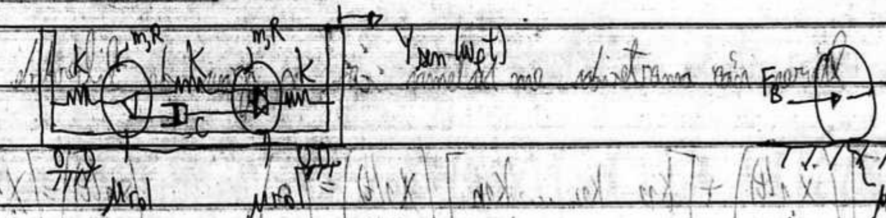


$$\begin{cases} \frac{3}{2} m \ddot{x}_A + c\dot{x}_A - c\dot{x}_B + m g \mu_{rol}\, \dot{x}_A/|\dot{x}_A| + 2k x_A - k x_B = m Y \omega_f^2 \cancel{\operatorname{sen}(\omega_f t)}\, e^{i\omega_f t} \\ \frac{3}{2} m \ddot{x}_B - c\dot{x}_A + c\dot{x}_B + m g \mu_{rol}\, \dot{x}_B/|\dot{x}_B| - k x_A + 2k x_B = m Y \omega_f^2 \cancel{\operatorname{sen}(\omega_f t)}\, e^{i\omega_f t} \end{cases}$$

$$Z = 4 m g \mu_{rol} X = \pi C_{eq} \omega_f X^2 \rightarrow \boxed{C_{eq} = \frac{4 m g \mu_{rol}}{\pi \omega_f X}}$$

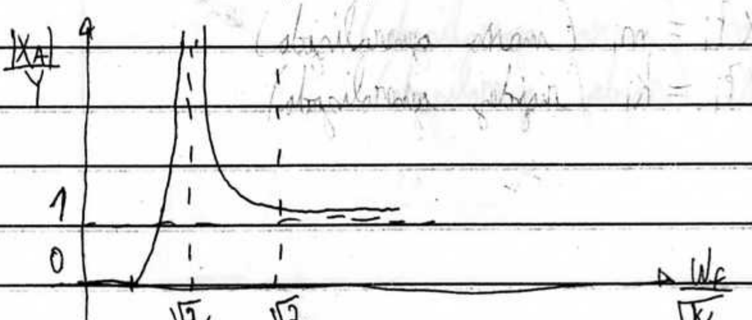
$$\begin{cases} x_A(t) = X_A e^{i\omega_f t} = |X_A| e^{i(\omega_f t - \psi_A)} \\ x_B(t) = X_B e^{i\omega_f t} = |X_B| e^{i(\omega_f t - \psi_B)} \end{cases}$$

$$\left[2k - \frac{3}{2} m\omega_f^2 + c\omega_f i + \frac{4 m g \mu_{rol}}{\pi |X_A|} i\right] X_A - (k + c\omega_f i) X_B = m Y \omega_f^2$$

$$-(k + c\omega_f i) X_A + \left[2k - \frac{3}{2} m\omega_f^2 + c\omega_f i + \frac{4 m g \mu_{rol}\, i}{\pi |X_B|}\right] X_B = m Y \omega_f^2$$

$$\begin{bmatrix} 2k - \frac{3}{2} m\omega_f^2 + c\omega_f i & -(k + c\omega_f i) \\ -(k + c\omega_f i) & 2k - \frac{3}{2} m\omega_f^2 + c\omega_f i \end{bmatrix} \begin{pmatrix} X_A \\ X_B \end{pmatrix} = \begin{pmatrix} m Y \omega_f^2 - 4 m g \mu_{rol} i/\pi \\ m Y \omega_f^2 - 4 m g \mu_{rol} i/\pi \end{pmatrix}$$

$$X_A = \frac{\quad}{(2k - \frac{3}{2} m\omega_f^2 + c\omega_f i)^2 - (k + c\omega_f i)^2} = \frac{\quad}{(k - \frac{3}{2} m\omega_f^2)(3k - \frac{3}{2}\omega_f^2 m + 2c\omega_f i)}$$



Vibrações livres não amortecidas em sistemas com n graus de liberdade

$$\begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ \vdots & & & \\ m_{n1} & \dots & & m_{nn}\end{bmatrix}\begin{Bmatrix}\ddot{x}_1(t)\\ \vdots \\ \ddot{x}_n(t)\end{Bmatrix} + \begin{bmatrix}k_{11} & k_{12} & \dots & k_{1n}\\ k_{n1} & \dots & & k_{nn}\end{bmatrix}\begin{Bmatrix}x_1(t)\\ x_n(t)\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix} \qquad \begin{Bmatrix}x_1(t)\\ \vdots \\ x_n(t)\end{Bmatrix} = \begin{Bmatrix}X_1\\ \vdots \\ X_n\end{Bmatrix}\text{sen}(\omega t+\phi)$$

