Hierarchy of correlation quantifiers comparable to negativity

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abstract

Quantum systems generally exhibit different kinds of correlations. In order to compare them on equal footing, one uses the so-called distance-based approach where different types of correlations are captured by the distance to different set of states. However, these quantifiers are usually hard to compute as their definition involves optimization aiming to find the closest states within the set. On the other hand, negativity is one of the few computable entanglement monotones, but its comparison with other correlations required further justification. Here we place negativity as part of a family of correlation measures that has a distance-based construction. We introduce a suitable distance, discuss the emerging measures and their applications, and compare them to relative entropy-based correlation quantifiers. This work is a step towards correlation measures that are simultaneously comparable and computable.

partial transpose distance

definition

$$d_T(\rho,\sigma) = \frac{1}{2}\left\|\rho^{T_B} - \sigma^{T_B}\right\|_1,$$

with trace norm $\|A\|_1 = \mathrm{Tr}\,|A|$.

properties

1. metric
2. contractive under PPT operations
3. Helstrom bound with PPT POVM

negativity

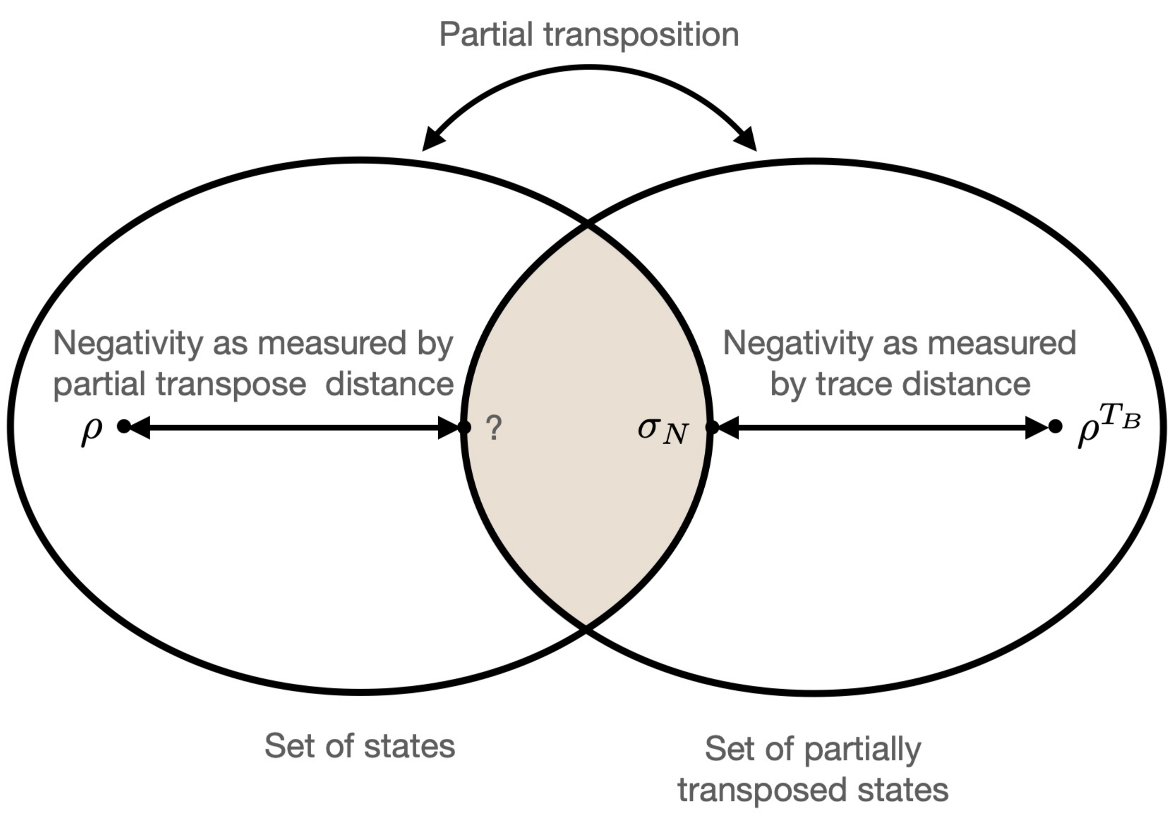
conjecture

$$\inf_{\sigma\in PPT} d_T(\rho,\sigma) = N(\rho)$$

$$N(\rho) = \inf_{\sigma_N\in\mathcal{S}} \frac{1}{2}\left\|\rho^{T_B} - \sigma_N\right\|_1 = \inf_{A\in\mathcal{S}^{T_B}} \frac{1}{2}\left\|\rho^{T_B} - A^{T_B}\right\|_1 \le \inf_{\sigma\in PPT} d_T(\rho,\sigma)$$

