Detector simulation and validation with dat

Abstract

We detail in this note the simulation method of the EASIER detector from the Radio Frequency (RF) signal to the ADC trace. We validate and compare our models with the data of the three different C-band detectors.

Introduction

The EASIER detectors goal is to measure the emission from EAS in the radio frequencies. Each EASIER detector is integrated in an Auger SD and is composed of antenna and an amplifier, a power detection level to transform the HF signal into its power level, and an adaptation board to fit the Auger SD acquistion front end (cf. 1).
We review here the method of the detector's simulation in the C-band. In order to understand our detector and verify that our simulation reproduce the data, we will often look at the ratio of the noise fluctuation over its mean. This parameter allows us to probe the simulation of the detector independently of parameters like the system temperature or the absolute gain.
We will first look at the power detection and adaptation board simulation. Then we will step back in the chain to focus on the HF waveform simulation from a frequency spectrum. Finally we will compare the simulation with the data.

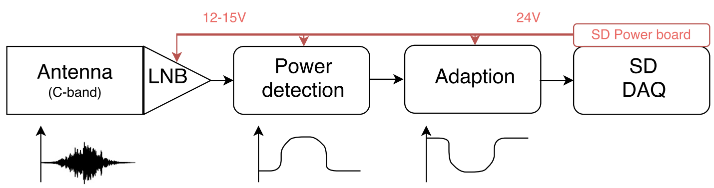


Figure 1

EASIER detectorS

Before going into the detail of the simulation, we remind the main differences between the three setups that were installed in the C-band. Three different detector type were installed, the main difference is the antenna, but there is also the presence or not of a capacitor after the power detector that was removed after the first setup. The table 1 shows the main difference between those setups.

Table 1 bandwidths

	EASIER 7 (setup 1)	EASIER61 (setup 2)	GIGADUCK (setup 3)
antenna type	GI301	DMX	Norsat
capacitor	yes	no	no

1 RF simulation

We detail here the way to simulate the RF noise signal, we will first show the ideal case of a flat spectrum and find numerically the formula of the ratio of the rms over the mean. Then we will see how to simulate a realistic spectrum.

1.1 Theory

One way to check our simulation is by looking at what is expected in term of signal distribution. In simple cases of input spectrum and detector, the RMS and the mean of the waveform can be derived analytically.
A derivation is given in the book "Tools of Radio Astronomy" (T.L Wilson, K. Rohlfs, S. Huttemeister). As we will see, a convenient value to look at is the ratio of the RMS over the mean, because some calibration parameters like the temperature and the gain will cancel out. Finally knowing the RMS of your signal is a good estimation of your sensitivity. The main steps of the derivation are the following:

- take a radio detector with a bandwidth B, the amplitude of the signal $v(t)$ follows a gaussian distribution.
- the signal goes through a square law detector $y = av(t)^2$. One can compute the mean and standard deviation and finds: $< y >= a\sigma_v^2$ and $\sigma_y = \sqrt{2}a\sigma_v^2$ (with $< v^2 >=< power >= kT_{sys}GB$). (this webpage might also help: `http://mathworld.wolfram.com/GaussianIntegral.html`)
- So we have: $\sigma_y = \sqrt{2} < y >$, with $\sigma_y = k_B \Delta T G B$ so $\Delta T = \sqrt{2} T_{sys}$
- now if we average over a time τ, that means we average over$N = f_{Nyquist} * \tau$ points where $f_{Nyquist} = 2B$, and we reduce the fluctuation by a factor $\sqrt{N}$
- we finally obtain: $\Delta T = \frac{T_{sys}}{\sqrt{B\tau}}$

To sum up, for a input gaussian signal followed by a square law detector we expect the ratio $\frac{\sigma}{\mu} = \sqrt{2}$. And this ratio is then modified by the later processing, averaging, filtering. The formula that is commonly used is:

$$\frac{\sigma}{\mu} = \frac{1}{\sqrt{\Delta B \cdot \tau}} \tag{1}$$

We verify in the next paragraph the validity of this expression.

1.2 flat spectrum

no filter We start with the case of a flat spectrum with a phase drawn randomly. We produce a waveform from the spectrum by inverse Fast Fourier Transform. An example of such a waveform and its spectrum is shown in the figure. 2. One can check that the $\frac{\sigma}{\mu}$ ratio is $\sqrt{2}$ as expected. We can also check that this ratio doesn't depend on the chosen bandwidth as shown in fig. 3 left.

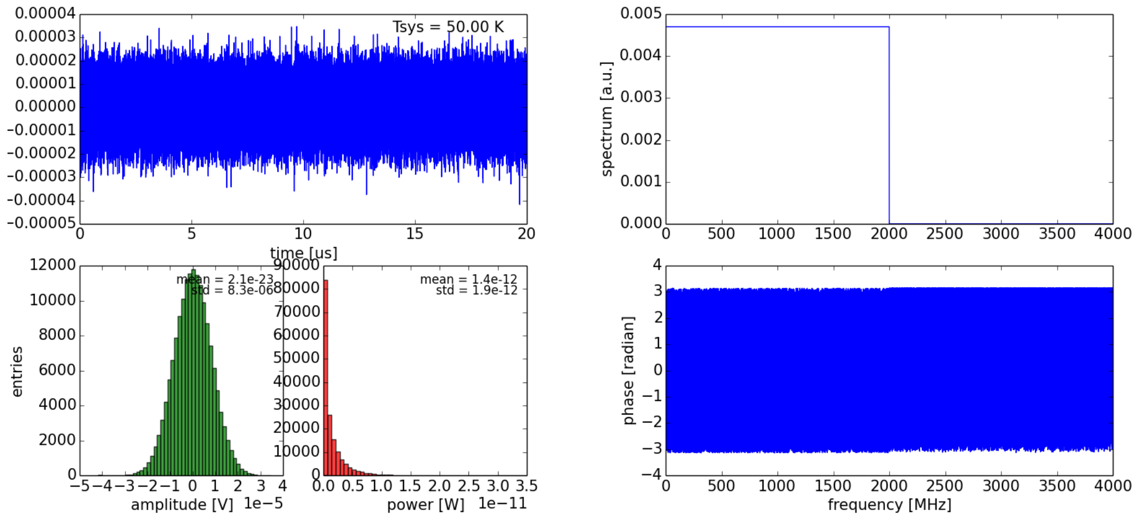


Figure 2 Left: top: waveform, bottom left: amplitude distribution, bottom right power distribution. Right: top: spectrum, bottom: phase

