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Section : O2

Part-B

1\. Prove $\overline{(A-B)} = \bar{A} \cup B$ Analytically

$\because A-B = A \cap B'$

let $x \in (A-B)$

$x \in A$ & $x \notin B$

$x \in (A \cap B')$

Now

$\because (A-B) = A \cap \bar{B}'$

Taking complement on Both sides

$\overline{(A-B)} = \overline{(A \cap \bar{B})}$

$= \bar{A} \cup \bar{\bar{B}}$ (According to De-Morgan's Law)

$= \bar{A} \cup B$

$\therefore \overline{(A-B)} = \bar{A} \cup B$

$\therefore$ Proved.

2\. $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4)$
$(3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

$R^{-1} = \{(1,1), (2,1), (3,1), (4,1), (5,1)\ (1,2), (2,2), (3,2), (4,2)$
$(1,3), (2,3), (3,3), (1,4), (2,4), (1,5)\}$

$\bar{R} = \{(2,5), (3,4), (3,5), (4,3), (4,4), (4,5), (5,2),$
$(5,3), (5,4), (5,5)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R^{-1}} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{\bar{R}} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

3. $f(x) = x+2$, $x \in R$
$g(x) = x-2$, $x \in R$
Since the range of $f \subset$ domain of g and range of $g \subseteq$ domain of f
$\therefore$ fog & gof exists

$fog = f(g(x))$
$= f(x-2)$

$= x-2+2 = x$

$gof = g(f(x))$
$= g(x+2)$

$= x+2-2$
$= x.$

$\therefore fog = gof$

4. $g: R \to R$
$f(x) = ax+b$
$g(x) = 1-x+x^2$

$gof(x) = 9x^2-9x+3$ — (i)

$g(f(x))$

$= g(ax+b)$

$= 1-(ax+b)+(ax+b)^2$
$= 1-ax-b+a^2x^2+b^2+2axb$
$= a^2x^2+(2ab-a)x+b^2-b+1$ (ii)

comparing eq^n (i) & (ii) we get

$a^2 = 9$
$a = \pm 3$

$2ab-a = -9$
taking $a=3$
we get $b = -1$

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taking $a = -3$
we get $b = 2$

$a = 3, -3$
$b = -1, 2$

5. Reflexive - $\because (a,a), (b,b), (c,c), (d,d) \in R$
$\therefore$ It is reflexive

symmetric - $\because (a,c), (c,a), (a,d), (d,a) \in R$
$\therefore$ It is symmetric

Transitive - $\because (a,a), (a,c) \in R$ & (a,c) also $\in R$
$\therefore$ It is Transitive.

6. $R = \{(1,1), (1,3), (2,2)\}$
$S = \{(1,2), (1,3), (2,1), (2,2), \cancel{(2,3)}, (3,3)\}$

$R \cup S = \{(1,1)\ (1,2), (1,3)\ (2,1), (2,2), \cancel{(2,3)}, (3,3)\}$

$R \cap S = \{(1,3), (2,2)\}$

$$M_{R\cup S} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}$$

$$M_{R\cap S} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 \end{array}$$

Part-C

1. **Demorgan's Law**

If A and B are any two sets then

i) $(A \cup B)' = A' \cap B'$
ii) $(A \cap B)' = A' \cup B'$

i) let x be an arbitary element of $(A \cup B)'$

$x \in (A \cup B)'$
$x \notin (A \cup B)$
$x \notin A$ & $x \notin B$
$x \in A'$ & $x \in B'$
$x \in A' \cap B'$
$\therefore (A \cap B)' \subseteq A' \cap B'$.

let y be an arbitary element of $(A' \cap B')$

$y \in A' \cap B'$
$y \in A'$ & $y \in B'$
$y \notin A$ & $y \notin B$
$y \notin A \cap B$
$y \in (A \cap B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$

Hence $(A \cup B)' = A' \cap B'$

(ii) Let x be an arbitrary element of $(A \cap B)'$

$x \in (A \cap B)'$
$x \notin (A \cap B)$
$x \notin A$ or $x \notin B$
$x \in A'$ or $x \in B'$
$x \in A' \cup B'$
$(A \cap B)' \subseteq A' \cup B'$

Let y be an arbitrary element of $A' \cup B'$

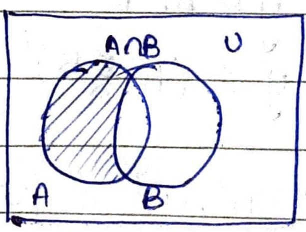
$y \in A' \cup B'$
$y \in A'$ or $y \in B'$
$y \notin A$ or $y \notin B$
$y \notin (A \cap B)$
$y \notin (A \cap B)'$