relation holds for

1. two qubits
2. Schmidt correlated states $\rho = \Sigma_{ij}\, a_{ij}\,|ii\rangle\langle jj|$
3. two mode Gaussian states
4. states with positive binegativity $|\rho^{T_B}|^{T_B} \ge 0$



structure of correlations

We quantify correlations by taking the distance to uncorrelated states:

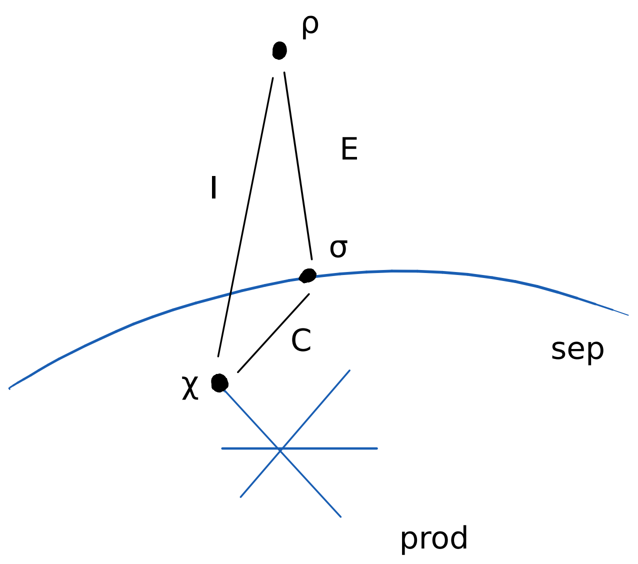
$$Q_X = \inf_{\sigma\in X} d_T(\rho,\sigma)$$

theorem

Let ψ be a pure quantum state. Then $Q_{PPT}(\psi) = Q_{CC}(\psi) = N(\psi)$.

theorem

Let $\sigma = \Sigma_i\, p_i\,|ii\rangle\langle ii|$ be a classically-correlated state, with p_i in non-increasing order. Let $m = \max\left\{n \mid \Sigma_{i<n}\sqrt{p_i} \le 1\right\}$. Then $Q_{Prod}(\sigma) = 1 - \Sigma_{i<m}\,p_i - \left(1 - \Sigma_{i<m}\sqrt{p_i}\right)^2$ and the closest product state is given by $\chi = \left(\Sigma_{i<m}\sqrt{p_i}\,|i\rangle\langle i| + \left(1 - \Sigma_{i<m}\sqrt{p_i}\right)|m\rangle\langle m|\right)^{\otimes 2}$.



$$E(\rho) = \inf_{\sigma\in SEP} d(\rho,\sigma)$$
$$C(\rho) = \inf_{\chi\in PROD} d(\sigma,\chi)$$
$$I(\rho) = \inf_{\chi\in PROD} d(\rho,\chi)$$

For relative entropy, $I(\psi) = E(\psi) + C(\psi)$.

theorem

For partial transpose distance, $I(\psi) \le E(\psi) + \frac{1}{2}C(\psi)$.

conclusion

Quantum mechanical correlations come in many flavours and ideally we would like to have a framework in which they all could be efficiently computed and meaningfully compared. In this spirit, we introduced partial transpose distance and explored its connection to negativity — a computable entanglement monotone. For a broad class of states we proved that negativity is equal to the partial transpose distance to the set of PPT states and we conjecture this relation holds in general. We then defined other types of correlations on equal footing to negativity (measured by the same distance) and derived their closed forms for selected classes of states. These findings allowed us to show a subadditivity relation where total correlations according to the partial transpose distance are less than quantum correlations summed with one half of classical correlations. The paper contains a number of open problems that we hope will stimulate further development of comparable and computable correlation quantifiers.

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