COS-D419 Factor Analysis and Structural Equation Models 2023, Assignment 4

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\mathbf{T}	he texts that reflect my understanding have been highlighted in red color.	

1 Task description

The first section is task description, which is copied from the assignment5.rmd. It is for communicating with future "me". Please skip it.

1.1 Exercise 5.1

Specify and estimate the initial baseline models for the two groups.

Present a brief summary of the model fit and make the first step of the modification by including (exceptionally, at the same time!) all the four parameters known to be required for improving the model fit of both models.

Fine-tune the models step by step following the guidelines given in the lecture material, i.e., implement the modifications (as usually, one change at a time) testing and studying each step.

Present the final baseline models of each group and draw the graphs

2 Preparation

##Read in the data set:

Start by downloading the two data files from Moodle to your Project folder!

```
#install the necessary pakages
if (!require("pacman")) install.packages("pacman")
pacman::p_load(here,
                expss,
               tidyverse,
                janitor,
               knitr,
                qualtRics,
                arules,
                arulesViz,
                sjlabelled,
               DT,
               stringr,
               labelled,
                ggstatsplot,
                ggcorplot)
library(tidyverse)
library(readr)
#This week's file name
latest.name1 <- "MBIELM1.CSV"</pre>
latest.name2 <- "MBISEC1.CSV"</pre>
#read in the data
mbi.elm <- #elementary school
  read_csv(
    file.path(
      here(),
      'data',
      latest.name1
    )
mbi.sec <- #secondary school
  read_csv(
    file.path(
      here(),
      'data',
      latest.name2
      )
```

2.1 Write functions

To control length of reports, codes already shown in the previous homework were not showing in the current report. Yet they are available in .rmd report.

- 2.1.1 To generate a function for calculating chi square difference was defined.
- 2.1.2 to generate CFA results with improved readability
- 2.1.3 Write a function to simplify plotting of merged tables for multi-group fit indicies

```
multi.fit.tab <- function(data, title, more.footnote = NULL){</pre>
data <- data |>
  rename(p = 'p value',
         p2 = 'RMSEA p value',
         chi = 'chi square') |>
  mutate(df = as.numeric(df) |> round(0),
         p = case_when(
           as.numeric(p) < 0.001 \sim "<0.001",
           as.numeric(p) >= 0.001 \sim p
           ),
         p2 = case_when(
           as.numeric(p2) < 0.001 \sim "<0.001",
           as.numeric(p2) \geq= 0.001 ~ p2
         ) |>
  mutate('Chi square (df, p)' =
           pasteO(chi, "(", df,", ", p, ")"),
         'RMSEA(p)'
           paste0(RMSEA, "(", p2, ")"
         ) |>
  select(
    Model,
    'Chi square (df, p)',
    CFI, TLI,
    'RMSEA(p)',
    SRMR,
    'CSF*'= CSF
#print the combined table with adjustment of aesthetics
data |>
  kable(booktabs = T,
        #format = "markdown",
        caption =
          title,
        align = "lrrrrrr"
        ) |>
  kable_styling(full_width = T) |>
  footnote(symbol =
             c("Chi square scaling factor",
               more.footnote)
           ) |>
  column_spec(1, width = "3.5cm") |>
  column_spec(2, width = "4cm")|>
  column_spec(3, width = "1cm")|>
  column spec(4, width = "1cm")|>
  column_spec(5, width = "2.5cm")|>
```

```
column_spec(6, width = "1cm") |>
column_spec(7, width = "1cm")
}
```

2.1.4 Write a function to simplify plotting aligned residual variance and co-variance tables

```
align.table <- function(data, num.no.header.col, title){</pre>
data |>
 kable(
    digits = 3,
    booktabs = T,
    #format = "markdown",
    caption = title,
    linesep = ""
    ) |>
  add_header_above(c(" " = num.no.header.col,
                      "Elementary level" = 5,
                      "Secondary level" = 5
                     )
                    ) |>
  kable_styling(
   latex_options = "striped"
  ) |>
  footnote(
           symbol = c(
             "Un-standardized estimates",
             "Standardized estimates"
                      )
           )
```

- 2.1.5 Write a function for correlation matrix with numbers
- 2.1.6 to generate a function for histogram overlapping with density plot
- 2.1.7 to generate a function for violin overlapping with box plot
- 2.1.8 To generate a function describing continuous data set
- 2.1.9 Write a function describing continuous data set
- 2.1.10 Write a function for histogram overlapping with density plot
- 2.1.11 Write a function to generate dot distribution plot

```
dot.dist <-
function(data, type, title){
  data |>
```

```
t() |>
as.data.frame() %>%
mutate(Item = rownames(.)) |>
rowwise() |>
mutate(Median = eval(parse(text = type))(V1:V580)) |>
ggstatsplot::ggdotplotstats(
    point.args = list(color = "red", size = 3, shape = 13),
    xlab = paste(type, "ratings"),
    title = title,
    x = Median,
    y = Item
)
}
```

2.1.12 Write a fuction to generate correlation matrix with statistical test

```
mycor <-
function(data, cols, title){
mbi.elm |>
    select(all_of(cols)) |>
    ggstatsplot::ggcorrmat(
        colors = c("#B2182B", "white", "#4D4D4D"),
        title = "(a) Items on emotional exhaustion,
        elementary school teacher",
        matrix.type = "lower"
    )
}
```

3 Inspect the data

3.1 Distribution

```
#generate the plots, by subgroup of teachers
p.dist.elm <-
    corr.density(
    mbi.elm,
    fig.num = "1(a)",
    group = "elementary school teacher"
    )
p.dist.sec <-
    corr.density(
    mbi.sec,
    fig.num = "1(b)",
    group = "secondary school teacher"
    )
#print the plot
library(patchwork); p.dist.elm/p.dist.sec</pre>
```

