Analyse anancée
Locally convex spaces

Question 1/10

$$A \subseteq X \text{ is convex}$$

Réponse 1/10

$$tA + (1 - t)A \subseteq A \text{ for all } t \in [0, 1]$$

Question 2/10

Properties of $\overline{A}$ and $\mathring{A}$ when A is convex

Réponse 2/10

$\overline{A}$ and $\mathring{A}$ are convex

If $\mathring{A} \neq \varnothing$ then $\mathring{\overline{A}} = \overline{A}$

Question 3/10

Properties of $A + U$ with $A \subseteq X$ and $U \subseteq X$ open

Réponse 3/10

$$A + U \text{ is open}$$

Question 4/10

Existence of symmetric neighbourhoods

Réponse 4/10

If W is a neighbourhood of 0 then there exists a neighbourhood V of 0 such as $V = -V$ and $V + V \subseteq W$

Question 5/10

Link between $\overline{A}$, $\overline{B}$ and $\overline{A+B}$

Réponse 5/10

$$\overline{A} + \overline{B} \subseteq \overline{A + B}$$

Question 6/10

Properties of $K + F$ with $K \subseteq X$ compact and $F \subseteq X$ closed

Réponse 6/10

$K + F$ is closed

Question 7/10

$$\text{Properties of } p_A : X \longrightarrow \mathbb{R}_+$$
$$u \longmapsto \inf(\{t > 0, u \in tA\})$$
$$\text{for } A \subseteq X \text{ absorbing}$$

Réponse 7/10

p_A is well defined and positively homogeneous,
$p_A(0) = 0$ and p_A is sub-additive
If A is convex, $\{p_A(x){<}1\} \subseteq A \subseteq \{p_A(x){\leqslant}1\}$
and if A is open then $A = \{p_A(x) < 1\}$
If A is convex and balanced, p_A is a semi-norm
If X is a normed space and A is a neighbourhood of the origin then there exists $K \geqslant 0$ such that $p_A(u) \leqslant K\|u\|_X$

Question 8/10

Separation of compac and closed sets in a TVS

Réponse 8/10

If X is a TVS, K is a compact subset of X and F a closed subset of X then there exists a neighbourhood V of 0 such that
$$(K+V)\cap(F+V)=\varnothing$$

Question 9/10

$A \subseteq X$ is balanced

Réponse 9/10

$$\forall |\lambda| \leqslant 1, \lambda A \subseteq A$$

Question 10/10

$A \subseteq X$ is absorbing

Réponse 10/10

$$\forall u \in X, \exists t > 0, tu \in A$$