Rahul Goel - RA1911030010094

Part (B)

(21.) For $(a * b)^{-1} = b^{-1} * a^{-1}$ for any $a, b \in G$

$(a * b) * (b^{-1} * a^{-1}) = a * (b * b^{-1}) * a^{-1}$

$= a * (e * a^{-1})$

$= a * e = e$

$\because b * b^{-1} = e$

$(b^{-1} * a^{-1}) * (a * b) = b^{-1} * (a^{-1} * a) * b$

$= (b^{-1} * e) * b$

$= e * b = e$

Thus the inverse of $(a * b)$ is $b^{-1} * a^{-1}$

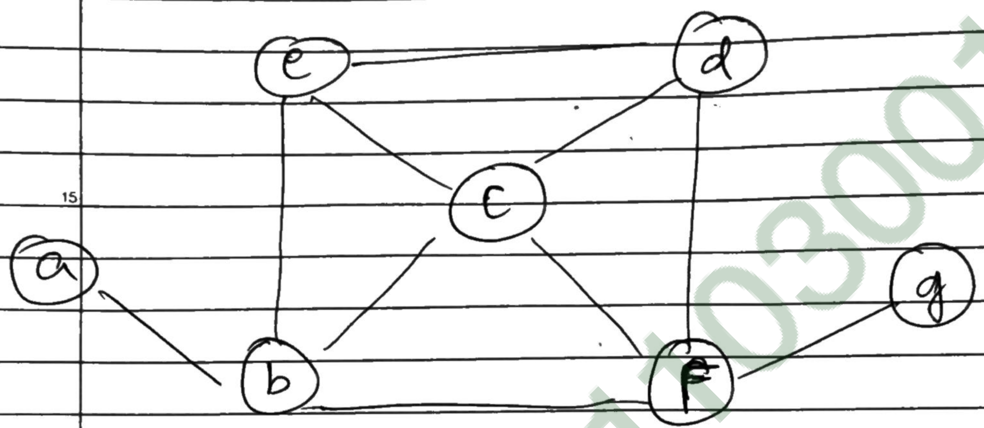
$\therefore (a * b)^{-1} = b^{-1} * a^{-1}$

Part B   Rahul Goel: RA1911030010094.

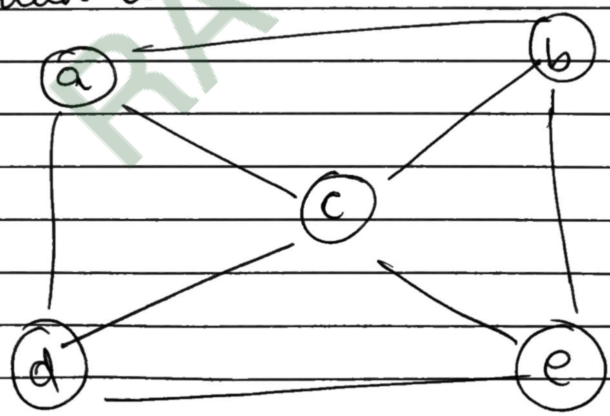
25. → A hamilton circuit is a circuit on a graph that visits each vertex exactly once.

→ An euler circuit is a graph that visits each each exactly once.

→ A graph that's neither Eulerian nor hamilton.



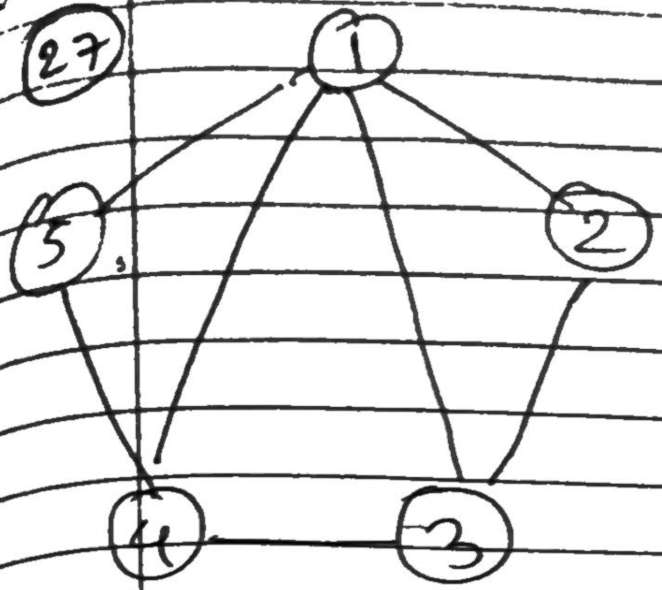
→ A graph that's hamilton but not eulerian is