Figure 1(a) Distribution of selected items for elementary school teacher

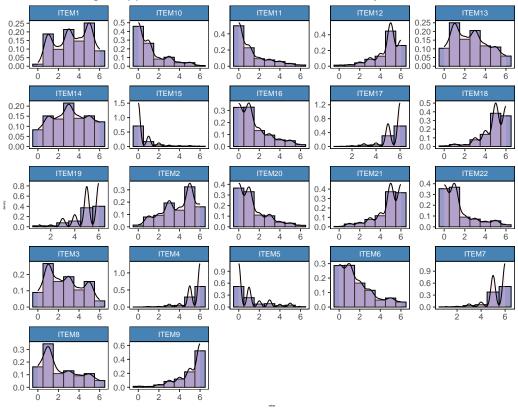


Figure 1(b) Distribution of selected items for secondary school teacher



```
#generate plot by subgroups of teachers
p.dot.elm <-
  dot.dist(
    data = mbi.elm,
    type = "median",
    title = "(a) Elementary school teacher"
p.dot.sec <-
  dot.dist(
    data = mbi.sec,
    type = "median",
    title = "(b) Secondary school teacher"
#plot layout
patchwork <- p.dot.elm|p.dot.sec</pre>
#print the plot with a genral title
patchwork+plot_annotation(
    title =
      'Figure 2 Distributions of median rating for each item',
      theme(plot.title =
              element_text(
                size = 16,
                face = "bold",
                vjust = -1.5,
                hjust =0.5)
            )
    )
```

Figure 2 Distributions of median rating for each item

(a) Elementary school teacher

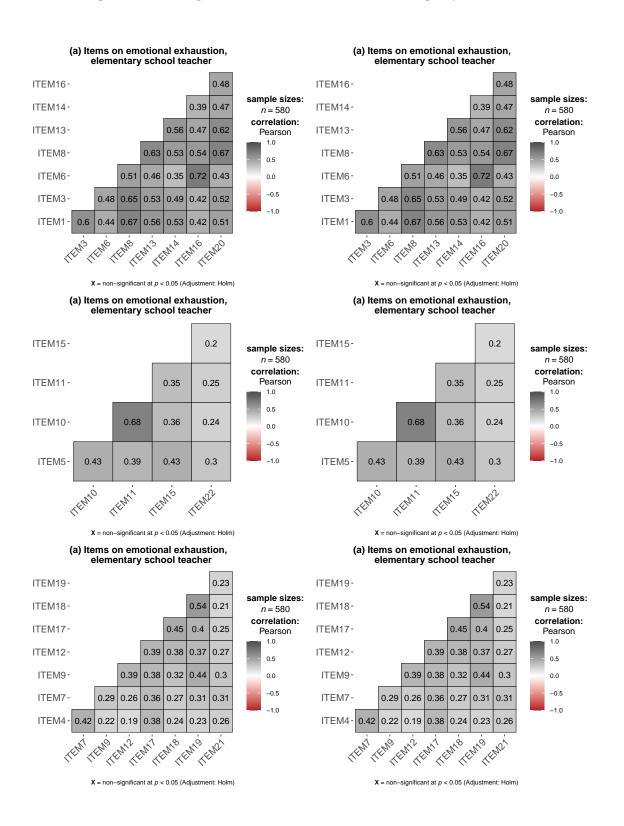
(b) Secondary school teacher

 $t_{\text{Student}}(21) = 4.74$, p = 1.11e-04, $\widehat{g}_{\text{Hedges}} = 0.97$, $\text{Cl}_{95\%}$ [0. $t_{\text{Student}}(21) = 7.06, p = 5.74e-07, \widehat{g}_{\text{Hedges}} = 1.45, \text{Cl}_{95\%} [0.8]$ $\widehat{\mu}_{mean} = 2.48$ $\widehat{\mu}_{mean} = 3.05$ ITEM9 -××××× **-** 100 ITEM7 - \boxtimes **-** 100 ITEM7 -ITEM4 -ITEM4 -ITEM17 -ITEM17 -ITEM21 - \boxtimes $\boxtimes \boxtimes \boxtimes$ ITEM21 -ITEM9 -ITEM19 -ITEM19 -**-** 75 **-** 75 ITEM18 -ITEM18 -ITEM12 -ITEM2 -ITEM2 - \boxtimes ITEM12 -ITEM1 -ITEM1 - \boxtimes ITEM13 -ITEM14 -- 50 - 50 ITEM16 -ITEM22 -ITEM13 -ITEM14 - $\bigotimes \bigotimes \bigotimes$ ITEM6 -ITEM8 -ITEM15 -ITEM5 -ITEM11 -ITEM3 -- 25 - 25 ITEM11 -ITEM10 -ITEM6 - 🔯 ITEM8 -ITEM20 - 💢 ITEM3 -ITEM16 - 💢 ITEM22 -ITEM15 - ⊠ ITEM20 -ITEM10 - 💢 - 0 ITEM5 - 💢 **-** 0 0 6 median ratings median ratings $log_{e}(BF_{01}) = -5.52$, $\widehat{\delta}_{difference}^{posterior} = 2.32$, $Cl_{95\%}^{ETI}$ [1.29, 3.43], $r_{Cauchy}^{JZS} = 0.71$ $\log_{e}(\mathrm{BF_{01}}) = -10.30, \, \widehat{\delta}_{difference}^{posterior} = 2.95, \, \mathrm{CI}_{95\%}^{ETI} \, [1.99, \, 3.82], \, r_{Cauchy}^{JZS} = 0.71$

```
fa.ee <- c("ITEM1", "ITEM3", "ITEM6", "ITEM8", "ITEM13", "ITEM14", "ITEM16", "ITEM20")</pre>
fa.dp <- c("ITEM5", "ITEM10", "ITEM11", "ITEM15", "ITEM22")
fa.pa <- c("ITEM4", "ITEM7", "ITEM9", "ITEM12", "ITEM17", "ITEM18", "ITEM19", "ITEM21")</pre>
#generate 6 plots, 3 factors X 2 subgroups of teachers
p.cor.elm.ee <-
       mycor(
         data= mbi.elm,
         cols = fa.ee,
         "(a) Items on emotional exhaustion,
         elementary school teacher"
p.cor.sec.ee <-
       mycor(
         data = mbi.sec,
         cols = fa.ee,
         "(b) Items on emotional exhaustion,
          secondary school teacher"
p.cor.elm.dp <-
       mycor(
         data = mbi.elm,
         cols = fa.dp,
         "(c) Items on depersonalization,
          elementary school teacher"
p.cor.sec.dp <-
       mycor(
         data = mbi.sec,
         cols = fa.dp,
         "(d) Items on depersonalization,
          secondary school teacher"
         )
p.cor.elm.pa <-
       mycor(
         data = mbi.elm,
         cols = fa.pa,
         "(e) Items on personal accomplishment,
         secondary school teacher"
p.cor.sec.pa <-
       mycor(
         data = mbi.sec ,
         cols = fa.pa,
         "(f) Items on personal accomplishment,
          secondary school teacher"
#plot sub-figure layout
patchwork <-
  p.cor.elm.ee/p.cor.elm.dp/p.cor.elm.pa|p.cor.sec.ee/p.cor.sec.dp/p.cor.sec.pa
#print the plot with a gernal title
patchwork+
  plot_annotation(
   title =
```

```
'Figure 3 Correlalogram for items on each factor for two groups of teachers',
theme =
    theme(plot.title =
        element_text(
            size = 16,
            face = "bold",
            vjust = -1.5,
            hjust = 0.5)
    )
)
```

