

COS-D419 Factor Analysis and Structural Equation Models 2023, Assignment 4

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The texts that reflect my understanding have been highlighted in red color.

1 Task description

The first section is task description, which is copied from the assignment5.rmd. It is for communicating with future “me”. Please skip it.

1.1 Exercise 5.1

Specify and estimate the initial baseline models for the two groups.

Present a brief summary of the model fit and make the first step of the modification by including (**exceptionally, at the same time!**) all the four parameters known to be required for improving the model fit of both models.

Fine-tune the models step by step following the guidelines given in the lecture material, i.e., implement the modifications (**as usually, one change at a time**) testing and studying each step.

Present the final baseline models of each group and draw the graphs

2 Preparation

##Read in the data set:

Start by downloading the **two data files** from Moodle to your Project folder!

```
#install the necessary packages
if (!require("pacman")) install.packages("pacman")
pacman::p_load(
  expss,
  tidyverse,
  janitor,
  knitr,
  qualtrics,
  arules,
  arulesViz,
  sjlabelled,
  DT,
  stringr,
  labelled,
  ggstatsplot,
  ggcorplot)

library(tidyverse)
library(readr)

#This week's file name
latest.name1 <- "MBIELM1.CSV"
latest.name2 <- "MBISEC1.CSV"
#read in the data
mbi.elm <- #elementary school
  read_csv(
    file.path(
      here(),
      'data',
      latest.name1
    )
  )

mbi.sec <- #secondary school
  read_csv(
    file.path(
      here(),
      'data',
      latest.name2
    )
  )
```

2.1 Write functions

To control length of reports, codes already shown in the previous homework were not showing in the current report. Yet they are available in .rmd report.

2.1.1 To generate a function for calculating chi square difference was defined.

2.1.2 to generate CFA results with improved readability

2.1.3 Write a function to simplify plotting of merged tables for multi-group fit indices

```
multi.fit.tab <- function(data, title){
data <- data |>
  rename(p = 'p value',
         p2 = 'RMSEA p value',
         chi = 'chi square') |>
  mutate(df = as.numeric(df) |> round(0),
         p = case_when(
           as.numeric(p) < 0.001 ~ "<0.001",
           as.numeric(p) >= 0.001 ~ p
         ),
         p2 = case_when(
           as.numeric(p2) < 0.001 ~ "<0.001",
           as.numeric(p2) >= 0.001 ~ p2
         )
  ) |>
  mutate('Chi square (df, p)' =
    paste0(chi, "(", df, ", ", p, ")"),
         'RMSEA(p)' =
    paste0(RMSEA, "(", p2, ")")
  )
  ) |>
  select('Chi square (df, p)',
        CFI, TLI,
        'RMSEA(p)',
        SRMR,
        'CSF*' = CSF
  )
#print the combined table with adjustment of aesthetics
data |>
  kable(booktabs = T,
        #format = "markdown",
        caption =
          title,
        align = "r"
  ) |>
  kable_styling(full_width = T) |>
  footnote(symbol =
    "Chi square scaling factor"
  ) |>
  column_spec(1, width = "3cm") |>
  column_spec(2, width = "4cm") |>
  column_spec(3, width = "1cm") |>
  column_spec(4, width = "1cm") |>
  column_spec(5, width = "2.5cm") |>
  column_spec(6, width = "1cm") |>
  column_spec(7, width = "1cm")
}
```

2.1.4 Write a function to simplify plotting aligned residual variance and co-variance tables

```
align.table <- function(data, num.no.header.col, title){  
  data |>  
  kable(  
    digits = 3,  
    booktabs = T,  
    #format = "markdown",  
    caption = title,  
    linesep = ""  
  ) |>  
  add_header_above(c(" " = num.no.header.col,  
                     "Elementary level" = 5,  
                     "Secondary level" = 5  
                     )  
  ) |>  
  kable_styling(  
    latex_options = "striped"  
  ) |>  
  footnote(  
    symbol = c(  
      "Un-standardized estimates",  
      "Standardized estimates"  
    )  
  )  
}
```

2.1.5 Write a function for correlation matrix with numbers

2.1.6 to generate a function for histogram overlapping with density plot

2.1.7 to generate a function for violin overlapping with box plot

2.1.8 To generate a function describing continuous data set

2.1.9 Write a function describing continuous data set

2.1.10 Write a function for histogram overlapping with density plot

2.1.11 Write a function to generate dot distribution plot

```
dot.dist <-  
function(data, type, title){  
  data |>  
  t() |>  
  as.data.frame() %>%  
  mutate(Item = rownames(.)) |>  
  rowwise() |>  
  mutate(Median = eval(parse(text = type))(V1:V580)) |>  
  ggstatsplot::ggdotplotstats(  
    data = data,  
    type = type,  
    title = title  
  )  
}
```

```

    point.args = list(color = "red", size = 3, shape = 13),
    xlab = paste(type, "ratings"),
    title = title,
    x = Median,
    y = Item
  )
}

```

2.1.12 Write a function to generate correlation matrix with statistical test

```

mycor <-
function(data, cols, title){
  mbi.elm |>
    select(all_of(cols)) |>
    ggstatsplot::ggcorrmat(
      colors = c("#B2182B", "white", "#4D4D4D"),
      title = "(a) Items on emotional exhaustion,
        elementary school teacher",
      matrix.type = "lower"
    )
}

