Given: $K = 1.99$ Type 1

Pole: $-0.45 \pm 0.43j$

$\zeta : 0.723$

$OS\% : 3.75$

Step 1

OLTF: $G_c G_p H = \dfrac{20}{s^4 + 21s^3 + 121s^2 + 100s}$

$t_s = \dfrac{4}{\sigma} = \dfrac{4}{.45} = 8.89s$
$_{uncomp}$

$t_{P\,uncomp} = \dfrac{\pi}{w_d} = \dfrac{\pi}{0.43} = 7.3$

$SSE = 0$ Step, $u(t)$ $K_p = \infty$

Step 2

$t_{pcomp} = .75 (t_{p\,uncomp}) = .75(7.3) = 5.5$

$W_d = \dfrac{\pi}{t_{pcomp}} = \dfrac{\pi}{5.5} = 0.571$

$\alpha = \cos^{-1}(\zeta)$

$\tan \alpha = \dfrac{w_d}{\sigma}$

$\sigma = \dfrac{w_d}{\tan \alpha} = \dfrac{w_d}{\tan(\cos^{-1}\zeta)} = \dfrac{.571}{\tan(\cos^{-1}(.723))} = 0.597$

$s = -0.597 \mp 0.571j$

$S = -10 + 1.0 \, j$

$S = -10 - 1.0 \, j$
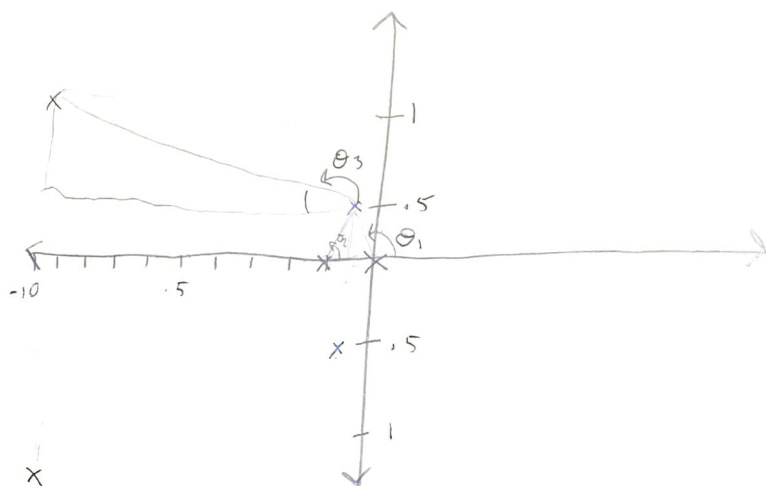
$S = -0.988 \approx -1.$

$S = 0$

$180 - \tan^{-1}\left(\frac{1 - .571}{10 - .597}\right) = 177°$

$\tan^{-1}\left(\frac{0.571}{1 - .597}\right) = 55°$

$180 - \tan^{-1}\left(\frac{0.571}{0.597}\right) = 136°$

$\theta_1 + \theta_2 + \theta_3 + \theta_4 - \theta_c = 180°$

$\theta_1 + \theta_2 + \theta_3 - 180 = \theta_c$

$177 + 55 + 136 - 180 = 188° = \theta_c$

$\tan \theta_c = \frac{wd}{z_c - \sigma}$

$G_{pp} = K(s + 4.66)$

$z_c = \frac{wd}{\tan \theta_c}, \quad \sigma = \frac{157}{\tan(188)} + .597 = -4.66$

Step 3

$$G_{pp} \quad \frac{20(s+4.66)}{s^4 + 21s^3 + 121s^2 + 100s} = \frac{20s + 93.2}{s^4 + 21s^3 + 121s^2 + 100s}$$

plot on root locus → find zeta
@ 0.725

Gain: 0.516

Pole: $-0.501 + 0.477i$

OS: 3.67
freq: 0.692

SSE: Type 1

$$t_s = \frac{4}{.501} = 7.984$$

$$t_p = \frac{\pi}{w_d} = \frac{\pi}{.477} = 6.59$$