Figure 3 Correlatogram for items on each factor for two groups of teachers



4 Testing the factorial invariance of MBI inventory between elementary and secondary school teachers

4.1 Define and estimate initial models for both subgroups

The postulated three-factor structure of the MBI that was tested in the previous assignments were re-tested as the initial model for establishing a baseline model.

4.1.1 Define the initial model

Cited from Byrne: It is important to note that measuring instruments are often group specific in the way they operate, and, thus, it is possible that baseline models may not be completely identical across groups.

4.1.2 Estimate indices to examine factorial validity

(1) Estimate factorial validity for the elementary teacher subgroup

```
cfa.elm <-
  cfa(
   initial.model,
  data = mbi.elm,
  estimator = "MLM",
  mimic = "Mplus"
)</pre>
```

(2) Estimate factorial validity for the secondary teacher subgroup

```
cfa.sec <-
cfa(
  initial.model,
  data = mbi.sec,
  estimator = "MLM",
  mimic = "Mplus"
)</pre>
```

Table 1: Fit indices for two subgroups, basline models

Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Elementary level Secondary level	826.573(206, <0.001) 999.359(206, <0.001)	0.857 0.836	0.840 0.816	$\begin{array}{c} 0.072(<0.001) \\ 0.075(<0.001) \end{array}$	$0.068 \\ 0.077$	1.225 1.284

^{*} Chi square scaling factor

4.1.3 Evaluate model

(1) Fit indices

```
library(knitr); library(kableExtra)
#combine fit indices of both levels
initial.elm.fit <-</pre>
  cfa.summary.mlm.a(cfa.elm) |>
  t() |>
  as.data.frame()
initial.sec.fit <-</pre>
  cfa.summary.mlm.a(cfa.sec) |>
  t() |>
  as.data.frame()
initial.both <-</pre>
  rbind(
    initial.elm.fit[2,],
    initial.sec.fit[2,]
names(initial.both) <-</pre>
  initial.elm.fit[1,]
rownames(initial.both) <- NULL
initial.both <-
  initial.both |>
  mutate(Model = c("Elementary level",
    "Secondary level")) |>
  select(Model, everything())
#print the table
multi.fit.tab(initial.both, "Fit indices for two subgroups, basline models")
```

See table 1. Goodness-of-fit statistics for this baseline model (three factor) reveals that the indices are less than optimal for both elementary (MLM Chi-square[206] = 826.573; CFI = 0.857; RMSEA = 0.072; SRMR = 0.068) and secondary (MLM Chi-square[206] = 999.359; CFI = 0.836; RMSEA = 0.075; SRMR = 0.077) levels.

(2) factor loading

Factor loading of elementary level were extracted.

```
fl.elm <- cfa.summary.b (cfa.elm) #fl is for factor loading)
colnames(fl.elm)[2] <- "Beta*"</pre>
```

Factor loading of secondary level were extracted.

Factor loading of both levels were merged in one table and printed.

```
fl.both <- left_join(fl.elm,</pre>
                     fl.sec,
                     by = "Parameter")
fl.both |>
 kable(
    digits = 3,
   booktabs = T,
    #format = "markdown",
    caption = "Factor loadings for both levels",
    linesep = ""
    ) |>
  add_header_above(c(" " = 1,
                     "Elementary level" = 4,
                     "Secondary level" = 4
                     )
                   ) |>
  kable_styling() |>
  row_spec(1:9,
           background = "#E5E4E2"
           ) |>
  row_spec(15:22,
           background = "#E5E4E2"
           ) |>
 row_spec(c(1,10,15), bold = T) >
  footnote(general =
             "Rows with coeffcient estimates fixed to 1 are highligted in bold ",
           symbol = c(
             "Standardized estimates"
                      )
```

the cross-loading involved the loading of Item 12 on Factor 1 (Emotional Exhaustion) in addition to its targeted Factor 3 (Personal Accomplishment)

(3) Variance

Variance of elementary level were extracted.

Table 2: Factor loadings for both levels

	Elementary level				Secondary level				
Parameter	Beta*	SE	Z	p-value	Beta*	SE	Z	p-value	
EE→ITEM1	0.776	0.000	NA	NA	0.756	0.000	NA	NA	
EE→ITEM2	0.754	0.032	28.561	< 0.001	0.736	0.031	30.236	< 0.001	
$EE \rightarrow ITEM3$	0.740	0.045	21.984	< 0.001	0.722	0.043	24.030	< 0.001	
$EE \rightarrow ITEM6$	0.631	0.051	16.064	< 0.001	0.626	0.046	18.669	< 0.001	
$EE \rightarrow ITEM8$	0.855	0.042	28.448	< 0.001	0.833	0.046	25.968	< 0.001	
$EE \rightarrow ITEM13$	0.754	0.045	22.474	< 0.001	0.762	0.045	23.619	< 0.001	
$EE \rightarrow ITEM14$	0.655	0.046	19.939	< 0.001	0.634	0.045	20.685	< 0.001	
$EE \rightarrow ITEM16$	0.640	0.047	15.992	< 0.001	0.596	0.047	15.261	< 0.001	
$EE \rightarrow ITEM20$	0.734	0.045	18.371	< 0.001	0.707	0.048	17.421	< 0.001	
$\mathrm{DP}{ o}\mathrm{ITEM5}$	0.576	0.000	NA	NA	0.453	0.000	NA	NA	
$DP \rightarrow ITEM10$	0.794	0.115	11.968	< 0.001	0.820	0.188	10.259	< 0.001	
$DP \rightarrow ITEM11$	0.793	0.122	11.588	< 0.001	0.808	0.197	9.666	< 0.001	
$DP \rightarrow ITEM15$	0.505	0.072	9.287	< 0.001	0.472	0.098	10.295	< 0.001	
$DP \rightarrow ITEM22$	0.351	0.091	6.997	< 0.001	0.447	0.131	8.226	< 0.001	
$PA \rightarrow ITEM4$	0.447	0.000	NA	NA	0.340	0.000	NA	NA	
$PA \rightarrow ITEM7$	0.516	0.148	7.308	< 0.001	0.545	0.221	7.495	< 0.001	
$PA \rightarrow ITEM9$	0.581	0.280	6.629	< 0.001	0.681	0.365	7.432	< 0.001	
$PA \rightarrow ITEM12$	0.611	0.303	6.214	< 0.001	0.586	0.283	7.398	< 0.001	
$PA \rightarrow ITEM17$	0.681	0.185	7.796	< 0.001	0.546	0.187	7.486	< 0.001	
$PA \rightarrow ITEM18$	0.628	0.276	6.628	< 0.001	0.698	0.294	7.431	< 0.001	
$PA \rightarrow ITEM19$	0.643	0.255	6.844	< 0.001	0.706	0.324	7.565	< 0.001	
$\mathrm{PA}{\rightarrow}\mathrm{ITEM21}$	0.425	0.187	7.018	< 0.001	0.410	0.242	6.808	< 0.001	

