Investigate the Relationship Between Fuel Consumption and Transmission Type of Automobiles

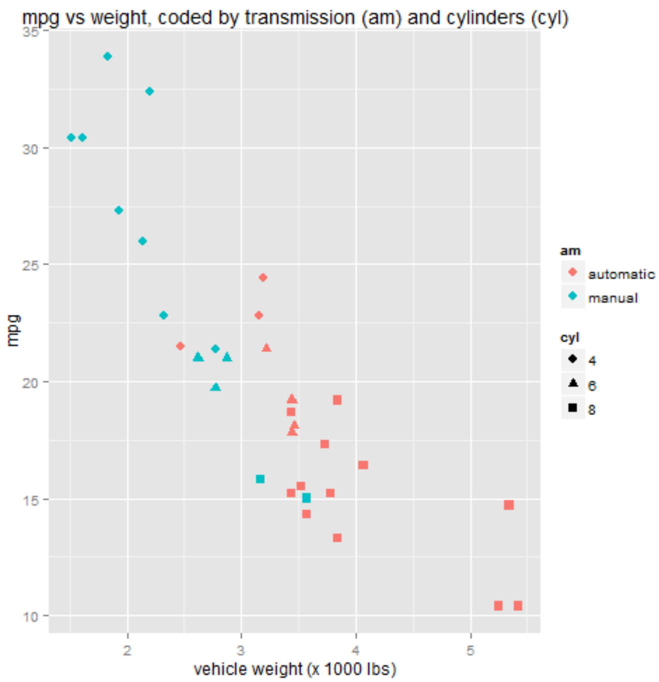
Abstract

This report investigates the relationship between fuel consumption and transmission type for automobiles. The report analyzes fuel consumption along with various design and performance metrics for 32 vehicles, all 1973-1974 models. The data suggest that automatic transmissions have better fuel performance for heavier vehicles, whereas manuals have better fuel performance for lighter vehicles. However the results are questionable due to high correlation between transmission type and vehicle weight in the data. The 95% confidence intervals for the two transmission types show a great deal of overlap, which indicates that the difference in fuel consumption between transmission types may be negligible.

Exploratory Analysis

For this analysis we'll explore the “mtcars” dataset provided by the “datasets” library in R. The mtcars data was extracted from a *Motor Trend* magazine. It contains 11 variables measuring various aspects of design, performance, and fuel consumption for 32 automobiles, all of which are 1973 - 1974 models. We will focus our analysis on fuel consumption (mpg) versus transmission (am).

First let's explore the data to get an idea of what it looks like. The first scatter plot below plots fuel consumption (mpg) vs weight (wt). The points are color-coded by transmission (am) and shape-coded by cylinders (cyl).



The chart reveals a few things. First there's a clear, negative correlation between weight and mpg. Second, there's also a clear correlation between transmission and weight: notice that manual transmissions (blue dots) are correlated with light-weight vehicles, while automatics (red dots) are correlated with heavier vehicles. We must take this into account in our model, given the clearly negative correlation between between weight and mpg.

There also appears to be a correlation between cylinders and weight, so we can exclude cylinders from our model, since much of the information it contributes is already captured by the weight variable.

Regression Analysis

Let's begin our analysis with a linear model that compares fuel consumption (mpg) with two regressor variables: transmission (am) and weight (wt). Since wt=0 is a meaningless value in this context (there's no such thing as a zero-weight vehicle), we'll shift the weight variable by its mean. This will give us meaningful values for the Y-intercept coefficients in our model.

```
fit.2 <- lm( mpg ~ am + I(wt-mean(wt)), data=mtcars )
summary(fit.2)$coefficients
```

```
##                     Estimate Std. Error     t value     Pr(>|t|)
## (Intercept)      20.10021868  0.8331837 24.12459551 9.639957e-21
## ammanual         -0.02361522  1.5456453 -0.01527855 9.879146e-01
## I(wt - mean(wt)) -5.35281145  0.7882438 -6.79080719 1.867415e-07
```

The regression equation for this model is:

```
mpg = B0 + B1 * (am == manual) + B2 * (wt-mean(wt))

mpg[am=automatic] = B0 + B2 * wt
mpg[am=manual] = (B0 + B1) + B2 * wt
```

Interpreting the Coefficients

The coefficients of our regression equation are interpreted as follows:

- B0=20.1, corresponds to base factor level am == automatic. It represents the mpg value for automatics when weight = mean(weight).
- B1=-0.02, corresponds to factor level am == manual. It represents the difference in mpg value between manuals and automatics when weight = mean(weight).
- B2=-5.35, represents the change in mpg per unit change in weight, holding transmission constant.

