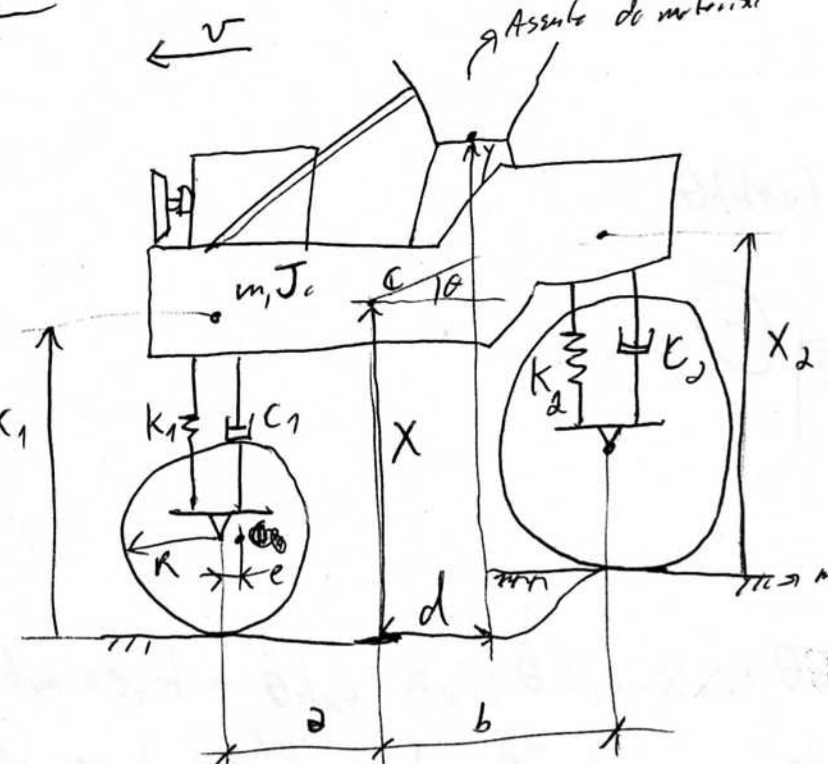
PME 2352 → Montar formulário de estudo (c/ sol. da eq. dif.) @30/11/11

Ex: Provas passadas: (3Q)(P3-2006) → PME 2341 → No site

Trator: c/ roda dianteira c/ excentricidade e



$m = 70\,kg$

$J_c = 12\,kg\,m^2$

$R = 0{,}15\,m$

$a = 0{,}6\,m$

$b = 0{,}3\,m$

$d = 0{,}2\,m$

$k_1 = 1000\,N/m$

$c_1 = 250\,Ns/m$

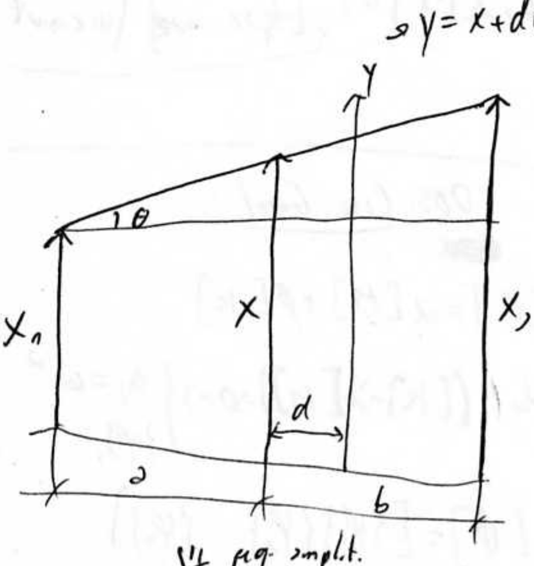
$e = 0{,}02\,m$

a) Eq. diferencial

b) K_2 e c_2 p/ desacoplar

c) Mov. vertical do assento do motorista

→ $y = x + d\theta$

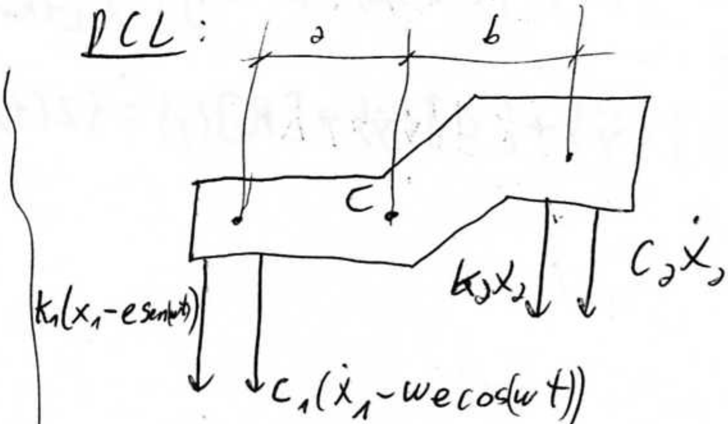


⇓ peq. amplit.

