COS-D419 Factor Analysis and Structural Equation Models 2023, Assignment 6

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Contents

1 Read me

The texts that reflect my understanding/questions/doubts have been highlighted in red color. The texts that describes important steps/results or that corresponds to certain exercise requirement have been highlighted in blue color.

2 Preparation

2.1 Read in the data set

```
library(tidyverse)
library(readr)
library(here)

#This week's file name
```

```r
latest.name1 <- "ELEMIND1.CSV"
latest.name2 <- "ELEMIND2.CSV"
#read in the data
ele.cali <-  #elementary school
  read_csv(
    file.path(
      here(),
      'data',
      latest.name1
      ),
      show_col_types = FALSE
    )

ele.vali <- #secondary school
  read_csv(
    file.path(
      here(),
      'data',
      latest.name2
      ),
      show_col_types = FALSE
    )
```

2.2 Write functions

To control length of reports, codes of fucntions were not showing in the current report. Yet they are available in .rmd report.

2.2.1 To generate a function for calculating chi square difference was defined.

2.2.2 Write a function to print a table with concerned parameters

############3

2.2.3 Write a function to print a table with concerned parameters

2.2.4 to generate CFA results with improved readability

2.2.5 Write a function to simplify plotting of merged tables for multi-group fit indicies

2.2.6 Write a function to simplify plotting of merged tables for multi-group fit indicies with chi square difference statistics

2.2.7 Write a function to simplify plotting aligned residual variance and co-variance tables

2.2.8 Write a function for correlation matrix with numbers

2.2.9 to generate a function for histogram overlapping with density plot

2.2.10 to generate a function for violin overlapping with box plot

2.2.11 To generate a function describing continuous data set

2.2.12 Write a function describing continuous data set

2.2.13 Write a function for histogram overlapping with density plot

2.2.14 Write a function to generate dot distribution plot

2.2.15 Write a fuction to generate correlation matrix with statistical test

3 Inspect the data

3.1 Distribution of values

```
#generate the plots, by subgroup of teachers
p.dist.elm <-
  corr.density(
    ele.cali,
    fig.num = "1(a)",
    group = "calibration dataset"
    )

p.dist.sec <-
  corr.density(
    ele.vali,
    fig.num = "1(b)",
    group = "validation dataset"
    )
#print the plot
library(patchwork); p.dist.elm/p.dist.sec
```

Figure 1(a) Distribution of the indicators for calibration dataset

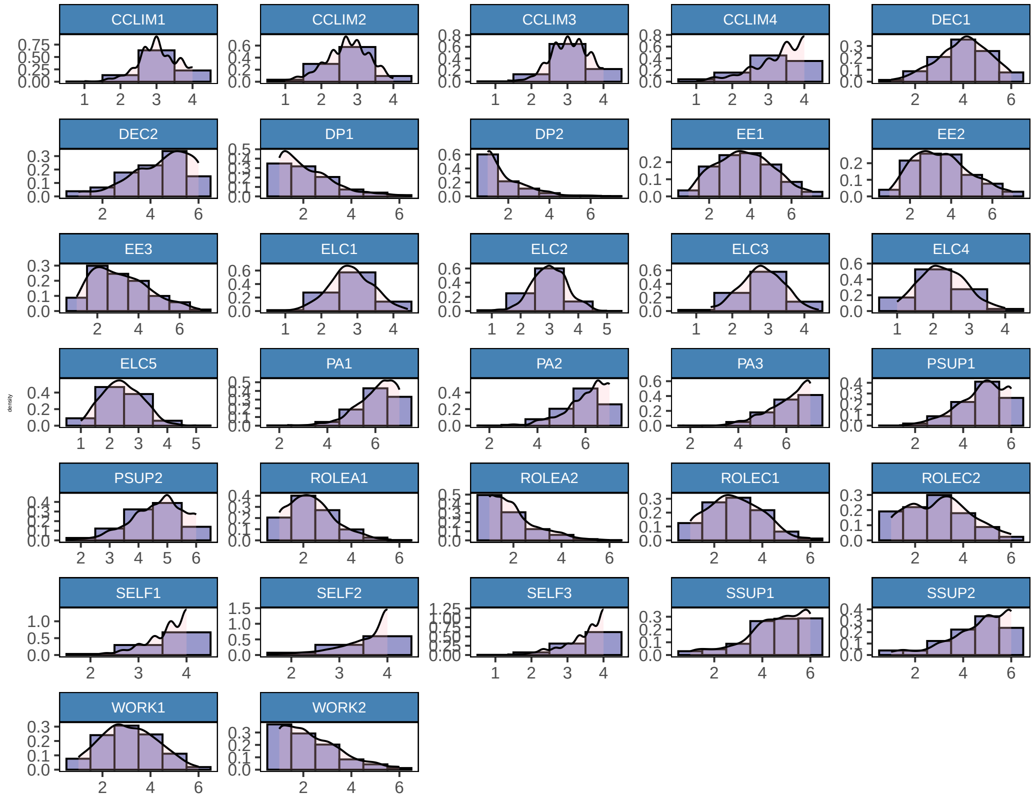
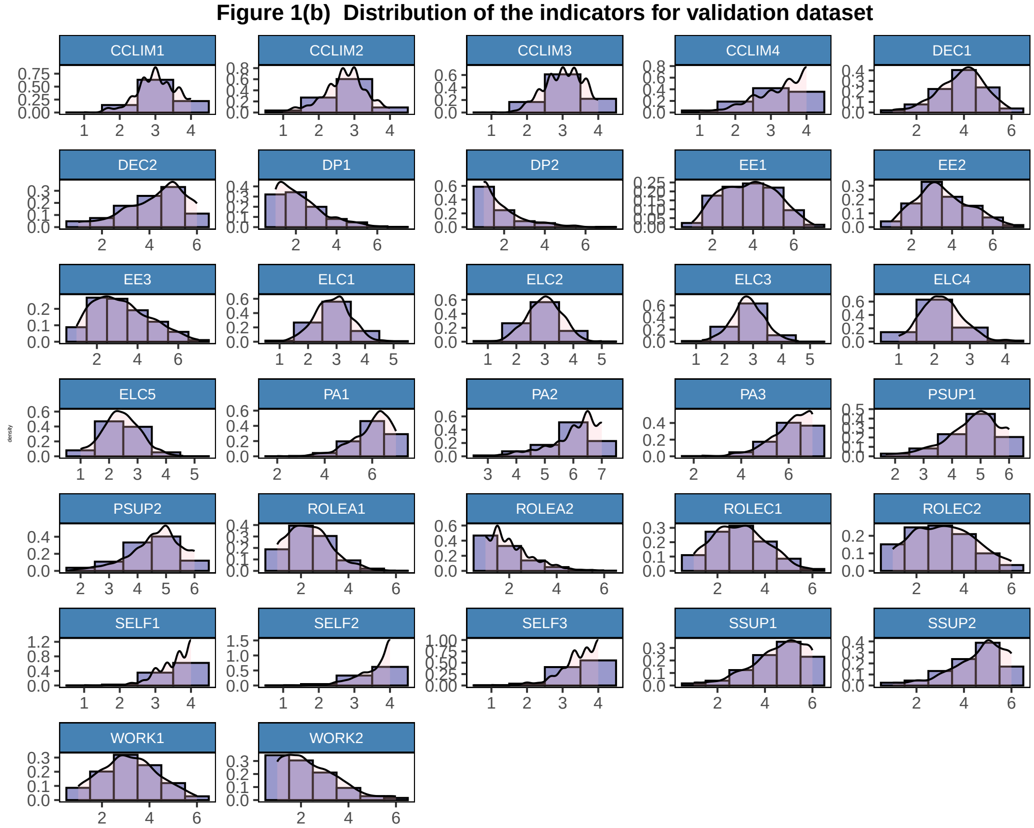


