

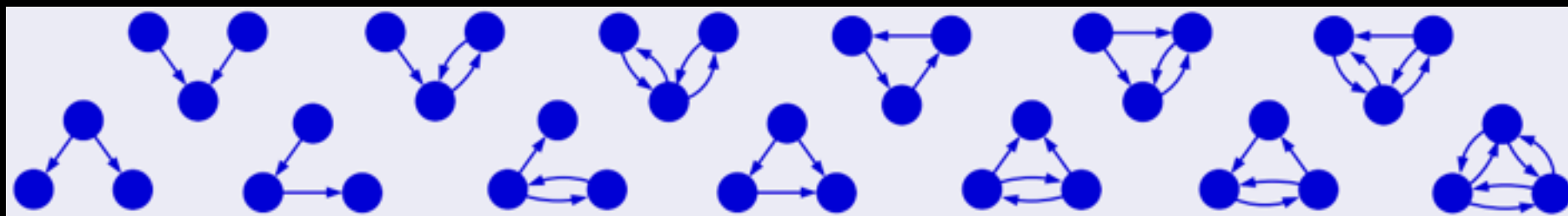
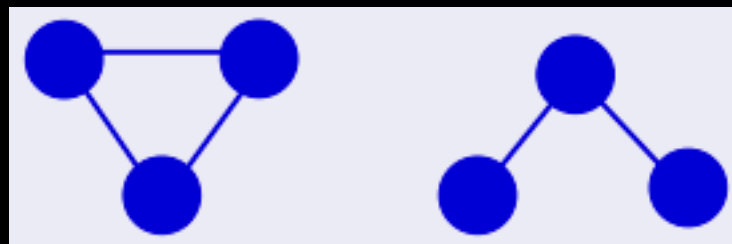
ROSS FLIEGER-ALLISON

# EXTRACTING TRIAD MOTIF SIGNIFICANCE PROFILES FROM COMPLEX NETWORKS



# NETWORK MOTIFS

- **motif**: a connected non-isomorphic induced subgraph with a predefined link configuration
- **triad motif**: motif consisting of 3 nodes
- 2 undirected, 13 directed:



Images taken from lecture slides.



# WHY THEY MATTER

- Help determine **network properties**.
- Play a key role in **system performance** of certain tasks.
- **Triadic > Dyadic**



# WHAT IS A SIGNIFICANCE PROFILE?

- **significance profile**: quantifier of motif over- or underrepresentation in a network
- Uses **normalized z-scores**.

$$SP_i = \frac{z_i}{\sqrt{\sum_j z_j^2}}$$

$$z_i = \frac{N_{\text{original},i} - \langle N_{\text{rand},i} \rangle}{\sigma_{\text{rand},i}}$$