$\therefore A' \cup B' \subseteq (A \cap B)'$
$\therefore (A \cap B)' = A' \cup B'$

2. $(A-B)-C = A-(B \cup C)$

LHS

$(A-B)-C$



$\because A-B = A \cap B'$
$\Rightarrow (A \cap B')-C$
$\Rightarrow (A \cap B') \cap C'$
$\Rightarrow A \cap B' \cap C'$

RHS

$A-(B\cup C)$

$\Rightarrow A\cap(B\cup C)'$ $\{\because A-B = A\cap B'\}$

$\Rightarrow A\cap B'\cap C'$ $\{$from demorgan's law $\because (A\cup B)' = A'\cap B'$

$A\cap B'\cap C' = A\cap B'\cap C'$

LHS = RHS

$\therefore$ Proved.

3. $(a,b)\in R$ if and only if $3a+4b=7n$.

Reflexive

let $(a,a)\in R$

$= 3a+4a$.

$= 7a$

$\because$ $7a$ is divisible by 7 where $a\in R$.

$\therefore$ It is Reflexive.

symmetric

let (a,b) & $(b,a)\in R$.

$3a+4b = 7n$. ① given.

$3b+4a$

$= 4a+3b$

$= 7(a+b) - 3a+4b$.

$= 7(a+b) - 7n$ - from eq^n ①

$\therefore$ It is divisible by 7

$\therefore$ $(b,a)\in R$

$\therefore$ It is symmetric

Transitive

let (a,b) & $(b,c) \in R$.

then $3a+4b=7n$ —(i)
$3b+4c=7k$ —(ii) where $n, k \in R$.

Adding we get -

$3a+7b+4c = 7n+7k$
$\Rightarrow 3a+4c = 7n+7k-7b$
$\Rightarrow 3a+4c = 7(n+k-b)$
$\therefore 3a+4c$ is divisible by 7
$\therefore (a,c) \in R$
$\therefore$ It is transitive.

4. $R = \{(1,2),(1,4),(2,4),(1,6),(2,6),(1,8),(2,8),$
$(4,8),(1,1),(2,2),$ ~~(3~~ $(4,4),(6,6),(8,8)\}$

Reflexive - $\because (1,1),(2,2),(4,4),(6,6),(8,8) \in R$
$\therefore$ It is reflexive.

Transitive :- $\because (1,2)$ & $(2,4) \in R$ & $(1,4)$ also belongs to R
Hence it is transitive

Antisymmetric :- for $a=1, b=2$ $(1,2) \in R$ but
$(2,1) \notin R$
$\therefore$ It is antisymmetric
since the relation is reflexive, transitive & antisymmetric hence it is partial order relation

5. $R = \{(1,1), (1,3), (1,5), (2,3), (2,4), (3,3), (3,5), (4,2), (4,4), (5,4)\}$

$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

	I	II	III	IV	V
C	$\{1\}$	$\{4\}$	$\{1\,2\,3\}$	$\{2\,4\,5\}$	$\{1\,3\}$
R	$\{1\,3\,5\}$	$\{3\,4\}$	$\{3\,5\}$	$\{2\,4\}$	$\{4\}$
	(1,1) (1,3) (1,5)	(4,3) (4,4)	(1,3) (1,5) (2,3) (2,5) (3,3) (3,5)	(2,2) (2,4) (4,2) (4,4) (5,2) (5,4)	(1,4) (3,4)

Transitive closure $= \{(1,1), (1,3), (1,5), (4,3), (4,4), (2,3), (2,5), (3,3), (3,5), (2,2), (2,4), (4,2), (5,2), (5,4), (1,4), (3,4)\}$

6. show that the composition of invertible function is invertible

let $f: A \to B$ and $g: B \to C$ are bijections

then $gof: A \to C$ will exist and is bijection.

