Géométrie avancée
Abstract differential varieties

Question 1/29

X a differnetial variety, G a discrete group
$G \curvearrowright X$ smoothly

Réponse 1/29

$$\forall g, x \mapsto g \cdot x \text{ is a diffeomorphism}$$

Question 2/29

Differential atlas of a topological variety X
Maximal differential atlas

Réponse 2/29

An atlas such that any two maps are always compatible
It is said to be maximal if any chart that is compatible with all charts in the atlas is in the atlas

Question 3/29

$M \subseteq X$ is a sub-variety with X a differential varieties of dimension p

Réponse 3/29

For all $x \in M$, there exists a chart $(U \ni x, \varphi)$ such that $\varphi(U \cap M)$ is a sub-variety of $\mathbb{R}^p$

Question 4/29

Covering space of a topological variety

Réponse 4/29

Data of a topological fibration, with fibers endowed with the discrete topology

Data of $p : E \to B$ wuch that for all $x \in B$, there exists a neighbourhood U of x such that

$$p^{-1}(U) \simeq \coprod_{\alpha \in A} V_\alpha$$

The same definition can be given in the *category of differential varieties*

Question 5/29

Isomorphism between two differential fiber bundles $f_1 : E_1 \to B$ and $f_2 : E_2 \to B_1$

Réponse 5/29

The data of a homeomorphism φ such that the following diagram commutes

$$\begin{array}{ccc} E_1 & \xrightarrow[\varphi]{} & E_2 \\ \downarrow{\scriptstyle f_1} & & {\scriptstyle f_2}\downarrow \\ B & \xrightarrow{\mathrm{id}} & B \end{array}$$

Question 6/29

Topological variety of dimension n

Réponse 6/29

Hausdorff topological space such that every points admits an open neighbourhood that is homeomorphic to an open subset of $\mathbb{R}^n$

Question 7/29

Chart of a topological variety X of dimension n

Réponse 7/29

$$(U, \varphi) \text{ where } U \subseteq X \text{ is open and}$$
$$\varphi : U \xrightarrow{\sim} V \subseteq \mathbb{R}^n$$

Question 8/29

X is paracompact

Réponse 8/29

Every covering of X admits a sub-covering which is locally finite

Question 9/29

Partition of the unity on a differential variety M

Réponse 9/29

Family of smooth functions $\varphi_i : M \to \mathbb{R}_+$ such that $\{\mathrm{supp}(\varphi_i)\}$ is locally finite and $\sum \varphi_i \equiv 1$

Such a family always exists

Question 10/29

Properties of E/G for E a locally compact topological variety, G discrete and $G \curvearrowright E$ continuous

Réponse 10/29

E/G is locally compact and $\pi : E \to E/G$ is open

Question 11/29

Lie group

Réponse 11/29

Group that is a differential variety and in which the product and inverse are smooth

Question 12/29

$f : M \to N$ continuous is smooth with M and N two differential varieties

Réponse 12/29

For all $a \in M$, there exists a chart $(U \ni a, \varphi)$ of M and a chart $(V \ni f(a), \psi)$ such that $\psi \circ f \circ \varphi^{-1} : \varphi\big(U \cap f^{-1}(V)\big) \to \psi(V)$ is smooth

Question 13/29

$f : M \to N$ smooth is a diffeomorphism

Réponse 13/29

f is bijective and f^{-1} is smooth

Question 14/29

Differential fiber bundle of base B and total space E

Réponse 14/29

Smooth fibration f of base B and total space E

$f^{-1}(b)$ is called the fiber over b

Question 15/29

Discrete group

Réponse 15/29

Group endowed with the discrete topology

Question 16/29

The partition of unity $(\varphi_i)_{i\in I}$ is subordinate to the cover $(U_\alpha)_{\alpha\in A}$

The partition of unity $(\varphi_i)_{i\in I}$ is subordinate with same index to the cover $(U_\alpha)_{\alpha\in A}$

Réponse 16/29

For all $i \in I$, there exists $\alpha \in A$ such that $\operatorname{supp}(\varphi) \subseteq U_\alpha$

It is subordinate with same index if $I = A$ and $\operatorname{supp}(\varphi_i) \subseteq U_i$

Question 17/29

$$G \curvearrowright X \text{ is free}$$

Réponse 17/29

$$\text{For all } g \in G \setminus \{1\},\ \{x \in X, g \cdot x = x\} = \varnothing$$

Question 18/29

X is a second countable topological space

Réponse 18/29

There exists a compact countable covering of X

Question 19/29

G acts on X proprtly and continuously

Réponse 19/29

X is locally compact, G is discrete, $G \curvearrowright X$ is continuous and for all K and L compacts in X, $\{g \in G, g \cdot K \cap L \neq \varnothing\}$ is finite

Equivalently, $\{g \in G, g \cdot K \cap K \neq \varnothing\}$ is finite

Question 20/29

Properties of the fibers when the base is connected

Réponse 20/29

The fibers of a differential fiber bundle are diffeomorphic

Question 21/29

Trivializable fiber bundle of base B and fiber F

Réponse 21/29

Fiber bundle isomorphic to $\mathrm{pr}_1 : F \times B \to B$

Question 22/29

Coordinates centered at x

Réponse 22/29

$$\varphi(x) = 0$$

Question 23/29

Sub-covering of $(U_i)_{i \in I}$

Réponse 23/29

$(V_j)_{j\in J}$ such that, for all $j \in J$, there exists $i \in I$ such that $V_j \subseteq U_i$

Question 24/29

$(A_\alpha)_\alpha$ is locally finite

Réponse 24/29

$$\forall x \in X, \exists U \ni x, |\{\alpha, A_\alpha \cap U \neq \varnothing\}| < +\infty$$

Question 25/29

(U_1, φ_1) and (U_2, φ_2) are compatible charts

Réponse 25/29

$$U_1 \cap U_2 = \varnothing \text{ or}$$
$$\varphi_2 \circ \varphi_1^{-1} : \varphi_1(U_1 \cap U_2) \to \varphi_2(U_1 \cap U_2) \text{ is a diffeomorphism}$$

Question 26/29

Smooth fibration of base B and total space E

Réponse 26/29

Map $f : E \to B$ such that, for all $x \in B$, there exists an open neighbourhood U of x, a differential variety F and a homeomorphism φ such that the following diagram commutes

$$\begin{array}{ccc} U \times F & \xrightarrow[\varphi]{} & f^{-1}(U) \\ \downarrow{\scriptstyle \mathrm{pr}_1} & & {\scriptstyle f}\downarrow \\ U & \xrightarrow{\mathrm{id}} & U \end{array}$$

Question 27/29

Atlas of a topological variety X

Réponse 27/29

Family of charts $(U_i, \varphi_i)_{i \in I}$ such that

$$X = \bigcup_{i \in I} U_i$$

Question 28/29

Properties of X/G for X a differential variety, G discrete and $G \curvearrowright X$ is smooth, proper and free

Réponse 28/29

X/G has a unique differential structure for which $\pi : E \to E/G$ is smooth

π is a covering of X/G

Question 29/29

Differential n-variety

Réponse 29/29

Topological variety endowed with a maximal differential atlas