THE PROBLEM

FIND AN EFFICIENT MEANS OF  
EXTRACTING TRIAD MOTIF  
EXPRESSION FROM A NETWORK

# MY SOLUTION (MOTIF COUNTING)

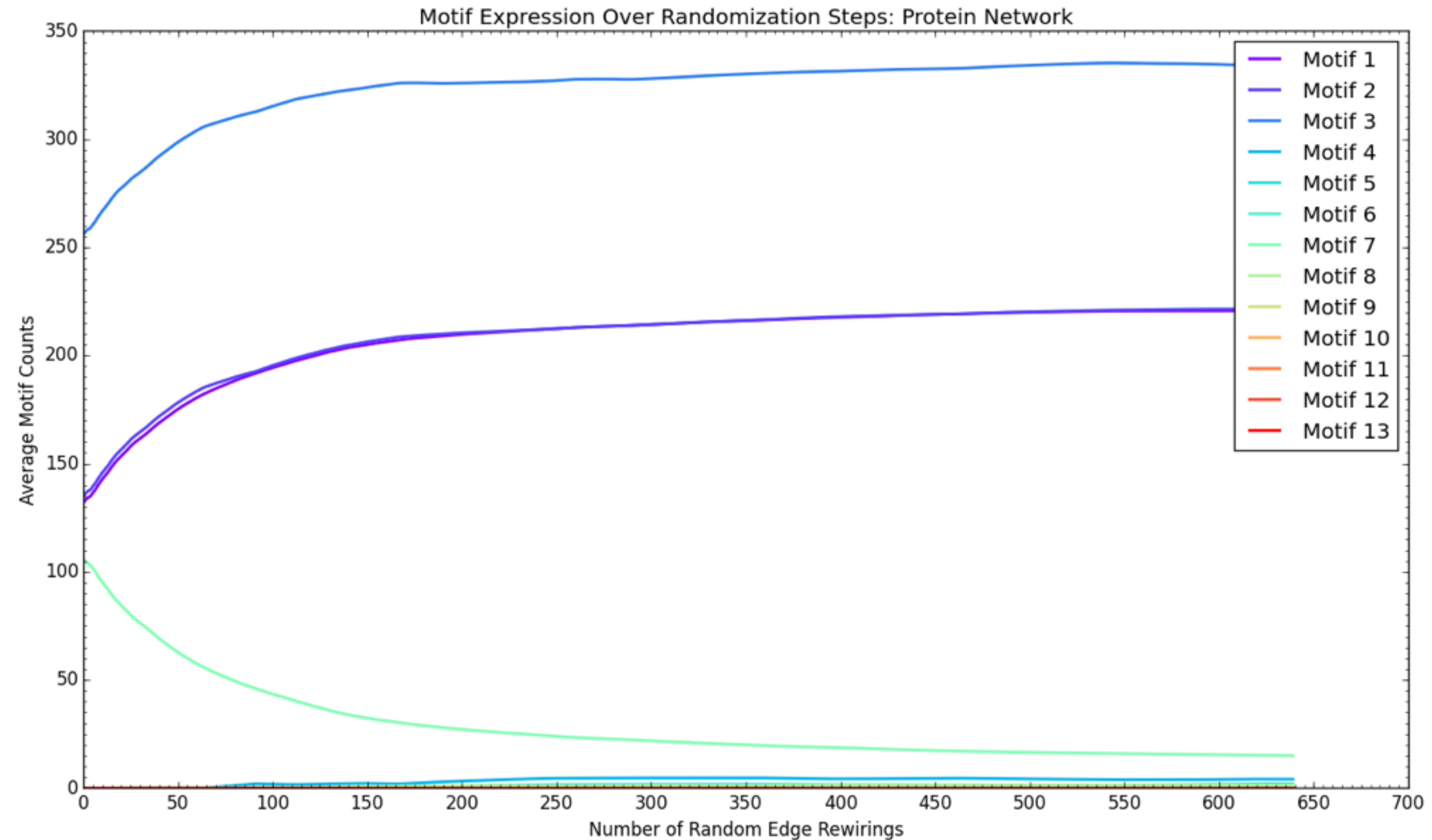
- Iterate through edges  $(a, b)$ .
- Find all unique neighbors of  $a$  and  $b$  (call them  $c$ ).
- Search for motifs in  $(a, b, c)$  triads such that every connected node triplet is searched exactly once.
- Return aggregated counts.

# MY SOLUTION (Z-SCORES)

- Count the motifs in the original network.
- Randomize the network such that its degree sequence is maintained.
- Count the motif occurrences in randomized instances.
- Average the random network motif counts (expected value).
- Compute the standard deviation of randomized counts.
- Use this to compute the z-score.

