Rory Fox

120445052

Experiment 10: Fourier Analysis and Bandpass Filter

06/04/22

Introduction

In this experiment the Fourier Series representations of various waveforms were examined. This was determined both analytically and using the FFT functionality of LabVIEW. A bandpass filter circuit was designed to capture the second harmonic of a square wave outputted from the MiniGen. (Second Harmonic is a Sine Wave at 3x the fundamental frequency). The circuit had to be designed from scratch to select the desired resonant frequency and have a decently high Q factor. This led to a high gain which had to be managed by reducing the input signal with a voltage divider. The output was read using the standalone Bipolar-to-Unipolar converter (B2U) circuit constructed for previous experiments.

Pre-Lab

The location of the second harmonic of a square wave was determined by calculating the Fourier Series representation of the wave. Indeed, the Fourier Series representation of a square wave, triangle wave and sine/cosine wave will all be given in subsection 3.1.

3.1 Fourier Series

All workings are included in the pdf “Experiment 10 Fourier Series Rory Fox”.

The series were as follows for a square wave , triangle wave and sine wave with amplitude 1 and frequency f

Square

$$f(x) = \frac{4}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\frac{\sin((2\pi fn)x)}{n}$$

Triangle

$$f(x) = \frac{8}{\pi^2}\sum_{n=1,3,5\ldots}^{\infty}\frac{\cos((2\pi fn)x)}{n^2}$$

Sine

$$f(x) = \sin((2\pi f)x)$$

From the workings it is clear that if you want to extract the sine wave which is the second harmonic of a square wave then the square wave should have a frequency one third that of the sine wave.

3.2 LTspice design and simulation of bandpass filter

The bandpass filter was constructed in LTSpice according to the schematic in Section 9.4.5 of the notes.

As opposed to using trial and error and manipulating the equations for gain, resonant frequency and quality by hand, a python script was written. This tests 125,000 different sets of resistances and capacitances. The capacitances range from 1nF to 1µF. The resistances range from 100Ω to 1MΩ. The values are spaced logarithmically. The program computes the Gain, resonant frequency, Quality factor and the full width half maximum frequency spread. The power of using the program is that certain minimum or maximum standards may be imposed. For example, it was stipulated that the max gain of the filter not exceed 1000. It was also stipulated that the resonant frequency be between 300Hz and 100KHz. The data is written to a .csv file. This could then be opened in Excel and filtered and sorted as required (e.g. highest Q to lowest Q).

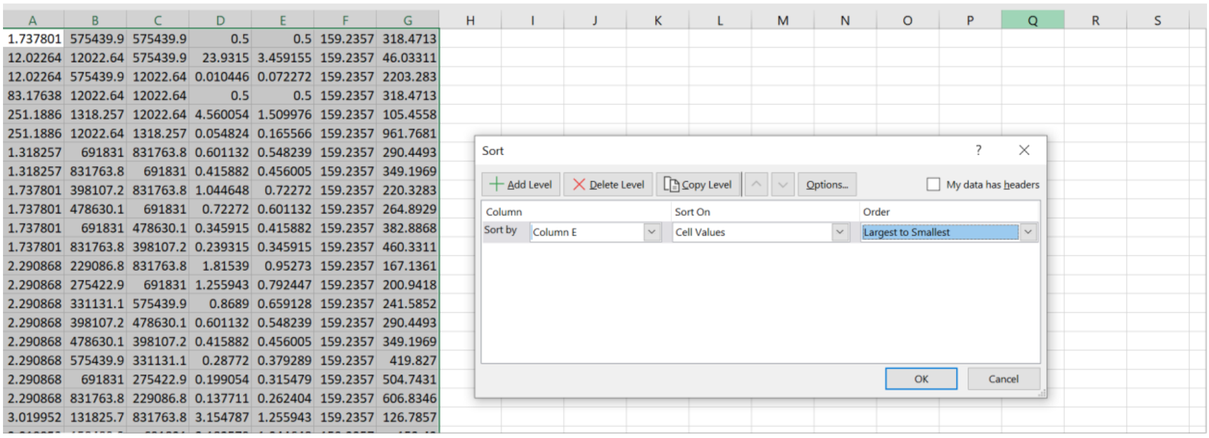
```
import math
diff1 = math.log(1000000,10) - math.log(100,10)
diff2 = math.log(1000000,10) - math.log(1,10)
k=50
resistances = []
capacitances = []
for i in range(1,k+1):
    x = 10 ** (math.log(100,10)+((i-1)*diff1)/k)
    resistances.append(x)

for i in range(1,k+1):
    x = 10 ** (math.log(1,10)+((i-1)*diff2)/k)
    capacitances.append(x)
f = open("RCvalues.csv", "w")
for C in capacitances:
```

```
    c = C * 1e-9
    for R1 in resistances:
        for R2 in resistances:
            G = (R2/(2*R1))
            W0 = 1/(6.28*c*(R1*R2)**0.5)
            Q = 0.5*(R2/R1)**0.5
            DW = 2/(6.28*(R2*c))
            if (W0 < 150):
                continue
            if (W0 > 100000):
                continue
            if (G > 1000):
                continue
            f.write(f"{C},{R1},{R2},{G},{Q},{W0},{DW}\n")

f.close()
```

This figure shows the data being sorted in Excel according to Q-factor (column F).