Rows with coeffcient estimates fixed to 1 are highligted in bold * Standardized estimates

```
var.elm <- cfa.summary.c(cfa.elm, fa.num = 3, item.num = 22)
names(var.elm)[3] <- "Beta*"
names(var.elm)[4]<- "Beta†"</pre>
```

Variance of secondary level were extracted.

```
var.sec <- cfa.summary.c(cfa.sec, fa.num = 3, item.num = 22)
var.sec <- var.sec[,-1]
names(var.sec) <-
c("Indicator",
    "Beta* ",
    "Beta† ",
    "SE ",
    "Z ",
    "p-value "
    )</pre>
```

Variance of both levels were merged in one table and printed.

(3) Co-variance

Co-variance of elementary level were extracted.

```
cov.elm <- cfa.summary.d(cfa.elm, fa.num = 3, item.num = 22)
colnames(cov.elm)[2:3] <- c("Beta*", "Beta†")</pre>
```

Co-variance of secondary level were extracted.

```
cov.sec <- cfa.summary.d(cfa.sec, fa.num = 3, item.num = 22)
colnames(cov.sec) <- c("Parameter", "Beta* ", "Beta† ", "SE ", "Z ", "p-value ")</pre>
```

Co-variance of both levels were merged in one table and printed.

Table 3: Residual variance for both levels

			Ele	mentary	level		Secondary level				
Parameter	Indicator	Beta*	Beta†	SE	Z	p-value	Beta*	Beta†	SE	Z	p-value
Residual	ITEM1	1.095	0.398	0.062	17.641	< 0.001	1.078	0.429	0.056	19.329	< 0.001
Residual	ITEM2	1.067	0.432	0.063	16.832	< 0.001	1.071	0.459	0.053	20.373	< 0.001
Residual	ITEM3	1.322	0.452	0.089	14.773	< 0.001	1.383	0.479	0.083	16.704	< 0.001
Residual	ITEM6	1.655	0.602	0.098	16.924	< 0.001	1.656	0.609	0.084	19.730	< 0.001
Residual	ITEM8	0.886	0.269	0.068	13.044	< 0.001	0.890	0.306	0.061	14.560	< 0.001
Residual	ITEM13	1.281	0.431	0.087	14.663	< 0.001	1.167	0.419	0.075	15.574	< 0.001
Residual	ITEM14	1.897	0.571	0.113	16.728	< 0.001	1.883	0.599	0.110	17.084	< 0.001
Residual	ITEM16	1.363	0.591	0.066	20.746	< 0.001	1.353	0.645	0.071	19.024	< 0.001
Residual	ITEM20	0.954	0.461	0.093	10.210	< 0.001	0.983	0.500	0.057	17.125	< 0.001
Residual	ITEM5	1.459	0.669	0.119	12.289	< 0.001	1.711	0.795	0.100	17.052	< 0.001
Residual	ITEM10	0.806	0.370	0.094	8.530	< 0.001	0.803	0.328	0.090	8.944	< 0.001
Residual	ITEM11	0.848	0.372	0.101	8.404	< 0.001	0.854	0.347	0.095	9.013	< 0.001
Residual	ITEM15	0.934	0.745	0.119	7.870	< 0.001	1.562	0.778	0.112	13.964	< 0.001
Residual	ITEM22	2.086	0.877	0.143	14.538	< 0.001	2.052	0.800	0.124	16.598	< 0.001
Residual	ITEM4	0.696	0.800	0.066	10.568	< 0.001	1.074	0.884	0.104	10.372	< 0.001
Residual	ITEM7	0.562	0.734	0.058	9.605	< 0.001	0.907	0.703	0.064	14.108	< 0.001
Residual	ITEM9	1.176	0.662	0.115	10.247	< 0.001	1.194	0.536	0.097	12.297	< 0.001
Residual	ITEM12	1.039	0.627	0.079	13.108	< 0.001	1.177	0.657	0.076	15.418	< 0.001
Residual	ITEM17	0.418	0.536	0.048	8.653	< 0.001	0.649	0.701	0.063	10.319	< 0.001
Residual	ITEM18	0.894	0.606	0.109	8.170	< 0.001	0.703	0.512	0.068	10.329	< 0.001
Residual	ITEM19	0.753	0.587	0.062	12.153	< 0.001	0.847	0.501	0.080	10.595	< 0.001
Residual	ITEM21	1.360	0.819	0.124	10.949	< 0.001	1.889	0.832	0.111	17.056	< 0.001
Total	EE	1.657	1.000	0.114	14.585	< 0.001	1.436	1.000	0.097	14.854	< 0.001
Total	DP	0.723	1.000	0.111	6.515	< 0.001	0.442	1.000	0.085	5.188	< 0.001
Total	PA	0.174	1.000	0.046	3.814	< 0.001	0.141	1.000	0.034	4.108	< 0.001

