Analysis of Covariance Structures; Invariance

Part 5 of the material is based on the Chapter 7 of the book
Byrne, Barbara M. (2012). Structural Equation Modeling with Mplus.
It gives a typical example of the **multiple-group analyses** of SEM: testing the **factorial invariance** (equivalence) of a measuring instrument.

Subject: the relations among the three dimensions of burnout (cf. ch4 & ch6) and their invariance across elementary and secondary teachers.

- **Data:** samples of elementary (N=580) and secondary (N=692) teachers,
  22 MBI items (details in previous parts and elsewhere).
- **Aim:** to test the invariance of MBI across the samples.

Five questions related to the invariance (p.193)

The following five questions are typical, when seeking evidence of multigroup invariance:

1. Do the **items** operate equivalently across different populations or groups (based on, e.g., gender, age, ability, culture)?
2. Is the **factorial structure** equivalent across populations? (*construct validity*, measurement and/or structural models)
3. Are **certain paths** in a specified causal structure equivalent across populations? *(ch9)*
4. Are the **latent means** of particular constructs different across populations? *(see ch8)*
5. Does the factorial structure of a measuring instrument **replicate** across independent samples drawn from the same population? *(cross-validation) (ch9)*

In this part, we focus on questions 1 and 2.

Testing multigroup invariance, some remarks (p.194–195)

In principle, testing the invariance begins from a global test concerning the population covariance matrices of the groups. The null hypothesis is

$$H_0 : \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \cdots = \boldsymbol{\Sigma}_G,$$

where G is the number of groups. If H_0 cannot be rejected, the groups are considered to have equivalent covariance structures, and no further invariance tests are needed. Then, the group data could be pooled and analysed with single-group methods.

The problem is that there is no baseline model, and hence the testing might be too restrictive and the results contradictory (e.g. differences in parameters although H_0 not rejected or no differences although H_0 rejected – cf. ANOVA and F test).

Testing multigroup invariance, testing strategy (p.195–196)

A more realistic testing strategy is based on setting up a well fitting *multigroup baseline model*, also called a *configural model*. With that model, sets of parameters are put to a test in a logically ordered and increasingly restrictive fashion.

The most common order of questions and related tests is 1) factor loadings, 2) factor covariances, 3) structural regression paths and 4) latent factor means. The list could be continued by 5) residual variances and 6) residual covariances, but in most cases the last two steps are nowadays considered as historical and not relevant for the invariance studies.

Setting up the configural model begins with the determination of a *separate baseline model* for each group. These models are first optimized, both substantially and statistically, and then combined to form the initial multigroup model, i.e., the configural model.

Testing multigroup invariance, testing strategy (p.196–197)

As soon as the configural model is established, the testing continues as follows (iteratively in each phase):

1. testing the group-invariance of the measurement model parameters
2. specifying the cross-group equality constraints for particular parameters of the measurement model
3. testing the group-invariance of the structural model parameters
4. specifying the cross-group equality constraints for particular parameters of the structural model

In this example, the focus is solely on covariance structures (invariance of the factor structures across the two groups), and hence no means or intercepts are involved. Factor loadings, variances and covariances are of the primary interest.

Hypothesized, initial structure (graph, p.97)

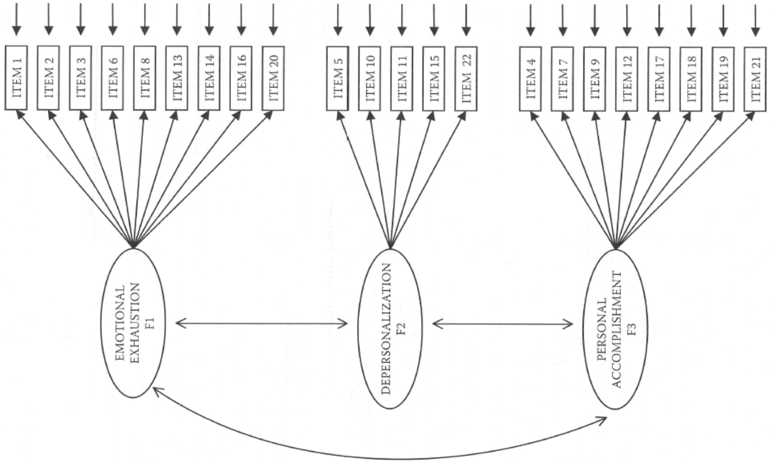

Figure 4.1. Hypothesized CFA model of factorial structure for the Maslach Burnout Inventory (MBI).

