Modeling Classification

HI 743

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What is Classification?

Classification is a predictive modeling task where the **response variable is categorical**.

- Unlike regression, classification predicts discrete labels instead of continuous values.
- Example: Predicting whether a patient has a disease (Yes/No) based on symptoms.
- Applications: Medical diagnosis, fraud detection, spam filtering, etc.

Examples of Classification Problems:

- **Medical Diagnosis:** Does a patient have diabetes? (Yes/No)
- **Fraud Detection:** Is a bank transaction fraudulent? (Fraud/Not Fraud)
- **Genetics:** Classifying a genetic mutation as harmful or benign.
- **Customer Churn:** Will a customer continue using a service? (Stay/Leave)

Example: Default Data Set

Objective: Predict whether an individual will default on a credit card payment.

- Response Variable: `Default` (Yes/No)
- Predictors: Income, Balance, Student Status

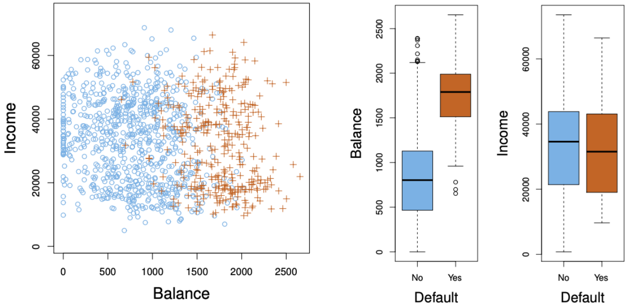


Figure: Credit card default dataset visualization.

Why Not Linear Regression?

- Linear regression assumes a continuous response variable, making it unsuitable for predicting discrete class labels.
- Predictions from a linear model can extend beyond the valid probability range of $[0, 1]$, which is problematic for classification.

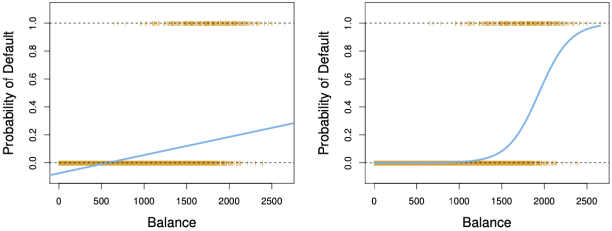


FIGURE 4.2. *Classification using the* `Default` *data.* Left: *Estimated probability of* `default` *using linear regression. Some estimated probabilities are negative! The orange ticks indicate the 0/1 values coded for* `default` *(*`No` *or* `Yes`*).* Right: *Predicted probabilities of* `default` *using logistic regression. All probabilities lie between* 0 *and* 1.

Logistic Regression

Logistic regression models the probability that a given observation belongs to a particular class. The logistic function ensures predictions stay within $[0, 1]$:

$$P(Y = 1|X) = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}} \tag{1}$$

Unlike linear regression, logistic regression predicts probabilities instead of continuous values. $P(Y = 1|X)$ reads as "The probability of Y equal to 1 given X".

Logistic Regression

The model uses the log-odds (logit) transformation to relate predictors to the probability of an event occurring:

$$log\left(\frac{P(Y=1|X)}{1-P(Y=1|X)}\right)=\beta_0+\beta_1 X \tag{2}$$

Maximum likelihood estimation (MLE) is used to estimate the coefficients, rather than least squares. The coefficients indicate how much the *log-odds* of the outcome change with a one-unit increase in the predictor.

Interpreting Logistic Regression

- A positive coefficient β_1 implies an increase in the predictor increases the probability of the event.
- A negative coefficient implies a decrease in probability.
- The odds ratio e^{β_1} represents how the odds change with a one-unit increase in the predictor.
- Example: If $\beta_1 = 0.3$, then $e^{0.3} \approx 1.35$, meaning the odds increase by 35%.