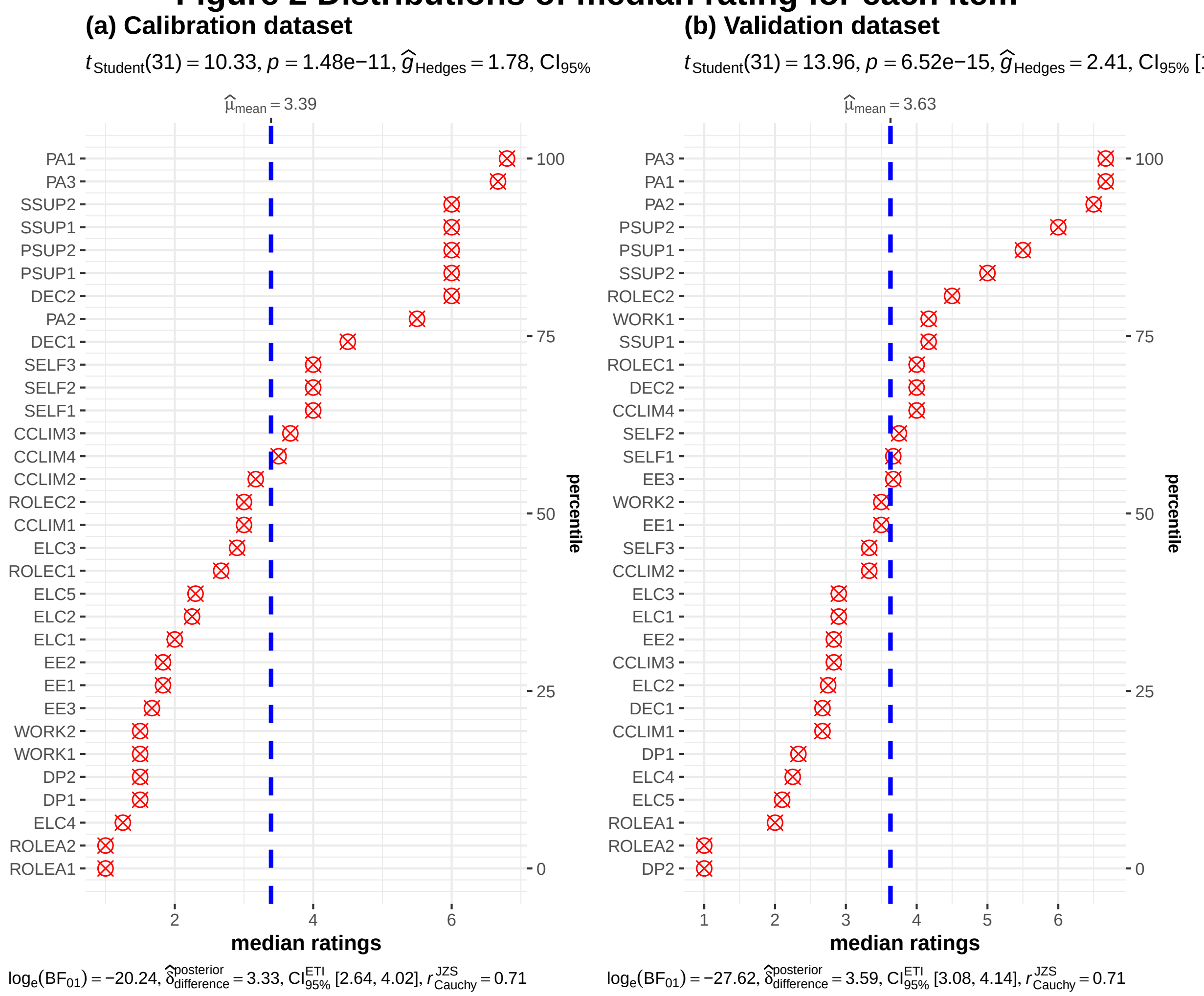
Figure 1(b) Distribution of the indicators for validation dataset



3.2 Distributions of Item statistics (median)

```
#generate plot by subgroups of teachers
p.dot.elm <-
  dot.dist(
    data = ele.cali, type = "median",
    title = "(a) Calibration dataset"
    )
p.dot.sec <-
  dot.dist(
    data = ele.vali,  type = "median",
    title = "(b) Validation dataset"
    )
#plot layout
patchwork <- p.dot.elm|p.dot.sec
#print the plot with a general title
patchwork+plot_annotation(
    title =
      'Figure 2 Distributions of median rating for each item',
    theme =
      theme(plot.title =
              element_text(
                size = 16,
                face = "bold",
                vjust = -1.5,
                hjust =0.5)
            )
    )
```

Figure 2 Distributions of median rating for each item



3.3 Correlation

```r
#save variable names of MBI indicators to object
indi.EE <- paste0("EE", 1:3)
indi.DP <- paste0("DP", 1:2)
indi.PA <- paste0("PA", 1:3)
scale.MBI <-
  c(indi.EE,
    indi.DP,
    indi.PA)
#save variable names of TSS indicators to object
indi.ROLEC <- paste0("ROLEC", 1:2)
indi.ROLEA <- paste0("ROLEA", 1:2)
indi.WORK <- paste0("WORK", 1:2)
indi.CLC <- paste0("CCLIM", 1:4)
indi.DEC <- paste0("DEC", 1:2)
indi.SUPS <- paste0("SSUP", 1:2)
```

