

# Econometria espacial com R - Aula 03

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## Modelos espaciais

### Pacote

```
library(spdep)
```

```
## Loading required package: sp
```

```
## Loading required package: Matrix
```

### Shapefile

```
# Pacotes
```

```
library(rgdal)
```

```
## rgdal: version: 1.2-8, (SVN revision 663)
```

```
## Geospatial Data Abstraction Library extensions to R successfully loaded
```

```
## Loaded GDAL runtime: GDAL 1.11.3, released 2015/09/16
```

```
## Path to GDAL shared files: /usr/share/gdal/1.11
```

```
## Loaded PROJ.4 runtime: Rel. 4.9.2, 08 September 2015, [PJ_VERSION: 492]
```

```
## Path to PROJ.4 shared files: (autodetected)
```

```
## Linking to sp version: 1.2-5
```

```
# Abra o arquivo 'gm10.shp'
```

```
fp_mg.shp <- readOGR("data", "FP_MG", encoding = "ISO-8859-1")
```

```
## OGR data source with driver: ESRI Shapefile
```

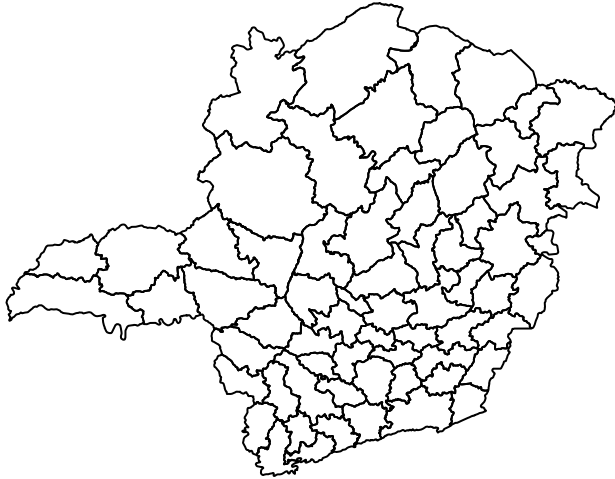
```
## Source: "data", layer: "FP_MG"
```

```
## with 66 features
```

```
## It has 41 fields
```

```
# Plotar o mapa
```

```
plot(fp_mg.shp)
```



## Matriz de vizinhança

```
w1 <- nb2listw(poly2nb(fp_mg.shp, queen = TRUE))
```

## Especificação do modelo

```
esp <- Q ~ AC + LP
```

## OLS

$$y = \alpha + X\beta + \varepsilon$$

```
mod1 <- lm(formula = esp, data = fp_mg.shp@data)
summary(mod1)
```

```
##
## Call:
## lm(formula = esp, data = fp_mg.shp@data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -790620 -125347  -58729   20382 1226351
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.022e+04  8.251e+04   0.851   0.3980
## AC           6.453e+00  8.943e-01   7.216 8.4e-10 ***
## LP          -2.022e+06  9.561e+05  -2.115  0.0384 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 338500 on 63 degrees of freedom
## Multiple R-squared:  0.4562, Adjusted R-squared:  0.4389
## F-statistic: 26.42 on 2 and 63 DF, p-value: 4.648e-09
```

## Multiplicador de Lagrange

```
mod1.lagrange <- lm.LMtests(model = mod1, listw = w1,
                           test = c("LMerr", "RLMerr", "LMlag", "RLMlag",
                                    "SARMA"))
mod1.lagrange

##
##  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = esp, data = fp_mg.shp@data)
## weights: w1
##
## LMerr = 1.7258, df = 1, p-value = 0.1889
##
##
##  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = esp, data = fp_mg.shp@data)
## weights: w1
##
## RLMerr = 1.3668, df = 1, p-value = 0.2424
##
##
##  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = esp, data = fp_mg.shp@data)
## weights: w1
##
## LMlag = 6.638, df = 1, p-value = 0.009983
##
##
##  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = esp, data = fp_mg.shp@data)
## weights: w1
##
## RLMlag = 6.279, df = 1, p-value = 0.01222
##
##
##  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = esp, data = fp_mg.shp@data)
## weights: w1
##
## SARMA = 8.0048, df = 2, p-value = 0.01827
```