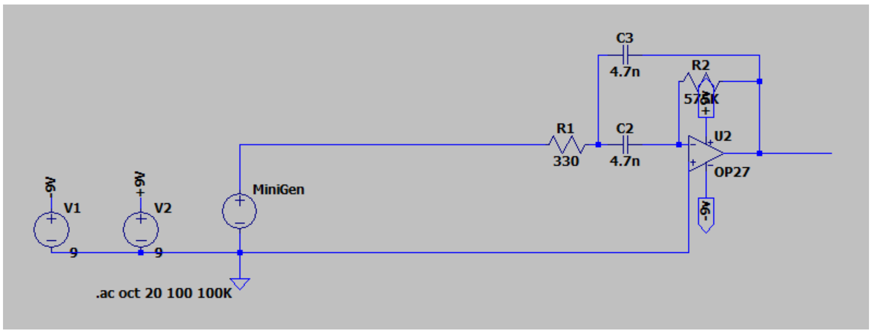


A	B	C	D	E	F	G
1.737801	575439.9	575439.9	0.5	0.5	159.2357	318.4713
12.02264	12022.64	575439.9	23.9315	3.459155	159.2357	46.03311
12.02264	575439.9	12022.64	0.010446	0.072272	159.2357	2203.283
83.17638	12022.64	12022.64	0.5	0.5	159.2357	318.4713
251.1886	1318.257	12022.64	4.560054	1.509976	159.2357	105.4558
251.1886	12022.64	1318.257	0.054824	0.165566	159.2357	961.7681
1.318257	691831	831763.8	0.601132	0.548239	159.2357	290.4493
1.318257	831763.8	691831	0.415882	0.456005	159.2357	349.1969
1.737801	398107.2	831763.8	1.044648	0.72272	159.2357	220.3283
1.737801	478630.1	691831	0.72272	0.601132	159.2357	264.8929
1.737801	691831	478630.1	0.345915	0.415882	159.2357	382.8868
1.737801	831763.8	398107.2	0.239315	0.345915	159.2357	460.3311
2.290868	229086.8	831763.8	1.81539	0.95273	159.2357	167.1361
2.290868	275422.9	691831	1.255943	0.792447	159.2357	200.9418
2.290868	331131.1	575439.9	0.8689	0.659128	159.2357	241.5852
2.290868	398107.2	478630.1	0.601132	0.548239	159.2357	290.4493
2.290868	478630.1	398107.2	0.415882	0.456005	159.2357	349.1969
2.290868	575439.9	331131.1	0.28772	0.379289	159.2357	419.827
2.290868	691831	275422.9	0.199054	0.315479	159.2357	504.7431
2.290868	831763.8	229086.8	0.137711	0.262404	159.2357	606.8346
3.019952	131825.7	831763.8	3.154787	1.255943	159.2357	126.7857

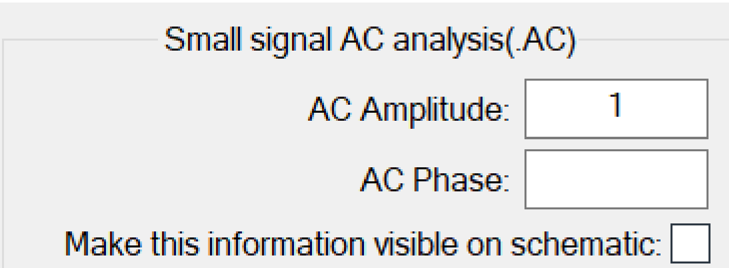
Ultimately the values chosen were $R_1 = 302\Omega$, $R_2 = 575439\Omega$ and the capacitance of the Capacitors $C = 3.98nF$. This gives a gain of 952, which had to be managed later in the lab. However, it also yielded a very high Q factor (21) at a reasonable resonant frequency (3KHz).

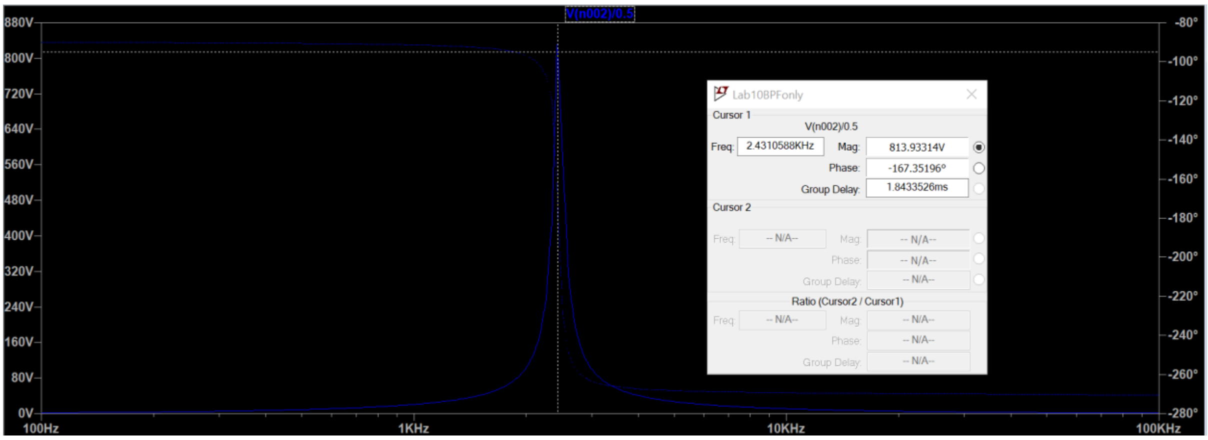
I could not use these exact values as I only had access to the standard components in the lab. Thus, the circuit was constructed with $R_1 = 330\Omega$, $R_2 = 575K\Omega$ and the capacitance of the Capacitors $C = 4.7nF$.

Calculating the gain, resonant frequency and Q-factor for this updated value set gives Gain = 871, Resonant Frequency = 2450Hz, and Q-factor = 20.87



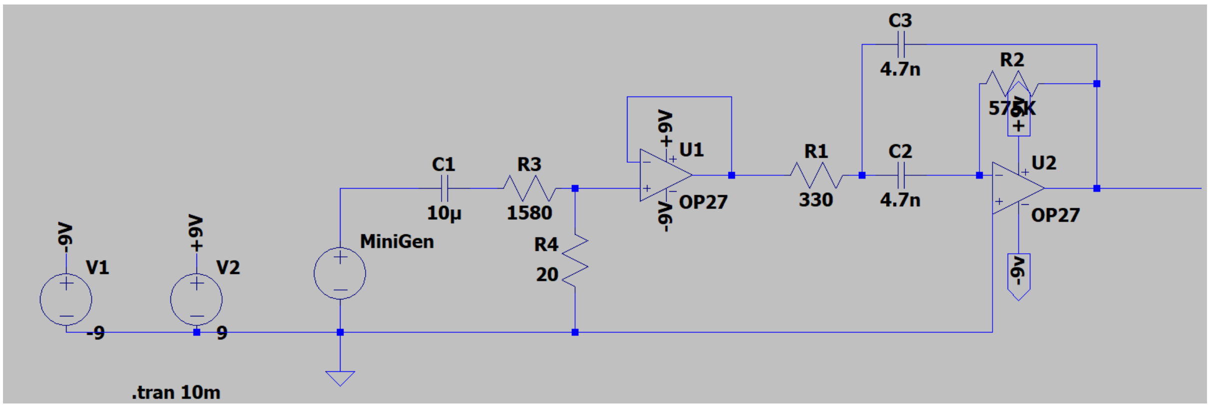
An AC analysis was performed in LTspice to simulate the expected performance of this bandpass filter.