Note that this model has two regression lines: one for automatics, and another for manuals. The Y-intercept of the line for am=automatic is B0. The Y-intercept of the line for am=manual is B0+B1. The slope of both lines is the same, B2.

B1 is small, meaning the difference in mpg between automatics and manuals at the average weight is negligible. The p value, 0.988, indicates that the difference is not very significant either. The p value associated with the weight variable however is very significant, which is expected given the negative correlation we observed in the exploratory plot.

So this model doesn't really help us quantify the difference in mpg between automatics and manuals.

Applying a Different Model

Let's try another model that considers the interaction between weight and transmission:

```
fit.3 <- lm(mpg ~ am * I(wt-mean(wt)), data=mtcars)
summary(fit.3)$coefficients
```

```
##                             Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)                19.235844  0.7356848 26.146856 3.243563e-21
## ammanual                   -2.167728  1.4188862 -1.527767 1.377893e-01
## I(wt - mean(wt))           -3.785908  0.7856478 -4.818836 4.551182e-05
## ammanual:I(wt - mean(wt)) -5.298360  1.4446993 -3.667449 1.017148e-03
```

This model results in a regression equation that looks like:

```
mpg = B0 + B1 * (am==manual) + B2 * wt + B3 * wt * (am == manual)

mpg[am=automatic] = B0 + B2 * wt
mpg[am=manual] = (B0 + B1) + (B2 + B3) * wt
```

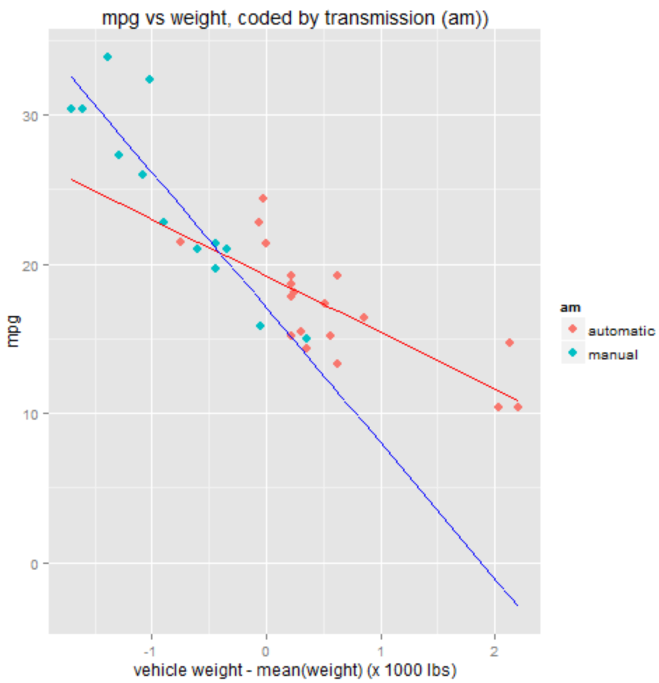
…where the coefficients are interpreted as:

- B0=19.24 represents the mpg value for automatics when weight = mean(weight).
- B1=-2.17, represents the difference in mpg value between manuals and automatics when weight = mean(weight).
- B2=-3.79, represents the slope of mpg vs. weight for automatics.
- B3=-5.3, represents the difference in the slope of mpg vs. weight between manuals and automatics.

The value of B1=-2.17 is a little bigger than our first model, but again has a small p value and is not very significant. However the value of B3=-5.3 is statistically significant with p=0.001. And finally the effect of weight, B2=-3.79 is negative and significant.

Visualizing the Model

Let's overlay our linear model on the scatter plot. Note that we've shifted the weight axis by the mean weight.



The model suggests that manuals have better mpg than automatics at lower weights, but worse mpg at higher weights.

Model Comparison

Let's compare the two models, fit.2 and fit.3. Since the models are nested – fit.3 merely adds another variable (the interaction am : wt) to fit.2 – we can do an anova comparison.

