Analyse avancée
Hahn-Banach theorems

Question 1/5

Structure of complex-valued linear functionals on a TVS X

Réponse 1/5

A linear functional $\varphi\colon X \to \mathbb{C}$ is of the form $\psi(\cdot) - \mathrm{i}\psi(\mathrm{i}\cdot)$ for $\psi\colon X \to \mathbb{R}$ a linear functional Conversely, any function of this form defines a linear functional $X \to \mathbb{C}$

Question 2/5

Geometric Hahn-Banach theorems

Réponse 2/5

If X is a TVS and $A, B \subseteq X$ are convex disjoint, if A is open, there exists a continuous functional φ on X and $\gamma \in \mathbb{R}$ such that $\mathrm{Re}(\varphi(u)) < \gamma \leqslant \mathrm{Re}(\varphi(v))$ for all $u \in A$, $v \in B$
If X is an LCTVS, A is closed and and B compact then there exists $\gamma_1 < \gamma_2 \in \mathbb{R}$ such that $\mathrm{Re}(\varphi(u)) < \gamma_1 < \gamma_2 < \mathrm{Re}(\varphi(v))$ for all $u \in A$, $v \in B$

Question 3/5

Corollary of Hahn-Banach theorem in $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$ on the existence of specific functionals on a TVS X

Réponse 3/5

If Y is a closed subspace and $v \notin Y$ then there exists a functional $\varphi \in X^*$ such that $\varphi \equiv 0$ on Y, $\varphi(v) = d(v, Y) > 0$ and $\|\varphi\|_{X^*} = 1$

If $u \in X$, there exists $\varphi \in X^*$ such that $\|\varphi\|_{X^*} = 1$ and $\varphi(u) = \|u\|_X$

$J : u \mapsto \mathrm{ev}_u$ is a isometry on its image

Question 4/5

Analytic Hahn-Banach theorems for functionals in $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$

Réponse 4/5

If $p\colon X \to \mathbb{R}$ is a semi-norm, if Y is a subspace of X and $\varphi\colon Y \to \mathbb{K}$ is a functional such that $|\varphi| \leqslant p$ on Y then φ can be extended to a linear functional on X with its modulus still dominated by p

Question 5/5

Analytic Hahn-Banach theorem for functionals in $\mathbb{R}$

Réponse 5/5

If $p\colon X \to \mathbb{R}$ is a positively homogeneous and sub-additive, if Y is a subspace of X and $\varphi\colon Y \to \mathbb{R}$ is a functional such that $\varphi \leqslant p$ on Y then φ can be extended to a linear functional on X still dominated by p