

$$\begin{cases} \frac{3}{2} m \ddot{x}_A + c \dot{x}_A - c \dot{x}_B + mg \mu_{\text{rol}} \dot{x}_A / |x_A| + 2k x_A - k x_B = m Y w_f^2 e^{i w_f t} \\ \frac{3}{2} m \ddot{x}_B - c \dot{x}_A + c \dot{x}_B + mg \mu_{\text{rol}} \dot{x}_B / |x_B| - k x_A + 2k x_B = m Y w_f^2 e^{i w_f t} \end{cases}$$

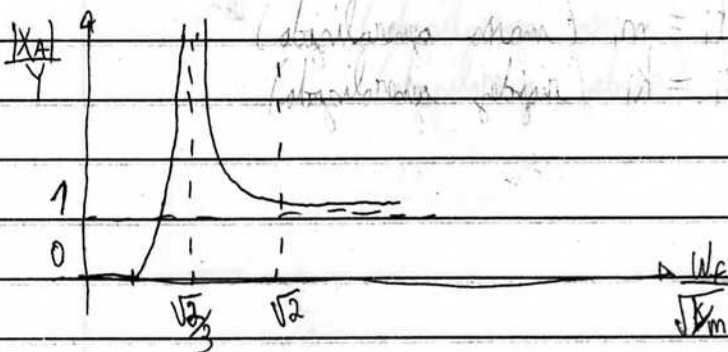
$$Z = 4 mg \mu_{\text{rol}} X = \pi C_{\text{eq}} w_f X^2 \rightarrow C_{\text{eq}} = \frac{4 mg \mu_{\text{rol}}}{\pi w_f X}$$

$$\begin{cases} x_A(t) = X_A e^{i w_f t} = |X_A| e^{i(w_f t - \psi_A)} \\ x_B(t) = X_B e^{i w_f t} = |X_B| e^{i(w_f t - \psi_B)} \end{cases}$$

$$\begin{cases} \left[2k - \frac{3}{2} m w_f^2 + c w_f i + \frac{4 mg \mu_{\text{rol}} i}{\pi |X_A|} \right] X_A - (k + c w_f i) X_B = m Y w_f^2 \\ -(k + c w_f i) X_A + \left[2k - \frac{3}{2} m w_f^2 + c w_f i + \frac{4 mg \mu_{\text{rol}} i}{\pi |X_B|} \right] X_B = m Y w_f^2 \end{cases}$$

$$\begin{bmatrix} 2k - \frac{3}{2} m w_f^2 + c w_f i & -(k + c w_f i) \\ -(k + c w_f i) & 2k - \frac{3}{2} m w_f^2 + c w_f i \end{bmatrix} \begin{pmatrix} X_A \\ X_B \end{pmatrix} = \begin{pmatrix} m Y w_f^2 - 4 mg \mu_{\text{rol}} i / \pi \\ m Y w_f^2 - 4 mg \mu_{\text{rol}} i / \pi \end{pmatrix}$$

$$X_A = \frac{(2k - \frac{3}{2} m w_f^2 + c w_f i)^2 - (k + c w_f i)^2}{(k - \frac{3}{2} m w_f^2)^2 (3k - \frac{3}{2} w_f^2 m + 2c w_f i)} =$$



Vibrações: sistemas com múltiplos graus de liberdade

$$\begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{bmatrix} \begin{pmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \\ \vdots \\ \ddot{x}_n(t) \end{pmatrix} + \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} \sin(\omega t + \phi)$$

$$[M] \ddot{X} + [K] X = 0 \quad \text{ou} \quad \ddot{X} + [A] X = 0 \quad \text{com} \quad [A] = [M]^{-1} [K]$$

$$\{-\omega^2 [M] + [K]\} X = \{0\}$$

$$\{-\omega^2 [I] + [A]\} X = \{0\} \quad \text{com} \quad X = X_0 \sin(\omega t + \phi) \quad \text{ou} \quad X = X_0 e^{i\omega t}$$

$$\{[A] - \lambda [I]\} X = \{0\}$$

$$|[A] - \lambda [I]| = 0 \quad \text{Equação característica para } \lambda$$

P/ cada autovalor λ_i , obtenha autovetores X_i :