$$\tan\theta \cong \theta = \frac{x_2 - x_1}{a+b} \quad (1)$$

$$\frac{(x - x_1)}{\ldots} = \frac{(x_2 - x_1)}{\ldots}$$

DCL:



$$\text{TMB}: \; m\ddot{x} = -k_1(x_1 - e\,\mathrm{sen}\,\omega t) - c_1(\dot{x}_1 - \omega e\cos\omega t) - k_2 x_2 - c_2\dot{x}_2 \quad (2)$$

$$\text{TMA})_c: \; J_c\ddot{\theta} = +k_1(x_1 - e\,\mathrm{sen}\,\omega t)a + c_1(\dot{x} - \omega e \cos\omega t \ldots$$

① → $x_1 = x_2 + (a+b)\theta = \frac{(a+b)}{a}x - \frac{b}{a}x_1 + (a+b)\theta$

$(a+b)x - (a+b)x_1 = a x_2 - a x_1$

$\Rightarrow x_2 = \left(\frac{a+b}{a}\right)x - \frac{b}{a}x_1$

$\frac{(a+b)}{a}x_1 = \frac{(a+b)x}{a} + (a+b)\theta$

$\Rightarrow \boxed{x_1 = x - a\theta}$ $\boxed{x_2 = x + b\theta}$ (4)

De (4) em (2) e (3):

$$m\ddot{x} + k_1 x - k_1 a\theta + k_2 x + k_2 b\theta + c_1\dot{x} - c_1 a\dot{\theta} + c_2\dot{x} + c_2 b\dot{\theta} = k_1 e \,\mathrm{sen}\,\omega t + c_1 e\omega\cos\omega t$$

$$J_c\ddot{\theta} - k_1 a x - k_1 a^2\theta + k_2 b x + k_2 b^2\theta - c_1 a\dot{x} - k_1 a^2\dot{\theta} + c_2 b\dot{x} + c_2 b^2\dot{\theta} = -k_1 a e\,\mathrm{sen}\,\omega t - c_1 a\omega c$$

Matricialmente:

$$\overbrace{\begin{bmatrix} m & 0 \\ 0 & J_c \end{bmatrix}}^{M}\begin{Bmatrix}\ddot{x}\\ \ddot{\theta}\end{Bmatrix} + \overbrace{\begin{bmatrix} (c_1+c_2) & -(c_1 a - c_2 b) \\ -(c_1 a - c_2 b) & -(c_1 a^2 - c_2 b^2)\end{bmatrix}}^{C}\begin{Bmatrix}\dot{x}\\ \dot{\theta}\end{Bmatrix} + \overbrace{\begin{bmatrix} k_1+k_2 & -(k_1 a - k_2 b) \\ -(k_1 a - k_2 b) & -(k_1 a^2 - k_2 b^2)\end{bmatrix}}^{K}\begin{Bmatrix}x\\ \theta\end{Bmatrix} = \begin{bmatrix} k_1 e & c_1 e \\ -k_1 a e & -c_1 a e\end{bmatrix}\begin{Bmatrix}\mathrm{sen}\,\omega t\\ \omega\cos\omega t\end{Bmatrix}$$

$$([M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{z(t)\}$$

OBS: Caso Geral:

$[C] = \alpha[M] + \beta[K]$

$\det([K] - \lambda[M]) = 0 \Rightarrow \begin{cases}\lambda_i = \omega_i^2 \\ \{\varphi\}_i\end{cases}$

$[\Phi] = [\{\varphi_1\}\{\varphi_2\}\cdots\{\varphi_n\}]$

$\Rightarrow [\diagdown_{m}\diagdown] = [\Phi][M][\Phi]$

$[\diagdown\ \diagdown] = [\Phi][M][\Phi]$

b) P/ desacoplar: $[M]$, $[C]$ e $[k]$ devem ser diagonalizados.

$[M] \to$ Já é diag.

Não precisa usar caso geral pois $[C]$ e $[k]$ é da forma $\begin{bmatrix} A & B \\ B & C \end{bmatrix}$

são iguais! $\Rightarrow$ basta fazer B

$$-(k_1 a - k_2 b) = 0 \Rightarrow \boxed{k_2 = \frac{a}{b} k_1}$$

$(i.e.\ k_1 a = k_2 b)$

$$-(c_1 a - c_2 b) = 0 \Rightarrow \boxed{c_2 = \frac{a}{b} c_1}$$

$(i.e.\ c_1 a = c_2 b)$

$$\Rightarrow \boxed{m\ddot{x} + (c_1 + c_2)\dot{x} + (k_1 + k_2)x = k_1 e \sin\omega t + c_1 e\omega\cos\omega t} \quad (A)$$

$$J_c\ddot{\theta} - (c_1 a^2 - c_2 b^2)\dot{\theta} - (k_1 a^2 - k_2 b^2)\theta = -k_1 a e \sin\omega t - c_1 a e\omega\cos\omega t \quad (B)$$

desacoplados!