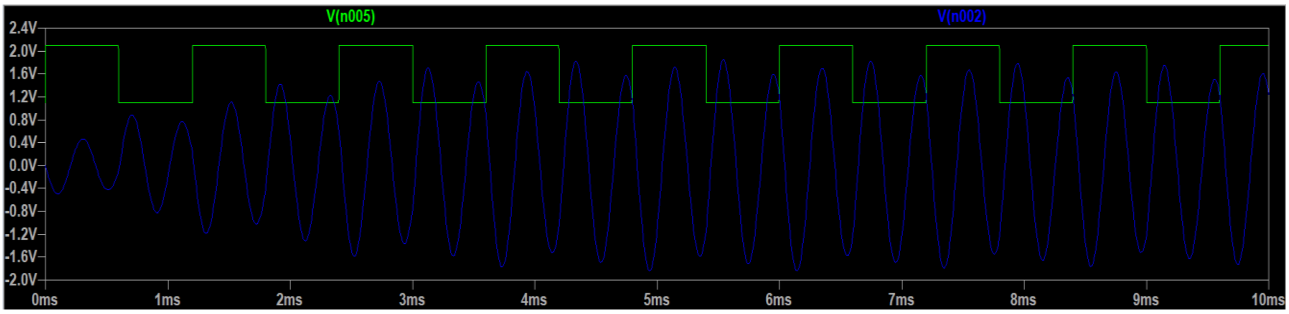


According to LTspice, this configuration gives a resonant frequency of 2.4KHz, with a maximum gain of 814 at this frequency. The Q-factor is given by finding the full-width-half-maximum of the peak at half the maximum gain squared. This is 120Hz in this case. The q factor is given by $\frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} = \frac{2500}{120} = 20.83$

Obviously, this gain was too massive for the IMB4 to be able to handle. The resultant voltages would have been too large. As such a voltage divider was inserted into the circuit. A voltage follower was also added to isolate the MiniGen input and divider from the bandpass filter which may drive a signal back into that segment of the circuit. The voltage divider was set up with $R_3 = 1580\Omega$ and $R_4 = 20\Omega$. The voltage flowing into the + input of the follower OpAmp is then given by the formula $\frac{R_4}{R_4+R_3}V_{in} = \frac{1}{80}V_{in}$. Combined with the gain of 814 from the bandpass filter, the overall gain should be something around 10. The amplitude of the MiniGen signal is 0.5V, thus, the largest signal outputted should be something around 5V. Note that this will be much lower in practice as the filter selects the second harmonic of the square wave which has an amplitude ($0.5\text{V} \times \frac{4}{3\pi} = 0.21\text{V}$). Thus, the output signal amplitude is expected to be ~ 2.1V. This can be measured easily with the B2U converter assembled in previous labs which does ±5V

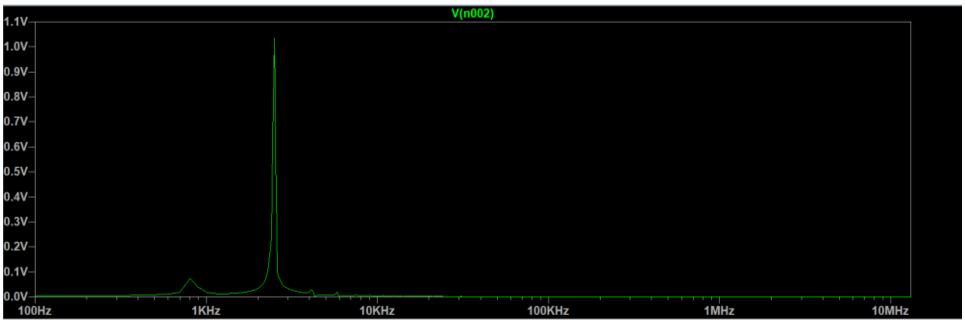


A transient analysis was performed.



The circuit was shown to perform as expected; the input in green is a square wave at 830Hz. The output waveform in blue seemed to be dominated by the second harmonic component of the square wave. This is based on a visual examination, no FFT was performed at this stage. The amplitude of the sign wave was ~1.6V which is slightly lower than expected but still close to the expected amplitude. Further, accounting for the phase shift there is in general roughly 3 peak-to-peak cycles of the sine wave for each period of the square wave.

The FFT view of the waveform in LTspice yielded the following



There is a large spike at 2.5KHz, this is where the second harmonic was expected to lie. This clearly dominates the waveform, with the next largest contribution being at ~830Hz (the fundamental frequency).

