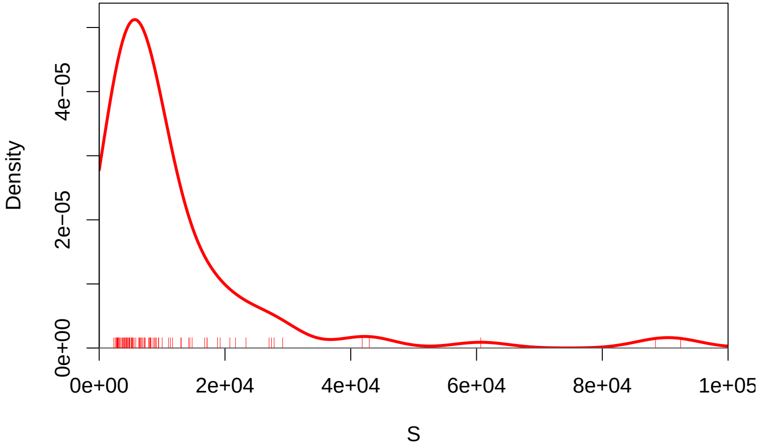
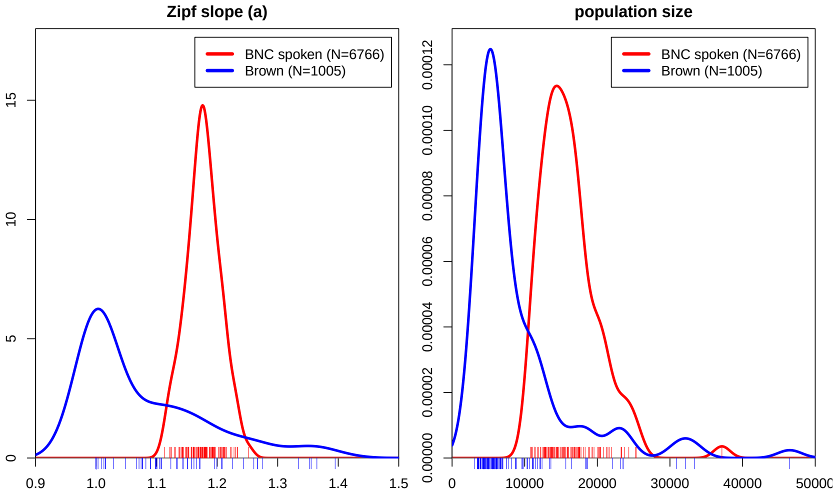


Sample size matters!

Brown corpus is too small for reliable LNRE parameter estimation (bare singulars)



Thank you!

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