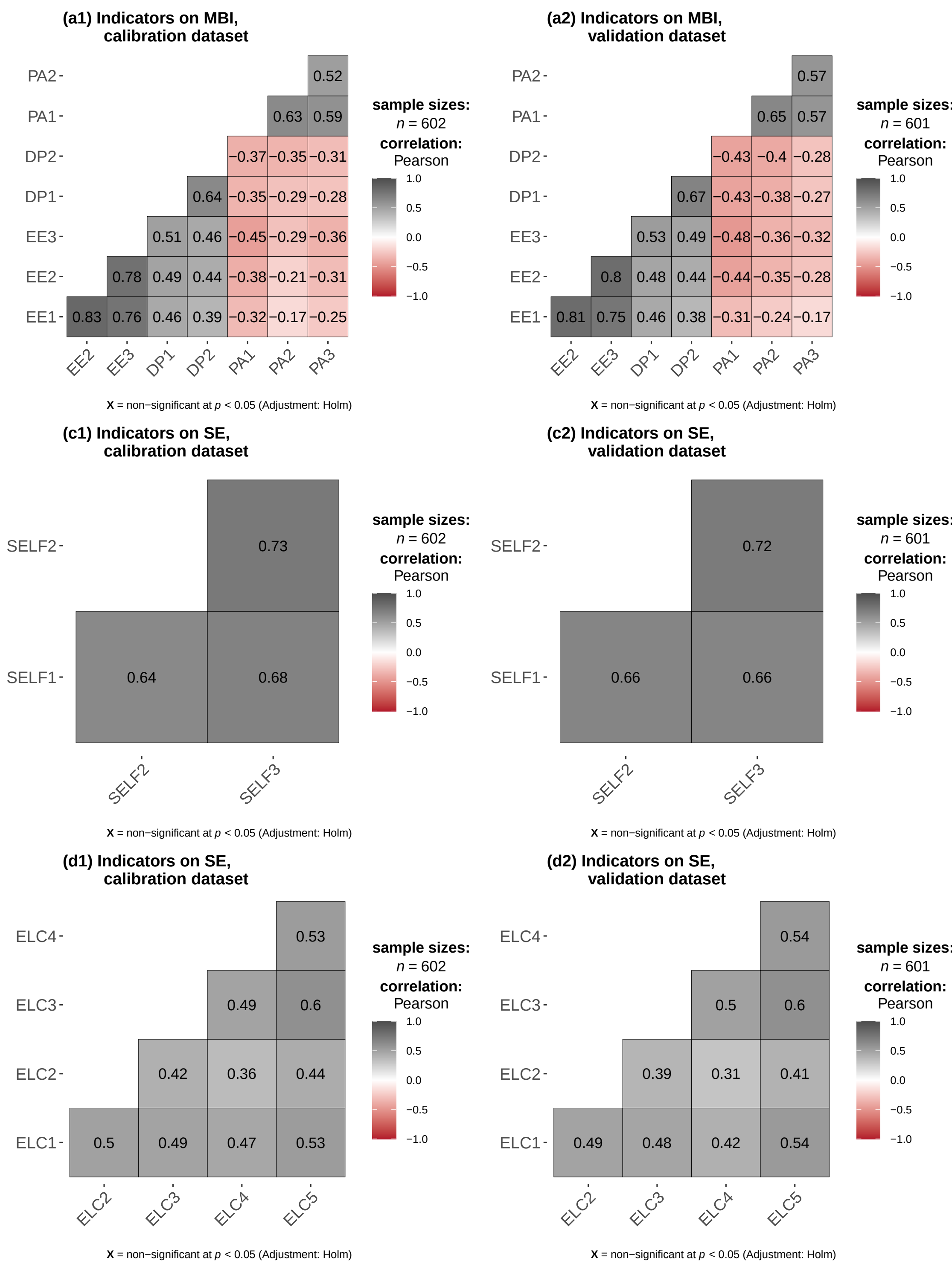
```r
indi.PEERS <- paste0("PSUP", 1:2)
scale.TSS <-
  c(indi.ROLEC,
    indi.ROLEA,
    indi.WORK,
    indi.CLC,
    indi.DEC,
    indi.SUPS,
    indi.PEERS)
```

```r
#save variable names of other indicators to object
scale.SE <- paste0("SELF", 1:3)
scale.ELC <- paste0("ELC", 1:5)

#generate the correlation plots scale-wise
p.cor.MBI.cali <-
       mycor(
         data = ele.cali,
         cols = scale.MBI,
         "(a1) Indicators on MBI,
         calibration dataset"
         )

p.cor.MBI.vali <-
       mycor(
         data = ele.vali,
         cols = scale.MBI,
         "(a2) Indicators on MBI,
         validation dataset"
         )

p.cor.TSS.cali <-
       mycor(
         data = ele.cali,
         cols = scale.TSS,
         "(b1) Indicators on TSS, calibration dataset"
         )

p.cor.TSS.vali <-
       mycor(
         data = ele.vali,
         cols = scale.TSS,
         "(b2) Indicators on TSS, validation dataset"
         )

p.cor.SE.cali <-
       mycor(
         data = ele.cali,
         cols = scale.SE,
         "(c1) Indicators on SE,
         calibration dataset"
         )
```

```r
p.cor.SE.vali <-
       mycor(
         data = ele.vali,
         cols = scale.SE,
         "(c2) Indicators on SE,
         validation dataset"
         )

p.cor.ELC.cali <-
       mycor(
         data = ele.cali,
         cols = scale.ELC,
         "(d1) Indicators on SE,
         calibration dataset"
         )

p.cor.ELC.vali <-
       mycor(
         data = ele.vali,
         cols = scale.ELC,
         "(d2) Indicators on SE,
         validation dataset"
         )
```

```r
#plot sub-figure layout
patchwork1 <-
  p.cor.MBI.cali/p.cor.SE.cali/p.cor.ELC.cali|
  p.cor.MBI.vali/p.cor.SE.vali/p.cor.ELC.vali


patchwork2 <-
  p.cor.TSS.cali/p.cor.TSS.vali

patchwork1+
  plot_annotation(
    title =
      'Figure 3-1 Correlalogram for indicators of TSS scale',
    theme =
      theme(plot.title =
              element_text(
                size = 16,
                face = "bold",
                vjust = -1.5,
                hjust =0.5
                )
            )
    )
```

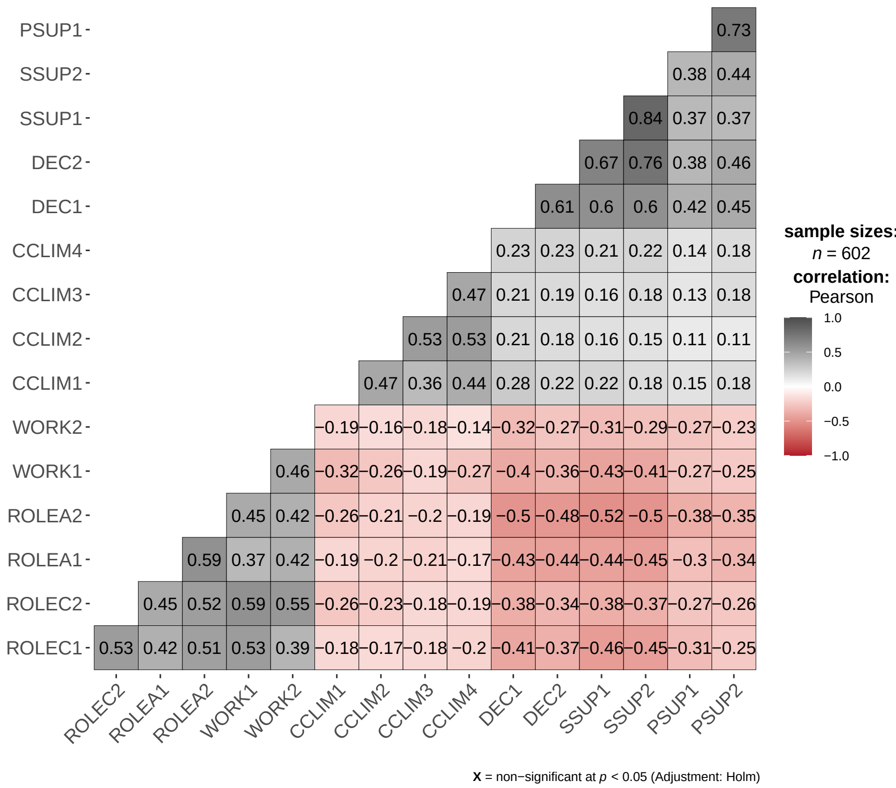
Figure 3−1 Correlalogram for indicators of TSS scale