Input specifications, baseline models (p.197–202)

Initially, these are similar for both samples.

```
MBIELM1.INP
TITLE: CFA of MBI for Elementary Teachers
       Initial Baseline Model (Byrne 2012, p. 198-)

DATA:  FILE IS "MBIELM1.DAT";

VARIABLE: NAMES ARE ITEM1 - ITEM22;
   USEVARIABLES ARE ITEM1 - ITEM22;

ANALYSIS: ESTIMATOR = MLM;

MODEL:
  F1 BY ITEM1 - ITEM3 ITEM6 ITEM8 ITEM13 ITEM14 ITEM16 ITEM20;
  F2 BY ITEM5 ITEM10 ITEM11 ITEM15 ITEM22;
  F3 BY ITEM4 ITEM7 ITEM9 ITEM12 ITEM17 - ITEM19 ITEM21;

OUTPUT: MODINDICES;

MBISEC1.INP
TITLE: CFA of MBI for Secondary Teachers
       Initial Baseline Model (Byrne 2012, p. 198-)

DATA:  FILE IS "MBISEC1.DAT";

VARIABLE: NAMES ARE ITEM1 - ITEM22;
   USEVARIABLES ARE ITEM1 - ITEM22;

ANALYSIS: ESTIMATOR = MLM;

MODEL:
  F1 BY ITEM1 - ITEM3 ITEM6 ITEM8 ITEM13 ITEM14 ITEM16 ITEM20;
  F2 BY ITEM5 ITEM10 ITEM11 ITEM15 ITEM22;
  F3 BY ITEM4 ITEM7 ITEM9 ITEM12 ITEM17 - ITEM19 ITEM21;

OUTPUT: MODINDICES;
```

Baseline models for the two samples (p.198–202)

The goodness-of-fit statistics (initial baseline models):

- Elementary: MLM $\chi^2_{[206]}$ 826, CFI 0.857, RMSEA 0.072
- Secondary: MLM $\chi^2_{[206]}$ 999, CFI 0.836, RMSEA 0.075

Modification indices quite similar with both samples: one cross-loading and three residual covariances *(output omitted)*. Based on this pattern and the knowledge for male elementary teachers (ch4), we *(exceptionally)* include all four parameters in the post-hoc models at once (not one at a time as usual).

The goodness-of-fit statistics (Model 2):

- Elementary: MLM $\chi^2_{[202]}$ 477, CFI 0.936, RMSEA 0.049
- Secondary: MLM $\chi^2_{[202]}$ 587, CFI 0.920, RMSEA 0.053

Modification indices of Model 2 (selected output)

```
MODEL MODIFICATION INDICES (Elementary teachers)
                           M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM11        12.045     0.250      0.311        0.205
F2       BY ITEM16        12.849     0.310      0.299        0.197
F2       BY ITEM17        10.444    -0.173     -0.166       -0.188
F3       BY ITEM14        19.957     0.864      0.373        0.205

ITEM7    WITH ITEM4       31.795     0.174      0.174        0.284
ITEM12   WITH ITEM3       19.583    -0.250     -0.250       -0.227
ITEM13   WITH ITEM12      16.737     0.231      0.231        0.211
ITEM14   WITH ITEM2       13.428     0.245      0.245        0.163
ITEM14   WITH ITEM13      12.119     0.281      0.281        0.180
ITEM19   WITH ITEM18      31.643     0.266      0.266        0.333

MODEL MODIFICATION INDICES (Secondary teachers)
                           M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM5         13.781    -0.306     -0.345       -0.235
F1       BY ITEM10        11.592    -0.221     -0.249       -0.159
F1       BY ITEM11        52.558     0.472      0.532        0.339
F1       BY ITEM15        12.532    -0.283     -0.319       -0.225
F2       BY ITEM13        12.575     0.339      0.294        0.176
F2       BY ITEM14        17.329    -0.490     -0.424       -0.239
F2       BY ITEM16        14.020     0.330      0.286        0.198
F3       BY ITEM14        12.912     0.693      0.266        0.150

ITEM11   WITH ITEM5       14.332    -0.244     -0.244       -0.162
ITEM13   WITH ITEM3       10.714    -0.212     -0.212       -0.169
ITEM14   WITH ITEM2       14.998     0.234      0.234        0.153
ITEM15   WITH ITEM5       27.843     0.416      0.416        0.310
ITEM15   WITH ITEM10      13.305     0.227      0.227        0.162
ITEM15   WITH ITEM11      22.708    -0.297     -0.297       -0.206
ITEM15   WITH ITEM12      12.013     0.191      0.191        0.168
ITEM18   WITH ITEM7       14.456    -0.160     -0.160       -0.203
ITEM18   WITH ITEM17      16.888     0.147      0.147        0.219
ITEM19   WITH ITEM9       34.189     0.355      0.355        0.357
ITEM19   WITH ITEM17      16.230    -0.159     -0.159       -0.217
ITEM20   WITH ITEM2       15.666    -0.171     -0.171       -0.162
ITEM20   WITH ITEM6       10.441    -0.173     -0.173       -0.137
ITEM20   WITH ITEM16      22.614     0.227      0.227        0.201
ITEM21   WITH ITEM7       16.722     0.247      0.247        0.191
```

