Measures of Productivity and Lexical Diversity

7 October 2018

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http://zipfr.r-forge.r-project.org/

JAECS
Japan Association for English Corpus Studies

Kallimachos

Outline

Outline

Research questions in computational corpus linguistics

- How many words did Shakespeare know?
- What is the coverage of my treebank grammar on big data?
- How many typos are there on the Internet?
- Is *-ness* more productive than *-ity* in English?
- Are there differences in the productivity of nominal compounds between academic writing and novels?
- Does Dickens use a more complex vocabulary than Rowling?
- Can a decline in lexical complexity predict Alzheimer's disease?
- How frequent is a hapax legomenon from the Brown corpus?
- What is appropriate smoothing for my n-gram model?
- Who wrote the Bixby letter, Lincoln or Hay?
- How many different species of . . . are there? (Brainerd 1982)

Research questions in computational corpus linguistics

-
- coverage estimates
-
-
- productivity
-
- lexical complexity & stylometry
-
- prior & posterior distribution
-
- unexpected applications
-

Type-token statistics

- These applications relate token and type counts
  - **tokens** = individual instances (occurrences)
  - **types** = distinct items
- Type-token statistics different from most statistical inference
  - not about probability of a specific event
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- Relatively little work in statistical science
- Nor a major research topic in computational linguistics
  - very specialized, usually plays ancillary role in NLP
- Corpus linguistics: TTR & simple productivity measures
  - often applied without any statistical inference

Zipf's law (Zipf 1949)

A) Frequency distributions in natural language are highly skewed

B) Curious relationship between rank & frequency

word	r	f	$r \cdot f$
the	1.	142,776	142,776
and	2.	100,637	201,274
be	3.	94,181	282,543
of	4.	74,054	296,216

(Dickens)

C) Various explanations of Zipf's law

- principle of least effort (Zipf 1949)
- optimal coding system, MDL (Mandelbrot 1953, 1962)
- random sequences (Miller 1957; Li 1992; Cao *et al.* 2017)
- Markov processes ➜ n-gram models (Rouault 1978)

D) Language evolution: birth-death-process (Simon 1955)

☞ not the main topic today!

Outline

Tokens & types

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size**
  (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

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  (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

type-frequency list

w	f_w
recently	1
very	5
not	3
otherwise	1
much	2
merely	2
now	1

Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

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Zipf ranking

w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1

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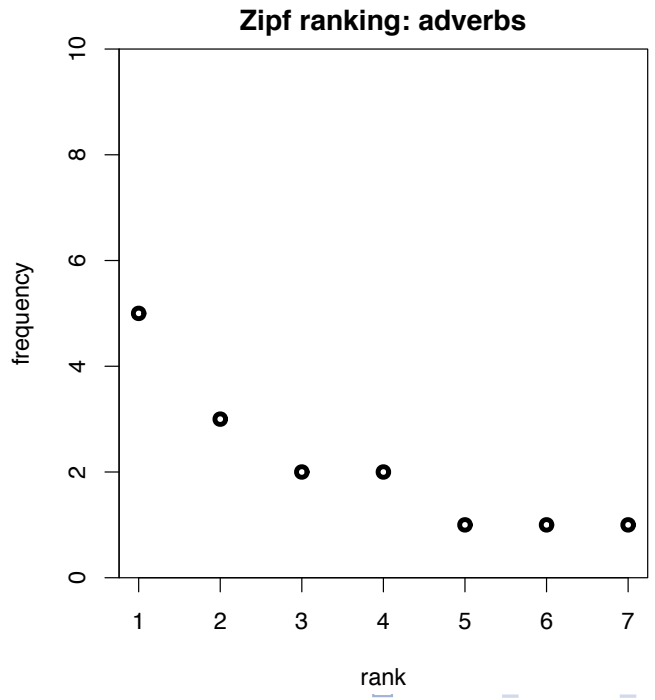

A realistic Zipf ranking: the Brown corpus

top frequencies			bottom frequencies		
r	***f***	**word**	**rank range**	***f***	**randomly selected examples**
1	69836	the	7731 – 8271	10	schedules, polynomials, bleak
2	36365	of	8272 – 8922	9	tolerance, shaved, hymn
3	28826	and	8923 – 9703	8	decreased, abolish, irresistible
4	26126	to	9704 – 10783	7	immunity, cruising, titan
5	23157	a	10784 – 11985	6	geographic, lauro, portrayed
6	21314	in	11986 – 13690	5	grigori, slashing, developer
7	10777	that	13691 – 15991	4	sheath, gaulle, ellipsoids
8	10182	is	15992 – 19627	3	mc, initials, abstracted
9	9968	was	19628 – 26085	2	thar, slackening, deluxe
10	9801	he	26086 – 45215	1	beck, encompasses, second-place

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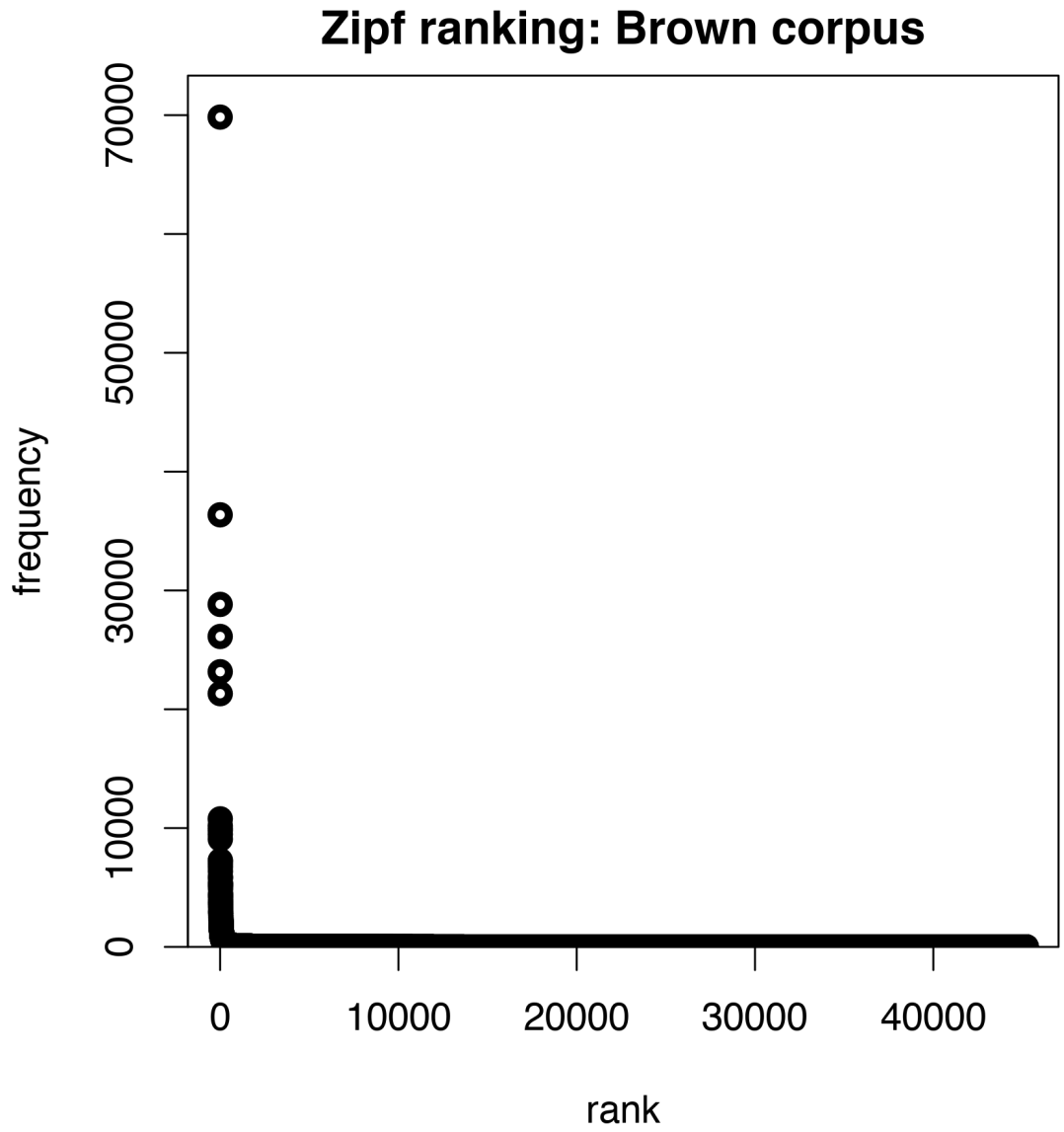

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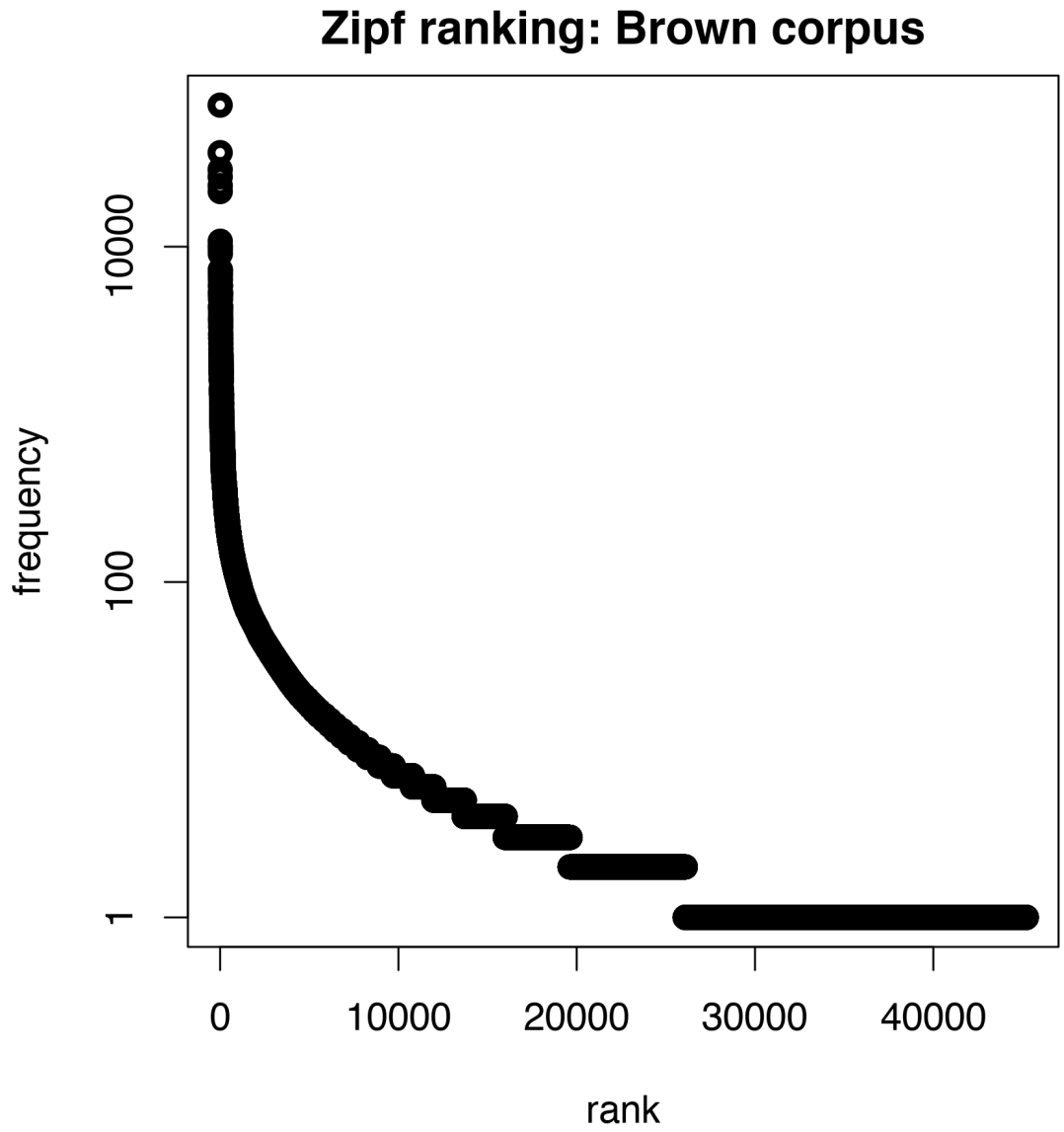

Frequency spectrum

- pool types with $f = 1$ (hapax legomena), types with $f = 2$ (dis legomena), ..., $f = m$, ...
- $V_1 = 3$: number of hapax legomena (*now, otherwise, recently*)
- $V_2 = 2$: number of dis legomena (*merely, much*)
- general definition: $V_m = |\{w \mid f_w = m\}|$

Zipf ranking

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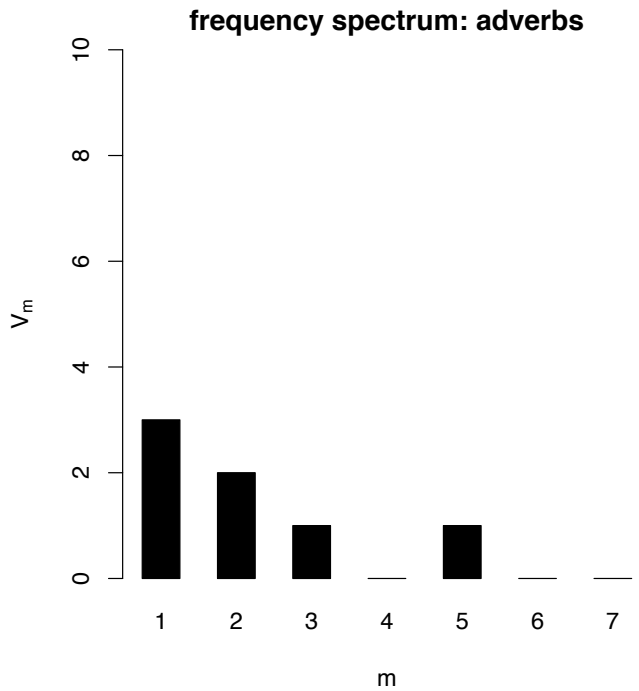

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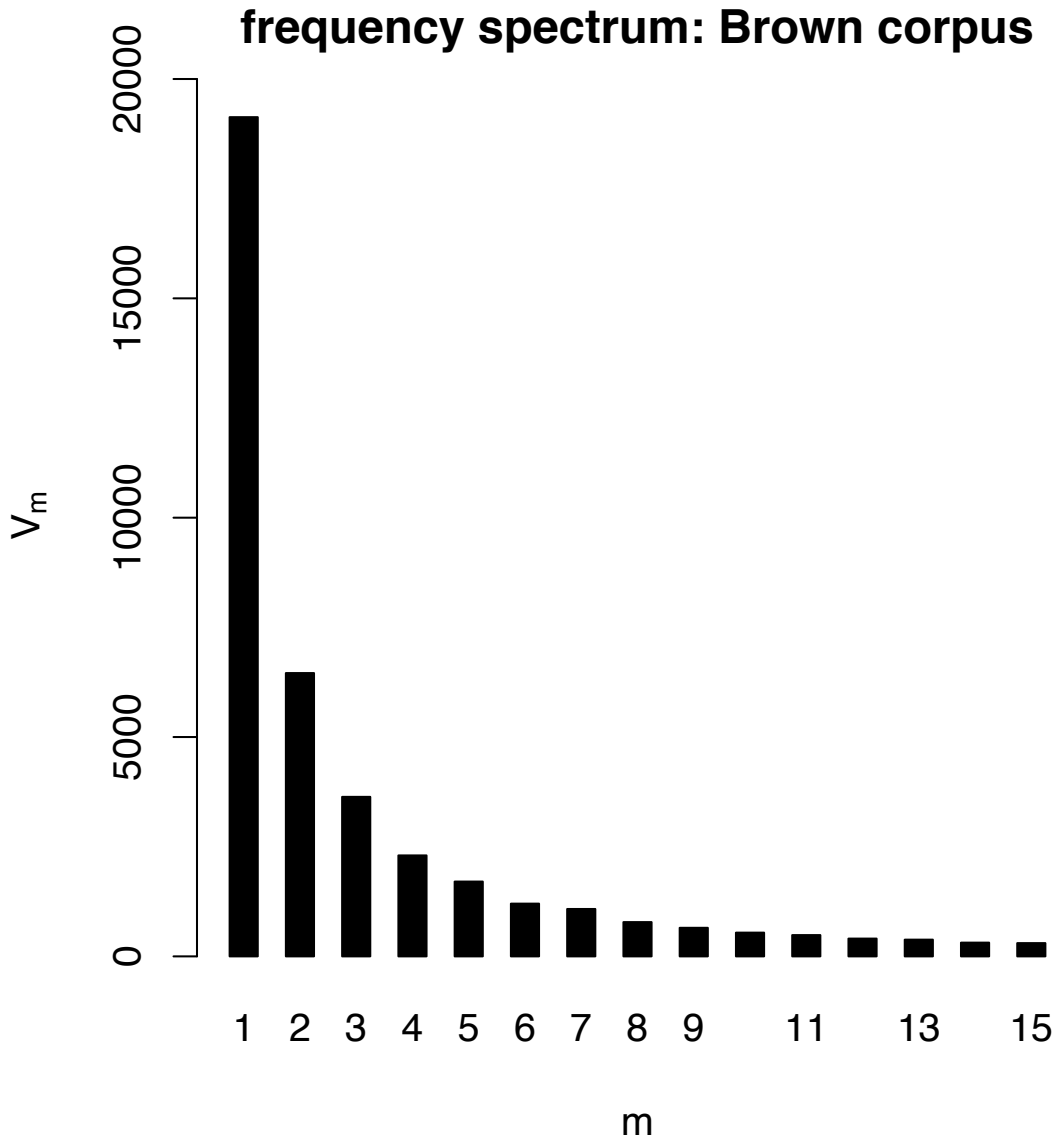

Vocabulary growth curve

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 1,\ V(N) = 1,\ V_1(N) = 1$

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- $N = 15,\ V(N) = 7,\ V_1(N) = 3$

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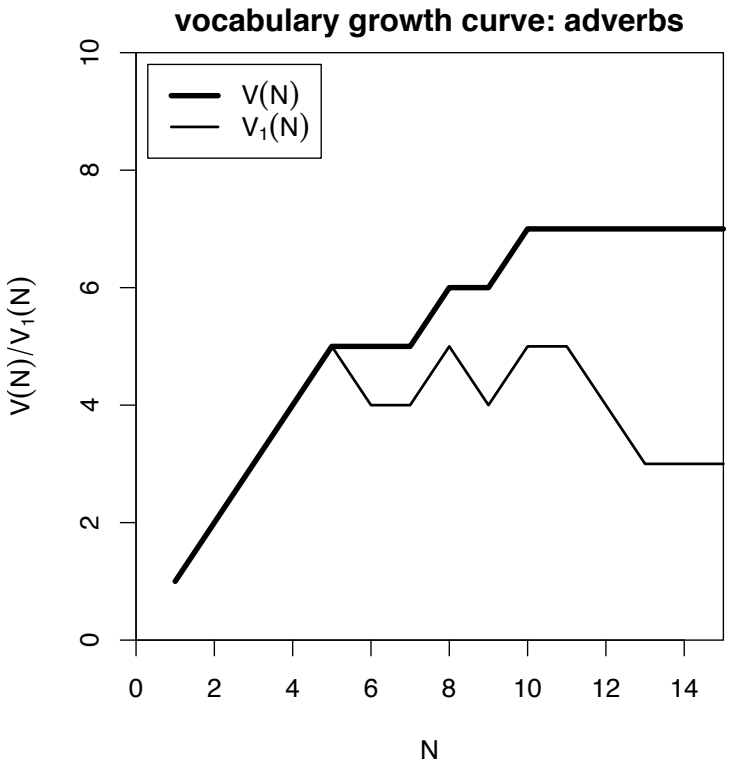

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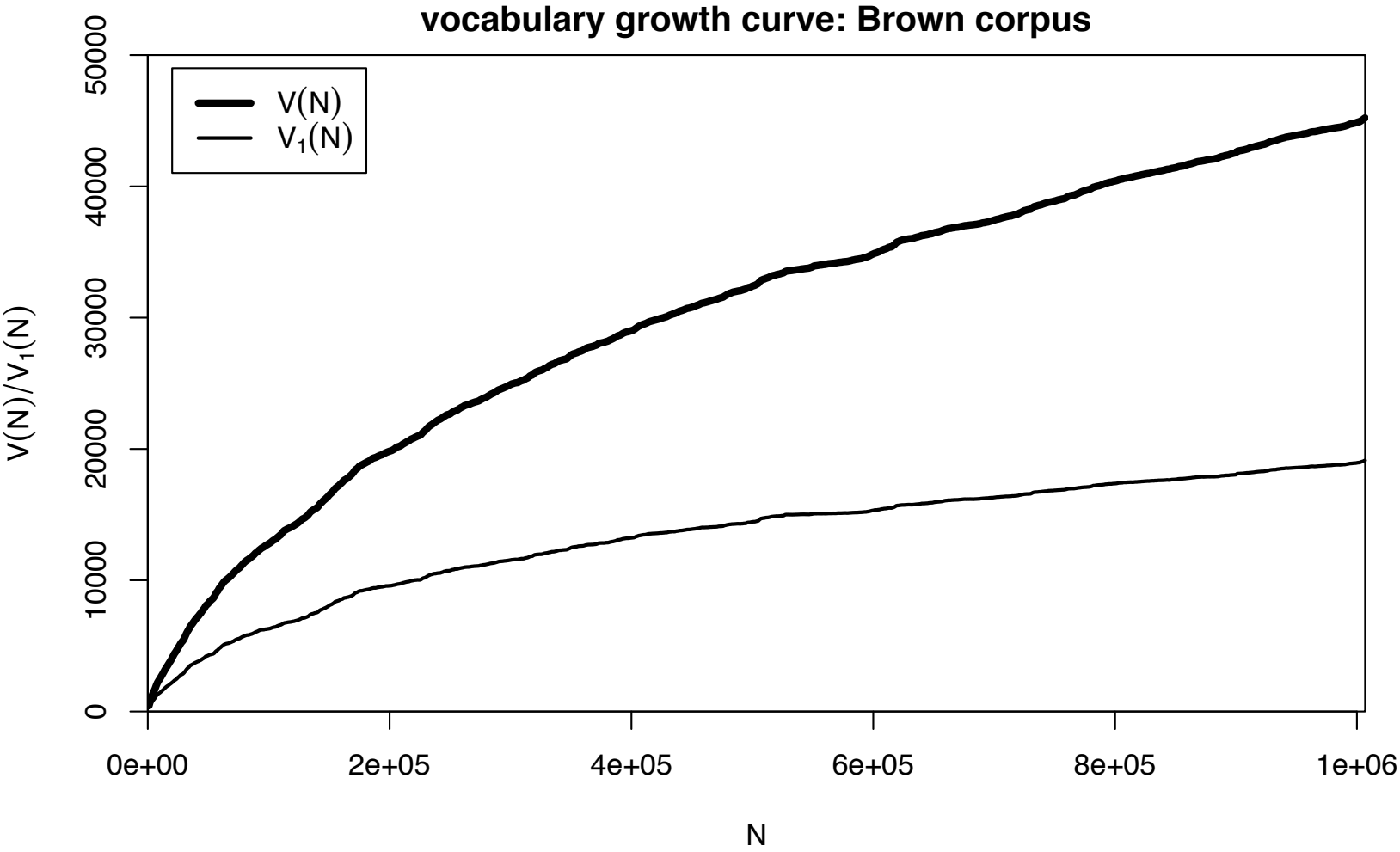
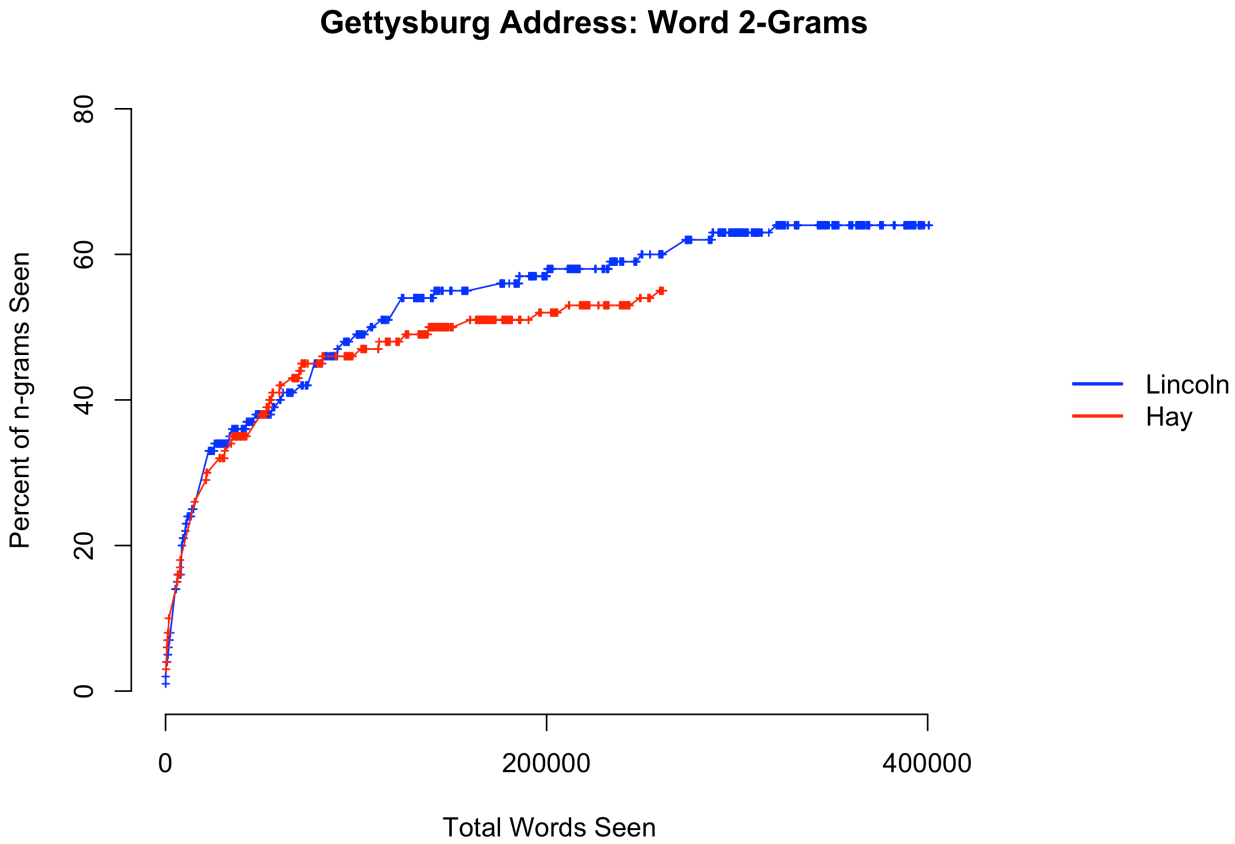


Vocabulary growth in authorship attribution

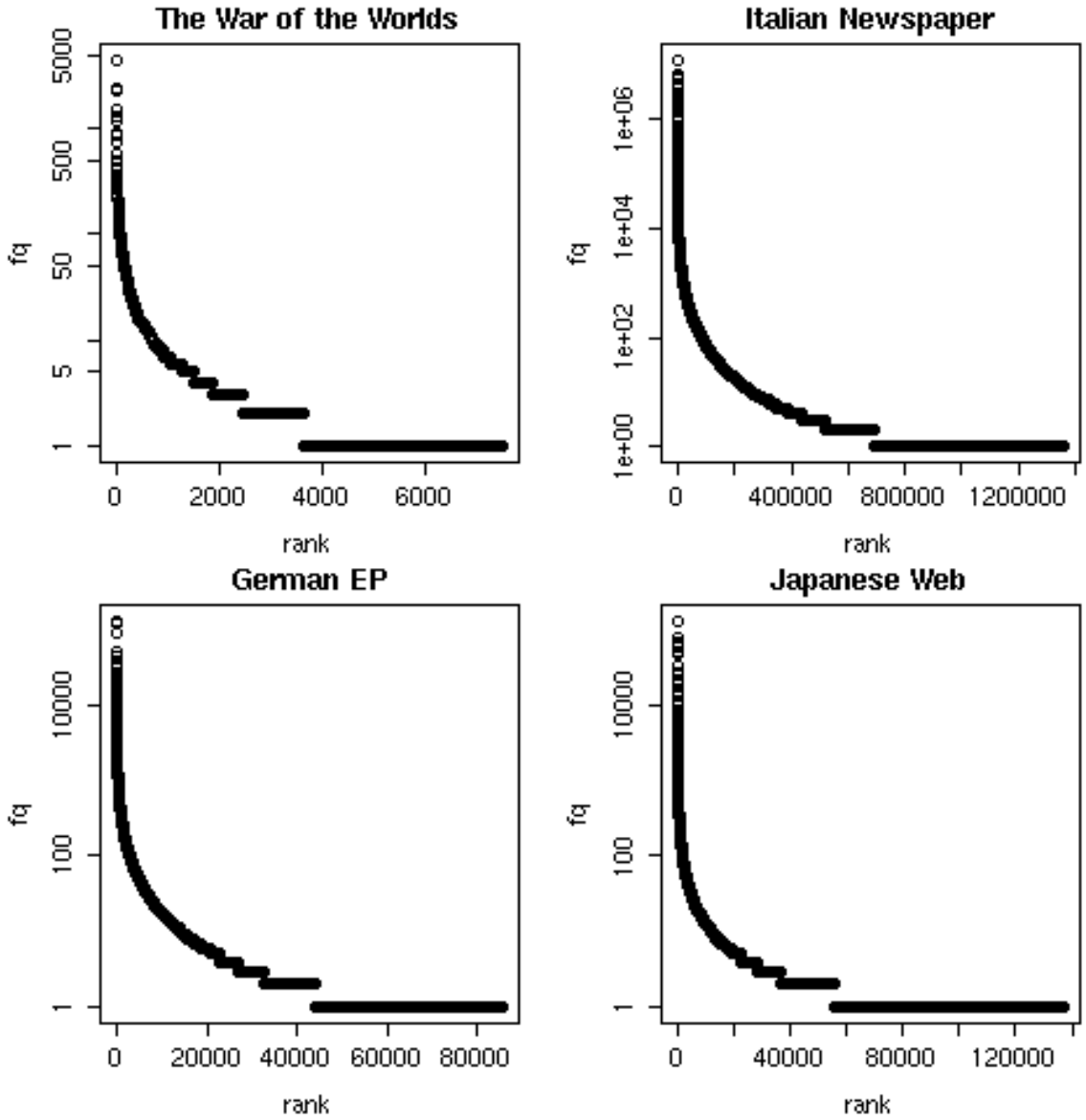
- Authorship attribution by n-gram tracing applied to the case of the Bixby letter (Grieve *et al.* submitted)
- Word or character n-grams in disputed text are compared against large "training" corpora from candidate authors



Outline

Observing Zipf's law

across languages and different linguistic units



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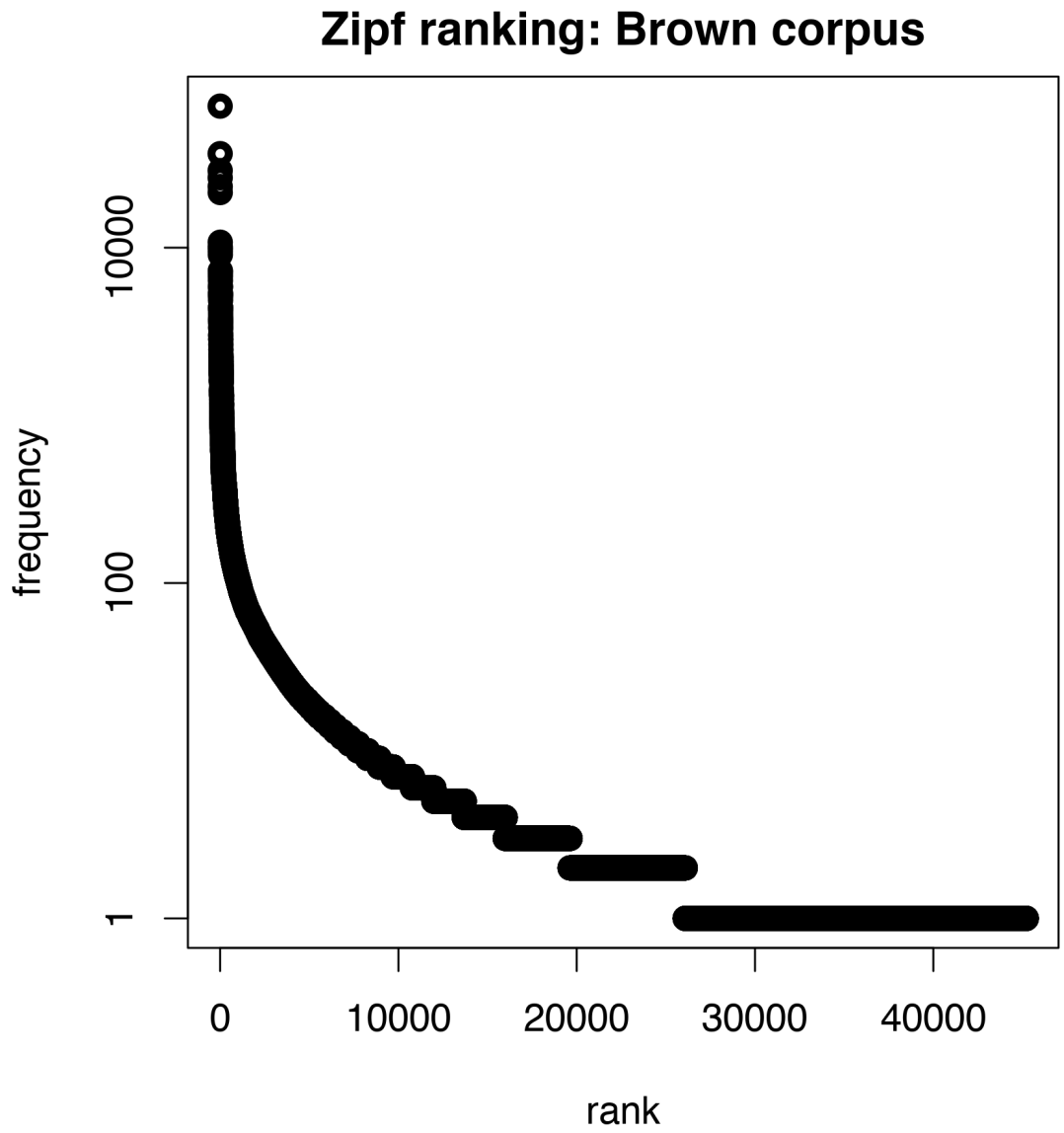

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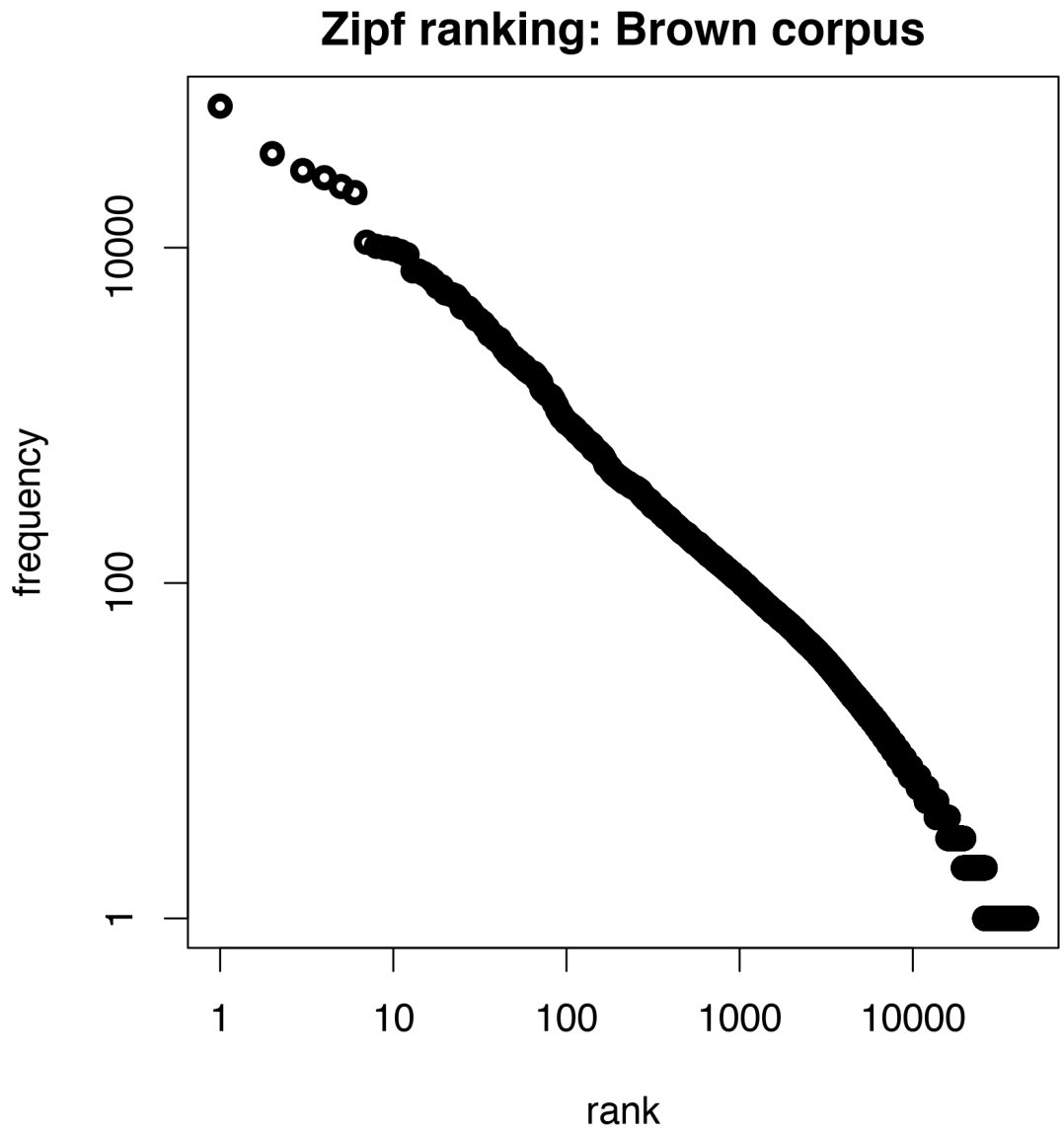