^{*} Un-standardized estimates

Table 4: Residual co-variance for both levels

	Elementary level					Sec	ondary	level		
Parameter	Beta*	Beta†	SE	Z	p-value	Beta*	Beta†	SE	Z	p-value
$\text{EE} \longleftrightarrow \text{DP}$	0.688	0.628	0.075	9.171	< 0.001	0.451	0.566	0.057	7.928	< 0.001
$\mathrm{EE} \longleftrightarrow \mathrm{PA}$	-0.254	-0.473	0.037	-6.952	< 0.001	-0.177	-0.393	0.029	-6.193	< 0.001

^{*} Un-standardized estimates

[†] Standardized estimates

 $^{^{\}dagger}$ Standardized estimates

4.1.4 Model re-specification

(1) Search for mis-specified parameters

To establish baseline models for both panels of teachers that represent good model fit and parsimony, I further investigated the modification indices of the hypothesized models, respectively for two levels.

MIs of elementary level panel were calculated.

MIs of secondary level panel were calculated.

MI tables with 10 largest MI parameters was printed in descending order of MI. Potential mis-specification of most concerns were highlighted in red.

```
MI.both <- rbind(initial.MI.elm, initial.MI.sec)
MI.both
            |>
  mutate(
    op = case\_when(op == "~~"~"\leftarrow \rightarrow ",
                    op == "=\sim"\sim"\to"),
    Parameter =
            paste(lhs, op, rhs)
          ) |>
  select(Parameter,
         MI = mi,
          EPC = epc,
          "std EPC" = sepc.all
         )|>
  kable(digits = 3,
        booktab = T,
        linesep = "",
        caption =
           "Selected modification indices for determining baseline model") |>
  kable_styling(
    latex_options = "striped"
    ) |>
  row_spec(
    c(1:4, 11:14),
    color = "red"
```

Table 5: Selected modification indices for determining baseline model

	Parameter	MI	EPC	std EPC
Elemen	tary level			
183	$ITEM6 \longleftrightarrow ITEM16$	180.298	0.893	0.595
120	$\text{ITEM1} \longleftrightarrow \text{ITEM2}$	103.177	0.534	0.494
84	$\mathrm{EE} ightarrow \mathrm{ITEM12}$	81.319	-0.400	-0.400
285	$\text{ITEM10} \longleftrightarrow \text{ITEM11}$	67.743	0.688	0.832
348	$ITEM18 \longleftrightarrow ITEM19$	43.669	0.279	0.340
323	$\text{ITEM4} \longleftrightarrow \text{ITEM7}$	42.833	0.184	0.294
175	$\text{ITEM3} \longleftrightarrow \text{ITEM12}$	28.187	-0.287	-0.245
275	$\text{ITEM5} \longleftrightarrow \text{ITEM15}$	25.815	0.273	0.234
96	$DP \rightarrow ITEM16$	25.652	0.459	0.257
185	$ITEM6 \longleftrightarrow ITEM5$	23.753	0.337	0.217
Second	ary level			
1201	$\text{ITEM1} \longleftrightarrow \text{ITEM2}$	171.647	0.627	0.583
2851	$\text{ITEM10} \longleftrightarrow \text{ITEM11}$	135.841	1.181	1.426
1831	$ITEM6 \longleftrightarrow ITEM16$	127.756	0.686	0.458
841	$\mathrm{EE} ightarrow \mathrm{ITEM12}$	118.156	-0.468	-0.419
2751	$\text{ITEM5} \longleftrightarrow \text{ITEM15}$	77.216	0.580	0.355
296	$\text{ITEM11} \longleftrightarrow \text{ITEM15}$	60.947	-0.485	-0.420
147	$\text{ITEM2} \longleftrightarrow \text{ITEM20}$	53.024	-0.324	-0.316
274	$\text{ITEM5} \longleftrightarrow \text{ITEM11}$	48.297	-0.446	-0.369
339	$\text{ITEM9} \longleftrightarrow \text{ITEM19}$	46.617	0.360	0.358
77	$\text{EE} \rightarrow \text{ITEM10}$	45.623	-0.394	-0.302

Note:

Rows highlighted in red are of special concerns

See table 5. Three exceptionally large residual co-variances and one cross-loading contributed to the misfit of the model for both teacher panels. The residual co-variances involved Items 1 and 2, Items 6 and 16, and Items 10 and 11; the cross-loading involved the loading of Item 12 on Factor 1 (Emotional Exhaustion) in addition to its targeted Factor 3 (Personal Accomplishment).

In reviewing both the MIs and expected parameter change (EPC) statistics for elementary teachers (table 5, upper part), it is clear that all four parameters are contributing substantially to model misfit, with the residual covariance between Item 6 and Item 16 exhibiting the most profound effect.

We see precisely the same pattern on secondary teachers, albeit the effect would appear to be even more pronounced than it was for elementary teachers. One slight difference between the two groups of teachers regards the impact of these four parameters on model misfit. Whereas the residual covariance between Items 6 and 16 was found to be the most seriously misfitting parameter for elementary teachers; for secondary teachers, the residual covariance between Items 1 and 2 was most pronounced.

(2) Re-specify initial model to model 2

The good practice is relaxing one parameter each time. Nonetheless, according to the knowledge derived from our previous work, I included all four mis-specified parameters in a post-hoc model (common to the groups).

First, the 4 parameters were relaxed in model statement.