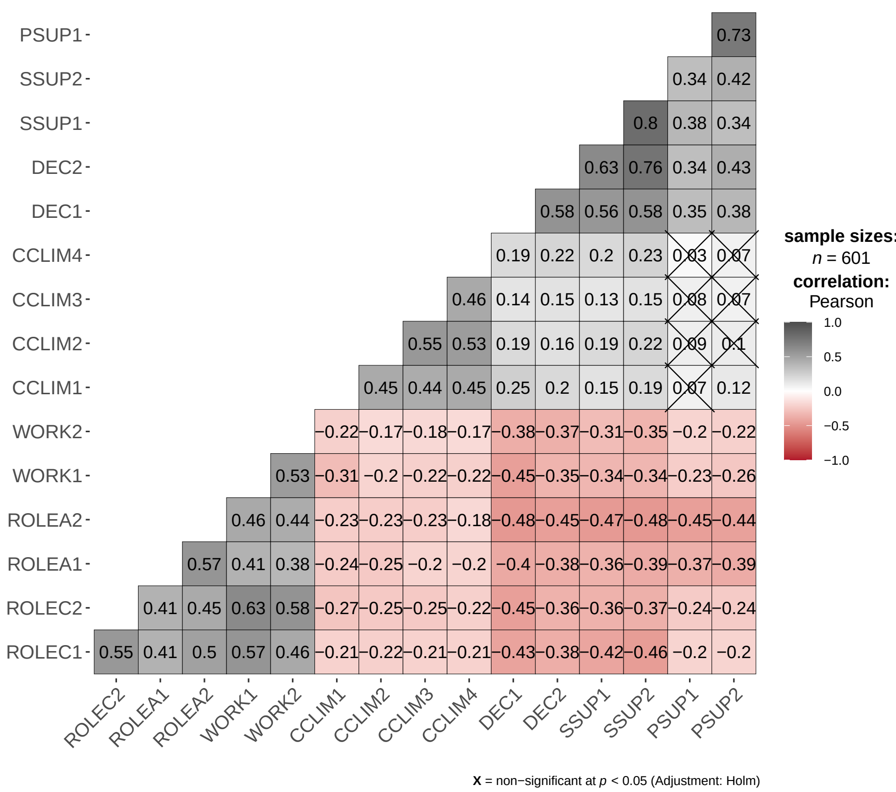
```
patchwork2+
  plot_annotation(
    title =
      'Figure 3-2 Correlalogram for indicators of MBI, self-esteem, external
    locus of control scales',
    theme =
      theme(plot.title =
              element_text(
                size = 16,
                face = "bold",
                vjust = -1.5,
                hjust =0.5
                )
              )
    )
```

Figure 3−2 Correlalogram for indicators of MBI, self−esteem, external locus of control scales

(b1) Indicators on TSS, calibration dataset



(b2) Indicators on TSS, validation dataset



4 Test the equivalence of causal structure involving the impact of organizational and personality factors on three facets of burnout for elementray teachers between calibration and validation datasets

This involves three steps:

(a) Define, modify and estimate a baseline model for the calibration group:

(b) Form and test the multi-group configural model with no parameter constraints.

(c) to test for the in-variance of common structural regression (or causal) paths across calibration and validation groups.

4.1 Define and estimate the baseline model for the calibration group

4.1.1 Establish and modify the hypothesized model (initial model) for calibration group

(1) Define the initial model for calibration group

```
initial.model <- '
# Burnout Factors:
# EE: EmotionalExhaustion;DP: Depersonalization;PA: PersonalAccomplishment
 F1ROLA =~ ROLEA1 + ROLEA2
 F2ROLC =~ ROLEC1 + ROLEC2
 F3WORK =~ WORK1 + WORK2
 F4CLIM =~ CCLIM1 + CCLIM2 + CCLIM3 + CCLIM4
 F5DEC =~ DEC1 + DEC2
 F6SSUP =~ SSUP1 + SSUP2
 F7PSUP =~ PSUP1 + PSUP2
 F8SELF =~ SELF1 + SELF2 + SELF3
 F9ELC =~ ELC1 + ELC2 + ELC3 + ELC4 + ELC5
 F10EE =~ EE1 + EE2 + EE3
 F11DP =~ DP1 + DP2
 F12PA =~ PA1 + PA2 + PA3
# Regression paths:
 F8SELF ~ F5DEC + F6SSUP + F7PSUP
 F9ELC ~ F5DEC
 F10EE ~ F2ROLC + F3WORK + F4CLIM
 F11DP ~ F2ROLC + F10EE
 F12PA ~ F1ROLA + F8SELF + F9ELC + F10EE + F11DP
'
```

(2) Visualize the initial model for calibration group

To approximate the visual effect on slides, the coordinates for each nodes were defined on a 60 by 72 matrix.

```
library(semPlot)
#generate a matrix
m <- matrix(NA, 60, 72)
#define positions of the factors
```