Understanding Odds and Odds Ratios

- **Odds:** The odds of an event occurring is defined as:

$$\text{Odds} = \frac{P(Y=1)}{1 - P(Y=1)} \tag{3}$$

- If $P(Y=1) = 0.75$, then the odds are $\frac{0.75}{0.25} = 3$, meaning the event is three times as likely to occur than not.

Understanding Odds and Odds Ratio

- **Odds Ratio (OR):** Compares the odds of an event occurring for different values of a predictor variable.
- The odds ratio is given by:

$$\mathrm{OR} = \frac{P(Y = 1|X + 1)/(1 - P(Y = 1|X + 1))}{P(Y = 1|X)/(1 - P(Y = 1|X))} \tag{4}$$

- If $OR > 1$, the event is more likely as the predictor increases; if $OR < 1$, the event is less likely.
- Example: If $OR = 2.5$, the event is 2.5 times more likely for each unit increase in the predictor.

Understanding Odds and Odds Ratio

		OUTCOME	
		Disease (Case)	No Disease (Controls)
EXPOSURE	Exposed	a	b
	Unexposed	c	d

$$Odds\ of\ Exposure\ in\ Cases = \frac{Number\ of\ Cases\ with\ Exposure}{Number\ of\ Cases\ without\ Exposure} = \frac{a}{c}$$

$$Odds\ of\ Exposure\ in\ Controls = \frac{Number\ of\ Controls\ with\ Exposure}{Number\ of\ Controls\ without\ Exposure} = \frac{b}{d}$$

$$Odds\ Ratio = \frac{Odds\ of\ Exposure\ in\ Cases}{Odds\ of\ Exposure\ in\ Controls} = \frac{a/c}{b/d} = \frac{a * d}{b * c}$$

Logistic Regression Model Output

Fitted Model:

$$\log\left(\frac{P(Y=1|X)}{1-P(Y=1|X)}\right) = -5.2 + 0.04X \tag{5}$$

Interpretation:

- Intercept (-5.2): Baseline log-odds of diabetes when glucose $= 0$.
- Coefficient (0.04): A one-unit increase in glucose increases log-odds of diabetes by 0.04.
- Normal glucose levels range between $70 - 99$, meaning individuals within this range generally have lower predicted probabilities of diabetes.

Calculating Predicted Probabilities

Example: Patient with Glucose Level = 150

$$\text{Log-Odds} = -5.2 + (0.04 \times 150) = 0.8 \tag{6}$$

$$\text{Probability} = \frac{e^{0.8}}{1 + e^{0.8}} \approx 0.69 \tag{7}$$

Interpretation: This patient has a 69% probability of having diabetes.

- A patient with glucose level $X = 85$ (mid-normal range) would have:

$$\text{Log-Odds} = -5.2 + (0.04 \times 85) = -1.8 \tag{8}$$

$$\text{Probability} = \frac{e^{-1.8}}{1 + e^{-1.8}} \approx 0.14 \tag{9}$$

- Meaning, a patient in the normal glucose range has a much lower probability of diabetes.

Thresholding for Classification

- A threshold (e.g., 0.5) is applied to classify patients.
- If $P(Y = 1|X) > 0.5$, classify as diabetic ($Y = 1$).
- If $P(Y = 1|X) \leq 0.5$, classify as non-diabetic ($Y = 0$).
- Example: With a probability of 0.69, the patient is classified as diabetic.
- Patients within the normal glucose range typically have probabilities well below 0.5, supporting non-diabetic classification.

Classification Model Evaluation - Basic Metrics

Metrics for Evaluating Model Performance:

- **Accuracy:** Measures the proportion of correctly classified cases.

