

# Uninformed Search

## Chapter 3

(Based on slides by Stuart Russell, Subbarao Kambhampati, Dan Weld, Oren Etzioni, Henry Kautz, Richard Korf, and other UW-AI faculty)

# What is a State?

- All information about the environment
- All information necessary to make a decision for the task at hand.

# Agent's Knowledge Representation

Type	State representation	Focus
Atomic	States are indivisible; No internal structure	Search on atomic states;
Propositional (aka Factored)	States are made of state variables that take values (Propositional or Multi- valued or Continuous)	Search+inference in logical (prop logic) and probabilistic (bayes nets) representations
Relational	States describe the objects in the world and their inter-relations	Search+Inference in predicate logic (or relational prob. Models)
First-order	+functions over objects	Search+Inference in first order logic (or first order probabilistic models)

# Illustration with Vacuum World

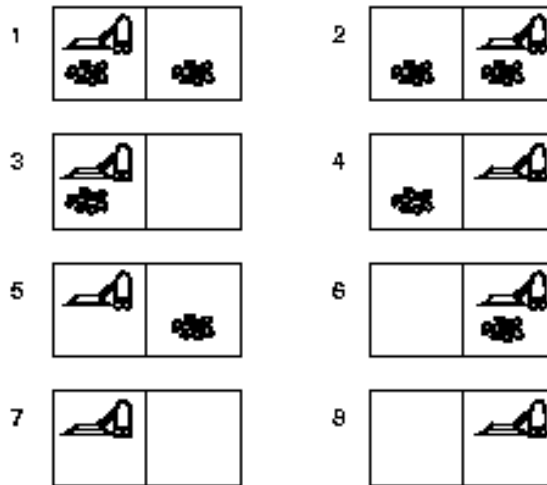
Atomic:

S1, S2.... S8

state is seen as an indivisible  
snapshot

All Actions are SXS matrices..

If you add a second roomba  
the state space *doubles*



Relational:

World made of objects: Roomba; L-room, R-room

Relations: In (<robot>, <room>); dirty(<room>)

If you add a second roomba, or more rooms, only the objects increase.

If you want to consider noisiness, you just need to add one other relation

Propositional/Factored:

States made up of 3 state variables

Dirt-in-left-room T/F

Dirt-in-right-room T/F

Roomba-in-room L/R

Each state is an assignment of  
Values to state variables

$2^3$  Different states

Actions can just mention the variables  
they affect

Note that the representation is  
compact (logarithmic in the  
size of the state space)

If you add a second roomba, the  
Representation increases by just one  
More state variable.

If you want to consider “noisiness” of  
rooms, we need *two* variables, one for

Each room

# Atomic Agent

## Input:

- Set of states
- Operators [and costs]
- Start state
- Goal state [test]

## Output:

- Path: start  $\Rightarrow$  a state satisfying goal test
- [May require shortest path]

# Why is search interesting?

- Many (all?) AI problems can be formulated as search problems!
- Examples:
  - Path planning
  - Games
  - Natural Language Processing
  - Machine learning
  - ...

# Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- states?
- actions?
- goal test?
- path cost?

# Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

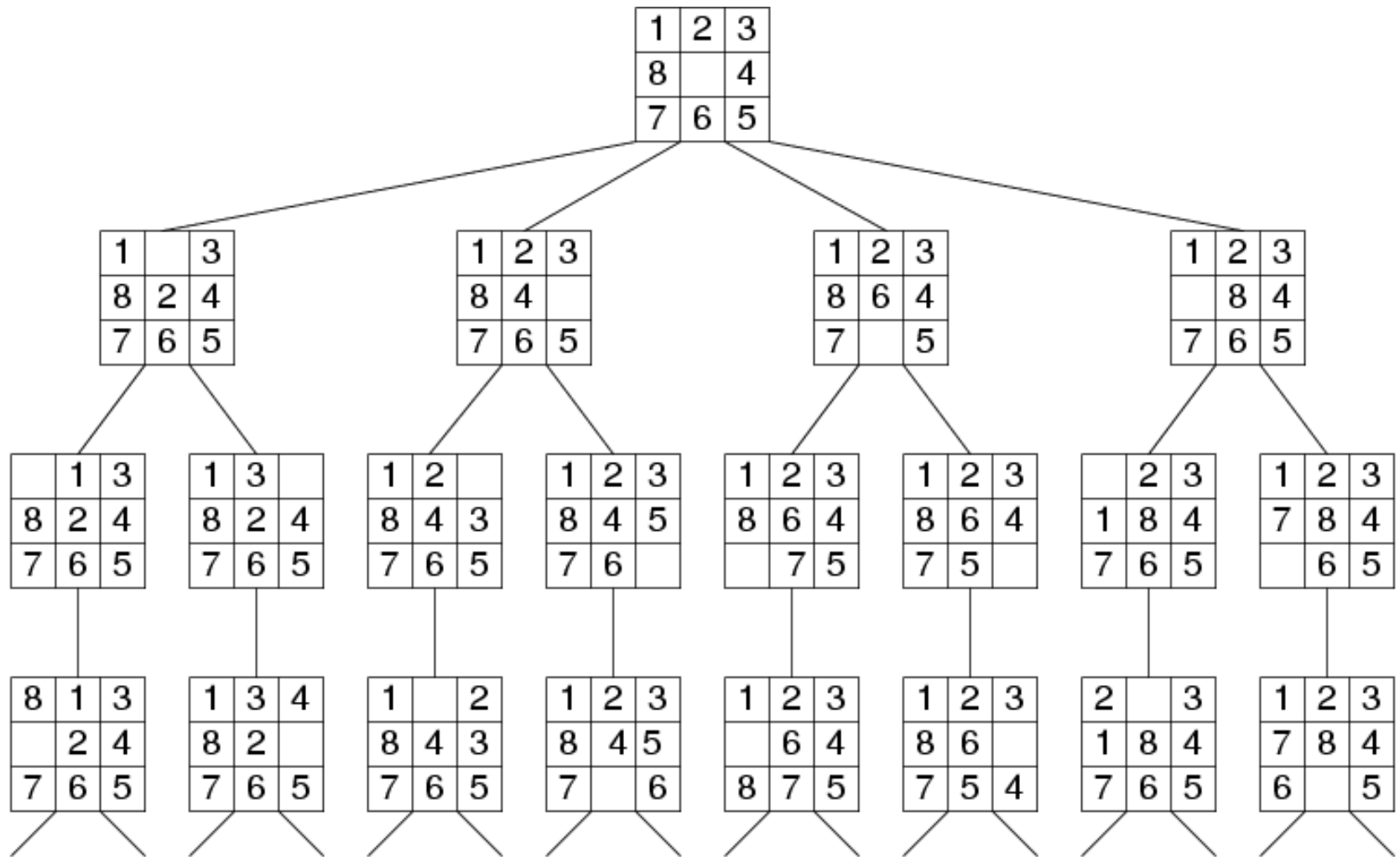
	1	2
3	4	5
6	7	8

Goal State

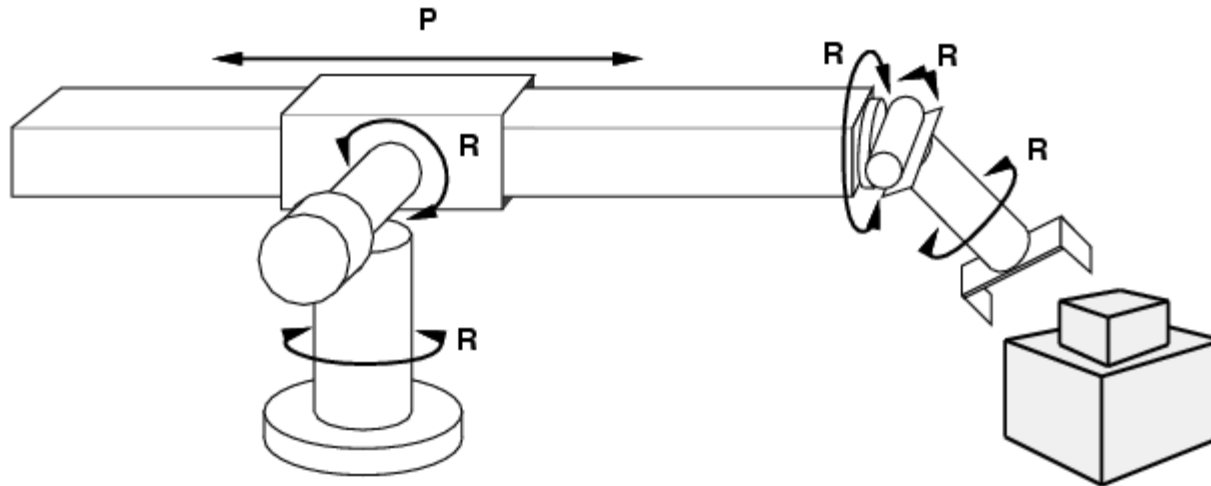
- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move
- 
- [Note: optimal solution of  $n$ -Puzzle family is NP-hard]



# Search Tree Example: Fragment of 8-Puzzle Problem Space



# Example: robotic assembly



- states?: real-valued coordinates of robot joint angles parts of the object to be assembled
- 
- actions?: continuous motions of robot joints
- 
- goal test?: complete assembly
- 
- path cost?: time to execute
-

# Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- 
- **Formulate goal:**
  - be in Bucharest
  -
- **Formulate problem:**
  - **states:** various cities
  - **actions:** drive between cities
  -
- **Find solution:**
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
  -

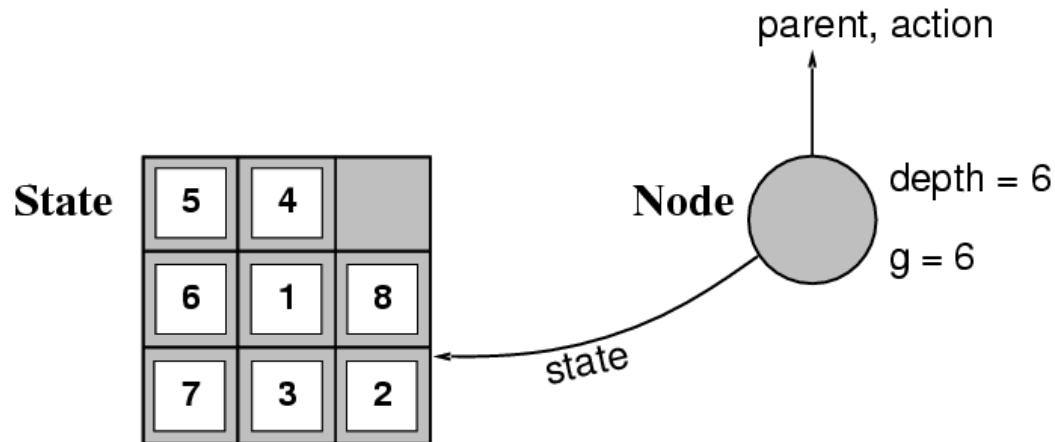
## Example: N Queens

- Input:
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state (test)
- Output

		Q	
Q			
			Q
	Q		

# Implementation: states vs. nodes

- A **state** is a (representation of) a physical configuration
- A **node** is a data structure constituting part of a search tree includes **state**, **parent node**, **action**, **path cost  $g(x)$** , **depth**



