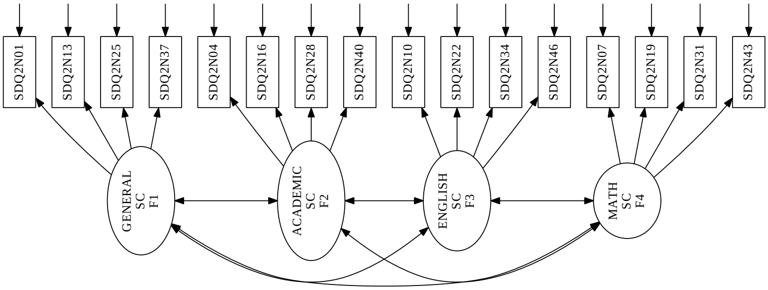
First-Order Confirmatory Factor Analysis (CFA)

Part 2 of the material is based on the Chapter 3 of the book
Byrne, Barbara M. (2012). Structural Equation Modeling with Mplus.
It gives a typical example of the **single-group analyses** of SEM: testing the **factorial validity** of a **theoretical construct**.

- **Hypothesis:** self-concept (SC) is a multidimensional construct composed of four factors:
  - General SC (GSC)
  - Academic SC (ASC)
  - English SC (ESC)
  - Mathematics SC (MSC)
- **Data:** 16 variables from grade 7 children (N=265)
- **Alternative hypotheses:** 1) SC has two factors (GSC, ASC), 2) SC is unidimensional (only one SC factor).

Hypothesized four-factor CFA model (graph)

Fig. 3.1 (p.44) (redrawn)



Hypothesized four-factor CFA model

Assumptions:

- four correlated SC factors
- 16 observed variables
- each loads on one and only one factor
- residuals of observed variables uncorrelated

The corresponding matrices include three types of parameters:

- $\# =$ parameter to be (freely) estimated
- $0 =$ parameter fixed to 0.0 (restrictions to model)
- $1 =$ parameter fixed to 1.0 (latent variable scaling, model identification)

Altogether to be estimated:

- 12 factor loadings (first of each congeneric set fixed)
- 16 observed variable residual variances
- 4 factor variances
- 6 factor covariances (two-headed arrows in the graph)

38 parameters (+ 16 observed variable intercepts, ignored)

Hypothesized four-factor CFA model (matrices)


```
1) Factor loadings: (LAMBDA)

                Factors
           GSC  ASC  ESC  MSC
Variable   (F1) (F2) (F3) (F4)
SDQ2N01     1    0    0    0
SDQ2N13     #    0    0    0
SDQ2N25     #    0    0    0
SDQ2N37     #    0    0    0
SDQ2N04     0    1    0    0
SDQ2N16     0    #    0    0
SDQ2N28     0    #    0    0
SDQ2N40     0    #    0    0
SDQ2N10     0    0    1    0
SDQ2N22     0    0    #    0
SDQ2N34     0    0    #    0
SDQ2N46     0    0    #    0
SDQ2N07     0    0    0    1
SDQ2N19     0    0    0    #
SDQ2N31     0    0    0    #
SDQ2N43     0    0    0    #

2) Factor variance-covariance matrix: (PSI)

GSC         #
ASC         #    #
ESC         #    #    #
MSC         #    #    #    #
```

Hypothesized four-factor CFA model (matrices)


```
3) (Measurement) Error variance-covariance matrix: (THETA)

SDQ2N01     #
SDQ2N13     0    #
SDQ2N25     0    0    #
SDQ2N37     0    0    0    #
SDQ2N04     0    0    0    0    #
SDQ2N16     0    0    0    0    0    #
SDQ2N28     0    0    0    0    0    0    #
SDQ2N40     0    0    0    0    0    0    0    #
SDQ2N10     0    0    0    0    0    0    0    0    #
SDQ2N22     0    0    0    0    0    0    0    0    0    #
SDQ2N34     0    0    0    0    0    0    0    0    0    0    #
SDQ2N46     0    0    0    0    0    0    0    0    0    0    0    #
SDQ2N07     0    0    0    0    0    0    0    0    0    0    0    0    #
SDQ2N19     0    0    0    0    0    0    0    0    0    0    0    0    0    #
SDQ2N31     0    0    0    0    0    0    0    0    0    0    0    0    0    0    #
SDQ2N43     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    #
```