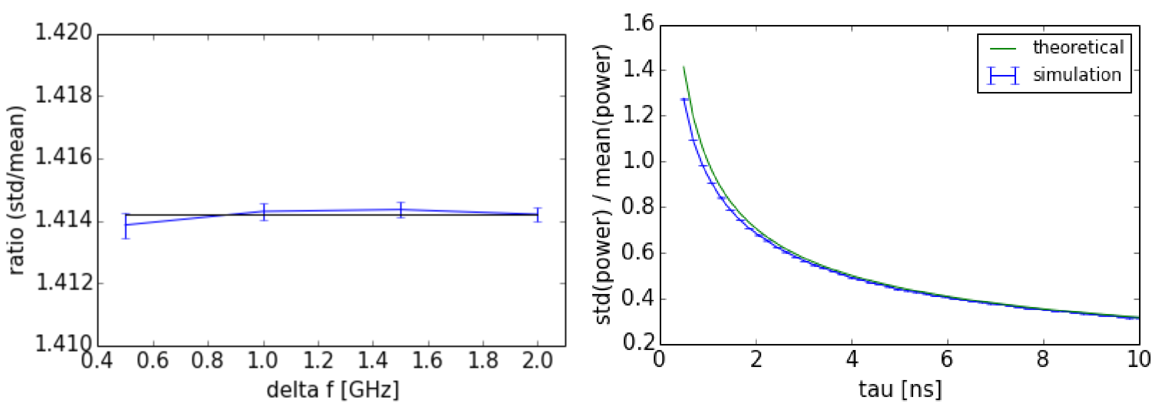


Figure 3 left: ratio $\frac{\sigma}{\mu}$ versus the lower frequency f_min of the noise spectrum. rigt: the ratio when we apply a low pass filter of frequency cut $f_{cut} = 1/\tau$ to the power. It is shown for three different f_{min}

numerical filter Now we filter the waveform (after converting it to power) with a numerical filter (we set the fft coefficient to 0 for the filtered frequencies.). The orignal waveform is issued from a spectrum between 0 and 2GHz. We show in fig. 3 (right) $\frac{\sigma}{\mu}$ the ratio for different frequency cuts of the filter where the x-axis is the time constant tau $= \frac{1}{\mathrm{f_{cut}}}$. We see that it follows very well the expected formula.

Now if we vary the bandwidth and then apply the filter we obtain the result in the fig. 4 (left). We see now a discrepancy between the expected ratio and the simulated one. We see that if we reduce the bandwidth, the ratio is lower than expected and plateaus at for the high frequency filters (low τ) but matches the formulas at the low frequency filter (high τ). This can be understood when we think about the FFTs (or look at them on figure 4). The fourier transform of the power for a signal at frequencies in [f1 - f2] has a contribution between [0 ; $\Delta B/2$], where $\Delta B = f2 - f1$ and another one between [2*f1 ; 2*f2]. On the figure we see an example for an amplitude input spectrum of [1.5 ; 2 GHz]. If we filter the power spectrum with a frequency f_{cut}smaller than $\Delta B/2$ then we follow the equation because we have a full or a continuous bandwidth from [0 ; f_{cut}], if f_{cut} is larger then we go into the hole in the power spectrum and we don't add any frequency in the average, that's why the ratio goes to a constant.

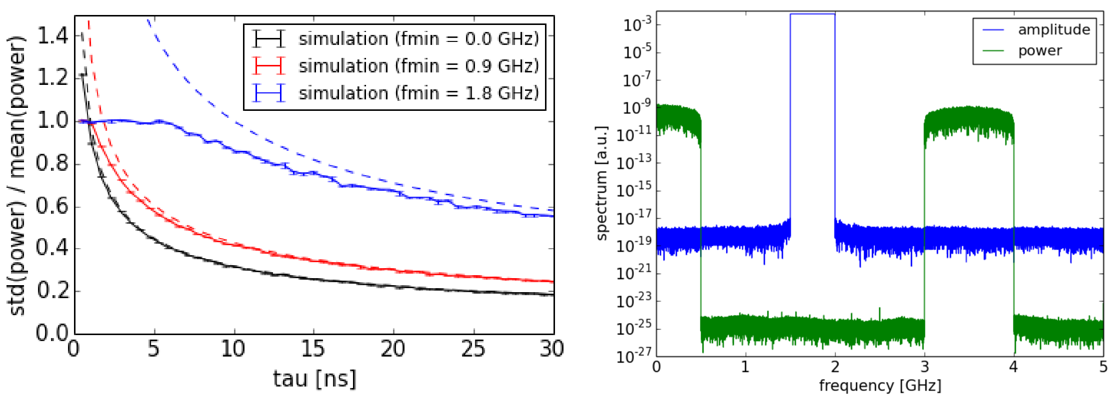


Figure 4 left: ratio $\frac{\sigma}{\mu}$ versus the lower frequency f_min of the noise spectrum. rigt: the ratio when we apply a low pass filter of frequency cut $f_{cut} = 1/\tau$ to the power. It is shown for three different f_{min}

sliding window In this paragraph, instead of filtering by cutting the frequency spectrum we average over several time bins, i.e. we use a sliding window average. If we consider τ as the window size we see (fig 5) that the ratio doesn't follow the equation 1. There is an additional factor to be added.

We showed that the formula 1 can not be always used in this form. Some conditions have to be matched and the processing technique (filtering or other sort of averaging) can change the ratio.
It can still be used as a figure of merit but it is not anymore representative of the minimum detectable signal (or the one sigma deviation from the mean).

1.3 realistic spectrum

Our detectors have not a flat spectrum (cf fig. 7). We first show how we simulate it and then we see how to account for this in the equation 1.

detectors gain The spectrum that we need to simulate our signal is the spectrum at the input of the power detector. We show here that this spectrum depends only on the antenna and LNB gain.