Then, the model fit were re-estimated for both group, respectively

```
#for elementary
cfa2.elm <-
    cfa(
        model2,
        data = mbi.elm,
        estimator = "MLM",
        mimic = "Mplus"
    )

#for secondary
cfa2.sec <-
    cfa(
        model2,
        data = mbi.sec,
        estimator = "MLM",
        mimic = "Mplus"
    )</pre>
```

4.1.5 Examine Model 2

(1) Inspect fit indices of model2 (comparing to initial model)

```
#combine fit indices of both levels
model2.elm.fit <-
    cfa.summary.mlm.a(
    cfa2.elm
    ) |>
    t() |>
    as.data.frame()

model2.sec.fit <-
    cfa.summary.mlm.a(
    cfa2.sec
    ) |>
    t() |>
    as.data.frame()
```

Table 6: Fit indices for two subgroups, model 2, comparing to initial model

Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Initial model						
Elementary level	826.573(206, < 0.001)	0.857	0.840	0.072(<0.001)	0.068	1.225
Secondary level	999.359(206, < 0.001)	0.836	0.816	0.075 (< 0.001)	0.077	1.284
Model 2						
Elementary level	477.667(202, < 0.001)	0.936	0.927	0.049(0.679)	0.050	1.224
Secondary level	587.538(202, <0.001)	0.920	0.909	$0.053(\ 0.168)$	0.056	1.278

^{*} Chi square scaling factor

```
model2.both <-
  rbind(
    model2.elm.fit[2,],
    model2.sec.fit[2,]
    )
names(model2.both) <- model2.elm.fit[1,]</pre>
rownames(model2.both) <- NULL
model2.both <-
 model2.both |>
  mutate(Model = c("Elementary level",
    "Secondary level")) |>
  select(Model, everything())
#combine model 1 and 2 tables
compare12 <- rbind(initial.both, model2.both)</pre>
#print the table
multi.fit.tab(compare12,
              "Fit indices for two subgroups, model 2, comparing to initial model") |>
  pack rows(index = c(
    "Initial model" = 2,
    "Model 2" = 2
  )
```

Estimation of this re-specified model, for each teacher group, yielded greatly improved model fit statistics than initial model. See table 6. However, we should note that several statistics, albeit improved comparing to initial model, still fall below the preferable value. For example, CFI from both groups were <0.95.

(2) Modification indices of model 2

To establish baseline models for both panels of teachers that represent good model fit and parsimony, I further investigated the modification indices of model 2, respectively for two groups, to decide if there was any more model mis-fit and mis-specification

MIs of elementary level panel were calculated.

MIs of secondary level panel were calculated.

MI tables with 10 largest MI parameters was printed in descending order of MI. Potential mis-specification of most concerns were highlighted in red.

```
MI2.both <- rbind(model2.MI.elm, model2.MI.sec)</pre>
MI2.both
            1>
  mutate(
    op = case\_when(op == "~~"~"\leftarrow ",
                    op == "= \sim " \sim " \rightarrow "),
    Parameter =
           paste(lhs, op, rhs)
         ) |>
  select(Parameter,
         MI = mi,
         EPC = epc,
          "std EPC" = sepc.all
         )|>
  kable(digits = 3,
        booktab = T,
        linesep = "",
        caption =
           "Selected modification indices for determining baseline model") |>
  kable_styling(
    latex_options = "striped"
    ) |>
  row_spec(
    c(1:2, 11:12),
    color = "red"
    ) |>
  footnote(general =
              "Rows highlighted in red are of special concerns") |>
  pack_rows(index = c(
    "Elementary level" = 10,
    "Secondary level" = 10
```

Table 7: Selected modification indices for determining baseline model

	Parameter	MI	EPC	std EPC
Eleme	ntary level			
323	$ITEM4 \longleftrightarrow ITEM7$	38.931	0.174	0.284
348	$\text{ITEM18} \longleftrightarrow \text{ITEM19}$	38.744	0.266	0.333
115	$PA \rightarrow ITEM14$	24.435	0.864	0.205
177	$\text{ITEM3} \longleftrightarrow \text{ITEM12}$	23.978	-0.250	-0.227
227	$\text{ITEM13} \longleftrightarrow \text{ITEM12}$	20.493	0.231	0.211
147	$\text{ITEM2} \longleftrightarrow \text{ITEM14}$	16.441	0.245	0.163
99	$DP \rightarrow ITEM16$	15.733	0.310	0.197
216	$\text{ITEM13} \longleftrightarrow \text{ITEM14}$	14.838	0.281	0.180
82	$\text{EE} \rightarrow \text{ITEM11}$	14.750	0.250	0.206
105	$DP \rightarrow ITEM17$	12.788	-0.173	-0.188
Second	dary level			
821	$\mathrm{EE} o \mathrm{ITEM11}$	67.177	0.472	0.339
339	$\text{ITEM9} \longleftrightarrow \text{ITEM19}$	43.690	0.355	0.357
276	$\text{ITEM5} \longleftrightarrow \text{ITEM15}$	35.576	0.416	0.310
296	$\text{ITEM11} \longleftrightarrow \text{ITEM15}$	29.016	-0.297	-0.206
247	$ITEM16 \longleftrightarrow ITEM20$	28.900	0.227	0.201
98	$DP \rightarrow ITEM14$	22.145	-0.490	-0.239
345	$\text{ITEM17} \longleftrightarrow \text{ITEM18}$	21.583	0.147	0.219
335	$\text{ITEM7} \longleftrightarrow \text{ITEM21}$	21.370	0.247	0.191
346	$\text{ITEM17} \longleftrightarrow \text{ITEM19}$	20.742	-0.159	-0.217
149	$\text{ITEM2} \longleftrightarrow \text{ITEM20}$	20.020	-0.171	-0.162

Note:

Rows highlighted in red are of special concerns

See table 7. In reviewing this information for elementary teachers, we observe two MIs larger than all other MIs (ITEM7 with ITEM4; ITEM19 with ITEM18); both represent residual co-variances. I followed Byrne's step in addressing these parameters. According to Byrne, of the two, only the residual covariance between Items 7 and 4 is substantively viable in that there is a clear overlapping of item content. In contrast, the content of Items 19 and 18 exhibits no such redundancy, and, thus, there is no reasonable justification for including this parameter in a succeeding Model 3.

However, in checking the MI for secondary teachers, the decision was made: more work is needed in establishing an appropriate baseline model. Two parameters were of special concern due to their large MI and substantive meaningfulness. They are Item 11 cross-loads onto factor EE, and item 19 co-varies with item 9. This time I operated by the good practice of specifying one parameter each time. Given the substantially large MI representing the cross-loading of Item 11 on factor EE, this parameter alone was included in our next post-hoc model (Model 3 for secondary teachers).

Byrne noted the reasons for making this decision (to further re-specifying model secondary teachers), which I quoted here for future reflection: (a) The model does not yet reflect a satisfactorily good fit to the data (CFI=0.920); and (b) in reviewing the MIs in Table 7.2, we observe one very large mis-specified parameter representing the loading of Item 11 on Factor 1 (F1 by ITEM11), as well as another substantially large MI representing a residual covariance between Items 19 and 9, both of which can be substantiated as substantively meaningful parameters.