Part B

27

Rahul Goel - RA1911030010094



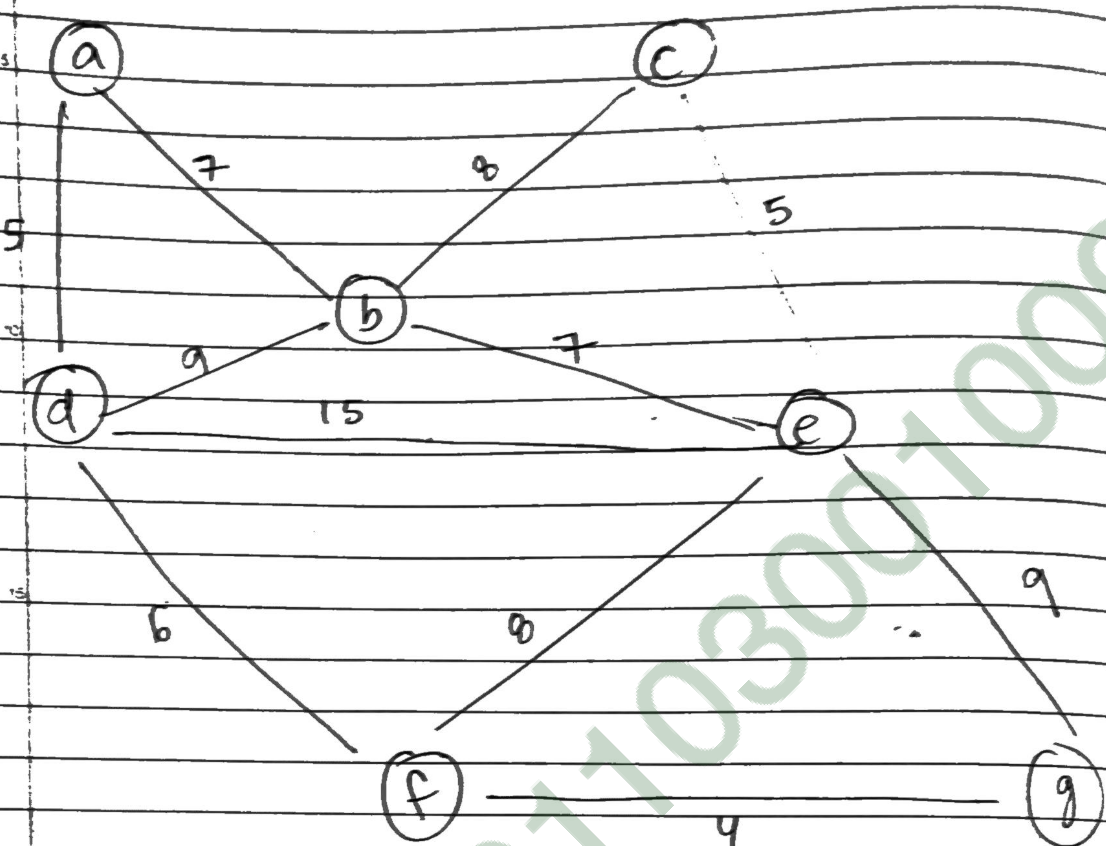
M_A (Adjacent Matrix) =

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Rahul [illegible] - RA19110030100[illegible]

(29) **Part C**

Given graph is :



The graph has 7 vertices and 11 edges
MSP will have (7-1) = 6 edge

After sorting we have

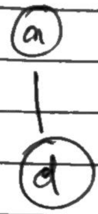
Weight	Source	Destination
5	a	d
5	c	e
6	d	f
7	a	b
7	b	e
8	b	c
8	d	e
9	e	g