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.
-

# Search strategies

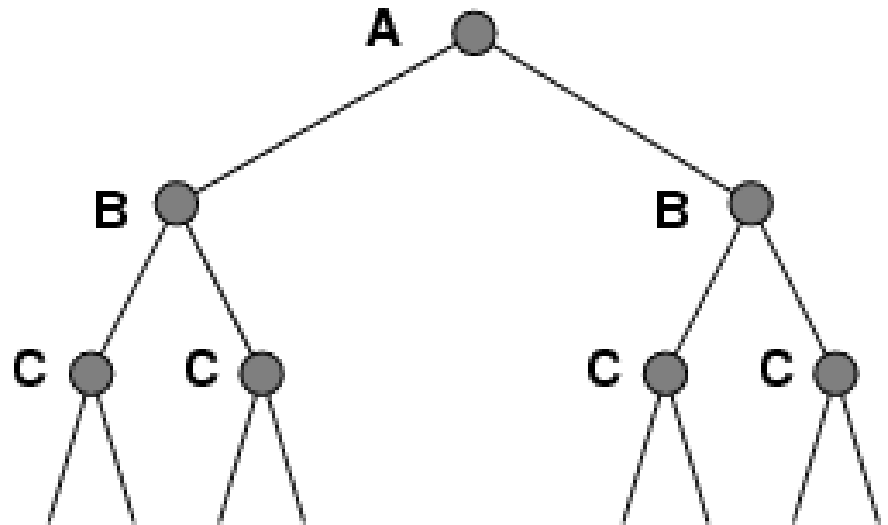
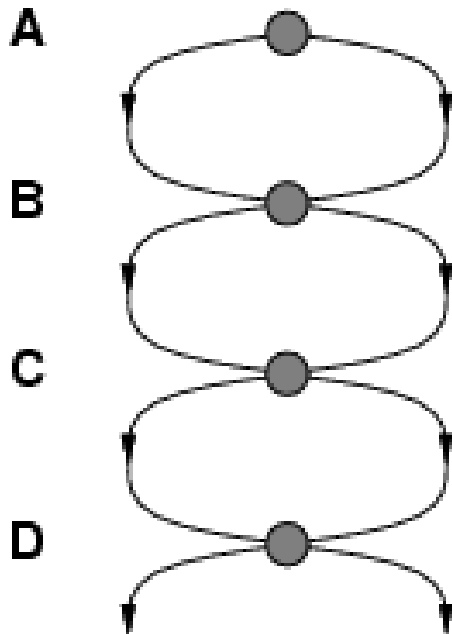
- A search strategy is defined by picking the **order of node expansion**
- Strategies are evaluated along the following dimensions:
  - **completeness**: does it always find a solution if one exists?
  - **time complexity**: number of nodes generated
  - **space complexity**: maximum number of nodes in memory
  - **optimality**: does it always find a least-cost solution?
  - **systematicity**: does it visit each state at most once?
- Time and space complexity are measured in terms of
  - *b*: maximum branching factor of the search tree
  - *d*: depth of the shallowest solution
  - *m*: maximum depth of the state space (may be  $\infty$ )

# Uninformed search strategies

- **Uninformed** search strategies use only the information available in the problem definition
- Breadth-first search
- Depth-first search
- Depth-limited search
- Iterative deepening search

# Repeated states

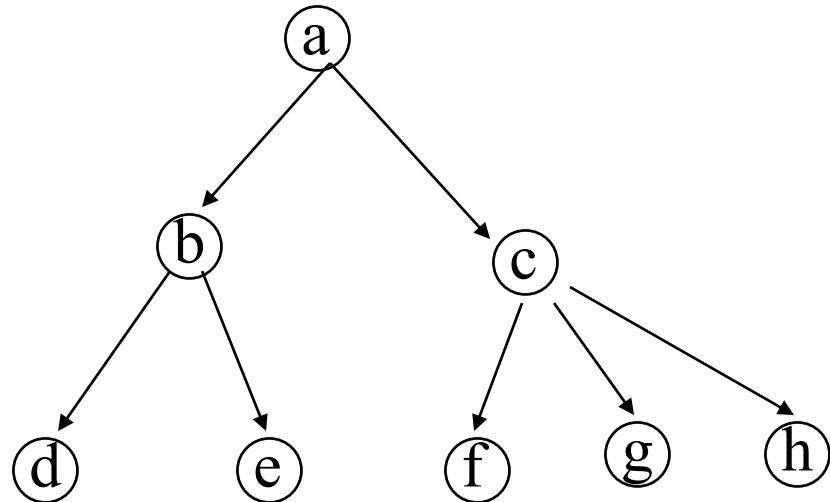
- Failure to detect repeated states can turn a linear problem into an exponential one!





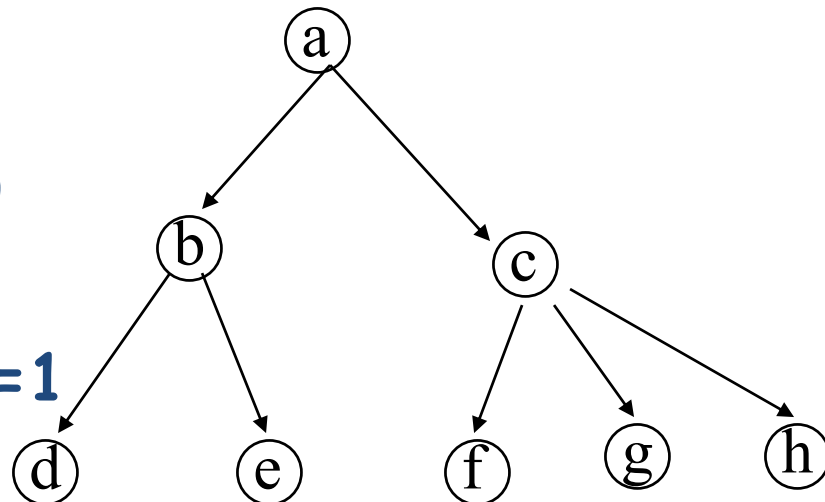
# Depth First Search

- Maintain stack of nodes to visit
- Evaluation
  - Complete? **No**
  - Time Complexity?  $O(b^m)$
  - Space Complexity?  $O(bm)$