3.3 Spectral Analysis

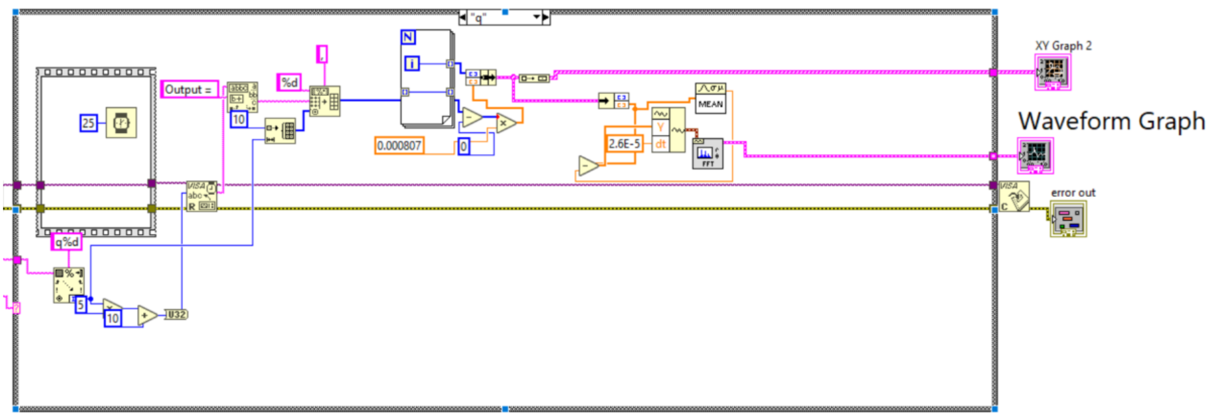
The Fourier analysis was carried out as in the Panopto tutorials.

Based on the theoretical Fourier series derived for the square wave in 3.1, we expected to see a large spike at some fundamental frequency of an amplitude around $0.5 \times \frac{4}{\pi} = 0.63\text{V}$. We then expected to see subsequent spikes at each odd multiple of the fundamental f (3f,5f etc) of amplitudes $0.5 \times \frac{4}{3\pi} = 0.21\text{V}$, $0.5 \times \frac{4}{5\pi} = 0.123\text{V}$. and so on.

With regards to the filtered wave, there will likely be a large spike at the second harmonic frequency, $3f$ where f is the fundamental. We also expect to see much a smaller spike at the fundamental frequency f and likely a very small spike at the third harmonic $5f$

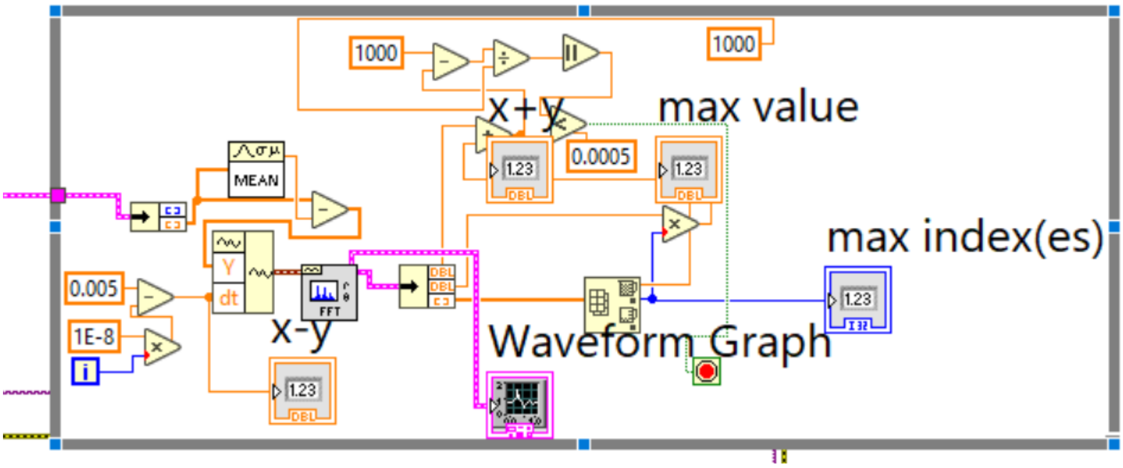
Method

4.1/4.2 Confirm Operational LabVIEW code and spectral analysis



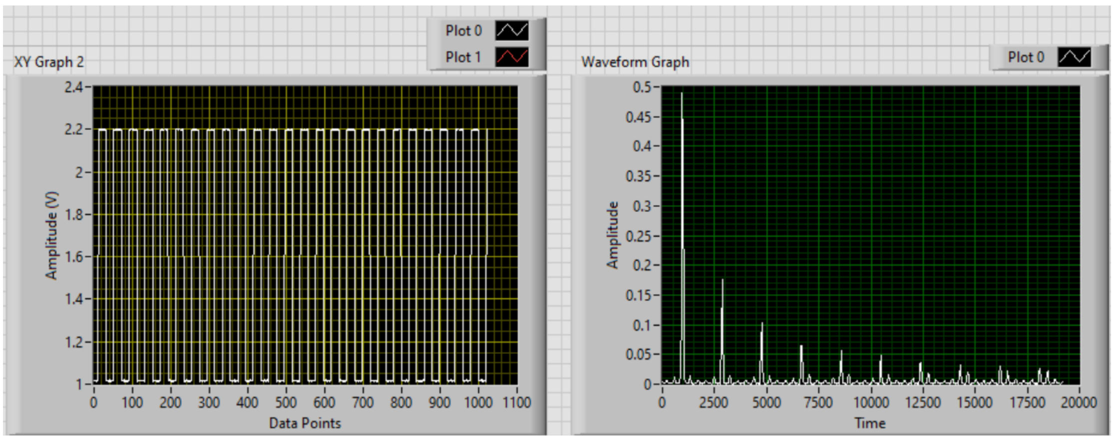
The spectral analysis was carried out in much the same manner as in the LabVIEW tutorial provided. Rather than manually copying the data into a control block, a second graph was added to the WaveformVisualiser VI, and this displays the FFT of whatever wave is shown in the main display.

While a time constant dt was provided for use in building the waveform, this is only accurate for the IMB4, and indeed may even differ from system to system. Therefore, it is useful to have a VI to quickly calculate this time constant. This was done by taking advantage of the single large peak of the FFT of a sine wave. Different time constants were used until the peak of the FFT of a 1KHz sine wave was within a 0.1% of 1KHz. This gave a time constant for the IMB4 of 2.68E-5. This is in line with the expected value provided. The program TimeConstant.vi is provided with the report.



