

# **Analyse avancée**

## ***General topology***

## Question 1/15

Initial topology on a non-empty set  $X$   
associated to  $(Y_i, \mathcal{T}_i, \varphi_i: X \rightarrow Y_i)$

## Réponse 1/15

The coarsest topology on  $X$  such that all of the  $\varphi_i$  are continuous

## Question 2/15

$p$  is a norm

## Réponse 2/15

$p$  is positive, sub-additive and homogeneous  
and  $p(u) = 0$  iff  $u = 0$

## Question 3/15

$(X, \mathcal{T})$  is a topological vector space

## Réponse 3/15

$X$  is a vector space such that

$X \times X \longrightarrow X$  and  $\mathbb{K} \times X \longrightarrow X$  are

$(x, y) \longmapsto x + y$        $(\lambda, x) \longmapsto \lambda x$

continuous and all singletons are closed

## Question 4/15

Topology induced by a family semi-norms



## Réponse 4/15

$\emptyset \in \mathcal{T}$ ,  $X \in \mathcal{T}$  and  $U \in \mathcal{T}$  if for all  $u \in U$ , there exists  $n_1, \dots, n_k$  and  $r > 0$  such that

$$u \in \bigcap_{i=1}^k B_{p_{n_i}}(u, r) \subseteq U$$

If each  $p_n$  is separating then  $(X, \mathcal{T})$  is metrizable

## Question 5/15

Neighbourhood basis for a topology  $\mathcal{T}$

## Réponse 5/15

$\mathcal{B} \subseteq \mathcal{P}(X)$  such that for all  $U \in \mathcal{T} \setminus \{\emptyset\}$ ,  
there exists  $V \in \mathcal{B}$  such that  $u \in V \subseteq U$

In particular, for each  $u \in X$ , there exists  
 $V \in \mathcal{B}$  such that  $u \in V$  and for all  $V, V' \in \mathcal{B}$ ,  
if  $u \in V \cap V'$ , then there exists  $W \in \mathcal{B}$  such  
that  $u \in W \subseteq V \cap V'$

Conversly, if  $\mathcal{T} \subseteq \mathcal{P}(X)$  and  $\mathcal{B}$  and  $\mathcal{T}$  verify  
the three previous properties,  $\mathcal{T}$  is a topology

## Question 6/15

$p$  is homogeneous

## Réponse 6/15

$$\forall t \in \mathbb{K}, p(tu) = |t|p(u)$$

## Question 7/15

Link between dense sets and neighbourhood  
bases

## Réponse 7/15

If  $X$  has a countable dense set then  $X$  has a countable neighbourhood basis

## Question 8/15

$A$  is a neighbourhood of  $u \in X$



## Réponse 8/15

There exists an open set  $U$  such that

$$u \in U \subseteq A$$

## Question 9/15

Final topology on a non-empty set  $X$   
associated to  $(Z_i, \mathcal{T}_i, \varphi_i: Z_i \rightarrow X)$

## Réponse 9/15

The finest topology on  $X$  such that all of the  $\varphi_i$  are continuous

## Question 10/15

$p$  is a semi-norm

## Réponse 10/15

$p$  is positive, sub-additive and homogeneous

## Question 11/15

Neighbourhood basis of  $u \in X$

## Réponse 11/15

$\mathcal{B}_u \subseteq \mathcal{T}$  such that for all neighbourhood  $A$  of  $u$ , there exists  $U \in \mathcal{B}_u$  such that  $U \subseteq A$

## Question 12/15

Basis for  $X$



## Réponse 12/15

$\bigcup_{x \in X} \mathcal{B}_x$  with  $\mathcal{B}_x$  a neighbourhood basis of  $x$

## Question 13/15

$\mathcal{T}$  is finer than  $\mathcal{S}$   
 $\mathcal{S}$  is coarser than  $\mathcal{T}$

## Réponse 13/15

$$\mathcal{S} \subseteq \mathcal{T}$$

## Question 14/15

$p$  is positively homogeneous

## Réponse 14/15

$$\forall t \in \mathbb{R}_+, p(tu) = tp(u)$$

## Question 15/15

$p$  is sub-additive

## Réponse 15/15

$$p(u + v) \leq p(u) + p(v)$$