Analyse avancée
General topology

Question 1/15

p is a semi-norm

Réponse 1/15

p is positive, sub-additive and homogeneous

Question 2/15

p is sub-additive

Réponse 2/15

$$p(u+v) \leqslant p(u) + p(v)$$

Question 3/15

Link between dense sets and neighbourhood bases

Réponse 3/15

If X has a countable dense set then X has a countable neighbourhood basis

Question 4/15

p is a norm

Réponse 4/15

p is positive, sub-additive and homogeneous and $p(u) = 0$ iff $u = 0$

Question 5/15

Neighbourhood basis for a topology $\mathcal{T}$

Réponse 5/15

$\mathcal{B} \subseteq \mathcal{P}(X)$ such that for all $U \in \mathcal{T} \setminus \{\varnothing\}$, there exists $V \in \mathcal{B}$ such that $u \in V \subseteq U$

In particular, for each $u \in X$, there exists $V \in \mathcal{B}$ such that $u \in V$ and for all $V, V' \in \mathcal{B}$, if $u \in V \cap V'$, then there exists $W \in \mathcal{B}$ such that $u \in W \subseteq V \cap V'$

Conversly, if $\mathcal{T} \subseteq \mathcal{P}(X)$ and $\mathcal{B}$ and $\mathcal{T}$ verify the three previous properties, $\mathcal{T}$ is a topology

Question 6/15

Basis for X

Réponse 6/15

$$\bigcup_{x \in X} \mathcal{B}_x \text{ with } \mathcal{B}_x \text{ a neighbourhood basis of } x$$

Question 7/15

$(X, \mathcal{T})$ is a topological vector space

Réponse 7/15

X is a vector space such that

$$\begin{array}{ccc} X \times X \longrightarrow X & \text{and } \mathbb{K} \times X \longrightarrow X & \text{are} \\ (x, y) \longmapsto x + y & (\lambda, x) \longmapsto \lambda x & \end{array}$$

continuous and all singletons are closed

Question 8/15

A is a neighbourhood of $u \in X$

Réponse 8/15

There exists an open set U such that

$$u \in U \subseteq A$$

Question 9/15

Final topology on a non-empty set X associated to $(Z_i, \mathcal{T}_i, \varphi_i\colon Z_i \to X)$

Réponse 9/15

The finest topology on X such that all of the φ_i are continuous

Question 10/15

$$\mathcal{T} \text{ is finer than } \mathcal{S}$$
$$\mathcal{S} \text{ is coarser than } \mathcal{T}$$

Réponse 10/15

$$\mathcal{S} \subseteq \mathcal{T}$$

Question 11/15

Topology induced by a family semi-norms

Réponse 11/15

$\varnothing \in \mathcal{T}$, $X \in \mathcal{T}$ and $U \in \mathcal{T}$ if for all $u \in U$, there exists $n_1, \cdots, n_k$ and $r > 0$ such that

$$u \in \bigcap_{i=1}^{k} B_{p_{n_i}}(u, r) \subseteq U$$

If each p_n is separating then $(X, \mathcal{T})$ is metrizable

Question 12/15

p is homogeneous

Réponse 12/15

$$\forall t \in \mathbb{K},\ p(tu) = |t|p(u)$$

Question 13/15

Neighbourhood basis of $u \in X$

Réponse 13/15

$\mathcal{B}_u \subseteq \mathcal{T}$ such that for all neighbourhood A of u, there exists $U \in \mathcal{B}_u$ such that $U \subseteq A$

Question 14/15

Initial topology on a non-empty set X associated to $(Y_i, \mathcal{T}_i, \varphi_i : X \to Y_i)$

Réponse 14/15

The coarsest topology on X such that all of the φ_i are continuous

Question 15/15

p is positively homogeneous

Réponse 15/15

$$\forall t \in \mathbb{R}_+, \, p(tu) = tp(u)$$