Hypothesized four-factor CFA model (input file)

```
ASC7INDM.INP
TITLE: CFA of Academic SC Structure for Grade 7 Adolescents
       (Byrne 2012, p. 56)
DATA:
  FILE IS "ASC7INDM.DAT";
  FORMAT IS 40F1.0,X,6F2.0;

VARIABLE:
NAMES ARE
SPPCN08 SPPCN18 SPPCN28 SPPCN38 SPPCN48 SPPCN58 SPPCN01 SPPCN11
SPPCN21 SPPCN31 SPPCN41 SPPCN51 SPPCN06 SPPCN16 SPPCN26 SPPCN36
SPPCN46 SPPCN56 SPPCN03 SPPCN13 SPPCN23 SPPCN33 SPPCN43 SPPCN53

SDQ2N01 SDQ2N13 SDQ2N25 SDQ2N37 SDQ2N04 SDQ2N16 SDQ2N28 SDQ2N40
SDQ2N10 SDQ2N22 SDQ2N34 SDQ2N46 SDQ2N07 SDQ2N19 SDQ2N31 SDQ2N43

MASTENG1 MASTMAT1 TENG1 TMAT1 SENG1 SMAT1;

USEVARIABLES ARE
SDQ2N01 SDQ2N13 SDQ2N25 SDQ2N37 SDQ2N04 SDQ2N16 SDQ2N28 SDQ2N40
SDQ2N10 SDQ2N22 SDQ2N34 SDQ2N46 SDQ2N07 SDQ2N19 SDQ2N31 SDQ2N43;

ANALYSIS:
  TYPE IS GENERAL;
  ESTIMATOR IS ML;
  ITERATIONS = 1000;
  CONVERGENCE = 0.00005;

MODEL:
F1 by SDQ2N01-SDQ2N37;
F2 by SDQ2N04-SDQ2N40;
F3 by SDQ2N10-SDQ2N46;
F4 by SDQ2N07-SDQ2N43;

OUTPUT: SAMPSTAT MODINDICES STANDARDIZED TECH1;
```

Some comments and points of the analysis (p.64–)

Assessment of the model

- primary interest: the extent to which a model fits the data
- several criteria for a) model as a whole, b) parameter estimates

Model fitting

- Let $\boldsymbol{\Sigma}$ be the population covariance matrix, $\boldsymbol{S}$ the sample covariance matrix and $\boldsymbol{\theta}$ the vector of model parameters.
- Now, $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ is the restricted covariance matrix implied by the model (the specified structure).
- Null hypothesis $H_0 : \boldsymbol{\Sigma} = \boldsymbol{\Sigma}(\boldsymbol{\theta})$, i.e., the postulated model holds in the population
- We hope *not* to reject H_0 (cf. traditional procedures).

Statistical significance

- As always, merely a starting point, not a result as such!
- Practical significance is important (values of estimates, effect sizes, confidence intervals etc.)

Hypothesized four-factor CFA model (selected output)


```
SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         265

Number of dependent variables                                   16
Number of independent variables                                  0
Number of continuous latent variables                            4

Observed dependent variables

  Continuous
   SDQ2N01     SDQ2N13     SDQ2N25     SDQ2N37     SDQ2N04     SDQ2N16
   SDQ2N28     SDQ2N40     SDQ2N10     SDQ2N22     SDQ2N34     SDQ2N46
   SDQ2N07     SDQ2N19     SDQ2N31     SDQ2N43

Continuous latent variables
   F1          F2          F3          F4


Estimator                                                       ML
Information matrix                                        OBSERVED
Maximum number of iterations                                  1000
Convergence criterion                                    0.500D-04
Maximum number of steepest descent iterations                   20
```

Hypothesized four-factor CFA model (selected output)


```
TESTS OF MODEL FIT

Chi-Square Test of Model Fit
          Value                              159.112
          Degrees of Freedom                      98
          P-Value                             0.0001

Chi-Square Test of Model Fit for the Baseline Model
          Value                             1703.155
          Degrees of Freedom                     120
          P-Value                             0.0000

CFI/TLI
          CFI                                  0.961
          TLI                                  0.953

Loglikelihood
          H0 Value                         -6562.678
          H1 Value                         -6483.122

Information Criteria
          Number of Free Parameters               54
          Akaike (AIC)                     13233.356
          Bayesian (BIC)                   13426.661
          Sample-Size Adjusted BIC         13255.453
            (n* = (n + 2) / 24)

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.049
          90 Percent C.I.                      0.034  0.062
          Probability RMSEA <= .05             0.556

SRMR (Standardized Root Mean Square Residual)
          Value                                0.045
```

