1. Given X be a discrete random variable with the following PMF

0.1 for X = 0.2 0.2for X = 0.4 0.2 for X = 0.5 0.3 for X = 0.8 0.2 for X = 1 0 otherwise

1. Find the range RX of the random variable X.

Solution:- The range RX of the random variable X is the set of all possible values that X can take. In this case, the range is {0.2, 0.4, 0.5, 0.8, 1}.

1. Find P(X ≤ 0.5)

Solution:- To find P(X ≤ 0.5), we sum the probabilities of all values of X that are less than or equal to 0.5. P(X ≤ 0.5) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) = 0.1 + 0.2 + 0.2 = 0.5.

1. Find P(0.25<X<0.75)

Solution:- To find P(0.25 < X < 0.75), we sum the probabilities of all values of X that are greater than 0.25 and less than 0.75. P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5) = 0.2 + 0.2 = 0.4.

1. P(X = 0.2|X<0.6)

Solution:- To find P(X = 0.2 | X < 0.6), we need to calculate the conditional probability of X being equal to 0.2 given that X is less than 0.6. P(X = 0.2 | X < 0.6) = P(X = 0.2 and X < 0.6) / P(X < 0.6).

The probability P(X = 0.2 and X < 0.6) is equal to P(X = 0.2) since X cannot take any other value less than 0.6. P(X = 0.2 and X < 0.6) = P(X = 0.2) = 0.1.

To calculate P(X < 0.6), we sum the probabilities of all values of X that are less than 0.6. P(X < 0.6) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) = 0.1 + 0.2 + 0.2 = 0.5.

Now we can calculate the conditional probability: P(X = 0.2 | X < 0.6) = (P(X = 0.2 and X < 0.6)) / P(X < 0.6) = 0.1 / 0.5 = 0.2.

So, P(X = 0.2 | X < 0.6) = 0.2.

1. Two equal and fair dice are rolled, and we observed two numbers X and Y.
2. Find RX, RY, and the PMFs of X and Y.

The range RX of X is {1, 2, 3, 4, 5, 6}, and the range RY of Y is also {1, 2, 3, 4, 5, 6}. Since both dice are fair, the PMFs of X and Y are uniform, meaning each value has an equal probability of occurring. PMF of X: P(X = 1) = 1/6 P(X = 2) = 1/6 P(X = 3) = 1/6 P(X = 4) = 1/6 P(X = 5) = 1/6 P(X = 6) = 1/6

PMF of Y: P(Y = 1) = 1/6 P(Y = 2) = 1/6 P(Y = 3) = 1/6 P(Y = 4) = 1/6 P(Y = 5) = 1/6 P(Y = 6) = 1/6

1. Find P(X = 2,Y = 6).

To find P(X = 2, Y = 6), we multiply the probabilities of X being 2 and Y being 6 since the two events are independent. P(X = 2, Y = 6) = P(X = 2) *P(Y = 6) = (1/6)* (1/6) = 1/36.

1. Find P(X>3|Y = 2).

To find P(X > 3 | Y = 2), we need to calculate the conditional probability of X being greater than 3 given that Y is 2. Since Y is fixed at 2, X can take values {4, 5, 6}. P(X > 3 | Y = 2) = P(X = 4 | Y = 2) + P(X = 5 | Y = 2) + P(X = 6 | Y = 2).

Since the two dice are independent, the probability of each value is unaffected by the other die, so: P(X = 4 | Y = 2) = P(X = 4) = 1/6 P(X = 5 | Y = 2) = P(X = 5) = 1/6 P(X = 6 | Y = 2) = P(X = 6) = 1/6

Therefore, P(X > 3 | Y = 2) = P(X = 4 | Y = 2) + P(X = 5 | Y = 2) + P(X = 6 | Y = 2) = 1/6 + 1/6 + 1/6 = 1/2.