Observing Zipf's law

- Straight line in double-logarithmic space corresponds to **power law** for original variables
- This leads to Zipf's (1949; 1965) famous law:

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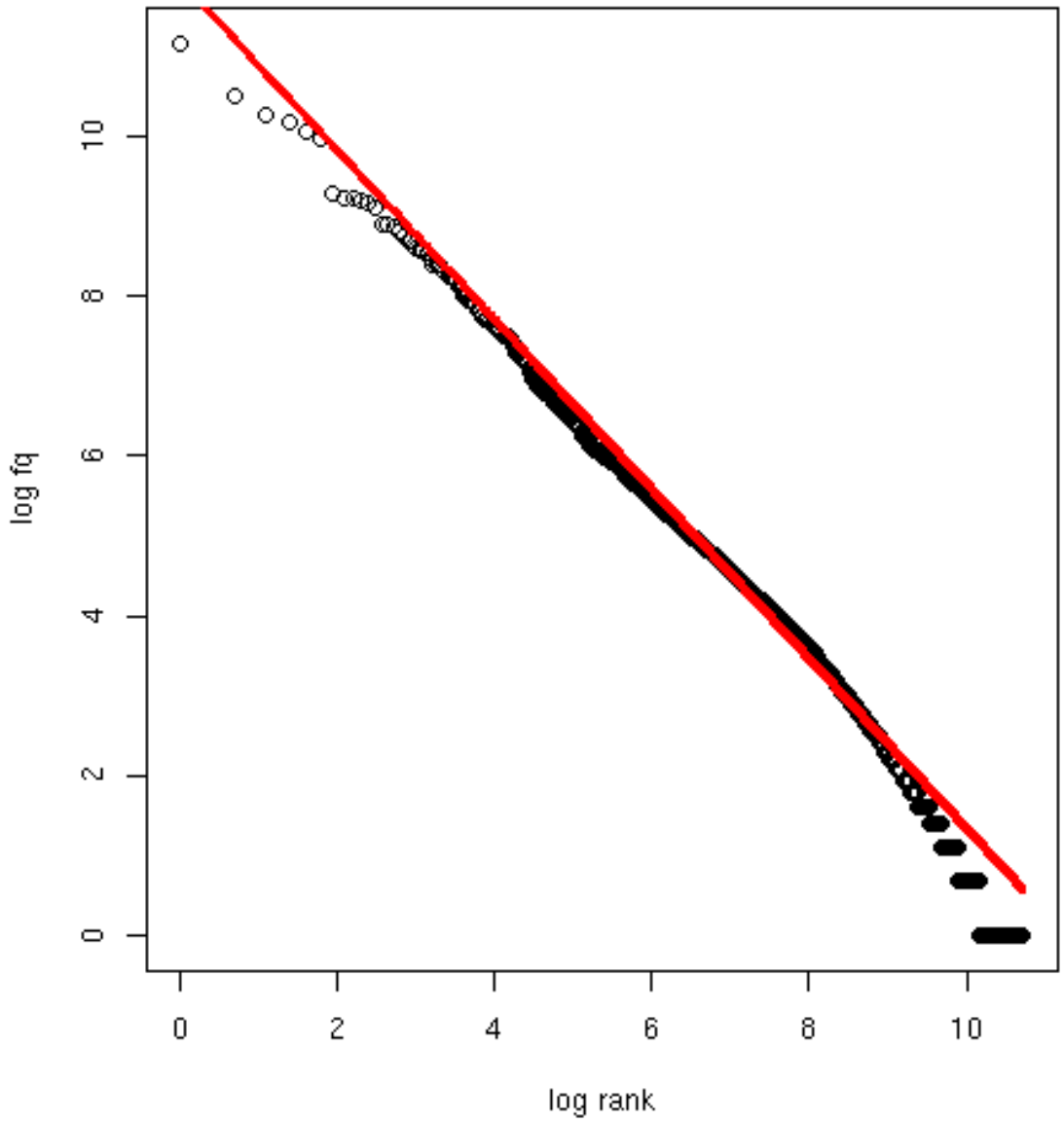
- Intuitive interpretation of a and C:
  - a is **slope** determining how fast log frequency decreases
  - $\log C$ is **intercept**, i.e. log frequency of most frequent word ($r = 1 \rightarrow \log r = 0$)

Observing Zipf's law

Least-squares fit = linear regression in log-space (Brown corpus)



Zipf-Mandelbrot law

Mandelbrot (1953, 1962)

- Mandelbrot's extra parameter:

$$f_r = \frac{C}{(r+b)^a}$$

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- Zipf's law is special case with $b = 0$
- Assuming $a = 1$, $C = 60{,}000$, $b = 1$:
  - For word with rank 1, Zipf's law predicts frequency of 60,000; Mandelbrot's variation predicts frequency of 30,000
  - For word with rank 1,000, Zipf's law predicts frequency of 60; Mandelbrot's variation predicts frequency of 59.94

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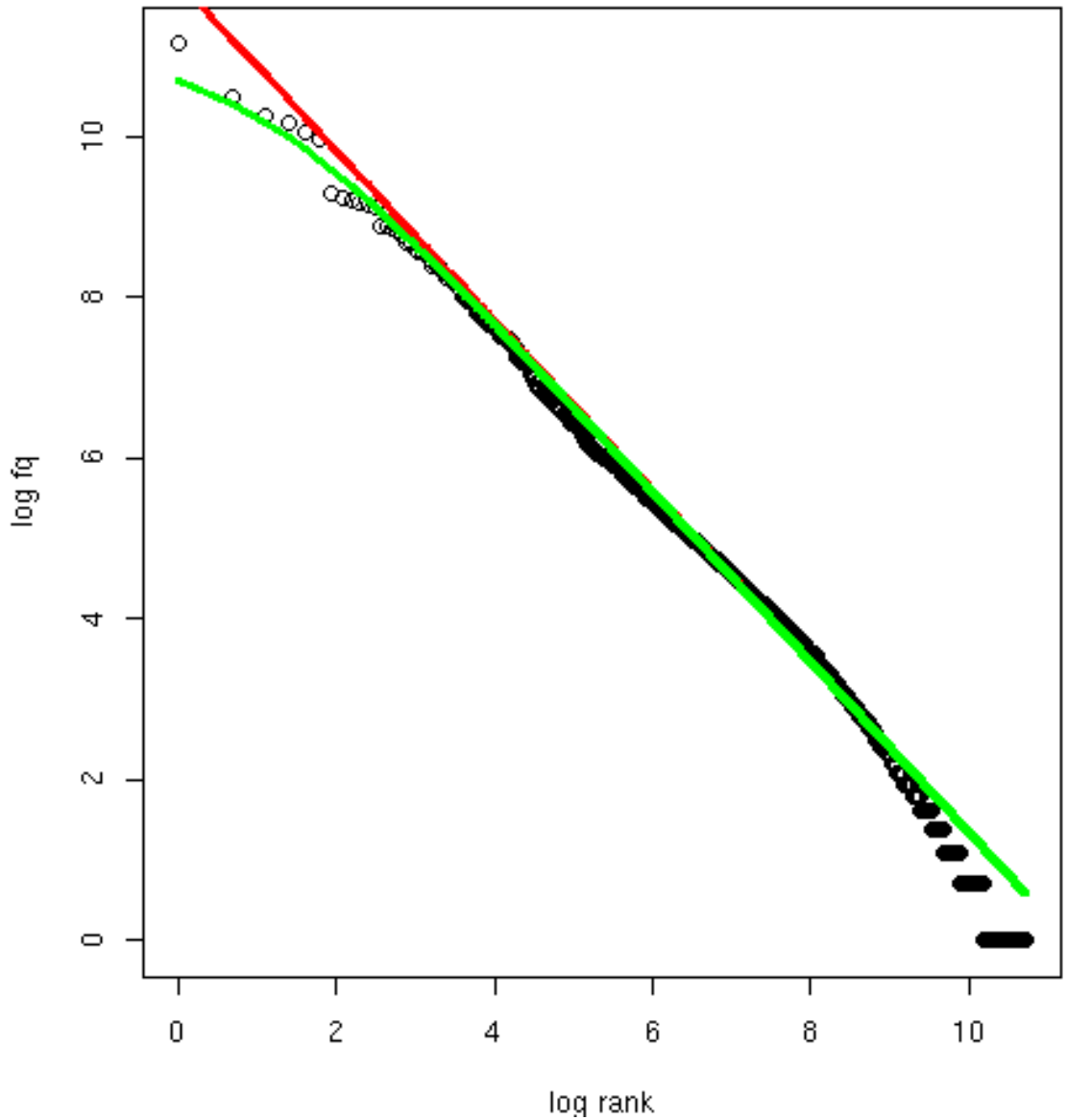
- Zipf-Mandelbrot law forms basis of statistical LNRE models
  - ZM law derived mathematically as limiting distribution of vocabulary generated by a character-level Markov process

Zipf-Mandelbrot law

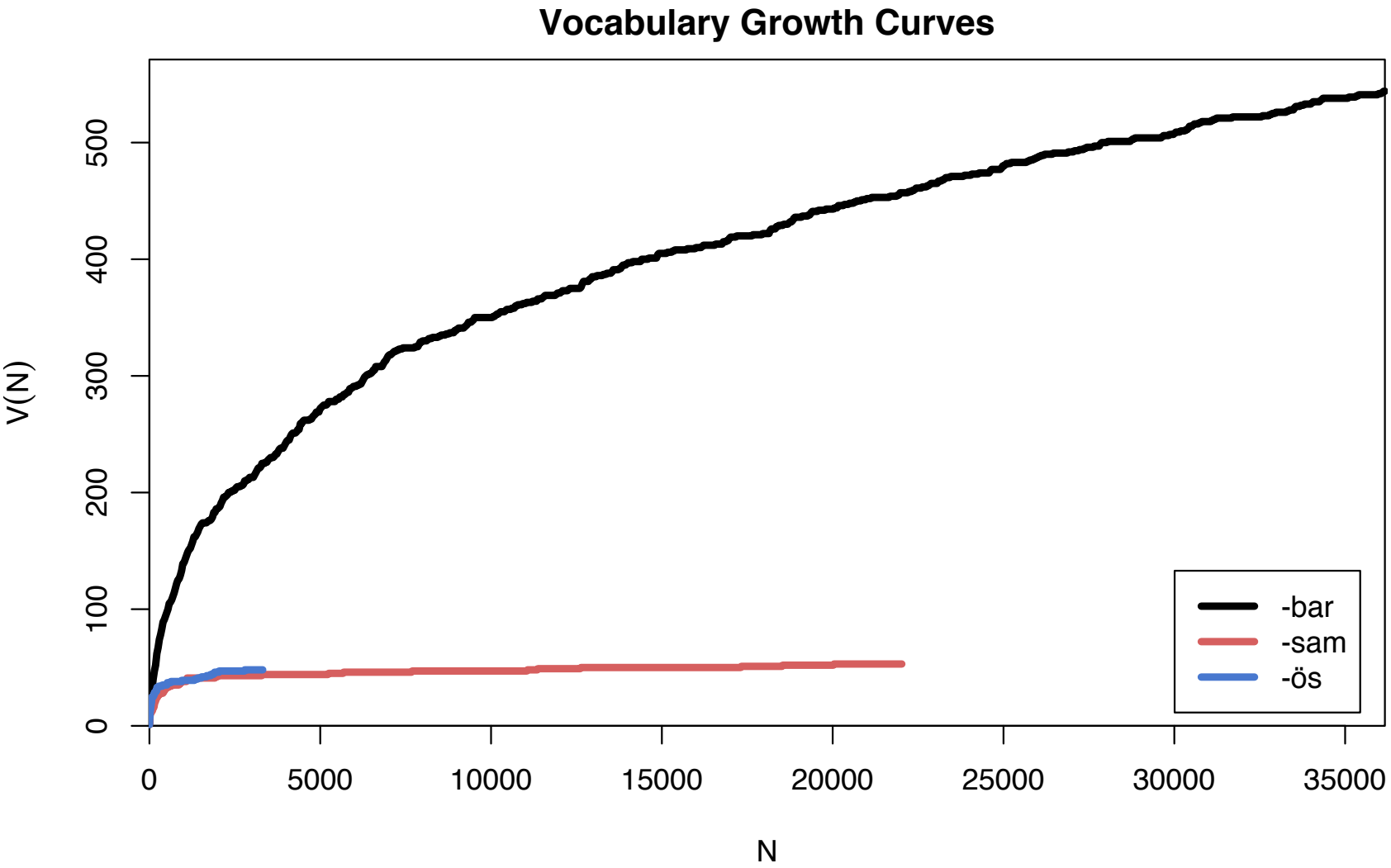
Non-linear least-squares fit (Brown corpus)



Outline

Measuring morphological productivity

example from Evert and Lüdeling (2001)



Quantitative measures of productivity

(Tweedie and Baayen 1998; Baayen 2001)

- Baayen's (1991) productivity index $\mathcal{P}$

$$\mathcal{P} = \frac{V_1}{N}$$

- TTR = type-token ratio

$$\text{TTR} = \frac{V}{N}$$

- Slope a of Zipf-Mandelbrot law
- Population size

$$S = \lim_{N \to \infty} V(N)$$

- Herdan's law (1964)

$$C = \frac{\log V}{\log N}$$

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$$C = \frac{\log V}{\log N}$$

- Yule (1944) / Simpson (1949)

$$K = 10\,000 \cdot \frac{\sum_m m^2 V_m - N}{N^2}$$

- Guiraud (1954)

$$R = \frac{V}{\sqrt{N}}$$

- Sichel (1975)

$$S = \frac{V_2}{V}$$

- Honoré (1979)

$$H = \frac{\log N}{1 - \frac{V_1}{V}}$$

Productivity measures for bare singulars in the BNC

	spoken	written
V	2,039	12,876
N	6,766	85,750
K	86.84	28.57
R	24.79	43.97
S	0.13	0.15
C	0.86	0.83
$\mathcal{P}$	0.21	0.08
TTR	0.301	0.150
a	1.18	1.27
pop. S	15,958	36,874

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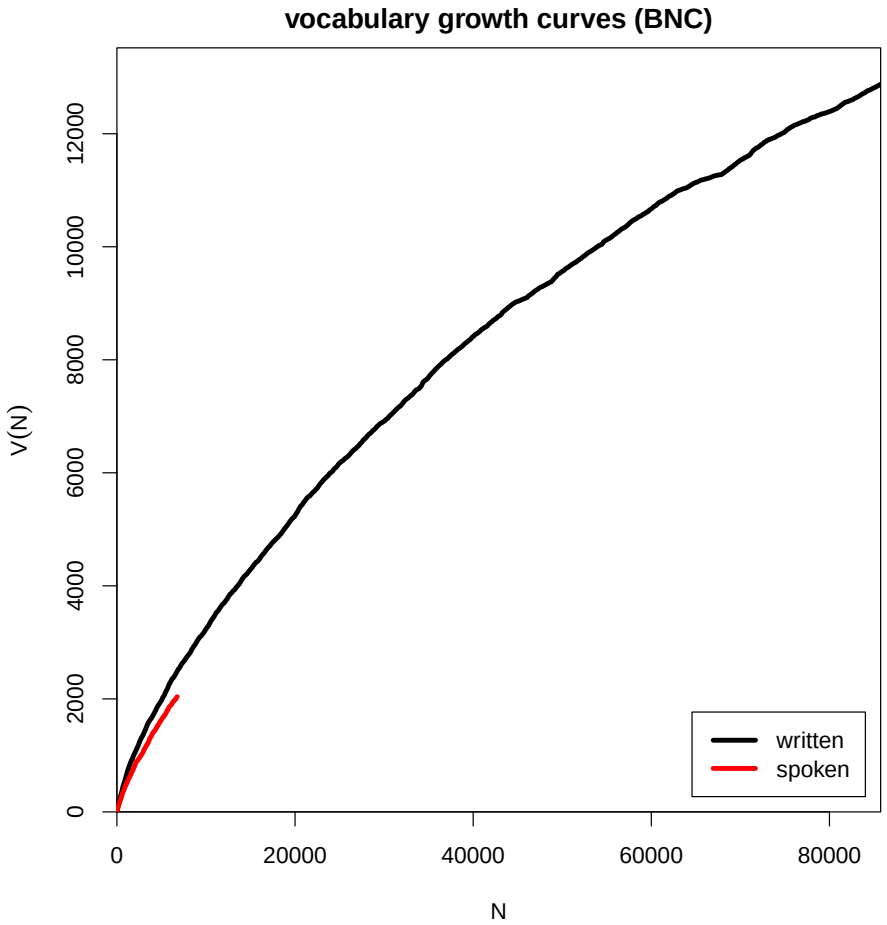
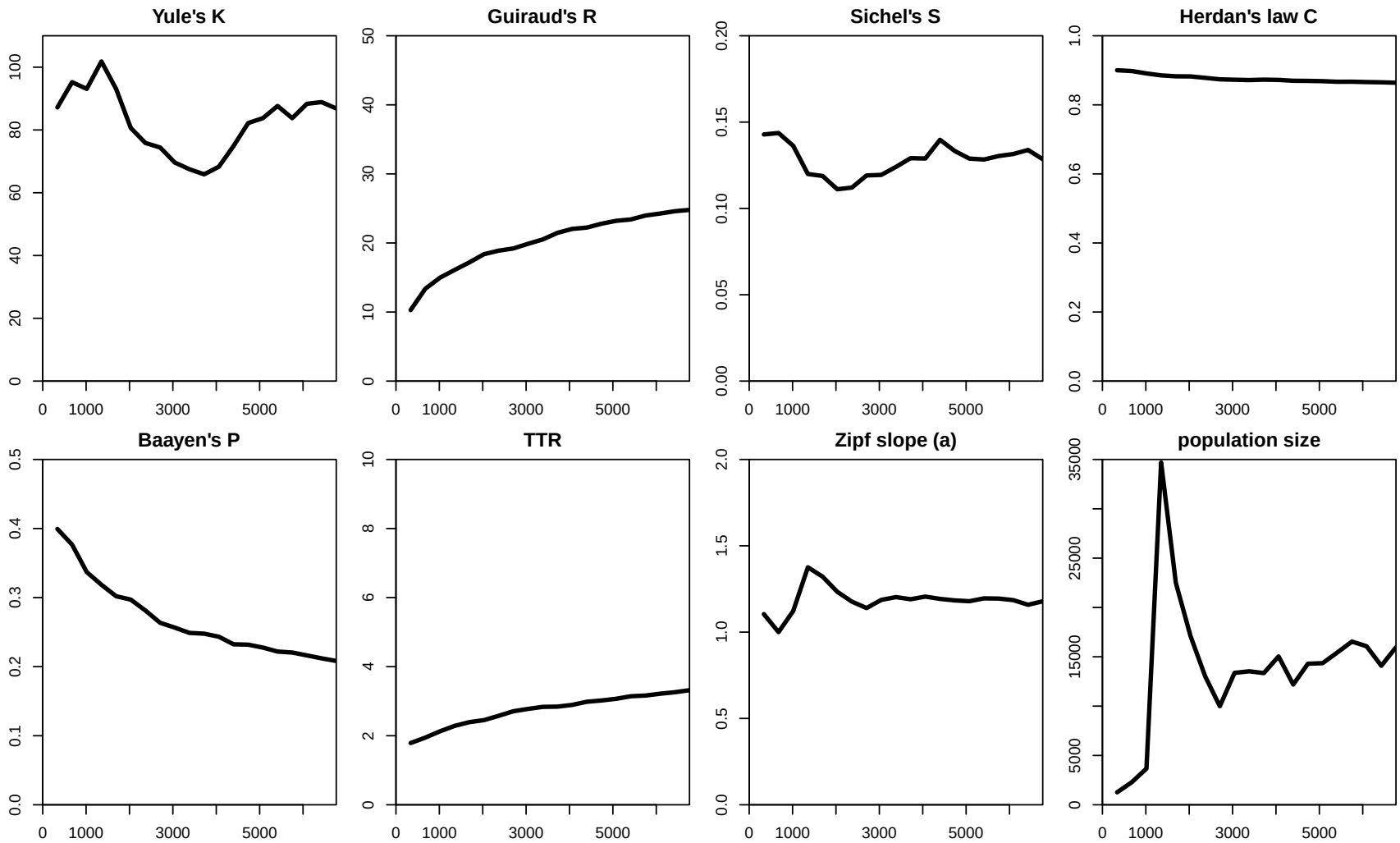


Are these "lexical constants" really constant?



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- Often need to compare samples of different sizes
  - ☞ extrapolation of VGC & productivity measures

- ➥ Specialized LNRE models (Baayen 2001)
  - LNRE = Large Number of Rare Events

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  - ☞ significance of differences in productivity
- Discrete frequency counts are difficult to capture with generalizations such as Zipf's law
  - ☞ Zipf's law predicts many impossible types with $1 < f_r < 2$
  - ☞ population does not suffer from such quantization effects

➥ Specialized LNRE models (Baayen 2001)

- LNRE = Large Number of Rare Events

The LNRE population

- Population: set of S types w_i with occurrence **probabilities** π_i
- $S =$ **population diversity** can be finite or infinite ($S = \infty$)
- Not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$

- Need **parametric** statistical **model** to describe full population (esp. for $S = \infty$), i.e. a function $i \mapsto \pi_i$
  - type probabilities π_i cannot be estimated reliably from a sample, but parameters of this function can
  - NB: population index $i \neq$ Zipf rank r

Zipf-Mandelbrot law as a population model

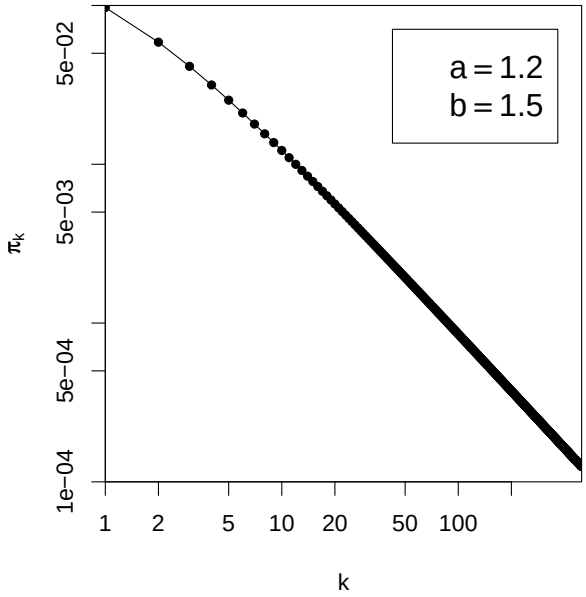
- Zipf-Mandelbrot law for type probabilities:

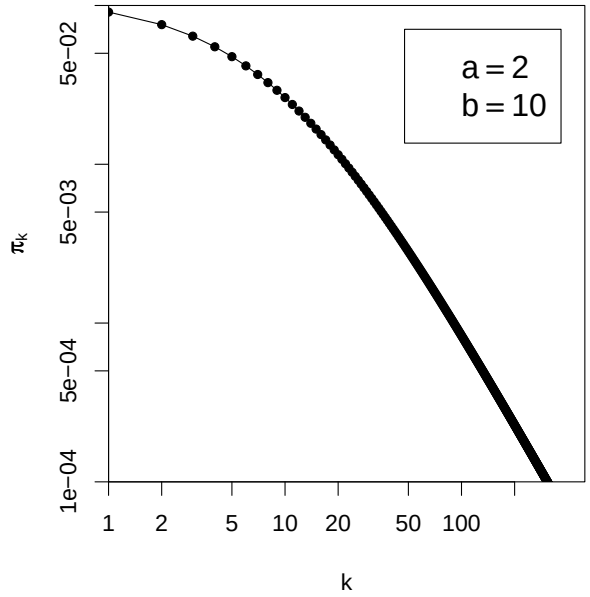
$$\pi_i := \frac{C}{(i+b)^a}$$

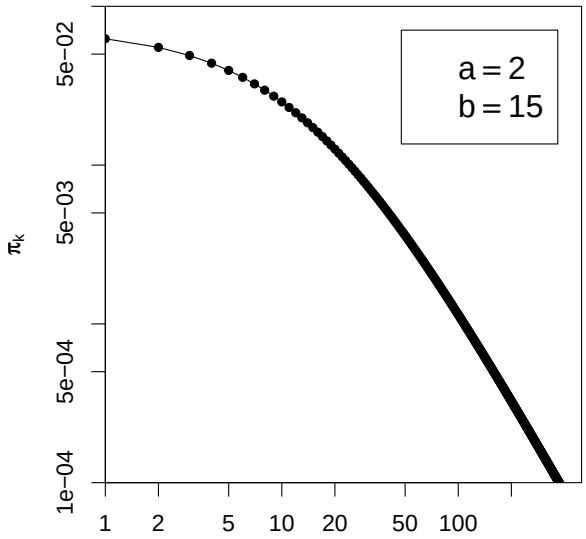
- Two free parameters: $a > 1$ and $b \geq 0$
  - ☞ C is not a parameter but a normalization constant, needed to ensure that $\sum_i \pi_i = 1$
- Third parameter: $S > 0$ or $S = \infty$

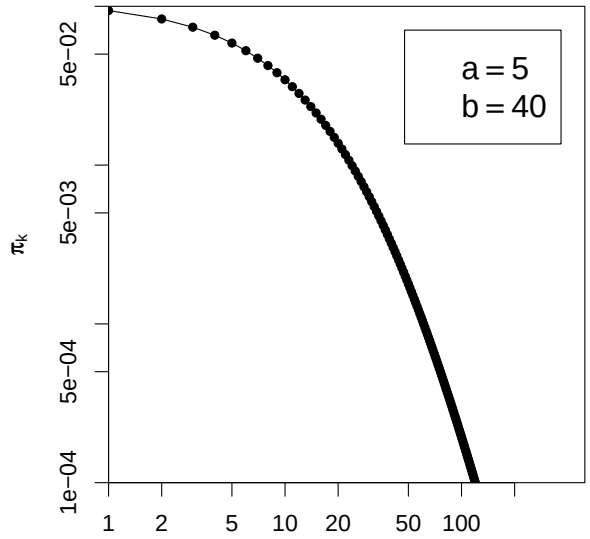
- This is the **Zipf-Mandelbrot** population model (Evert 2004)
  - **ZM** for Zipf-Mandelbrot model ($S = \infty$)
  - **fZM** for finite Zipf-Mandelbrot model

The parameters of the Zipf-Mandelbrot model









Sampling from a population model

#1: 1 42 34 23 108 18 48 18 1 ...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...
#3:	2	11	105	21	11	17	17	1	16	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...
#3:	2	11	105	21	11	17	17	1	16	...
#4:	44	3	110	34	223	2	25	20	28	...
#5:	24	81	54	11	8	61	1	31	35	...
#6:	3	65	9	165	5	42	16	20	7	...
#7:	10	21	11	60	164	54	18	16	203	...
#8:	11	7	147	5	24	19	15	85	37	...
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Samples: type frequency list & spectrum

rank r	f_r	type i
1	37	6
2	36	1
3	33	3
4	31	7
5	31	10
6	30	5
7	28	12
8	27	2
9	24	4
10	24	16
11	23	8
12	22	14
⋮	⋮	⋮

m	V_m
1	83
2	22
3	20
4	12
5	10
6	5
7	5
8	3
9	3
10	3
⋮	⋮

sample #1

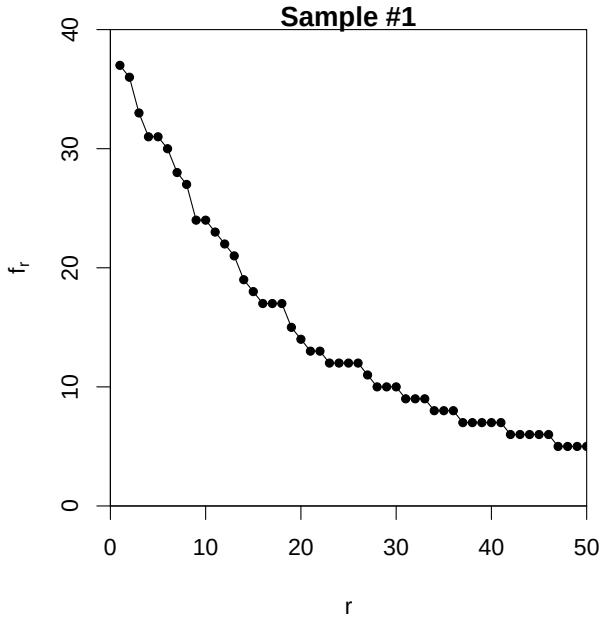
Samples: type frequency list & spectrum

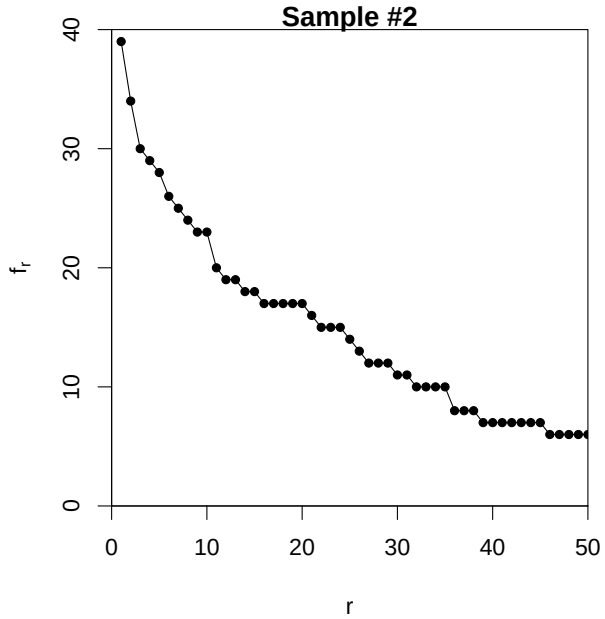
rank r	f_r	type i
1	39	2
2	34	3
3	30	5
4	29	10
5	28	8
6	26	1
7	25	13
8	24	7
9	23	6
10	23	11
11	20	4
12	19	17
⋮	⋮	⋮

m	V_m
1	76
2	27
3	17
4	10
5	6
6	5
7	7
8	3
10	4
11	2
⋮	⋮

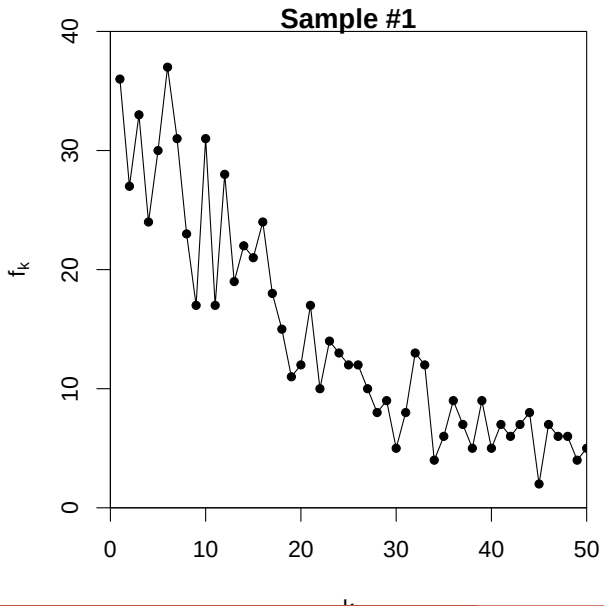
sample #2

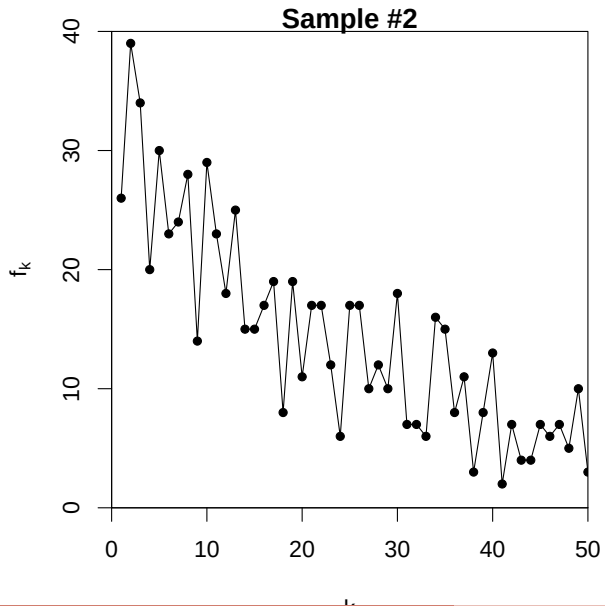
Random variation in type-frequency lists