Some comments and points of the analysis

Estimation process

- minimize the discrepancy (residual) between $\boldsymbol{S}$ and $\boldsymbol{\Sigma}(\boldsymbol{\theta})$

The goodness-of-fit statistics

Chi-Square Test of Model Fit

- traditional Likelihood Ratio Test statistic, expressed as a chi-square (χ^2) statistic
- *p*-value represents the likelihood of obtaining a χ^2 value that exceeds the χ^2 value when H_0 is true, so the higher the p, the closer the fit.
- always reported but rarely used as the sole index of model fit

Here, $\chi^2 = 159.112$, with 98 degrees of freedom (df) and a *p*-value of less than 0.0001 suggests that the fit of the data to the model is not adequate and H_0 should be rejected. However, χ^2 is quite sensitive to sample size (it is based on large sample theory) and nonnormal data. It is still important in model comparisons.

Some comments and points of the analysis

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Chi-Square Test of Model Fit for the Baseline Model

- Usually the so called "null model", but in Mplus a slight difference: in addition to the variances, also the means of the observed items are included (although they may not be relevant in the model).
- In any baseline case, the observed items are assumed uncorrelated (no structure). The point: χ^2 value of the H_0 is much less than the value for the baseline, that is, the model is superior to the unstructured baseline model. (But this is not a remarkable or surprising result, it is just a good start.)

Some comments and points of the analysis (p.69–)

Alternative (subjective) indices of fit: more pragmatic approach, e.g. Comparative Fit Index (CFI), Tucker–Lewis Fit Index (TLI), Akaike's Information Criterion (AIC), Bayes Information Criterion (BIC). First two represent the most typical incremental indices measuring the proportionate improvement in fit with *nested* models. The latter two are predictive or parsimony-corrected criteria for *non-nested* models.

A careful consideration of these is essential when fitting models. Use of multiple indices highly recommended. Here, only four, perhaps the most typical ones. **Warning:** model may fit well, and still be incorrectly specified! These are merely giving information on the model's lack of fit. Researcher must know if the model is plausible or not.

Therefore the assessment of model adequacy must be based on multiple criteria that take into account a) theoretical, b) statistical, and c) practical considerations!

Some comments and points of the analysis

- CFI:
  - compares the hypothesized (H) and the baseline model (B):
  - $CFI = 1 - [(\chi^2_H - \mathrm{df}_H)/(\chi^2_B - \mathrm{df}_B)]$
  - range: $[0, 1]$, well-fitting models have CFI > 0.95
- TLI:
  - quite similar as CFI, but nonnormed (may extend $[0, 1]$):
  - $TLI = [(\chi^2_B/\,\mathrm{df}_B) - (\chi^2_H/\,\mathrm{df}_H)]/[(\chi^2_B/\,\mathrm{df}_B) - 1]$
  - includes a penalty function for overly complex models (H)
  - well-fitting models have TLI close to 1

The Loglikelihood values (H0 for H, H1 for B) are used in computation of information criteria AIC and BIC. AIC is the more commonly used one in SEM (they are quite similar):
$AIC = -2(\text{loglikelihood}) + 2a$,
where loglikelihood refers to the H_0 value and a is the number of estimated parameters (again including observed item intercepts in Mplus). Smaller value indicates a good fit when comparing across different models or samples.

Some comments and points of the analysis (p.72–)

The last two fit indices are RMSEA and SRMR, both absolute ones, sometimes termed "absolute misfit indices". They do not compare models, but depend only on determining how well the model fits the data. Therefore these decrease as the fit improves. Values of RMSEA less than 0.05 indicate good fit, 0.08 to 0.10 mediocre/reasonable fit, and greater than 0.10 poor fit.

RMSEA is sensitive to the number of estimated parameters (model complexity). Routine use of RMSEA is strongly recommended in literature. Here: 0.049 (with a 90 % C.I. [0.034, 0.062]), indicating a good precision.