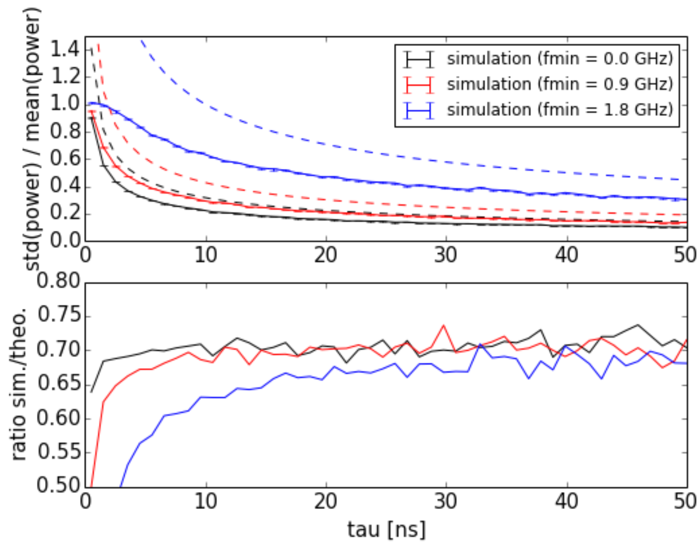


Figure 5 $\frac{\sigma}{\mu}$ ratio after applying a sliding window of size τ for different f_{min}

What comes out of the antenna + LNB is:

$$\mathrm{P}(\nu) = \frac{1}{2}\int_{\Omega} \mathrm{F}(\nu)\mathrm{A_{eff}}(\nu)\mathrm{G_{LNB}}(\nu)\mathrm{d}\Omega \tag{2}$$

with

$$\mathrm{F}(\nu) = \frac{2\mathrm{k_B}\mathrm{T}\nu^2}{\mathrm{c}^2} \tag{3}$$

and

$$\mathrm{A_{eff}}(\nu) = \frac{\mathrm{G_{ant}}(\nu)\mathrm{c}^2}{4\pi\nu^2} \tag{4}$$

So the ν^2 cancels out and we have only:

$$\mathrm{P}(\nu) = \frac{1}{4\pi}\int_{\Omega} \mathrm{k_B}\mathrm{T}\mathrm{G_{ant}}(\nu)\mathrm{G_{LNB}}(\nu)\mathrm{d}\Omega = \mathrm{k_B}\mathrm{T_{ant}}\mathrm{G_{LNB}}(\nu) \tag{5}$$

Then the only dependence in frequency is given by the antenna gain and the amplifier gain. For the three antenna and LNB that we have installed we have this data with:

- for EASIER antennas (DMX and GI301):spectrum measurement (in the field or at room temperature)
- for GIGADuck antenna (Norsat) : IMEP measurement of the total gain antenna+LNB

simulation with realistic spectrum To simulate a waveform with the detector spectrum, we set first the fft with the **square root of the power gain** and the phase is again drawn randomly from a uniform distribution between $[-\pi;\pi]$.

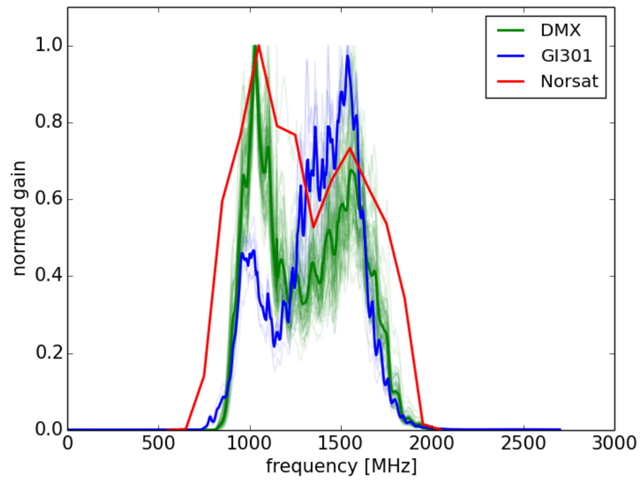


Figure 6 gain spectra for the three types of LNB in the C-band.

what is the bandwidth ? Now we want to know how to relate the realistic bandwith to the equation 1. Usually we take the bandwith as 800 MHz because of the definition of the C-band (3.4-4.2 GHz). However we see that the gain is not flat so what quantity should we take to enter the equation 1 ? One can think about the following definition:

$$\Delta \mathrm{B} = \frac{1}{\mathrm{G}_{\mathrm{max}}} \int \mathrm{G}(\nu) \cdot \mathrm{d}\nu \tag{6}$$

This definition is usefull for instance to estimate the total power in a band. For instance this bandwidth will enter the calculation of the average power (see second point of the derivation section1.1)
In fact we need to compute the bandwidth as:

$$\Delta \mathrm{B} = \frac{1}{\sqrt{\mathrm{G}_{\mathrm{max}}}} \int \sqrt{\mathrm{G}(\nu)} \cdot \mathrm{d}\nu \tag{7}$$

To see this we can look at the $\frac{\sigma}{\mu}$ ratio from a real waveform of noise coming from an antenna (a DMX antenna) and compare it with the $\frac{\sigma}{\mu}$ ratio of a flat spectrum generated waveform with ΔB computed as eq 6 or eq 7. (We need to apply a filter so that the ratio is different from $\sqrt{2}$). At the end the bandwidth are given is the table 2.

Table 2 bandwidths

antenna type	GI301	DMX	Norsat
voltage bandwidth	704 $\pm$29	689 $\pm$ 46	901
power bandwidth	437 $\pm$30	445 $\pm$ 56	684

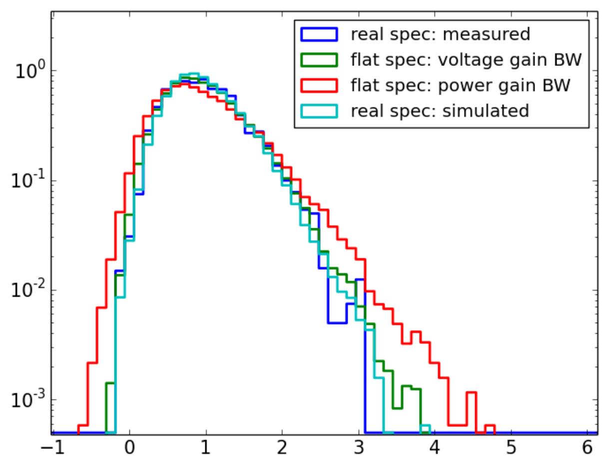


Figure 7 noise distribution after power filtering in 4 cases: blue: real waveform from a DMX antenna. green: flat spectrum generated waveform with bandwidth according eq 7, red:flat spectrum generated waveform with bandwidth according eq 6, light blue: waveform generated from the spectrum in fig 7

2 Adaptation electronics simulation

Here we consider the adaptation electronics starting from the power detection up to the analog to digital conversion and the final data see fig. 1. In this chain, the main components are: the power detector, the adaptation board, the SD front end filter and the ADC. Some other minor elements (minor in the sense they won't modify the signal shape or SNR but only its amplitude, like cables, 75-50 adapter or other lossy element) won't be simulated.