```

3 Inspect the data

3.1 Distribution

```

#generate the plots, by subgroup of teachers
p.dist.elm <-
  corr.density(
    mbi.elm,
    fig.num = "1(a)",
    group = "elementary school teacher"
  )
p.dist.sec <-
  corr.density(
    mbi.sec,
    fig.num = "1(b)",
    group = "secondary school teacher"
  )
#print the plot
library(patchwork); p.dist.elm/p.dist.sec

```

Figure 1(a) Distribution of selected items for elementary school teacher

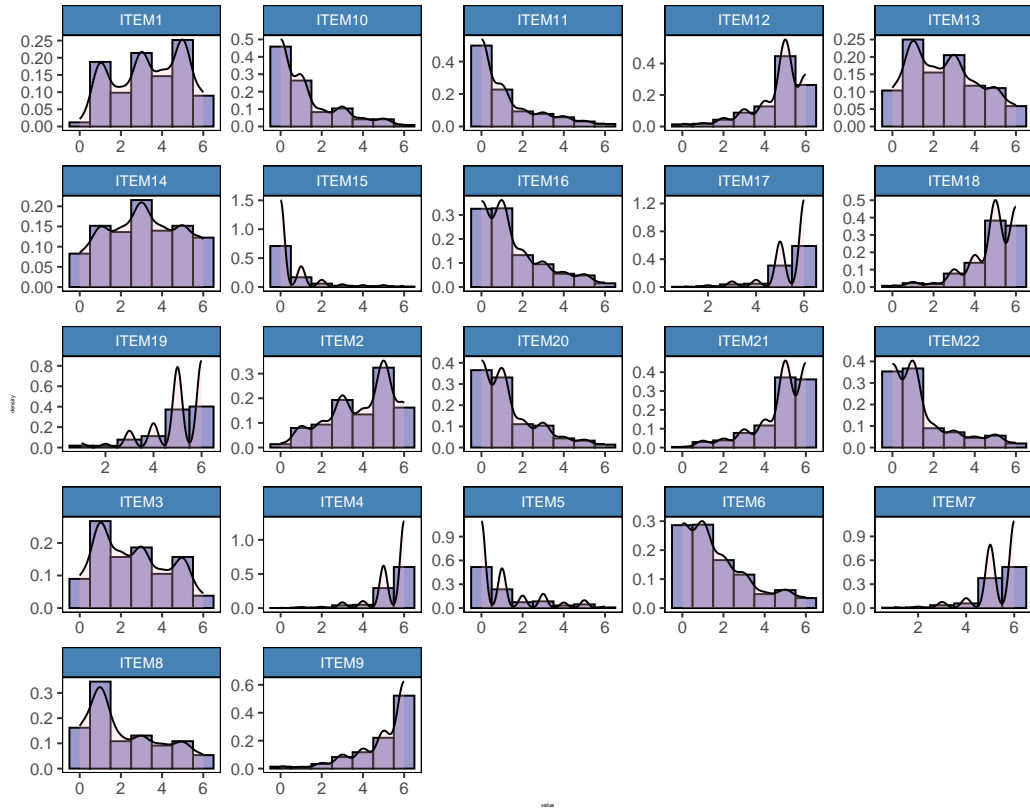
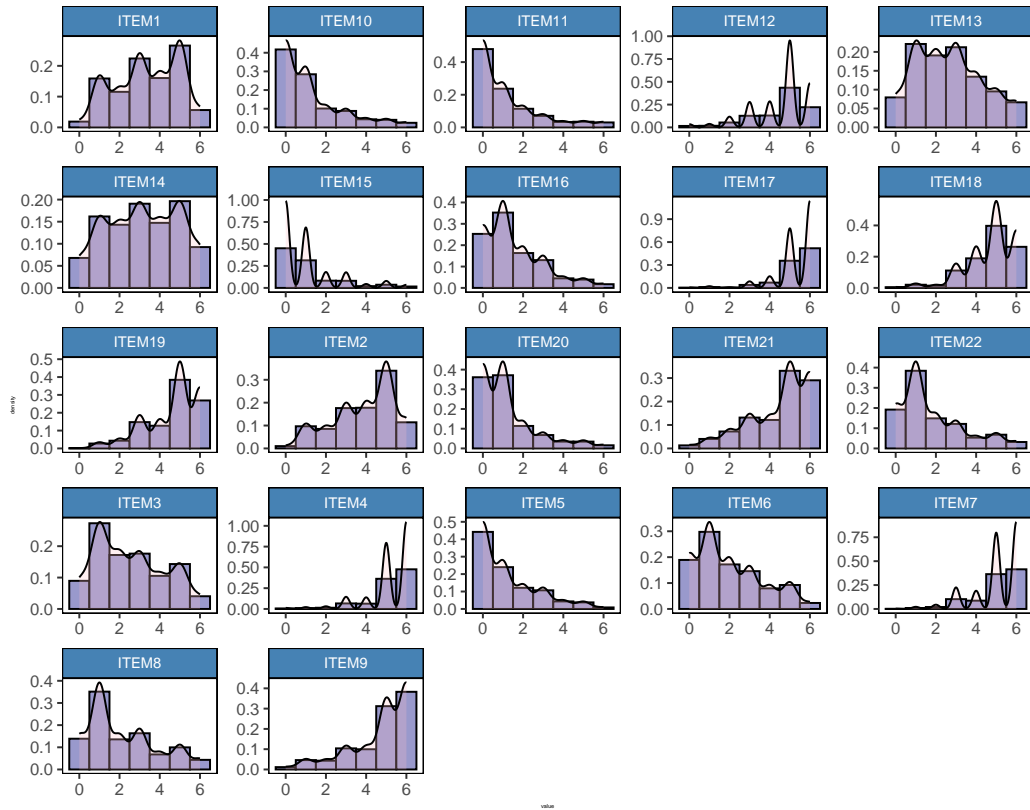