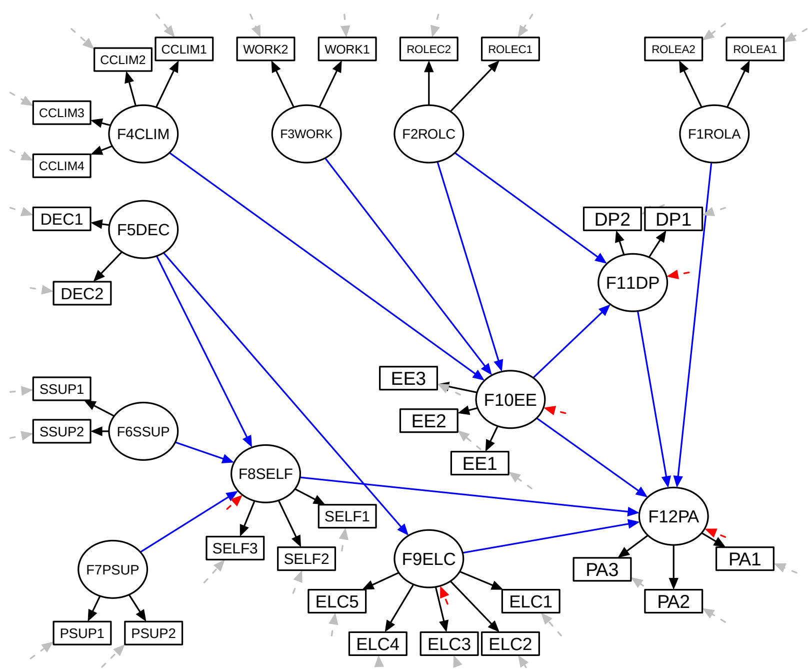
```
m[12, 68] <- "F1ROLA"
m[12, 40] <- "F2ROLC"
m[12, 28] <- "F3WORK"
m[12,12] <- "F4CLIM"
m[21,12] <-"F5DEC"
m[40,12] <-"F6SSUP"
m[53,9] <-"F7PSUP"
m[44,24] <-"F8SELF"
m[52,40] <-"F9ELC"
m[37,48] <-"F10EE"
m[26,60] <-"F11DP"
m[48,64] <-"F12PA"
#define the positions of the indicators (parcelled items)
m[4, 72] <- "ROLEA1"
m[4, 64] <- "ROLEA2"
m[4, 48] <- "ROLEC1"
m[4, 40] <- "ROLEC2"
m[4, 32] <- "WORK1"
m[4, 24] <- "WORK2"
m[4, 16] <- "CCLIM1"
m[5, 10] <- "CCLIM2"
m[10, 4] <- "CCLIM3"
m[15, 4] <- "CCLIM4"
m[20, 4] <- "DEC1"
m[27, 6] <- "DEC2"
m[36, 4] <- "SSUP1"
m[40, 4] <- "SSUP2"
m[59, 6] <- "PSUP1"
m[59, 13] <- "PSUP2"
m[48, 32] <- "SELF1"
m[52, 28] <- "SELF2"
m[51, 21] <- "SELF3"
m[56, 50] <- "ELC1"
m[60, 48] <- "ELC2"
m[60, 42] <- "ELC3"
m[60, 35] <- "ELC4"
m[56, 31] <- "ELC5"
m[43, 45] <- "EE1"
m[39, 40] <- "EE2"
m[35, 38] <- "EE3"
m[20, 64] <- "DP1"
m[20, 58] <- "DP2"
m[52, 71] <- "PA1"
m[56, 64] <- "PA2"
m[53, 57] <- "PA3"
```

The diagram of the initial model was generated.

```
semPaths(semPlotModel(initial.model),
         style = "lisrel",
         rotation = 2,
         sizeLat = 6,
         sizeLat2 = 5,
```

```
         sizeMan = 5,
         sizeMan2 = 2,
         residScale = 4,
         shapeMan = "rectangle",
         edge.color = c(rep("black", 32), #34
                        rep("blue", 14),
                        rep("gray", 32),
                        rep("red", 5)),
         residuals = T,
         layout = m,
         nCharNodes=0,
         optimizeLatRes = T,
         exoVar = F)
title(main = list("Figure 4. Hypothesized model of elementary teacher burnout",
                  cex = 1.5, font =1),
     outer = F, line = -1)
title(
  sub =
  "Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
  blue arrow indicates regression path; black arrow indicates factor loading",
  ine = 0, adj = 0.7
  )
```

Figure 4. Hypothesized model of elementary teacher burnout



Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
blue arrow indicates regression path; black arrow indicates factor loading

(3) Estimate the initial model for calibration group

Table 1: Fit indices for calibration dataset(initial model)

Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Initial model	897.816(429, <0.001)	0.949	0.941	0.043(1.000)	0.055	1.092

* Chi square scaling factor

```
library(lavaan)
library(knitr)
library(kableExtra)
model1 <- initial.model # defined above
# Estimate the model with the robust (MLM) estimator:
sem1 <-
  sem(
    model1,
    data = ele.cali,
    estimator = "MLM",
    mimic = "Mplus"
  )
# Numerical summary of the model:
sem1.fit <-
  cfa.summary.mlm.a(sem1) |>
  t() |>
  as.data.frame()

names(sem1.fit) <- sem1.fit[1,]
sem1.fit <- sem1.fit[-1,]
rownames(sem1.fit) <- NULL

sem1.fit <-
  sem1.fit |>
  mutate(Model = "Initial model") |>
  select(Model, everything())
#print the table
multi.fit.tab(sem1.fit, "Fit indices for calibration dataset(initial model)")
```

The values of fit indices were basically acceptable, though most of them were still fell a little below/above the required cutoff. See table 1. However, residual variance and co-variance still needed to be checked for any anomaly.

```
#print concern table for model 1
concern.table(sem1,
              nofpath = 14,
              nofpredictor = 7,
              "model1")
```

See table 2. I can readily see a couple of structural regression paths were not significant. I left these aberrant parameters untreated for the current stage.

The correlation between Factors 3 (workload) and 2 (role conflict) exceeds a value of 1.00, which are Heywood cases. This finding indicated a definite overlapping of variance between the factors of Role Conflict and Work Overload such that divergent (i.e., discriminant) validity between these two constructs is in-distinctive. It needed to be addressed.

Table 2: Residual variance of structural regression path and select factors for model1

Parameter*	B†	Beta‡	SE	Z	p-value
Regression paths (Residual variance)					
F5DEC→F8SELF	0.777	1.647	0.162	4.788	0
F6SSUP→F8SELF	-0.404	-1.216	0.096	-4.210	0
F7PSUP→F8SELF	-0.049	-0.106	0.050	-0.978	0.328
F5DEC→F9ELC	-0.246	-0.45	0.027	-9.146	0
F2ROLC→F10EE	15.857	10.299	28.587	0.555	0.579
F3WORK→F10EE	-14.277	-10.114	27.143	-0.526	0.599
F4CLIM→F10EE	-3.764	-1.07	6.284	-0.599	0.549
F2ROLC→F11DP	0.115	0.096	0.068	1.685	0.092
F10EE→F11DP	0.456	0.588	0.046	9.924	0
F1ROLA→F12PA	-0.135	-0.131	0.065	-2.089	0.037
F8SELF→F12PA	0.318	0.164	0.102	3.120	0.002
F9ELC→F12PA	-0.088	-0.053	0.065	-1.350	0.177
F10EE→F12PA	-0.054	-0.092	0.038	-1.410	0.158
F11DP→F12PA	-0.25	-0.331	0.055	-4.516	0
Endogenous factors(Residual variance)					
F8SELF	0.093	0.705	0.012	8.052	0
F9ELC	0.142	0.798	0.014	10.262	0
F10EE	3.457	2.371	5.074	0.681	0.496
F11DP	0.511	0.583	0.058	8.728	0
F12PA	0.334	0.672	0.036	9.266	0
Exogenous factors (Residual covariance)					
F2ROLC←→F1ROLA	0.43	0.802	0.041	10.456	0
F3WORK←→F1ROLA	0.47	0.804	0.042	11.230	0
F4CLIM←→F1ROLA	-0.088	-0.375	0.015	-6.033	0
F5DEC←→F1ROLA	-0.415	-0.789	0.040	-10.302	0
F6SSUP←→F1ROLA	-0.501	-0.67	0.052	-9.539	0
F7PSUP←→F1ROLA	-0.28	-0.52	0.031	-9.063	0
F3WORK←→F2ROLC	0.674	1.005	0.050	13.388	0
F4CLIM←→F2ROLC	-0.104	-0.387	0.016	-6.359	0
F5DEC←→F2ROLC	-0.419	-0.694	0.042	-10.047	0
F6SSUP←→F2ROLC	-0.49	-0.572	0.051	-9.519	0
F7PSUP←→F2ROLC	-0.256	-0.415	0.034	-7.619	0
F4CLIM←→F3WORK	-0.135	-0.46	0.020	-6.781	0
F5DEC←→F3WORK	-0.456	-0.692	0.042	-10.721	0
F6SSUP←→F3WORK	-0.537	-0.575	0.051	-10.439	0
F7PSUP←→F3WORK	-0.278	-0.413	0.036	-7.615	0
F5DEC←→F4CLIM	0.1	0.379	0.017	5.993	0
F6SSUP←→F4CLIM	0.107	0.285	0.022	4.897	0
F7PSUP←→F4CLIM	0.066	0.246	0.015	4.289	0
F6SSUP←→F5DEC	0.798	0.95	0.060	13.364	0
F7PSUP←→F5DEC	0.403	0.665	0.039	10.376	0
F7PSUP←→F6SSUP	0.433	0.503	0.046	9.476	0

