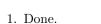# Answer to the referee for: *Duality for the multispecies stirring process with open boundaries*

May 15, 2024

## Answer to general comments

1. We expanded the sentence in the introduction to explain our choice of the boundary generators. One could in principle consider other choices of boundaries but then this would imply that dual particles could possibly change their color before getting absorbed at the boundaries. In view of the application of duality to the computation of the non-equilibrium steady state we are interested in we limit ourselves to the choice that avoid this color-changing.

2. When the rates are zero, then the generator of the process in (2.2.6) [DOUBLE CHECK] is zero and therefore we get indeed for the closed system the expected duality in product form.

3. A citation has been added on page [ADD PAGE NUMBER]

4. As the computations in section 4 are rather lengthy and technical, we believe it is useful to keep a small introductory subsection where we explain the main ideas and show that the matrix product ansatz simplies after the similarity transformations. We considered the option of moving section 4.2 at the end of 4.1, but we believe it is useful to explain the strategy of proof right at the beginning.

## Minor issues

1. Done.

2. Done.

3. Done.

4. Done.

5. Done.

6. Yes, $N$ is the number of species.

7. We deleted line 52.

8. Done.

9. Done.

10. The reason is that when $\nu = 1$ the we have the fundamental (defining) representation of the $gl(N)$ Lie algebra. Therefore, the matrices $E_{AB}$ coincide with the elementary matrices $e_{AB}$ such that $(e_{AB})_{CD} = \delta_{AC}\delta_{BD}$ for all $A, B, C, D \in \{1, \ldots, N\}$. A footnote has been added.

11. Done.

12. Done.

13. Done.

14. We agree with this suggestion. $E$ and $\mathcal{D}_{u(x)}$ have been added in the suggested equations.

15. Done.

16. Equations (3.3.30), (3.3.31) and (3.3.32) have been added at page 15. Moreover, they have been used to add an intermediate step in equation (3.3.35) at page 15.

17. Done.

18. We made a more precise statement.

19. Done.

20. We replaced everywhere the symbol $t_i$ with $s_i$.

21. Done.

22. NOT CLEAR

23. GOOD NOW

24. Done.

25. Done.

26. Fixed, the index is now $A$.

27. NOT CLEAR

28. Done.

29. Done.