

Analyse avancée

Locally convex spaces

Question 1/24

Properties of \overline{A} and \mathring{A} when A is convex

Réponse 1/24

\overline{A} and \mathring{A} are convex
If $\mathring{A} \neq \emptyset$ then $\mathring{A} = \overline{A}$

Question 2/24

$A \subseteq X$ is balanced

Réponse 2/24

$$\forall |\lambda| \leq 1, \lambda A \subseteq A$$

Question 3/24

Properties of a non-trivial linear functional

$$\varphi : X \rightarrow \mathbb{K}$$

Réponse 3/24

φ is open

Question 4/24

Bounded map

Réponse 4/24

A map is bounded if it maps bounded sets to bounded sets

Question 5/24

Properties of a surjective linear map

$$T:X \rightarrow Y$$

Réponse 5/24

T is open

If moreover $\ker(T)$ is closed the T is continuous

Question 6/24

Bounded set

Réponse 6/24

A is bounded if for all neighbourhood V of 0,
there exists $t_0 > 0$ such that, for all $t \geq t_0$,

$$A \subseteq tV$$

A is bounded if for all neighbourhood V of 0,
there exists $t > 0$ such that $A \subseteq tV$

Question 7/24

Topologies on a finite-dimensional TVS

Réponse 7/24

A finite TVS has a unique topology, associated to the final topology of $\Phi: \mathbb{K}^d \rightarrow X$,

$$(a_i) \rightarrow \sum_{i=1}^k a_i w_i$$

Question 8/24

Properties of $p_A : X \rightarrow \mathbb{R}_+$
 $u \longmapsto \inf(\{t > 0, u \in tA\})$
for $A \subseteq X$ absorbing

Réponse 8/24

p_A is well defined and positively homogeneous,

$p_A(0) = 0$ and p_A is sub-additive

If A is convex, $\{p_A(x) < 1\} \subseteq A \subseteq \{p_A(x) \leq 1\}$

and if A is open then $A = \{p_A(x) < 1\}$

If A is convex and balanced, p_A is a semi-norm

If X is a normed space and A is a

neighbourhood of the origin then there exists

$K \geq 0$ such that $p_A(u) \leq K \|u\|_X$

Question 9/24

Separation of compact and closed sets in a TVS

Réponse 9/24

If X is a TVS, K is a compact subset of X and F a closed subset of X then there exists a neighbourhood V of 0 such that

$$(K + V) \cap (F + V) = \emptyset$$

Question 10/24

Basic properties/example of bounded sets

Réponse 10/24

A subset of a normed space is bounded iff it is contained in $B(0, R)$ for $R > 0$

$\{u_n, n \in \mathbb{N}\}$ for (u_n) convergent is bounded

A finite union/sum of bounded sets is bounded

A compact set is bounded

A subset A of a Fréchet space of semi-norms (p_n) is bounded iff there exists (M_n) such that for all $u \in A$, $p_n(u) \leq M_n$

Question 11/24

Necessary condition for X to be a Fréchet space

Réponse 11/24

If X is endowed with a family of separating semi-norms for which the space is complete
then X is a Fréchet space

Question 12/24

Topological properties of hyperplanes

Réponse 12/24

Hyperplanes are dense or closed
Dense iff the associated linear map is
continuous

Question 13/24

Properties of a linear map $T:X \rightarrow Y$ with X a finite-dimensional TVS

Réponse 13/24

T is continuous

Question 14/24

Link between \overline{A} , \overline{B} and $\overline{A + B}$

Réponse 14/24

$$\overline{A} + \overline{B} \subseteq \overline{A + B}$$

Question 15/24

$A \subseteq X$ is convex

Réponse 15/24

$$tA + (1 - t)A \subseteq A \text{ for all } t \in [0, 1]$$

Question 16/24

$A \subseteq X$ is absorbing

Réponse 16/24

$$\forall u \in X, \exists t > 0, tu \in A$$

Question 17/24

Properties finite-dimensional subspace Y of a
TVS X

Réponse 17/24

Y is closed

Question 18/24

Existence of symmetric neighbourhoods

Réponse 18/24

If W is a neighbourhood of 0 then there exists a neighbourhood V of 0 such as $V = -V$ and $V + V \subseteq W$

Question 19/24

Link between bounded and continuous linear maps

Réponse 19/24

A continuous linear map is bounded

A bounded linear map of a normed vector space is bounded

In a space with a compatible metric invariant by translation, a bounded linear map is continuous iff it is sequentially continuous at 0

Question 20/24

Properties of $K + F$ with $K \subseteq X$ compact
and $F \subseteq X$ closed

Réponse 20/24

$K + F$ is closed

Question 21/24

Properties of $A + U$ with $A \subseteq X$ and $U \subseteq X$
open

Réponse 21/24

$A + U$ is open

Question 22/24

Hyperplane

Réponse 22/24

Kernel of a non-trivial linear map
It has dimension 1 (the converse holds)

Question 23/24

Linear functional

Réponse 23/24

Linear map $X \rightarrow \mathbb{K}$ with X a \mathbb{K} -vector space

Question 24/24

Fréchet space

Réponse 24/24

Topological vector space with a compatible metric, that is invariant by translation and with which the space is complete