Rahul Urad - RA1911030010094

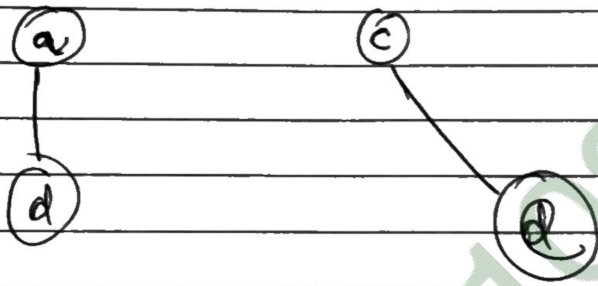
11	f	g
15	d	e

Starting from the smallest

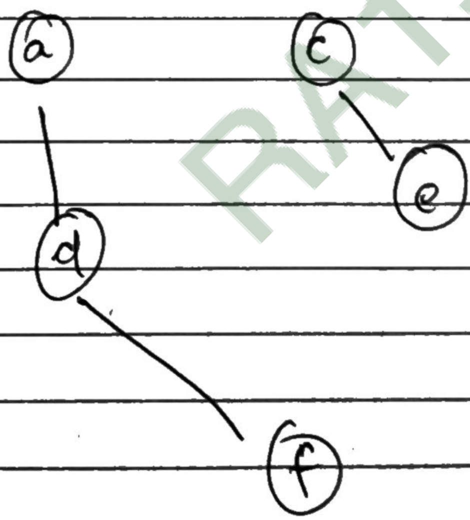
① Picking a → d : No cycle formed



② Picking c → e : No cycle Formed

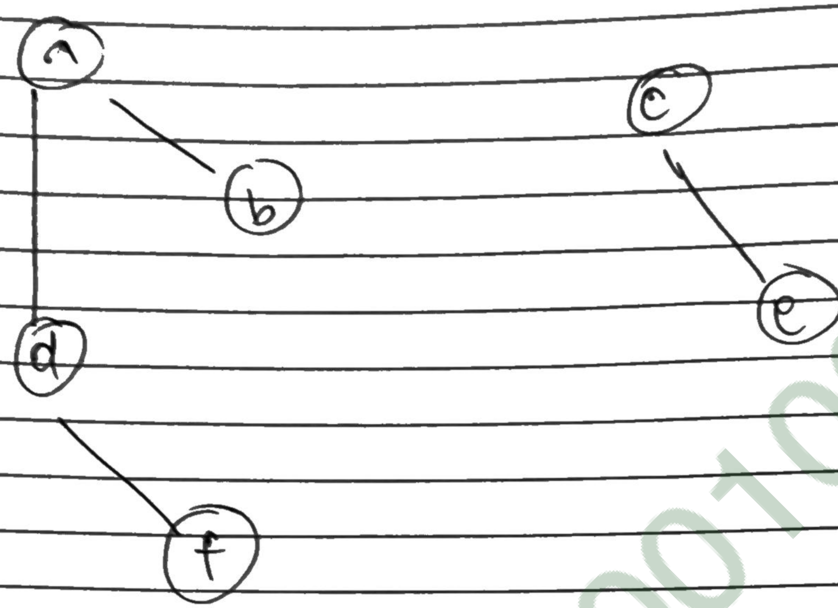


③ Picking d → F : No cycle formed

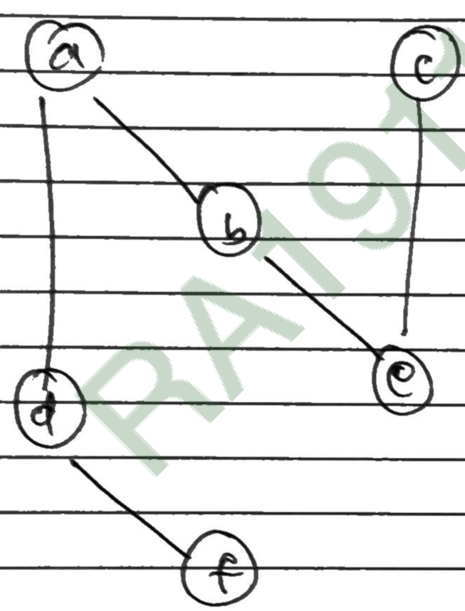


Rahul Goel - RA1911030010094

(4) Picking a → b : No cycle formed

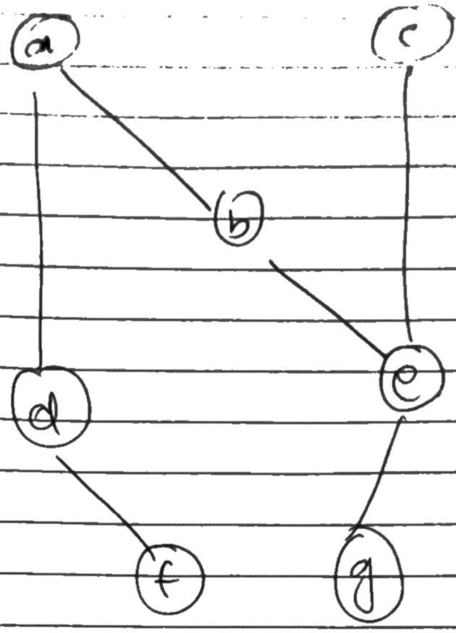


(5) Picking b → e : No cycle formed



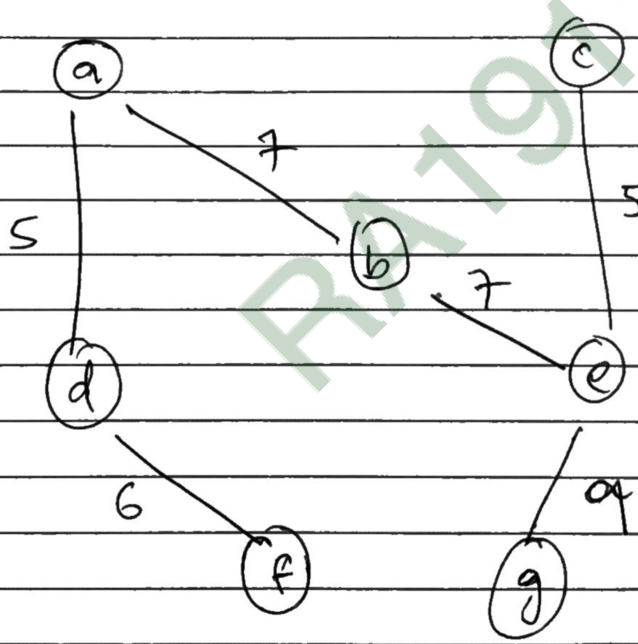
(6) Picking b → c : cycle formed discarded
(7) Picking d → e : cycle formed discarded
(8) Picking e → g : no cycle formed

Rahul Prasad, RA1911030010094



9\. Rest are discarded because cycles are formed and $(V-1) = 6$ edges are reached.

∴ minimum spanning tree



minimum weight $= 5 + 6 + 7 + 7 + 5 + 9$
$= 39$

Rahul Goel - RA1911030010094

Part (B)

(23) $G = \{1, -1, i, -i\}$