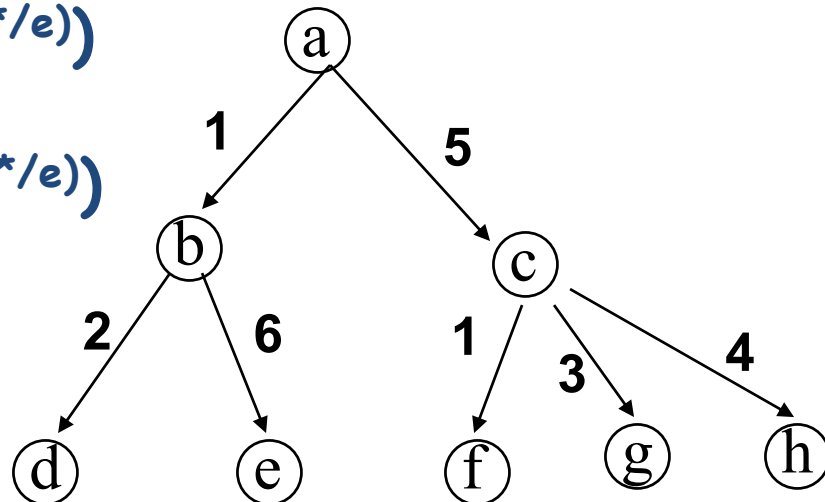
# Breadth First Search: shortest first

- Maintain queue of nodes to visit
- Evaluation
  - Complete? **Yes (b is finite)**
  - Time Complexity?  **$O(b^d)$**
  - Space Complexity?  **$O(b^d)$**
  - Optimal? **Yes, if stepcost=1**



# Uniform Cost Search: cheapest first

- Maintain queue of nodes to visit
- Evaluation
  - Complete? **Yes (b is finite)**
  - Time Complexity?  **$O(b^{(C^*/e)})$**
  - Space Complexity?  **$O(b^{(C^*/e)})$**
  - Optimal? **Yes**



# DFS



<http://www.youtube.com/watch?v=dtoFAvtVE4U>

# UCS



<http://www.youtube.com/watch?v=z6lUnb9kthE>

# Memory Limitation

- Suppose:
  - 2 GHz CPU
  - 1 GB main memory
  - 100 instructions / expansion
  - 5 bytes / node
- 200,000 expansions / sec
- Memory filled in 100 sec ... < 2 minutes

# Time vs. Memory

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	$10^6$	1.1 seconds	1 gigabyte
8	$10^8$	2 minutes	103 gigabytes
10	$10^{10}$	3 hours	10 terabytes
12	$10^{12}$	13 days	1 petabyte
14	$10^{14}$	3.5 years	99 petabytes
16	$10^{16}$	350 years	10 exabytes

**Figure 3.13** Time and memory requirements for breadth-first search. The numbers shown assume branching factor  $b = 10$ ; 1 million nodes/second; 1000 bytes/node.

# Idea 1: Beam Search

- Maintain a constant sized frontier
- Whenever the frontier becomes large
  - Prune the worst nodes