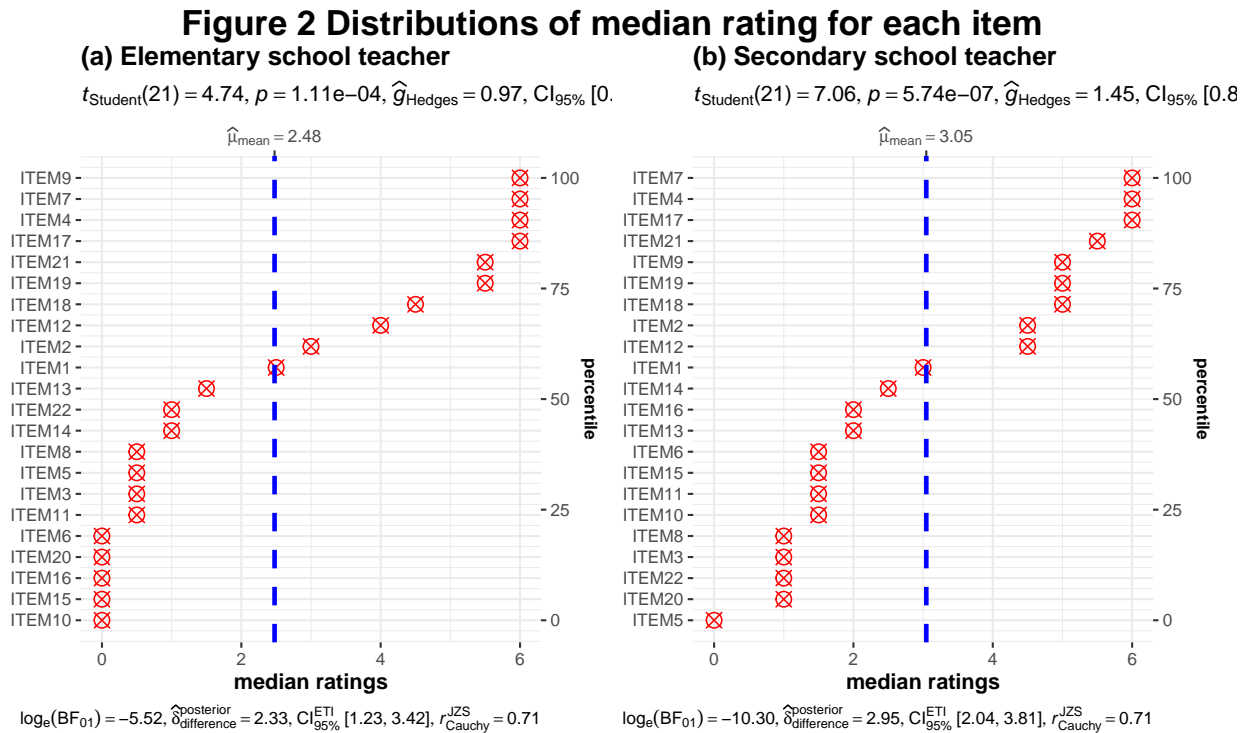
Figure 1(b) Distribution of selected items for secondary school teacher



```

#generate plot by subgroups of teachers
p.dot.elm <-
  dot.dist(
    data = mbi.elm,
    type = "median",
    title = "(a) Elementary school teacher"
  )
p.dot.sec <-
  dot.dist(
    data = mbi.sec,
    type = "median",
    title = "(b) Secondary school teacher"
  )
#plot layout
patchwork <- p.dot.elm|p.dot.sec
#print the plot with a genral title
patchwork+plot_annotation(
  title =
    'Figure 2 Distributions of median rating for each item',
  theme =
    theme(plot.title =
      element_text(
        size = 16,
        face = "bold",
        vjust = -1.5,
        hjust = 0.5)
    )
)