2.1 power detection

The step of the power detection is the most specific to EASIER system. We present here three methods to simulate the power detector. They are closely related, two of them are based on a convolution in a simular way I had done in my thesis. The third one is based on the frequency response. To test the methods we have calibration data we had taken back in 2013. The setup is composed of a C-band antennna followed by a power detector. The signal is recorded with the large bandwidth oscilloscope simultaneously after the antenna and the after the power detector. We have data with only backgroud noise (fig. 8 left) and data with short pulses emitted with an electronic lighter (fig. 8 right).

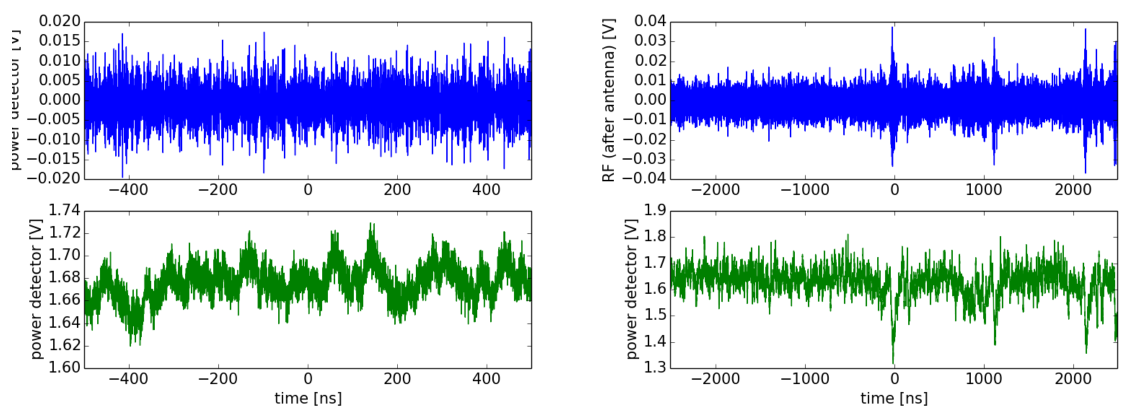


Figure 8 example of calibration data, with only background noise (left), and with pulses (right)

2.1.1 DC calibration

We first look at the calibration of the DC component. We will use the noise data to determine it. The method is very simple we just plot the average of the power detector waveform against the RF average in dBm. The only interesting detail here is the difference between the average of the log ($< \mathrm{P} >= \frac{1}{\mathrm{N}}\Sigma \mathrm{P}_{\mathrm{dBm}}$) and the log of the average ($< \mathrm{P} >= \frac{10}{\mathrm{N}}\log_1 0\Sigma \mathrm{P}_{\mathrm{mW}}$). We see these characteristics on the figure 9. The main difference is the offset, the slope found from the fit is approximately the same. This characteristics is only valid to relate the baseline value, i.e. mean values. If we want to reproduce the waveform, we need a bit more complicated methods. We present some example in the following paragraphs.

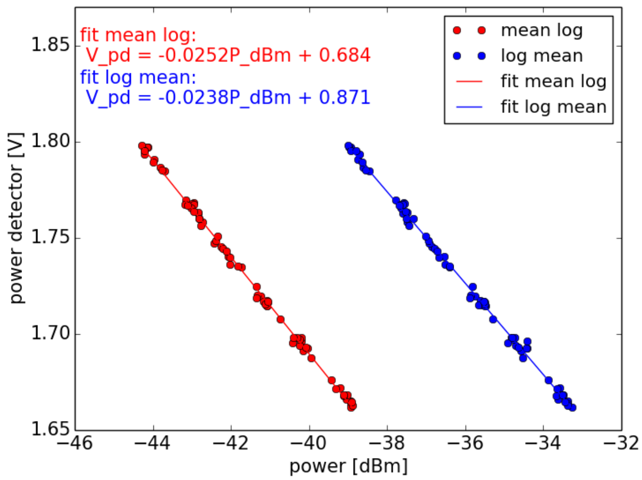


Figure 9 caracteristics of the power detector in the case we take the log of the mean power or in the case we take the mean of the log power

2.1.2 convolution with floating parameters (method 1)

We found that a good way to simulate the power detector is by performing a convolution of the signal in dBm with an exponential decay function.
We want to reproduce the power detector signal from the RF signal. Here are the steps we follow:

- transform RF signal in dBm
- do the convolution with $f(t) = A \cdot \exp \frac{t}{\tau}$
- transform linearly the result to obtain the power detector waveform: $V_{sim} = a \cdot V_{conv} + b$

In order to find the parameters τ, a and b, we scan a range of values of τ, and then fit V_{conv} versus V_{PD}. Then we search for τ, a and b that minimize the distance $\mathrm{d} = \Sigma(\mathrm{V_{sim}} - \mathrm{V_{PD}})^2$ between the two waveforms. This procedure is perfomed on 20 waveforms and the average parameters are chosen (cf table 3). An example of a resulting simulated signal overlayed with

Table 3 parameters for the power detection

	τ [ns]	a	b
with capacitor	4.7 ± 0.2	-0.019 ± 0.001	0.88 ± 0.04
without capacitor	35.2 ± 2.2	-0.022 ± 0.001	0.79 ± 0.04

the original one is shown in the fig. 10 (middle) and the difference is shown in the fig. 10 (bottom). When using the average parameters to all waveforms, we obtain the residuals distribution in the figure 10. The standard deviation for the no capacitor case is 22mV and 11mV for the capacitor.

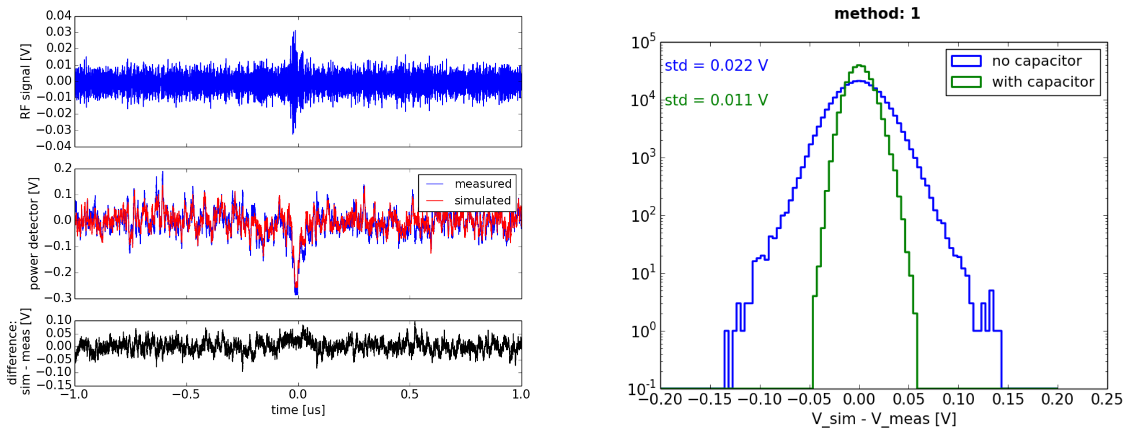


Figure 10 top: RF waveform, middle: power detector measured and simulated, bottom: difference

