Answer to general comments

All the page and equation cited in the answers do refer to the new file.

1. *P3, line 25: Could you provide more detail on the "choice of the boundary reservoirs"? Is this selection the most general case that allows for the determination of an absorbing dual process?*

   We expanded the sentence in the introduction to explain our choice of the boundary generators. One could in principle consider other choices but then this would imply that dual particles could possibly change their color before getting absorbed at the boundaries. In view of the application of duality to the computation of the non-equilibrium steady state we are interested in, we limit ourselves to the choice that avoids this color-changing.

2. *P4, line 33: The rates α_A are strictly positive, what happens when they are zero? Does the corresponding duality degenerates to nontrivial duality for closed system?*

   When the rates α_A are zero, then the generator of the process in (2.2.5) is zero and therefore we get indeed for the closed system the expected duality in product form.

3. *P14, line 20: In my understanding, the Hadamard formula is a special case of the Baker–Campbell–Hausdorff formula, which was first utilized for duality in the paper arXiv:2209.03531. The authors should consider citing that paper as well.*

   A citation has been added on page 14.

4. *P20, Section 4.2: "In fact, such simplification can always be achieved if there exists an absorbing dual process as established in Section 3": It looks like this method is general, could you be more precise about the conditions for simplification and maybe analogues for (4.2.1)? Additionally, I find equations (4.2.2) to (4.2.3) distracting in this context. It's better to keep the introduction as simple as possible. In fact, I find this section redundant with the steps outlined on page 22. Please consider rewriting these two parts together.*

   As the computations in section 4 are rather lengthy and technical, we believe it is useful to keep a small introductory subsection where we explain the main ideas and show that the matrix product ansatz simplies after the similarity transformations. We considered the option of moving section 4.2 at the end of 4.1, but we believe it is useful to explain the strategy of the proof right at the beginning of the section.

Minor issues

1. Done.

2. Done.

3. Done.

4. Done.

5. Done.

6. We fixed it. N is the number of species.

7. We deleted line 52.

8. Done.

9. Done.

10. The reason is that when $\nu = 1$ the we have the fundamental (defining) representation of the $gl(N)$ Lie algebra. We added a footnote at page 8.

11. Done.

12. Done.

13. Done.

14. E and $\mathcal{D}_{u(x)}$ have been added in the suggested equations.

15. Done.

16. Equations (3.3.30), (3.3.31) and (3.3.32) have been added at page 15. Moreover, they have been used to add an intermediate step in equation (3.3.35) at page 15.

17. Done.

18. We made a more precise statement at page 16.

19. Done.

20. We replaced everywhere the symbol t_i with s_i. Notice also that, to avoid confusion, the s_a^x in equation (3.3.18) have been replaced by r_a^x.

21. Done.

22. Details about $\langle\langle W|$ have been added at page 21.

23. Details and equations (4.4.17), (4.4.18) have been added at page 25.

24. Done.

25. Done.

26. Fixed, the index is now A.

27. Done, probably the sentence was confusing by the fact that we used (4.4.32) in the second equality and (4.4.37) in the third and not just (4.4.37) in the second equality (as it was mistakenly written). We clarified which equations are used.

28. Done.

29. Done.

30. $c_j = \delta_{\sigma_j, N}$ has been added at page 29.

31. Details have been added in Remark 9 at page 30. See the bracket in the first sentence and the reference to equation (4.4.48) just below equation (4.4.63).

32. Words removed in the sentence just above Remark 12 at page 32.

33. Words removed in the sentence just above the paragraph with title "Correlations of SSEP" at page 33.

34. In the third line of the paragraph with title "Proof of formula (4.5.14)" at page 34, in the second curly bracket there was $|\tau|$ instead of $|\boldsymbol{\tau}|$. We fixed it.

35. Words removed in the sixth line after the title of Section 5 at page 35.

36. Done.

37. Done.

38. More details added in the paragraph just above equation (5.2.2) at page 37. Bold removed in equation (5.2.7) at page 38.

39. Brackets added in equation (5.2.11), page 38.

40. Subscripts fixed in equation (5.3.20) page 41.

41. We put a consistent notation for the absorption probabilities in Appendix C, pages 44-45.

42. We put a consistent notation for the boundary densities in Appendix B, pages 43-44.

43. Done.

44. For the sake of clarity we prefer to keep this part like it was.

Further corrections

1. **Equation (5.2.10)**: the boundary dual generator of the thermalized process was wrong. Compare with the old (ArXiv) equation (5.2.10) with the new equation (5.2.10).

2. **Equations (5.1.5), (5.1.6)**: the absorption probabilities of the dual particles should be written for $s_1, \ldots, s_{N-1}$ and not until s_N as they were before. Indeed, s_a denotes the number of particles of species a that we absorbed at 0 and we can only absorb particles, not the holes.

3. **Section 5.1**: we can talk about m-point correlations only if we choose $1 \leq x_1 < \ldots < x_m \leq L$ otherwise we obtain the mixed moments in the non-equilibrium steady state. I have fixed equation (5.1.3) and above and I have added Remark 13 to clarify.