```



```

fa.ee <- c("ITEM1", "ITEM3", "ITEM6", "ITEM8", "ITEM13", "ITEM14", "ITEM16", "ITEM20")
fa.dp <- c("ITEM5", "ITEM10", "ITEM11", "ITEM15", "ITEM22")
fa.pa <- c("ITEM4", "ITEM7", "ITEM9", "ITEM12", "ITEM17", "ITEM18", "ITEM19", "ITEM21")
#generate 6 plots, 3 factors X 2 subgroups of teachers
p.cor.elm.ee <-
  mycor(
    data= mbi.elm,
    cols = fa.ee,
    "(a) Items on emotional exhaustion,
    elementary school teacher"
  )
p.cor.sec.ee <-
  mycor(
    data = mbi.sec,
    cols = fa.ee,
    "(b) Items on emotional exhaustion,
    secondary school teacher"
  )
p.cor.elm.dp <-
  mycor(
    data = mbi.elm,
    cols = fa.dp,
    "(c) Items on depersonalization,
    elementary school teacher"
  )
p.cor.sec.dp <-
  mycor(
    data = mbi.sec,
    cols = fa.dp,
    "(d) Items on depersonalization,
    secondary school teacher"
  )
p.cor.elm.pa <-
  mycor(
    data = mbi.elm,
    cols = fa.pa,
    "(e) Items on personal accomplishment,
    secondary school teacher"
  )
p.cor.sec.pa <-
  mycor(
    data = mbi.sec ,
    cols = fa.pa,
    "(f) Items on personal accomplishment,
    secondary school teacher"
  )
#plot sub-figure layout
patchwork <-
  p.cor.elm.ee/p.cor.elm.dp/p.cor.elm.pa|p.cor.sec.ee/p.cor.sec.dp/p.cor.sec.pa
#print the plot with a gernal title
patchwork+
  plot_annotation(
    title =

```

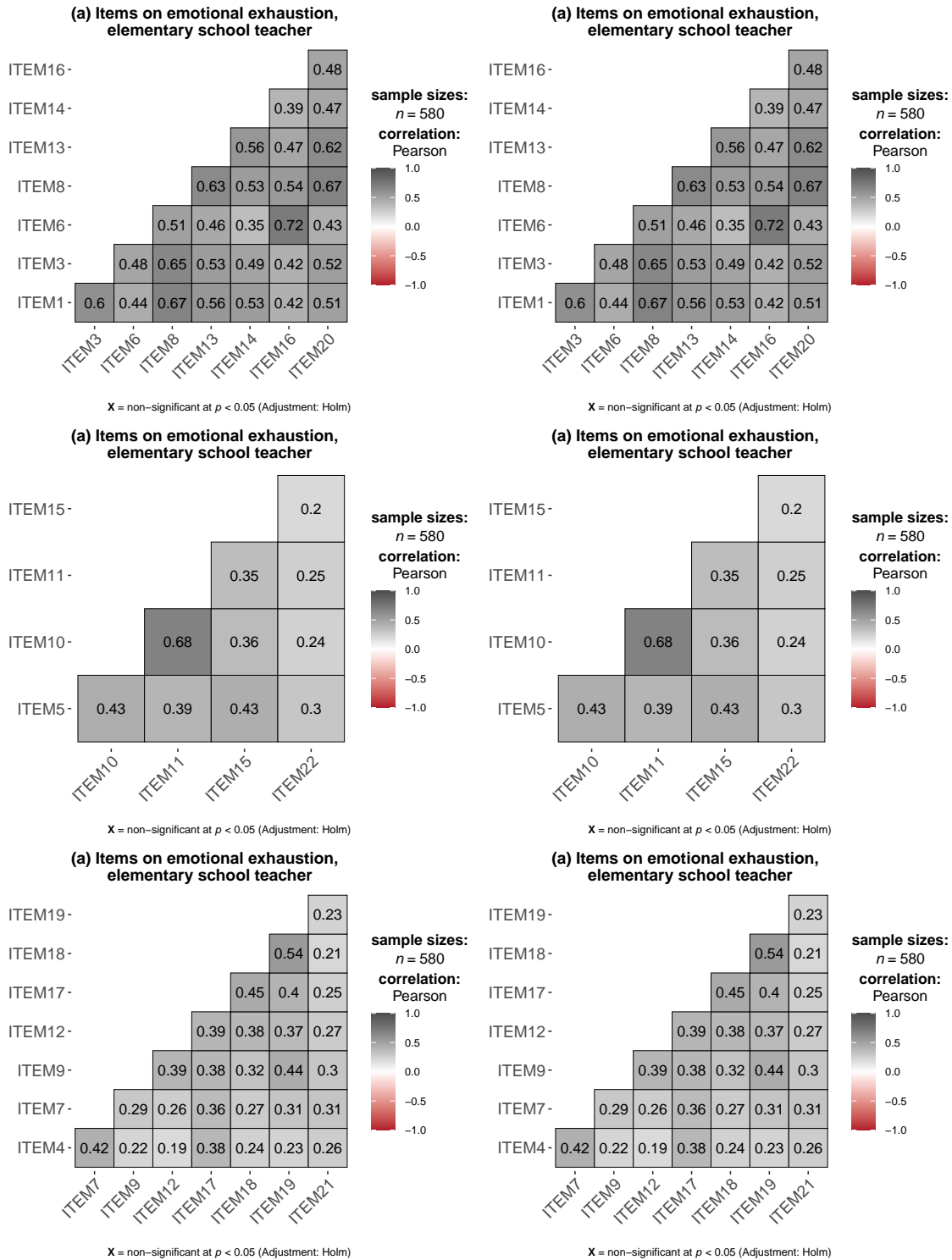


```

    'Figure 3 Correlalogram for items on each factor for two groups of teachers',
  theme =
    theme(plot.title =
      element_text(
        size = 16,
        face = "bold",
        vjust = -1.5,
        hjust =0.5)
    )
)

```

Figure 3 Correlalogram for items on each factor for two groups of teachers



4 Testing the factorial invariance of MBI inventory between elementary and secondary school teachers

4.1 Define and estimate initial models for both subgroups

The postulated three-factor structure of the MBI that was tested in the previous assignments were re-tested as the initial model for establishing a baseline model.