Note:

Values highlighted in red should be taken note of

* → indicates regression path

† Crude estimates

‡ Standardized estimates

(4) Re-specification of initial model to model 2

Given the two factors in the Heywood case are different factors comprising TSS construct, one approach is to combine these two factors into one, leading to 12-1=11 factors in the structure. I did this and refit the model (model 2).

```
#replace the old parameters with new one
library(stringr)
model2 <-
  initial.model |>
  str_replace(".F3WORK.=~.WORK1.+.WORK2\n", "") |>
  str_replace(".F2ROLC.=~.ROLEC1.+.ROLEC2",
              " F2ROWO =~ ROLEC1 + ROLEC2 + WORK1 + WORK2") |>
  str_replace_all("F3WORK", "F2ROWO") |>
  str_replace_all("F2ROLC", "F2ROWO") |>
  str_replace_all("F2ROWO.+.F2ROWO", "F2ROWO")

#update the factor indexing
for (i in 4:12){
  original <- paste0("\\sF", i) # \\s is  regex for white-space
  new <- paste0(" F", i-1)
  model2 <- model2 |>
    str_replace_all(original, new)
}
```

4.1.2 Establish and modify the model 2 for calibration group

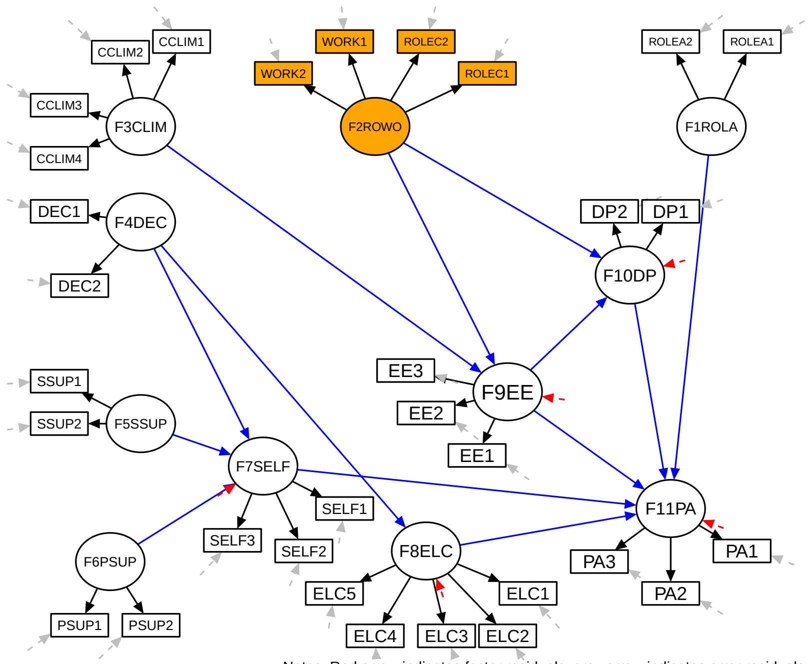
(1) Visualize model 2

```
m[12, 40] <- NA
m[12, 28] <- NA
m[12, 35] <- "F2ROWO"
m[12,12] <- "F3CLIM"
m[21,12] <-"F4DEC"
m[40,12] <-"F5SSUP"
m[53,9] <-"F6PSUP"
m[44,24] <-"F7SELF"
m[52,40] <-"F8ELC"
m[37,48] <-"F9EE"
m[26,60] <-"F10DP"
m[48,64] <-"F11PA"
m[4, 24] <- NA
m[4, 48] <- NA
m[7, 26] <- "WORK2"
m[7, 46] <- "ROLEC1"
```

```
grps <- list(
  c("F2ROWO"),
  c(
    "F3CLIM",
    "F4DEC",
    "F5SSUP",
```

```
    "F6PSUP",
    "F7SELF",
    "F8ELC",
    "F9EE",
    "F10DP",
    "F11PA",
    "F1ROLA"
  )
)
semPaths(semPlotModel(model2),
         style = "lisrel",
         rotation = 2,
         sizeLat = 6,
         sizeLat2 = 5,
         sizeMan = 5,
         sizeMan2 = 2,
         residScale = 4,
         shapeMan = "rectangle",
         edge.color = c(rep("black", 32), #34
                        rep("blue", 13),
                        rep("gray", 32),
                        rep("red", 5)),
         residuals = T,
         layout = m,
         nCharNodes=0,
         optimizeLatRes = T,
         exoVar = F,
         group = grps,
         color = c("orange", "white"))
title(main = list("Figure 5. Model 2 of teacher burnout, modified from initial model",
                  cex = 1.5, font =1),
     outer = F, line = -1)
title(sub =
"Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
        Blue arrow indicates regression path; black arrow indicates factor loading;
                                         Newly merged factor is highlighted in orange",
     line = 0, adj = 0.7)
```

Figure 5. Model 2 of teacher burnout, modified from initial model



Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
Blue arrow indicates regression path; black arrow indicates factor loading;
Newly merged factor is highlighted in orange

(2) Estimate model2 for calibration group

```
sem2 <-
  sem(
    model2,
    data = ele.cali,
    estimator = "MLM",
    mimic = "Mplus"
  )
```

```
# Numerical summary of the model:
sem2.fit <-
  cfa.summary.mlm.a(sem2) |>
  t() |>
  as.data.frame()

#combine with preceding fit indices
names(sem2.fit) <- sem2.fit[1,]
sem2.fit <- sem2.fit[-1,]
rownames(sem2.fit) <- NULL

sem2.fit <-
  sem2.fit |>
  mutate(Model = "Model2") |>
```

Table 3: Fit indices for calibration dataset, model2 comparing with preceding model

Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Initial model	897.816(429, <0.001)	0.949	0.941	0.043(1.000)	0.055	1.092
Model2	955.863(436, <0.001)	0.943	0.935	0.045(0.994)	0.060	1.091

* Chi square scaling factor

```
  select(Model, everything())

sem12.fit <- rbind(sem1.fit, sem2.fit)

#print the table
multi.fit.tab(sem12.fit,
              "Fit indices for calibration dataset, model2 comparing with preceding model")
```

See table 3. Goodness-of-fit statistics for this modified model 2 were as follows: chi-square(436) = 955.863, CFI= 0.943, RMSEA = 0.045, suggesting relatively well fit.