Model misspecification and modification (p.202–204)

According to Byrne, there is a clear overlap of content between Item7 and Item4 but not between Item19 and Item18.

More work is needed in establishing an appropriate baseline model for secondary teachers. Let us begin from the cross-loading of Item11 on Factor1.

We specify Model 3 for both samples and see the new MIs.

The goodness-of-fit statistics (Model 3):

- Elementary: MLM $\chi^2_{[201]}$ 451, CFI 0.942, RMSEA 0.046
- Secondary: MLM $\chi^2_{[201]}$ 535, CFI 0.931, RMSEA 0.049

Note that baseline models may not be completely identical across groups – multigroup analyses still possible with a condition called *partial measurement invariance*, see p.198 and pp.254–255.

Modification indices of Model 3 (selected output)

MODEL MODIFICATION INDICES (Elementary teachers)

			M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
F1	BY	ITEM11	12.212	0.251	0.312	0.206
F2	BY	ITEM16	12.970	0.310	0.299	0.197
F2	BY	ITEM17	11.420	-0.181	-0.174	-0.197
F3	BY	ITEM14	20.992	0.977	0.383	0.210
ITEM12	WITH	ITEM3	19.547	-0.248	-0.248	-0.226
ITEM13	WITH	ITEM12	17.224	0.232	0.232	0.213
ITEM14	WITH	ITEM2	13.599	0.245	0.245	0.163
ITEM14	WITH	ITEM13	12.266	0.282	0.282	0.180
ITEM17	WITH	ITEM4	11.706	0.096	0.096	0.174
ITEM19	WITH	ITEM18	26.859	0.247	0.247	0.319

MODEL MODIFICATION INDICES (Secondary teachers)

			M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
F1	BY	ITEM22	15.920	0.321	0.360	0.225
F2	BY	ITEM13	12.617	0.293	0.270	0.162
F2	BY	ITEM14	15.794	-0.404	-0.372	-0.210
F2	BY	ITEM16	11.700	0.261	0.240	0.166
F3	BY	ITEM14	13.338	0.707	0.271	0.153
ITEM13	WITH	ITEM3	10.197	-0.206	-0.206	-0.164
ITEM14	WITH	ITEM2	15.599	0.239	0.239	0.155
ITEM15	WITH	ITEM5	12.889	0.304	0.304	0.245
ITEM15	WITH	ITEM12	10.877	0.178	0.178	0.164
ITEM18	WITH	ITEM7	14.240	-0.159	-0.159	-0.202
ITEM18	WITH	ITEM17	17.212	0.148	0.148	0.221
ITEM19	**WITH**	**ITEM9**	**33.467**	**0.351**	**0.351**	**0.355**
ITEM19	WITH	ITEM17	16.338	-0.160	-0.160	-0.218
ITEM20	WITH	ITEM2	14.476	-0.164	-0.164	-0.155
ITEM20	WITH	ITEM6	10.363	-0.171	-0.171	-0.136
ITEM20	WITH	ITEM16	22.169	0.223	0.223	0.199
ITEM21	WITH	ITEM7	16.938	0.248	0.248	0.192

Model misspecification and modification (p.204–206)

Model 3 will serve as the baseline for the elementary teachers.

