PME2352

26 de outubro de 2011

1 Vibrações com 2 graus de liberdade

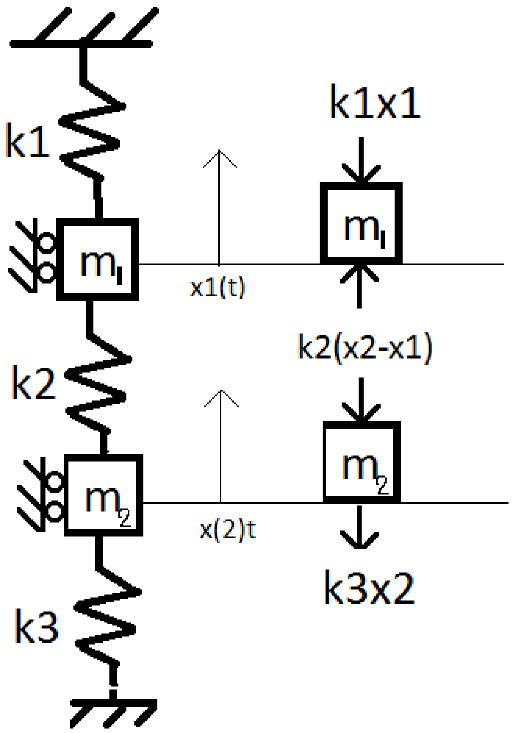

Figura 1: 1

1:

$$m_1\ddot{x}_1 = k_2(x_2 - x_1) - k_1x_1$$
$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) - k_3x_2$$

2:

$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0$$
$$m_2\ddot{x}_2 - k_2x_1 + (k_2+k_3)x_2 = 0$$

Soluções:

$$x_1(t) = A_1\sin(\omega t+\phi)$$
$$x_2(t) = A_2\sin(\omega t+\phi)$$

$$x_{11}(t) = A_{11}\sin(\omega_1 t+\phi_1)$$
$$x_{21}(t) = \alpha A_{11}\sin(\omega_1 t+\phi_1)$$

$$x_{12}(t) = A_{12}\sin(\omega_2 t+\phi_2)$$
$$x_{22}(t) = \beta A_{12}\sin(\omega_2 t+\phi_2)$$

Assim:

$$[-m_1\omega^2 + (k_1+k_2)A_1 - k_2A_2]\sin(\omega t+\phi) = 0$$
$$[-m_2\omega^2 A_2 - k_2A_2 - (k_2+k_{3)}A_2]\sin(\omega t+\phi) = 0$$

$$(k_1+k_2-m_1\omega^2)A_1 - k_2A_2 = 0$$
$$-k_2A_1 + (k_2+k_3-m_2\omega^2)A_2 = 0$$

$$\begin{pmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2+k_3-m_2\omega_2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Soluiçao nao trivial:

$$\Delta = (k_1+k_2-m_1\omega_2)(k_2+k_3-m_1\omega^2) - k_2^2 = 0$$
$$m_1m_2\omega^4 - [(k_2+k_3)m_1 + (k_1+k_2)m_2]\omega^2 + k_1k_2 + k_1k_3 + k_2k_3 = 0$$

$$\omega_{1,2}^2 = \frac{(k_2+k_3)m_1 + (k_1+k_2)m_2}{2m_1m_2} + -\sqrt{()^2 - bla}$$

$$\omega_1^2 > 0$$

$$\omega_2^2 > 0$$

Para $\omega^2 = \omega_1^2$

$$(k_1 + k_2 - m_1\omega_1^2)A_{11} - k_2 A_{21} = 0$$
$$A_{21} = \alpha A_{11}$$

Para $\omega^2 = \omega_2^2$

$$(k_1 + k_2 - m_1\omega_2^2)A_{12} - k_2 A_{22} = 0$$
$$A_{21} = \beta A_{12}$$

$$\begin{pmatrix} 2k - m\omega^2 & -k \\ -k & 2k - m\omega_2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Solução não-trivial

$$\Delta = (2k - m\omega^2) - k^2 = 0$$
$$(2k - m\omega^2) = + - k$$
$$\omega_1^2 = \frac{k}{m}$$
$$\omega_2^2 = \frac{3k}{m}$$

Para $\omega^2 = \omega_1^2$

$$(2k - k)A_{11} - kA_{21} = 0$$

Portanto $A_{21} = A_{11}$

$$-kA_{11} + (2k - k)A_{21} = 0$$
$$x_{11}(t) = A_{11}\sin(\sqrt{\frac{k}{m}}t + \phi_1)$$
$$x_{21}(t) = A_{11}\sin(\sqrt{\frac{k}{m}}t + \phi_1)$$