c) $R\omega = v \Rightarrow \omega = \frac{v}{R}$

Forma Geral da excitação

$(A) = F_1 \sin(\omega t + \varphi_1)$

$(B) = F_2 \sin(\omega t + \varphi_2)$

Lembrando: p/ $m\ddot{x} + kx + c\dot{x} = F(t)$

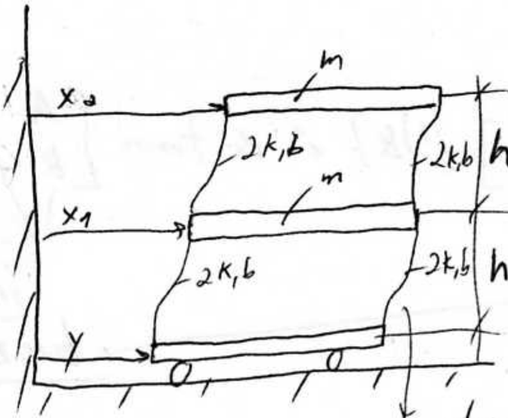
$$x(t) = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \cdot \sin(\Omega t)$$

Eq dif 1GL: $\Rightarrow x(t) = \frac{F_1/(k_1+k_2)}{\sqrt{(1-r_1^2)^2 + (2\zeta_1 r_1)^2}} \sin(\omega t + \varphi_1 + \psi_1)$;

$$\omega_1 = \sqrt{\frac{k_2+k_1}{m}}; \quad r_1 = \frac{\omega}{\omega_1}; \quad \zeta_1 = \frac{c_1 + c_2}{2\sqrt{(k_1+k_2)m}}$$

(B) $\to$ Análogo

Ex: PSub-2009 (PME 2352)



$y(t) = Y_0 \, sen(\Omega t)$

$Y_0 = 5 \, mm$

$\Omega = 2\pi \, rad/s$

$m = 800000 \, kg$

$k = 2 \cdot 10^5 \, N/m$

$b = 0,2$

amortec. por histerese

$k(1+jb)$ — $\boxed{1^{a} \text{ Forma}}$

amortecimento complexo

encontrar amort. viscoso equivalente (c_{eq}) — $\boxed{2^{a} \text{ Forma}}$

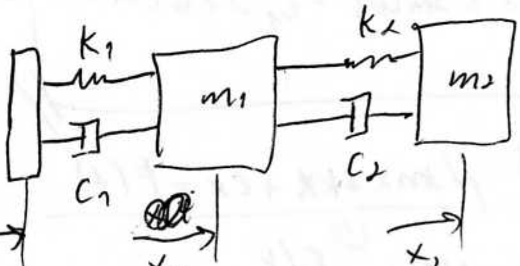
a) Eqs. dif.

b) $\omega_1 = ?$; $\varphi_1 = ?$

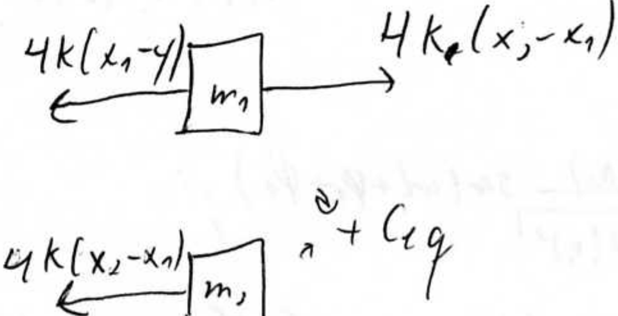
freq. natural (formas de vibrar)

c) X_2, X_1 (amplit.)

$\equiv$

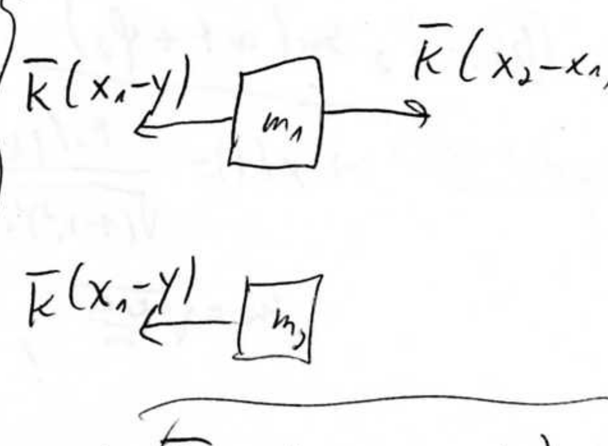


DCL:



$\boxed{2^{a} \text{ Forma}}$

* 1^{a} Forma: + simples



onde $\boxed{\bar{K} = 4k(1-jb)}$

$$\Rightarrow \begin{cases} m_1 \ddot{x}_1 = -\bar{k}(x_1 - y) + \bar{k}(x_2 - x_1) \\ m_2 \ddot{x}_2 = -\bar{k}(x_2 - x_1) \end{cases}$$

matricialmente:

$$\Rightarrow \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2\bar{k} & -\bar{k} \\ -\bar{k} & +\bar{k} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \bar{k}\cdot y(t) \\ 0 \end{Bmatrix}$$

$$[M]\{\ddot{y}\} + [\bar{K}]\{y\} = \{F(t)\}$$

$$([\bar{K}] - \lambda[M])\{y\} = \{0\}$$

$$\det([\bar{K}] - \lambda[M]) = 0$$

$$\Rightarrow \begin{vmatrix} 2k - \lambda m & -\bar{k} \\ -\bar{k} & \bar{k} - \lambda m \end{vmatrix} = 0 \Rightarrow \cancel{(2\bar{k} - \lambda m) - \bar{k}^2 = 0}$$

$$(2\bar{k} - \lambda m)(\bar{k} - \lambda m) - \bar{k}^2 = 0$$

$$2\bar{k}^2 - 3\bar{k}m\lambda + m^2\lambda^2 - \bar{k}^2 = 0$$

$$\lambda^2 - \frac{3\bar{k}}{m}\lambda + \frac{\bar{k}^2}{m^2} = 0$$

$$\lambda = \frac{3\bar{k}}{2m} \pm \frac{1}{2}\sqrt{\left(\frac{3k}{m}\right)^2 - 4\left(\frac{\bar{k}}{m}\right)^2}$$

$$\lambda = \frac{3\bar{k}}{2m} \pm \frac{1}{2}\frac{\bar{k}}{m}\sqrt{5}$$

$$\Rightarrow \begin{cases} \lambda_1 = \frac{3-\sqrt{5}}{2}\,\frac{\bar{k}}{m} = \frac{3-\sqrt{5}}{2}\cdot\frac{4k}{m}(1+jb) \\ \lambda_2 = \frac{3+\sqrt{5}}{2}\,\frac{\bar{k}}{m} = \frac{3+\sqrt{5}}{2}\cdot\frac{4k}{m}(1+jb) \end{cases} \rightarrow \begin{matrix} w_1 = \sqrt{2(3-\sqrt{5})\frac{k}{m}} \\ e \\ w_2 = \sqrt{2(3+\sqrt{5})\frac{k}{m}} \end{matrix}$$

- $\underline{\lambda_1 = \frac{3-\sqrt{5}}{2}\frac{\bar{k}}{m}}$:

$$\Rightarrow \begin{bmatrix} 2\bar{k} - \frac{3-\sqrt{5}}{2}\bar{k} & -\bar{k} \\ -\bar{k} & \bar{k} - \frac{3-\sqrt{5}}{2}\bar{k} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \boxed{\left(\frac{1+\sqrt{5}}{2}\right) x_1 = x_2} \Rightarrow \boxed{\varphi_1 = \begin{Bmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{Bmatrix}} \;//$$

1º Modo/Forma de vibr

- $\underline{\lambda_2 = \frac{3+\sqrt{5}}{2}\frac{\bar{k}}{m}}$:

$$\therefore \boxed{\left(\frac{1-\sqrt{5}}{2}\right) x_1 = x_2} \Rightarrow \boxed{\varphi_2 = \begin{Bmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \end{Bmatrix}} \;//$$

2º modo de vibra

c) desacopl:

$$[M]\{\ddot{x}\} + [\bar{k}]\{y\} = \{F(t)\}$$

$$\{y\} = [\Phi]\cdot\{q\} = \begin{bmatrix} 1 & 1 \\ \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$\Rightarrow \underbrace{[\Phi]^T[M][\Phi]}_{[\diagdown m \diagdown]}\{\ddot{q}\} + \underbrace{[\Phi]^T[\bar{k}][\Phi]}_{[\diagdown \bar{k} \diagdown]}\{q\} = \underbrace{[\Phi]^T\{F(t)\}}_{\{p(t)\}}$$

i.e., 2 eq de 1ª ordem sem amortecimento, desc

$$m\begin{bmatrix} 1 & \frac{1+\sqrt{5}}{2} \\ 1 & \frac{1-\sqrt{5}}{2} \end{bmatrix}\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \end{bmatrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \Rightarrow \begin{cases} m_1\ddot{q}_1 + \bar{k}_1 q_1 = p_1 \\ \ddot{q}_2 + \bar{k} \ldots \end{cases}$$

Se escrevem soluções

q_1: ...

q_2: ...