## SAR

$$y = \rho W y + X\beta + \varepsilon$$

```
mod1.sar <- lagsarlm(formula = esp, data = fp_mg.shp@data, listw = w1)
```

```
## Warning in lagsarlm(formula = esp, data = fp_mg.shp@data, listw = w1): inversion of asymptotic covar  
## condição recíproca número = 1.94547e-23 - using numerical Hessian.
```

```
summary(mod1.sar)
```

```
##  
## Call:lagsarlm(formula = esp, data = fp_mg.shp@data, listw = w1)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -679044 -122530  -52916   39396 1067523   
##  
## Type: lag  
## Coefficients: (numerical Hessian approximate standard errors)  
##              Estimate Std. Error z value Pr(>|z|)      
## (Intercept)  1.4887e+04  7.9158e+04  0.1881   0.8508      
## AC           5.0128e+00  1.0133e+00  4.9469  7.542e-07      
## LP          -1.4283e+06  9.1359e+05 -1.5634   0.1180      
##  
## Rho: 0.31068, LR test value: 5.624, p-value: 0.017716  
## Approximate (numerical Hessian) standard error: 0.12471  
##      z-value: 2.4913, p-value: 0.012728  
## Wald statistic: 6.2065, p-value: 0.012728  
##  
## Log likelihood: -929.6401 for lag model  
## ML residual variance (sigma squared): 9.8356e+10, (sigma: 313620)  
## Number of observations: 66  
## Number of parameters estimated: 5  
## AIC: 1869.3, (AIC for lm: 1872.9)
```

```
impacts(mod1.sar, listw = w1)
```

```
## Impact measures (lag, exact):  
##           Direct      Indirect      Total  
## AC  5.125193e+00  2.146888e+00  7.272082e+00  
## LP -1.460320e+06 -6.117123e+05 -2.072032e+06
```

## SEM

$$y = X\beta + u$$

$$u = \lambda W u + \varepsilon$$

```
mod1.sem <- errorsarlm(formula = esp, data = fp_mg.shp@data, listw = w1)
```

```
## Warning in errorsarlm(formula = esp, data = fp_mg.shp@data, listw = w1): inversion of asymptotic covar  
## condição recíproca número = 8.07554e-23 - using numerical Hessian.
```

```
summary(mod1.sem)
```

```
##  
## Call:errorsarlm(formula = esp, data = fp_mg.shp@data, listw = w1)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -648297 -111924  -67640   13670 1170268
##
## Type: error
## Coefficients: (asymptotic standard errors)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  9.2648e+04 9.3093e+04  0.9952  0.3196
## AC          5.7217e+00 9.9629e-01  5.7430 9.302e-09
## LP          -1.6699e+06 1.0221e+06 -1.6337  0.1023
##
## Lambda: 0.28501, LR test value: 2.0548, p-value: 0.15172
## Approximate (numerical Hessian) standard error: 0.18804
##      z-value: 1.5157, p-value: 0.12959
## Wald statistic: 2.2974, p-value: 0.12959
##
## Log likelihood: -931.4247 for error model
## ML residual variance (sigma squared): 1.0419e+11, (sigma: 322780)
## Number of observations: 66
## Number of parameters estimated: 5
## AIC: 1872.8, (AIC for lm: 1872.9)
mod1.semGMM <- GMerrorsar(formula = esp, data = fp_mg.shp@data, listw = w1)
summary(mod1.semGMM)
```

```
##
## Call:GMerrorsar(formula = esp, data = fp_mg.shp@data, listw = w1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -710221 -135191  -66280   14109 1284646
##
## Type: GM SAR estimator
## Coefficients: (GM standard errors)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  8.8104e+04 9.0373e+04  0.9749  0.3296
## AC          5.8764e+00 9.7442e-01  6.0307 1.633e-09
## LP          -1.7468e+06 1.0073e+06 -1.7341  0.0829
##
## Lambda: 0.23082 (standard error): 0.4651 (z-value): 0.49628
## Residual variance (sigma squared): 1.0557e+11, (sigma: 324920)
## GM argmin sigma squared: 1.0544e+11
## Number of observations: 66
## Number of parameters estimated: 5
```