Optimal: no

Complete: no



# Idea 2: Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or fail-  
ure  
  inputs: problem, a problem  
  for depth  $\leftarrow$  0 to  $\infty$  do  
    result  $\leftarrow$  DEPTH-LIMITED-SEARCH( problem, depth)  
    if result  $\neq$  cutoff then return result
```

# Iterative deepening search / =0

Limit = 0



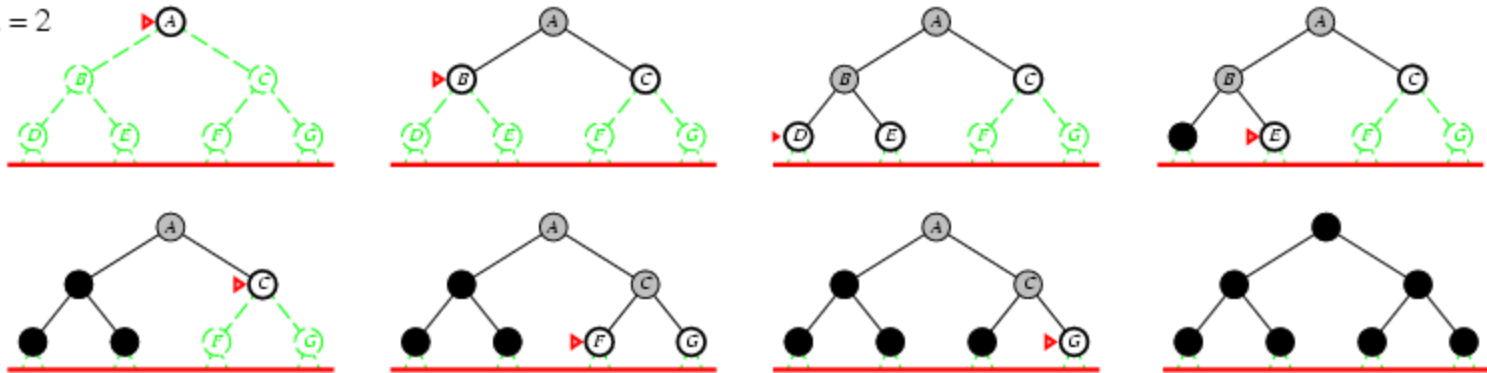
# Iterative deepening search / =1

Limit = 1



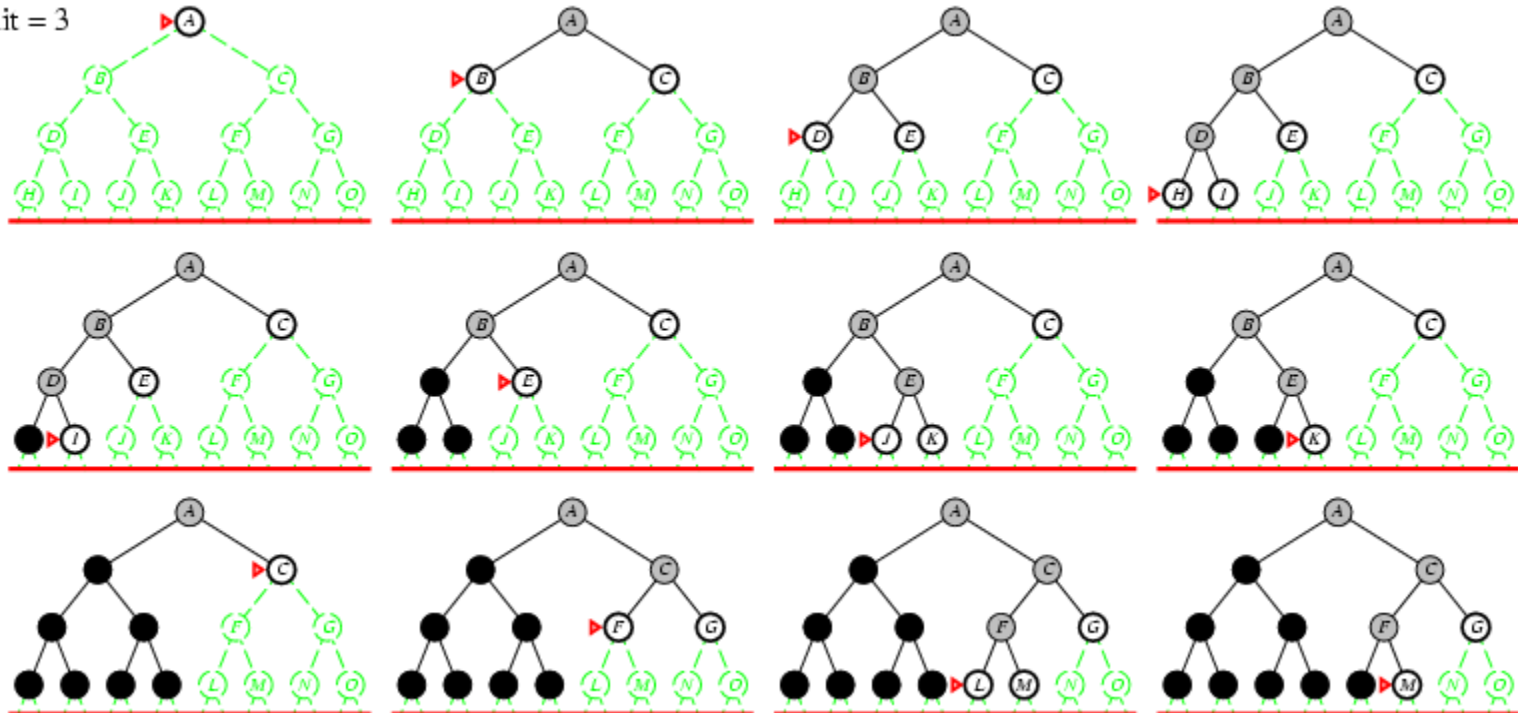
# Iterative deepening search / =2

Limit = 2



# Iterative deepening search / =3

Limit = 3



# Iterative deepening search

- Number of nodes generated in a depth-limited search to depth  $d$  with branching factor  $b$ :
  - $$N_{DLS} = b^0 + b^1 + b^2 + \dots + b^{d-2} + b^{d-1} + b^d$$
- Number of nodes generated in an iterative deepening search to depth  $d$  with branching factor  $b$ :
  - $$N_{IDS} = (d+1)b^0 + d b^1 + (d-1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d$$
- Asymptotic ratio:  $(b+1)/(b-1)$
- For  $b = 10, d = 5$ ,