4.1.1 Define the initial model

```
library(lavaan)
# Define a CFA model using the lavaan (Latent Variable Analysis) syntax:
# see https://lavaan.ugent.be/tutorial/syntax1.html
initial.model <- '
# CFA model for the burnout, the baseline model:
  EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 +
      ITEM13 + ITEM14 + ITEM16 + ITEM20
  DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
  PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 +
      ITEM17 + ITEM18 + ITEM19 + ITEM21
'
```

Cited from Byrne: *It is important to note that measuring instruments are often group specific in the way they operate, and, thus, it is possible that baseline models may not be completely identical across groups.*

4.1.2 Estimate indices to examine factorial validity

- (1) Estimate factorial validity for the elementary teacher subgroup

```
cfa.elm <-
  cfa(
    initial.model,
    data = mbi.elm,
    estimator = "MLM",
    mimic = "Mplus"
  )
```

- (2) Estimate factorial validity for the secondary teacher subgroup

```
cfa.sec <-
  cfa(
    initial.model,
    data = mbi.sec,
    estimator = "MLM",
    mimic = "Mplus"
  )
```

Table 1: Fit indices for two subgroups, baseline models

	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Value	826.573(206, <0.001)	0.857	0.840	0.072(<0.001)	0.068	1.225
Value1	999.359(206, <0.001)	0.836	0.816	0.075(<0.001)	0.077	1.284

* Chi square scaling factor

4.1.3 Evaluate model

(1) Fit indices

```
library(knitr);library(kableExtra)
#combine fit indices of both levels
initial.elm.fit <-
  cfa.summary.mlm.a(cfa.elm) |>
  t() |>
  as.data.frame()

initial.sec.fit <-
  cfa.summary.mlm.a(cfa.sec) |>
  t() |>
  as.data.frame()
initial.both <-
  rbind(
    initial.elm.fit[2,],
    initial.sec.fit[2,]
  )

names(initial.both) <-
  initial.elm.fit[1,]

initial.both |>
  mutate(Model = c("Elementary level",
    "Secondary level")) |>
  select(Model, everything())
```

```
##           Model chi square      df p value      CFI      TLI      RMSEA
## Value Elementary level    826.573 206.000    0.000    0.857    0.840    0.072
## Value1 Secondary level    999.359 206.000    0.000    0.836    0.816    0.075
##           RMSEA p value      SRMR      CSF
## Value           0.000    0.068    1.225
## Value1          0.000    0.077    1.284
```

```
#print the table
multi.fit.tab(initial.both, "Fit indices for two subgroups, baseline models")
```

See table 1. Goodness-of-fit statistics for this baseline model (three factor) reveals that the indices are less than optimal for both elementary (MLM Chi-square[206] = 826.573; CFI = 0.857; RMSEA = 0.072 ; SRMR = 0.068) and secondary (MLM Chi-square[206] = 999.359; CFI = 0.836; RMSEA = 0.075; SRMR = 0.077) levels.

(2) factor loading

Factor loading of elementary level were extracted.

```
fl.elm <- cfa.summary.b (cfa.elm) #fl is for factor loading
colnames(fl.elm)[2] <- "Beta*"
```

Factor loading of secondary level were extracted.

```
fl.sec <- cfa.summary.b (cfa.sec) #fl is for factor loading
colnames(fl.sec) <- c("Parameter",
                     "Beta* ",
                     "SE ",
                     "Z ",
                     "p-value ")
```

Factor loading of both levels were merged in one table and printed.

```
fl.both <- left_join(fl.elm,
                    fl.sec,
                    by = "Parameter")

fl.both |>
  kable(
    digits = 3,
    booktabs = T,
    #format = "markdown",
    caption = "Factor loadings for both levels",
    linesep = ""
  ) |>
  add_header_above(c(" " = 1,
                    "Elementary level" = 4,
                    "Secondary level" = 4
                    )
  ) |>
  kable_styling() |>
  row_spec(1:9,
    background = "#E5E4E2"
  ) |>
  row_spec(15:22,
    background = "#E5E4E2"
  ) |>
  row_spec(c(1,10,15), bold = T) |>
  footnote(general =
    "Rows with coefficient estimates fixed to 1 are highlighted in bold ",
    symbol = c(
      "Standardized estimates"
    )
  )
```

the cross-loading involved the loading of Item 12 on Factor 1 (Emotional Exhaustion) in addition to its targeted Factor 3 (Personal Accomplishment)

(3) Variance

Variance of elementary level were extracted.