$$\lambda_i [M] X_i = [K] X_i \quad \text{ou} \quad \lambda_i [I] X_i = [A] X_i$$

$$\lambda_i X_i^T [M] X_j = X_j^T [K] X_i \quad \text{ou} \quad \lambda_i X_i^T [I] X_j = X_j^T [A] X_i$$

$$\lambda_i [M] X_i = [K] X_i$$

$$\lambda_j [M] X_j = [K] X_j$$

$$(\lambda_i - \lambda_j) X_i^T [M] X_j = 0$$

Autovalores são ortogonais em relação às matrizes de massa

$$X_i^T [K] X_j = 0 \quad \text{ou} \quad X_i^T [M] X_j = 0 \quad \text{para} \quad i \neq j$$

$$\text{P/ } i=j \quad X_i^T [M] X_i = m_i \quad (\text{massa generalizada})$$

$$X_i^T [K] X_i = k_i \quad (\text{rigidez generalizada})$$

$$\{M\} \ddot{x} = \{F\} - \{K\}x + \{F\}$$

$$[\Phi] = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

$$\{F\} = \{F_1\} \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} + \{F_2\} \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + \{F_3\} \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

$$[\Phi]^T [M] [\Phi] = \begin{bmatrix} m_{11} & \dots & 0 \\ 0 & \dots & m_{nn} \end{bmatrix}$$

$$[\Phi]^T [K] [\Phi] = \begin{bmatrix} k_{11} & \dots & 0 \\ 0 & \dots & k_{nn} \end{bmatrix}$$

$$[M] \ddot{x}(t) + [K] x(t) = \{0\}$$

$$\ddot{x}(t) = [\Phi] \ddot{y}(t)$$

$$\ddot{y}(t) = [\Phi]^{-1} \ddot{x}(t)$$

$$[\Phi]^T [M] [\Phi] \ddot{y}(t) + [\Phi]^T [K] [\Phi] y(t) = [\Phi]^T \{0\} = \{0\}$$

$$\begin{bmatrix} m_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & m_n \end{bmatrix} \begin{pmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \\ \vdots \\ \ddot{y}_n(t) \end{pmatrix} + \begin{bmatrix} k_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_n \end{bmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\lambda_1 = \omega_1^2 = \frac{k_1}{m_1}$$

$$\lambda_i = \omega_i^2 = \frac{k_i}{m_i}$$

Vibrações em sistemas com n graus de liberdade

$$[M] \ddot{x}(t) + [C] \dot{x}(t) + [K] x(t) = \{F(t)\}$$

Se $[C] = \alpha [M] + \beta [K]$ amortecimento proporcional

Resolvendo o sistema homogêneo livre

$$[M] \ddot{x}(t) = [K] x(t) = \{0\}$$

$$\lambda_i = \{x_i\} \rightarrow [\Phi] = \{x_1, x_2, \dots, x_n\}$$

$$\text{Definir } \ddot{y}(t) = [\Phi]^{-1} \ddot{x}(t)$$

$$[\Phi]^T [M] [\Phi] \ddot{y}(t) + [\Phi]^T [C] [\Phi] \dot{y}(t) + [\Phi]^T [K] [\Phi] y(t) = [\Phi]^T F(t)$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_n \end{bmatrix} \ddot{y}(t) + \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \dot{y}(t) + \begin{bmatrix} k_1 & 0 \\ 0 & k_n \end{bmatrix} y(t) = \begin{bmatrix} f_1(t) \\ f_n(t) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = [\Phi]^T [M] [\Phi]$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = [\Phi]^T [K] [\Phi]$$

$$f_0(t) = f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + f(t) \times \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f_0(t) = f_0(t) [\Phi] = f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [\Phi] + f(t) \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} [\Phi]$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ f(t) \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{pmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{pmatrix} f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ f(t) \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{pmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$x = w = x$$

$$x = w = x$$

$$x = w = x$$

Abbildung der Summe von zwei Vektoren

$$f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

und nachfolgend mit der Ableitung

$$f_0(t) = f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(t) \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = f(t) \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$