For secondary teachers, the residual covariance of Item19 and Item9 is freed for estimation, leading to Model 4 with fit indices MLM $\chi^2_{[200]}$ 505, CFI 0.937, RMSEA 0.047, and no large MIs:

```
MODEL MODIFICATION INDICES (Secondary teachers)
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM22          15.715     0.317      0.357        0.223
F2       BY ITEM13          12.497     0.292      0.268        0.161
F2       BY ITEM14          15.839    -0.404     -0.371       -0.209
F2       BY ITEM16          11.933     0.263      0.242        0.167
F3       BY ITEM14          13.689     0.685      0.279        0.157

ITEM13   WITH ITEM3         10.245    -0.206     -0.206       -0.164
ITEM14   WITH ITEM2         15.600     0.239      0.239        0.155
ITEM15   WITH ITEM5         12.778     0.301      0.301        0.243
ITEM15   WITH ITEM12        12.395     0.190      0.190        0.176
ITEM18   WITH ITEM7         25.806    -0.227     -0.227       -0.296
ITEM19   WITH ITEM18        13.568     0.168      0.168        0.208
ITEM20   WITH ITEM2         14.511    -0.164     -0.164       -0.155
ITEM20   WITH ITEM6         10.352    -0.171     -0.171       -0.136
ITEM20   WITH ITEM16        22.287     0.224      0.224        0.199
ITEM21   WITH ITEM7         15.752     0.242      0.242        0.189
ITEM21   WITH ITEM18        12.230    -0.208     -0.208       -0.186
```

Model 4 will serve as the baseline for the secondary teachers.

Hypothesized configural model (graph, p.207)

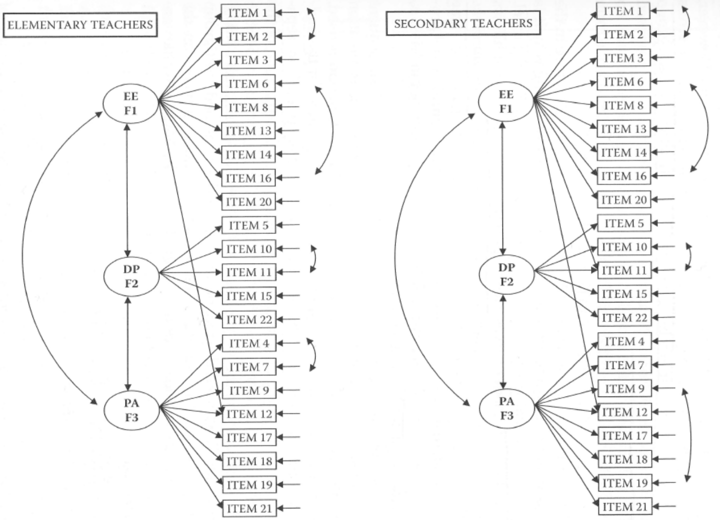

Figure 7.1. Hypothesized multigroup baseline model of MBI structure for elementary and secondary teachers.

Testing invariance: the configural model (p.206–212)

Now we get to the point of this part: testing the invariance. It is only required that the baseline models of the groups have the same number of factors and the same factor-loading pattern, which is clearly the case here. They have certain differences that we specified above, and we must just incorporate these differences in the common baseline model.

Now, the same parameters as above will be estimated *simultaneously* for the both groups. That requires combining the separate input files into one input file (see next pages) for the *configural model*, i.e., the common baseline model for both groups. The input file refers to two datasets, but as they are now analyzed at the same time, we will only get one χ^2 value (and other fit indices).

The MODEL command gives the factor loadings and residual covariances that were found to be the same across the groups. The parameter specifications unique to the groups are given for each group separately.

Input specification, configural model (p.209)

The initial configural model is a bit more complicated:

```
MBIINV1.INP
TITLE: Testing for Invariance of MBI across Elementary/Secondary Teachers
       Common Baseline (Configural) Model (Byrne 2012, p. 209)

DATA:
  FILE (Elem) IS "MBIELM1.DAT";
  FILE (Sec)  IS "MBISEC1.DAT";

VARIABLE: NAMES ARE ITEM1 - ITEM22;
   USEVARIABLES ARE ITEM1 - ITEM22;

ANALYSIS: ESTIMATOR = MLM;

MODEL: ! "overall" model section
  F1 BY ITEM1 - ITEM3 ITEM6 ITEM8 ITEM13 ITEM14 ITEM16 ITEM20;
  F1 BY ITEM12; ! common cross-loading (from baseline models)
  F2 BY ITEM5 ITEM10 ITEM11 ITEM15 ITEM22;
  F3 BY ITEM4 ITEM7 ITEM9 ITEM12 ITEM17 - ITEM19 ITEM21;

  ! common modifications (from baseline models)
  ITEM6 WITH ITEM16;
  ITEM1 WITH ITEM2;
  ITEM10 WITH ITEM11;

  [F1@0 F2@0 F3@0]; ! factor means constrained to zero
```