1. If Z = X + Y. Find the range and PMF of Z.

If Z = X + Y, the range of Z is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. To find the PMF of Z, we need to calculate the probabilities of each possible value of Z. PMF of Z: P(Z = 2) = P(X = 1, Y = 1) = (1/6) *(1/6) = 1/36 P(Z = 3) = P(X = 1, Y = 2) + P(X = 2, Y = 1) = (1/6)* (1/6) + (1/6) *(1/6) = 2/36 P(Z = 4) = P(X = 1, Y = 3) + P(X = 2, Y = 2) + P(X = 3, Y = 1) = (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) = 3/36 P(Z = 5) = P(X = 1, Y = 4) + P(X = 2, Y = 3) + P(X = 3, Y = 2) + P(X = 4, Y = 1) = (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) = 4/36 P(Z = 6) = P(X = 1, Y = 5) + P(X = 2, Y = 4) + P(X = 3, Y = 3) + P(X = 4, Y = 2) + P(X = 5, Y = 1) = (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) = 5/36 P(Z = 7) = P(X = 1, Y = 6) + P(X = 2, Y = 5) + P(X = 3, Y = 4) + P(X = 4, Y = 3) + P(X = 5, Y = 2) + P(X = 6, Y = 1) = (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) = 6/36 P(Z = 8) = P(X = 2, Y = 6) + P(X = 3, Y = 5) + P(X = 4, Y = 4) + P(X = 5, Y = 3) + P(X = 6, Y = 2) = (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) = 5/36 P(Z = 9) = P(X = 3, Y = 6) + P(X = 4, Y = 5) + P(X = 5, Y = 4) + P(X = 6, Y = 3) = (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) + (1/6)* (1/6) = 4/36 P(Z = 10) = P(X = 4, Y = 6) + P(X = 5, Y = 5) + P(X = 6, Y = 4) = (1/6) *(1/6) + (1/6)* (1/6) + (1/6) *(1/6) = 3/36 P(Z = 11) = P(X = 5, Y = 6) + P(X = 6, Y = 5) = (1/6)* (1/6) + (1/6) *(1/6) = 2/36 P(Z = 12) = P(X = 6, Y = 6) = (1/6)* (1/6) = 1/36

1. Find P(X = 4|Z = 8).

To find P(X = 4 | Z = 8), we need to calculate the conditional probability of X being 4 given that Z is 8. Since Z = X + Y, and we know that X = 4, we can find the corresponding Y value by subtracting X from Z. Y = Z - X = 8 - 4 = 4. P(X = 4 | Z = 8) = P(X = 4, Y = 4) = (1/6) * (1/6) = 1/36.

1. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?

Ans:- The student's score, X, can range from 0 to 20. Let's calculate the PMF for each possible value of X.

To find P(X = k), we need to consider the combinations of questions the student knows and the questions he guesses.

P(X = k) = C(10, k) *(1/44)^(k)* (43/44)^(10-k)

where C(10, k) represents the number of ways to choose k questions out of the 10 questions the student knows, (1/44)^(k) represents the probability of guessing k correct answers out of the unknown questions, and (43/44)^(10-k) represents the probability of guessing (10-k) incorrect answers out of the unknown questions.

Now, let's calculate the PMF for each value of X:

P(X = 0) = C(10, 0) *(1/44)^0* (43/44)^10 = (43/44)^10 P(X = 1) = C(10, 1) *(1/44)^1* (43/44)^9 P(X = 2) = C(10, 2) *(1/44)^2* (43/44)^8 ... P(X = 19) = C(10, 19) *(1/44)^19* (43/44)^1 P(X = 20) = C(10, 20) *(1/44)^20* (43/44)^0 = (1/44)^20

To find P(X > 15), we need to sum the probabilities of X being greater than 15:

P(X > 15) = P(X = 16) + P(X = 17) + ... + P(X = 20)

You can calculate the values of the PMF and P(X > 15) using the formula above and the appropriate combination calculations.

1. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

Ans:- the number of students arriving in the time interval from 10 am to 11:30 am, Y, follows a Poisson distribution with an average arrival rate of 10 students per hour. However, the given time interval is not exactly one hour, so we need to adjust the average arrival rate accordingly.

The time interval from 10 am to 11:30 am is 1.5 hours. Therefore, the average number of students arriving in this time interval would be λ = 10 students/hour * 1.5 hours = 15 students.

To find P(10 < Y ≤ 15), we can use the Poisson distribution formula:

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + ... + P(Y = 15)

The probability mass function (PMF) of the Poisson distribution is given by:

P(Y = k) = (e^(-λ) * λ^k) / k!

where e is the base of the natural logarithm (approximately 2.71828), and k! represents the factorial of k.

Using this formula, we can calculate the probabilities for each value of Y and sum them up to find P(10 < Y ≤ 15).

1. Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

Ans:- To find the probability mass function (PMF) of the random variable Z = X + Y, where X ~ Poisson(α) and Y ~ Poisson(β), we can use the concept of the convolution of two probability distributions.

The PMF of the Poisson distribution is given by:

P(X = k) = (e^(-α) * α^k) / k!

P(Y = m) = (e^(-β) * β^m) / m!

The PMF of Z can be calculated as the sum of the individual probabilities of all possible values of Z:

P(Z = z) = ∑[P(X = k) * P(Y = z - k)], for k = 0 to z

where z represents the possible values that Z can take.

In other words, we sum up the product of the probabilities that X takes a particular value k and Y takes a value z - k, for all possible values of k.

The resulting PMF of Z will have values for all possible values of z, and the probabilities will be computed using the above formula.

Note that since X and Y are independent, we can compute their probabilities separately and then combine them to obtain the PMF of Z.

1. There is a discrete random variable X with the pmf.

If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.
2. Find the pmf of Y.

Ans:-- To find the range and PMF of the random variable Y = (X + 1)^2, where X follows a given PMF, we can proceed as follows:

Range of Y:

The range of Y can be determined by considering the possible values of X. Since X takes on non-negative integer values, the range of X + 1 is [1, ∞) When we square each value of X + 1 the range of Y becomes [1, ∞)as well

PMF of Y: To find the PMF of Y, we need to determine the probability of each value in the range [1, ∞)

For any given value y in the range, we can calculate its probability as follows:

P(Y = y) = P((X + 1)^2 = y) = P(X + 1 = √y) = P(X = √y - 1)

By substituting √y - 1 for X in the original PMF of X, we can find the corresponding probability for Y.

Please provide the PMF of X so that I can provide a more specific calculation for the PMF of Y. 2.Assuming X is a continuous random variable with PDF

1. Find EX and Var(X).
2. Find P(X ≥).
3. If X is a continuous random variable with pdf

4. If X~Uniform and Y = sin(X), then find fY(y).

5. If X is a random variable with CDF

6. What kind of random variable is X: discrete, continuous, or mixed?

7. Find the PDF of X, fX(x).
8. Find E(eX).
9. Find P(X = 0|X≤0.5).
10. There are two random variables X and Y with joint PMF given in Table below
11. Find P(X≤2, Y≤4).
12. Find the marginal PMFs of X and Y.
13. Find P(Y = 2|X = 1).
14. Are X and Y independent?

15. A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

Ans: - To find the joint PMF (Probability Mass Function) of X (number of white shirts) and Y (number of black shirts), we can use the hypergeometric distribution since we are sampling without replacement from a finite population.

Let's define X as the number of white shirts and Y as the number of black shirts.

The PMF of X and Y can be calculated as follows:

P(X = x, Y = y) = (C(40, x) * C(60, y)) / C(100, 10)

where C(n, r) represents the number of ways to choose r items from a set of n items.