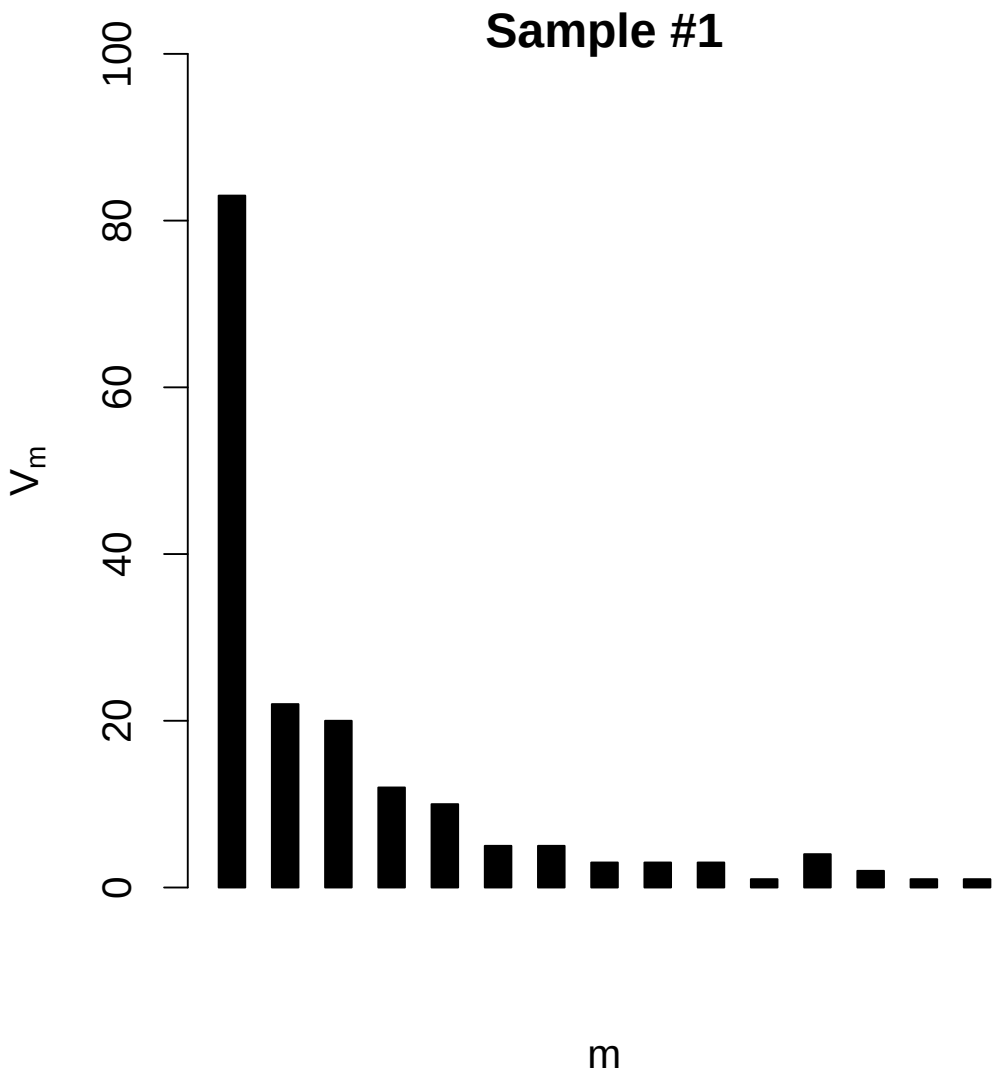
$r \leftrightarrow f_r$



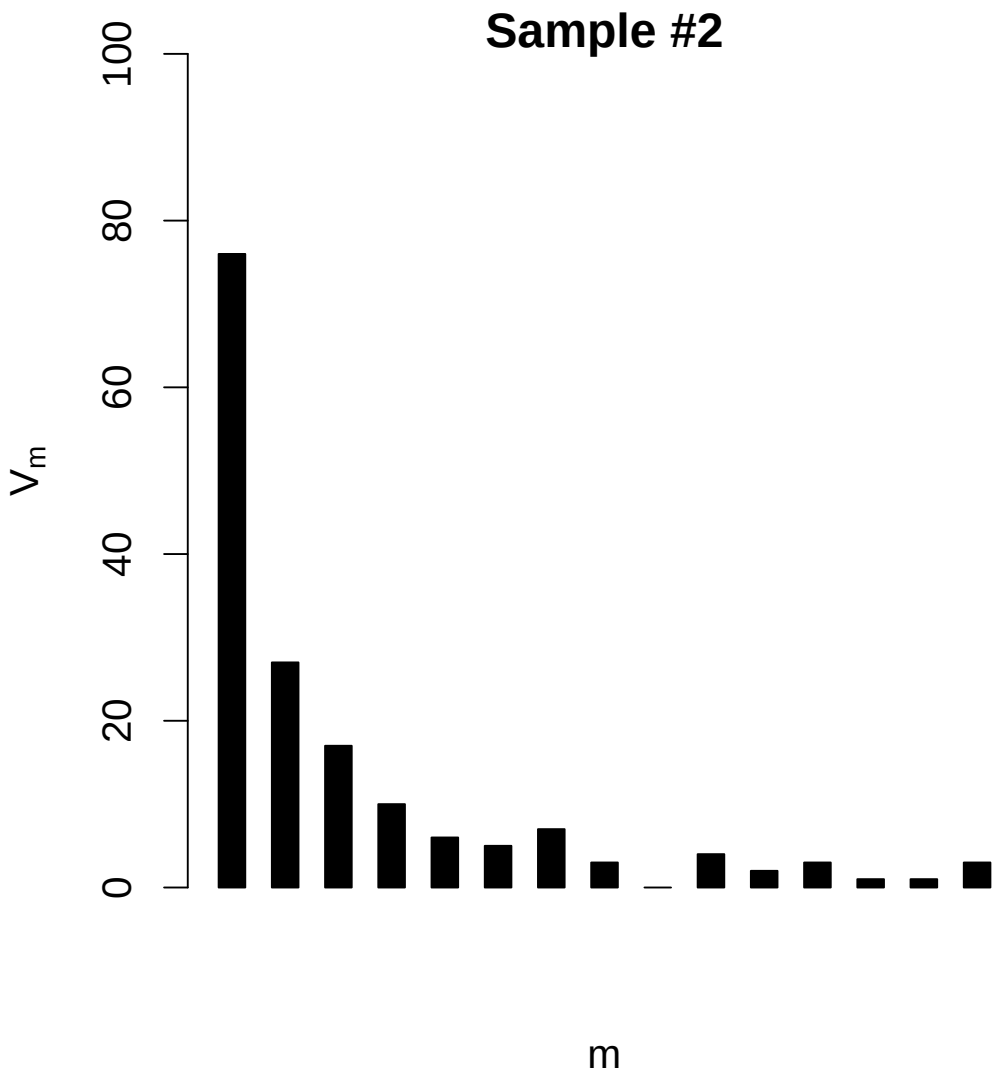


$i \leftrightarrow f_i$

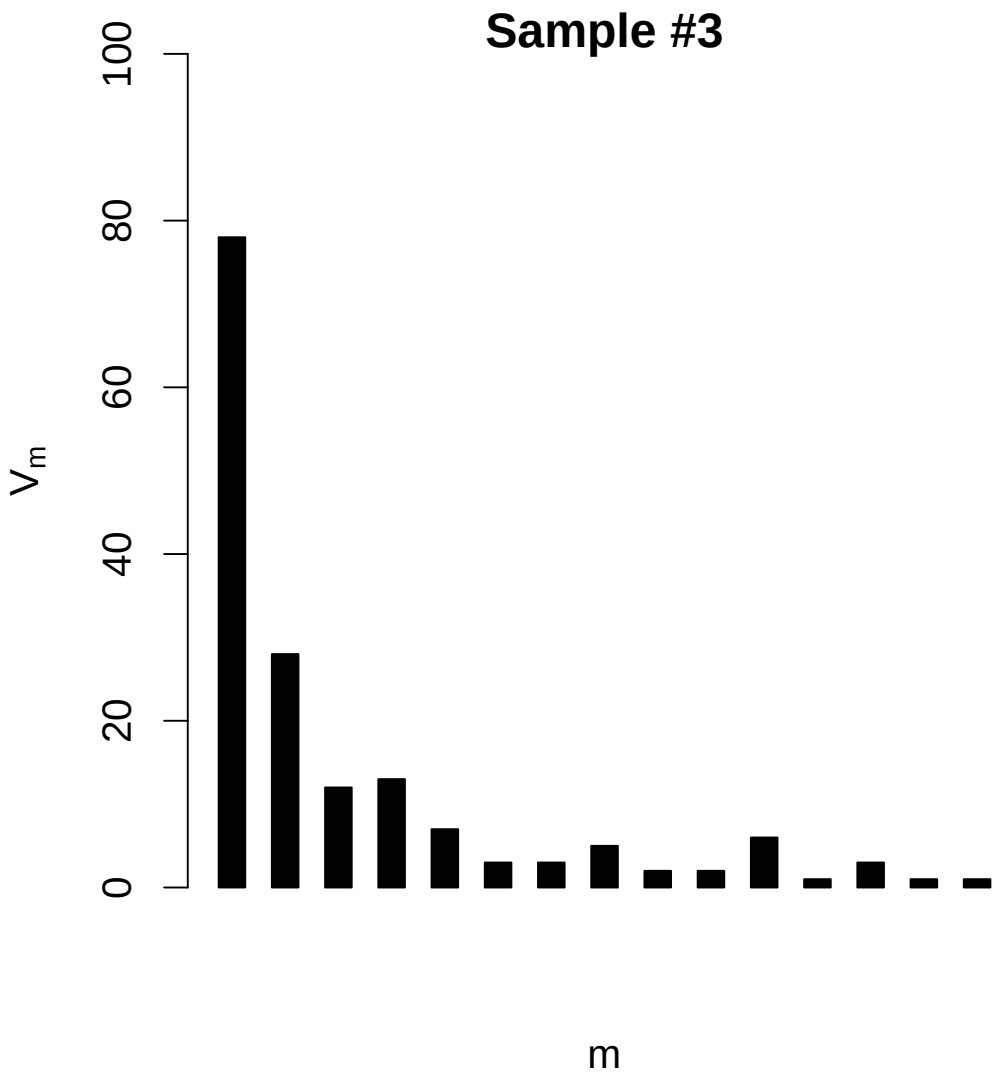
Random variation: frequency spectrum



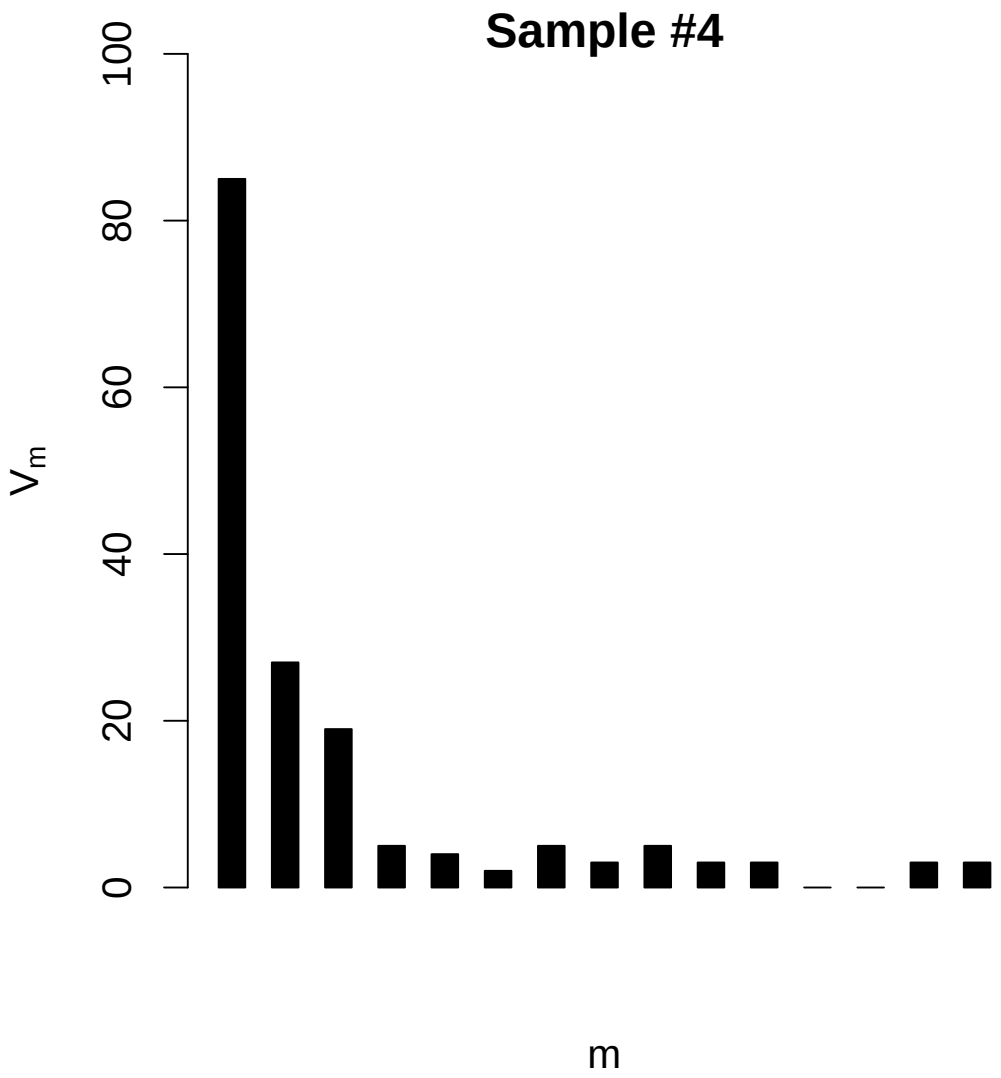
Random variation: frequency spectrum



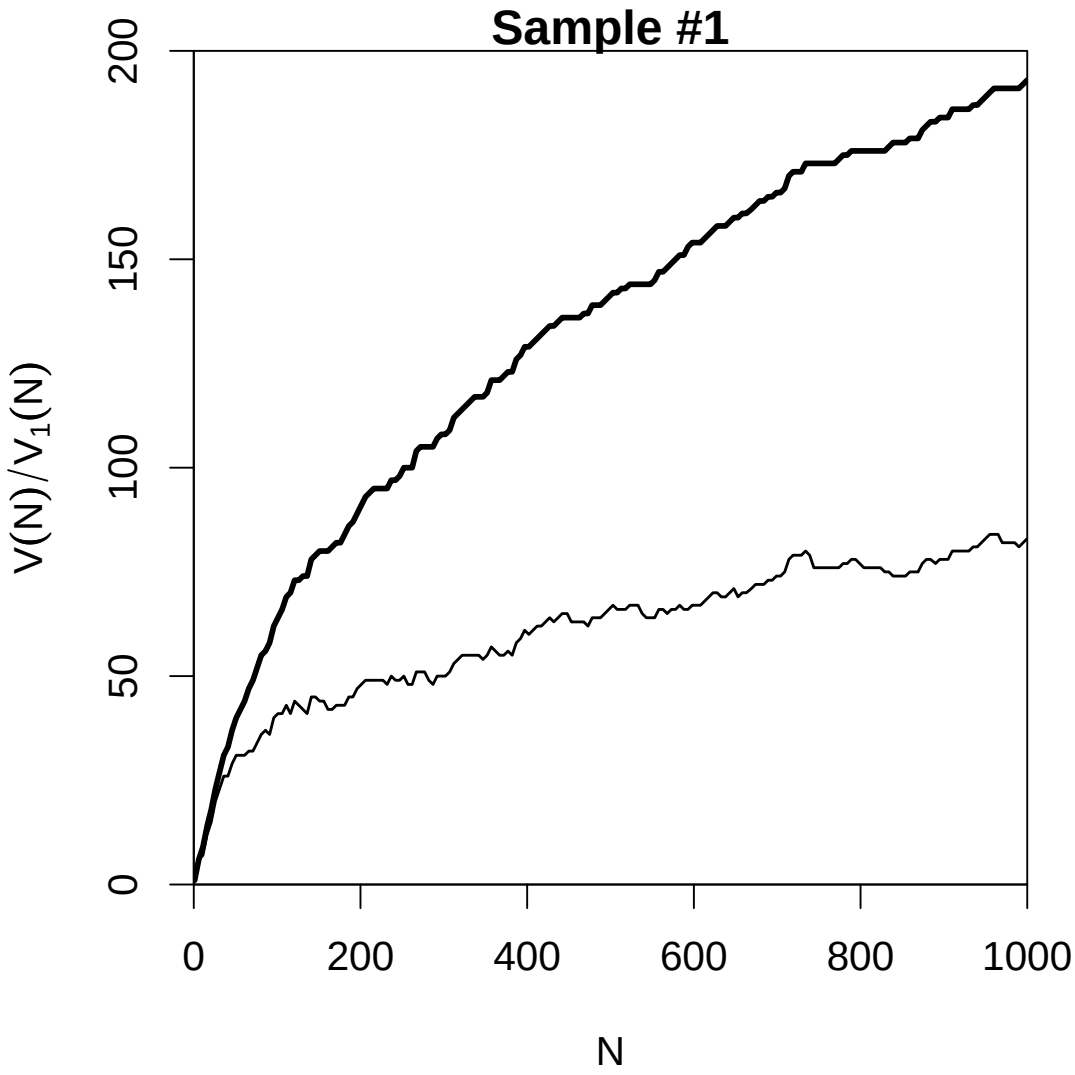
Random variation: frequency spectrum



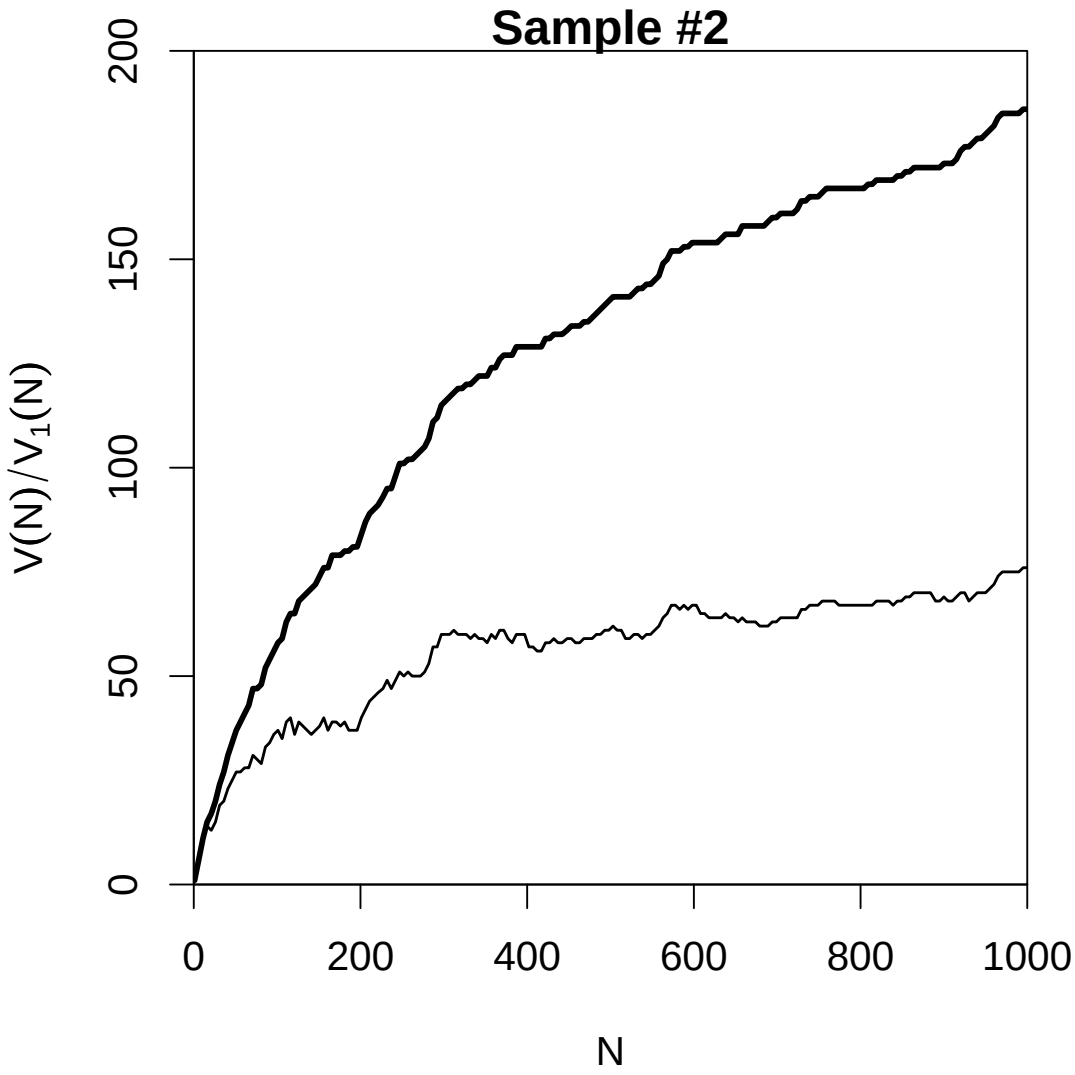
Random variation: frequency spectrum



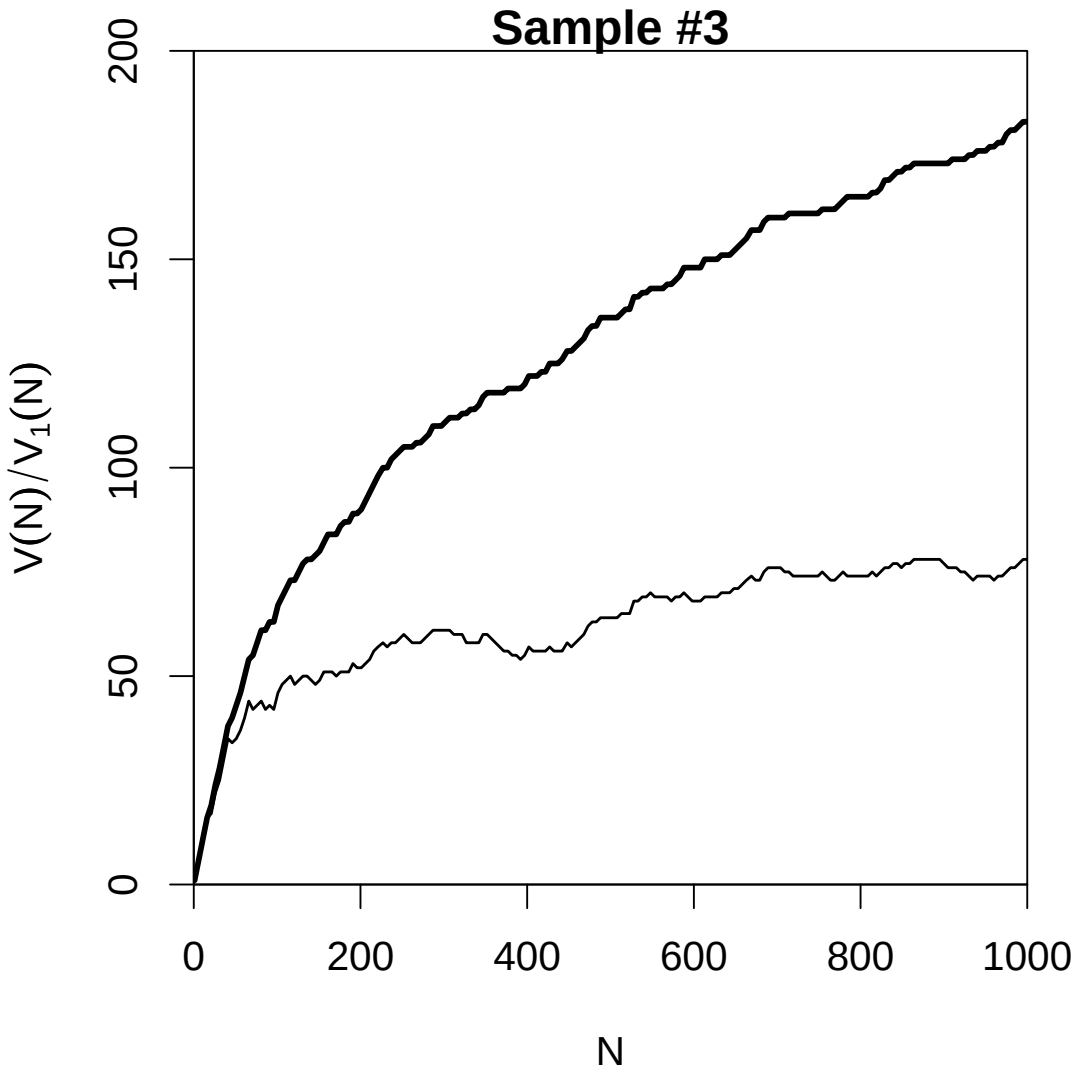
Random variation: vocabulary growth curve



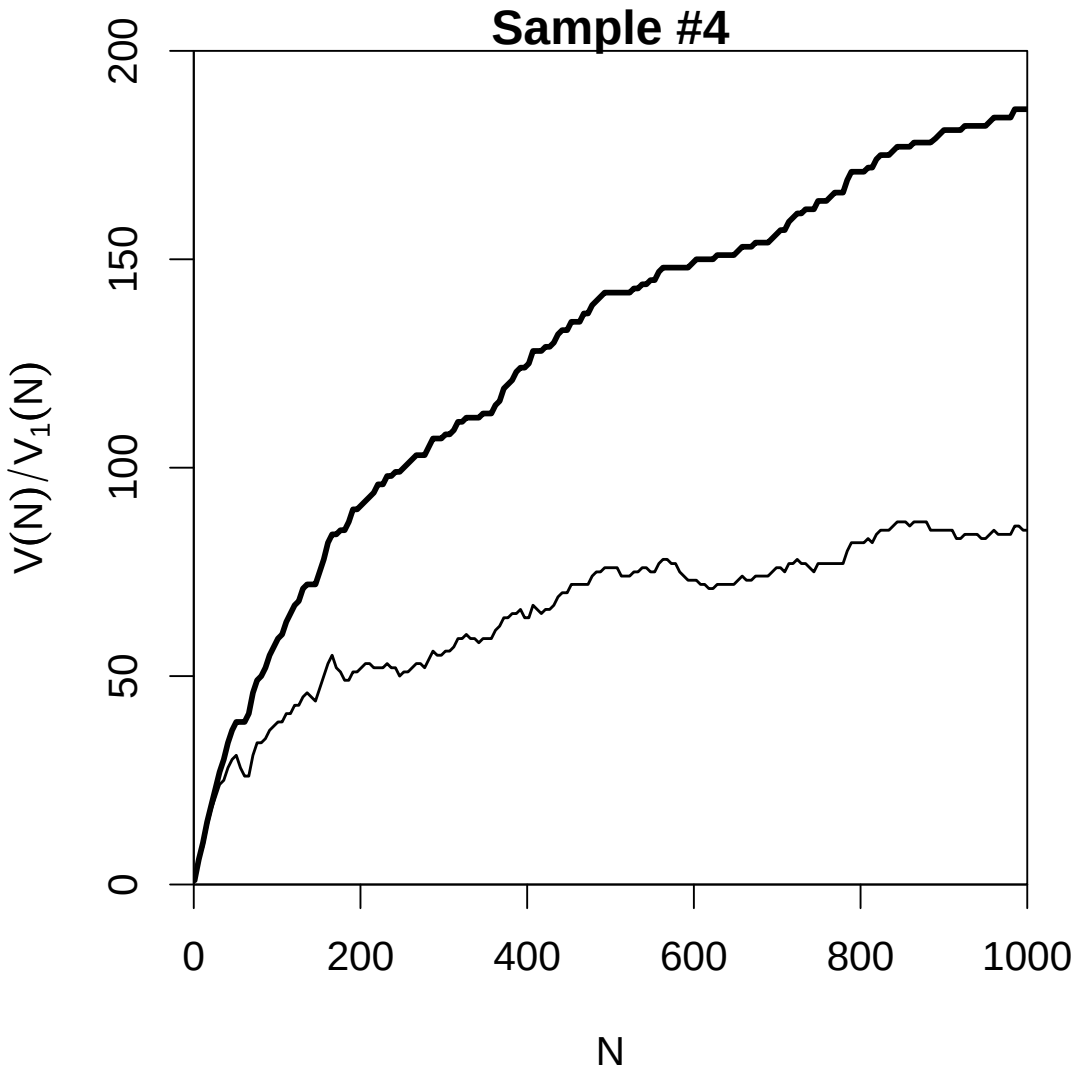
Random variation: vocabulary growth curve



Random variation: vocabulary growth curve



Random variation: vocabulary growth curve



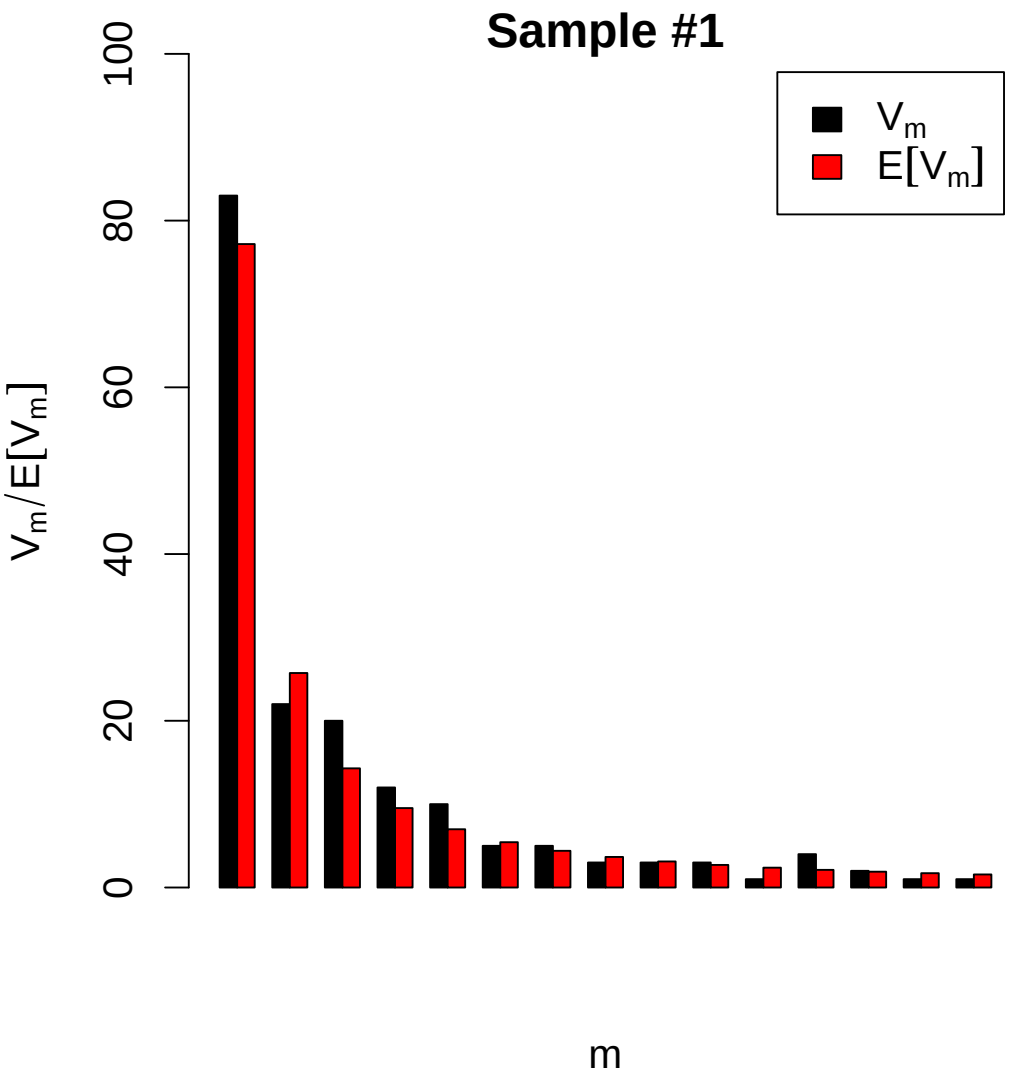
Expected values

- There is no reason why we should choose a particular sample to compare to the real data or make a prediction – each one is equally likely or unlikely
- Take the average over a large number of samples, called **expected value** or **expectation** in statistics

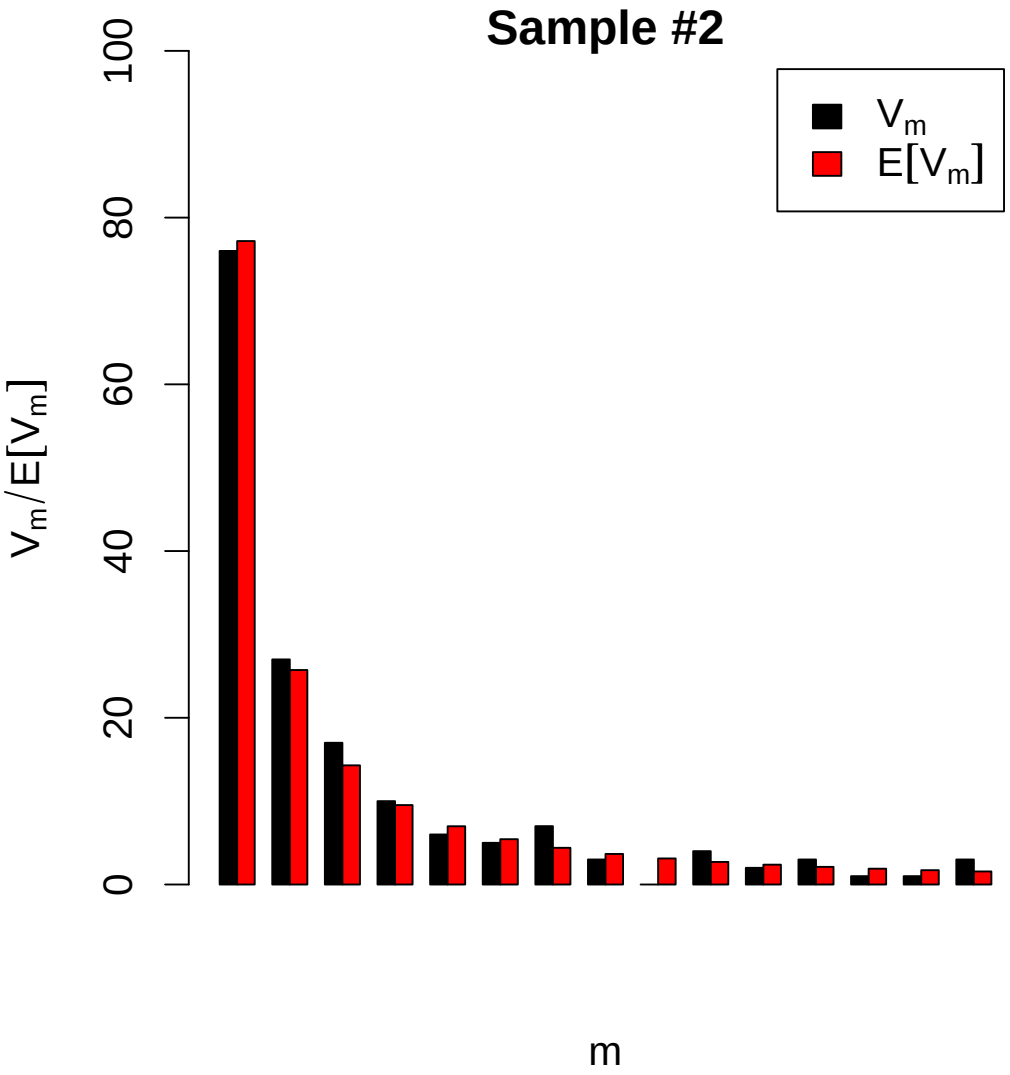
- Notation: $\mathrm{E}\big[V(N)\big]$ and $\mathrm{E}\big[V_m(N)\big]$
  - indicates that we are referring to expected values for a sample of size N
  - rather than to the specific values V and V_m observed in a particular sample or a real-world data set

- Expected values can be calculated efficiently *without* generating thousands of random samples