(continued on the next page)

Input specification, configural model (p.209)

(continued from the previous page)

MBIINV1.INP (continued)

```
! group-specific model parameters:

MODEL Elem:
  ITEM7 WITH ITEM4; ! specific residual covariance (from baseline model)

MODEL Sec:
  F1 BY ITEM11; ! specific cross-loading (from baseline model)
  ITEM19 WITH ITEM9; ! specific residual covariance (from baseline model)

  ! RELAX the following equality constraints, i.e., specify those
  ! parameters that are NOT constrained equal:
  ! (OBS: the first items excluded because of their special role)

  F1 BY       ITEM2 ITEM3 ITEM6 ITEM8 ITEM13 ITEM14 ITEM16 ITEM20;
  F1 BY ITEM12; ! common cross-loading (from baseline models)
  F2 BY       ITEM10 ITEM11 ITEM15 ITEM22;
  F3 BY       ITEM7 ITEM9 ITEM12 ITEM17 - ITEM19 ITEM21;

  [ITEM1 - ITEM22]; ! intercepts NOT constrained equal

OUTPUT: MODINDICES (3.84); ! cutpoint (instead of default 10, see below)

! Why exactly "3.84"?
! Because it corresponds (approximately) to the 5 % point of the
! chi-square distribution with 1 df: 1-chi2.F(1,3.84)=0.0500435212487
! (by using the cumulative distribution function)
! or the other way around: chi2.G(1,1-0.05)=3.8414588206941
! (by using the inverse distribution function)
!
```

Testing invariance: the configural model (p.208–212)

Under the MODEL, there is a new specification
[F1@0 F2@0 F3@0]
which fixes the factor means at zero for both groups. With this model, we are only interested in the analysis of covariance structures and hence no means are needed. (By default, Mplus would fix the means to zero for the *first* group only.)

In the group-specific MODEL commands, which refer to the group labels given in the FILE command, are the parameters that differ from the overall MODEL. These are exactly the modifications that we specified for the group baseline models one at a time. **The invariance of these parameters will not be tested.** This is how we apply the condition of the *partial measurement invariance*.

On the next lines of the input file, we have the factor-loading parameters again, but now without the first items of each factor. Again, this is related to Mplus... *(cont'd...)*

Testing invariance: the configural model (p.208–212)

...as by default in CFA models (based on continuous data), all factor loadings (and intercepts) are constrained equal across groups, while all residual covariances are freely estimated. We must **relax** these default constraints by specifying them under one of the MODEL-specific commands.

Similarly, after the factor loadings, there is a specification
[ITEM1 – ITEM22]
which relaxes the observed variable intercepts for free estimation (so that they are not constrained equal across groups).

Of course, the first items of the congeneric factors are not mentioned here, because they are fixed at one (for the usual identication purpose) and hence are not free to vary.

In summary, the input file specifies a configural model in which **no equality constraints** are imposed. The model fit represents a multigroup version of the combined baseline models for the elementary and secondary teachers:
MLM $\chi^2_{[401]}$ 958.341, CFI 0.939, RMSEA 0.047, SRMR 0.051

Testing invariance: the measurement model (p.212–219)

The key measurement model parameters of interest here are the factor loadings and the commonly specified residual covariances.

1) Factor loadings

Now we begin to study the equality constraints so we **remove the lines** that were used to relax the factor loading parameters.

The fit statistics suggest that the model does not fit the data quite as well as it did with no factor-loading constrains imposed: MLM $\chi^2_{[421]}$ 1015.228, CFI 0.935, RMSEA 0.047, SRMR 0.057. (Note: 22-3=19 factor loadings and 1 cross-loading were constrained equal, thus *increasing* the df by 20.)