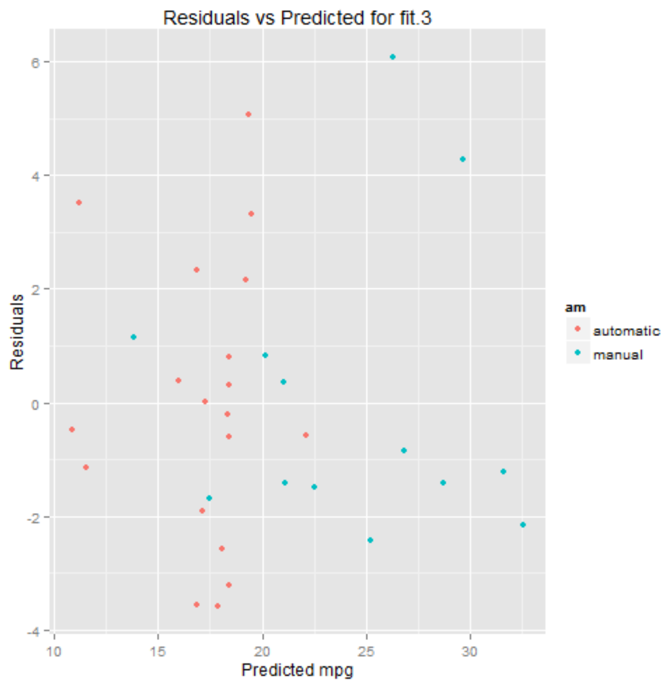
```
anova(fit.2, fit.3)
```

```
## Analysis of Variance Table
##
## Model 1: mpg ~ am + I(wt - mean(wt))
## Model 2: mpg ~ am * I(wt - mean(wt))
##   Res.Df    RSS Df Sum of Sq     F   Pr(>F)
## 1     29 278.32
## 2     28 188.01  1    90.312 13.45 0.001017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The p value, 0.001, indicates a statistically significant difference between the two models, which suggests that the interaction term is indeed significant and should be included in the model.

Residuals and Diagnostics

First let's plot the model residuals vs. the model fit (predicted values) to look for patterns in the residuals.



The residuals appear to be randomly distributed. There does not appear to be a discernable pattern, although there are a few outliers.

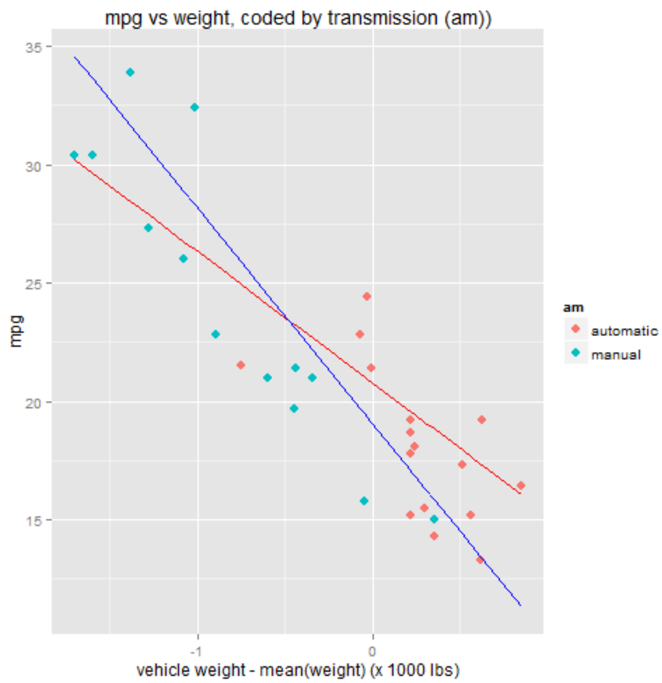
One problem with this model is that there aren't many observations across transmission types that overlap in terms of weight. In other words, most of our automatics are heavy, most of our manuals are light, and there aren't many observations across the two transmissions at similar weights. Also there are a few outliers in the data, particularly for heavy automatics, that may be exerting a lot of influence on the result. Let's see if we can account for that.

First let's quickly look at which data points exert the most leverage on the results by analyzing their hat values:

```
##    mpg    wt        am      hats
## 1 15.0 3.570    manual 0.3709866
## 2 10.4 5.424 automatic 0.3044512
## 3 14.7 5.345 automatic 0.2809856
## 4 10.4 5.250 automatic 0.2542871
## 5 30.4 1.513    manual 0.2534565
## 6 30.4 1.615    manual 0.2156307
```

As expected, three of the top four data points that have the most leverage correspond to our three heavy outliers, all of which have wt > 5 (they are the only observations in the data with wt > 5). Let's filter out these heavy automatics and re-apply our linear model:

```
mtcars.2 <- mtcars %>% filter(wt < 5)
fit.4 <- lm(mpg ~ am * I(wt-mean(wt)), data=mtcars.2)
```



Notice that the two slopes are less different once the heavy automatics are removed. Let's take a look at the coefficients.

```
summary(fit.4)$coefficients
```

```
##                           Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)              20.781027   1.073928 19.350486 1.471239e-16
## ammanual                 -1.718365   1.474379 -1.165484 2.548231e-01
## I(wt - mean(wt))         -5.532531   1.797864 -3.077281 5.011139e-03
## ammanual:I(wt - mean(wt)) -3.551737   2.167804 -1.638403 1.138640e-01
```

Note that slope coefficient for automatics, B2=-5.53 is larger (in absolute terms) than it was in fit.3 (which included the heavy automatics). The difference between the two slopes, captured by B3=-3.55, is smaller than before. Also notice that the difference between the slopes is no longer very significant, with a p value of 0.11.

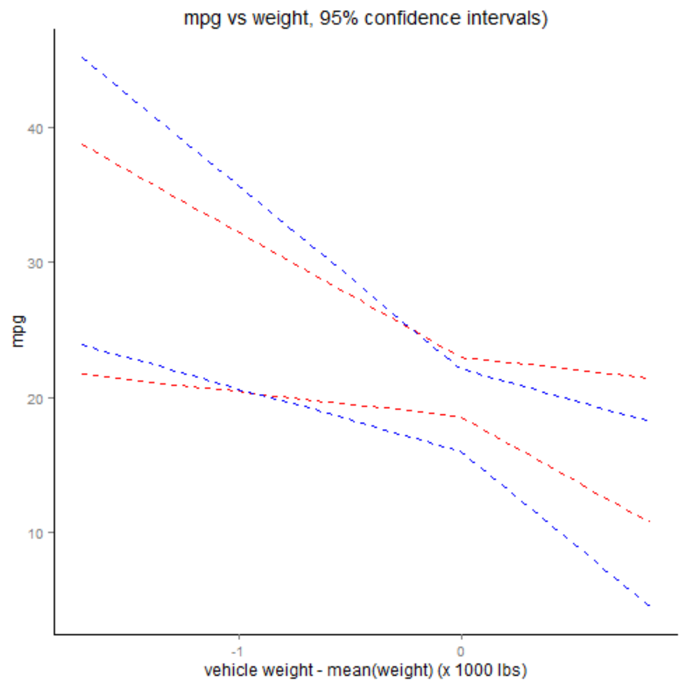
Confidence Intervals

Let's try to quantify the uncertainty in our model by plotting 95% confidence intervals for the two regression lines. The confidence intervals are governed by the standard errors of the coefficients, which are reported in the model summary.

```
## $B0
## [1] 18.56923 22.99282
##
## $B1
## [1] -4.754906  1.318176
##
## $B2
## [1] -9.235301 -1.829762
##
## $B3
## [1] -8.0164137  0.9129402
```

B1 and B3 both include 0 in their 95% confidence intervals, which is expected given that neither of them had a significant p value.

Let's plot the 95% confidence intervals on the chart for visual examination.
For the purpose of clarity, the plot below removes the data points and regression lines to leave just the confidence intervals.



As you can see the 95% confidence intervals show a great deal of overlap between the two transmission types, which suggests that there may not be a significant difference in mpg between them.

Conclusion

The data is inconclusive as to whether there's a significant difference in fuel consumption between automatic and manual transmissions. The data suggests that automatics have better fuel consumption than manuals for heavier vehicles, whereas manuals have better fuel consumption for lighter vehicles; however these results may not be significant due to the high correlation between transmission type and vehicle weight.

For the R code that produced this report, visit https://github.com/rga78/regmods-project.