- - $N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111$
  - 
  - $N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456$
  -
- Overhead =  $(123,456 - 111,111)/111,111 = 11\%$

# Iterative deepening search

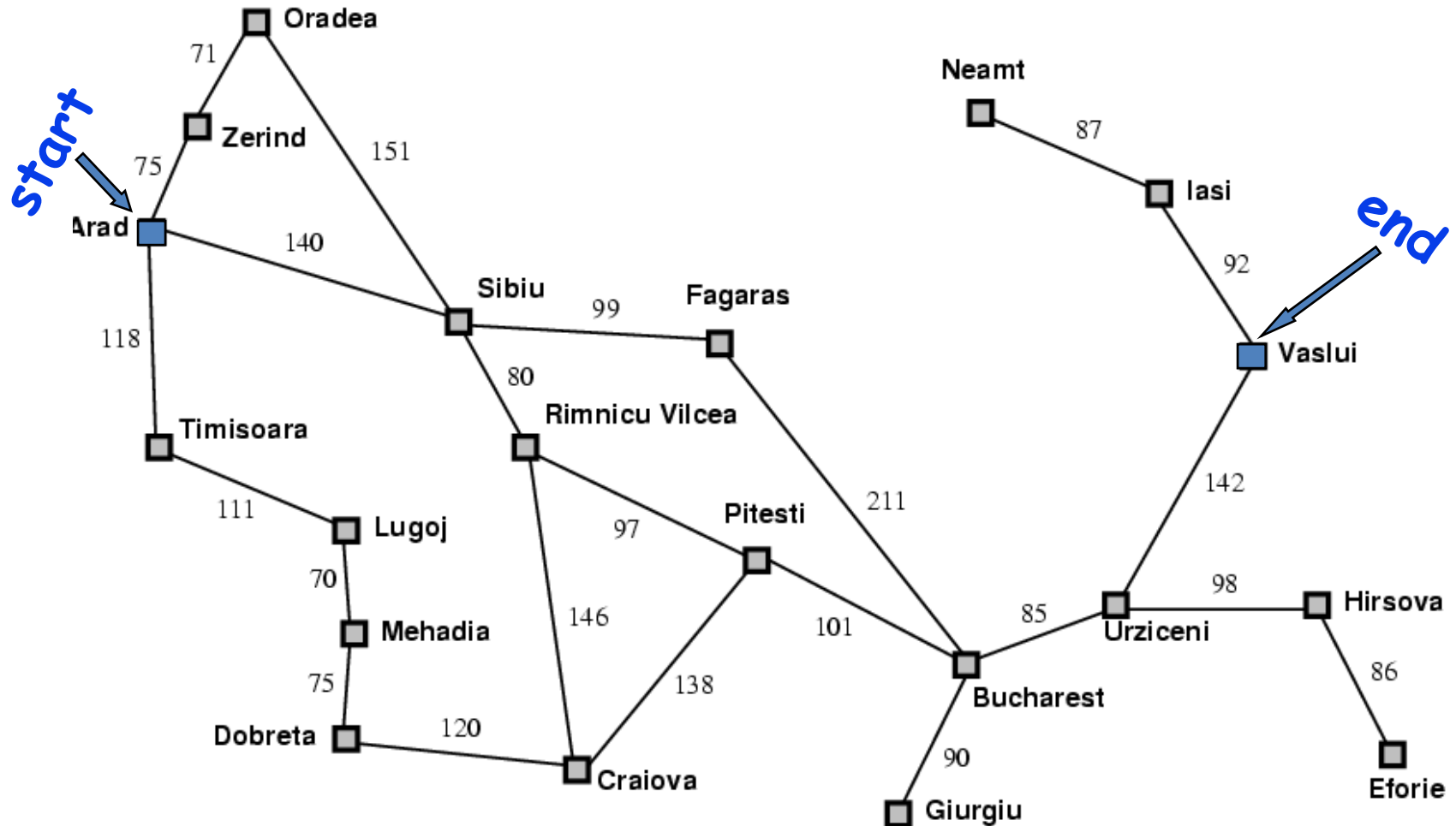
- Complete?
  - Yes
- Time?
  - $(d+1)b^0 + d b^1 + (d-1)b^2 + \dots + b^d = O(b^d)$
- Space?
  - $O(bd)$
- Optimal?
  - Yes, if step cost = 1
  - Can be modified to explore uniform cost tree (iterative lengthening)
- Systematic?

# Cost of Iterative Deepening

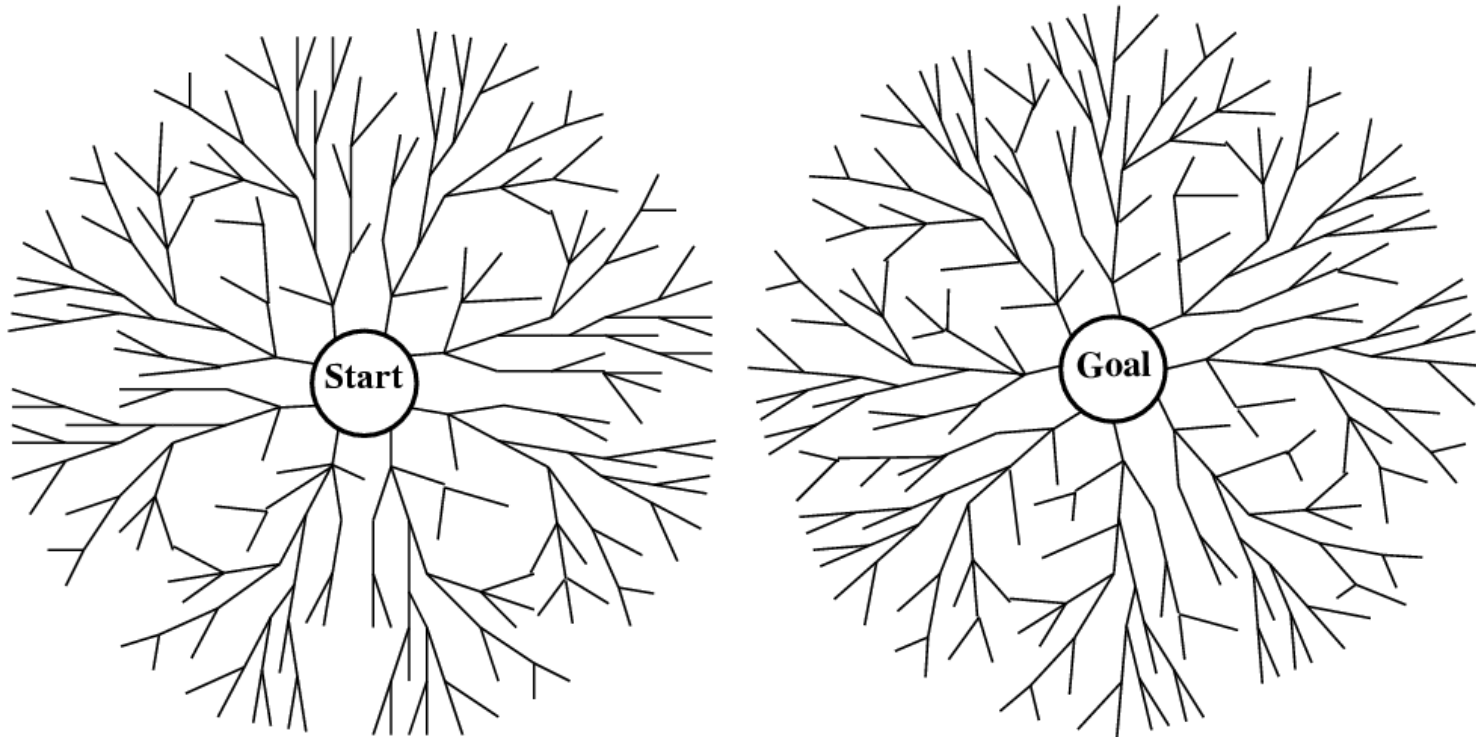
b	ratio ID to DLS
2	3
3	2
5	1.5
10	1.2
25	1.08
100	1.02



# Forwards vs. Backwards



## vs. Bidirectional



**When is bidirectional search applicable?**

- **Generating predecessors is easy**
- **Only 1 (or few) goal states**

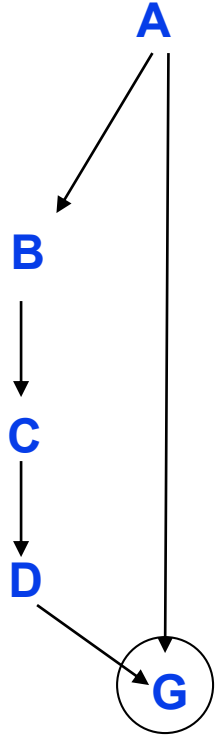
# Bidirectional search

- Complete? Yes
- Time?
  - $O(b^{d/2})$
- Space?
  - $O(b^{d/2})$
- Optimal?
  - Yes if uniform cost search used in both directions

# Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes <sup>a</sup>	Yes <sup>a,b</sup>	No	No	Yes <sup>a</sup>	Yes <sup>a,d</sup>
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes <sup>c</sup>	Yes	No	No	Yes <sup>c</sup>	Yes <sup>c,d</sup>

**Figure 3.21** Evaluation of tree-search strategies.  $b$  is the branching factor;  $d$  is the depth of the shallowest solution;  $m$  is the maximum depth of the search tree;  $\ell$  is the depth limit. Superscript caveats are as follows: <sup>a</sup> complete if  $b$  is finite; <sup>b</sup> complete if step costs  $\geq \epsilon$  for positive  $\epsilon$ ; <sup>c</sup> optimal if step costs are all identical; <sup>d</sup> if both directions use breadth-first search.

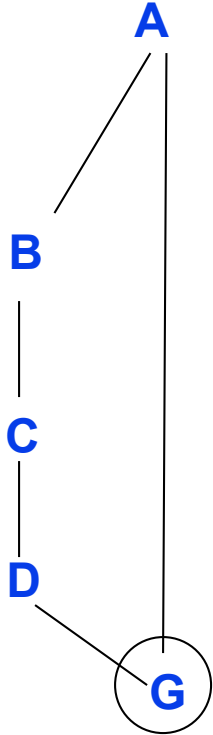


**BFS:        A,B,G**

**DFS:        A,B,C,D,G**

**IDDFS: (A), (A, B, G)**

**Note that IDDFS can do fewer  
expansions than DFS on a graph  
shaped search space.**



**BFS: A,B,G**

**DFS: A,B,A,B,A,B,A,B,A,B**

**IDDFS: (A), (A, B, G)**

**Note that IDDFS can do fewer  
expansions than DFS on a graph  
