Rahul Goel

12RA1911 C100100 94 __/__/____

(Q31). First digit must be 5, 6 or 7

There are a total of 7 digits out of which we have a total of 4 of 5s, 6s and 7s

Hence $4/7$ of the total number of positive integer that can be formed from the given digits obey the constraints.

The total number of positive Integers that can be formed from the given 7 digits noting that 4 and 5 appear twice.

$$= \frac{7!}{2!2!}$$

$\therefore$ The total number of positive integers satisfying the constraint.

$$= \frac{4}{7} \times \frac{7!}{2!2!}$$

$$= \frac{\cancel{4}}{\cancel{7}} \times \frac{\cancel{7} \times 6!}{\cancel{4}}$$

$$= 6!$$

$$= 720.$$

Rahul Goel
RA1911030010094

/ /

(Q37) $n! \geq 2^{n-1}$ $n = 1, 2, 3$

for $n=1$

$1 \geq 2^{1-1}$

$1 \geq 1$ (Holds true)

Let it hold true for $n=k$ (where k is a positive integer).

$E(k) = k! \geq 2^{k-1}$

for $n = k+1$

$E(k+1) = (k+1)! \geq 2^{k+1-1}$

$(k+1)k! \geq 2^k$

$(k+1)2^{k-1} \geq 2^k$

$(k+1) \geq 2$

$k \geq 1$

∴ The condition holds true for all $n \geq 1$ as verified by mathematical induction.

Rahul Goel

RA1911030010094

(Q3b) p → If there is rain

q → Growth of crops will be good.

Symbolic Representation $p \to q$

Converse $q \to p$

" Growth of crops will be good if there is rain".

Inverse : $\sim p \to \sim q$

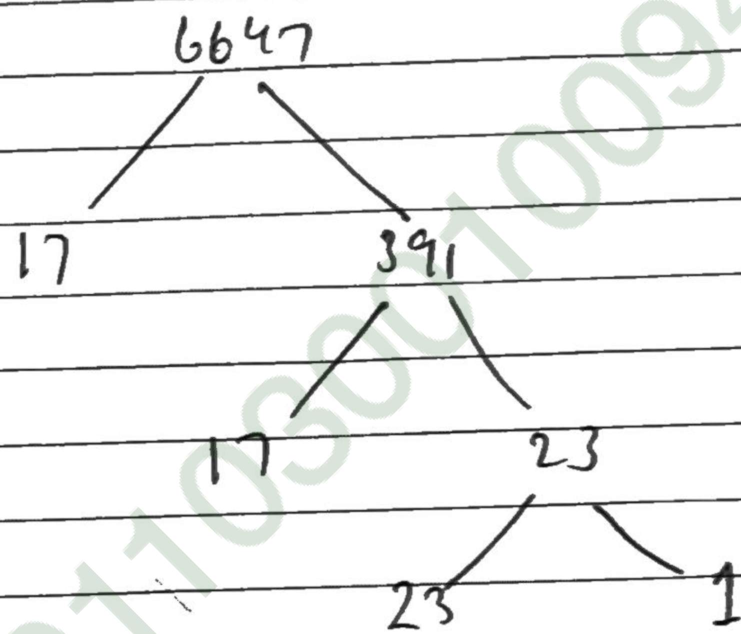
" If it does not rain the growth of crops will not be good."

Contrapositive : $\sim q \to \sim p$.

" If the growth of crops will not be good then it does not rain".

Rahul Raj
RA1911030010094

(Q33) Prime factorization of 6647
The factor tree of 6647 can be drawn as



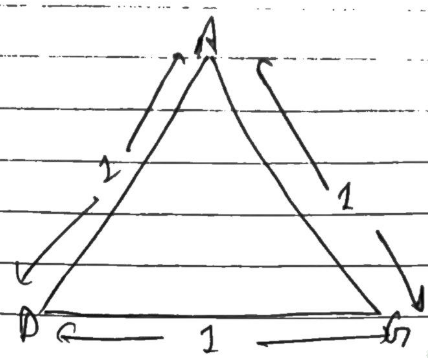
$$6647 = 1^1 \times 17^2 \times 23^1$$

Rahul Rao

RA1911030010094

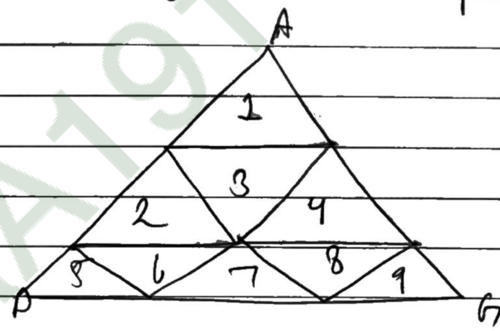
(Q32) We're to select 10 points in the interior of an equilateral triangle of side 1.

Let ADG be the given equilateral Δ^{le}.



The pair of points B, C; E, F; and H, I are the points of intersection of the sides AD, DG and GA respectively.

So, we have divided the Δ^{le} ADG onto 9 equilateral Δ^{le} each of side 1/3.



The 9 sub Δ^{les} may be considered as 9 pigeon holes and the 10 interior points may be regarded as 10 pigeons. Then by pigeon hole principle, at least the sub Δ^{le} must contain 2 interior points. The distance b/w any two interior points of any sub Δ^{le} cannot exceed the length of the side namely 1/3. Hence, proved.