$gof: A \to C$ is bijection $\Rightarrow (gof)^{-1}: C \to A$ exists

similarly

$f: A \to B$ is bijection $\Rightarrow f^{-1}: B \to A$ is bijection

$g: B \to C$ is bijection $\Rightarrow g^{-1}: C \to B$ is bijection

$\Downarrow$

$f^{-1} o g^{-1}: C \to A$

let $x \in A$, $y \in B$ & $z \in C$ such that $f(x) = y$ & $g(y) = z$
then

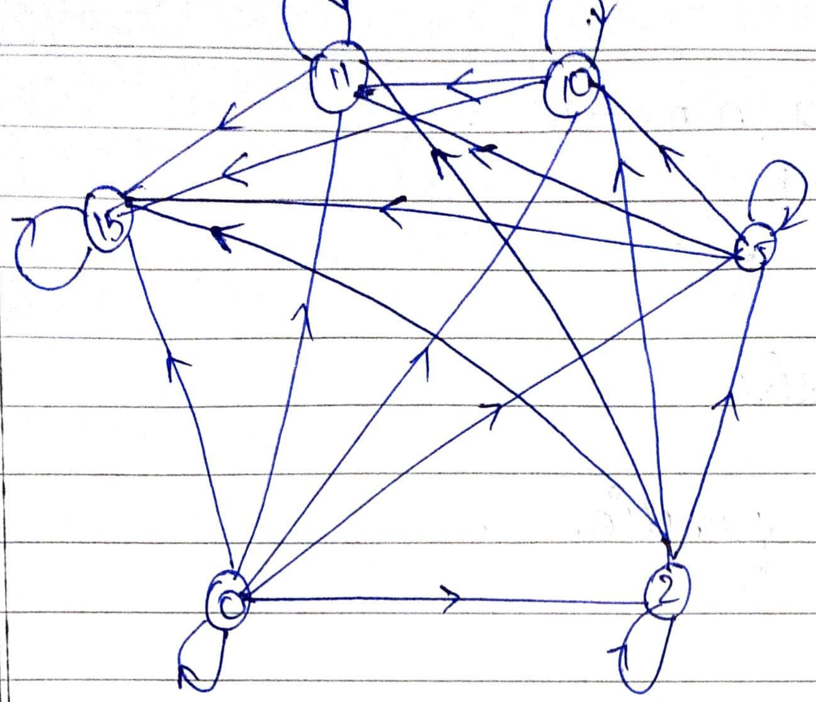
$gof(x) = g(f(x)) = g(y) = z$ — (I)
$(gof)^{-1}(z) = x$

$f(x) = y$ & $g(y) = z$
$\Rightarrow f^{-1}(y) = x$ & $g^{-1}(z) = y$
$\therefore (f^{-1} o g^{-1}) z = f^{-1}(g^{-1}(z)) = f^{-1}(y) = x$ — (II)

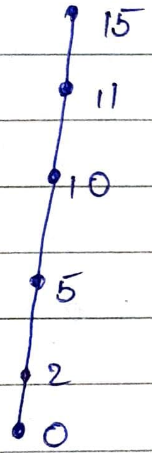
from eq^n (I) & (II)
$(gof)^{-1} z = (f^{-1} o g^{-1})(z) \quad \forall z \in C$
$\therefore (gof)^{-1} = f^{-1} o g^{-1}$

$\therefore$ composition of Invertible function is invertible

7.



Hasse diagram



8. $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$; $f(m,n) = 2m + 3n$

<u>one to one</u>

$\because$ $(0,2)$ & $(3,0)$ both equals 6

$\therefore$ The function is not one to one

<u>onto</u>

$f: A \to B$ is surjective if $\forall$ $b \in B$ there exists $a \in A$ such that $f(a) = b$

$\Rightarrow f(m,n) = b$

$\Rightarrow 2m + 3n = b$

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Take $m = 2b$ and $n = -b$

$2 \times 2b + 3 \times -b = b$

$\therefore (2b, -b) \in f$

$\therefore f$ is surjective

$\therefore$ It is onto function.

9. $f: Z \to N$, $f(x) = \begin{cases} 2x-1, & x > 0 \\ -2x, & x \leq 0 \end{cases}$

<u>case 1:-</u> $x > 0$

let $y \in N$.

$y = 2x - 1$

$y + 1 = 2x$

$x = \frac{y+1}{2}$

for $y \neq 1$ there does not exist any x in Z.

$\therefore$ It is not ~~one-one~~ onto

Hence f is not bijective

10. $3a+b$ is multiple of 4

let $3a+b = 4m$ where $m \in R$

<u>Reflexive</u> let $(a,a) \in R$

$3a + a$

$= 4a$

$\because$ It is divisible by 4 hence it is reflexive

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symmetric

let (a,b) ~~& (b~~ $(b,a) \in R$.

$3a+b = 4m$

$3b+a = 4a+4b-3a-b$

$= 4(a+b) - (3a+b)$

$= 4(a+b) - 4m$

$= 4(a+b-m)$

{ ∵ $3a+b$ is divisible by 4

∴ It is ~~divisible~~ multiple by 4

∴ It is symmetric

Transitive

let (a,b) & $(b,c) \in R$

$3a+b = 4m$

$3b+c = 4n$

Adding

$3a+4b+c = 4m+4n$

$3a+c = 4m+4n-4b$

$3a+c = 4(m+n-b)$

∴ It is a multiple of 4

∴ $(a,c) \in R$

∴ It is Transitive