2.1.3 transfer function method

Another way to simulate the power detector response can be done by looking at the transfer function in the frequency domain. This can be seen as a filter response. In practice, we need to use the DC caracteristics we already found (cf fig. 9) and then look at the frequency spectrum of the signal before and after the power detector. Note that the RF signal is first transformed in dBm, i.e. in logarithmic scale. We use the waveforms presented before for the convolution method. On the figures 11 we show the spectra before and after the power detector (left), the ratio (middle), an example of the phase difference (right). Fig. 11 is the case when the capacitor is used and fig. 12 is the no capacitor case. We see that the capacitor case filters at lower frequencies as expected.

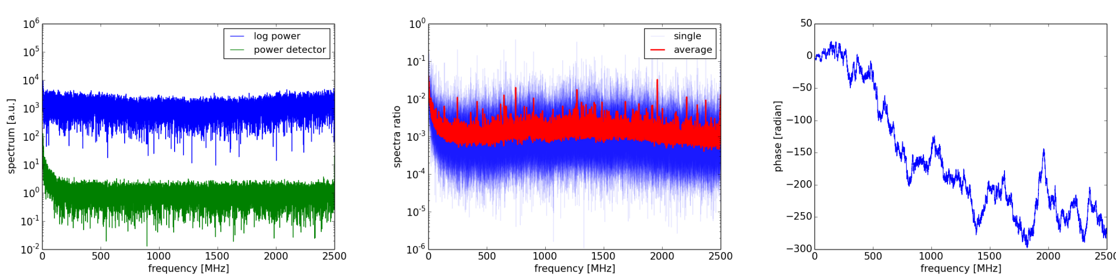


Figure 11 capacitor case (left: spectrum, middle: ratio after/before, right: phase difference)

We show an example of waveform for the no capacitor case and the distribution of the difference in the fig. 13. For this method, the residuals are larger than for the first method.

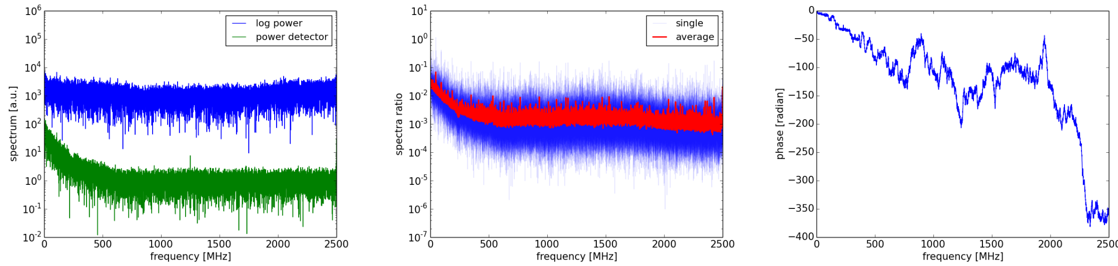


Figure 12 no capacitor case (left: spectrum, middle: ratio after/before, right: phase difference)

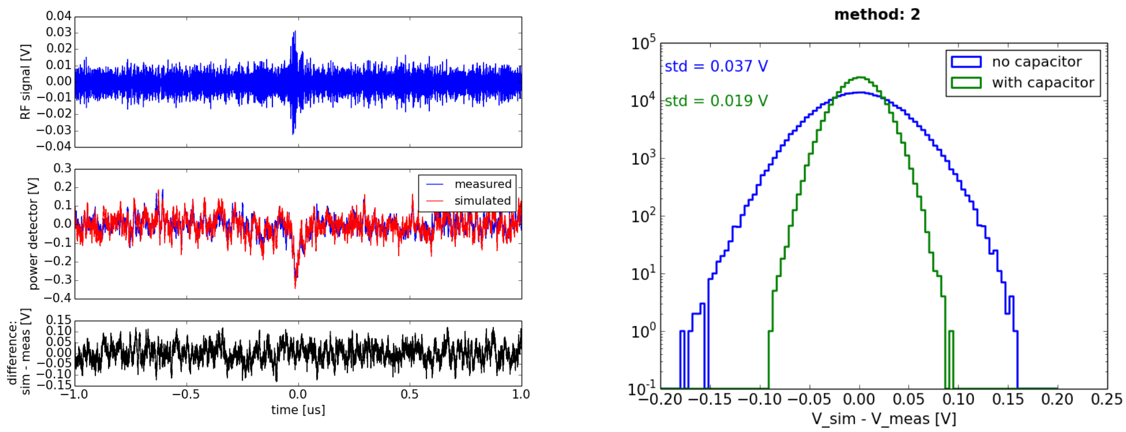
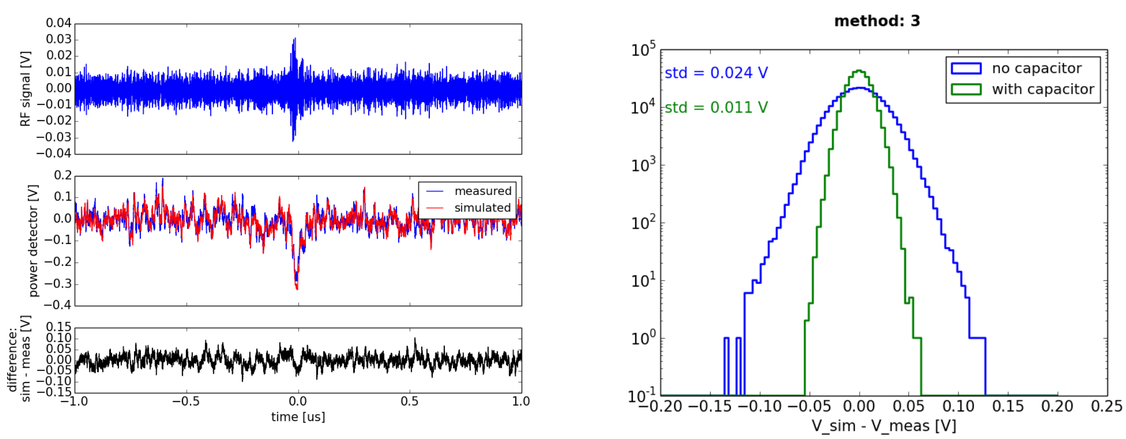