For all group elements.
Identify elements for $G = 1$

Order :-

a.) $1 \Rightarrow (1)^1 = 1$

1 is the order for 1

b.) $-1 \Rightarrow (-1)^1 = -1$

$(-1)^2 = 1$

$\therefore$ Order for -1 is 2

c.) $i \Rightarrow (i)^1 = i$

$(i)^2 = -1$

$(i)^3 = -i$

$(i)^4 = 1$

$\therefore$ Order for i is 4

d.) $-i \Rightarrow (-i)^1 = -i$

$(-i)^2 = 1$

$\therefore$ Order for $-i$ is 2

Part B

Rahul Goel RA1911030010094

22) Let $(G, *)$ be a group.

H_1, H_2 be subgroups of G.

To prove that $H_1 \cap H_2$ is a subgroup.

Since $e \in H_1$ and $e \in H_2$, $e \in H_1 \cap H_2$

$\Rightarrow H_1 \cap H_2 \neq \phi$.

To prove for any $a, b \in H_1 \cap H_2 \Rightarrow$

$a * b^{-1} \in H_1 \cap H_2$

Let $a, b \in H_1 \cap H_2$

$\Rightarrow a, b \in H_1$ and $a, b \in H_2$

$a * b^{-1} \in H_1$ and $a * b^{-1} \in H_2$

$a * b^{-1} \in H_1 \cap H_2$

Hence by the sufficient condition for subgroup.

$H_1 \cap H_2$ is also a subgroup.