SRMR represents the average residual value of the fit (standardized, i.e., range $[0, 1]$). In well-fitting models, it will be small, less than 0.05. Here: 0.045, that is, model explains the correlations to within an average error of 0.045.

RMSEA: Root Mean Square Error Of Approximation
SRMR: Standardized Root Mean Square Residual

Some comments and points of the analysis (p.77–)

Assessment of parameter estimates

1) appropriateness, 2) statistical significance

- Parameter estimates should exhibit the correct sign and size, and be consistent with the underlying theory.
- Any estimates falling outside the admissible range indicate that a) model is wrong or b) input matrix lacks sufficient information.
- Standard errors of parameters should not be excessively large or small.
- Assuming that the sample size is adequate, nonsignificant parameters (except the error variances) should be deleted from the model.

Hypothesized four-factor CFA model (selected output)

```
MODEL RESULTS
                                                            Two-Tailed
                         Estimate       S.E.  Est./S.E.    P-Value
                     (factor loadings)
 F1        BY
    SDQ2N01     fixed:  1.000          0.000    999.000    999.000
    SDQ2N13             1.083          0.156      6.939      0.000
    SDQ2N25             0.851          0.125      6.806      0.000
    SDQ2N37             0.934          0.141      6.607      0.000

 F2        BY
    SDQ2N04             1.000          0.000    999.000    999.000
    SDQ2N16             1.279          0.151      8.489      0.000
    SDQ2N28             1.247          0.155      8.022      0.000
    SDQ2N40             1.259          0.159      7.913      0.000

 F3        BY
    SDQ2N10             1.000          0.000    999.000    999.000
    SDQ2N22             0.889          0.104      8.561      0.000
    SDQ2N34             0.670          0.148      4.541      0.000
    SDQ2N46             0.843          0.118      7.160      0.000

 F4        BY
    SDQ2N07             1.000          0.000    999.000    999.000
    SDQ2N19             0.841          0.058     14.447      0.000
    SDQ2N31             0.952          0.048     19.905      0.000
    SDQ2N43             0.655          0.050     13.182      0.000
                     (factor covariances)
 F2        WITH
    F1                  0.415          0.078      5.325      0.000

 F3        WITH
    F1                  0.355          0.072      4.928      0.000
    F2                  0.464          0.080      5.825      0.000

 F4        WITH
    F1                  0.635          0.117      5.437      0.000
    F2                  0.873          0.134      6.507      0.000
    F3                  0.331          0.100      3.309      0.001
```

Hypothesized four-factor CFA model (selected output)

```
Intercepts
   SDQ2N01            4.408      0.083     53.312      0.000
   SDQ2N13            5.004      0.083     60.087      0.000
   SDQ2N25            5.098      0.075     67.766      0.000
   SDQ2N37            4.826      0.070     68.758      0.000
   SDQ2N04            4.521      0.086     52.630      0.000
   SDQ2N16            4.649      0.076     61.116      0.000
   SDQ2N28            4.691      0.082     57.419      0.000
   SDQ2N40            4.977      0.083     59.717      0.000
   SDQ2N10            4.623      0.071     65.454      0.000
   SDQ2N22            5.377      0.067     80.384      0.000
   SDQ2N34            3.891      0.104     37.256      0.000
   SDQ2N46            5.268      0.080     66.253      0.000
   SDQ2N07            4.321      0.109     39.560      0.000
   SDQ2N19            4.543      0.104     43.738      0.000
   SDQ2N31            4.740      0.096     49.225      0.000
   SDQ2N43            4.977      0.086     57.961      0.000

Variances
   F1                 0.613      0.141      4.342      0.000
   F2                 0.561      0.126      4.449      0.000
   F3                 0.668      0.116      5.744      0.000
   F4                 2.307      0.272      8.483      0.000

Residual Variances
   SDQ2N01            1.198      0.130      9.228      0.000
   SDQ2N13            1.119      0.124      9.000      0.000
   SDQ2N25            1.056      0.109      9.675      0.000
   SDQ2N37            0.771      0.089      8.621      0.000
   SDQ2N04            1.394      0.128     10.890      0.000
   SDQ2N16            0.616      0.070      8.856      0.000
   SDQ2N28            0.896      0.092      9.739      0.000
   SDQ2N40            0.952      0.095     10.061      0.000
   SDQ2N10            0.653      0.082      7.926      0.000
   SDQ2N22            0.657      0.076      8.703      0.000
   SDQ2N34            2.590      0.233     11.093      0.000
   SDQ2N46            1.201      0.118     10.164      0.000
   SDQ2N07            0.854      0.098      8.729      0.000
   SDQ2N19            1.228      0.125      9.808      0.000
   SDQ2N31            0.365      0.065      5.581      0.000
   SDQ2N43            0.964      0.093     10.410      0.000
```