Figure 13

convolution with fixed parameters (method 3) For the first method we had found that depending on the waveform the result for the linear transformation parameters would vary, which is not really meaningful. We implemented a third method which is very similar to the first one except that we fix the DC caracterisitcs and then we fit the best τ. We use the relation found between the average the RF waveform in dBm and the average of the power detector waveform (see fig 9) and produce the convolution the same way as describe before. The decay time constant we find in this case is slightly different, we find $\tau_{\text{capa}} = 41.5$ns and $\tau_{\text{capa}} = 6.3$ns. The results are shown in the fig. 14 The residuals found with this method are as good as for the first method.



RF signal [V]
power detector [V]
measured
simulated
difference:
sim - meas [V]
time [us]
method: 3
std = 0.024 V
std = 0.011 V
no capacitor
with capacitor
V_sim - V_meas [V]

Figure 14

2.2 easier board

This stage is an amplification of the power detector signal in order to choose the dynamic range we want to keep at the final stage. It is performed with an amplifier and adjustable voltage offset. Up to now, this step was simulated with a linear transformation. We will see that we need to account for the dependence in frequency of the amplifier.
To determine the characteristics of this stage, we had set up an experiment with a C-band antenna followed by the power detector and an EASIER board. The signal was recorded after the power detector and after the board (see figure 15).
First we look at the characteristics $< V_{\mathrm{board}} >= f(< V_{\mathrm{PD}} >)$ of the mean value of the waveforms (see figure 16 left). However, when we look at the characteristics for $V_{\mathrm{board}} = f(V_{\mathrm{PD}})$ we obtain a different result (see figure 16 right). That means that the response for the DC component is different from the higher frequencies, or equivalently that the amplifier gain is not flat in frequency.

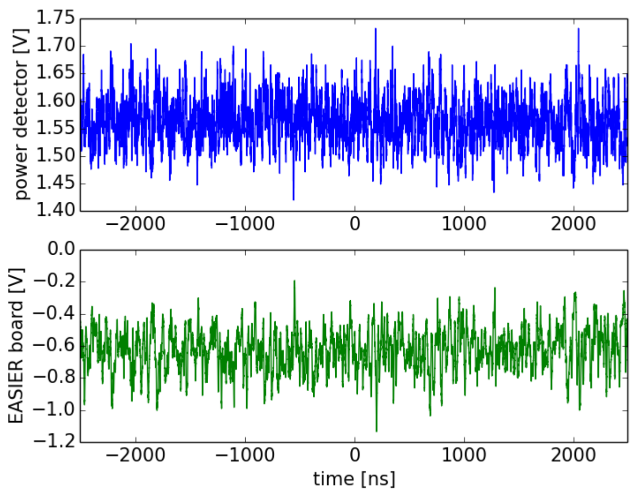


Figure 15 example of waveform after the power detector (top) and aftern the EASIER board (bottom)

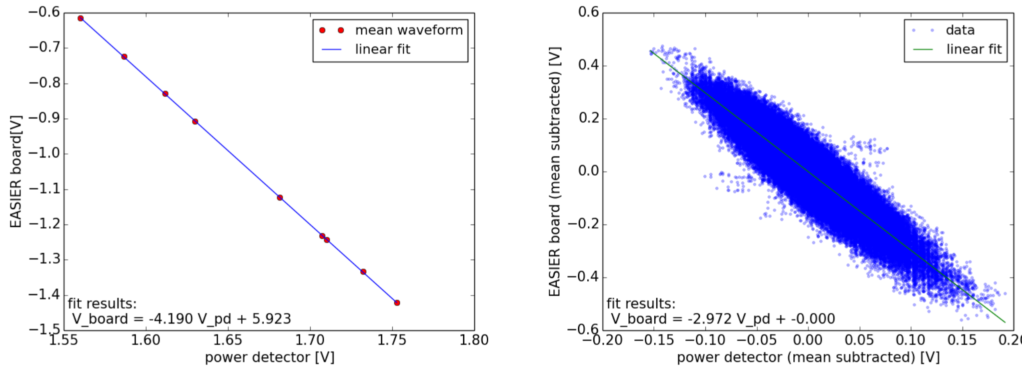


Figure 16 caracteristics of the easier board for the average of the waveform (left) or for all the points when the baseline is removed (right)

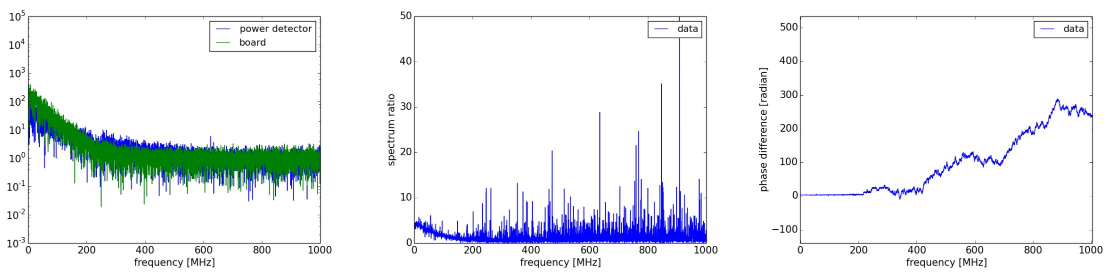


Figure 17 Left: example of spectra of the power detector and the easier board. Middle: the ratio of the spectra on the left. Right: phase difference between the two fft.

We determine the response in frequency of the board taking the ratio of the FFT of the waveforms. An example of the FFT of one waveform is shown in figure 17. We see that it is really noisy, that's because it is FFT of noise ! Most of the power is in the first 200MHz. For the spectrum we could average over multiple spectra, for the phase we encountered difficulties to average it due to phase jumps. So we fit an average spectrum and fit only one phase. Then to obtain the board signal from the power detector the first step is to make the non DC frequencies go through the response we found and then add the correct offset with the $< \mathrm{V_{board}} >= \mathrm{f}(< \mathrm{V_{PD}} >)$ characteristic.
The fitted curve shown in red on figure 18 are:

- spectrum: $\mathrm{s} = \mathrm{a} \cdot \mathrm{e}^{-\frac{(\mathrm{x}-\mu)^2}{2\sigma^2}} + \mathrm{k}$
  with $\mathrm{a} = 3.86$, $\mu = -40$, $\sigma = 75.1$, $\mathrm{k} = 1.0$ and x is the frequency in MHz
- phase: $\mathrm{p} = \mathrm{ax}^2 + \mathrm{bx} + \mathrm{c}$
  with a = 4.8 10^{-5}, b = -1.1 10^{-3}, c = 2.97

Now we can look at the comparison between the old simulation with a constant gain and the new method with a frequency dependent gain. An example of waveforms (measured, simulated with old and new method) is shown in figure 19 left and the difference histogram is given on the right.