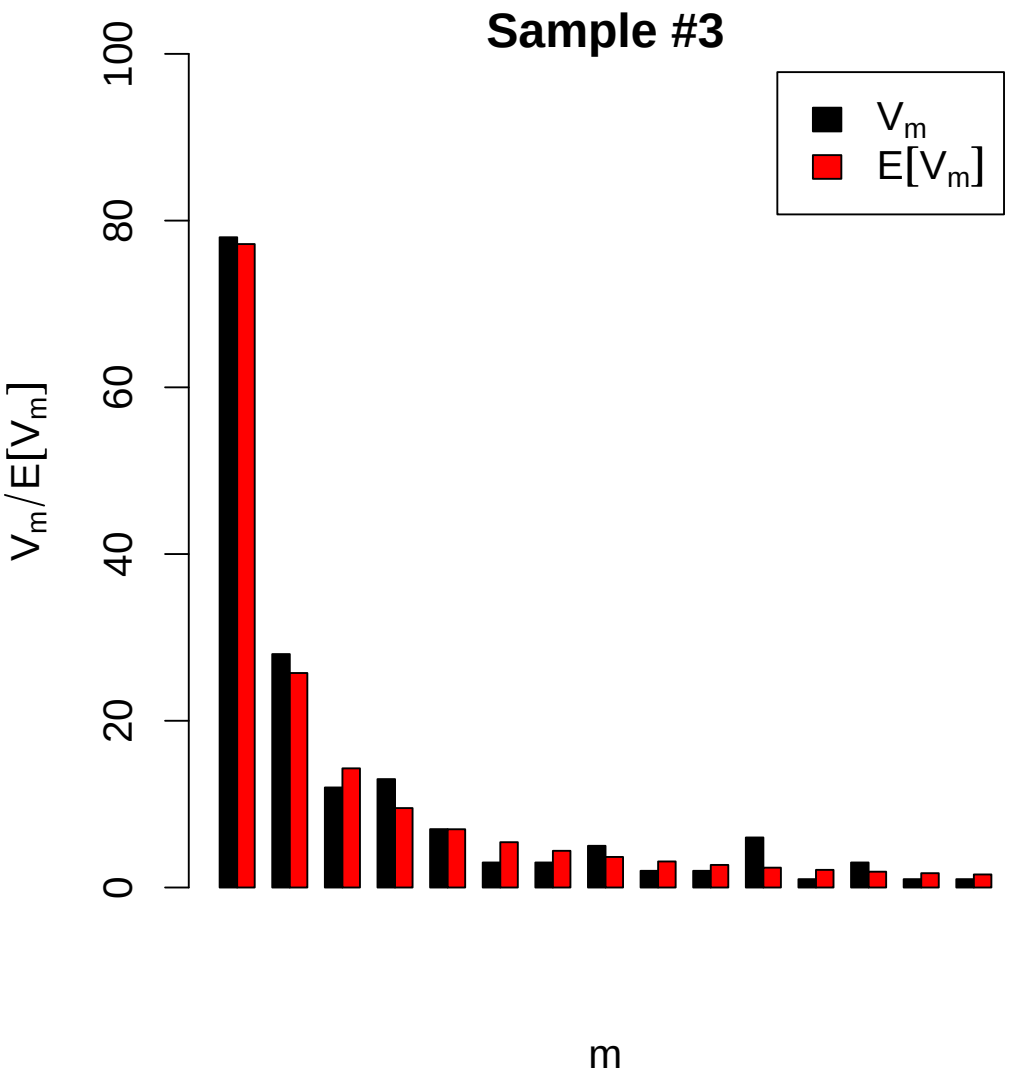
The expected frequency spectrum



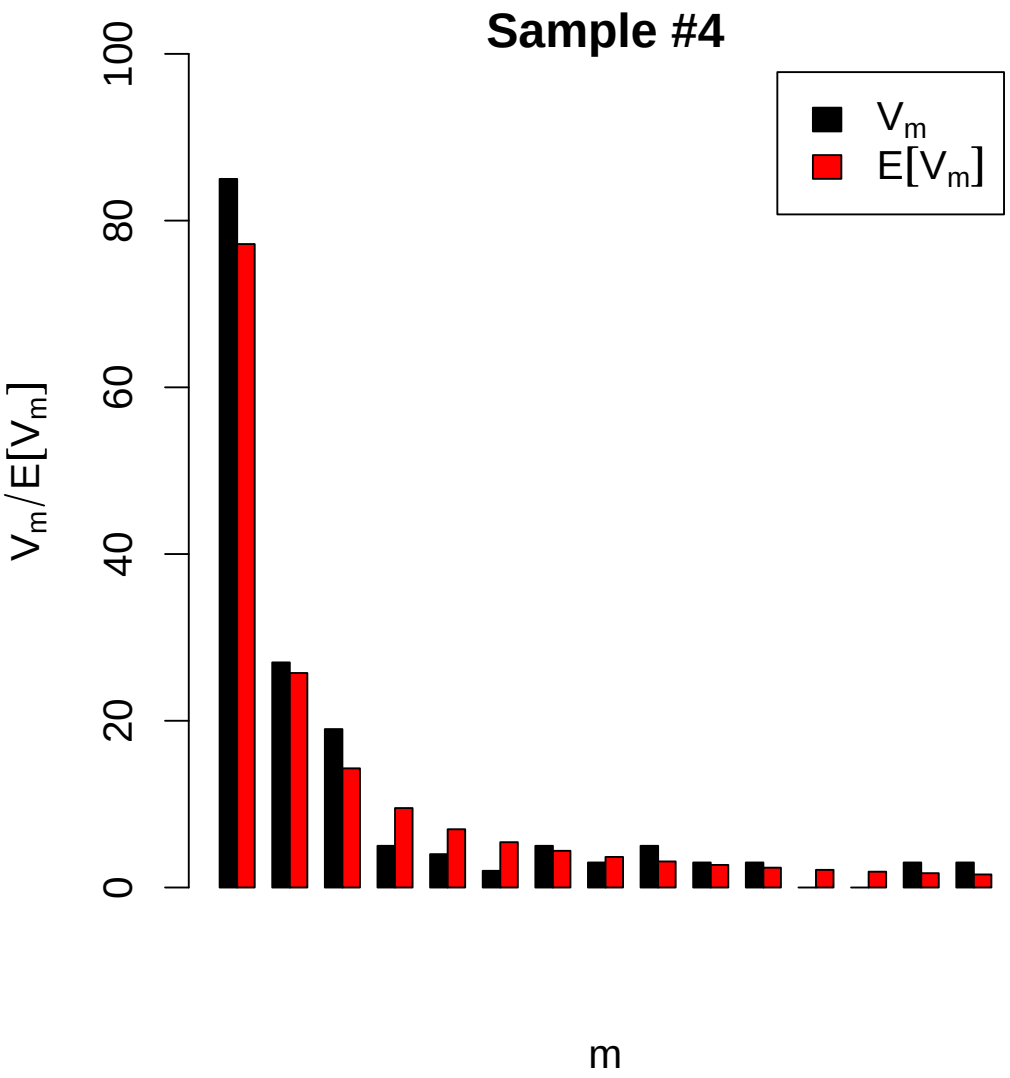
The expected frequency spectrum



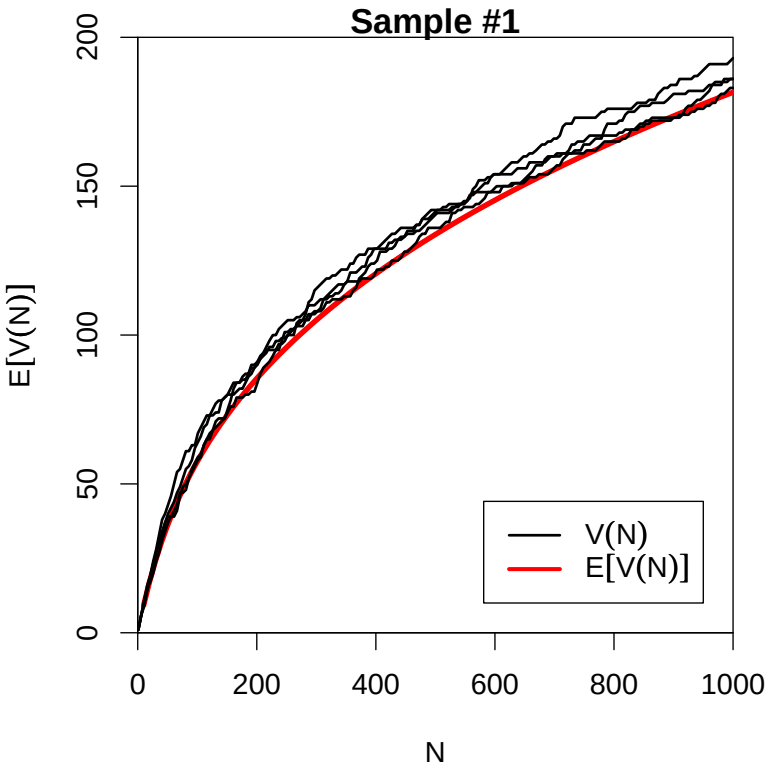
The expected frequency spectrum

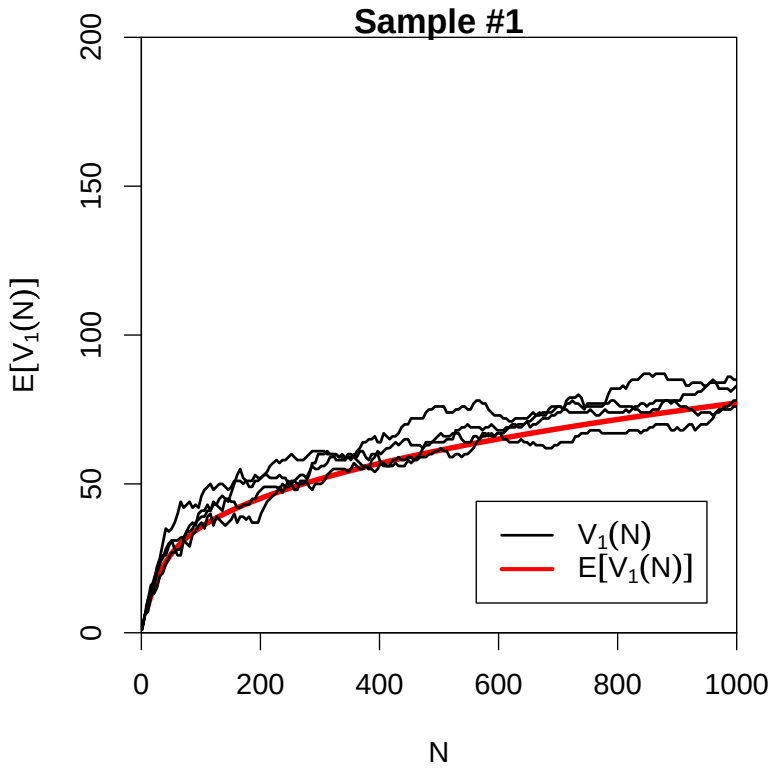


The expected frequency spectrum

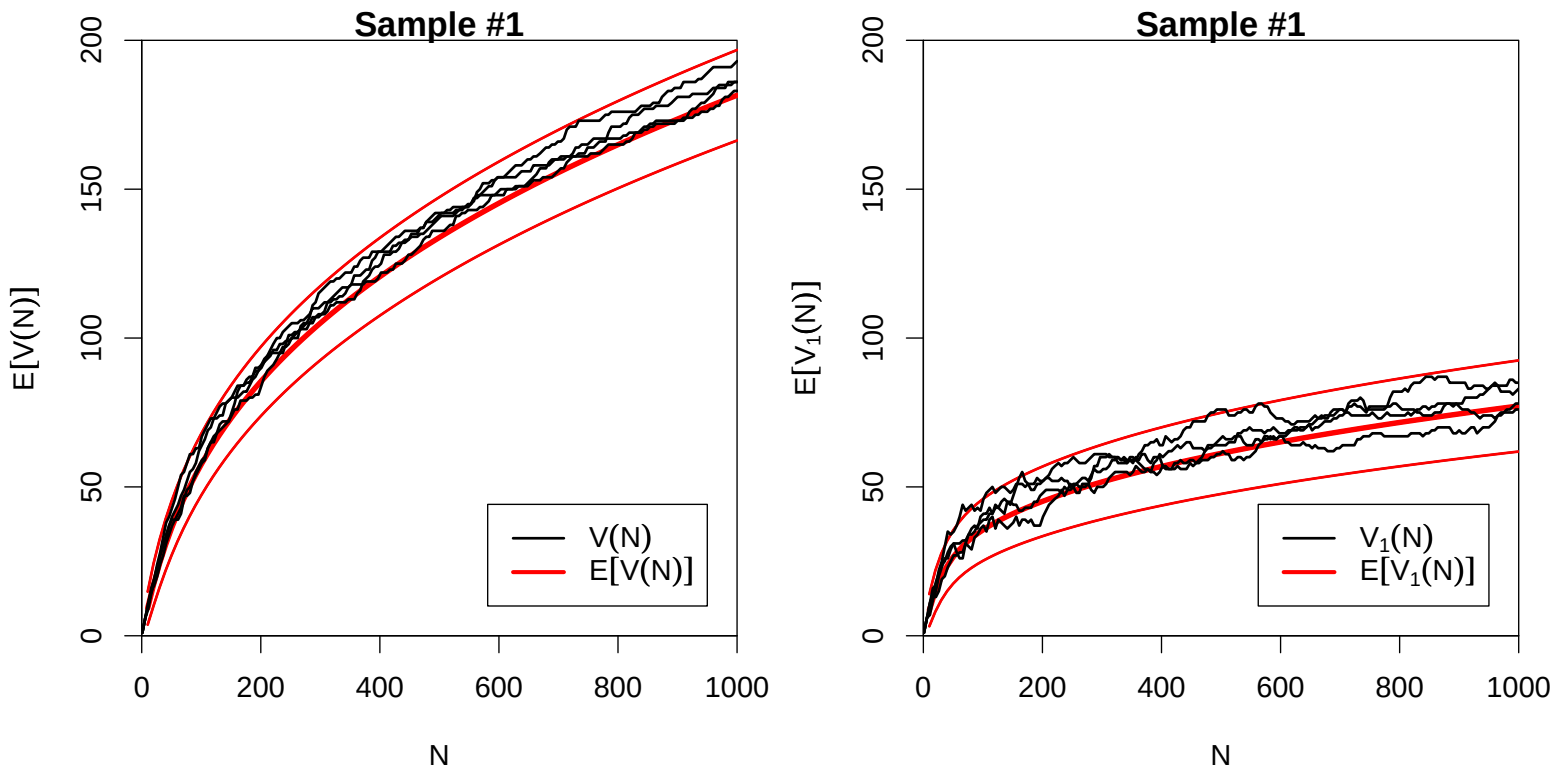


The expected vocabulary growth curve





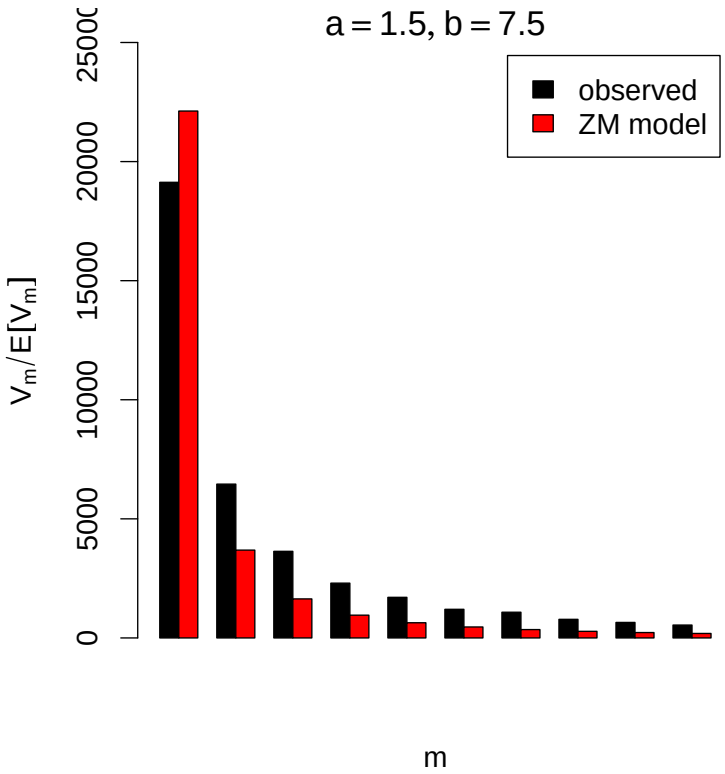
Prediction intervals for the expected VGC

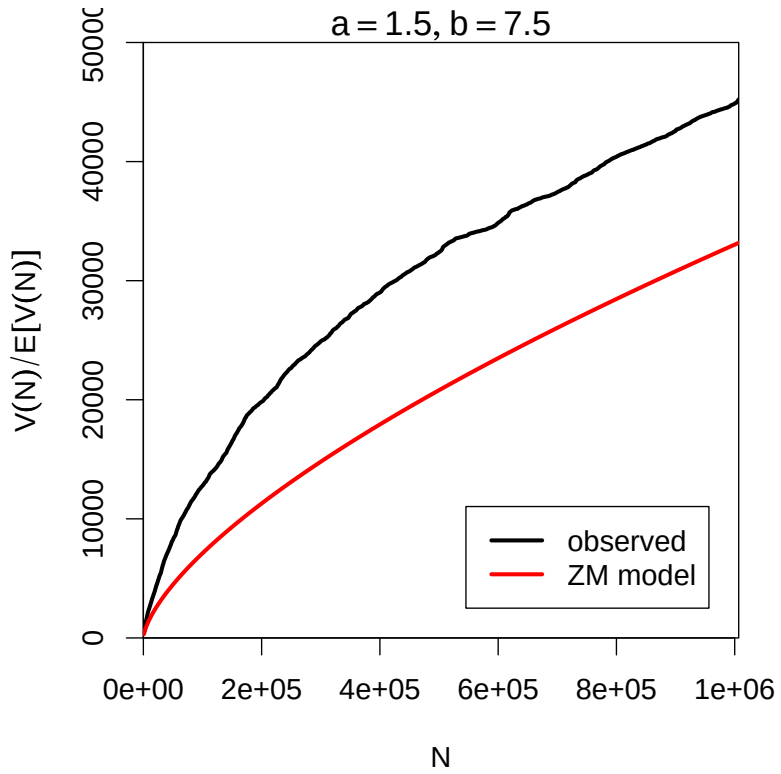


"Confidence intervals" indicate predicted sampling distribution:

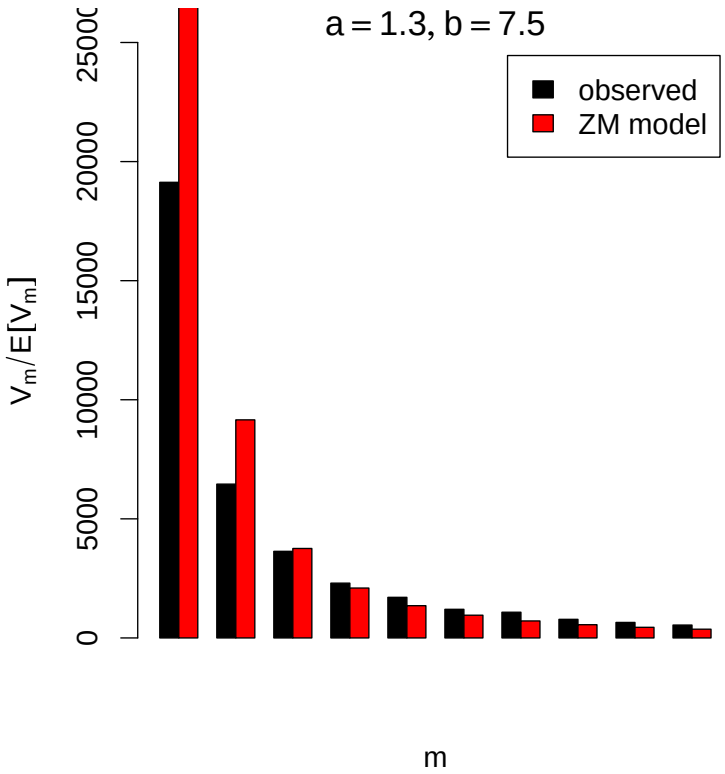
- ☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines

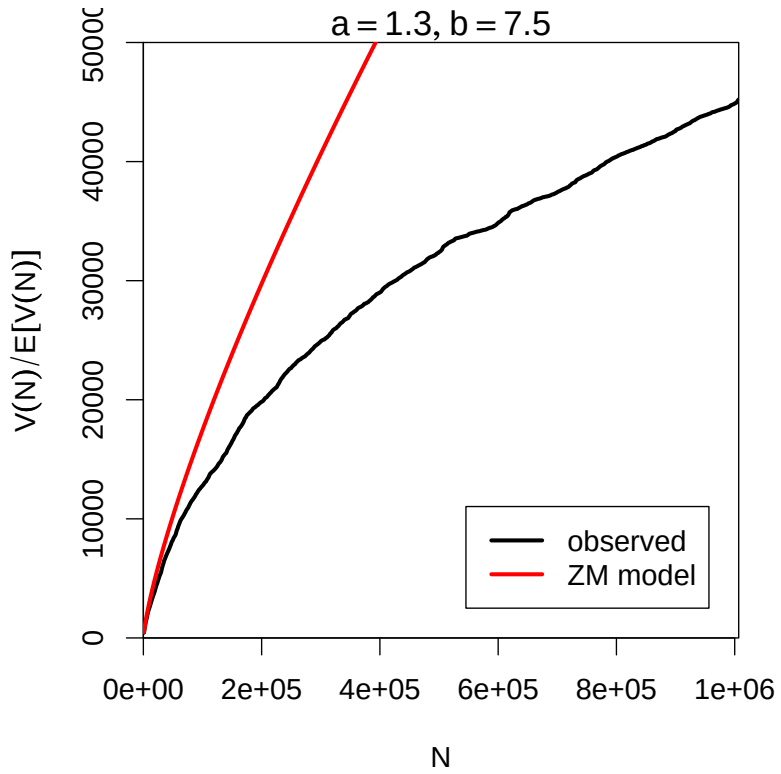
Parameter estimation by trial & error



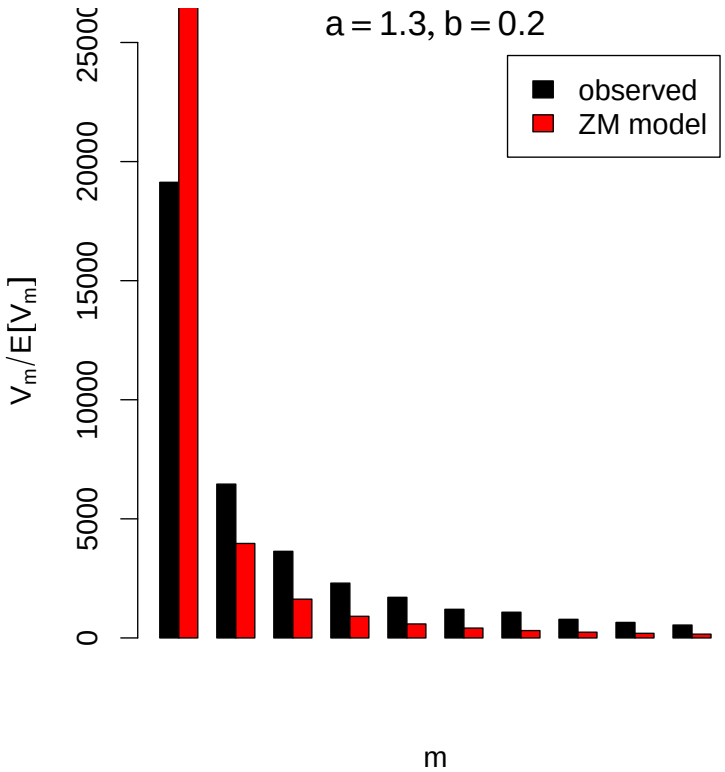


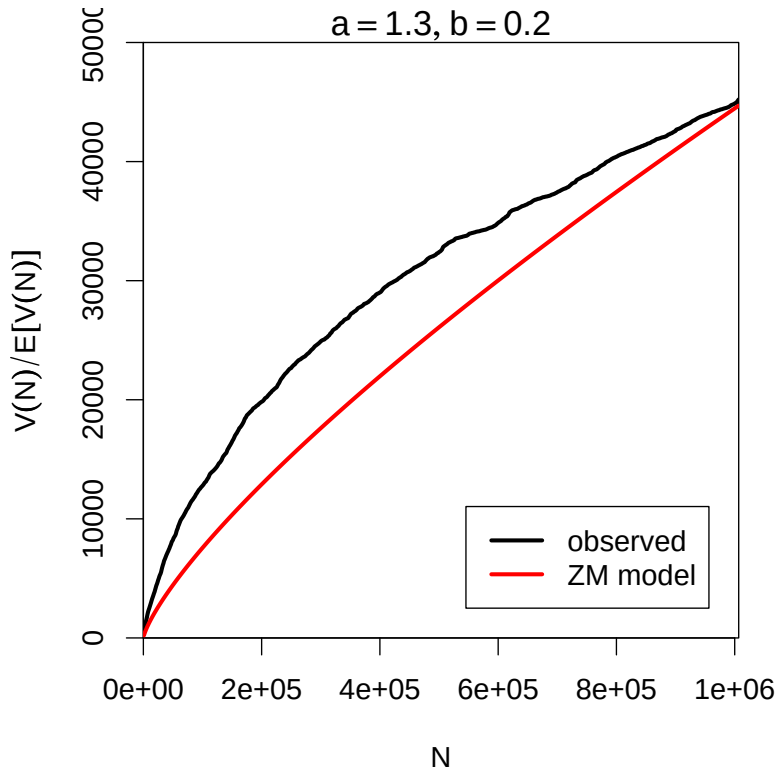
Parameter estimation by trial & error



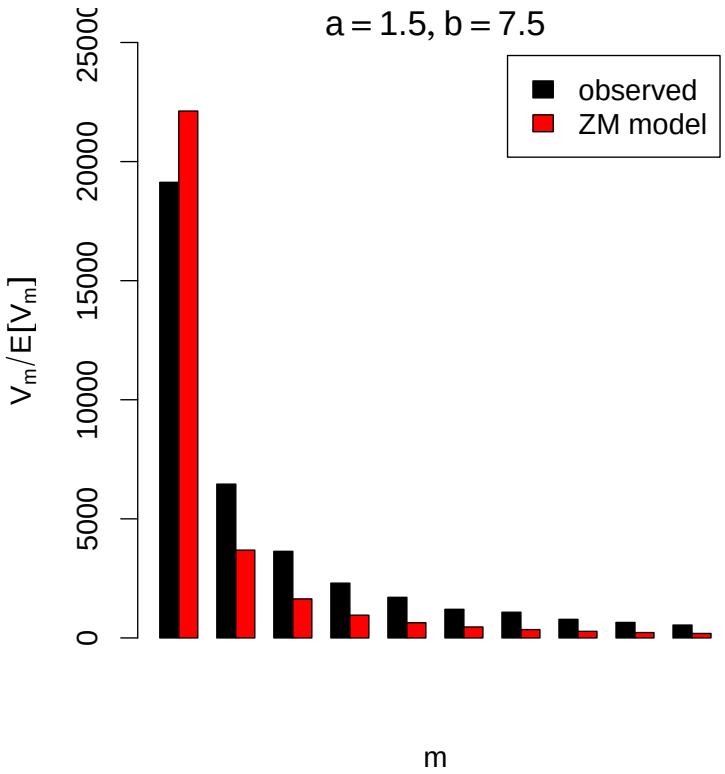


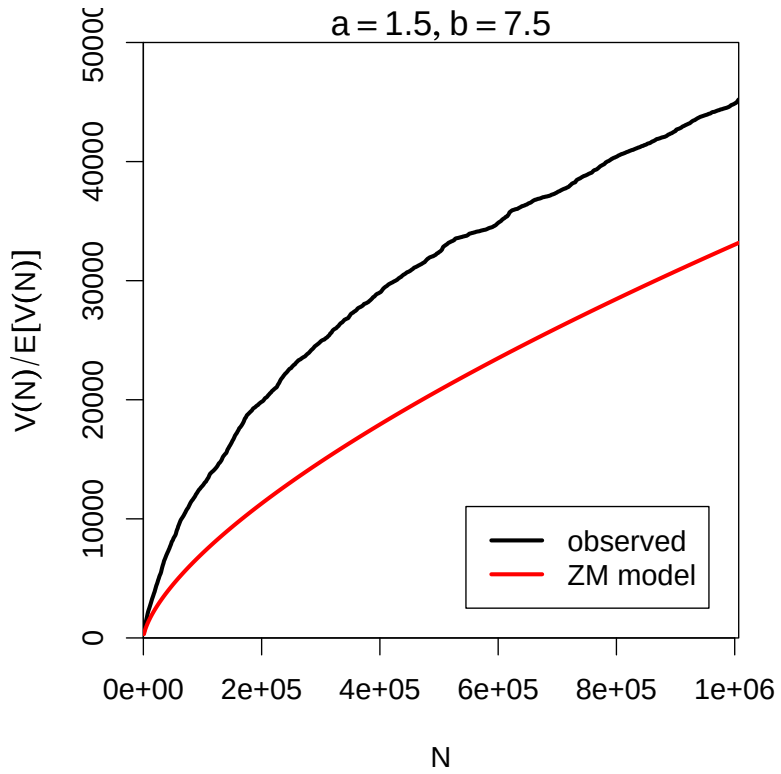
Parameter estimation by trial & error



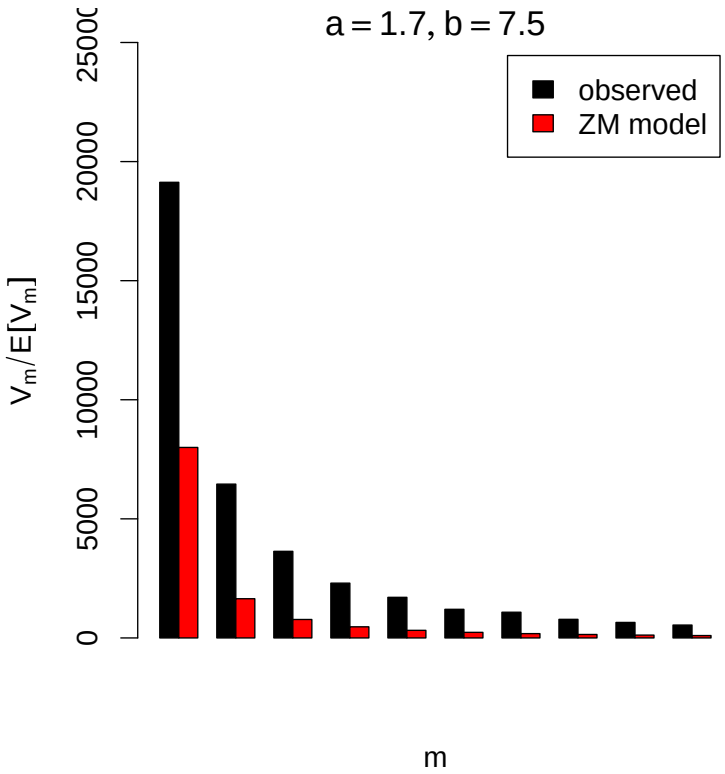


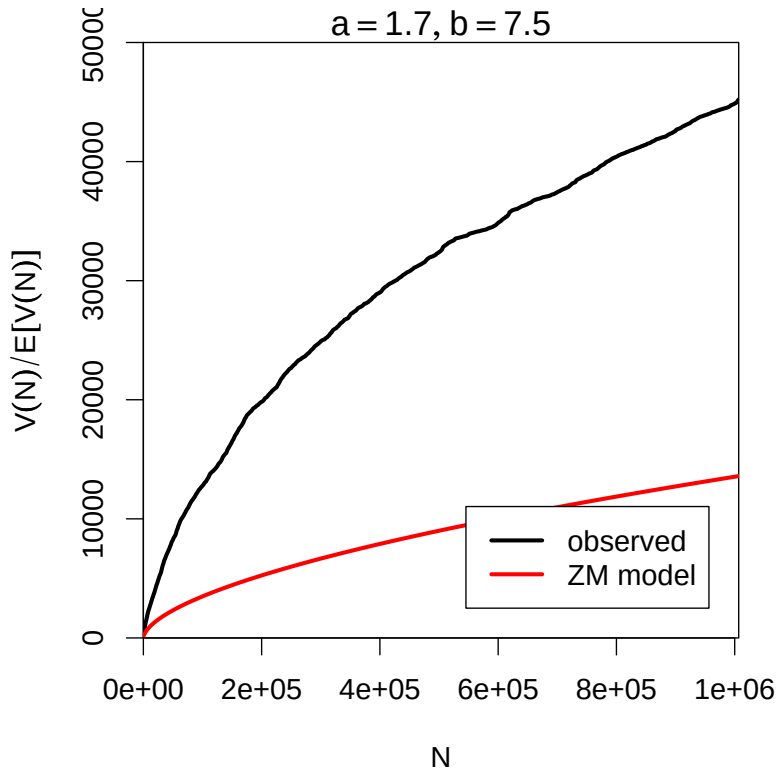
Parameter estimation by trial & error



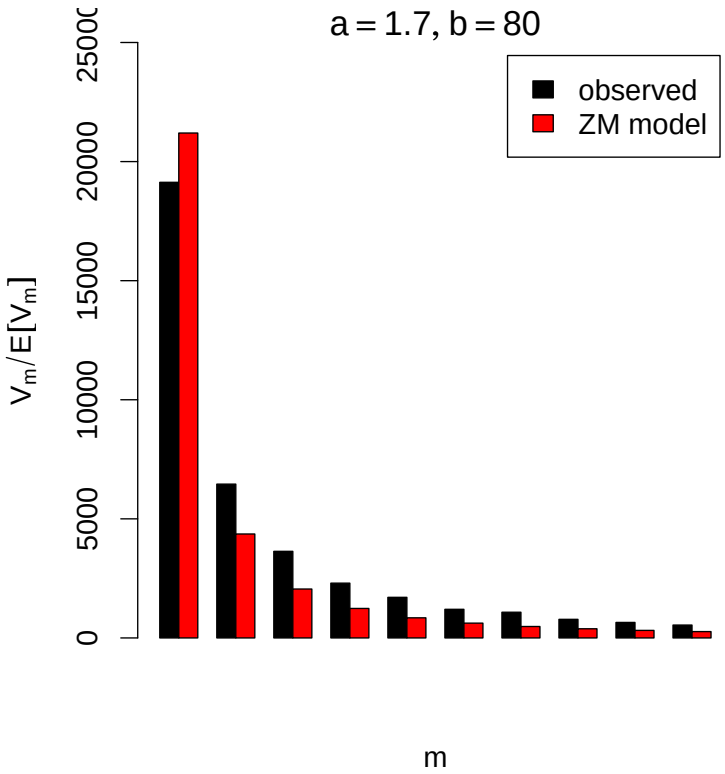


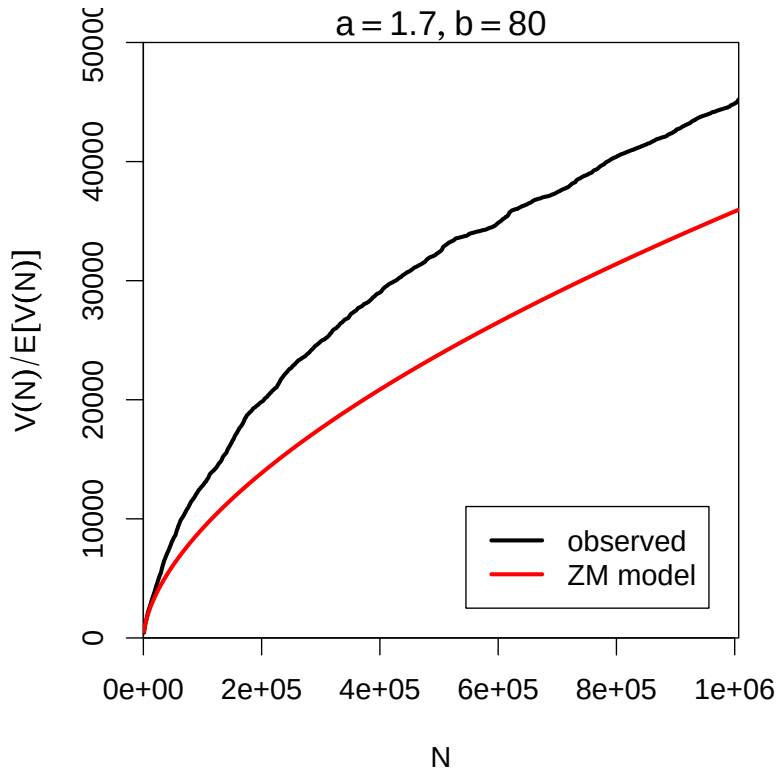
Parameter estimation by trial & error



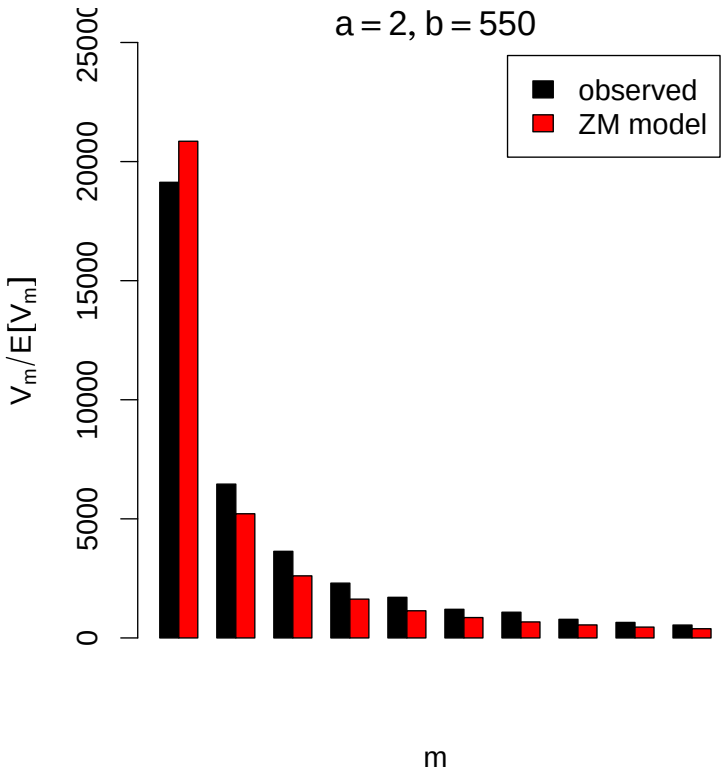


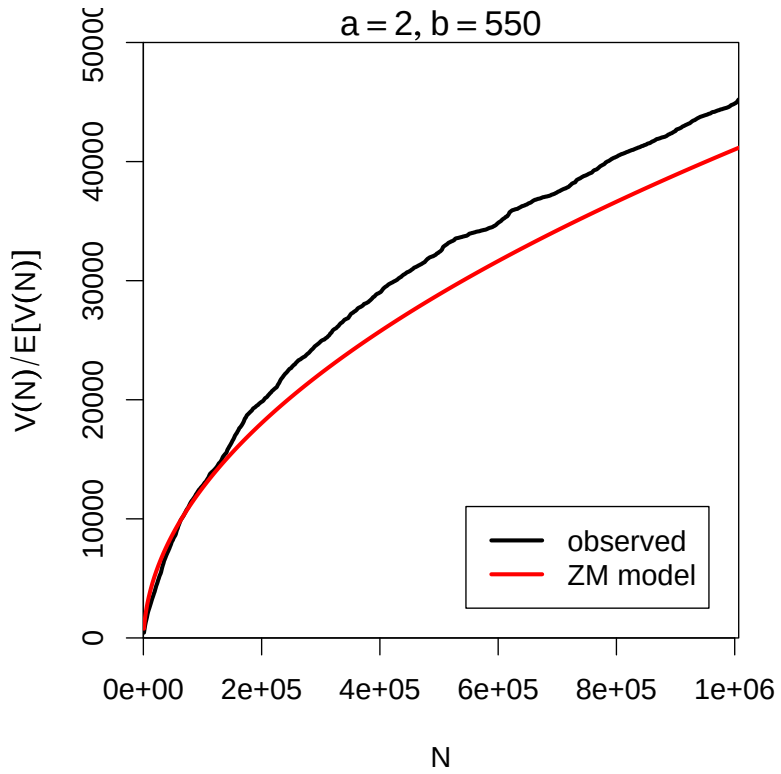
Parameter estimation by trial & error



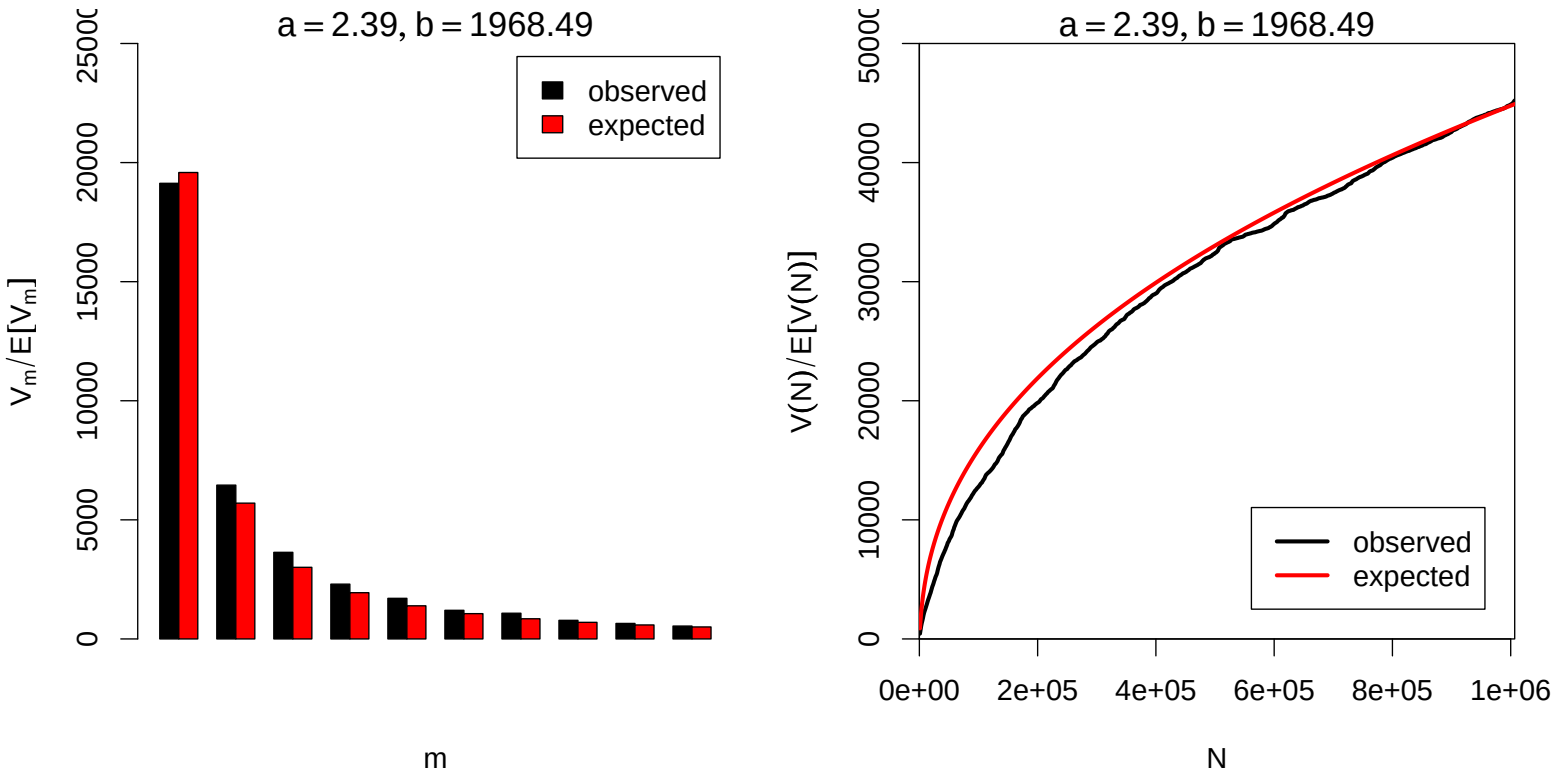


Parameter estimation by trial & error



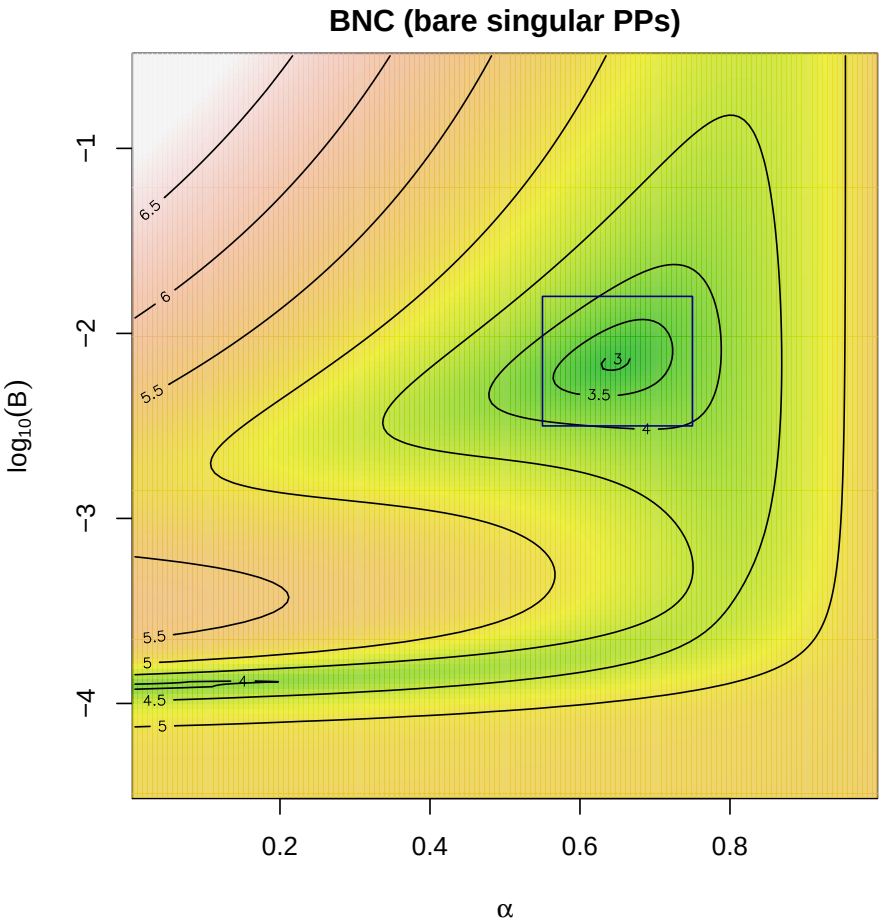


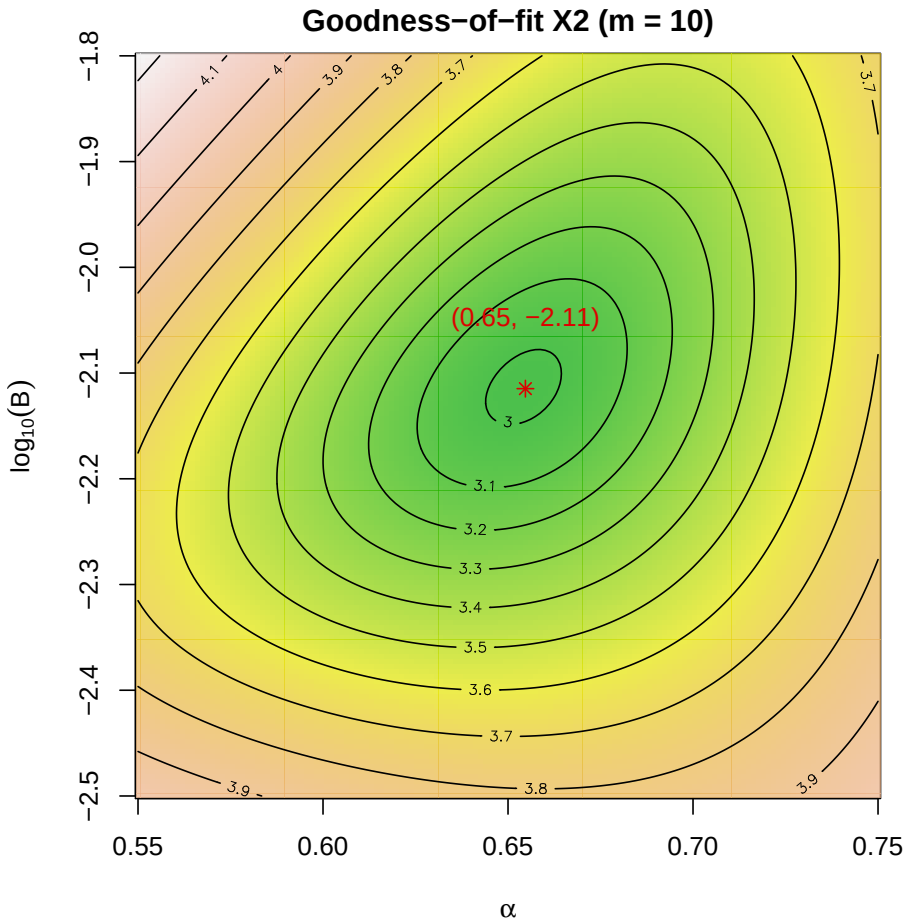
Automatic parameter estimation



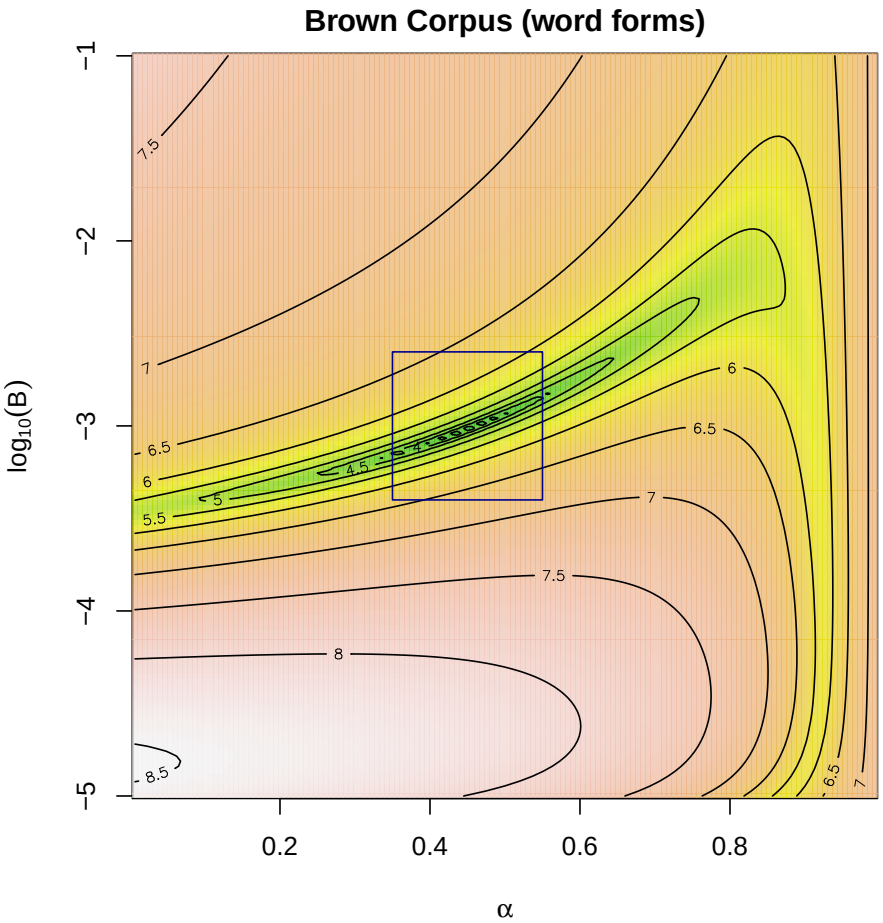
- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure: $a = 2.39$ and $b = 1968$

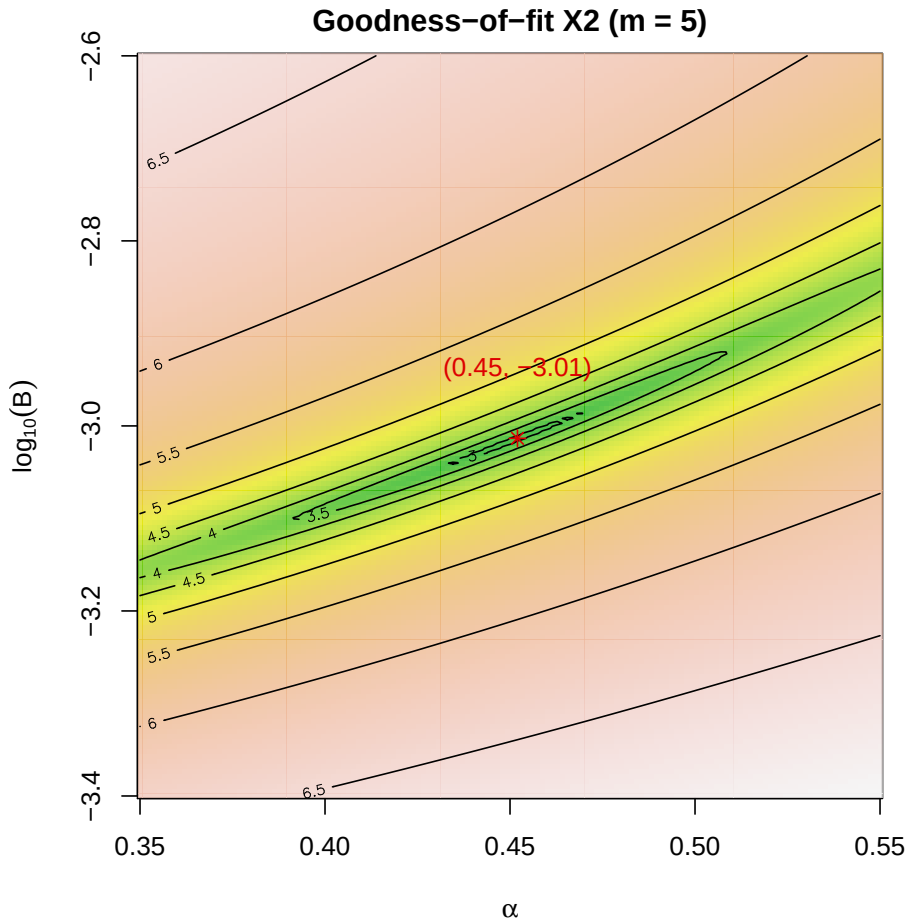
Automatic parameter estimation



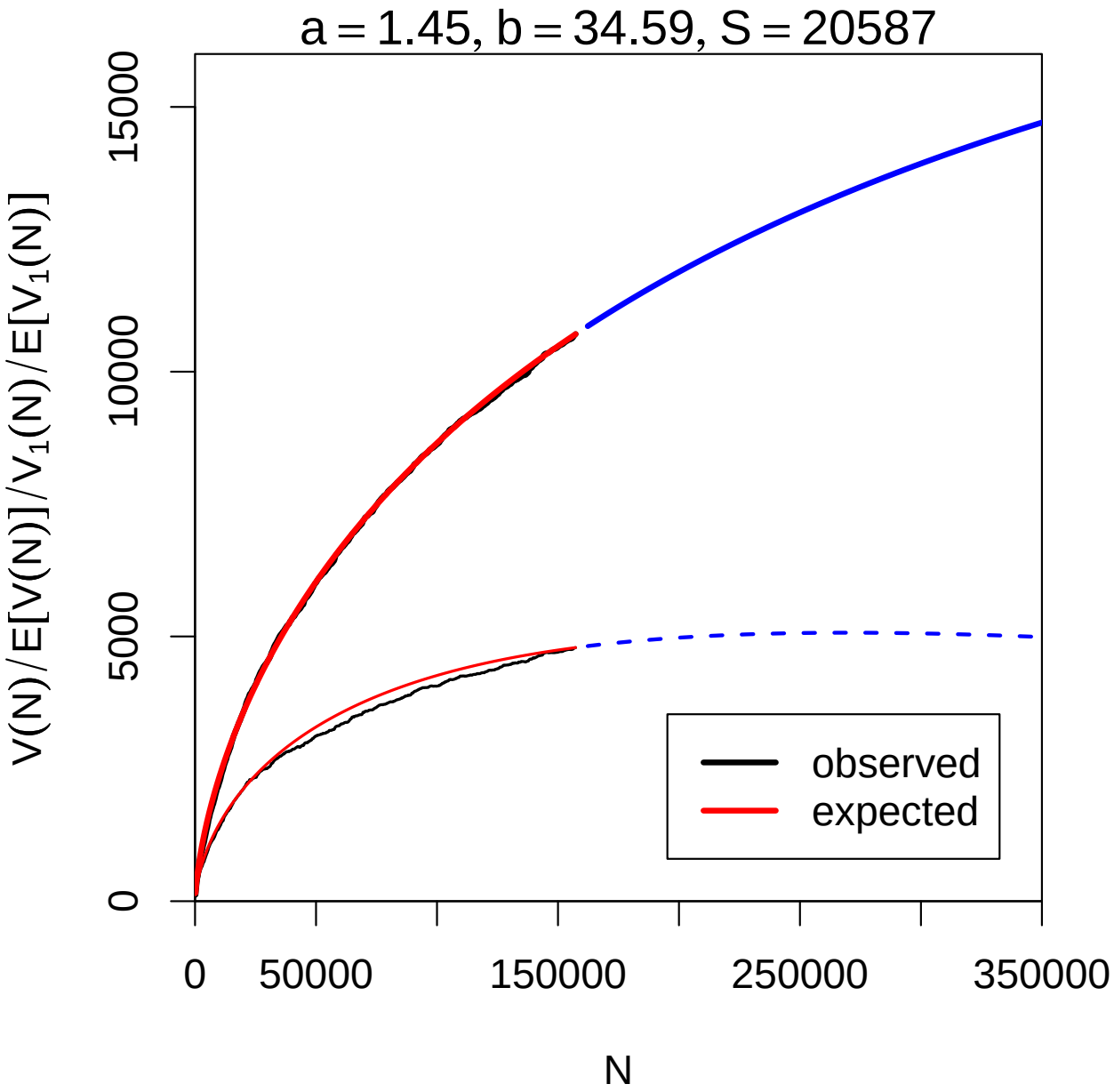


Automatic parameter estimation



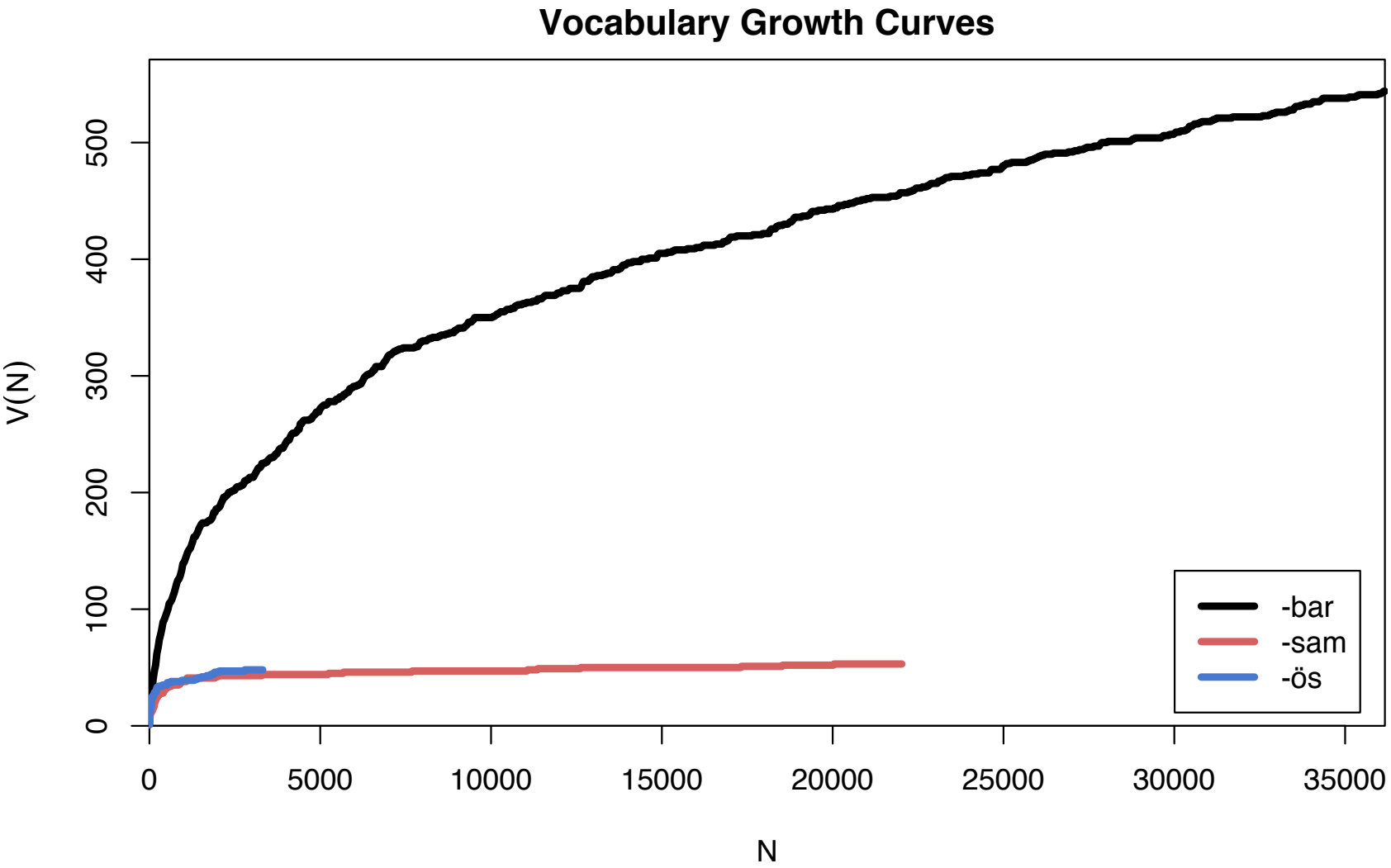


Extrapolation of vocabulary growth curves



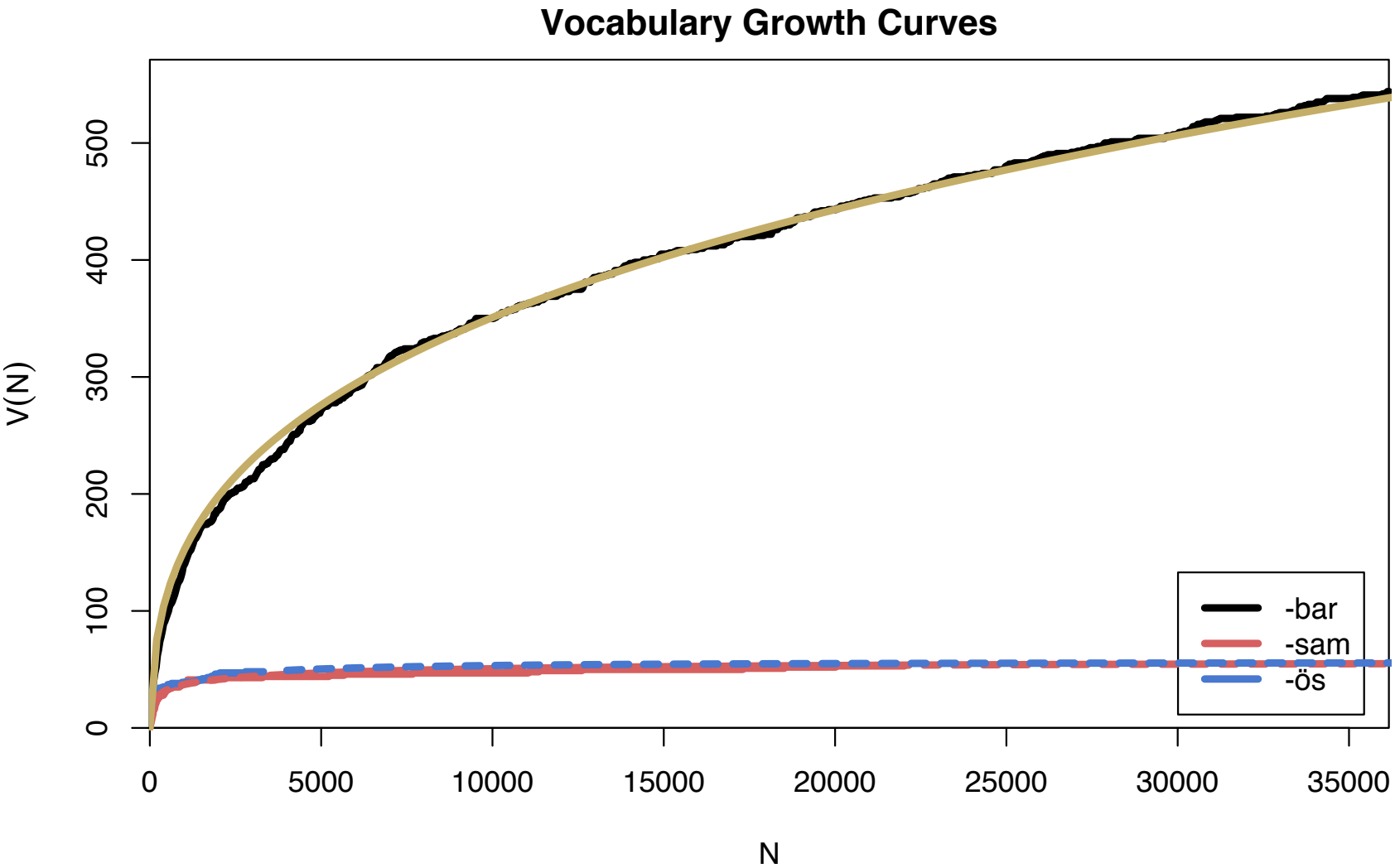
Measuring morphological productivity

example from Evert and Lüdeling (2001)



Measuring morphological productivity

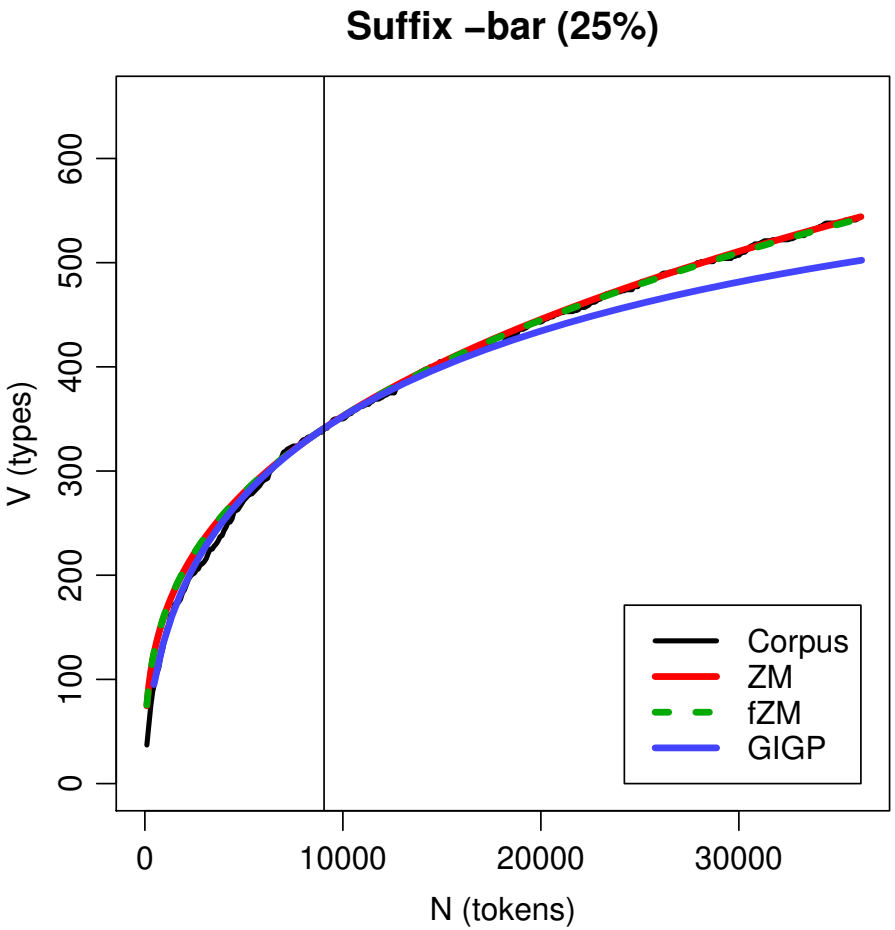
example from Evert and Lüdeling (2001)

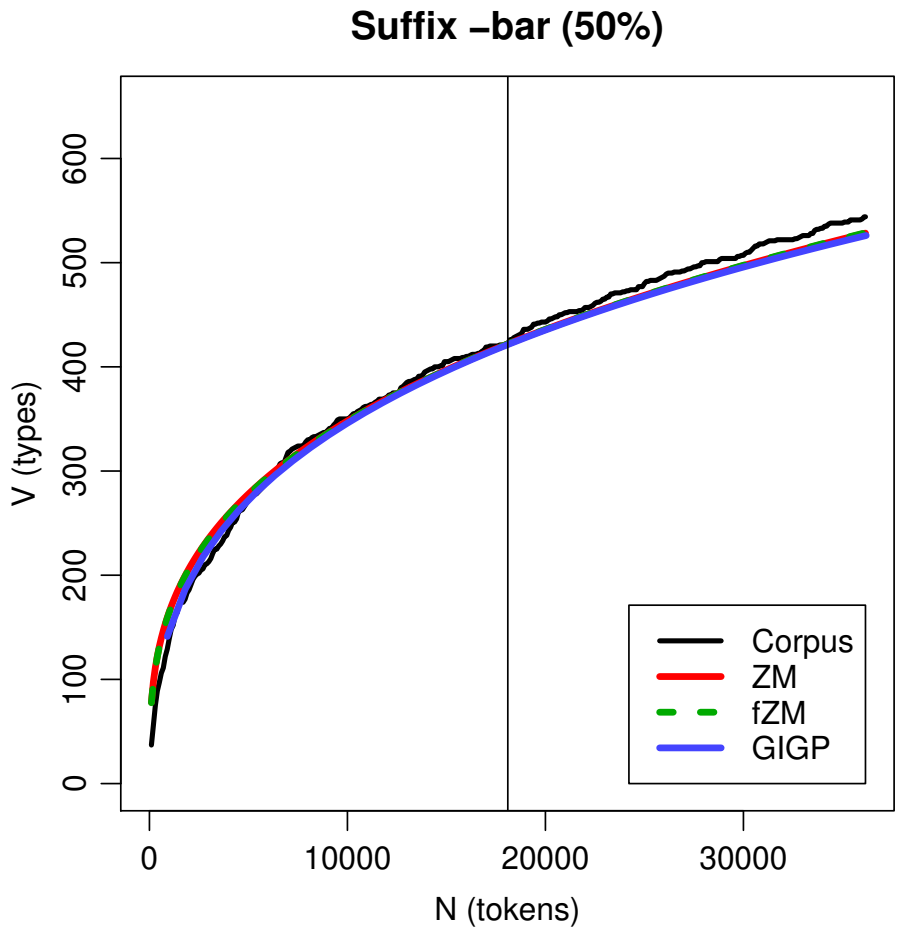


Outline

How accurate is LNRE-based extrapolation?

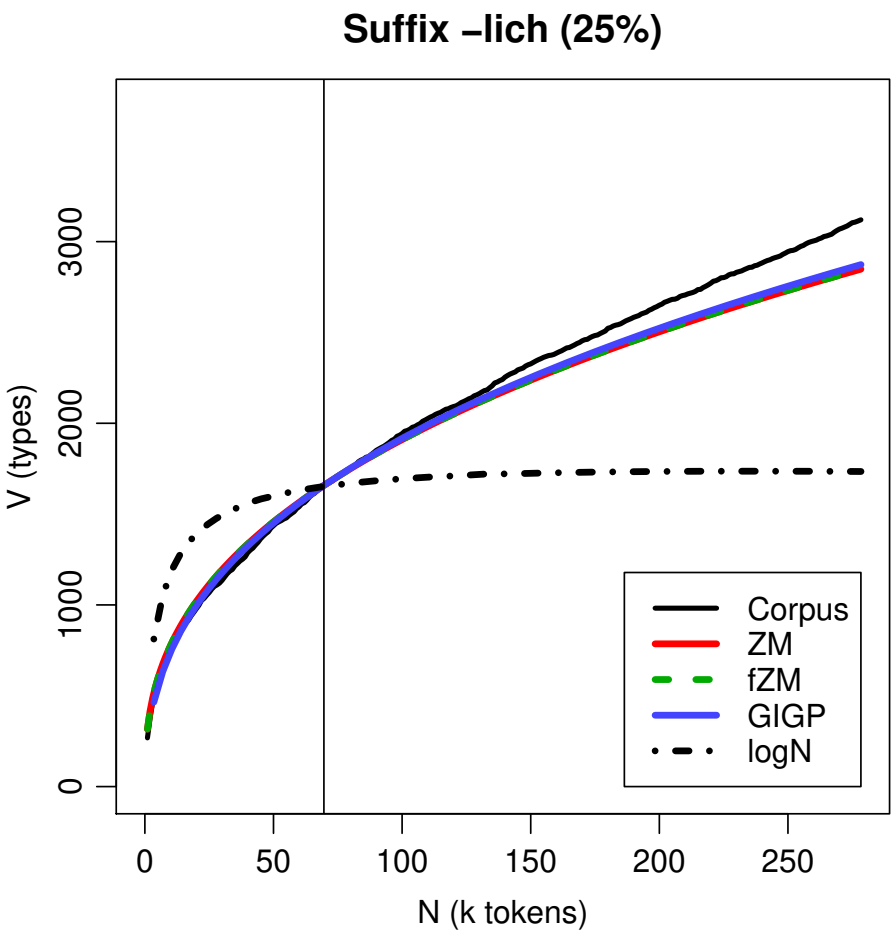
(Baroni and Evert 2005)

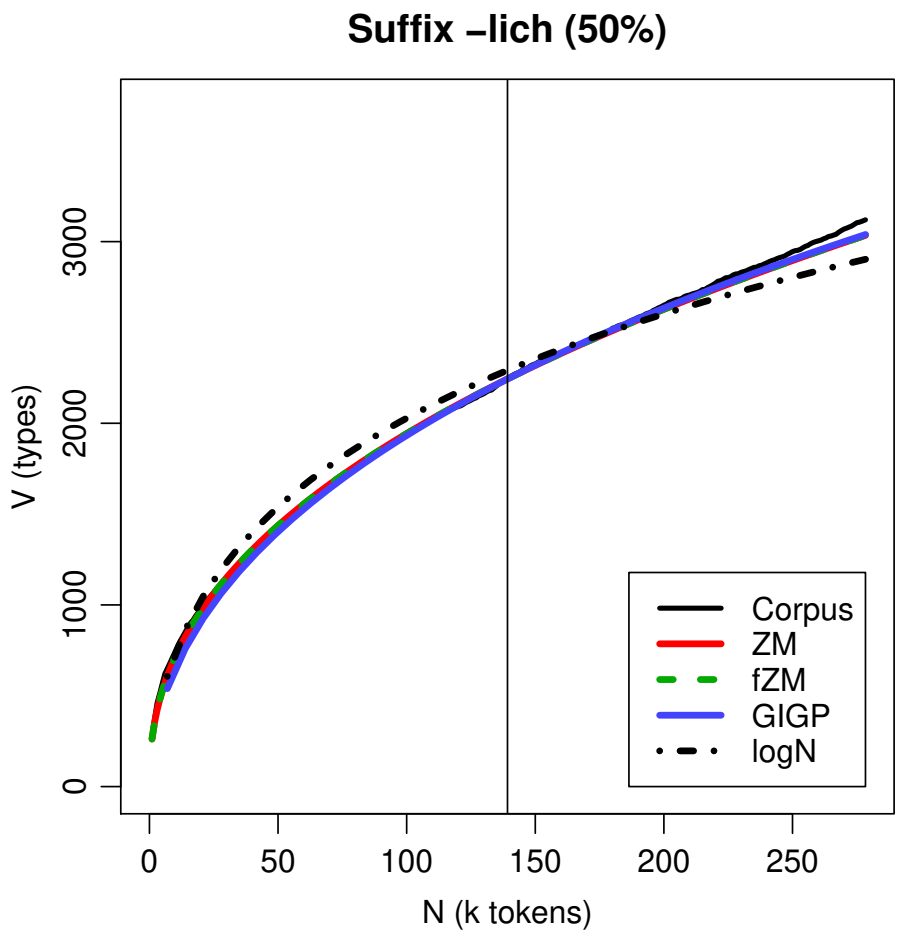




How accurate is LNRE-based extrapolation?