In this case, we are choosing x white shirts from a set of 40 white shirts and y black shirts from a set of 60 black shirts, and the total number of shirts to choose from is 100. The denominator C(100, 10) represents the total number of ways to choose 10 shirts from a set of 100.

The joint PMF will give us the probability of observing a specific combination of white and black shirts in the chosen sample of 10 shirts.

1. If A and B are two jointly continuous random variables with joint PDF

a. Find fX(a) and fY(b).

b. Are A and B independent of each other?

c. Find the conditional PDF of A given B = b, fA|B(a|b).

d. Find E[A|B = b], for 0 ≤ y ≤ 1.

e. Find Var(A|B = b), for 0 ≤ y ≤ 1.

1. There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

Ans:- To find the probability that the men's total weight on the ship exceeds 18,000, we need to calculate the probability of the sum of the men's weights being greater than 18,000.

Let's define the random variable X as the weight of a single man on the ship. Given that EX = μ = 170 and σX = σ = 30, we can assume that X follows a normal distribution with mean μ = 170 and standard deviation σ = 30.

The total weight of the 100 men can be represented as the sum of the individual weights:

Y = X1 + X2 + X3 + ... + X100

The mean of the total weight Y can be calculated as EY = 100 $\mu = 100$ 170 = 17,000.

The standard deviation of the total weight Y can be calculated as σY = sqrt(100) $\sigma = 10$ 30 = 300.

To find the probability that the men's total weight exceeds 18,000, we can use the properties of the normal distribution:

P(Y > 18,000) = P((Y - EY) / σY > (18,000 - 17,000) / 300)

Now, we can standardize the variable Y and calculate the probability using the standard normal distribution table or a statistical software.

P((Y - EY) / σY > (18,000 - 17,000) / 300) = P(Z > 1/3)

Using the standard normal distribution table, we can find the probability P(Z > 1/3) or use statistical software to calculate it.

Note that we are assuming that the weights of the men are independent and identically distributed, and their sum follows a normal distribution due to the Central Limit Theorem.

1. Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.

   Ans:- To estimate P(4 ≤ Y ≤ 6) using the Central Limit Theorem, we need to consider the sum of the random variables X1, X2, ..., Xn, where n = 25.

Given that X1, X2, ..., X25 are independent and identically distributed random variables, we can assume that each Xi follows the given PMF.

To apply the Central Limit Theorem, we need to calculate the mean and standard deviation of the sum Y = X1 + X2 + ... + Xn.

The mean of Y can be calculated as E(Y) = E(X1) + E(X2) + ... + E(Xn). Since each Xi has the same PMF, we can denote E(Xi) as μ. Therefore, E(Y) = n * μ.

The standard deviation of Y can be calculated as σ(Y) = sqrt(n) * σ, where σ is the standard deviation of each Xi.

In this case, the PMF provided doesn't mention the values of μ and σ. Hence, we cannot calculate the exact probabilities without knowing the values of μ and σ.

However, assuming that the PMF satisfies the conditions required for the Central Limit Theorem, we can make an approximation based on the theorem.

According to the Central Limit Theorem, when n is large enough, the sum of independent and identically distributed random variables approaches a normal distribution.

To estimate P(4 ≤ Y ≤ 6), we can approximate it as P(4 ≤ Y ≤ 6) ≈ P(3.5 ≤ Y ≤ 6.5) since we are dealing with discrete random variables.

Using the estimated mean and standard deviation, we can calculate the z-scores for the lower and upper bounds:

z1 = (3.5 - E(Y)) / σ(Y) z2 = (6.5 - E(Y)) / σ(Y)

Then, we can use the standard normal distribution table or statistical software to find the probabilities P(Z ≤ z1) and P(Z ≤ z2) and calculate P(3.5 ≤ Y ≤ 6.5) as P(Z ≤ z2) - P(Z ≤ z1).

Note that this is an approximation based on the Central Limit Theorem, and the accuracy of the estimation depends on the sample size and the underlying distribution of the random variables.