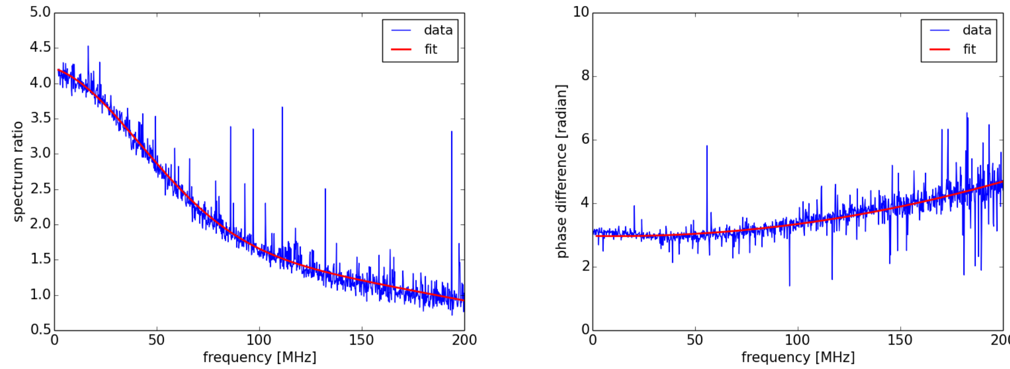


Figure 18 Fit of the board response (spectrum on the left, phase on the left), function and parameters are given in the text

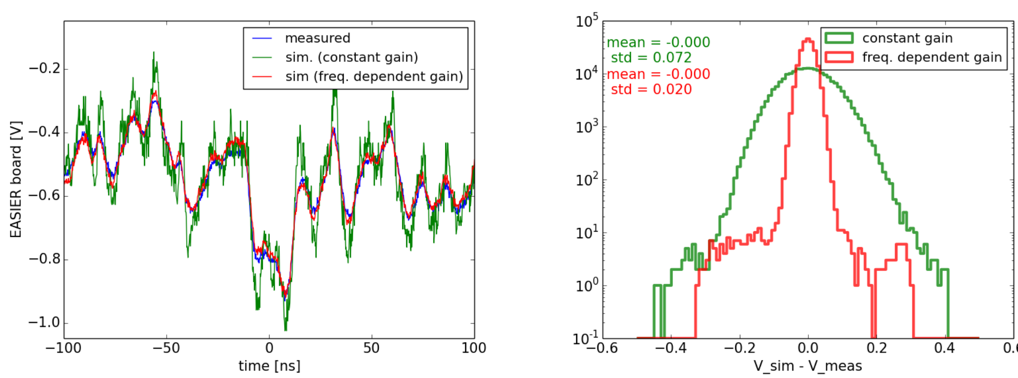


Figure 19 left:waveforms (measured and simulated), right: voltage difference of around 20 waveforms

2.3 SD front end

This is the last piece of the electronics chain. We don't have any data to look at its response. From the Auger technical report it is composed of a low pass filter (f_{cut} = 20 MHz). We simulate it with a butterworth filter of 4th order.

3 Comparison data simulation

Now that we have a good simulation of each step of the electronics chain, we want to test the simulation with the data. One way to do that is by looking at the fluctuations of the radio signal that we measure in the recorded events. So we will carry out a full simulation and compare the distribution of the radio signal in ADC units.

3.1 data

To measure the RMS of the detectors installed in the field, we select events with EASIER stations hit and just histogram the radio waveform in ADC (after removing the baseline). We have picked arbitrarely the month of April 2015 (it is the period when all the arrays are present). We show as an example 3 waveforms, one for each array of the C-band in the figure 25 (left) and the average RMS is shown for all the stations during the chosen month in the figure 25 (right). Most of the EASIER61 detector have their RMS around 50 ADCs and some of them are a little off and some close to zero. For the EASIER7 array the RMS is lower because the capacitor is present after the power detector. Finally all the GIGADUCK detectors (except one) have their RMS between 47 and 50 ADCs.

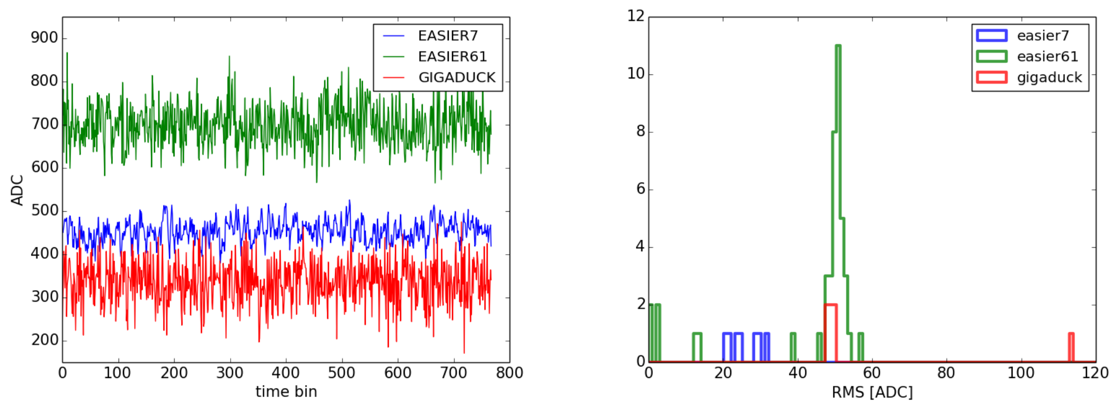


Figure 20 example of trace from each detector in the C-band. right: distribution of the average RMS of all detectors

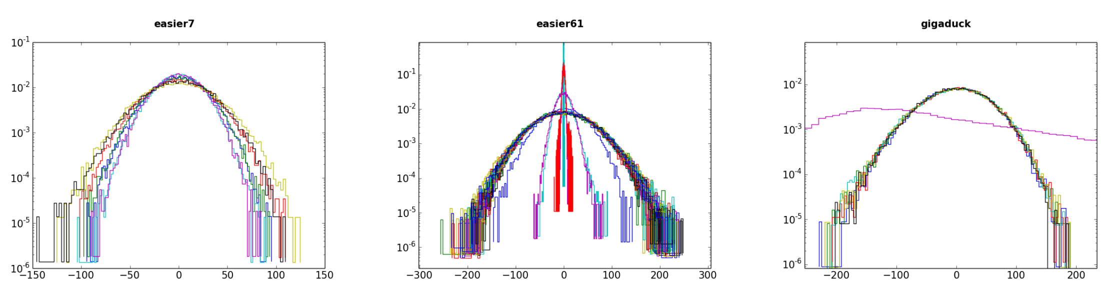


Figure 21 example of trace from each detector in the C-band. right: distribution of the average RMS of all detectors.