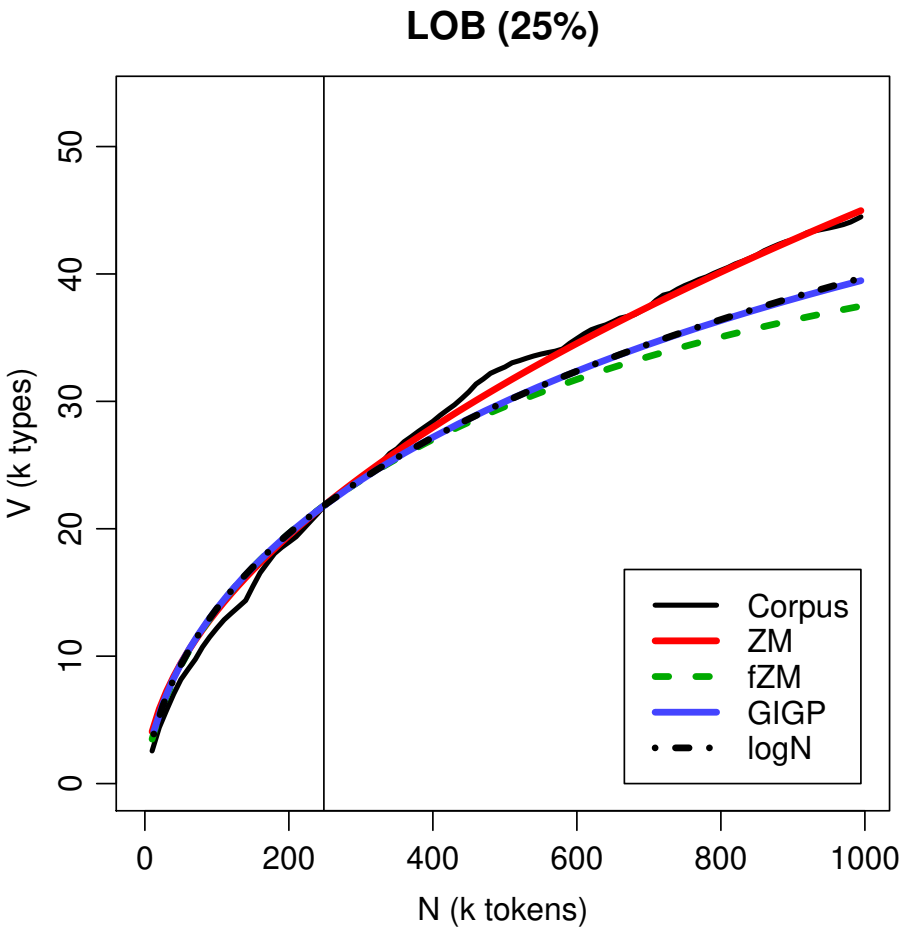
(Baroni and Evert 2005)

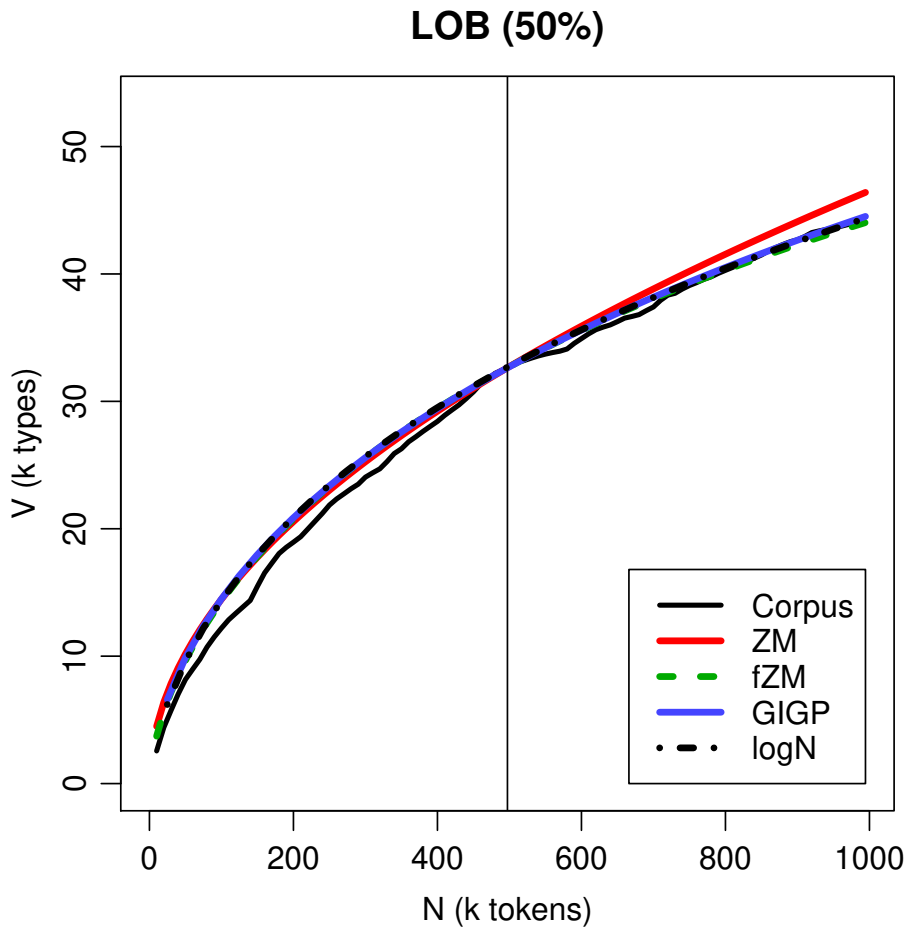




How accurate is LNRE-based extrapolation?

(Baroni and Evert 2005)





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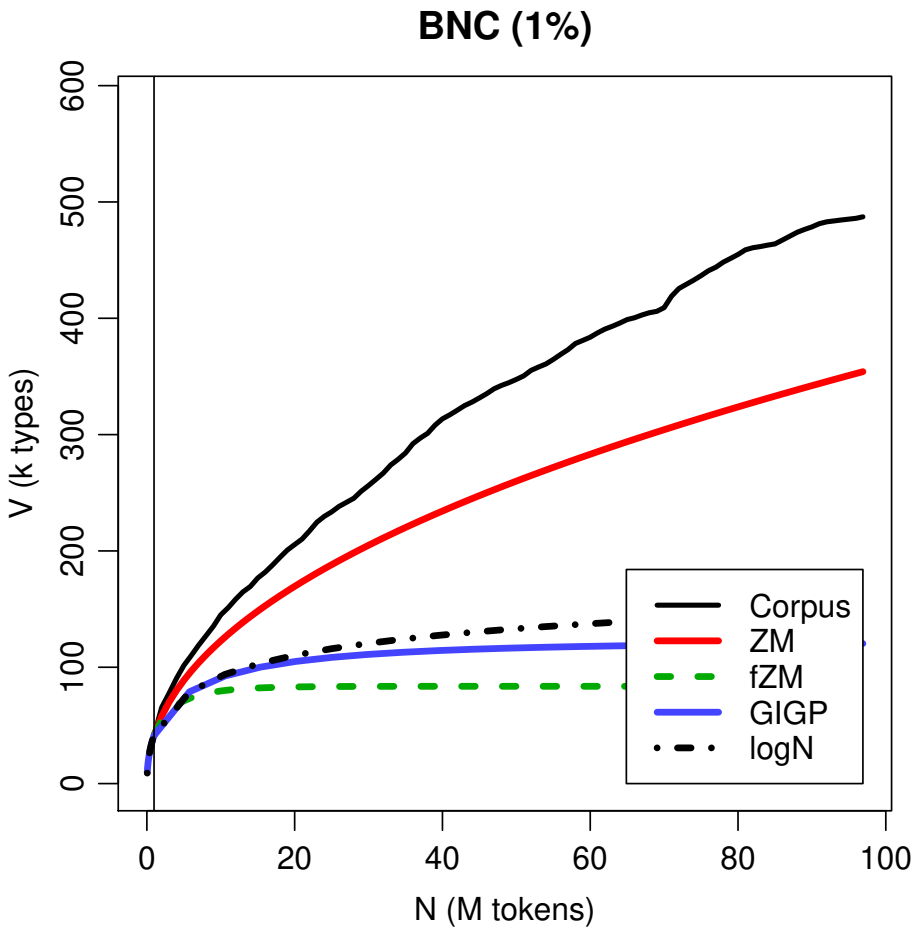

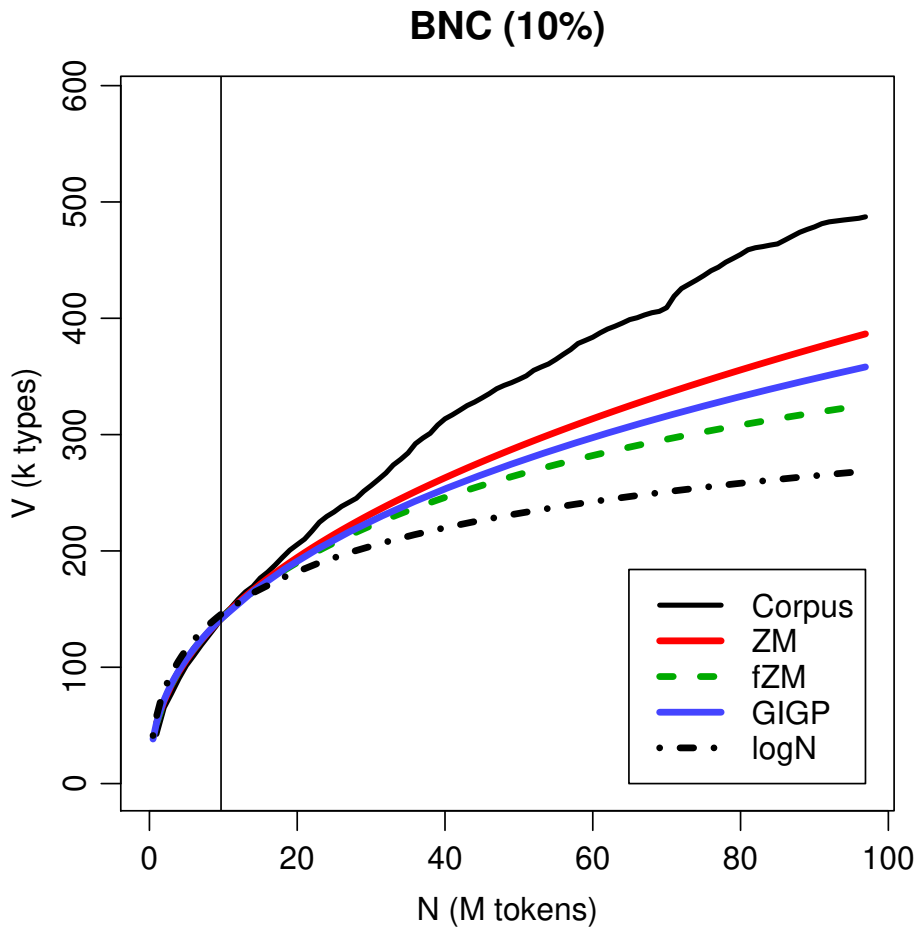

Reasons for poor extrapolation quality

- Major problem: **non-randomness** of corpus data
  - LNRE modelling assumes that corpus is random sample

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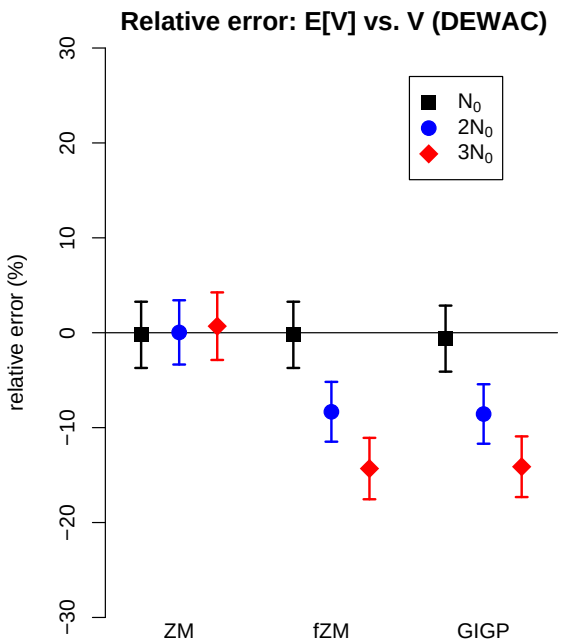
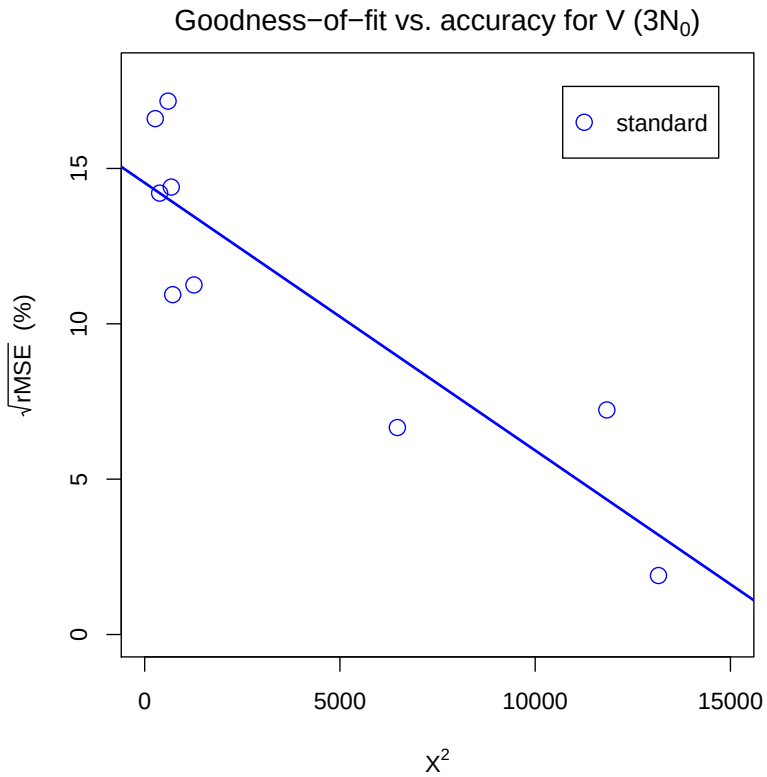
  - well-known in computational linguistics (Church 2000)
- Cause 2: **non-homogeneous** corpus
  - cannot extrapolate from spoken BNC to written BNC
  - similar for different genres and domains
  - also within single text, e.g. beginning/end of novel

The ECHO correction

(Baroni and Evert 2007)

- Empirical study: quality of extrapolation $N_0 \rightarrow 4N_0$ starting from random samples of corpus texts





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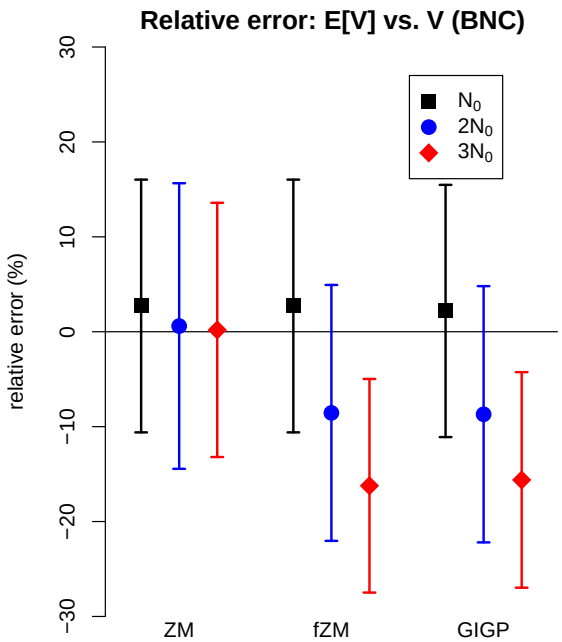

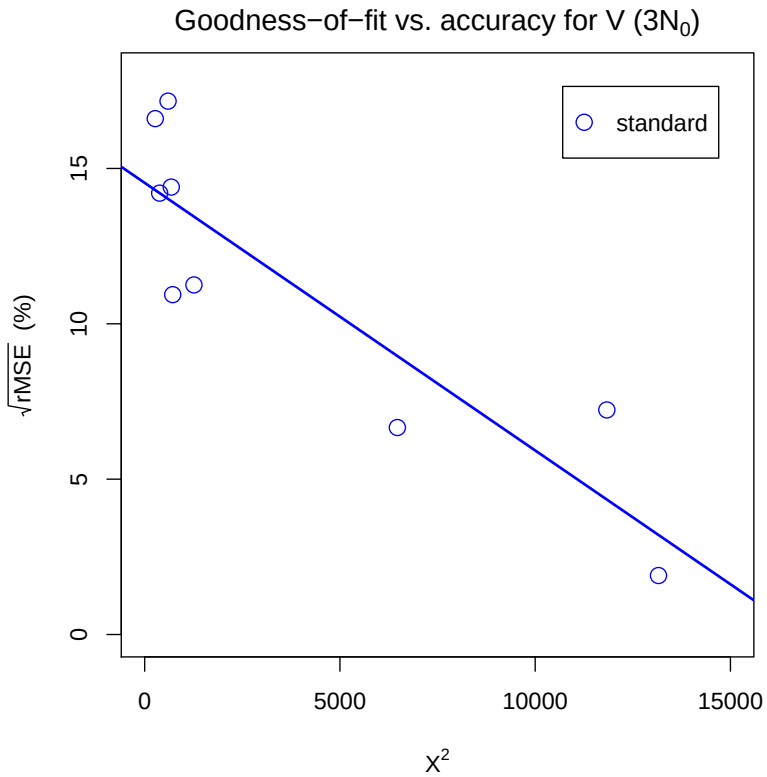

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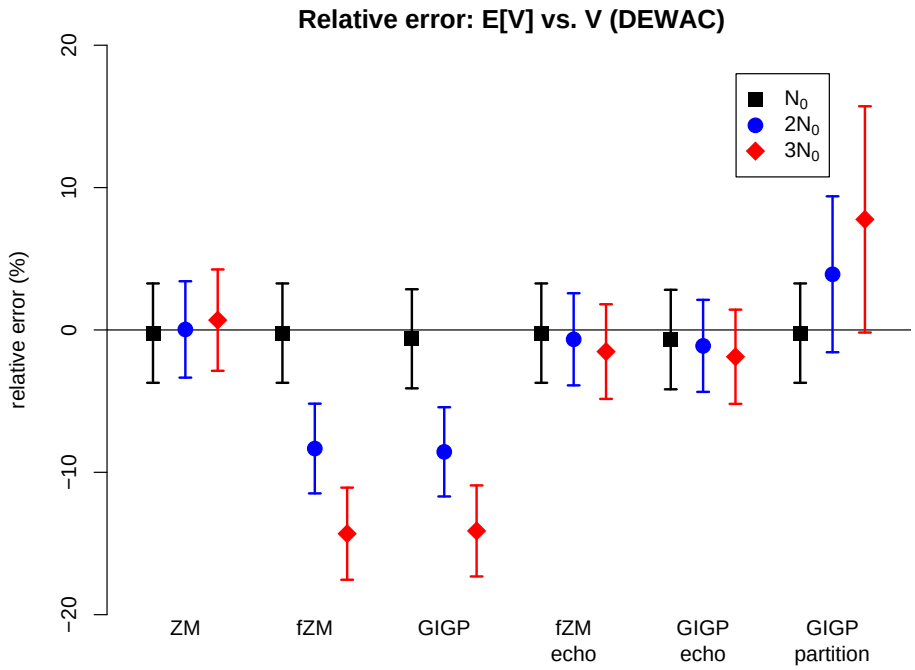

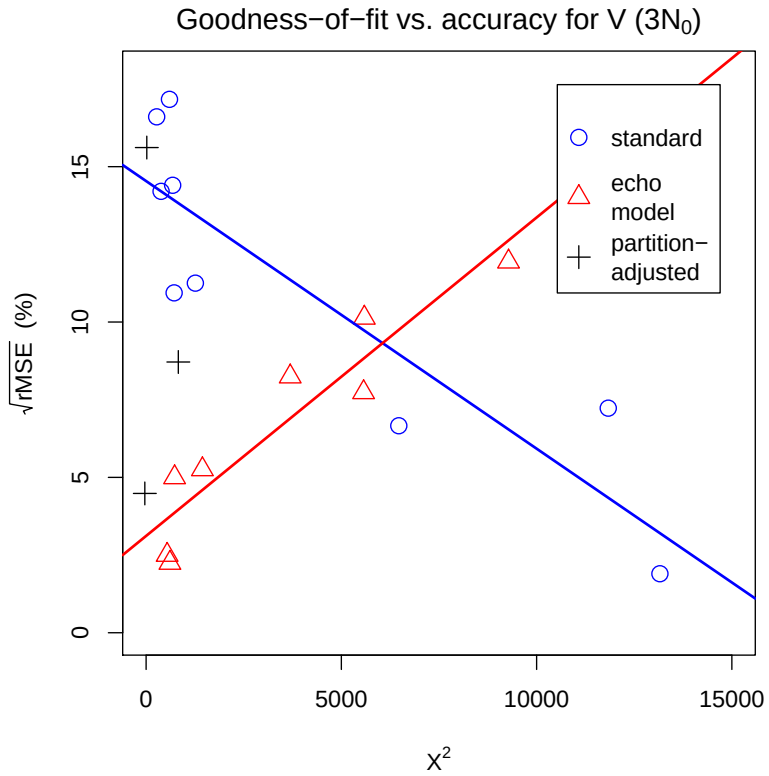

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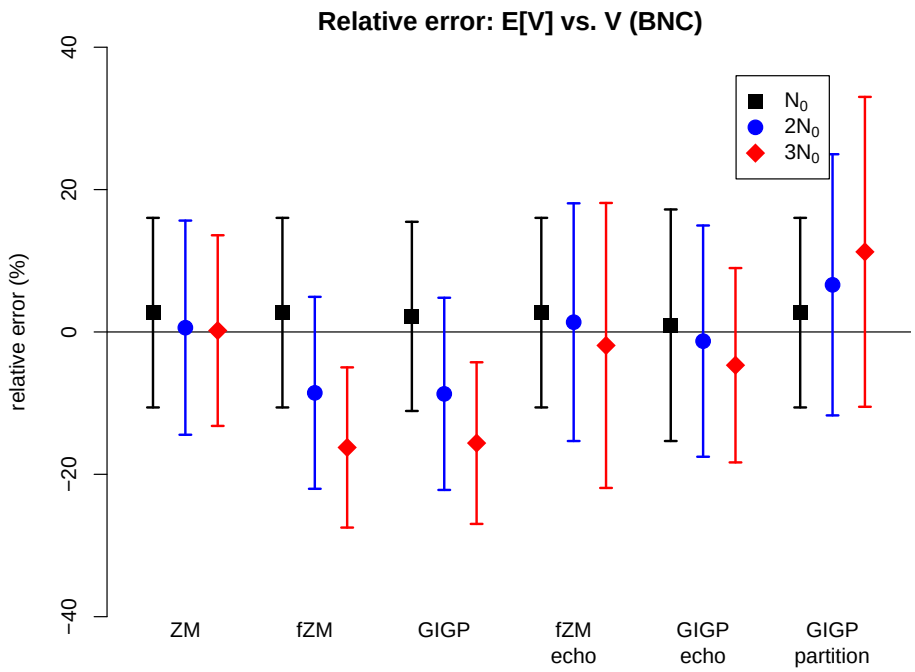

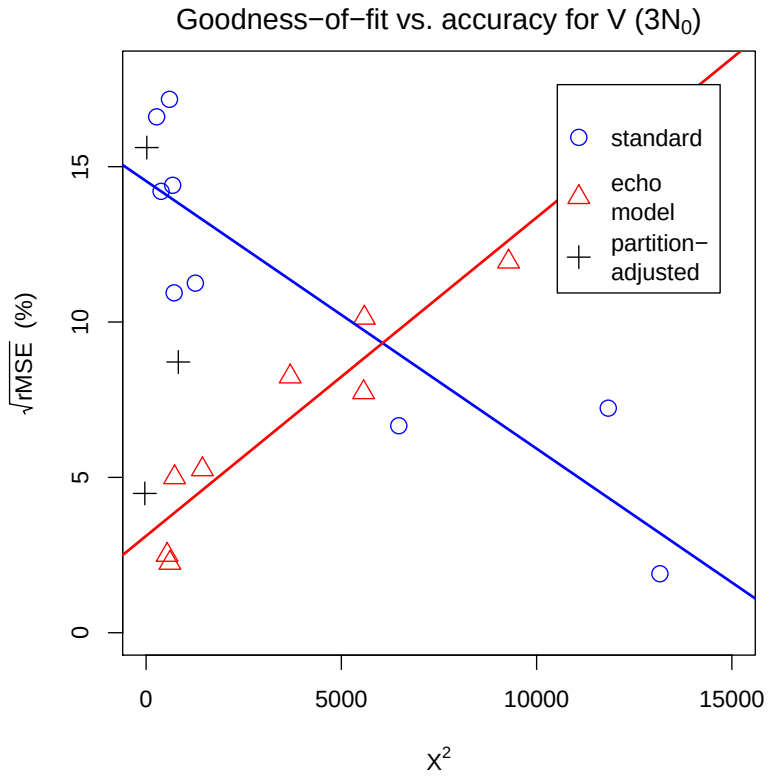

Outline

Bootstrapping

- An empirical approach to sampling variation:
  - take many random samples from the same population
  - train LNRE model on each sample
  - analyse distribution of model parameters, goodness-of-fit, etc. (mean, median, s.d., boxplot, histogram, ...)
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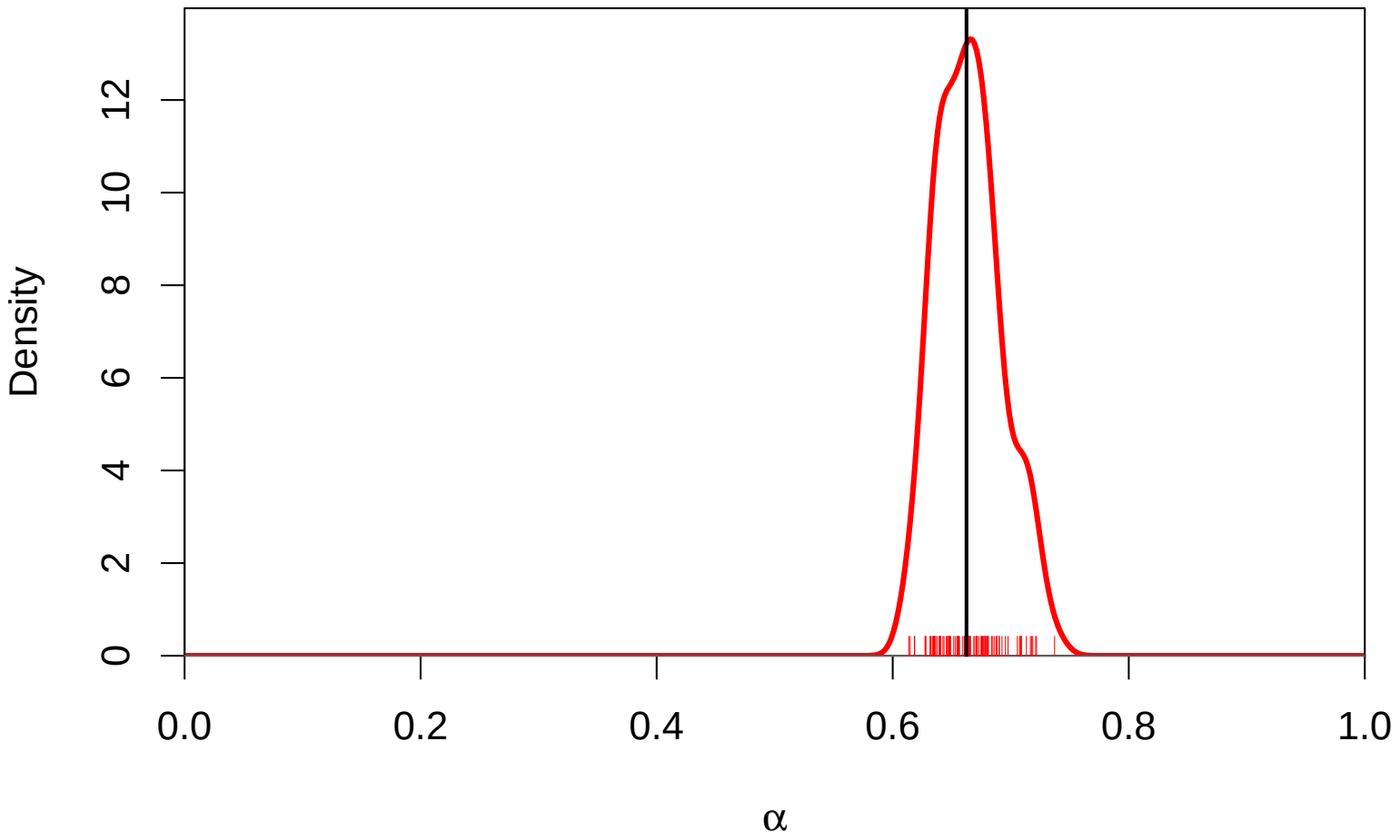
- Bootstrapping (Efron 1979)
  - resample from observed data *with replacement*
  - this approach is not suitable for type-token distributions (resamples underestimate vocabulary size V ➜ biased)
- Parametric bootstrapping
  - use fitted model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known

Bootstrapping

parametric bootstrapping with 100 replicates, fZM samples for $N = 3467$

Zipfian slope $a = 1/\alpha$



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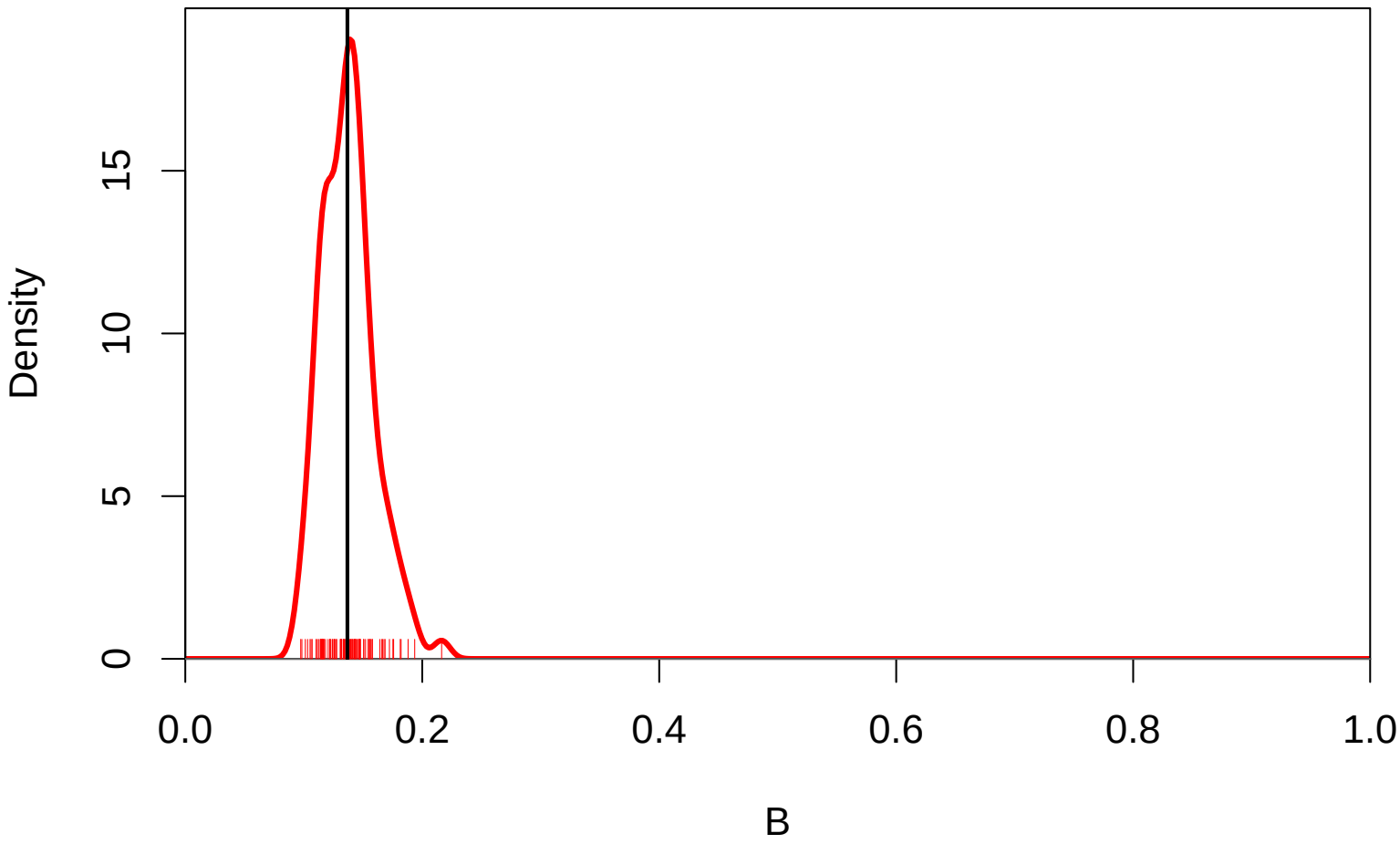
Offset $b = (1 - \alpha)/(B \cdot \alpha)$



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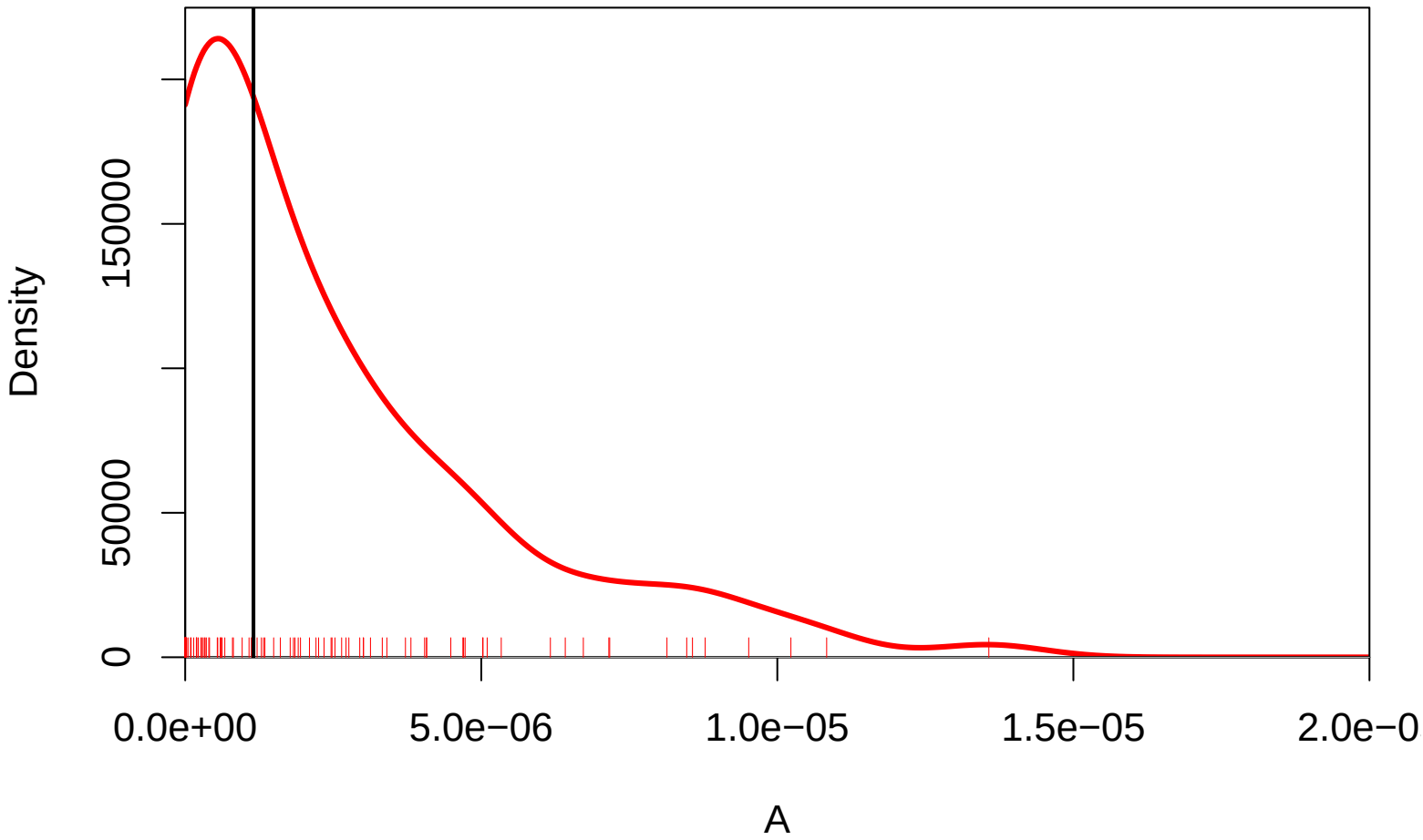
fZM probability cutoff $A = \pi_S$