Table 2: Factor loadings for both levels

Parameter	Elementary level				Secondary level			
	Beta*	SE	Z	p-value	Beta*	SE	Z	p-value
EE→ITEM1	0.776	0.000	NA	NA	0.756	0.000	NA	NA
EE→ITEM2	0.754	0.032	28.561	<0.001	0.736	0.031	30.236	<0.001
EE→ITEM3	0.740	0.045	21.984	<0.001	0.722	0.043	24.030	<0.001
EE→ITEM6	0.631	0.051	16.064	<0.001	0.626	0.046	18.669	<0.001
EE→ITEM8	0.855	0.042	28.448	<0.001	0.833	0.046	25.968	<0.001
EE→ITEM13	0.754	0.045	22.474	<0.001	0.762	0.045	23.619	<0.001
EE→ITEM14	0.655	0.046	19.939	<0.001	0.634	0.045	20.685	<0.001
EE→ITEM16	0.640	0.047	15.992	<0.001	0.596	0.047	15.261	<0.001
EE→ITEM20	0.734	0.045	18.371	<0.001	0.707	0.048	17.421	<0.001
DP→ITEM5	0.576	0.000	NA	NA	0.453	0.000	NA	NA
DP→ITEM10	0.794	0.115	11.968	<0.001	0.820	0.188	10.259	<0.001
DP→ITEM11	0.793	0.122	11.588	<0.001	0.808	0.197	9.666	<0.001
DP→ITEM15	0.505	0.072	9.287	<0.001	0.472	0.098	10.295	<0.001
DP→ITEM22	0.351	0.091	6.997	<0.001	0.447	0.131	8.226	<0.001
PA→ITEM4	0.447	0.000	NA	NA	0.340	0.000	NA	NA
PA→ITEM7	0.516	0.148	7.308	<0.001	0.545	0.221	7.495	<0.001
PA→ITEM9	0.581	0.280	6.629	<0.001	0.681	0.365	7.432	<0.001
PA→ITEM12	0.611	0.303	6.214	<0.001	0.586	0.283	7.398	<0.001
PA→ITEM17	0.681	0.185	7.796	<0.001	0.546	0.187	7.486	<0.001
PA→ITEM18	0.628	0.276	6.628	<0.001	0.698	0.294	7.431	<0.001
PA→ITEM19	0.643	0.255	6.844	<0.001	0.706	0.324	7.565	<0.001
PA→ITEM21	0.425	0.187	7.018	<0.001	0.410	0.242	6.808	<0.001

Note:

Rows with coefficient estimates fixed to 1 are highlighted in bold

* Standardized estimates

```
var.elm <- cfa.summary.c(cfa.elm, fa.num = 3, item.num = 22)
names(var.elm)[3] <- "Beta*"
names(var.elm)[4] <- "Beta†"
```

Variance of secondary level were extracted.

```
var.sec <- cfa.summary.c(cfa.sec, fa.num = 3, item.num = 22)
var.sec <- var.sec[,-1]
names(var.sec) <-
  c("Indicator",
    "Beta* ",
    "Beta† ",
    "SE ",
    "Z ",
    "p-value ")
)
```

Variance of both levels were merged in one table and printed.

```
var.both <- left_join(var.elm,
                      var.sec,
                      by = "Indicator")

align.table(data = var.both,
            num.no.header.col = 2,
            title = "Residual variance for both levels")
```

(3) Co-variance

Co-variance of elementary level were extracted.

```
cov.elm <- cfa.summary.d(cfa.elm, fa.num = 3, item.num = 22)
colnames(cov.elm)[2:3] <- c("Beta*", "Beta†")
```

Co-variance of secondary level were extracted.

```
cov.sec <- cfa.summary.d(cfa.sec, fa.num = 3, item.num = 22)
colnames(cov.sec) <- c("Parameter", "Beta* ", "Beta† ", "SE ", "Z ", "p-value ")
```

Co-variance of both levels were merged in one table and printed.

```
cov.both <- left_join(cov.elm,
                      cov.sec,
                      by = "Parameter")

align.table(data = cov.both,
            num.no.header.col = 1,
            title = "Residual co-variance for both levels")
```