3.1.1 simulation

Now we can produce HF waveforms from the measured spectra 1, then make the waveform go through the electronics simulation we described in section 2. We show here the results for the three methods of power detector simulation. Figure 22, 23, 24 show the measured and simulated distribution of amplitude in ADC counts, and figure **??** compares the average RMS (we removed the bad stations).

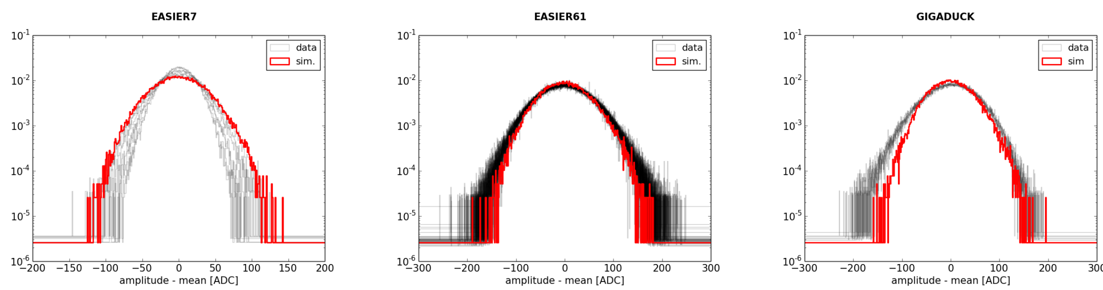


Figure 22 method 1: comparison of the measured distribution of amplitude and the simulated one.

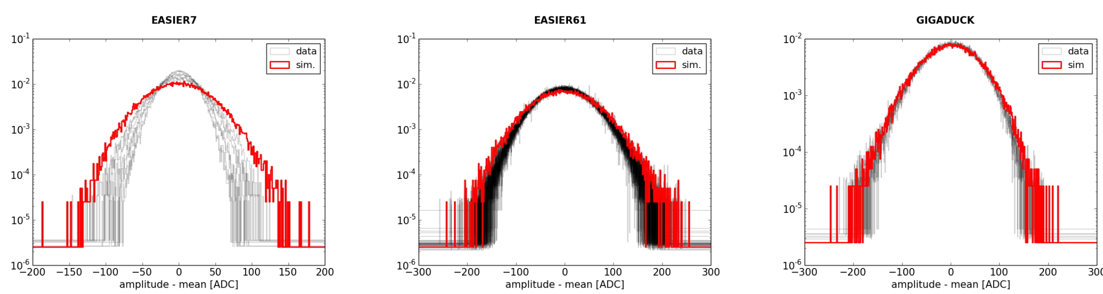


Figure 23 method 2: comparison of the measured distribution of amplitude and the simulated one.

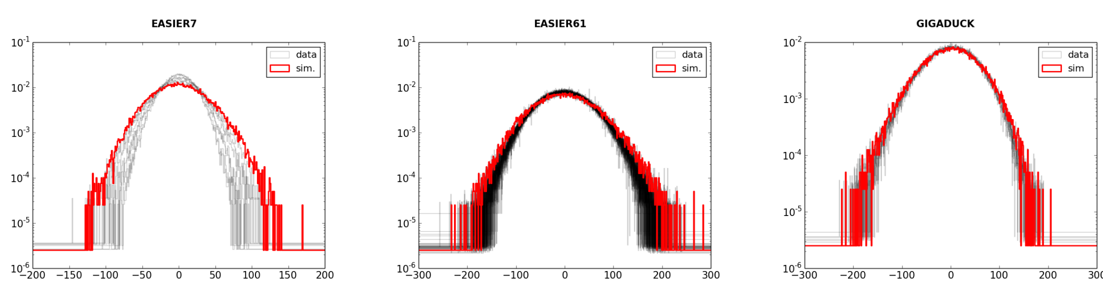


Figure 24 method 3: comparison of the measured distribution of amplitude and the simulated one.

At the end, the three method give very close results, the first one yields a larger RMS for the capacitor case, but a smaller one for the non capacitor case. The two other give consistantly a larger RMS for the three detector configurations. Note that the difference is

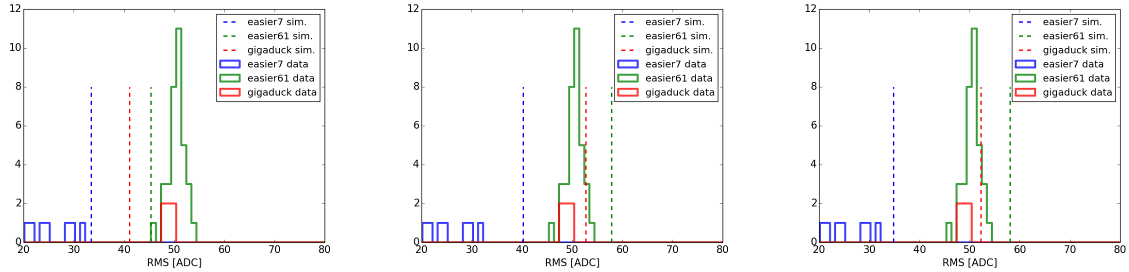


Figure 25 comparison of the measured distribution of amplitude and the simulated one.

quite small, only a few ADC counts. (50 ADC is 1dB and we have at most differences of around 15 ADC i.e. less than 10%) 242

conclusion

We have gone through the different step to simulate an EASIER waveform. In particular, we have detailed the generation of an RF waveform from a realistic spectrum. We also improved the simulation of the adaptation electronics. The full simulation is then compared against data looking at the traces fluctuation in ADC. We find a simulated waveform overestimated the fluctuation by a small number of ADC.