(3) Re-specification of model 2 to model 3&4

```
#extract needed variables
MI.model2 <- modindices(sem2,
                  standardized = TRUE,
                  sort. = TRUE,
                  maximum.number = 50) |>
  filter(op %in% c("~","~~"))

#adapt to publication style
MI.model2 <- MI.model2 |>
  mutate(op = ifelse(op == "~", "→","↔"),
    Parameter = paste(rhs, op, lhs)) |>
  select(
    'Parameter*' = Parameter,
    MI = mi,
    EPC = epc,
    "std EPC" = sepc.all
  ) |>
  filter(MI > 30)

#print the table
MI.model2 |>
  kable(digits = 3,
        booktab = T,
        linesep = "",
        caption = "Selected modification indices for model 2") |>
  kable_styling(latex_options = "striped") |>
  row_spec(c(1,2), color = "red") |>
  footnote(general =
             "Parameters highlighted in red is of special concern",
    symbol = c('"→" indicates regression path; "↔" indicates residual covariance'))
```

Table 4: Selected modification indices for model 2

Parameter*	MI	EPC	std EPC
F2ROWO → F8ELC	51.043	0.281	0.503
EE2 ←→ EE1	46.273	0.297	0.876
F5SSUP → F8ELC	39.419	0.384	0.994
F10DP → F9EE	34.264	-2.136	-1.657
F10DP ←→ F9EE	34.261	-1.091	-1.687
F3CLIM → F10DP	34.257	-0.796	-0.292
F10DP ←→ F3CLIM	31.063	-0.073	-0.297

Note:

Parameters highlighted in red is of special concern

* "→" indicates regression path; "←→" indicates residual covariance

See table 4. Two parameters with the highest values were substantively meaningful. They are (a) the structural path of F8 on F2 (External Locus of Control on Role Conflict/Work Overload) and (b) a covariance between residuals associated with the observed variables EE1 and EE2, both of which are highlighted and flagged in red. They were incorporated into the model consecutively. F8 on F2 went first. They were re-specified as follows:

```
model3 <- paste(model2, "F8ELC ~ F2ROWO\n")
model4 <- paste(model3, "EE1 ~~ EE2\n")
```

4.1.3 Establish and modify the model 3 and model 4 for calibration group, consecutively

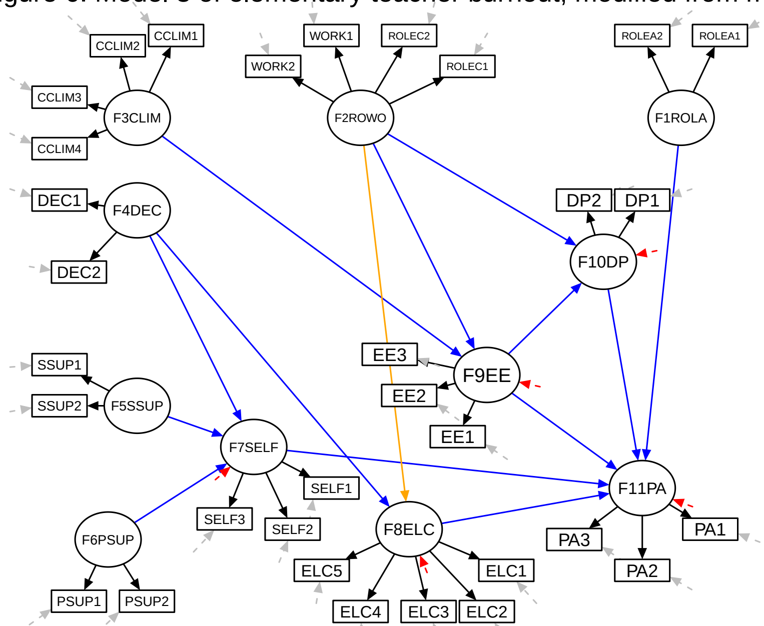
(1) Visualize model 2 and model 3

Model 3 was defined by re-specifying model. After model 3 was estimated, model 4 was defined by re-specifying model 3.

```
par(mfrow=c(2,1))
semPaths(semPlotModel(model3),
         style = "lisrel",
         rotation = 2,
         sizeLat = 6,
         sizeLat2 = 5,
         sizeMan = 5,
         sizeMan2 = 2,
         residScale = 4,
         shapeMan = "rectangle",
         edge.color = c(rep("black", 32), #34
                      rep("blue", 13),
                      rep("orange",1),
                      rep("gray", 32),
                      rep("red", 5)),
         residuals = T,
         layout = m,
         nCharNodes=0,
         optimizeLatRes = T,
         exoVar = F)
```

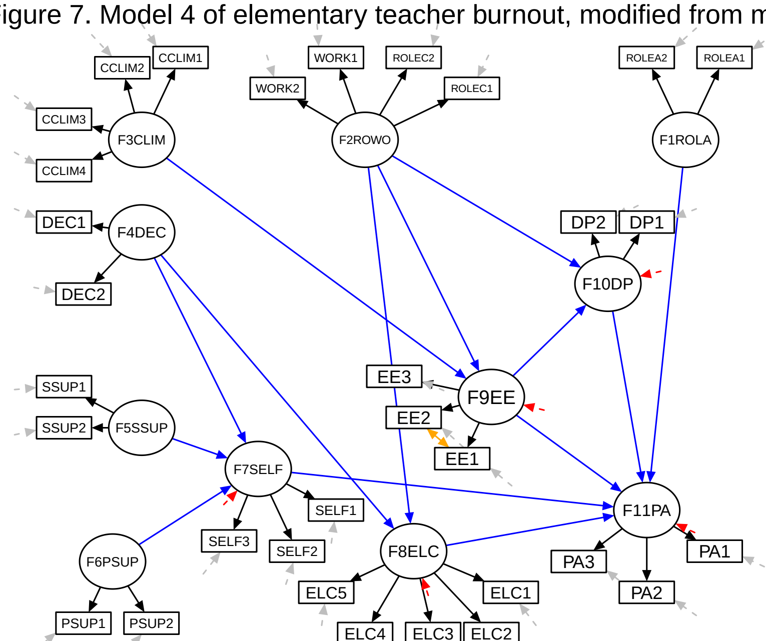
```
title(main = list(
  "Figure 6. Model 3 of elementary teacher burnout, modified from model 2",
                cex = 1.5, font =1
  ),
     outer = F, line = -1)
title(sub = "Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
     Blue arrow indicates regression path; black arrow indicates factor loading;
     Newly incorporated parameter is highlighted in orange",
     line = 1, adj = 0.7)

semPaths(semPlotModel(model4),
         style = "lisrel",
         rotation = 2,
         covAtResiduals = F,
         sizeLat = 6,
         sizeLat2 = 5,
         sizeMan = 5,
         sizeMan2 = 2,
         residScale = 4,
         shapeMan = "rectangle",
         edge.color = c(rep("black", 32), #34
                        rep("blue", 14),
                        rep("orange",1),
                        rep("gray", 32),
                        rep("red", 5)),
         residuals = T,
         layout = m,
         nCharNodes=0,
         optimizeLatRes = T,
         exoVar = F #if exogenous variables also has variance estimated
         )
title(main = list(
  "Figure 7. Model 4 of elementary teacher burnout, modified from model 3",
                cex = 1.5, font =1
  ),
     outer = F, line = -1)
title(sub = "Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
     blue arrow indicates regression path; black arrow indicates factor loading;
     Newly incorporated covariance is highlighted in orange",
     line = 1, adj = 0.7)
```