As the focus is on the factor loadings, only the MIs related to them are of interest. In addition, in testing for invariance, only those parameters that were constrained equal, are of relevance. It means that the output will include *several non-relevant suggestions.* **Be careful!** I have indicated the eligible parameters on next page by stars. Unfortunately, Mplus can not do that for you!

Modification indices of InvModel 2 (selected output)

```
MODEL MODIFICATION INDICES (Elementary teachers)
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM11           33.745    0.312      0.380         0.264
F1       BY ITEM15            7.453   -0.142     -0.173        -0.150
F1       BY ITEM17           12.029   -0.108     -0.132        -0.153
F1       BY ITEM19            3.869    0.079      0.096         0.085
F2       BY ITEM5      *      4.607    0.248      0.224         0.155
F2       BY ITEM11     *     25.480    0.208      0.188         0.131
F2       BY ITEM12            7.113    0.196      0.177         0.136
F2       BY ITEM14            5.067   -0.224     -0.202        -0.111
F2       BY ITEM15     *      9.361   -0.142     -0.129        -0.111
F2       BY ITEM16           11.846    0.244      0.221         0.147
F2       BY ITEM17           15.876   -0.193     -0.174        -0.202
F2       BY ITEM22     *      5.003   -0.153     -0.138        -0.087
F3       BY ITEM7      *      7.242   -0.157     -0.061        -0.068
F3       BY ITEM14           17.271    0.840      0.327         0.180
F3       BY ITEM17     *      4.041    0.151      0.059         0.068

MODEL MODIFICATION INDICES (Secondary teachers)
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM5            14.231   -0.239     -0.275        -0.184
F1       BY ITEM22           15.918    0.266      0.306         0.195
F2       BY ITEM1             7.001   -0.153     -0.147        -0.092
F2       BY ITEM5      *      4.607   -0.248     -0.238        -0.159
F2       BY ITEM6             4.033    0.140      0.134         0.082
F2       BY ITEM11     *     25.481   -0.306     -0.294        -0.181
F2       BY ITEM13           12.658    0.243      0.233         0.141
F2       BY ITEM14           12.667   -0.298     -0.286        -0.161
F2       BY ITEM15     *      9.360    0.156      0.150         0.109
F2       BY ITEM16            6.989    0.165      0.158         0.108
F2       BY ITEM20            7.769    0.166      0.159         0.114
F2       BY ITEM22     *      5.003    0.116      0.112         0.071
F3       BY ITEM1             6.759    0.295      0.127         0.079
F3       BY ITEM6             4.005   -0.277     -0.119        -0.073
F3       BY ITEM7      *      7.240    0.257      0.111         0.100
F3       BY ITEM14           12.587    0.585      0.252         0.142
F3       BY ITEM15            4.311   -0.325     -0.140        -0.102
F3       BY ITEM16            5.359   -0.286     -0.123        -0.084
F3       BY ITEM17     *      4.041   -0.132     -0.057        -0.058
```

Testing invariance: the measurement model (p.212–219)

We relax the factor loading with the largest MI by specifying F2 BY ITEM11 under one of the MODEL-specific commands. (It remains also under the general MODEL command, otherwise it would not be estimated at all.) The fit statistics of InvModel 3: MLM $\chi^2_{[420]}$ 989.427, CFI 0.938, RMSEA 0.046, SRMR 0.054. Indeed, a review of the estimated parameters reveals a fairly substantial difference for the specified parameter (1.095 for elementary and 0.581 for secondary teachers).

Of prime interest now is whether the difference in model fit between InvModel 3 and and the configural model (InvModel 1) is or is not statistically significant. This requires the corrected χ^2 test (see PART4, p.14–15). *Note a difference here:* now the second model represents the nested (more restrictive) model.

Here, $TR_d = (989.427 \times 1.239 - 958.341 \times 1.242)/1.176 = 30.3$ and as 1-chi2.F(420-401,30.3)=0.048, we can conclude that further improvement of the model is required.