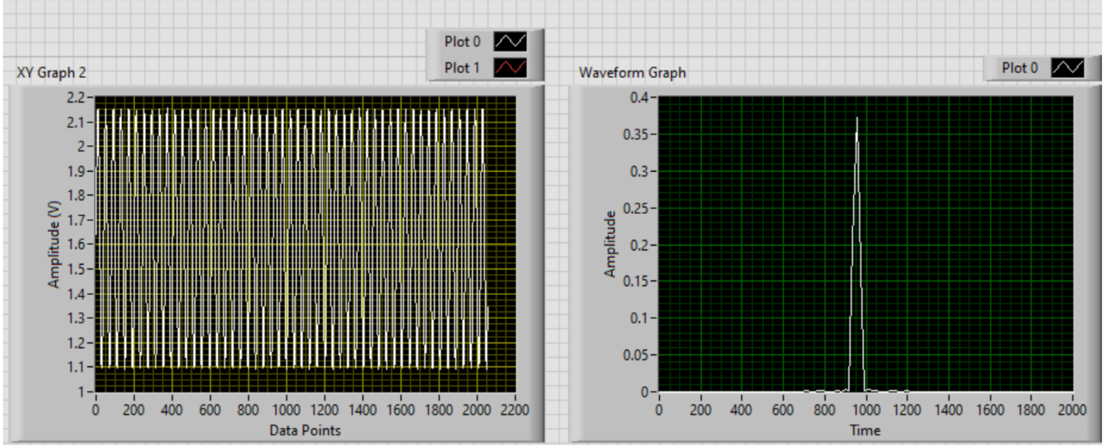
The following FFT graphs were produced for the Square, Sine and Triangle waves respectively.

Square Wave – 930Hz



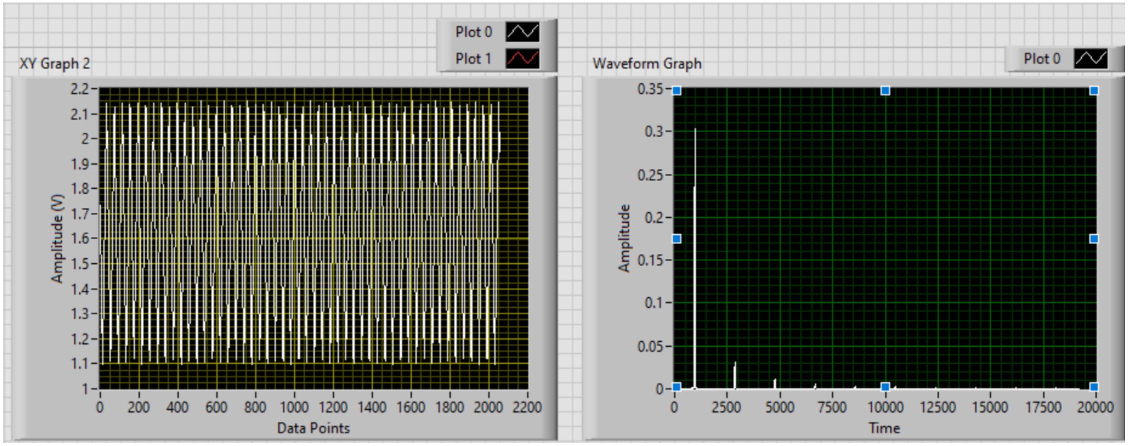
Here the amplitude of the fundamental frequency is 0.5V. This is lower than the 0.63V predicted from theory, however given that the amplitudes of the frequencies around the peak are non-zero (unlike the theoretical Fourier series) this is to be expected. In any regard the qualitative behaviour is as expected. With the second harmonic (3f) and third harmonic (5f) having amplitudes of approximately one third and one fifth of the fundamental.

Sine Wave – 930Hz



The FFT of the sine wave shows one peak at the fundamental frequency of 930Hz. The maximum amplitude is again lower than the predicted value of 0.5V. Although this can again be explained by the not insignificant spread of the frequencies around the peak.

Triangle Wave – 930Hz

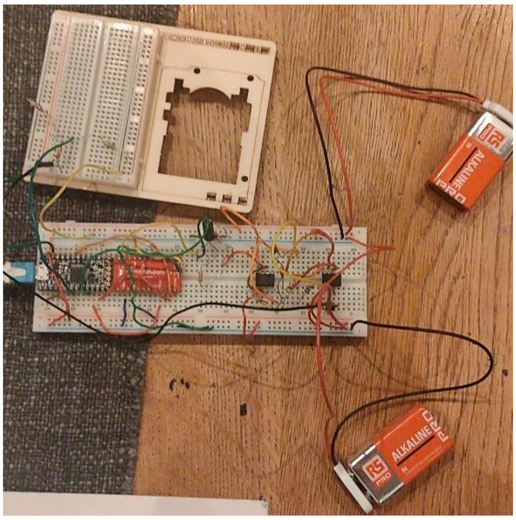


The predicted amplitude of the fundamental frequency peak was $0.5 \times \frac{8}{\pi^2} = 0.42\text{V}$. This is again higher than the observed peak. The FFT of the triangle wave still behaves qualitatively as expected based on the theory. There is another peak at the second harmonic which has an amplitude of roughly one ninth that of the fundamental. There is also a peak at the third harmonic which has an amplitude of one-twenty-fifth of the fundamental as expected