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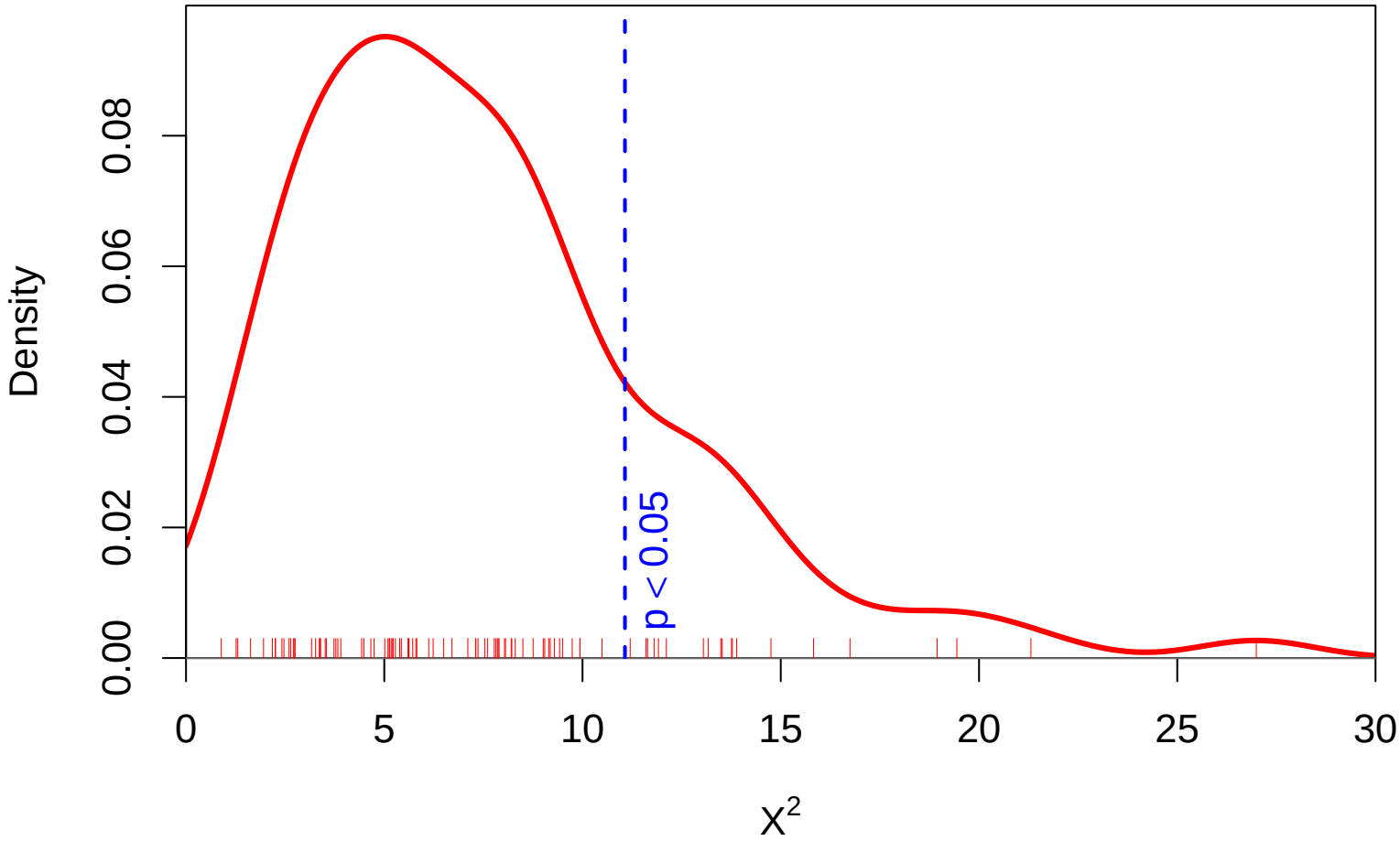
Goodness-of-fit statistic X^2 (model not plausible for $X^2 > 11$)



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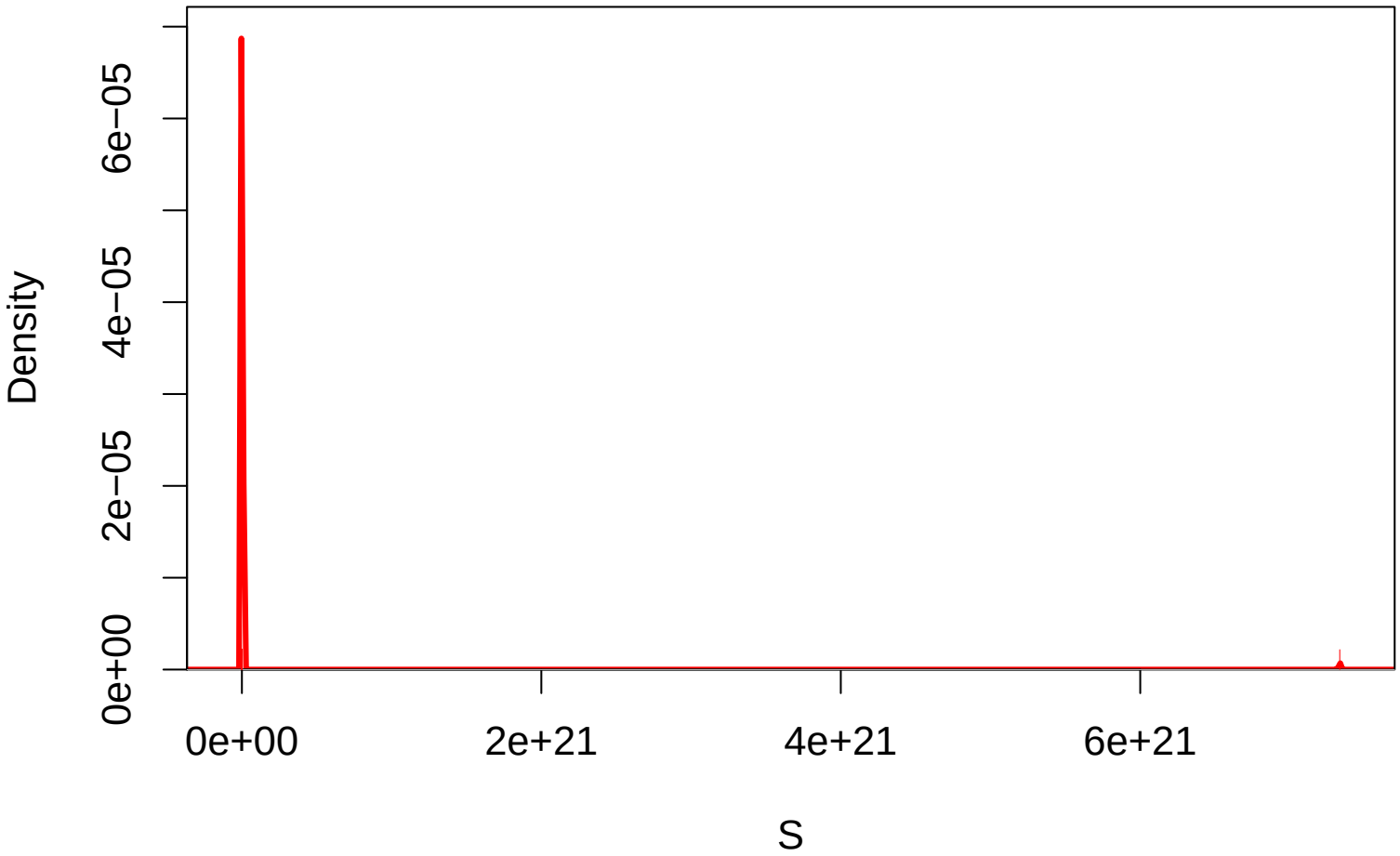

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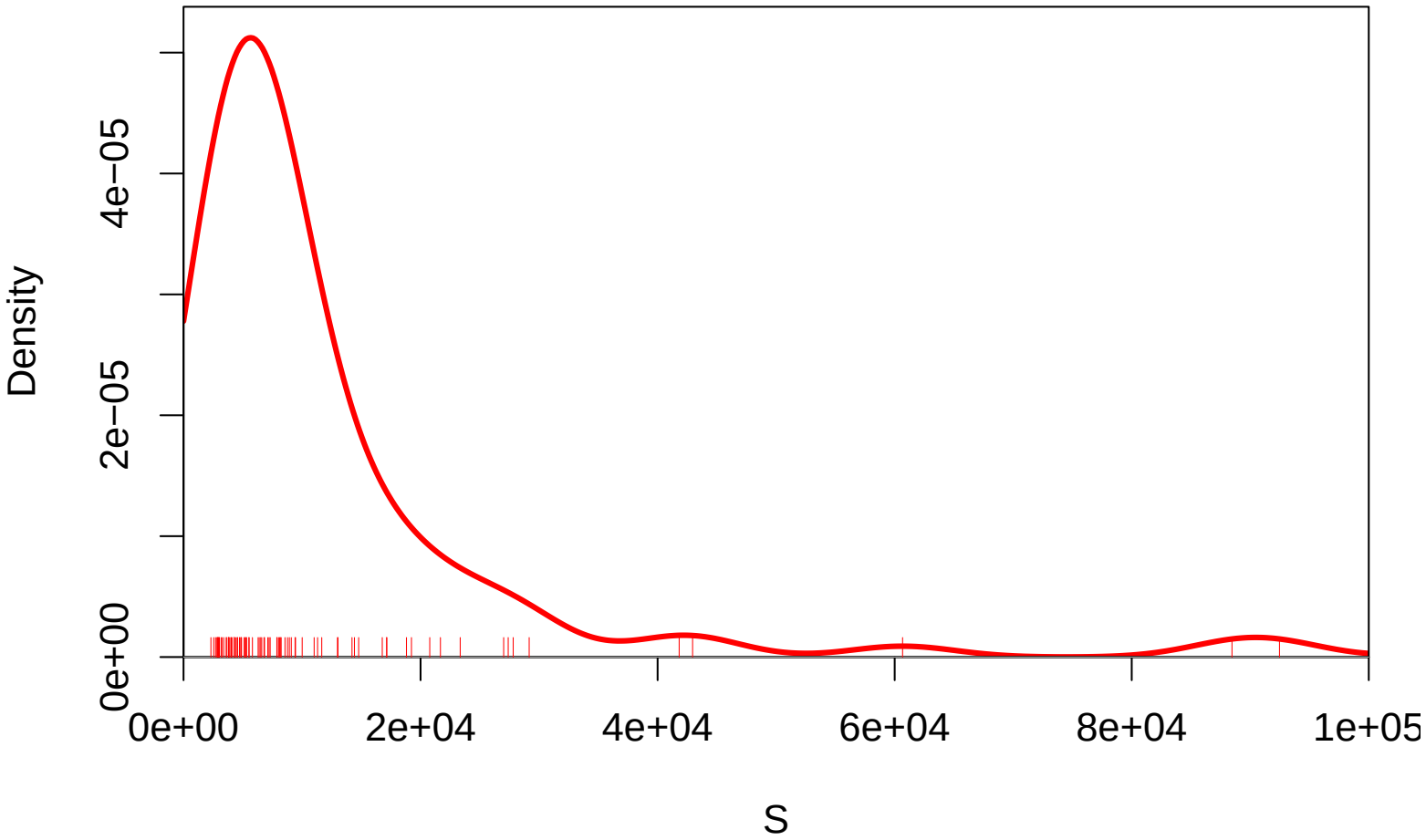
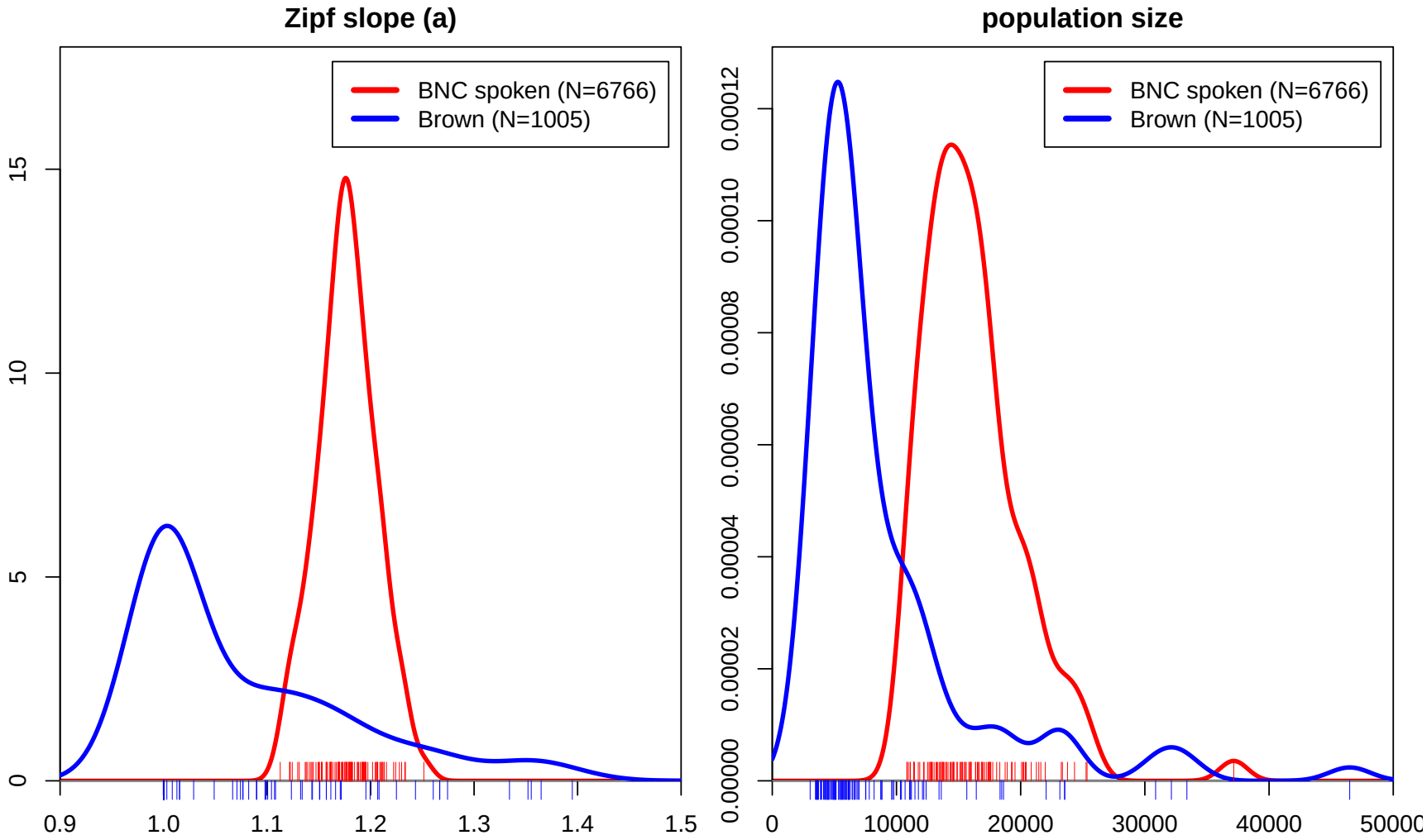


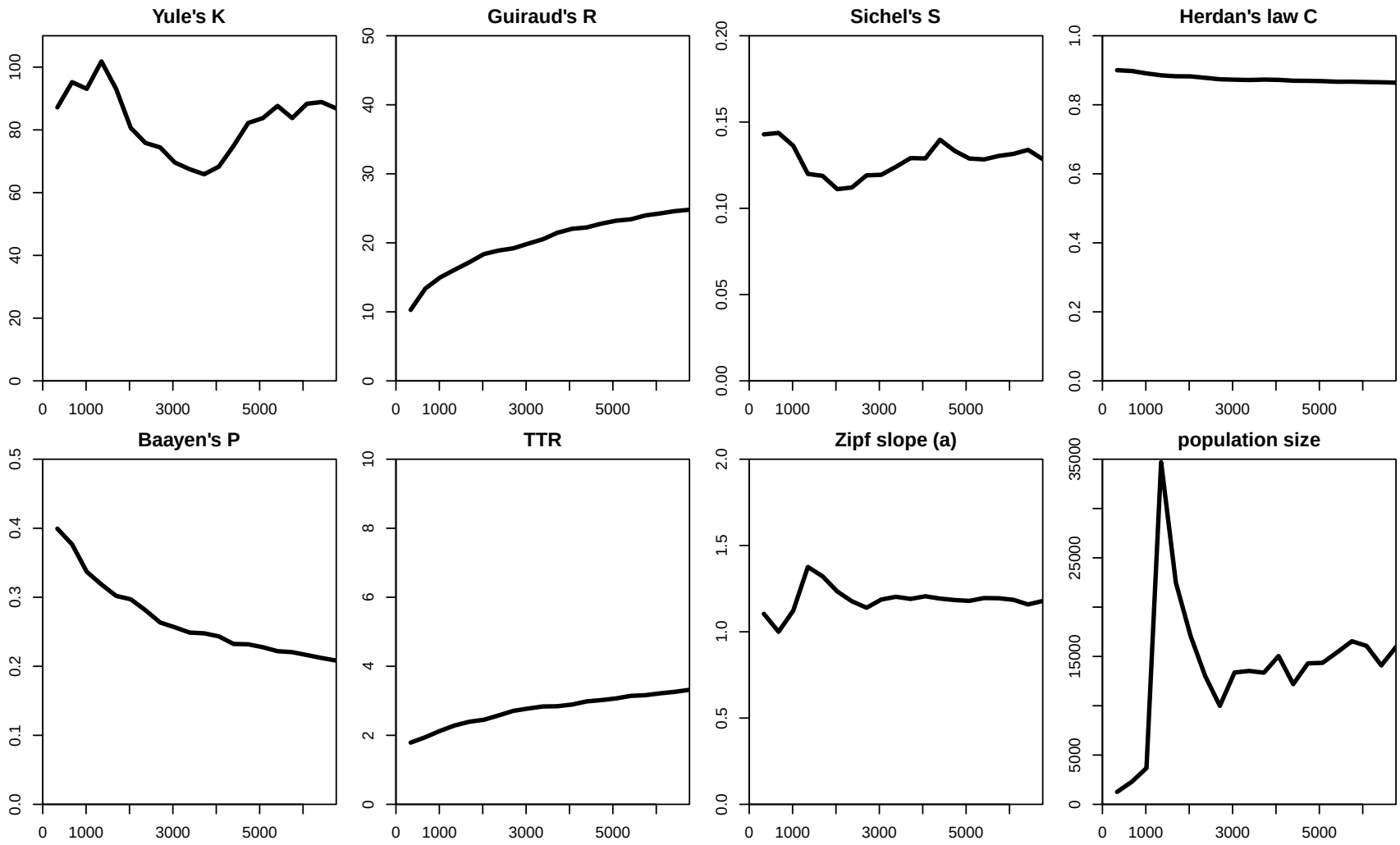
Sample size matters!

Brown corpus too small for reliable LNRE parameter estimation on bare singulars



Outline

Empirical observations



Parametric bootstrapping with LNRE models

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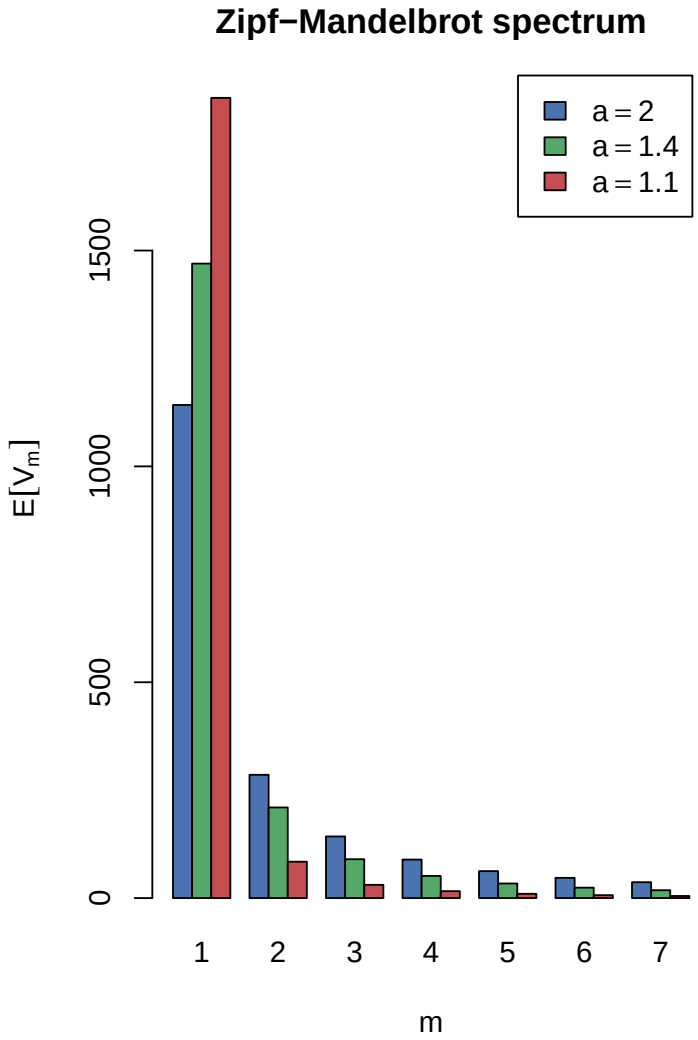
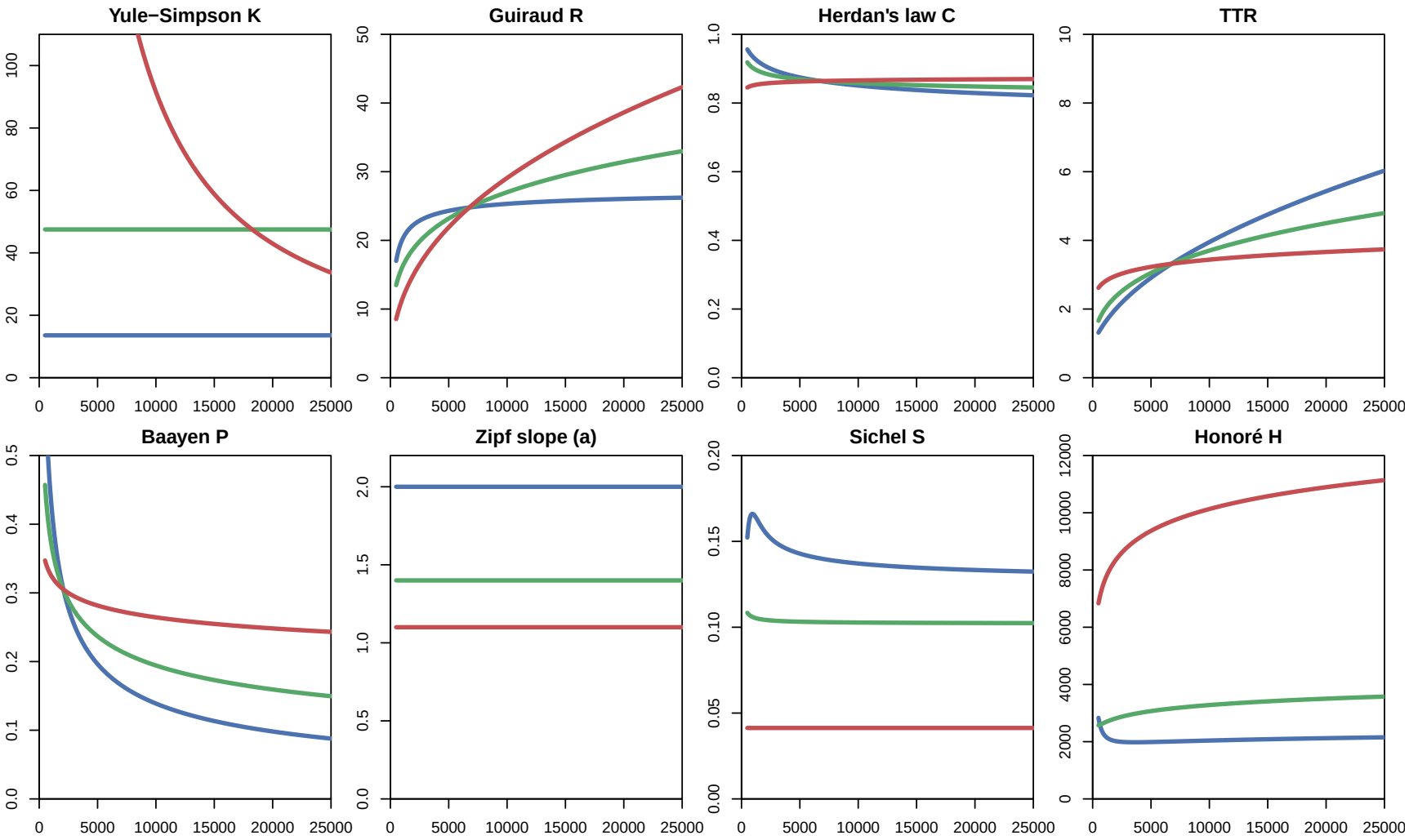
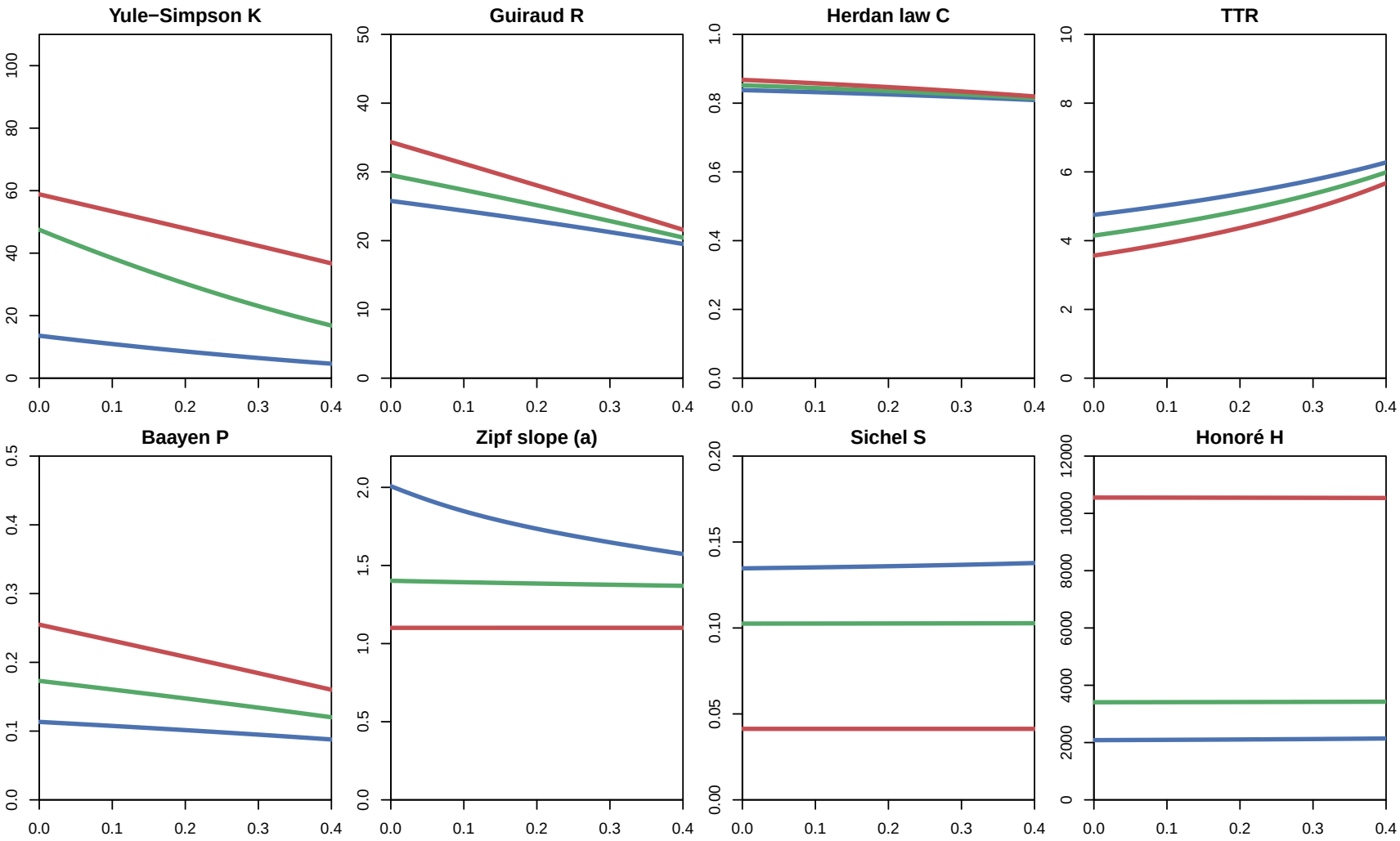
- Use simulation experiments to gain better understanding of quantitative measures
- Resampling (bootstrapping) leads to biased type counts
- ➥ Parametric bootstrapping based on LNRE population
  - arbitrary sample size
  - intuitive notion of productivity ➜ parameters
  - controlled manipulation of confounding factors



Parametric bootstrapping: sample size



Parametric bootstrapping: frequent lexicalized types



Outline

Case study: Iris Murdoch & early symptoms of AD

(Evert *et al.* 2017)

- Renowned British author (1919–1999)
- Published a total of 26 novels, mostly well received by critics
- Murdoch experienced unexpected difficulties composing her last novel, received "without enthusiasm" (Garrard *et al.* 2005)
- Diagnosis of Alzheimer's disease shortly after publication

Murdoch novel reveals Alzheimer's

The last novel by the author Iris Murdoch reveals the first signs of Alzheimer's disease, experts say.

Experts analysed three of Dame Iris's books

A team from University College London say their examination of works from throughout Dame Iris's career could be used to help diagnose others.

They found the structure and grammar of her novels was relatively unchanged, but her language was noticeably simpler in her last novel, 'Jackson's Dilemma'.

The study is published online by the journal Brain.

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Conflicting results:

- Decline of lexical diversity in last novel (Garrard *et al.* 2005; Pakhomov *et al.* 2011)
- No clear effects found (Le *et al.* 2011)

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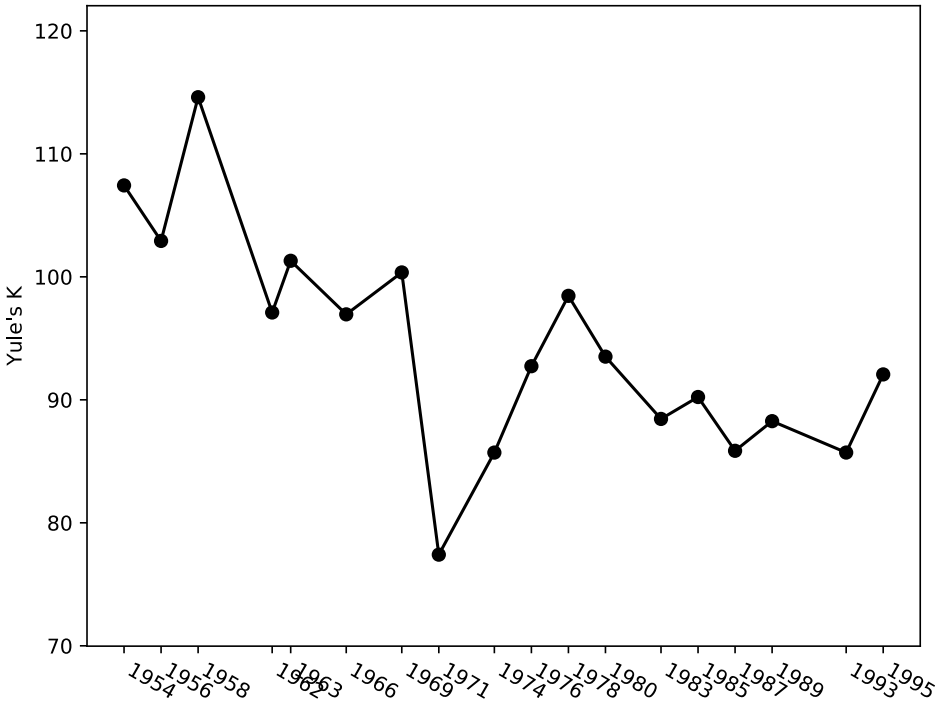
Case study: Iris Murdoch & early symptoms of AD

(Evert *et al.* 2017)

- Corpus data
  - 19 out of 26 novels written by Iris Murdoch
  - including 9 last novels, spanning a period of almost 20 years
  - acquired as e-books (no errors due to OCR)
- Pre-processing and annotation
  - Stanford CoreNLP (Manning *et al.* 2014) for tokenization, sentence splitting, POS tagging, and syntactic parsing
  - exclude dialogue based on typographic quotation marks (following Garrard *et al.* 2005; Pakhomov *et al.* 2011)
- The challenge
  - ☞ assess significance of differences in productivity for single texts
  - ☞ might explain conflicting results in prior work

Measures of vocabulary diversity = productivity

(Evert *et al.* 2017)



Yule's κ

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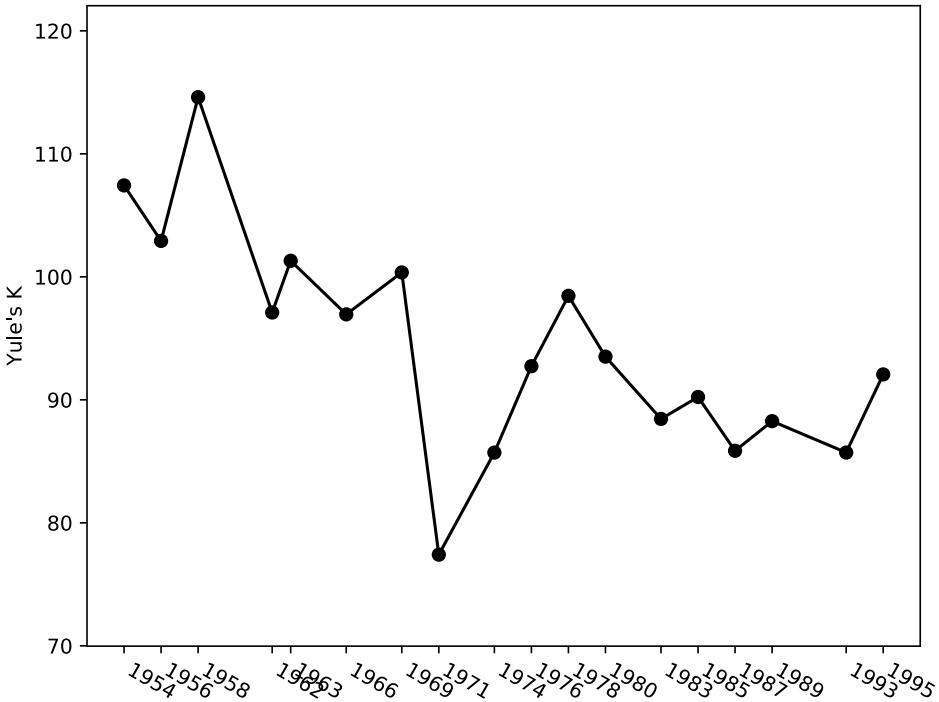

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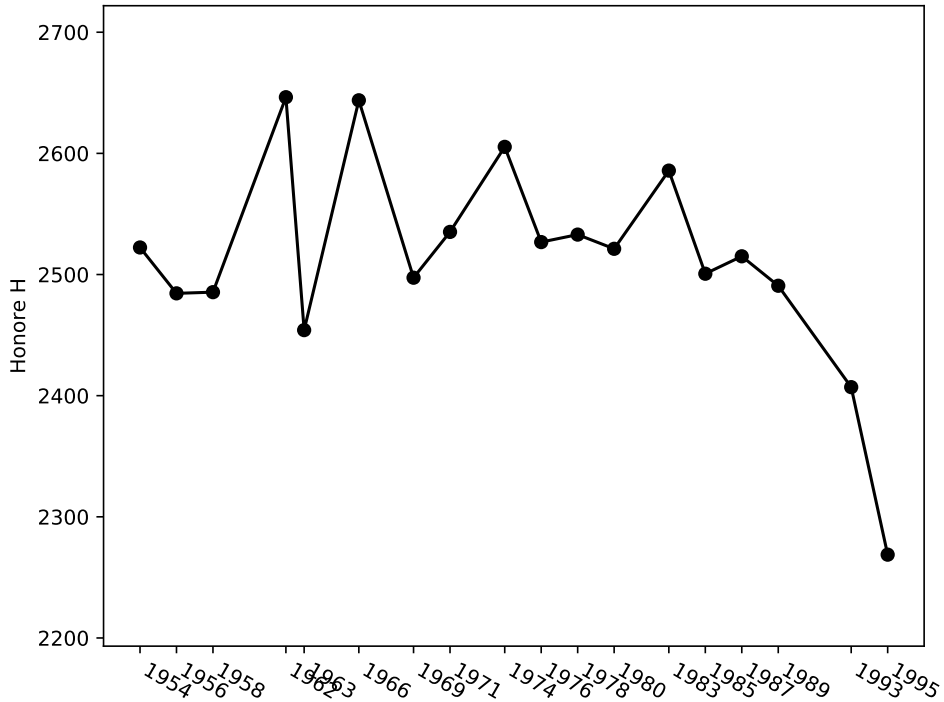