Modification indices of InvModel 3 (selected output)


```
MODEL MODIFICATION INDICES (Elementary teachers)
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM11          11.625     0.253      0.308        0.205
F1       BY ITEM15           8.172    -0.149     -0.181       -0.157
F1       BY ITEM17          12.042    -0.108     -0.132       -0.153
F1       BY ITEM19           3.923     0.079      0.096        0.085
F2       BY ITEM5    *       5.634     0.276      0.241        0.168
F2       BY ITEM12           6.286     0.190      0.167        0.128
F2       BY ITEM14           4.524    -0.220     -0.192       -0.106
F2       BY ITEM15   *       8.393    -0.145     -0.127       -0.109
F2       BY ITEM16          11.325     0.248      0.217        0.144
F2       BY ITEM17          14.970    -0.192     -0.168       -0.195
F3       BY ITEM7    *       7.185    -0.156     -0.061       -0.067
F3       BY ITEM10           4.372    -0.350     -0.136       -0.094
F3       BY ITEM14          17.168     0.838      0.326        0.179
F3       BY ITEM17   *       3.994     0.150      0.058        0.068

MODEL MODIFICATION INDICES (Secondary teachers)
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
F1       BY ITEM5           14.144    -0.236     -0.271       -0.181
F1       BY ITEM22          16.972     0.274      0.315        0.200
F2       BY ITEM1            7.495    -0.153     -0.151       -0.094
F2       BY ITEM5    *       5.634    -0.276     -0.272       -0.181
F2       BY ITEM6            4.188     0.138      0.136        0.083
F2       BY ITEM13          11.994     0.228      0.225        0.136
F2       BY ITEM14          12.823    -0.290     -0.285       -0.161
F2       BY ITEM15   *       8.393     0.139      0.137        0.100
F2       BY ITEM16           6.193     0.150      0.148        0.101
F2       BY ITEM20           6.697     0.149      0.146        0.105
F3       BY ITEM1            7.020     0.301      0.130        0.081
F3       BY ITEM6            3.865    -0.272     -0.117       -0.072
F3       BY ITEM7    *       7.183     0.256      0.110        0.100
F3       BY ITEM10           7.045     0.385      0.166        0.104
F3       BY ITEM11           5.912    -0.350     -0.150       -0.096
F3       BY ITEM14          13.002     0.596      0.256        0.144
F3       BY ITEM15           3.894    -0.308     -0.133       -0.096
F3       BY ITEM16           5.016    -0.277     -0.119       -0.082
F3       BY ITEM17   *       3.993    -0.132     -0.057       -0.058
```

Testing invariance: the measurement model (p.218–221)

Adding the line F2 BY ITEM15 under the MODEL-specific command and re-estimating the model InvModel 4 we obtain the following fit statistics:
MLM $\chi^2_{[419]}$ 981.189, CFI 0.939, RMSEA 0.046, SRMR 0.054.
Again, a review of the estimated parameters reveals a fairly substantial difference for the specified parameter (0.684 for elementary and 0.963 for secondary teachers).

Final comparison of this model with the configural model yields a corrected MLM χ^2 difference of 21.975, which is n.s., as 1-chi2.F(419-401,21.975)$=$0.23.

Hence, we know that **all items on the MBI**, except for items 11 and 15, both of which load on Factor2, **are operating equivalently** across the two groups of teachers.

So let us move on to further testing of the invariance of this instrument.

Testing invariance: the measurement model (p.219–221)

2) Residual covariances

We have three commonly specified residual covariances. As they are not constrained equal by default, the process in specifying their invariance differs from the models analyzed so far.

For the input file of InvModel 5, we need to accompany each of the three specified residual covariances with parenthesized values (1,2,3), i.e., to give them on *separate lines* under the overall MODEL command as follows:

ITEM6 WITH ITEM16 (1);
ITEM1 WITH ITEM2 (2);
ITEM10 WITH ITEM11 (3);

This will ensure that each of these will be constrained equal across the two groups.

Testing invariance: the measurement model (p.219–221)

In total, 21 parameters in InvModel 5 are constrained equal across groups: 17 factor loadings, 1 cross-loading and 3 residual covariances, leading to the following fit statistics:
MLM $\chi^2_{[422]}$ 992.614, CFI 0.938, RMSEA 0.046, SRMR 0.054.
(It seems that Byrne has an error on p.220–221: the comparison with InvModel 4 gives a corrected MLM χ^2 difference of 10.62, which is significant, as 1-chi2.F(422-419,10.62)=0.014. However, we must continue as it was n.s. – PLEASE CHECK THIS, as I might have an error, too!)

We consider the InvModel 5 as the final model in this phase and conclude that the three residual covariances are operating equivalently across the groups.

Having established invariance related to the measurement model, let us move further on to testing the invariance of the *structural parameters*. Here, they include only the factor variances and covariances.