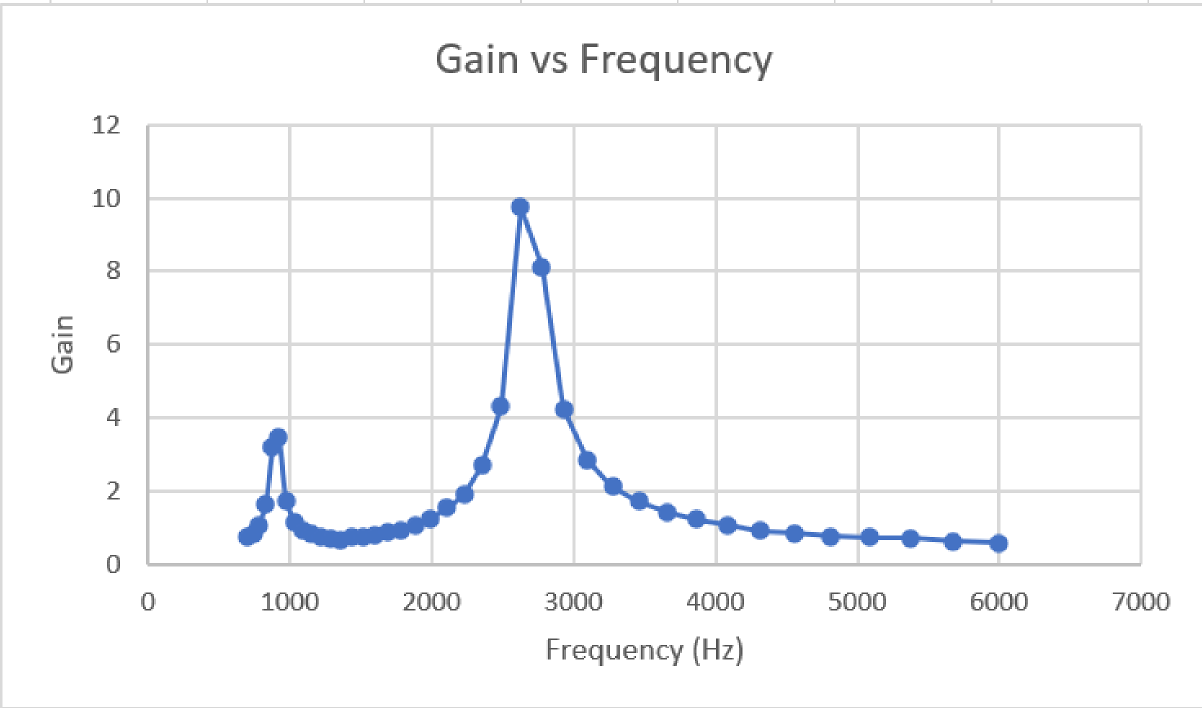
4.3/4.4 Build and confirm operation of the bandpass filter/ Build and refine your square wave generator

Due to the results of the simulation, it was clear that at the resonant frequency the gain would render the output voltage much too high to measure with the IMB4. Therefore, the resistive network was added from the beginning, in accordance with the LTSpice schematic from Section 3.2.

The finished circuit looked as such

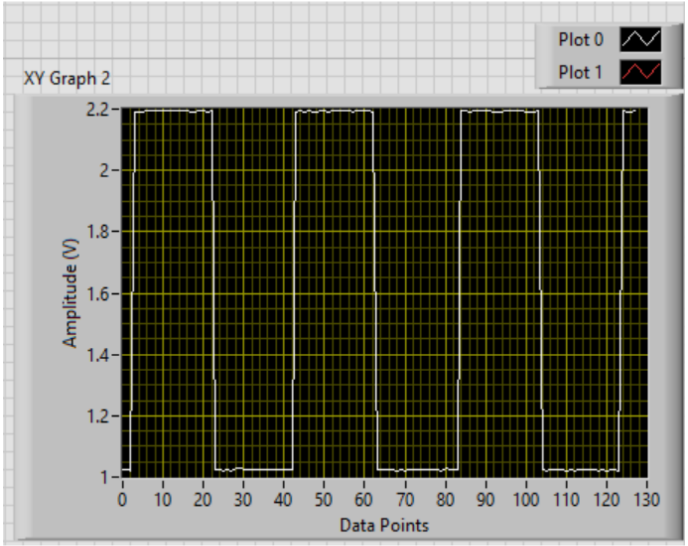


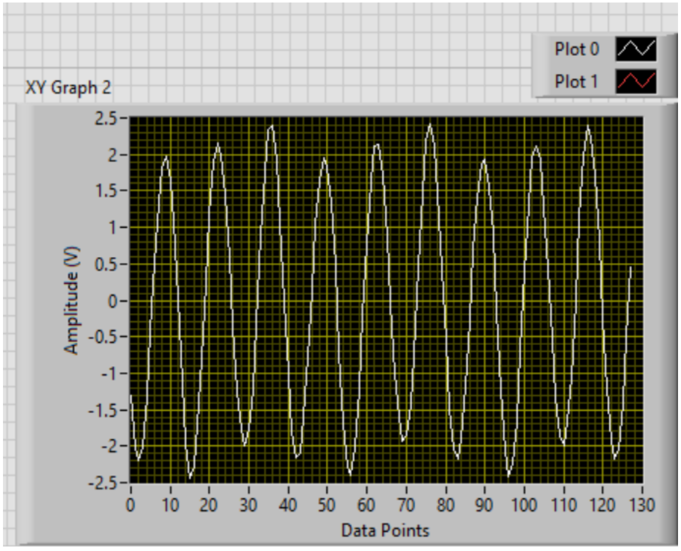
The frequency response was carried out as in previous labs using the included VI. An initial sweep was performed from 800Hz to 6000Hz. 30 datapoints were taken. It was interesting to note here that if the MiniGen was not set to Sine, then the frequency response would show a series of peaks in Gain, the peaks increased in amplitude as the frequency increased towards the resonant frequency, after this frequency the gain rolled off as normal. This presumably meant that one of the harmonics of the square wave was hitting the fundamental frequency. This would explain why the peaks increased in height. For low frequencies only the higher harmonics of the fundamental (which have lower amplitudes) will reach the resonant frequency.