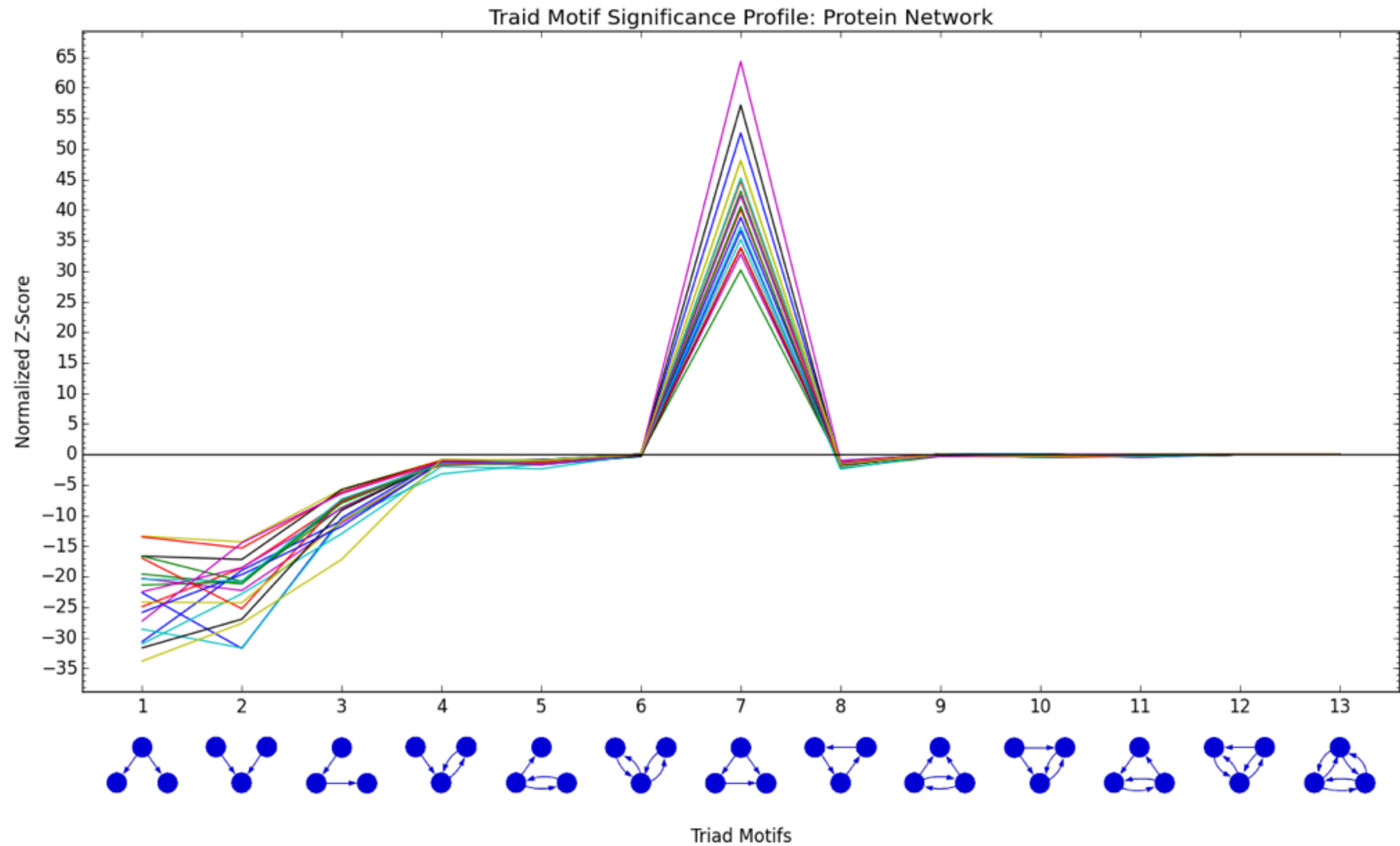
$$z_i = \frac{N_{\text{original},i} - \langle N_{\text{rand},i} \rangle}{\sigma_{\text{rand},i}}$$

