

Analyse anancée

Hahn-Banach theorems

Question 1/5

Analytic Hahn-Banach theorems for
functionals in $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$

Réponse 1/5

If $p:X \rightarrow \mathbb{R}$ is a semi-norm, if Y is a subspace of X and $\varphi:Y \rightarrow \mathbb{K}$ is a functional such that $|\varphi| \leq p$ on Y then φ can be extended to a linear functional on X with its modulus still dominated by p

Question 2/5

Geometric Hahn-Banach theorems

Réponse 2/5

If X is a TVS and $A, B \subseteq X$ are convex disjoint, if A is open, there exists a continuous functional φ on X and $\gamma \in \mathbb{R}$ such that

$$\text{Re}(\varphi(u)) < \gamma \leq \text{Re}(\varphi(v)) \text{ for all } u \in A, v \in B$$

If X is an LCTVS, A is closed and B compact then there exists $\gamma_1 < \gamma_2 \in \mathbb{R}$ such that $\text{Re}(\varphi(u)) < \gamma_1 < \gamma_2 < \text{Re}(\varphi(v))$ for all $u \in A, v \in B$

Question 3/5

Structure of complex-valued linear functionals
on a TVS X

Réponse 3/5

A linear functional $\varphi:X \rightarrow \mathbb{C}$ is of the form $\psi(\cdot) - i\psi(i\cdot)$ for $\psi:X \rightarrow \mathbb{R}$ a linear functional
Conversely, any function of this form defines a
linear functional $X \rightarrow \mathbb{C}$

Question 4/5

Corollary of Hahn-Banach theorem in $\mathbb{K} = \mathbb{R}$
or $\mathbb{K} = \mathbb{C}$ on the existence of specific
functionals on a TVS X

Réponse 4/5

If Y is a closed subspace and $v \notin Y$ then there exists a functional $\varphi \in X^*$ such that $\varphi \equiv 0$ on Y , $\varphi(v) = d(v, Y) > 0$ and $\|\varphi\|_{X^*} = 1$

If $u \in X$, there exists $\varphi \in X^*$ such that

$$\|\varphi\|_{X^*} = 1 \text{ and } \varphi(u) = \|u\|_X$$

$J: u \mapsto \text{ev}_u$ is a isometry on its image

Question 5/5

Analytic Hahn-Banach theorem for functionals
in \mathbb{R}

Réponse 5/5

If $p:X \rightarrow \mathbb{R}$ is a positively homogeneous and sub-additive, if Y is a subspace of X and $\varphi:Y \rightarrow \mathbb{R}$ is a functional such that $\varphi \leq p$ on Y then φ can be extended to a linear functional on X still dominated by p