Table 3: Residual variance for both levels

Parameter	Indicator	Elementary level					Secondary level				
		Beta*	Beta†	SE	Z	p-value	Beta*	Beta†	SE	Z	p-value
Residual	ITEM1	1.095	0.398	0.062	17.641	<0.001	1.078	0.429	0.056	19.329	<0.001
Residual	ITEM2	1.067	0.432	0.063	16.832	<0.001	1.071	0.459	0.053	20.373	<0.001
Residual	ITEM3	1.322	0.452	0.089	14.773	<0.001	1.383	0.479	0.083	16.704	<0.001
Residual	ITEM6	1.655	0.602	0.098	16.924	<0.001	1.656	0.609	0.084	19.730	<0.001
Residual	ITEM8	0.886	0.269	0.068	13.044	<0.001	0.890	0.306	0.061	14.560	<0.001
Residual	ITEM13	1.281	0.431	0.087	14.663	<0.001	1.167	0.419	0.075	15.574	<0.001
Residual	ITEM14	1.897	0.571	0.113	16.728	<0.001	1.883	0.599	0.110	17.084	<0.001
Residual	ITEM16	1.363	0.591	0.066	20.746	<0.001	1.353	0.645	0.071	19.024	<0.001
Residual	ITEM20	0.954	0.461	0.093	10.210	<0.001	0.983	0.500	0.057	17.125	<0.001
Residual	ITEM5	1.459	0.669	0.119	12.289	<0.001	1.711	0.795	0.100	17.052	<0.001
Residual	ITEM10	0.806	0.370	0.094	8.530	<0.001	0.803	0.328	0.090	8.944	<0.001
Residual	ITEM11	0.848	0.372	0.101	8.404	<0.001	0.854	0.347	0.095	9.013	<0.001
Residual	ITEM15	0.934	0.745	0.119	7.870	<0.001	1.562	0.778	0.112	13.964	<0.001
Residual	ITEM22	2.086	0.877	0.143	14.538	<0.001	2.052	0.800	0.124	16.598	<0.001
Residual	ITEM4	0.696	0.800	0.066	10.568	<0.001	1.074	0.884	0.104	10.372	<0.001
Residual	ITEM7	0.562	0.734	0.058	9.605	<0.001	0.907	0.703	0.064	14.108	<0.001
Residual	ITEM9	1.176	0.662	0.115	10.247	<0.001	1.194	0.536	0.097	12.297	<0.001
Residual	ITEM12	1.039	0.627	0.079	13.108	<0.001	1.177	0.657	0.076	15.418	<0.001
Residual	ITEM17	0.418	0.536	0.048	8.653	<0.001	0.649	0.701	0.063	10.319	<0.001
Residual	ITEM18	0.894	0.606	0.109	8.170	<0.001	0.703	0.512	0.068	10.329	<0.001
Residual	ITEM19	0.753	0.587	0.062	12.153	<0.001	0.847	0.501	0.080	10.595	<0.001
Residual	ITEM21	1.360	0.819	0.124	10.949	<0.001	1.889	0.832	0.111	17.056	<0.001
Total	EE	1.657	1.000	0.114	14.585	<0.001	1.436	1.000	0.097	14.854	<0.001
Total	DP	0.723	1.000	0.111	6.515	<0.001	0.442	1.000	0.085	5.188	<0.001
Total	PA	0.174	1.000	0.046	3.814	<0.001	0.141	1.000	0.034	4.108	<0.001

* Un-standardized estimates

† Standardized estimates

Table 4: Residual co-variance for both levels

Parameter	Elementary level					Secondary level				
	Beta*	Beta†	SE	Z	p-value	Beta*	Beta†	SE	Z	p-value
EE $\leftarrow \rightarrow$ DP	0.688	0.628	0.075	9.171	<0.001	0.451	0.566	0.057	7.928	<0.001
EE $\leftarrow \rightarrow$ PA	-0.254	-0.473	0.037	-6.952	<0.001	-0.177	-0.393	0.029	-6.193	<0.001

* Un-standardized estimates

† Standardized estimates

4.1.4 Model re-specification

- (1) Search for mis-specified parameters

To establish baseline models for both panels of teachers that represent good model fit and parsimony, I further investigated the modification indices of the hypothesized models, respectively for two levels.

MI's of elementary level panel were calculated.

```
#extract needed variables
initial.MI.elm <-
  modindices(cfa.elm,
             standardized = TRUE,
             sort. = TRUE,
             maximum.number = 10)
```

MI's of secondary level panel were calculated.

```
#extract needed variables
initial.MI.sec <-
  modindices(cfa.sec,
             standardized = TRUE,
             sort. = TRUE,
             maximum.number = 10)
```

MI tables with 10 largest MI parameters was printed in descending order of MI. Potential mis-specification of most concerns were highlighted in red.

```
MI.both <- rbind(initial.MI.elm, initial.MI.sec)

MI.both |>
  mutate(Parameter =
    paste(rhs, "→", lhs)
  ) |>
  select(Parameter,
    MI = mi,
    EPC = epc,
    "std EPC" = sepc.all
  ) |>
  kable(digits = 3,
    booktab = T,
    linesep = "",
    caption =
      "Selected modification indices for determining baseline model") |>
  kable_styling(
    latex_options = "striped"
  ) |>
  row_spec(
    c(1:4, 11:14),
    color = "red"
  ) |>
  footnote(general =
    "Rows highlighted in red are of special concerns") |>
```

Table 5: Selected modification indices for determining baseline model

	Parameter	MI	EPC	std EPC
Elementary level				
183	ITEM16 → ITEM6	180.298	0.893	0.595
120	ITEM2 → ITEM1	103.177	0.534	0.494
84	ITEM12 → EE	81.319	-0.400	-0.400
285	ITEM11 → ITEM10	67.743	0.688	0.832
348	ITEM19 → ITEM18	43.669	0.279	0.340
323	ITEM7 → ITEM4	42.833	0.184	0.294
175	ITEM12 → ITEM3	28.187	-0.287	-0.245
275	ITEM15 → ITEM5	25.815	0.273	0.234
96	ITEM16 → DP	25.652	0.459	0.257
185	ITEM5 → ITEM6	23.753	0.337	0.217
Secondary level				
1201	ITEM2 → ITEM1	171.647	0.627	0.583
2851	ITEM11 → ITEM10	135.841	1.181	1.426
1831	ITEM16 → ITEM6	127.756	0.686	0.458
841	ITEM12 → EE	118.156	-0.468	-0.419
2751	ITEM15 → ITEM5	77.216	0.580	0.355
296	ITEM15 → ITEM11	60.947	-0.485	-0.420
147	ITEM20 → ITEM2	53.024	-0.324	-0.316
274	ITEM11 → ITEM5	48.297	-0.446	-0.369
339	ITEM19 → ITEM9	46.617	0.360	0.358
77	ITEM10 → EE	45.623	-0.394	-0.302

Note:

Rows highlighted in red are of special concerns

```
pack_rows(index = c(
  "Elementary level" = 10,
  "Secondary level" = 10
))
```

See table 5. Three exceptionally large residual co-variances and one cross-loading contributed to the misfit of the model for both teacher panels. The residual co-variances involved Items 1 and 2, Items 6 and 16, and Items 10 and 11; the cross-loading involved the loading of Item 12 on Factor 1 (Emotional Exhaustion) in addition to its targeted Factor 3 (Personal Accomplishment).