# RANDOMIZATION STEPS

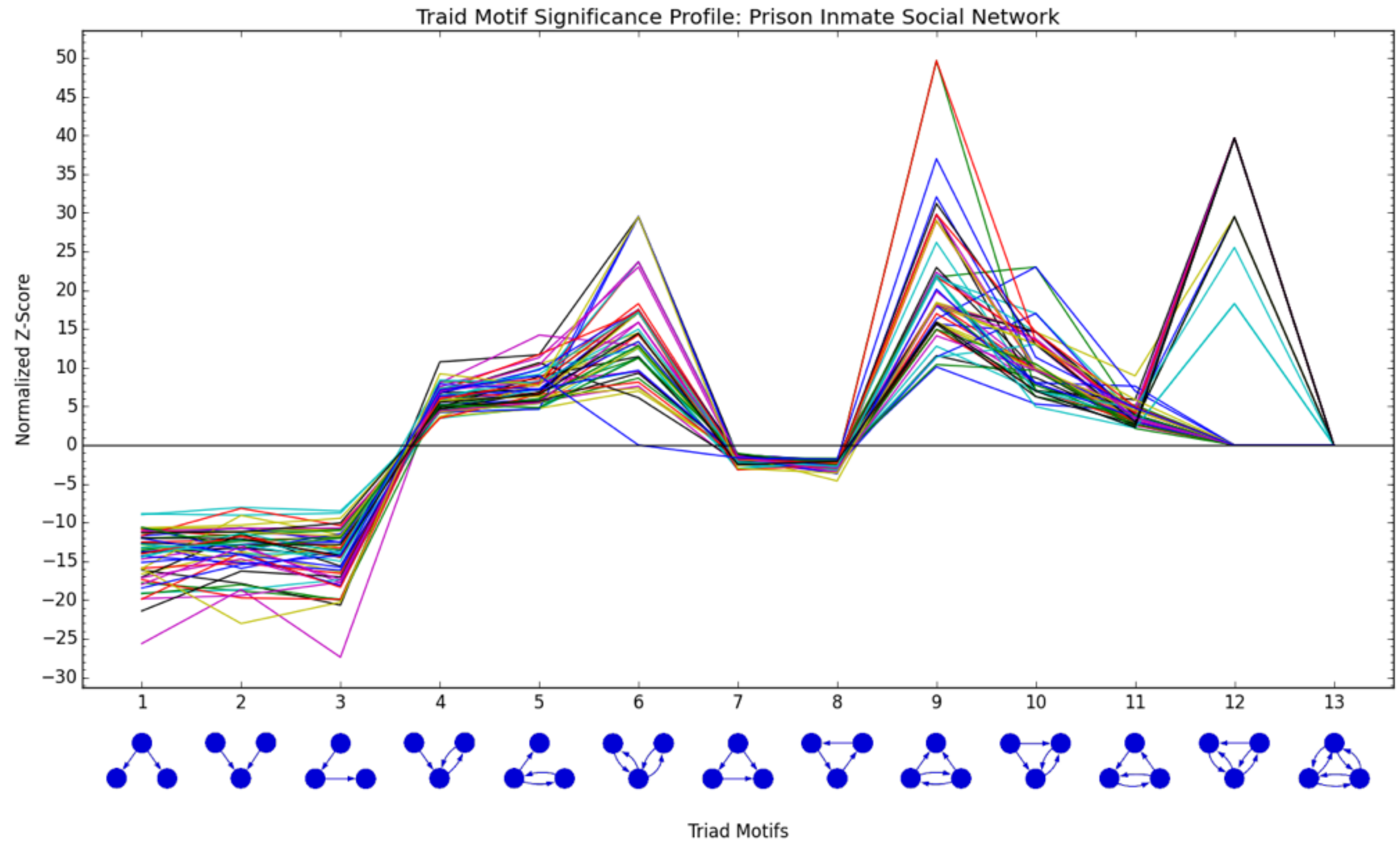




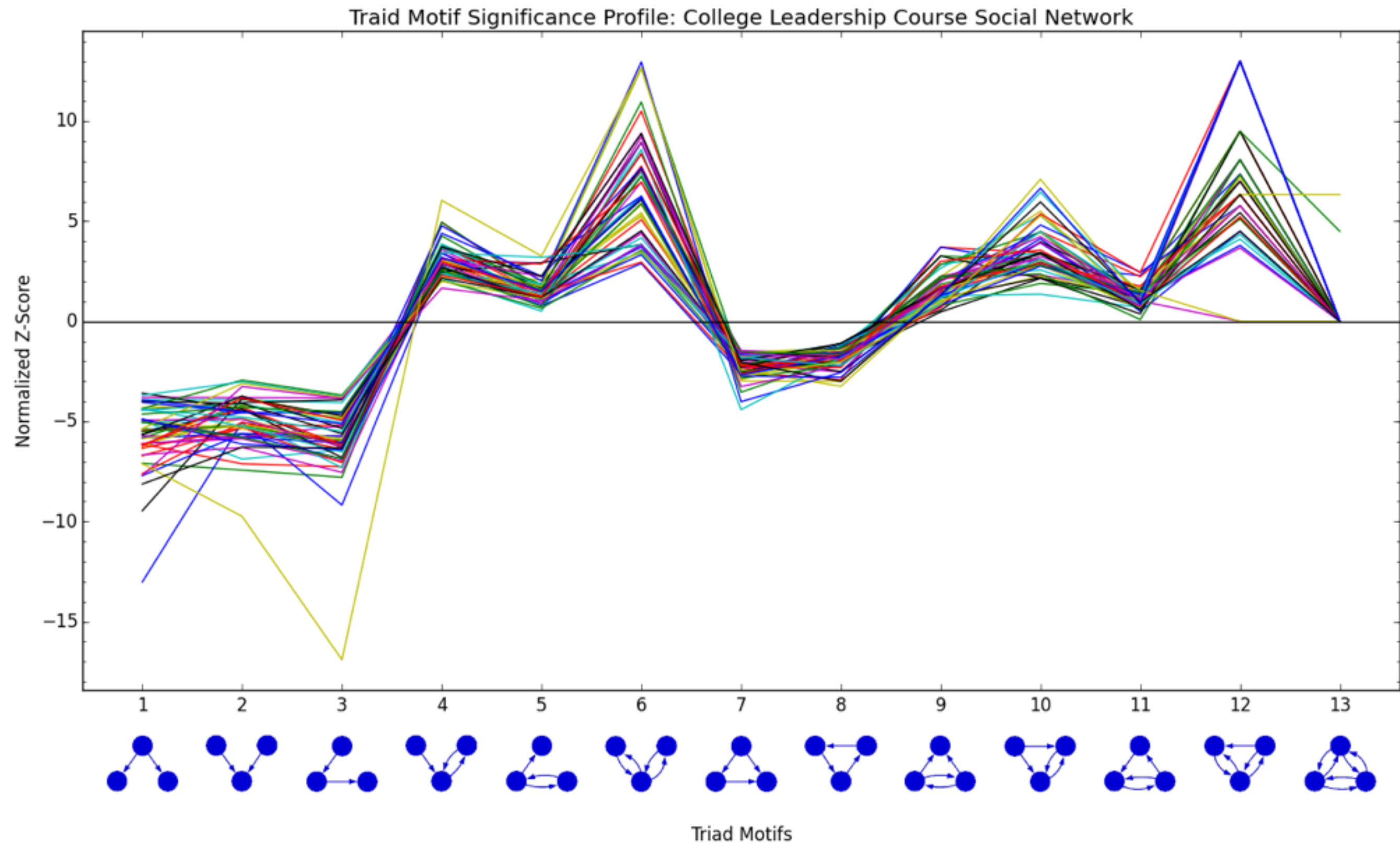
# THE RESULT



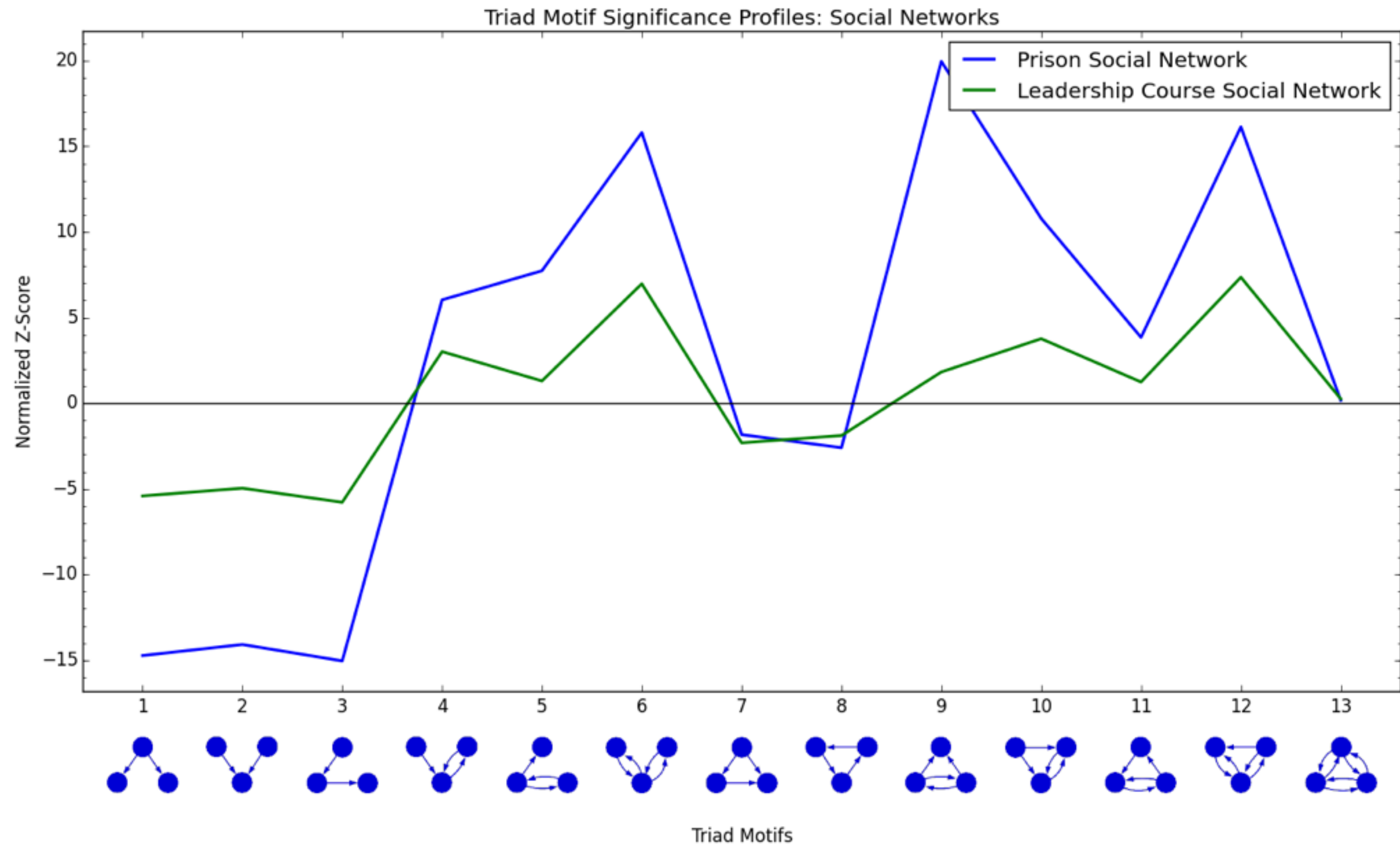
# PRISON INMATE SOCIAL NETWORK



# LEADERSHIP COURSE SOCIAL NETWORK



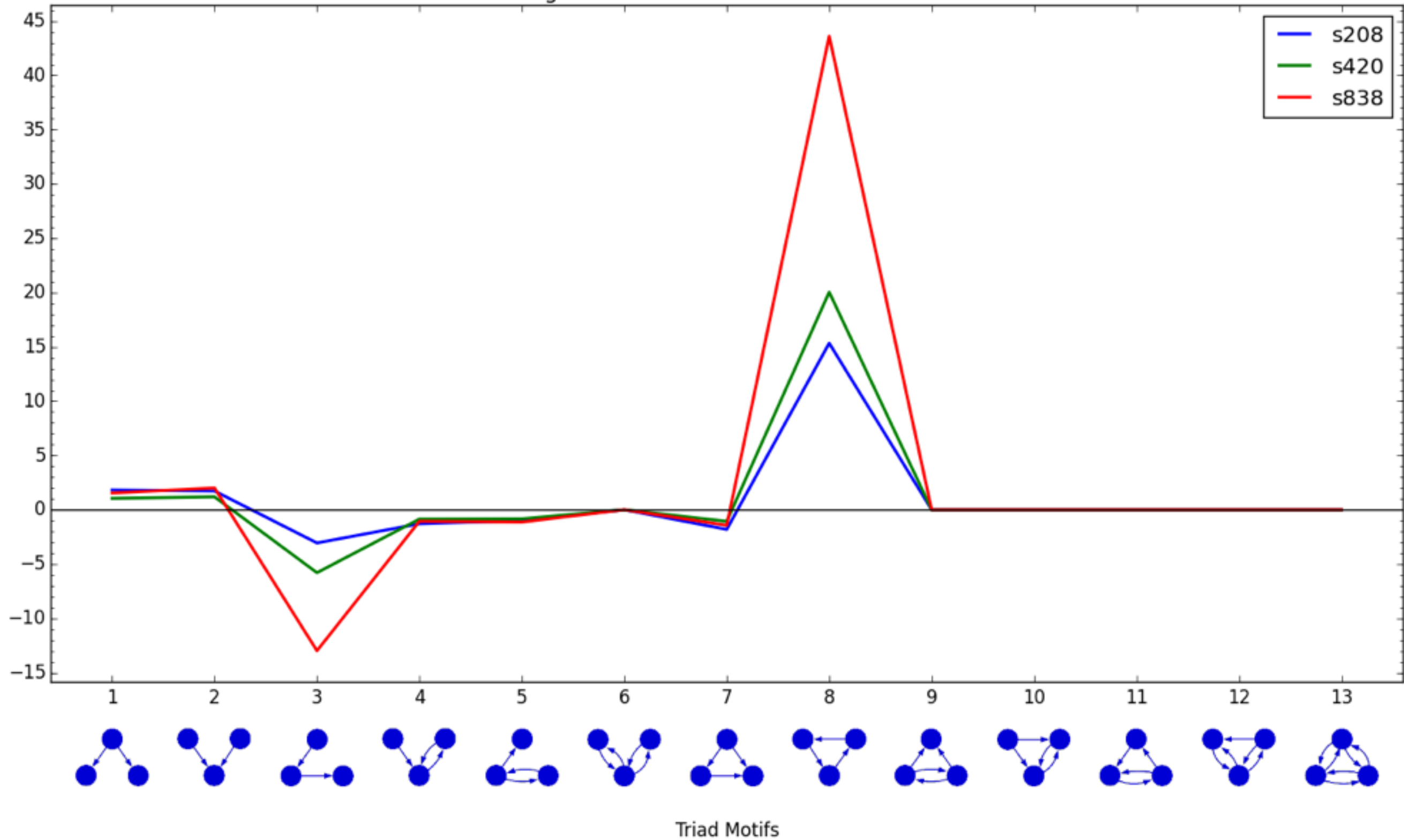
# SOCIAL NETWORKS





# ELECTRONIC NETWORKS

Triad Motif Significance Profiles: Electronic Circuit Networks



# REFERENCES

- Prison dataset: J. P. Eckmann, E. Moses, Proc. Natl. Acad. Sci. U.S.A. 99, 5825 (2002).
- Leadership course dataset: M. A. J. Van Duijn, M. Huisman, F. N. Stokman, F. W. Wasseur, E. P. H. Zeggelink, J. Math. Sociol. 27 (2003).
- Protein dataset: from lab
- Electronic circuit datasets: R.F. Cancho, C. Janssen, R.V. Sole, Phys. Rev. E 6404 (2001) 046119.

```

12
13 def count_triad_motifs(network, directed=None):