Testing invariance: the structural model (p.221–225)

We make the following additions to the overall MODEL:

F1 (4);
F2 (5);
F3 (6);
F1 WITH F2 (7);
F1 WITH F3 (8);
F2 WITH F3 (9);

Similarly with the residual covariances, the factor variances and covariances are not constrained equal by default and, thus, they must be separately noted in the input file.

The goodness-of-fit statistics for InvModel 6 are:
MLM $\chi^2_{[428]}$ 1004.731, CFI 0.937, RMSEA 0.046, SRMR 0.059.
Comparison with InvModel 5 yields a corrected MLM χ^2 difference of 12.117, which is n.s., as 1-chi2.F(428-422,12.117)$=$0.059.

The conclusion is that (despite of the presence of two noninvariant factor loadings and freely estimated item intercepts) the factor variances and covariances remain equivalent across the two groups.

A summary of all *noninvariant* (i.e., freely estimated) unstandardized estimates follows.

Summary of noninvariant parameters (p.223)


```
                                                    Two-Tailed
Group ELEM           Estimate       S.E.  Est./S.E.    P-Value

 F2        BY
    ITEM11              0.998      0.074     13.459      0.000
    ITEM15              0.684      0.069      9.867      0.000

 ITEM7     WITH
    ITEM4               0.169      0.056      3.000      0.003

 Intercepts
    ITEM1               3.409      0.069     49.484      0.000
    ITEM2               3.976      0.065     60.903      0.000
    ITEM3               2.572      0.071     36.229      0.000
    ITEM4               5.412      0.039    139.751      0.000
    ITEM5               1.053      0.061     17.177      0.000
    ITEM6               1.676      0.069     24.353      0.000
    ITEM7               5.338      0.036    146.968      0.000
    ITEM8               2.184      0.075     29.012      0.000
    ITEM9               5.031      0.055     90.946      0.000
    ITEM10              1.164      0.061     18.990      0.000
    ITEM11              1.122      0.063     17.889      0.000
    ITEM12              4.693      0.053     87.789      0.000
    ITEM13              2.548      0.072     35.621      0.000
    ITEM14              3.122      0.076     41.262      0.000
    ITEM15              0.545      0.047     11.713      0.000
    ITEM16              1.433      0.063     22.715      0.000
    ITEM17              5.416      0.037    147.600      0.000
    ITEM18              4.883      0.050     96.794      0.000
    ITEM19              5.007      0.047    106.468      0.000
    ITEM20              1.281      0.060     21.456      0.000
    ITEM21              4.841      0.054     90.467      0.000
    ITEM22              1.328      0.064     20.730      0.000
```

Summary of noninvariant parameters (p.224)


```
                                                    Two-Tailed
Group SEC            Estimate       S.E.  Est./S.E.    P-Value

 F1       BY
    ITEM11              0.399      0.060      6.691      0.000

 F2       BY
    ITEM11              0.588      0.089      6.584      0.000
    ITEM15              0.950      0.074     12.812      0.000

 ITEM19   WITH
    ITEM9               0.366      0.067      5.433      0.000

 Intercepts
    ITEM1               3.371      0.060     55.937      0.000
    ITEM2               3.890      0.058     66.973      0.000
    ITEM3               2.526      0.065     39.101      0.000
    ITEM4               5.168      0.042    123.343      0.000
    ITEM5               1.217      0.056     21.817      0.000
    ITEM6               1.999      0.063     31.871      0.000
    ITEM7               5.014      0.043    116.064      0.000
    ITEM8               2.143      0.065     33.039      0.000
    ITEM9               4.702      0.057     82.848      0.000
    ITEM10              1.275      0.059     21.431      0.000
    ITEM11              1.166      0.060     19.550      0.000
    ITEM12              4.527      0.051     88.954      0.000
    ITEM13              2.653      0.063     41.827      0.000
    ITEM14              3.147      0.067     46.680      0.000
    ITEM15              1.078      0.054     20.011      0.000
    ITEM16              1.548      0.055     28.112      0.000
    ITEM17              5.303      0.037    145.039      0.000
    ITEM18              4.705      0.045    105.646      0.000
    ITEM19              4.600      0.049     93.051      0.000
    ITEM20              1.211      0.053     22.713      0.000
    ITEM21              4.462      0.057     77.884      0.000
    ITEM22              1.790      0.061     29.412      0.000
```