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\}$$

$$\{-\omega^2[M]\{X\} + [K]\{X\}\}\,\text{sen}(\omega t+\phi) = \{0\}$$

$$\{-\underbrace{\omega^2}_{\lambda}[I] + \underbrace{[M]^{-1}[K]}_{[A]}\}\{X\} = \{0\}$$

$$\{[A] - \lambda[I]\}\{X\} = \{0\}$$

$$|[A] - \lambda[I]| = 0 \quad \{n \text{ autovalores } \lambda_i\}$$

P/ cada autovalor λ_i, obtenha autovetor $\{X\}_i$

$$\lambda_i[M]\{X\}_i = [K]\{X\}_i$$

$$\lambda_i\{X\}_j^T[M]\{X\}_i = \{X\}_j^T[K]\{X\}_i$$

$$\lambda_j[M]\{X\}_j = [K]\{X\}_j$$

$$\lambda_j\{X\}_i^T[M]\{X\}_j = \{X\}_i^T[K]\{X\}_j$$

$$(\lambda_i - \lambda_j)\{X\}_j^T[M]\{X\}_i = 0$$

autovetores são ortogonais em relação à matriz de massa

$$\{X\}_j^T[K]\{X\}_i = 0$$

P/ $i=j$ $\quad \{X\}_i^T[M]\{X\}_i = m_i$ (massa generalizada)

$\{X\}_i^T[K]\{X\}_i = k_i$ (rigidez generalizada)

Matriz dos modos

$$[\Phi] = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ \vdots & \vdots & & \vdots \\ X_{n1} & X_{n2} & \dots & X_{nn} \end{bmatrix}$$

$$[\Phi]^T [M][\Phi] = \begin{bmatrix} m_{11} & & 0 \\ & \ddots & \\ 0 & & m_{nn} \end{bmatrix}$$

$$[\Phi]^T [K] [\Phi] = \begin{bmatrix} k_{11} & & 0 \\ & \ddots & \\ 0 & & k_{nn} \end{bmatrix}$$

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = \{0\}$$

$$\{x(t)\} = [\Phi]\{y(t)\}$$

$$\{y(t)\} = [\Phi]^{-1}\{x(t)\}$$

$$[\Phi]^T[M][\Phi]\{\ddot{y}(t)\} + [\Phi]^T[K][\Phi]\{y(t)\} = [\Phi]\{0\} = \{0\}$$

$$\begin{bmatrix} m_1 & \text{---} & 0 \\ | & m_2 & | \\ 0 & \ddots & m_n \end{bmatrix} \begin{pmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \\ \vdots \\ \ddot{y}_n(t) \end{pmatrix} + \begin{bmatrix} k_1 & & 0 \\ & k_2 & \\ 0 & \ddots & k_n \end{bmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_1 = w_1^2 = \frac{k_1}{m_1}$$

$$\lambda_i = w_i^2 = \frac{k_i}{m_i}$$

Vibrações em sistemas com n graus de liberdade

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\}$$

Se $[C] = \alpha[M] + \beta[K]$ amortecimento proporcional

Resolvendo o sistema homogêneo livre

$$[M]\{\ddot{x}(t)\} = [K]\{x(t)\} = \{0\}$$

$$\lambda_i \rightarrow \{x_i\} \rightarrow [\Phi] = \{\{x\}_1 \quad \{x\}_2 \quad \dots \{x\}_n\}$$

Define $\{y(t)\} = [\Phi]^{-1}\{x(t)\}$

$$[\Phi]^T[M][\Phi]\{\ddot{y}(t)\} + [\Phi]^T[C][\Phi]\{\dot{y}(t)\} + [\Phi]^T[K][\Phi]\{y(t)\} = [\Phi]^T\{F(t)\}$$

$$\begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{bmatrix}\{\ddot{y}(t)\} + \left\{\alpha\begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{bmatrix} + \beta\begin{bmatrix} k_1 & & 0 \\ & \ddots & \\ 0 & & k_n \end{bmatrix}\right\}\{\dot{y}(t)\} + \begin{bmatrix} k_1 & & 0 \\ & \ddots & \\ 0 & & k_n \end{bmatrix}\{y(t)\} = [\Phi]^T\{F(t)\}$$