## SAC

$$y = \rho W y + X\beta + u$$

$$u = \lambda W u + \varepsilon$$

```
mod1.sac <- sacsarlml(formula = esp, data = fp_mg.shp@data, listw = w1)
```

```
## Warning in sacsarlml(formula = esp, data = fp_mg.shp@data, listw = w1): inversion of asymptotic covar.
##      condição recíproca número = 8.18274e-24 - using numerical Hessian.
```

```
summary(mod1.sac)
```

```
##
## Call: sacsarl原因m(formula = esp, data = fp_mg.shp@data, listw = w1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -689080 -115504  -51697   63475  975341
##
## Type: sac
## Coefficients: (numerical Hessian approximate standard errors)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.6044e+04  6.3499e+04 -0.2527 0.800526
## AC          4.1996e+00  1.1493e+00  3.6542 0.000258
## LP          -1.1547e+06  7.9726e+05 -1.4483 0.147533
##
## Rho: 0.49926
## Approximate (numerical Hessian) standard error: 0.17067
##      z-value: 2.9253, p-value: 0.0034412
## Lambda: -0.42264
## Approximate (numerical Hessian) standard error: 0.3322
##      z-value: -1.2723, p-value: 0.20328
##
## LR test value: 7.1272, p-value: 0.028336
##
## Log likelihood: -928.8885 for sac model
## ML residual variance (sigma squared): 8.9449e+10, (sigma: 299080)
## Number of observations: 66
## Number of parameters estimated: 6
## AIC: 1869.8, (AIC for lm: 1872.9)
```

```
impacts(mod1.sac, listw = w1)
```

```
## Impact measures (sac, exact):
##      Direct      Indirect      Total
## AC  4.483545e+00  3.903302e+00  8.386847e+00
## LP -1.232735e+06 -1.073199e+06 -2.305935e+06
mod1.sacGMM <- gsts原因s(formula = esp, data = fp_mg.shp@data, listw = w1)
summary(mod1.sacGMM)
```

```
##
## Call: gsts原因s(formula = esp, data = fp_mg.shp@data, listw = w1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -852547 -107324  -37609   45111  933539
##
## Type: GM SARAR estimator
## Coefficients: (GM standard errors)
##              Estimate Std. Error z value Pr(>|z|)
## Rho_Wy         6.7942e-01  1.4464e-01  4.6973 2.637e-06
## (Intercept) -3.5637e+04  5.7779e+04 -0.6168 0.537376
## AC          3.1093e+00  1.0714e+00  2.9020 0.003708
## LP          -7.4698e+05  7.5025e+05 -0.9957 0.319420
```

```
##
## Lambda: -0.5933
## Residual variance (sigma squared): 8.8604e+10, (sigma: 297660)
## GM argmin sigma squared: 8.5071e+10
## Number of observations: 66
## Number of parameters estimated: 6
```

```
impacts(mod1.sacGMM, listw = w1)
```

```
## Impact measures (lag, exact):
##           Direct      Indirect      Total
## AC          3.60077  6.098180e+00  9.698950e+00
## LP -865051.37608 -1.465031e+06 -2.330082e+06
```

## SMA

$$y = X\beta + u$$

$$u = \lambda W\varepsilon + \varepsilon$$

```
mod1.sma <- spautolm(formula = esp, data = fp_mg.shp@data, listw = w1, family = "SMA")
summary(mod1.sma)
```

```
##
## Call:
## spautolm(formula = esp, data = fp_mg.shp@data, listw = w1, family = "SMA")
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -652200 -108910  -61485   17458 1148085
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  8.9238e+04  9.2239e+04  0.9675  0.33331
## AC          5.7949e+00  9.8579e-01  5.8784 4.142e-09
## LP        -1.6866e+06  1.0238e+06 -1.6473  0.09949
##
## Lambda: 0.32339 LR test value: 2.154 p-value: 0.1422
## Numerical Hessian standard error of lambda: 0.22526
##
## Log likelihood: -931.3751
## ML residual variance (sigma squared): 1.0801e+11, (sigma: 328660)
## Number of observations: 66
## Number of parameters estimated: 5
## AIC: 1872.8
```