Honoré H

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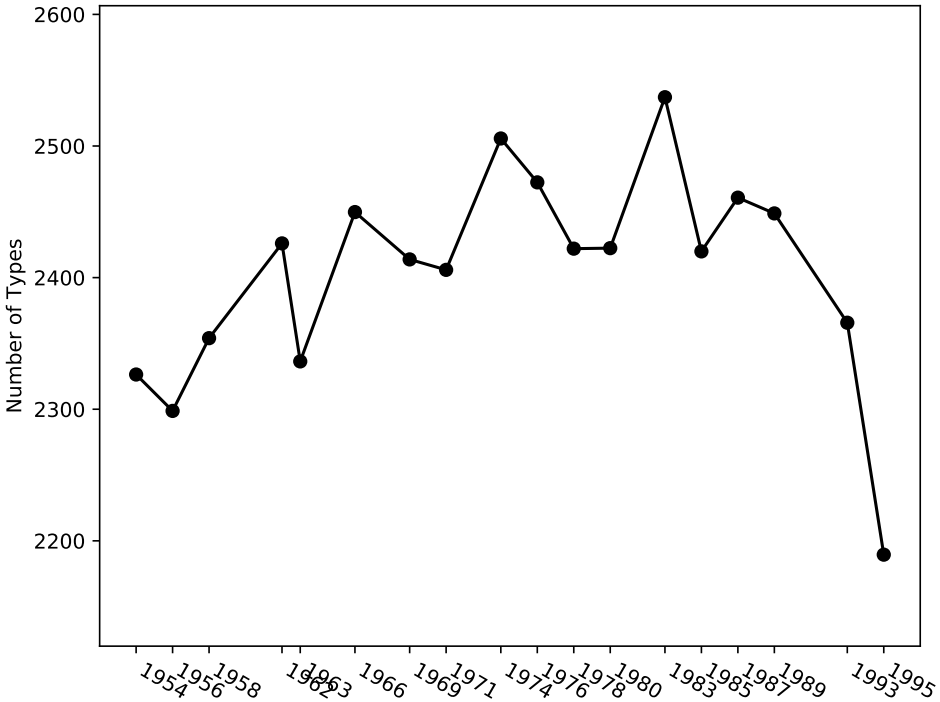
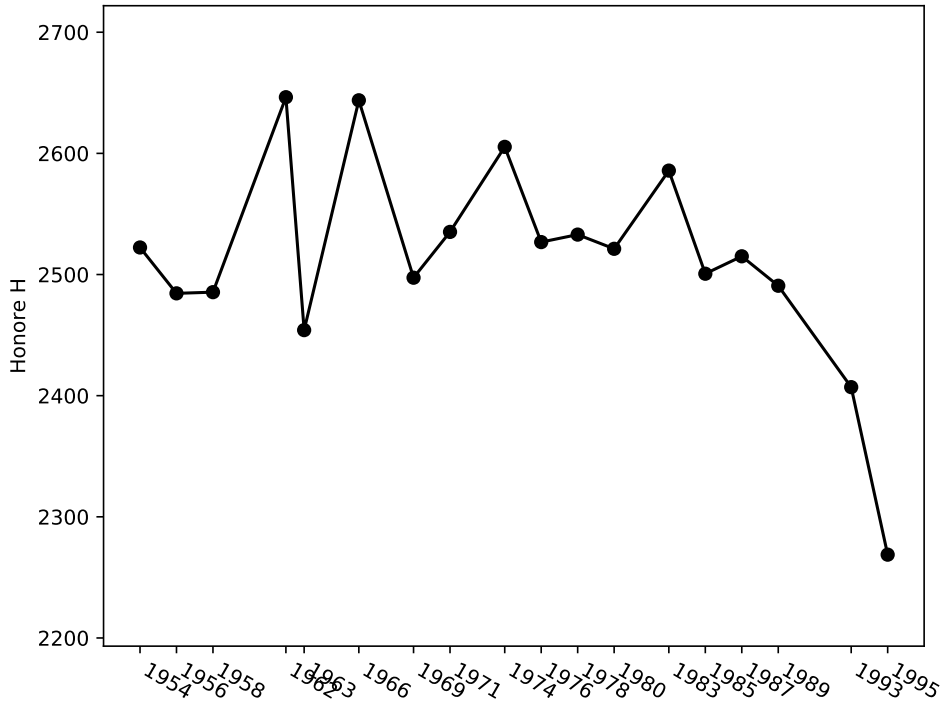


type count / TTR



Honoré *H*

Cross-validation for productivity measures

(Evert *et al.* 2017)

As a first step:

- Partition each novel into folds of 10,000 consecutive tokens
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$$y_1, \ldots, y_k$$

- Compute macro-average as overall measure for the entire text

$$\bar{y} = \frac{y_1 + \cdots + y_k}{k}$$

- Instead of value x obtained by evaluating measure on full text

Cross-validation for productivity measures

(Evert *et al.* 2017)

Significance testing procedure:

- Standard deviation σ of individual folds estimated from data

$$\sigma^2 \approx s^2 = \frac{1}{k-1} \sum_{i=1}^{k} (y_i - \bar{y})^2$$

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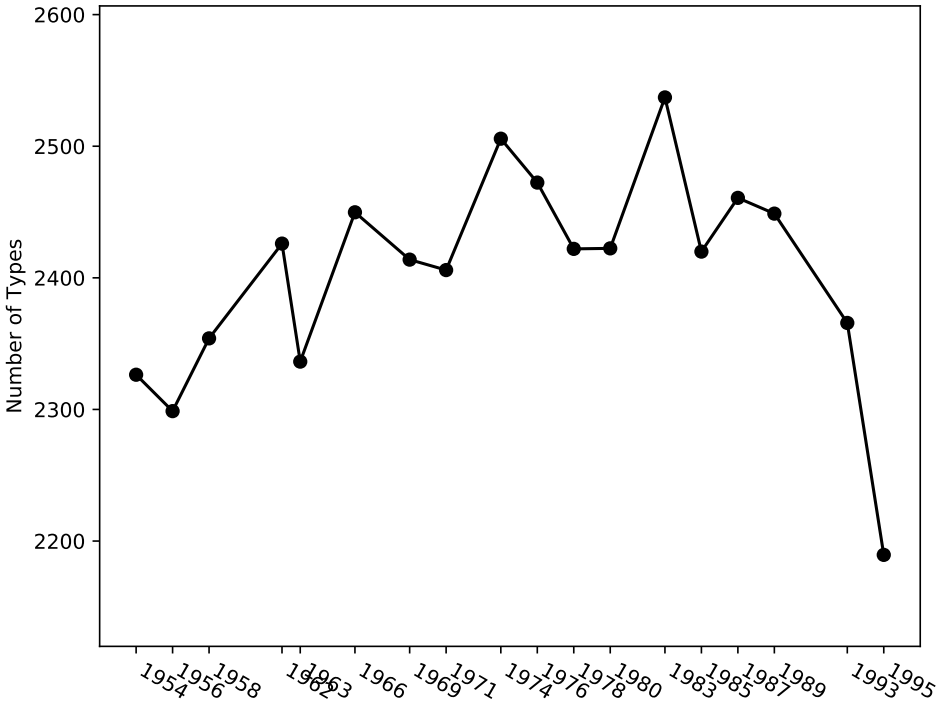
- Comparison of samples with Student's t-test, based on pooled cross-validation folds (feasible even for $n_1 = 1$)

Productivity measures with confidence intervals

(Evert *et al.* 2017)



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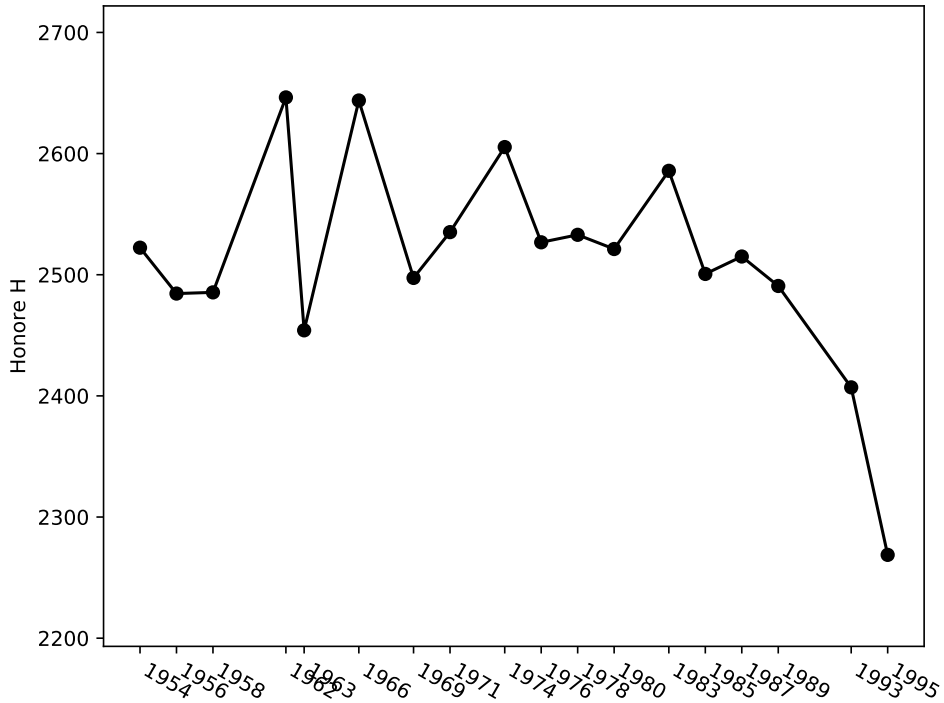

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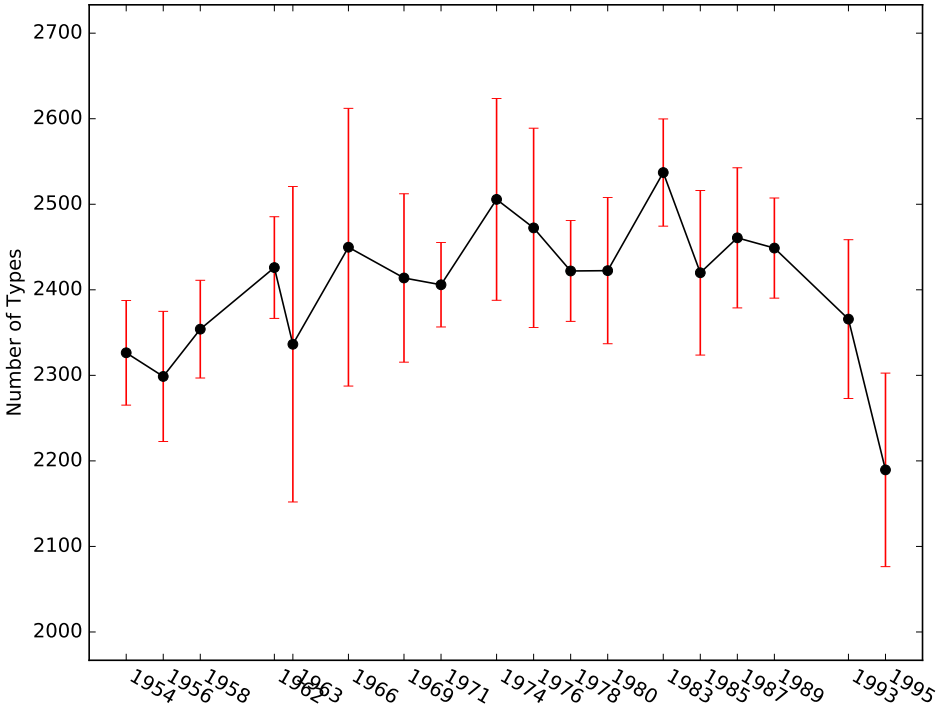

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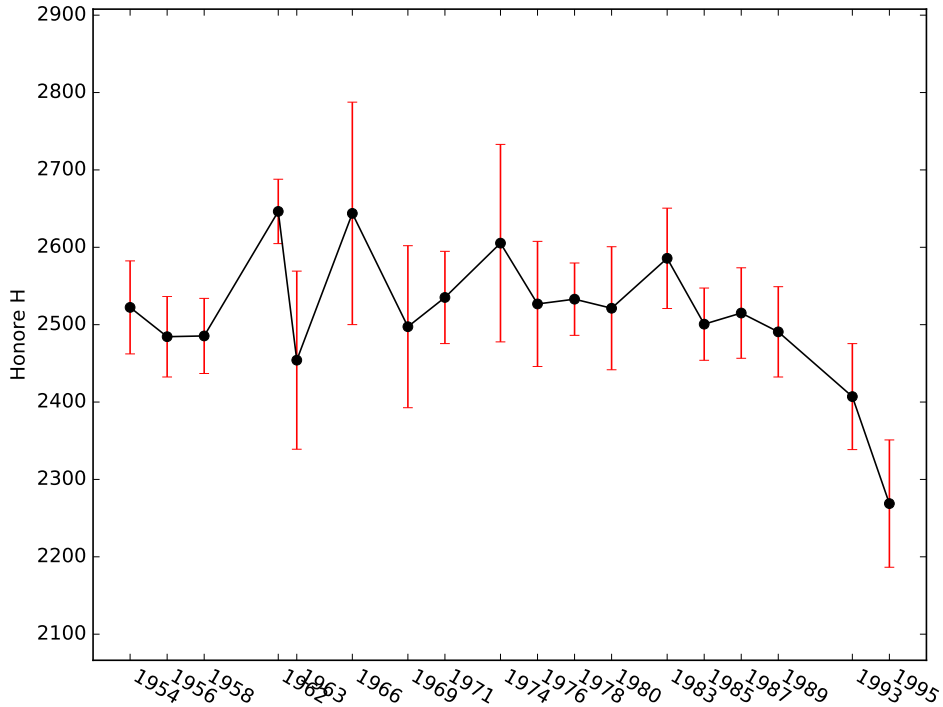

Honoré *H*

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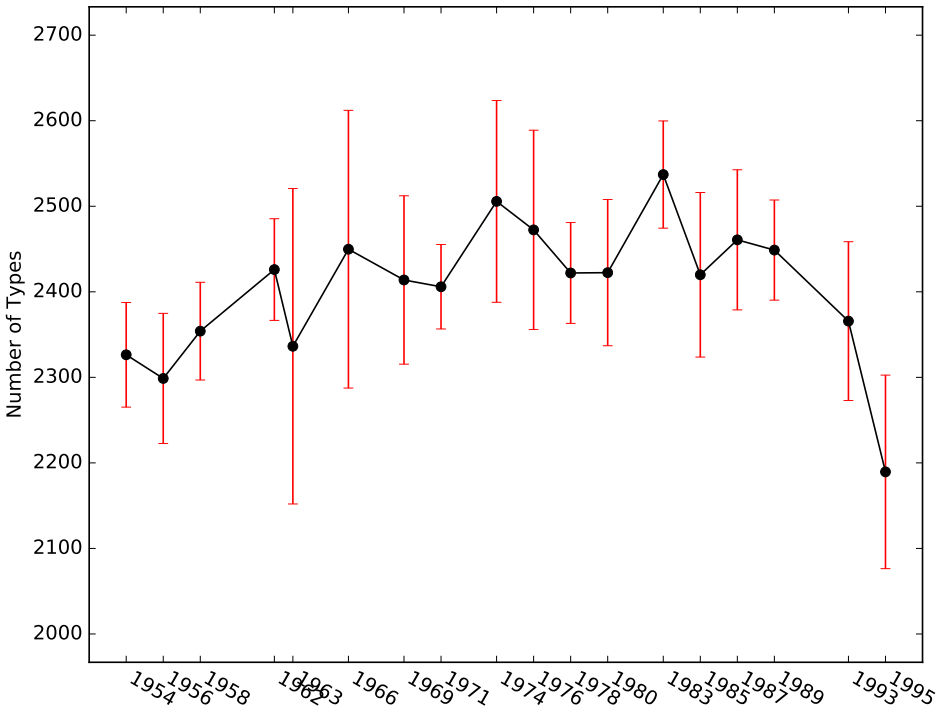

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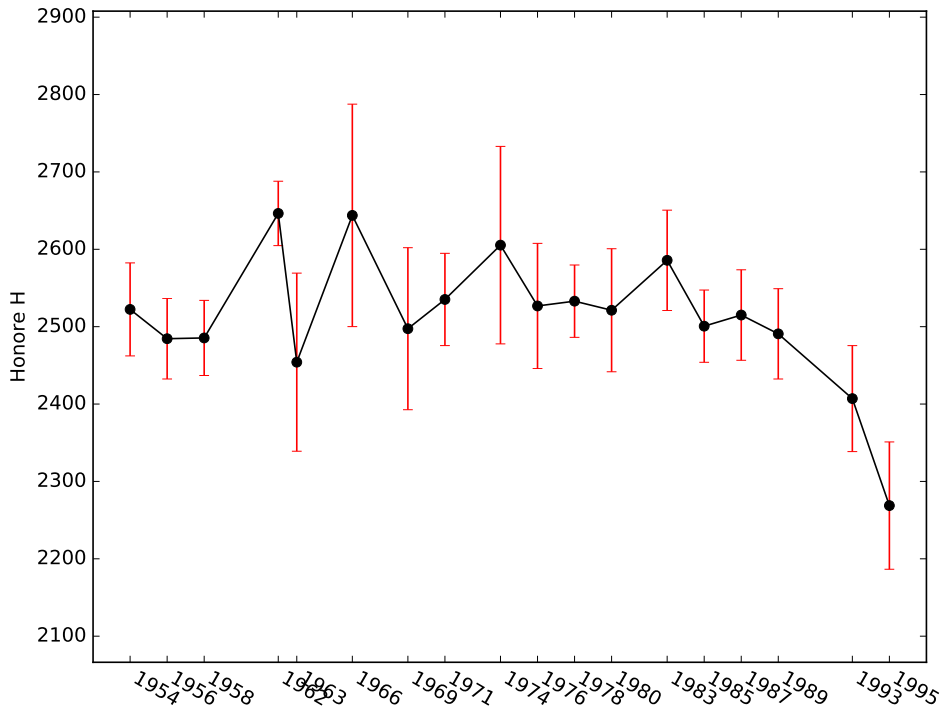

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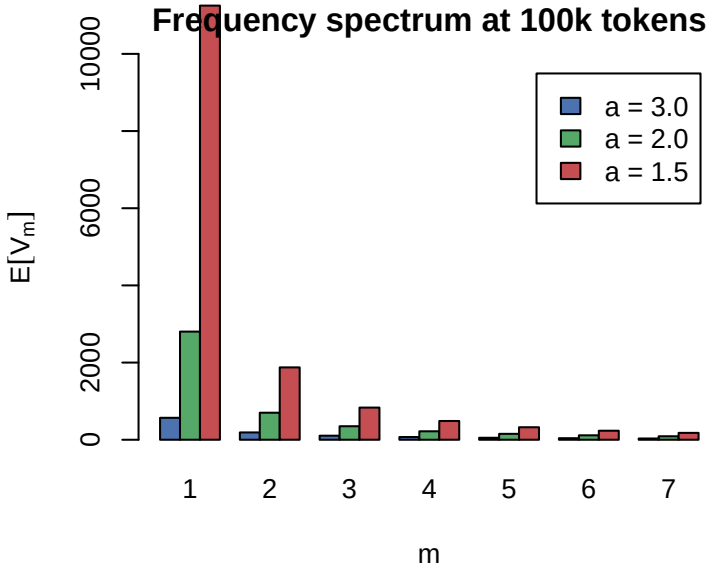
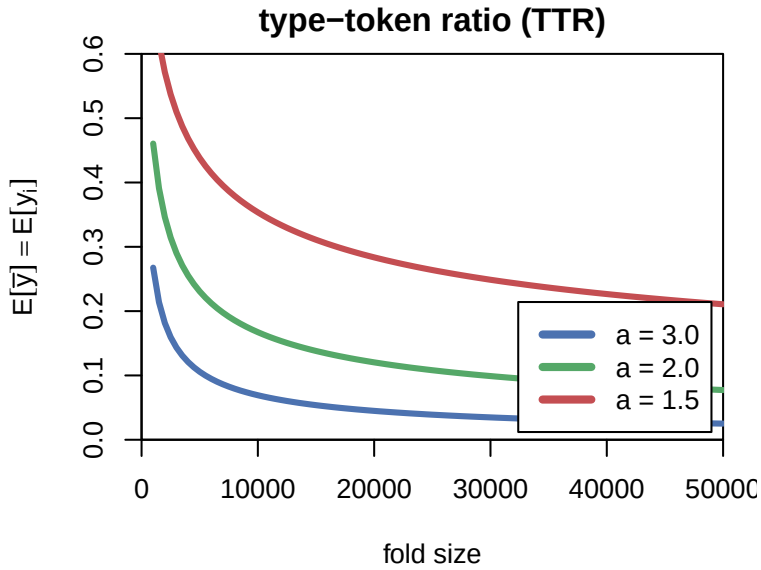
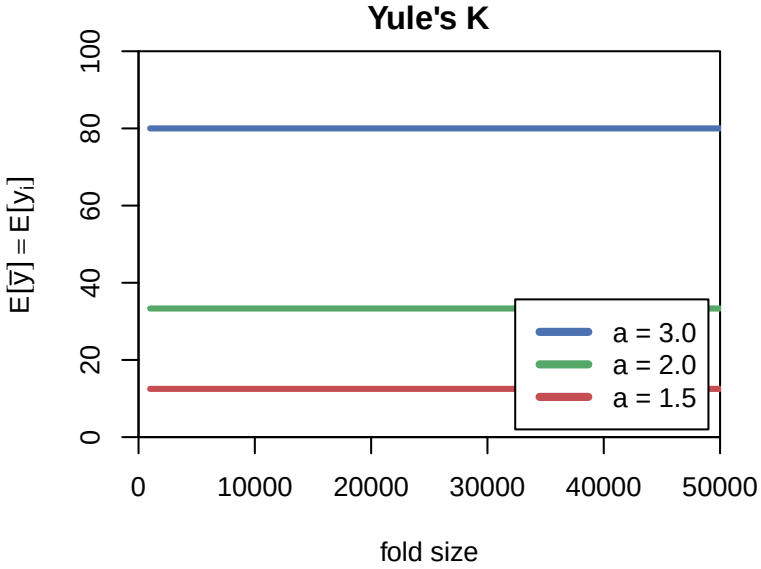
significance test vs. first 17 novels

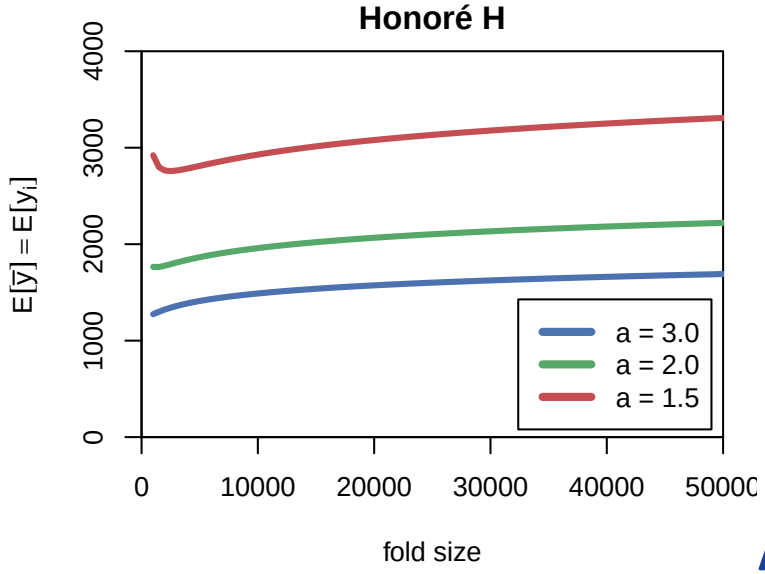
$t = -6.1$, df=5.52, $p = .0012^{**}$

Cross-validated measures depend on fold size!

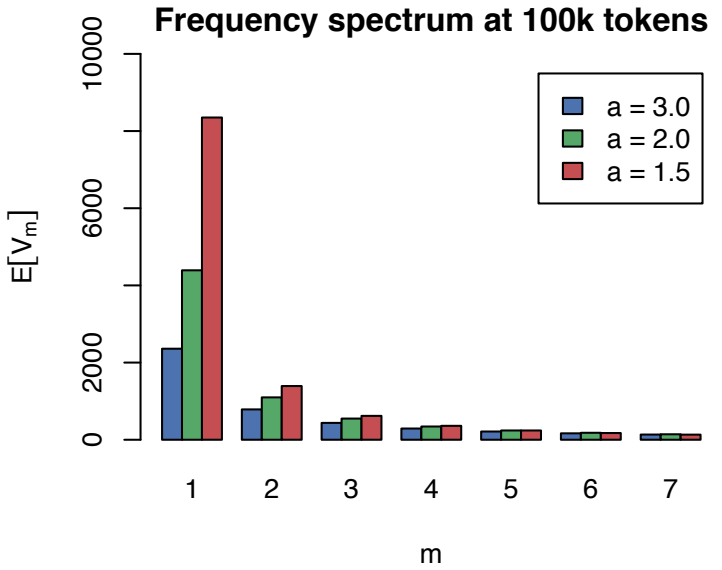


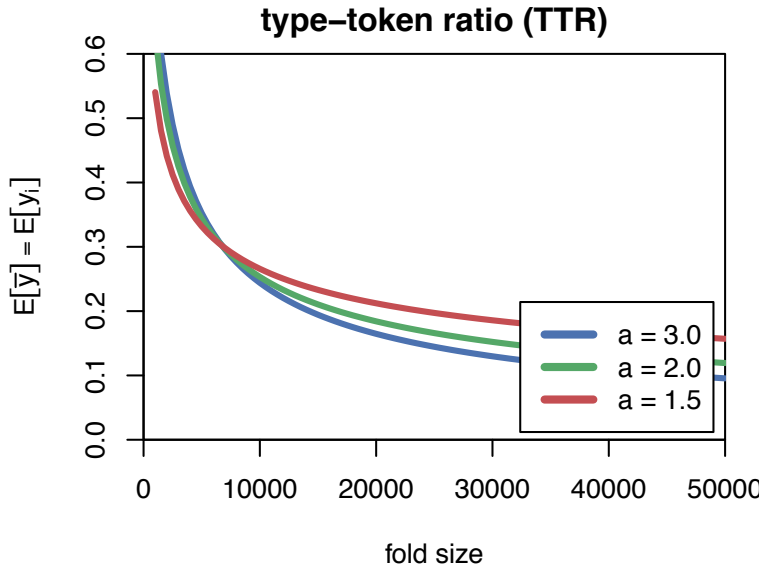


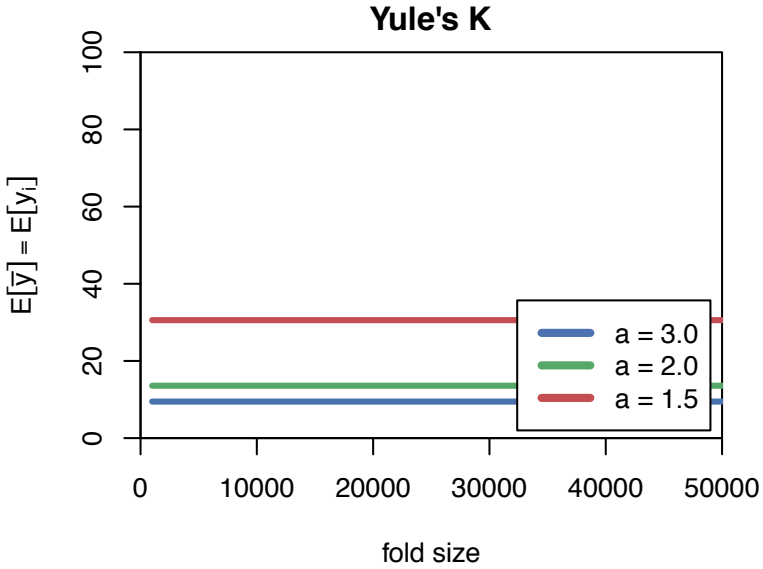


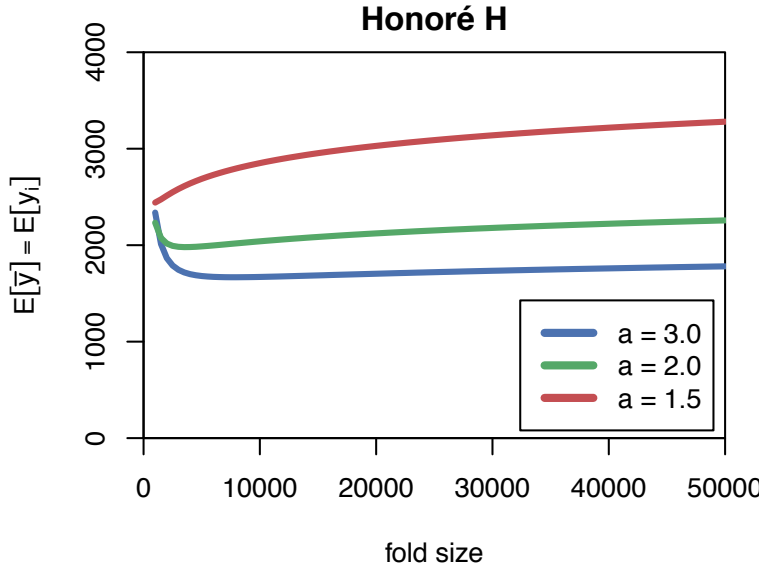


Cross-validated measures depend on fold size!

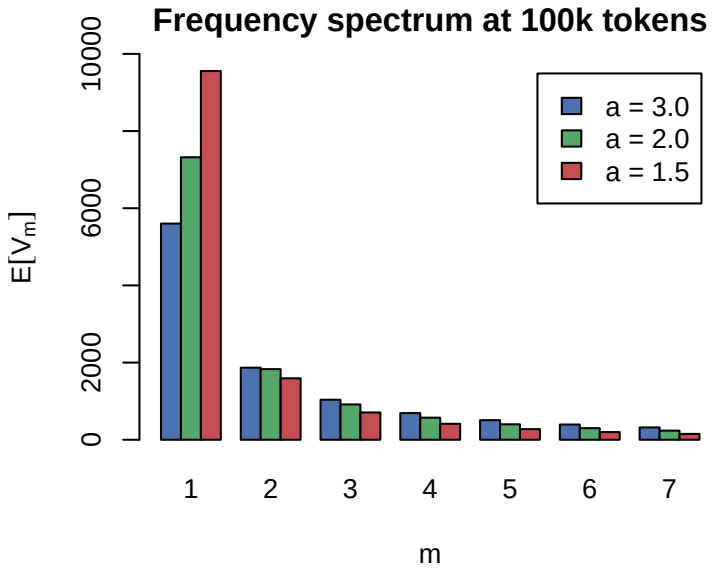


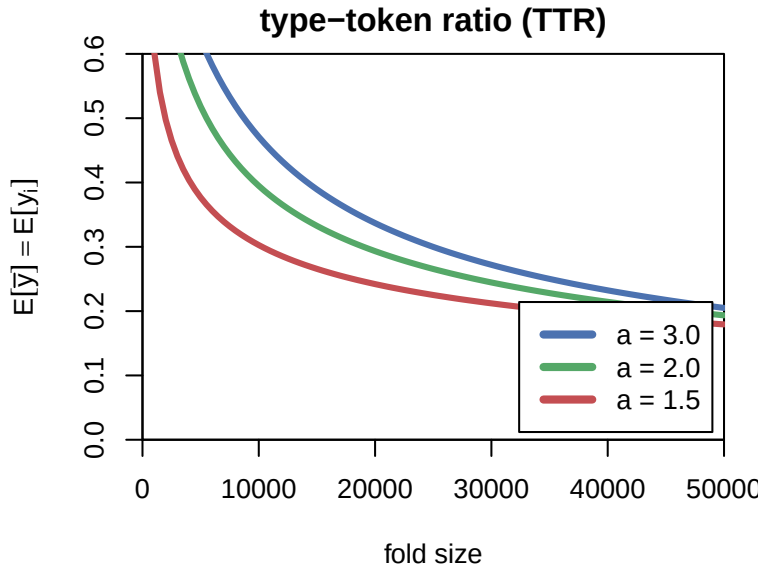


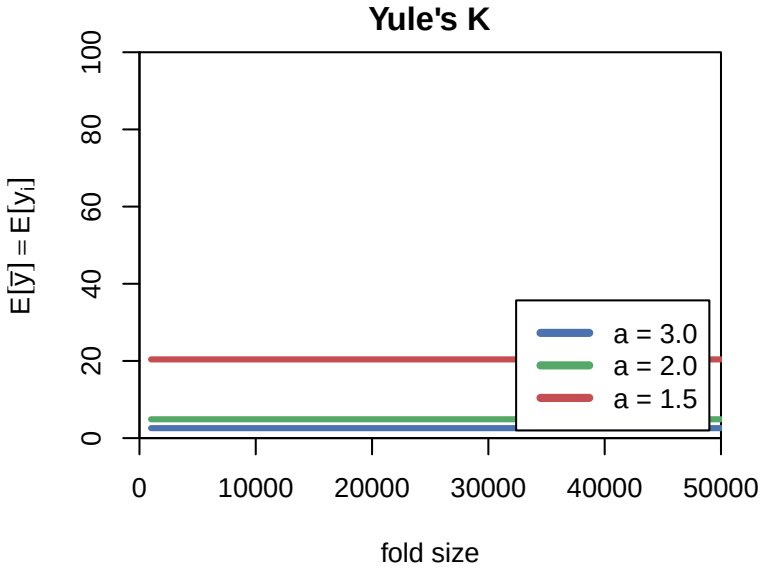


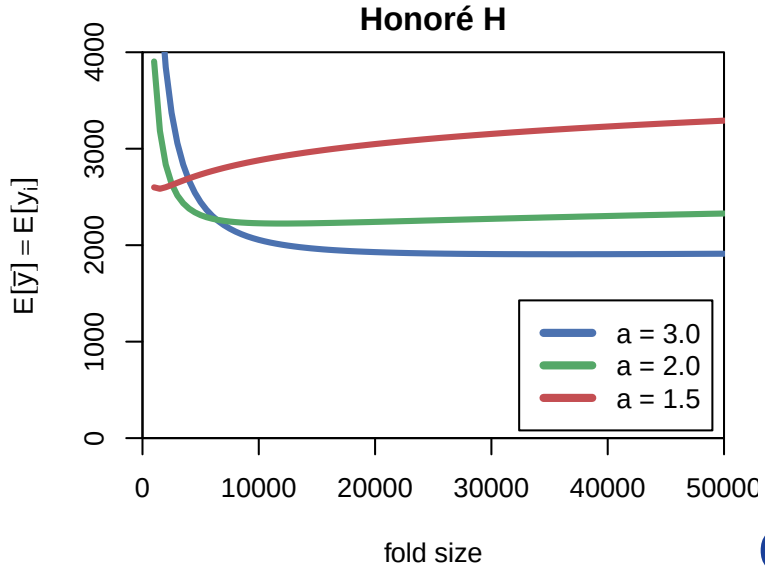


Cross-validated measures depend on fold size!









Conclusion

Thank you!

http://zipfR.R-Forge.R-Project.org/

- contains full LaTeX source code of this presentation
- R package `zipfR` (Evert and Baroni 2007)
  conveniently available from CRAN repository

My research programme for LNRE models

- Improve efficiency & numerical accuracy of implementation
  - numerical integrals instead of differences of Gamma functions
  - efficient generation of large random samples
- Analyze accuracy of LNRE approximations
  - comprehensive simulation experiments, esp. for small samples
- Specify more flexible LNRE population models
  - my favourite: piecewise Zipfian type density functions
  - flexible approximation, but no deep mathematical justification
- Develop hypothesis tests & confidence intervals
  - key challenge: goodness-of-fit vs. confidence region
  - prediction intervals for model-based extrapolation
- Simulation experiments for productivity measures
  - Can we find a quantitative measure that is robust against confounding factors and corresponds to intuitive notions of productivity & lexical diversity?

My research programme for LNRE models

- Is non-randomness a problem?
  - not for morphological productivity ➜ ECHO correction
  - tricky to include explicitly in LNRE approach
- Do we need LNRE models for practical applications?
  - better productivity measures + empirical sampling variation
  - based on cross-validation approach (Evert *et al.* 2017)
- How important is semantics & context?
  - Does it make sense to measure productivity and lexical diversity purely in terms of type-token distributions?
  - e.g. register variation for morphological productivity
  - type-token ratio $\neq$ complexity of author's vocabulary

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