Para $\omega^2 = \omega_2^2$

$$(2k - 3k)A_{12} - kA_{22} = 0$$

Portanto $A_{22} = -A_{12}$

$$x_{12}(t) = A_{12}\sin(\sqrt{\frac{3k}{m}}t + \phi_2)$$

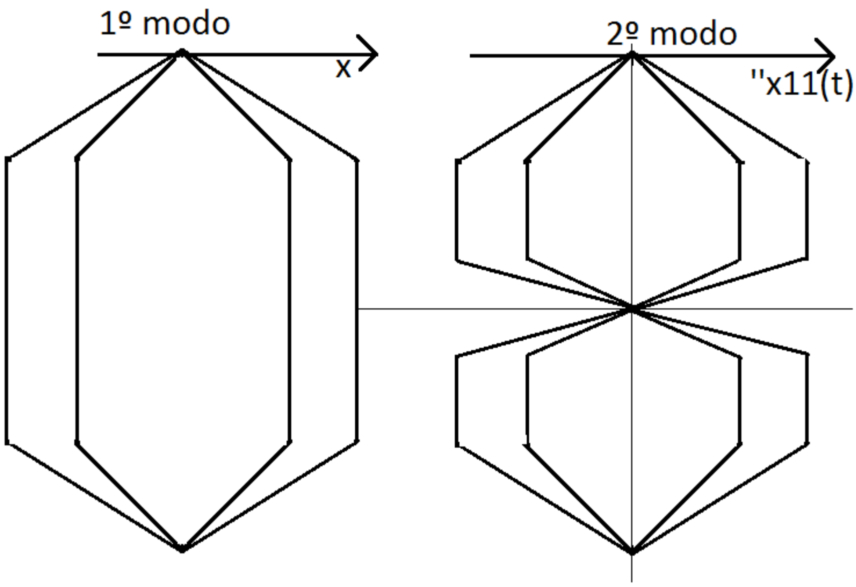

Figura 2: 2

$$x_{22}(t) = -A_{12}\sin(\sqrt{\frac{3k}{m}}t + \phi_2)$$

Soluções:

$$x_{11}(t) = A_{11}\sin(\sqrt{\frac{k}{m}}t + \phi_1)$$

$$x_{21}(t) = A_{11}\sin(\sqrt{\frac{k}{m}}t + \phi_1)$$

$$x_{12}(t) = A_{12}\sin(\sqrt{\frac{3k}{m}}t + \phi_2)$$

$$x_{22}(t) = -A_{12}\sin(\sqrt{\frac{3k}{m}}t + \phi_2)$$

$$x_1(t) = x_{11}(t) + x_{12}(t)$$

$$x_2(t) = x_{21}(t) + x_{22}(t)$$

$$x_1(t) = A_{11}\sin(\omega_1 t + \phi_1) + A_{12}\sin(\omega_2 t + \phi_2)$$

$$x_2(t) = A_{11}\sin(\omega_1 t + \phi_1) - A_{12}\sin(\omega_2 t + \phi_2)$$

$$x_1(t) = (A_{11}\cos(\phi_1))\sin(\omega_1 t) + (A_{11}\sin(\phi_1))\cos(\omega_1 t) + (A_{12}\cos(\phi_2))\sin(\omega_2 t) + (A_{12}\sin(\phi_2))\cos(\omega_2 t)$$

Onde:

$$(A_{11}\cos(\phi_1)) = A$$
$$(A_{11}\sin(\phi_1)) = B$$
$$(A_{12}\cos(\phi_2)) = C$$
$$(A_{12}\sin(\phi_2)) = D$$

Para $x_2(t)$, temos:

$$x_2(t) = A\sin(\omega_1 t) + B\cos(\omega_1 t) - C\sin(\omega_2 t) - D\cos(\omega_2 t)$$

Condições iniciais para t=0

- $x_1(0) = x_{10}$
- $x_2(0) = x_{20}$
- $\dot{x}_1(0) = \dot{x}_{10}$
- $\dot{x}_2(0) = \dot{x}_{20}$

$$\dot{x}_1(t) = A\omega_1\cos(\omega_1 t) - B\omega_1\sin(\omega_1 t) + C\omega_2\cos(\omega_2 t) - D\omega_2\sin(\omega_2 t)$$

$$\dot{x}_2(t) = A\omega_1\cos(\omega_1 t) - B\omega_1\sin(\omega_1 t) - C\omega_2\cos(\omega_2 t) + D\omega_2\sin(\omega_2 t)$$

$$x_{10} = B + D$$
$$x_{20} = x_2(0) = B - D$$
$$\dot{x}_1(0) = \dot{x}_{10} = A\omega_1 + C\omega_2$$
$$\dot{x}_2(0) = \dot{x}_{20} = A\omega_1 - C\omega_2$$

$$B = \frac{x_{10} + x_{20}}{2}$$
$$D = \frac{x_{10} - x_{20}}{2}$$
$$A = \frac{\dot{x}_{10} + \dot{x}_{20}}{2\omega_1}$$
$$C = \frac{\dot{x}_{10} - \dot{x}_{20}}{2\omega_2}$$