Figure 6. Model 3 of elementary teacher burnout, modified from model 2



Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
Blue arrow indicates regression path; black arrow indicates factor loading;
Newly incorporated parameter is highlighted in orange

Figure 7. Model 4 of elementary teacher burnout, modified from model 3



Notes: Red arrow indicates factor residuals; gray arrow indicates error residuals;
blue arrow indicates regression path; black arrow indicates factor loading;
Newly incorporated covariance is highlighted in orange

(3) Estimate model 3 and model 4 for calibration group

```
sem3 <-
  sem(
    model3,
    data = ele.cali,
    estimator = "MLM",
    mimic = "Mplus"
  )

sem4 <-
  sem(
    model4,
    data = ele.cali,
    estimator = "MLM",
    mimic = "Mplus"
  )
```

```
# Numerical summary of the model:
sem3.fit <-
  cfa.summary.mlm.a(sem3) |>
  t() |>
  as.data.frame()

sem4.fit <-
  cfa.summary.mlm.a(sem4) |>
  t() |>
  as.data.frame()

#combine with preceding fit indices
#model3
names(sem3.fit) <- sem3.fit[1,]
sem3.fit <- sem3.fit[-1,]
rownames(sem3.fit) <- NULL

sem3.fit <-
  sem3.fit |>
  mutate(Model = "Model3") |>
  select(Model, everything())
#model4
names(sem4.fit) <- sem4.fit[1,]
sem4.fit <- sem4.fit[-1,]
rownames(sem4.fit) <- NULL

sem4.fit <-
  sem4.fit |>
  mutate(Model = "Model4") |>
  select(Model, everything())


sem1234.fit <- rbind(sem1.fit, sem2.fit, sem3.fit, sem4.fit)

#print the table
multi.fit.tab(sem1234.fit,
```

Table 5: Fit indices for calibration dataset, model 3 and model 4 comparing with preceding models

Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Initial model	897.816(429, <0.001)	0.949	0.941	0.043(1.000)	0.055	1.092
Model2	955.863(436, <0.001)	0.943	0.935	0.045(0.994)	0.060	1.091
Model3	907.120(435, <0.001)	0.948	0.941	0.042(1.000)	0.050	1.090
Model4	866.557(434, <0.001)	0.953	0.946	0.041(1.000)	0.048	1.089

* Chi square scaling factor

```
               "Fit indices for calibration dataset, model 3 and model 4
                comparing with preceding models")
```

See table 5. Model had a chi-square[435] of 907.120, CFI of 0.948 and SRMR of 0.05; Fit of model 4 further improved in comparison to model 3, yielding a chi-square[434] of 866.557 with CFI of 0.953 and SRMR of 0.048, all of which met the numeric requirement for acceptable goodness-of-fit. I hence took model 4 as a well-fitting model.

Further, I checked the factor-loading, variance and co-variance residual estimates to check the state of aberrant parameters.

```
#print concern table for model 4
concern.table(sem4,
              nofpath = 14,
              nofpredictor = 6,
              "model4")
```

See table 6. The

(2) Estimate model2 for calibration group

4.1.4 Establish and modify the model 2 for calibration group

(1) Visualize model 2
(2) Estimate model2 for calibration group
(3) Re-specification of model2

Table 6: Residual variance of structural regression path and select factors for model4

Parameter*	B†	Beta‡	SE	Z	p-value
Regression paths (Residual variance)					
F4DEC→F7SELF	1.072	2.256	0.337	3.181	0.001
F5SSUP→F7SELF	-0.588	-1.772	0.203	-2.900	0.004
F6PSUP→F7SELF	-0.104	-0.226	0.083	-1.258	0.208
F4DEC→F8ELC	-0.047	-0.086	0.032	-1.473	0.141
F2ROWO→F9EE	0.838	0.577	0.077	10.895	0
F3CLIM→F9EE	-0.685	-0.213	0.136	-5.034	0
F2ROWO→F10DP	0.081	0.066	0.080	1.012	0.311
F9EE→F10DP	0.525	0.62	0.052	10.046	0
F1ROLA→F11PA	-0.107	-0.104	0.070	-1.532	0.126
F7SELF→F11PA	0.299	0.154	0.101	2.962	0.003
F8ELC→F11PA	-0.058	-0.034	0.082	-0.702	0.482
F9EE→F11PA	-0.115	-0.18	0.043	-2.661	0.008
F10DP→F11PA	-0.221	-0.293	0.059	-3.773	0
F2ROWO→F8ELC	0.276	0.498	0.036	7.708	0
Endogenous factors(Residual variance)					
F7SELF	0.095	0.721	0.013	7.325	0
F8ELC	0.121	0.686	0.013	9.124	0
F9EE	0.633	0.52	0.053	11.910	0
F10DP	0.485	0.557	0.058	8.404	0
F11PA	0.331	0.665	0.036	9.172	0
Exogenous factors (Residual covariance)					
EE2←→EE1	0.268	0.464	0.045	5.931	0
F2ROWO←→F1ROLA	0.42	0.808	0.042	10.078	0
F3CLIM←→F1ROLA	-0.088	-0.376	0.015	-5.922	0
F4DEC←→F1ROLA	-0.401	-0.768	0.041	-9.872	0
F5SSUP←→F1ROLA	-0.503	-0.672	0.053	-9.471	0
F6PSUP←→F1ROLA	-0.28	-0.52	0.031	-9.059	0
F3CLIM←→F2ROWO	-0.107	-0.412	0.016	-6.612	0
F4DEC←→F2ROWO	-0.398	-0.687	0.042	-9.486	0
F5SSUP←→F2ROWO	-0.474	-0.571	0.051	-9.296	0
F6PSUP←→F2ROWO	-0.262	-0.438	0.032	-8.066	0
F4DEC←→F3CLIM	0.097	0.369	0.017	5.705	0
F5SSUP←→F3CLIM	0.108	0.288	0.022	4.883	0
F6PSUP←→F3CLIM	0.068	0.253	0.015	4.433	0
F5SSUP←→F4DEC	0.806	0.967	0.061	13.252	0
F6PSUP←→F4DEC	0.398	0.662	0.039	10.217	0
F6PSUP←→F5SSUP	0.433	0.503	0.046	9.371	0

Note:

Values highlighted in red should be taken note of

* → indicates regression path

† Crude estimates

‡ Standardized estimates