shaped search space.**

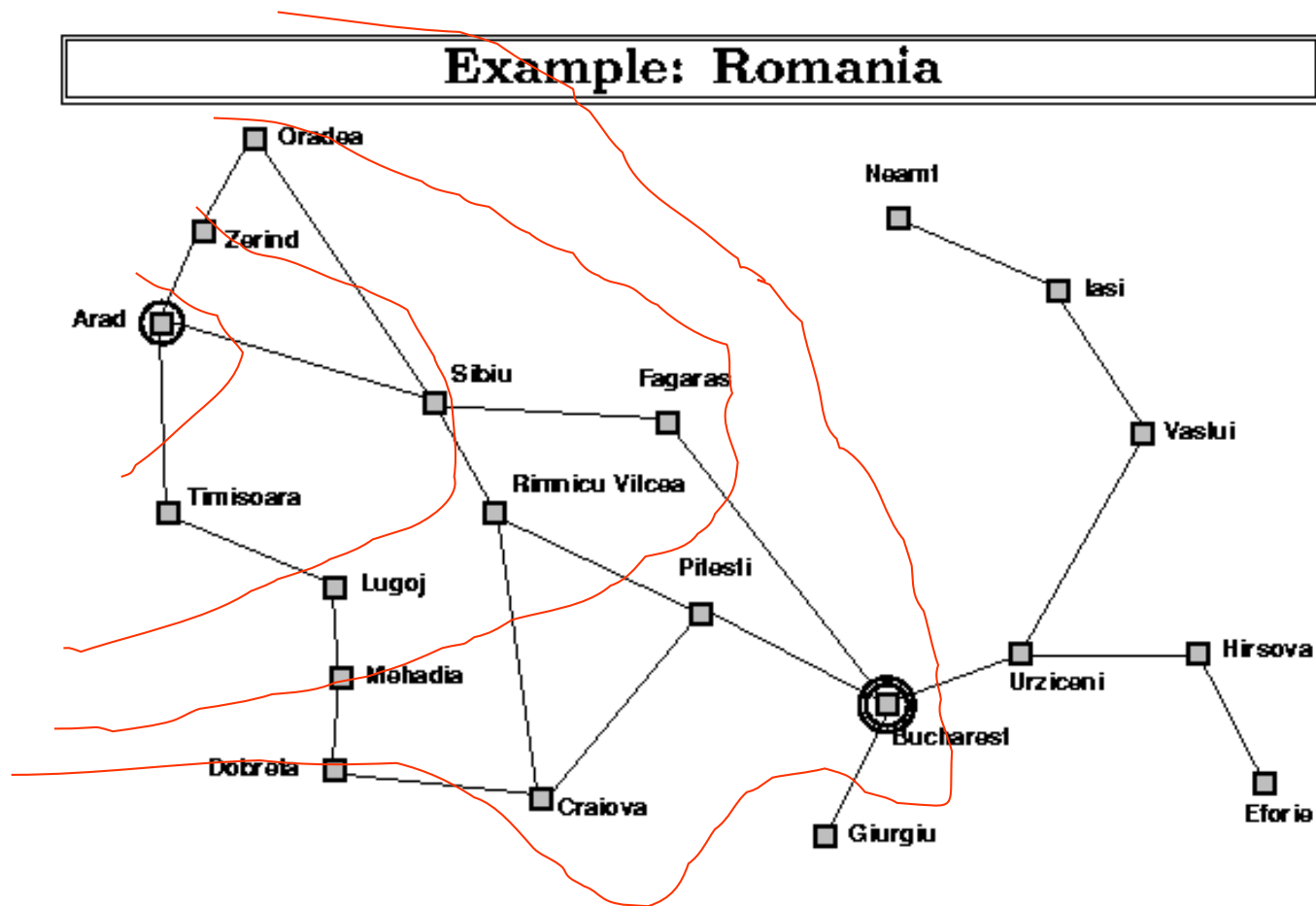
**Search on undirected graphs or directed graphs with cycles...**

**Cycles galore...**

# Graph (instead of tree) Search:

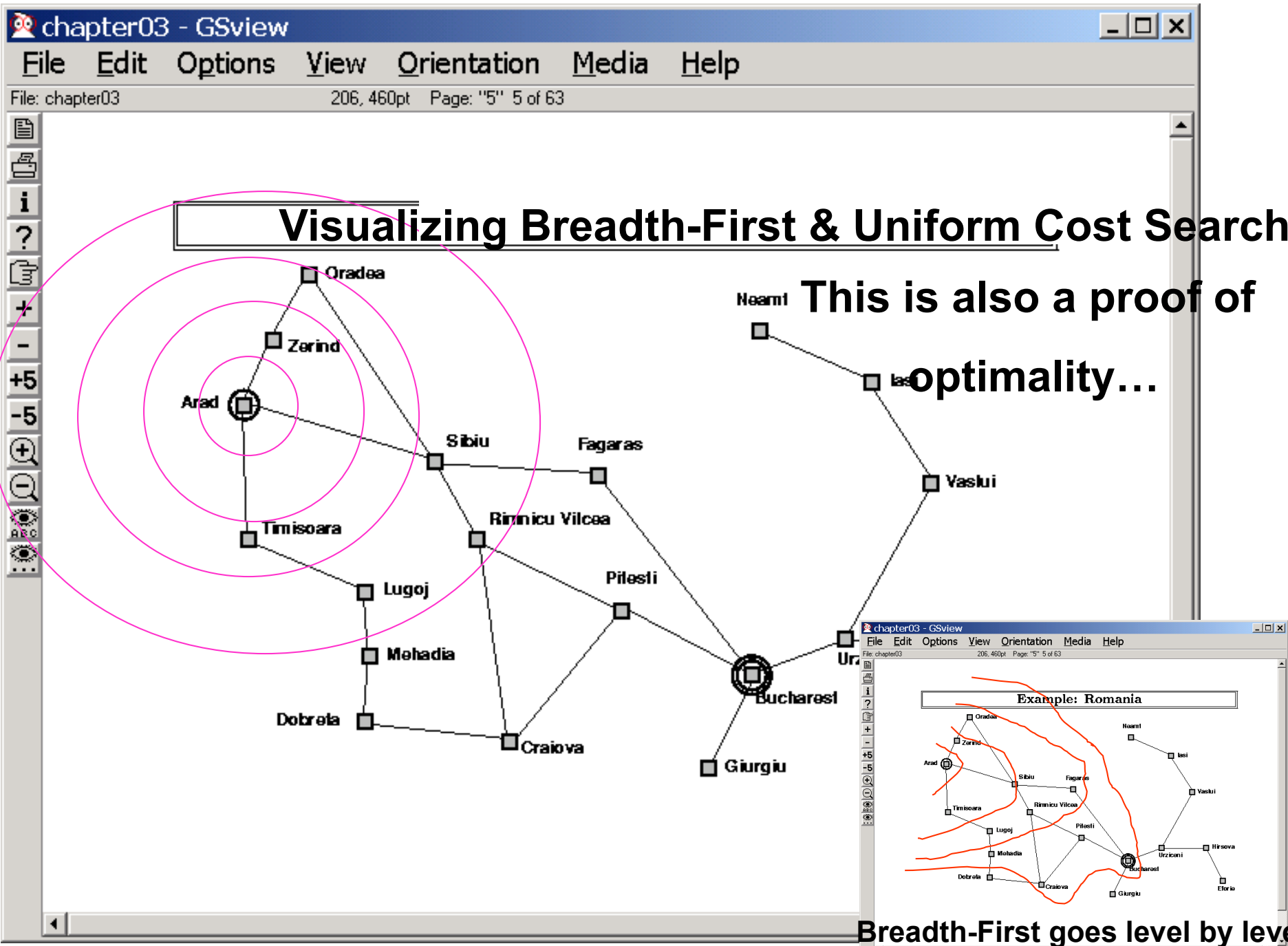
## Handling repeated nodes

- Repeated expansions is a bigger issue for DFS than for BFS or IDDFS
  - Trying to remember all previously expanded nodes and comparing the new nodes with them is infeasible
  - Space becomes exponential
  - duplicate checking can also be expensive
- Partial reduction in repeated expansion can be done by
  - Checking to see if any children of a node  $n$  have the same state as the parent of  $n$
  - Checking to see if any children of a node  $n$  have the same state as any ancestor of  $n$  (at most  $d$  ancestors for  $n$ —where  $d$  is the depth of  $n$ )



Breadth-First goes level by level





# Problem

- All these methods are slow (blind)



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- Solution → add guidance (“**heuristic estimate**”)  
→ “**informed search**”