Some comments and points of the analysis (p.81–)

Standardized estimates
Mplus offers three types of standardization: STDYX, STDY and STD. These correspond to various ways of conceptualizing standardization (there is no one right choice!). In this respect, SEM programs vary. Hence: check, compare and verify that particular parameter values are consistent with the literature, if you want to replicate some known results! (Here, I follow Byrne and consider only the STDYX option).

Two aspects of standardized values (compared with the unstandardized solution):

- parameters reported earlier as 1.0 have now new values
- factor variances are now reported as 1.0 (no matter which STD option is used)

Hypothesized four-factor CFA model (selected output)


```
STANDARDIZED MODEL RESULTS - STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 F1       BY
    SDQ2N01            0.582      0.055     10.613      0.000
    SDQ2N13            0.626      0.050     12.426      0.000
    SDQ2N25            0.544      0.056      9.644      0.000
    SDQ2N37            0.640      0.051     12.608      0.000

 F2       BY
    SDQ2N04            0.536      0.048     11.143      0.000
    SDQ2N16            0.774      0.031     24.983      0.000
    SDQ2N28            0.703      0.037     19.071      0.000
    SDQ2N40            0.695      0.036     19.043      0.000

 F3       BY
    SDQ2N10            0.711      0.044     16.167      0.000
    SDQ2N22            0.668      0.046     14.447      0.000
    SDQ2N34            0.322      0.064      5.002      0.000
    SDQ2N46            0.532      0.054      9.873      0.000

 F4       BY
    SDQ2N07            0.854      0.020     41.953      0.000
    SDQ2N19            0.755      0.030     24.825      0.000
    SDQ2N31            0.923      0.016     59.292      0.000
    SDQ2N43            0.712      0.033     21.287      0.000

 F2       WITH
    F1                 0.707      0.057     12.421      0.000

 F3       WITH
    F1                 0.555      0.072      7.733      0.000
    F2                 0.758      0.052     14.652      0.000

 F4       WITH
    F1                 0.534      0.061      8.756      0.000
    F2                 0.767      0.038     20.291      0.000
    F3                 0.266      0.073      3.652      0.000
```

Hypothesized four-factor CFA model (selected output)

```
Intercepts
    SDQ2N01            3.275       0.155      21.135       0.000
    SDQ2N13            3.691       0.172      21.498       0.000
    SDQ2N25            4.163       0.191      21.798       0.000
    SDQ2N37            4.224       0.193      21.831       0.000
    SDQ2N04            3.233       0.153      21.092       0.000
    SDQ2N16            3.754       0.174      21.544       0.000
    SDQ2N28            3.527       0.165      21.368       0.000
    SDQ2N40            3.668       0.171      21.481       0.000
    SDQ2N10            4.021       0.185      21.718       0.000
    SDQ2N22            4.938       0.223      22.132       0.000
    SDQ2N34            2.289       0.117      19.584       0.000
    SDQ2N46            4.070       0.187      21.746       0.000
    SDQ2N07            2.430       0.122      19.898       0.000
    SDQ2N19            2.687       0.132      20.372       0.000
    SDQ2N31            3.024       0.145      20.854       0.000
    SDQ2N43            3.561       0.166      21.396       0.000

Variances
    F1          fixed: 1.000       0.000     999.000     999.000
    F2                 1.000       0.000     999.000     999.000
    F3                 1.000       0.000     999.000     999.000
    F4                 1.000       0.000     999.000     999.000

Residual Variances
    SDQ2N01            0.661       0.064      10.366       0.000
    SDQ2N13            0.609       0.063       9.665       0.000
    SDQ2N25            0.704       0.061      11.467       0.000
    SDQ2N37            0.591       0.065       9.096       0.000
    SDQ2N04            0.713       0.051      13.847       0.000
    SDQ2N16            0.402       0.048       8.381       0.000
    SDQ2N28            0.506       0.052       9.784       0.000
    SDQ2N40            0.517       0.051      10.195       0.000
    SDQ2N10            0.494       0.063       7.905       0.000
    SDQ2N22            0.554       0.062       8.987       0.000
    SDQ2N34            0.896       0.042      21.583       0.000
    SDQ2N46            0.717       0.057      12.493       0.000
    SDQ2N07            0.270       0.035       7.763       0.000
    SDQ2N19            0.429       0.046       9.341       0.000
    SDQ2N31            0.148       0.029       5.170       0.000
    SDQ2N43            0.493       0.048      10.351       0.000
```