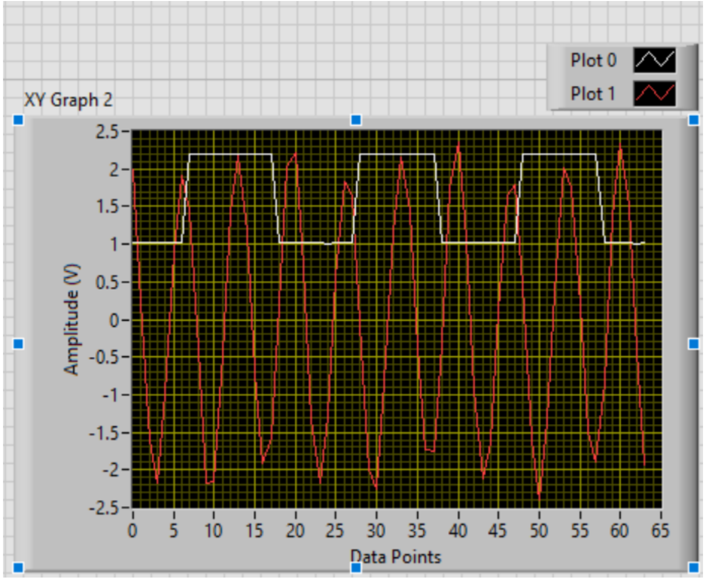
The first interesting thing to note is that there is an abnormality in the shape of the frequency response graph. There is a large peak at ~2700Hz, which is in line with the expected resonant frequency from the LTspice simulation. However, there is also a significant peak at ~900Hz. It is interesting to consider that this first, smaller peak has the resonant frequency very close to an integer multiple of its own frequency. (2700Hz is three times 900Hz). This does not appear to me to be incidental, and possible reasons for this abnormality will be included in the discussion. In any event, outside of this the bandpass filter performs very closely to the expected behaviour from the simulations. The Gain peaks at a manageable level of 10 at around 2700Hz. This is slightly higher than the 2500Hz predicted and as such a square wave of 930Hz will be used as the source instead of the 830Hz used in LTspice.

The input waveform and output waveform were captured.

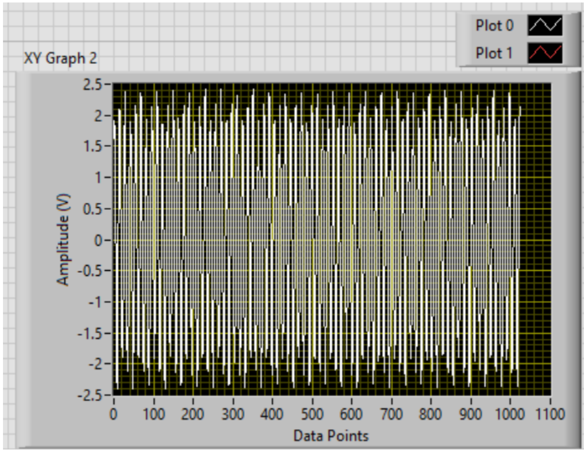


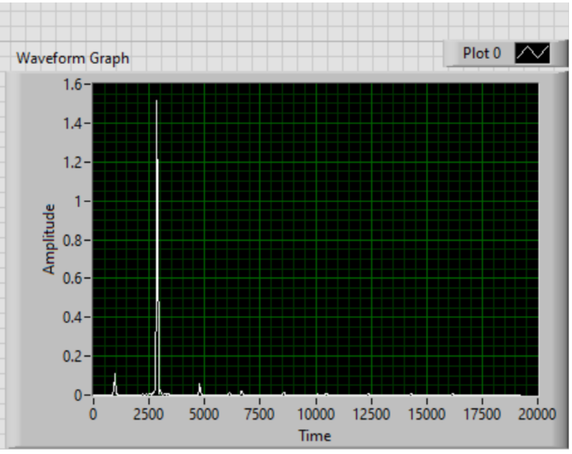


The output wave is clearly close to being a sinusoid. The amplitude of the wave is ~2V which is somewhat higher than expected from the simulation. When both traces are measured simultaneously (not particularly good resolution due to halving sample rate) the following graph was produced



Here it is clear that the output wave has a frequency approximately 3 times that of the input square wave. This is confirmed by the FFT graph shown below. The FFT is dominated by a spike at roughly 2700Hz with an amplitude of roughly 1.6. There is also a spike at 900Hz and 4500Hz as expected.





Analysis

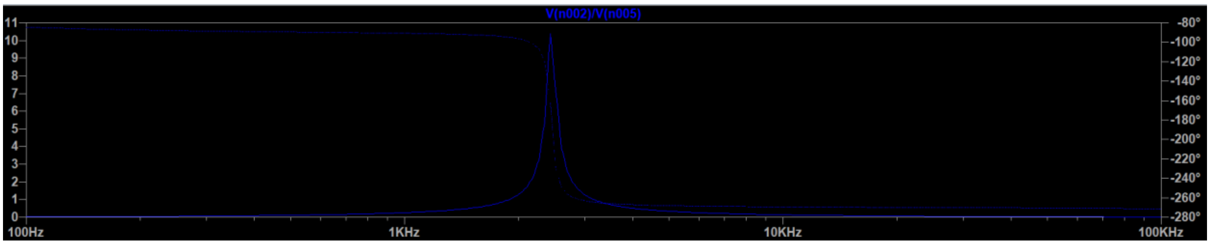
Analysis of Circuit Performance vs Theory

The circuit performed largely as expected based on both the theory and the simulation. The resonant frequency expected based on the theory was

$$\frac{1}{2\pi C\sqrt{R_1R_2}} = \frac{1}{2\pi(4.7\times 10^{-9})\sqrt{(330)(575\times 10^3)}} = 2458.28019 = 2450\text{Hz}$$

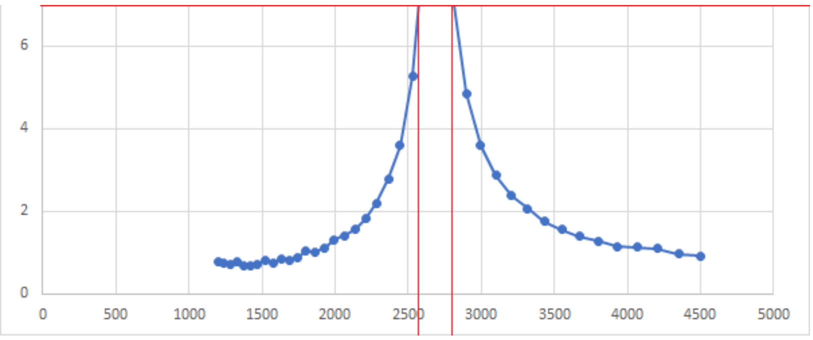
The resonant frequency observed was 2700Hz. This is within the same order of magnitude and is very close to the expected value.

