This generic model is compatible with the following1 linear model:

$$y_i = \mathbf{x}_i'\boldsymbol{\beta}(\tau) + \epsilon_i(\tau) \text{ for } i = 1, \ldots, n$$

where y_i is the response value, $\mathbf{x}_i$ is the explanatory covariates vector, and $\epsilon_i(\tau) = y_i - Q_{Y|\mathbf{x}_i}(\tau)$ is an unknown error.

L_1 regression, also known as median regression, is a natural extension of the sample median when the response is conditioned on the covariates. In L_1 regression, the least absolute residuals estimate $\hat{\boldsymbol{\beta}}_{LAR}$, referred to as the L_1*-norm estimate*, is obtained as the solution of the following minimization problem:

$$\min_{\boldsymbol{\beta} \in \mathbf{R}^p} \sum_{i=1}^{n} |y_i - \mathbf{x}_i'\boldsymbol{\beta}|$$

More generally, for quantile regression Koenker and Bassett (1978) defined the τ *regression quantile*, $0 < \tau < 1$, as any solution to the following minimization problem:

$$\min_{\boldsymbol{\beta} \in \mathbf{R}^p} \left[\sum_{i \in \{i: y_i \geq \mathbf{x}_i'\boldsymbol{\beta}\}} \tau |y_i - \mathbf{x}_i'\boldsymbol{\beta}| + \sum_{i \in \{i: y_i < \mathbf{x}_i'\boldsymbol{\beta}\}} (1-\tau)|y_i - \mathbf{x}_i'\boldsymbol{\beta}| \right]$$

The solution is denoted as $\hat{\boldsymbol{\beta}}(\tau)$, and the L_1-norm estimate corresponds to $\hat{\boldsymbol{\beta}}(1/2)$. The τ regression quantile is an extension of the τ sample quantile $\hat{\xi}(\tau)$, which can be formulated as the solution of

$$\min_{\xi \in \mathbf{R}} \left[\sum_{i \in \{i: y_i \geq \xi\}} \tau |y_i - \xi| + \sum_{i \in \{i: y_i < \xi\}} (1-\tau)|y_i - \xi| \right]$$

If you specify weights $w_i, i = 1, \ldots, n$, with the WEIGHT statement, weighted quantile regression is carried out by solving

$$\min_{\boldsymbol{\beta}_w \in \mathbf{R}^p} \left[\sum_{i \in \{i: y_i \geq \mathbf{x}_i'\boldsymbol{\beta}_w\}} w_i \tau |y_i - \mathbf{x}_i'\boldsymbol{\beta}_w| + \sum_{i \in \{i: y_i < \mathbf{x}_i'\boldsymbol{\beta}_w\}} w_i (1-\tau)|y_i - \mathbf{x}_i'\boldsymbol{\beta}_w| \right]$$

Weighted regression quantiles $\boldsymbol{\beta}_w$ can be used for L-estimation (Koenker and Zhao