In reviewing both the MIs and expected parameter change (EPC) statistics for elementary teachers (table 5, upper part), it is clear that all four parameters are contributing substantially to model misfit, with the residual covariance between Item 6 and Item 16 exhibiting the most profound effect.

We see precisely the same pattern on secondary teachers, albeit the effect would appear to be even more pronounced than it was for elementary teachers. One slight difference between the two groups of teachers regards the impact of these four parameters on model misfit. Whereas the residual covariance between Items 6 and 16 was found to be the most seriously misfitting parameter for elementary teachers; for secondary teachers, the residual covariance between Items 1 and 2 was most pronounced.

(2) Re-specified both models

The good practice is relaxing one parameter each time. Nonetheless, according to the knowledge derived

from our previous work, I included all four mis-specified parameters in a post-hoc model (common to the groups).

First, the 4 parameters were relaxed in model statement.

```
respecified4 <- 'EE =~ ITEM12
                ITEM6 ~~ ITEM16
                ITEM10 ~~ ITEM11
                ITEM1 ~~ ITEM2
                '
model2 <- paste(initial.model, respecified4)
```

Then, the model fit were re-estimated for both group, respectively

```
#for elementary
cfa2.elm <-
  cfa(
    model2,
    data = mbi.elm,
    estimator = "MLM",
    mimic = "Mplus"
  )
#for secondary
cfa2.sec <-
  cfa(
    model2,
    data = mbi.sec,
    estimator = "MLM",
    mimic = "Mplus"
  )

#combine fit indices of both levels
model2.elm.fit <-
  cfa.summary.mlm.a(
    cfa2.elm
  ) |>
  t() |>
  as.data.frame()

model2.sec.fit <-
  cfa.summary.mlm.a(
    cfa2.sec
  ) |>
  t() |>
  as.data.frame()

model2.both <-
  rbind(
    model2.elm.fit[2,],
    model2.sec.fit[2,]
  )

names(model2.both) <- model2.elm.fit[1,]
```

Table 6: Fit indices for two subgroups, model 2

	Chi square (df, p)	CFI	TLI	RMSEA(p)	SRMR	CSF*
Initial model						
Value	826.573(206, <0.001)	0.857	0.840	0.072(<0.001)	0.068	1.225
Value1	999.359(206, <0.001)	0.836	0.816	0.075(<0.001)	0.077	1.284
Model 2						
Value2	477.667(202, <0.001)	0.936	0.927	0.049(0.679)	0.050	1.224
Value11	587.538(202, <0.001)	0.920	0.909	0.053(0.168)	0.056	1.278

* Chi square scaling factor

```
model2.both |>
  mutate(Model = c("Elementary level",
    "Secondary level")) |>
  select(Model, everything())
```

```
##           Model chi square      df p value      CFI      TLI      RMSEA
## Value Elementary level  477.667 202.000  0.000  0.936  0.927  0.049
## Value1 Secondary level  587.538 202.000  0.000  0.920  0.909  0.053
##           RMSEA p value      SRMR      CSF
## Value           0.679  0.050  1.224
## Value1          0.168  0.056  1.278
```

```
#
compare12 <- rbind(initial.both, model2.both)

#print the table
multi.fit.tab(compare12,
  "Fit indices for two subgroups, model 2") |>
  pack_rows(index = c(
    "Initial model" = 2,
    "Model 2" = 2
  )
)
```

Estimation of this re-specified model, for each teacher group, yielded greatly improved model fit statistics. See table 6.

```
chisq_mlm <- function(fit_nested, fit_parent) {
  # scaling correction factors
  c0 <- fitMeasures(fit_nested, "chisq.scaling.factor") %>% as.numeric()
  c1 <- fitMeasures(fit_parent, "chisq.scaling.factor") %>% as.numeric()
  # scaling correction of the difference test
  d0 <- fitMeasures(fit_nested, "df") %>% as.numeric()
  d1 <- fitMeasures(fit_parent, "df") %>% as.numeric()
  cd <- ((d0 * c0) - (d1 * c1))/(d0 - d1)
  # MLM chi-square difference test
  T0 <- fitMeasures(fit_nested, "chisq.scaled") %>% as.numeric()
  T1 <- fitMeasures(fit_parent, "chisq.scaled") %>% as.numeric()
  TRd <- (T0*c0 - T1*c1)/cd
  # degrees of freedom
```

```
    df = d0 - d1
    return(c("TR_d" = round(TRd,3), "df" = round(df,0), "p_value" = pchisq(TRd, df, lower.tail = FALSE)
  }
```

```
a <- chi.diff(cfa.elm, cfa2.elm)
chisq_mlm(cfa.elm, cfa2.elm)
```

```
##      TR_d      df p_value
## 339.504    4.000    0.000
```

xie