(3) Model re-specification of model 2 to model 3

```
respecified3.elm <- 'ITEM4 ~~ ITEM7
respecified3.sec <- 'EE =~ ITEM11
model3.elm <- paste(model2, respecified3.elm)
model3.sec <- paste(model2, respecified3.sec)</pre>
```

Then, the model fit were re-estimated for both group, separately.

```
#for elementary
cfa3.elm <-
    cfa(
        model3.elm,
        data = mbi.elm,
        estimator = "MLM",
        mimic = "Mplus"
    )

#for secondary
cfa3.sec <-
    cfa(
        model3.sec,
        data = mbi.sec,
        estimator = "MLM",
        mimic = "Mplus"
    )</pre>
```

4.1.6 Examine Model 3

(1) Inspect fit indices of model (comparing to model 2)

```
#combine fit indices of both levels
model3.elm.fit <-</pre>
  cfa.summary.mlm.a(
    cfa3.elm
    ) |>
  t() |>
  as.data.frame()
model3.sec.fit <-</pre>
  cfa.summary.mlm.a(
    cfa3.sec
    ) |>
  t() |>
  as.data.frame()
model3.both <-
  rbind(
    model3.elm.fit[2,],
    model3.sec.fit[2,]
names(model3.both) <- model3.elm.fit[1,]</pre>
rownames(model3.both) <- NULL
model3.both <-
  model3.both |>
  mutate(Model = c("Elementary level",
    "Secondary level")) |>
  select(Model, everything())
#combine model 1 and 2 tables
compare123 <- rbind(initial.both, model2.both, model3.both)</pre>
#print the table
multi.fit.tab(compare123,
               "Fit indices for two subgroups, model 3, comparing to preceding models") |>
  pack_rows(index = c(
    "Initial model" = 2,
    "Model 2" = 2,
    "Model 3" =2
  )
```

See table 8. Results from the estimation of Model 3 for elementary teachers yielded goodness-of-fit statistics that represented a satisfactorily good fit to the data (MLM chi square [201] = 451.061; CFI = 0.942; RMSEA = 0.046; SRMR = 0.049). Although a review of Table 9 (find below) reveals several additional moderately large MIs, for balancing goodness-of-fit and parsimony, the decision was model 3 can serve as the baseline model for elementary teachers.

Results from the estimation of Model 3 for secondary teachers, on the other hand, further substantiated the residual covariance between Items 19 and 9 as representing an acutely mis-specified parameter in the model. Thus, for secondary teachers only, model 4 was put to the test with this residual covariance specified as a freely estimated parameter.

Table 8: Fit indices for two subgroups, model 3, comparing to preceding models

	0 1			<u> </u>		
Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF^*
Initial model						
Elementary level	826.573(206, < 0.001)	0.857	0.840	0.072(<0.001)	0.068	1.225
Secondary level	999.359(206, < 0.001)	0.836	0.816	0.075 (< 0.001)	0.077	1.284
Model 2						
Elementary level	477.667(202, < 0.001)	0.936	0.927	0.049(0.679)	0.050	1.224
Secondary level	587.538(202, <0.001)	0.920	0.909	0.053(0.168)	0.056	1.278
Model 3						
Elementary level	451.061(201, < 0.001)	0.942	0.934	0.046(0.876)	0.049	1.210
Secondary level	535.759(201, < 0.001)	0.931	0.920	0.049(0.629)	0.053	1.275

^{*} Chi square scaling factor

(2) Modification indices of model 3

MIs of model 3 for each groups were calculated.

 ${
m MI}$ tables with 10 largest MI parameters was printed in descending order of MI. Potential mis-specification of most concerns were highlighted in red.

```
MI3.both <- rbind(model3.MI.elm, model3.MI.sec)
MI3.both
  mutate(
    op = case\_when(op == "~~"~"\leftarrow \rightarrow ",
                     op == "=~"~"→"),
    Parameter =
            paste(lhs, op, rhs)
          ) |>
  select(Parameter,
         MI = mi,
          EPC = epc,
          "std EPC" = sepc.all
          )|>
  kable(digits = 3,
        booktab = T,
        linesep = "",
```

Table 9: Selected modification indices for determining baseline model

	Parameter	MI	EPC	std EPC
Eleme	ntary level			
348	$\text{ITEM18} \longleftrightarrow \text{ITEM19}$	32.503	0.247	0.319
116	$PA \rightarrow ITEM14$	25.403	0.977	0.210
178	$\text{ITEM3} \longleftrightarrow \text{ITEM12}$	23.654	-0.248	-0.226
228	$\text{ITEM13} \longleftrightarrow \text{ITEM12}$	20.844	0.232	0.213
148	$\text{ITEM2} \longleftrightarrow \text{ITEM14}$	16.457	0.245	0.163
100	$DP \rightarrow ITEM16$	15.696	0.310	0.197
217	$\text{ITEM13} \longleftrightarrow \text{ITEM14}$	14.844	0.282	0.180
83	$\text{EE} \rightarrow \text{ITEM11}$	14.780	0.251	0.206
326	$\text{ITEM4} \longleftrightarrow \text{ITEM17}$	14.165	0.096	0.174
106	$\mathrm{DP} \to \mathrm{ITEM17}$	13.820	-0.181	-0.197
Second	dary level			
339	$\text{ITEM9} \longleftrightarrow \text{ITEM19}$	42.687	0.351	0.355
247	$ITEM16 \longleftrightarrow ITEM20$	28.275	0.223	0.199
345	$\text{ITEM17} \longleftrightarrow \text{ITEM18}$	21.951	0.148	0.221
335	$\text{ITEM7} \longleftrightarrow \text{ITEM21}$	21.602	0.248	0.192
346	$\text{ITEM17} \longleftrightarrow \text{ITEM19}$	20.837	-0.160	-0.218
84	$\mathrm{EE} ightarrow \mathrm{ITEM22}$	20.306	0.321	0.225
98	$DP \rightarrow ITEM14$	20.142	-0.404	-0.210
147	$\text{ITEM2} \longleftrightarrow \text{ITEM14}$	19.895	0.239	0.155
149	$\text{ITEM2} \longleftrightarrow \text{ITEM20}$	18.463	-0.164	-0.155
333	$\text{ITEM7} \longleftrightarrow \text{ITEM18}$	18.163	-0.159	-0.202

Note:

Rows highlighted in red are of special concerns

(3) Re-specification of model 3 to model 4 (only for secondary teacher)

The parameter ITEM9 $\sim\sim$ ITEM19 was relaxed for estimation.

```
respecified4.sec <- 'ITEM9 ~~ ITEM19
model4 <- paste(model3.sec, respecified4.sec)</pre>
```

Then, the model fit were re-estimated for secondary group, only

```
cfa4.sec <-
  cfa(
    model4,
    data = mbi.sec,
    estimator = "MLM",
    mimic = "Mplus"
    )</pre>
```

4.1.7 Examine Model 4

Note that at this point I had taken model 3 as the baseline model for elementary teachers, and model 4 was to achieve the baseline model for secondary teachers.