## SLX

$$y = X\beta + WX\theta + \varepsilon$$

```
mod1.slx <- lmSLX(formula = esp, data = fp_mg.shp@data, listw = w1)
summary(mod1.slx)
```

```
##
## Call:
```

```
## lm(formula = y ~ x - 1, weights = weights)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -648361 -142243  -44940   73137 1088122
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## x(Intercept) -1.535e+04  1.336e+05  -0.115   0.9089
## xAC           4.231e+00  1.235e+00   3.425   0.0011 **
## xLP          -1.162e+06  1.250e+06  -0.930   0.3562
## xlag.AC       3.950e+00  1.623e+00   2.434   0.0179 *
## xlag.LP      -1.244e+06  2.117e+06  -0.588   0.5589
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 327300 on 61 degrees of freedom
## Multiple R-squared:  0.6753, Adjusted R-squared:  0.6487
## F-statistic: 25.37 on 5 and 61 DF,  p-value: 9.644e-14
```

```
impacts(mod1.slx)
```

```
## Impact measures (SLX, estimable):
##           Direct      Indirect      Total
## AC  4.230713e+00  3.949692e+00  8.180404e+00
## LP -1.161668e+06 -1.244138e+06 -2.405806e+06
```

## SDM

$$y = \rho W y + X \beta + W X \theta + \varepsilon$$

```
mod1.sdm <- lagsarlm(formula = esp, data = fp_mg.shp@data, listw = w1, type = "mixed")
```

```
## Warning in lagsarlm(formula = esp, data = fp_mg.shp@data, listw = w1, type = "mixed"): inversion of a
## condição recíproca número = 1.78024e-23 - using numerical Hessian.
```

```
summary(mod1.sdm)
```

```
##
## Call:
## lagsarlm(formula = esp, data = fp_mg.shp@data, listw = w1, type = "mixed")
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -734538 -129033  -43689   66992 1053670
##
## Type: mixed
## Coefficients: (numerical Hessian approximate standard errors)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.0478e+04  9.0349e+04 -0.1160 0.9076752
## AC           4.2407e+00  1.1776e+00  3.6010 0.0003169
## LP          -1.1118e+06  1.1815e+06 -0.9410 0.3466996
## lag.AC       2.6714e+00  2.1254e+00  1.2569 0.2087968
## lag.LP      -9.4061e+05  1.7928e+06 -0.5247 0.5998116
##
```



```
## Rho: 0.1514, LR test value: 0.6695, p-value: 0.41323
## Approximate (numerical Hessian) standard error: 0.1823
##      z-value: 0.83049, p-value: 0.40626
## Wald statistic: 0.68971, p-value: 0.40626
##
## Log likelihood: -928.8183 for mixed model
## ML residual variance (sigma squared): 9.7524e+10, (sigma: 312290)
## Number of observations: 66
## Number of parameters estimated: 7
## AIC: 1871.6, (AIC for lm: 1870.3)
```

```
impacts(mod1.sdm, listw = w1)
```

```
## Impact measures (mixed, exact):
##           Direct      Indirect      Total
## AC  4.347372e+00  3.797958e+00  8.145330e+00
## LP -1.147453e+06 -1.271079e+06 -2.418531e+06
```

## SDEM

$$y = X\beta + WX\theta + u$$

$$u = \lambda Wu + \varepsilon$$

```
mod1.sdem <- errorsarlm(formula = esp, data = fp_mg.shp@data, listw = w1, etype = "emixed")
```

```
## Warning in errorsarlm(formula = esp, data = fp_mg.shp@data, listw = w1, : inversion of asymptotic co
## condição recíproca número = 1.13909e-22 - using numerical Hessian.
```

```
summary(mod1.sdem)
```

```
##
## Call:errorsarlm(formula = esp, data = fp_mg.shp@data, listw = w1,
##      etype = "emixed")
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -689844 -131833  -47861   70135 1079092
##
## Type: error
## Coefficients: (asymptotic standard errors)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.5635e+03  1.3684e+05 -0.0260 0.9792252
## AC           4.3885e+00  1.1594e+00  3.7852 0.0001536
## LP          -1.1686e+06  1.1813e+06 -0.9892 0.3225470
## lag.AC       3.6271e+00  1.5669e+00  2.3148 0.0206215
## lag.LP      -1.2674e+06  2.0642e+06 -0.6140 0.5392220
##
## Lambda: 0.10118, LR test value: 0.24235, p-value: 0.62252
## Approximate (numerical Hessian) standard error: 0.20085
##      z-value: 0.50379, p-value: 0.61441
## Wald statistic: 0.25381, p-value: 0.61441
##
## Log likelihood: -929.0319 for error model
## ML residual variance (sigma squared): 9.8419e+10, (sigma: 313720)
## Number of observations: 66
```

```
## Number of parameters estimated: 7  
## AIC: 1872.1, (AIC for lm: 1870.3)
```