14     """
15     Counts the occurrences of triad motifs in a network.
16
17     Arguments:
18         network => The input network.
19         directed => Whether or not the network is directed.
20
21     Returns:
22         A fixed-size array with indices representing unique triad motifs and the values
23         representing their number of occurrences within the network.
24     """
25
26     if directed or nx.is_directed(network):
27
28         # Initialize an array for storing our motif counts. Each index represents the following motif:
29         # 0 => a <- b -> c
30         # 1 => a -> b <- c
31         # 2 => b -> a -> c
32         # 3 => a -> b <-> c
33         # 4 => c <-> a -> b
34         # 5 => a <-> b <-> c
35         # 6 => a <- b -> c <- a
36         # 7 => a -> b -> c -> a
37         # 8 => c <-> a -> b <- c
38         # 9 => c <- a -> b <-> c
39         # 10 => a <-> c -> b -> a
40         # 11 => a <-> b <-> c -> a
41         # 12 => a <-> b <-> c <-> a
42         motif_counts = np.zeros(shape=(13,), dtype=np.int)
43
44         # Store a set of visited node combinations so repeats
45         visited_triplets = []
46
47         # Iterate through the valid edges.
48         for a, b in sorted(nx.edges_iter(network)):
49
50             # Take all unique c nodes that form valid a, b, c
51             # By ensuring that all neighbors have node ID greater
52             # than node triplets from being repeated.
53             c_neighbors = set([neighbor for neighbor in
54                               nx.all_neighbors(a, network)
55                               if neighbor > a and neighbor
56                               != b])
57
58             # Iterate through the valid a, b, c triplets.
59             for c in c_neighbors:
60
61                 # Make sure unique node triplets aren't repeated.
62                 sorted_abc = sorted((a, b, c))
63                 if sorted_abc in visited_triplets:
64                     continue
65                 else:
66                     visited_triplets.append(sorted_abc)
67
68             # a <-> b
69             # a -> c
70             if network.has_edge(b, a):
71
72                 # a <-> b
73                 # a -> c
74                 if network.has_edge(a, c):
75
76                     # a <-> b
77                     # a <-> c
78                     if network.has_edge(c, a):
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# Questions?