(1) Inspect fit indices of model4 (comparing to 3)

```
model4.sec.fit <-</pre>
  cfa.summary.mlm.a(
    cfa4.sec
    ) |>
  t() |>
  as.data.frame()
names(model4.sec.fit ) <- model4.sec.fit[1,]</pre>
model4.sec.fit <- model4.sec.fit [-1,]</pre>
model4.sec.fit <-</pre>
  model4.sec.fit |>
  mutate(Model = "Secondary level") |>
  select(Model, everything())
rownames(model4.sec.fit ) <- NULL
#combine model 1 and 2 tables
model3.both[1,1] <- "Elementary level†"</pre>
model4.sec.fit[1,1] <- "Secondary level‡"</pre>
compare1234 <-
  rbind(initial.both,
        model2.both,
        model3.both,
        model4.sec.fit )
#print the table
multi.fit.tab(compare1234,
               "Fit indices for two subgroups, model 4, comparing to preceding models",
               c("Baseline model for elementary teachers",
                 "Baseline model for secondary teachers")) |>
```

Table 10: Fit indices for two subgroups, model 4, comparing to preceding models

Model	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Initial model						
Elementary level	826.573(206, < 0.001)	0.857	0.840	0.072(<0.001)	0.068	1.225
Secondary level	999.359(206, < 0.001)	0.836	0.816	0.075 (< 0.001)	0.077	1.284
Model 2						
Elementary level	477.667(202, < 0.001)	0.936	0.927	0.049(0.679)	0.050	1.224
Secondary level	587.538(202, < 0.001)	0.920	0.909	0.053(0.168)	0.056	1.278
Model 3						
Elementary level†	451.061(201, < 0.001)	0.942	0.934	0.046(0.876)	0.049	1.210
Secondary level	535.759(201, < 0.001)	0.931	0.920	0.049(0.629)	0.053	1.275
Model 4						
Secondary level‡	505.831(200, < 0.001)	0.937	0.927	$0.047(\ 0.859)$	0.052	1.273

^{*} Chi square scaling factor

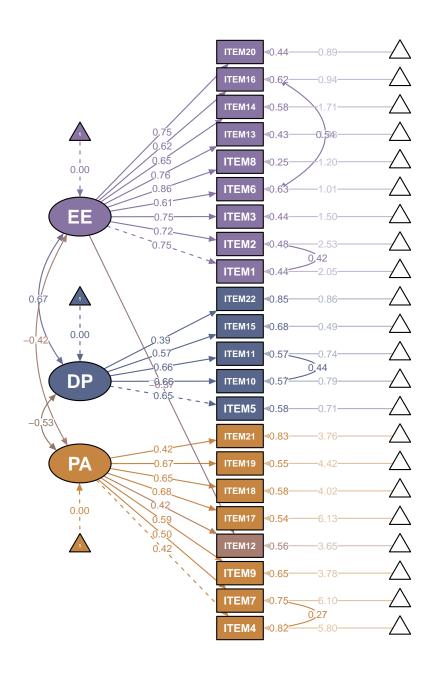
```
pack_rows(index = c(
    "Initial model" = 2,
    "Model 2" = 2,
    "Model 3" = 2,
    "Model 4" = 1
)
) |>
row_spec(c(5,7),
    color = "red"
)
```

See table 10. Based on a moderately satisfactory goodness-of-fit (MLM cgi-square [200] = 505.831; CFI = 0.937; RMSEA = 0.047; SRMR = 0.052) and to balance fit with parsimony, I consider Model 4 as the final baseline model for secondary teachers.

 $^{^{\}dagger}$ Baseline model for elementary teachers

[‡] Baseline model for secondary teachers

```
library(semPlot)
DP = c("ITEM5", "ITEM10", "ITEM11", "ITEM15","ITEM22"),
        PA = c("ITEM4", "ITEM7", "ITEM9", "ITEM12",
               "ITEM17", "ITEM18", "ITEM19", "ITEM21"))
order.manifest <- c("ITEM4", "ITEM7", "ITEM9", "ITEM12",
        "ITEM17", "ITEM18", "ITEM19", "ITEM21", "ITEM5", "ITEM10", "ITEM11", "ITEM15", "ITEM22",
        "ITEM1", "ITEM2", "ITEM3", "ITEM6", "ITEM8",
        "ITEM13", "ITEM14", "ITEM16", "ITEM20")
order.latent <- c("PA", "DP", "EE")
semPaths(cfa3.elm,
         "col", #un-weighted edges
         "std", #edge label is standarized
         reorder = F,
         latents = order.latent,
         manifest = order.manifest,
         sizeLat = 8,
         sizeLat2 = 5,
         sizeMan = 6,
         sizeMan2 = 3,
         residScale = 4,
         curveAdjacent = "cov", # if edge for adjacent nodes curly or not, "req"
         shapeMan = "rectangle",
         style = "lisrel",
         group = "latent",
         residuals = T,
         curve = 1.5,
         curvature = 0.2,
         #theme = "colorblind",
         #cardinal = "lat cov",
         curvePivot = F,
         layout = "tree",
         rotation = 2,
         posColor = c("red", "purple", "yellow", "black"),
         color = c("#c68642", "#58668b", "#8874a3"),
         #edge.color = "steelblue",
         shapeLat = "ellipse",
         label.font = 2,
         label.color = "white",
         #Label.scale =T,
         label.prop = 0.7,
         posCol = c("black", "black"),
         negCol = c("steelblue", "steelblue")
```



xie