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} \tag{10}$$

- **Precision:** Measures how many predicted positives are actually correct.

$$\text{Precision} = \frac{TP}{TP + FP} \tag{11}$$

- **Recall (Sensitivity):** Measures how many actual positives were correctly identified.

$$\text{Recall} = \frac{TP}{TP + FN} \tag{12}$$

Classification Model Evaluation - Advanced Metrics

Additional Performance Measures:

- **F1-Score:** Harmonic mean of precision and recall, balancing false positives and false negatives.

$$F1 = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \tag{13}$$

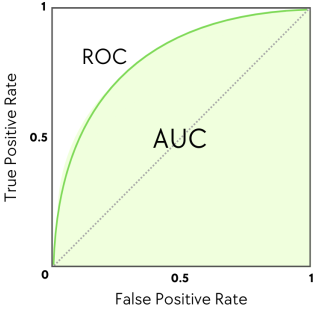
- **Specificity:** Measures the proportion of true negatives correctly identified.

$$\text{Specificity} = \frac{TN}{TN + FP} \tag{14}$$

AUC-ROC Curve for Classification

AUC-ROC Curve: Measures how well the model distinguishes between classes.

- The Area Under the Curve (AUC) quantifies performance:
    - $\text{AUC} = 1$: Perfect classifier.
    - $\text{AUC} = 0.5$: Random guessing.
- The ROC curve plots the true positive rate (recall) against the false positive rate at different threshold values.



Multiple Logistic Regression

Extending Logistic Regression to Multiple Predictors

- In multiple logistic regression, we model the probability of a binary outcome using multiple predictors.
- The model equation is:

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p \tag{15}$$

- This allows us to account for multiple factors simultaneously when predicting an outcome.
- Model evaluation methods are the same as from simple Logistic Regression.

Example: Predicting Credit Default

Using Multiple Predictors:

- Consider a dataset where we predict whether a person defaults on a loan.
- Predictors: Credit balance, income, and student status.
- Fitted model:

$$\log\left(\frac{p(X)}{1-p(X)}\right) = -10.869 + 0.0057 \times \text{Balance} + 0.0030 \times \text{Income} - 0.6468 \times \text{Student} \tag{16}$$

- Interpretation:
  - A one-unit increase in balance increases the log-odds of default.
  - Being a student decreases the probability of default, holding other factors constant.

Interpreting Coefficients

Understanding the Impact of Each Predictor:

- **Odds Ratio:** The exponentiated coefficient e^{β} represents the multiplicative change in the odds.
- Example:

$$e^{0.0057} \approx 1.0057 \tag{17}$$

- This means that for each additional dollar in balance, the odds of default increase by approximately 0.57%.
- Similarly, a student has lower odds of default by a factor of:

$$e^{-0.6468} \approx 0.523 \tag{18}$$

- This means students are about 48% less likely to default than non-students, controlling for other factors.

Multinomial Logistic Regression

Extending Logistic Regression to More than Two Classes (Outcomes)

- When the response variable has more than two categories, multinomial logistic regression is used.
- Unlike binary logistic regression, we model multiple class probabilities simultaneously.
- The model equation for class k is:

$$P(Y = k|X) = \frac{e^{\beta_{k0}+\beta_{k1}X_1+\cdots+\beta_{kp}X_p}}{\sum_{l=1}^{K} e^{\beta_{l0}+\beta_{l1}X_1+\cdots+\beta_{lp}X_p}} \tag{19}$$

Example: Predicting Disease Type

Classifying Patients into One of Three Conditions:

- Response Variable: Y (Disease type: Stroke, Drug Overdose, Epileptic Seizure).
- Predictors: Age, Blood Pressure, Medical History.
- A fitted model:

$$\log\left(\frac{P(Y = \text{Stroke}|X)}{P(Y = \text{Epileptic Seizure}|X)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \tag{20}$$

- Interpretation:
  - A one-unit increase in X_1 (e.g., blood pressure) changes the log-odds of having a stroke relative to an epileptic seizure.
  - Probabilities for all categories sum to one.

Interpreting Coefficients

Understanding the Impact of Each Predictor:

- Coefficients describe log-odds of one category relative to the baseline category.
- The exponentiated coefficient e^{β} represents the change in odds for a one-unit increase in the predictor.
- Example:

$$e^{0.3} \approx 1.35 \tag{21}$$

- This means that for each unit increase in the predictor, the odds of belonging to a given class (e.g., Stroke) relative to the baseline increase by 35%.