Some comments and points of the analysis (p.82–)

R^2 values

When standardized estimates are requested, R^2 values for the dependent variables are reported.

- R^2 is the proportion of variance accounted for by its related factor (*communality* in factor analytic terms).
- SDQ2N34 is the weakest indicator ($R^2 = 1 - 0.896 = 0.104$).

R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SDQ2N01	0.339	0.064	5.307	0.000
SDQ2N13	0.391	0.063	6.213	0.000
SDQ2N25	0.296	0.061	4.822	0.000
SDQ2N37	0.409	0.065	6.304	0.000
SDQ2N04	0.287	0.051	5.572	0.000
SDQ2N16	0.598	0.048	12.492	0.000
SDQ2N28	0.494	0.052	9.535	0.000
SDQ2N40	0.483	0.051	9.522	0.000
SDQ2N10	0.506	0.063	8.083	0.000
SDQ2N22	0.446	0.062	7.224	0.000
SDQ2N34	**0.104**	0.042	2.501	0.012
SDQ2N46	0.283	0.057	4.936	0.000
SDQ2N07	0.730	0.035	20.976	0.000
SDQ2N19	0.571	0.046	12.412	0.000
SDQ2N31	0.852	0.029	29.646	0.000
SDQ2N43	0.507	0.048	10.643	0.000

Some comments and points of the analysis (p.82–)

Model misspecification and Modification Indices (MI)
The function of MIs is to identify badly chosen parameter constraints (e.g. those fixed to a value of 0.00).
MIs are used to help answering to these questions:

- "What if a parameter would be freely estimated?"
- "How much would χ^2 value of the model decrease?"
- "Would the drop be significant?"
- "Would it lead to a better fitting model?"

A clue is given by the corresponding EPC (Expected Parameter Change) values. Again substantive knowledge and reasoning is required.

By default, Mplus reports only those parameters having $MI \geq 10$.

Some comments and points of the analysis (p.87–)

Here, we have four suggestions: one factor loading and three *residual* covariances (note: observed variables in CFA are always dependent variables; their (co)variances are not eligible for estimation).

```
MODEL MODIFICATION INDICES
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
BY Statements

F2       BY SDQ2N07         11.251     -0.563     -0.422      -0.237

WITH Statements

SDQ2N25  WITH SDQ2N01       17.054      0.359      0.359       0.319
SDQ2N31  WITH SDQ2N07       10.696      0.305      0.305       0.546
SDQ2N31  WITH SDQ2N19       17.819     -0.331     -0.331      -0.495
```

The factor loading (F2 BY SDQ2N07) represents a **cross-loading** that could be added. The problem is that from a substantive perspective, the EPC value has a wrong sign! (The relation should be positive, not negative.) Thus it would be questionable to free this parameter for estimation, no matter the MI and EPC.

Some comments and points of the analysis (p.87–)

Overall: the residual covariances seem to be rather small and not worthy of inclusion in a subsequently specified model. Remember the topic of scientific parsimony: avoid too many parameters and hence too complex models! **First of all: model must be substantively meaningful.**

In general, model respecification is commonly conducted in SEM. (We will get back to this issue many times!) However, it is important to realize that when we move to so called *Post Hoc* analyses, they will then be framed within *exploratory*, not anymore confirmatory modeling approach! Post hoc model fitting is also called *specification searches* (detection of misfitting parameters in the originally hypothesized model). Combination of 1) substantive and 2) statistical aspects is required (always in this order!).

"When to stop fitting a model?" – a good question...