The expected maximum gain of the full circuit from LT spice was 10. This agrees extremely closely with the maximum gain observed of 9.85



This agrees with the theoretical filter gain of $\frac{575000}{2\times 330} = 871$. When combined with the resistive network reduction of 80 times, the overall gain is $\frac{871}{80} = 10.89$. This is very close to the observed gain also.

With regards to Q factor, the full width half maximum was found to be 202Hz



The real Q factor was thus determined to be $Q = \frac{\omega_0}{\Delta\omega} = \frac{2700}{202} = 13.36$. This is markedly lower than the predicted Q factor of $Q = \frac{1}{2}\sqrt{\frac{575000}{330}} = 20.87$. However, it was still quite high and given that the filter still performed well it Is likely sufficient.

The spectral analysis of the square wave is detailed above and agreed closely with the results expected from the theory.

From the theory the second harmonic was expected to be a sine wave. The amplitude was expected to be one third of the square wave amplitude. The frequency was expected to be three times the fundamental frequency of the square wave. We predicted that the amplitude of the third harmonic would be $\frac{0.5\times 4}{3\pi} = 0.21$. After passing through the circuit, we expected an output signal of amplitude 2.1V. The measured value of 1.6V is lower than this, although seeing as the peak has a spread and is not concentrated at one frequency this is to be expected. The output wave is dominated by a component with an amplitude of roughly 1.6V. The frequency of this spike is around 3 times the fundamental; this was observed both visually and from the FFT.

Justification of Design Choices/ Potential Improvements

The resistors used were $R_2 = 575\text{K}\Omega$, $R_1 = 330\Omega$. The capacitors used were 4.7nF. These values were chosen out of all the possible sets of values from the python script as it gave the highest Q factor possible under a gain of 1000 and a resonant frequency under 3KHz. There was a fear that by going above this gain the voltage divider would result in too much loss of signal and the waveform could be lost to the ambient noise background in any measurements. Also, some trials showed that the greatest frequency that could be accurately measured was around 3KHz.

A potential improvement to the experimental setup would be the resistor values chosen for the Bipolar-to-Unipolar (B2U). The values used were the same as for the amplifier circuits where the signal to be measured ran between ±5V. In this experiment the output signal to be measured never went in excess of ±2.5V. By using a B2U circuit calibrated to reduce this wider range to 0V-3.3V the resolution of the signal is less than it could otherwise be. This was justified as being accurate enough for the purposes of this experiment which was largely about observing the gross shape of the waveform. However, for a more accurate FFT, a more tightly calibrated B2U circuit could be implemented.

With regards to other design choices, one choice that I cannot really justify is the inclusion of a 10µF capacitor in series with the MiniGen. This had utility in the non-inverting amplifier lab for stripping off the DC component. However, the bandpass filter already strips off this component by default and as such the capacitor is redundant and merely introduces needless complexity into the circuit analysis. It was left in merely for agreement with all the Pre-lab simulation and would not be included if the experiment was repeated.

Discussion

Second Peak of Frequency Response

Overall, the experiment was a success. As laid out in the analysis section, the performance of the bandpass filter agreed quite closely with both the simulation and the theory.

One interesting finding of the experiment was the strange secondary resonant peak in the bandpass filter. My first theory was that this was not a real second peak. It seemed possible to me that the input wave at 900Hz had a second harmonic that was being allowed to pass by the filter. As this was a higher harmonic (lower amplitude) the spike observed is lower than that at a frequency where the fundamental component of the wave is passed.

There are two obvious flaws in this theory, however. Firstly, if the problem was an issue of harmonics, then there would be some sort of spike around 450Hz, where the third harmonic would be passed by the filter. The more pressing flaw is that the FFT of the MiniGen sine wave shown in Section 4.2 had only one spike and showed very little amplitude at other frequencies, including multiples of the fundamental frequency which would be the prime candidates for harmonics.

As such, I was inclined to consider other possibilities. One possibility is that this peak was not present at all. Consider that the component of the square wave at 900Hz (The fundamental frequency) is supposed to undergo a gain of 4. The amplitude of this component is meant to be 0.63 according to Section 3.1. Thus, you would expect there to be a component to the output wave with a frequency of 900Hz and an amplitude of 2.4. This would visibly dominate the output waveform and more importantly, the FFT of the output wave. This is not observed. In fact, the FFT of the output waveform is very similar to that expected from the FFT of the LTspice simulation. This is even though the frequency response of the bandpass filter in LTspice does not show any sort of secondary peak at the frequency of 900Hz. Thus, there is the possibility that the bandpass filter is in fact performing as expected and that the spike is the result of some sort of error in measurement or processing of the data. I am not sure what might cause a problem such as this. Measuring the voltages on a different IMB4/PC and seeing if the strange measurement persists would help to determine whether this is in fact a software error.

Applications of Fourier Series and Bandpass filters

Having completed the lab. I have a deeper understanding of the practical application of Fourier Series. One real life application of Fourier transforms, bandpass filters and FFTs is noise cancellation technology. Often in an audio signal there may be noise; unwanted high frequencies are one example. This noise can be reduced in the signal by performing an FFT. The high frequency noise peaks can then be located and flattened. Another algorithm called an inverse-FFT can then be performed to obtain the noise-reduced audio signal.

Conclusion

In conclusion, all the learning objectives were met. A square wave signal was generated with the MiniGen. Spectral analysis of the signal was carried out in LabVIEW to understand its components and based on this an active-bandpass filter was designed to attempt to filter out this second harmonic. The resonant frequency of the bandpass filter was determined, and the fundamental frequency of the square wave was adjusted to match this. The second harmonic of the wave was then successfully extracted and spectral analysis performed on it in LabVIEW. Finally, I gained experience of performing Fourier Series expansions of odd and even functions.