Géométrie avancée
Abstract differential varieties

Question 1/17

Partition of the unity on a differential variety M

Réponse 1/17

Family of smooth functions $\varphi_i : M \to \mathbb{R}_+$ such that $\mathrm{supp}(\varphi_i)$ is locally finite and $\sum(\varphi_i) \equiv 1$

Such a family always exists

Question 2/17

Differential atlas of a topological variety X
Maximal differential atlas

Réponse 2/17

An atlas such that any two maps are always compatible
It is said to be maximal if any chart that is compatible with all charts in the atlas is in the atlas

Question 3/17

$(A_\alpha)_\alpha$ is locally finite

Réponse 3/17

$$\forall x \in X, \exists U \ni x, |\{\alpha, A_\alpha \cap U \neq \varnothing\}| < +\infty$$

Question 4/17

Chart of a topological variety X of dimension n

Réponse 4/17

$$(U, \varphi) \text{ where } U \subseteq X \text{ is open and } \varphi : U \xrightarrow{\sim} V \subseteq \mathbb{R}^n$$

Question 5/17

Sub-covering of $(U_i)_{i \in I}$

Réponse 5/17

$(V_j)_{j\in J}$ such that, for all $j \in J$, there exists $i \in I$ such that $V_j \subseteq U_i$

Question 6/17

Atlas of a topological variety X

Réponse 6/17

Family of charts $(U_i, \varphi_i)_{i \in I}$ such that

$$X = \bigcup_{i \in I} (U_i)$$

Question 7/17

(U_1, φ_1) and (U_2, φ_2) are compatible charts

Réponse 7/17

$$U_1 \cap U_2 = \varnothing \text{ or}$$
$$\varphi_2 \circ \varphi_1^{-1} : \varphi_1(U_1 \cap U_2) \to \varphi_2(U_1 \cap U_2) \text{ is a diffeomorphism}$$

Question 8/17

X is paracompact

Réponse 8/17

Every covering of X admits a sub-covering which is locally finite

Question 9/17

$M \subseteq X$ is a sub-variety with X a differential varieties of dimension p

Réponse 9/17

For all $x \in M$, there exists a chart $(U \ni x, \varphi)$ such that $\varphi(U \cap M)$ is a sub-variety of $\mathbb{R}^p$

Question 10/17

Topological variety of dimension n

Réponse 10/17

Hausdorff topological space such that every points admits an open neighbourhood that is homeomorphic to an open subset of $\mathbb{R}^n$

Question 11/17

Lie group

Réponse 11/17

Group that is a differential variety and in which the product and inverse are smooth

Question 12/17

X is a second countable topological space

Réponse 12/17

There exists a compact countable covering of X

Question 13/17

Coordinates centered at x

Réponse 13/17

$$\varphi(x) = 0$$

Question 14/17

$f : M \to N$ continuous is smooth with M and N two differential varieties

Réponse 14/17

For all $a \in M$, there exists a chart $(U \ni a, \varphi)$ of M and a chart $(V \ni f(a), \psi)$ such that $\psi \circ f \circ \varphi^{-1} : \varphi\big(U \cap f^{-1}(V)\big) \to \psi(V)$ is smooth

Question 15/17

Differential n-variety

Réponse 15/17

Topological variety endowed with a maximal differential atlas

Question 16/17

The partition of unity $(\varphi_i)_{i\in I}$ is subordinate to the cover $(U_\alpha)_{\alpha\in A}$

The partition of unity $(\varphi_i)_{i\in I}$ is subordinate with same index to the cover $(U_\alpha)_{\alpha\in A}$

Réponse 16/17

For all $i \in I$, there exists $\alpha \in A$ such that $\operatorname{supp}(\varphi) \subseteq U_\alpha$

It is subordinate with same index if $I = A$ and $\operatorname{supp}(\varphi_i) \subseteq U_i$

Question 17/17

$f : M \to N$ smooth is a diffeomorphism

Réponse 17/17

f is bijective and f^{-1} is smooth