Catalytic and asymptotic equivalence for quantum entanglement

Can your bank help you to distill more entanglement?
arxiv:2305.03488

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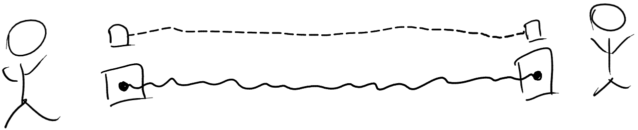
Joint work with Tulja Varun Kondra and Alexander Streltsov

Entanglement



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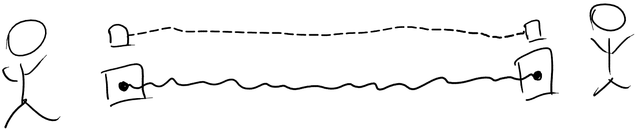

LOCC: $\Lambda(\rho) = \sum_{ij} (A_i \otimes B_{ij})\rho(A_i \otimes B_{ij})^\dagger$
with $\sum_i A_i^\dagger A_i = \mathbb{I}, \sum_j B_{ij}^\dagger B_{ij} = \mathbb{I}$

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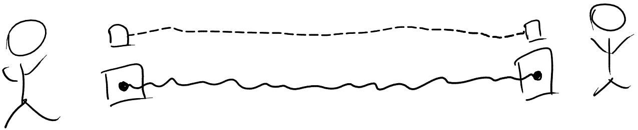

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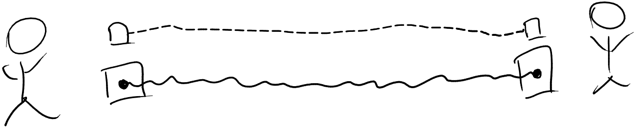

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Entanglement is useful

Entanglement distillation

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Distillable state: $E_D(\rho) > 0$

Bound entangled state: ρ entangled but $E_D(\rho) = 0$

Why limit ourselves to LOCC?

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What if we can “borrow” entanglement?

Catalysis

definition (exact)

$\rho \xrightarrow{ec} \sigma$ if there exists a catalyst state τ and an LOCC protocol Λ s.t. $\Lambda(\rho \otimes \tau) = \sigma \otimes \tau$

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Jonathan, Plenio (PRL 1999): yes

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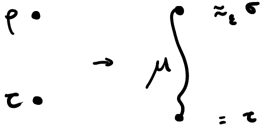

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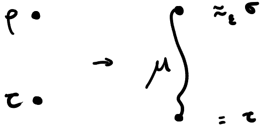

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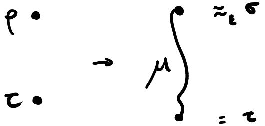

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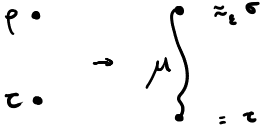

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  Catalytic-asymptotic equivalence (for bipartite pure states)!
- Duan construction can be extended to mixed states

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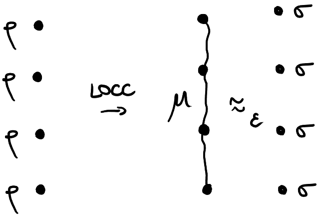
This work: exactly as powerful as asymptotics**

Marginal asymptotics



standard

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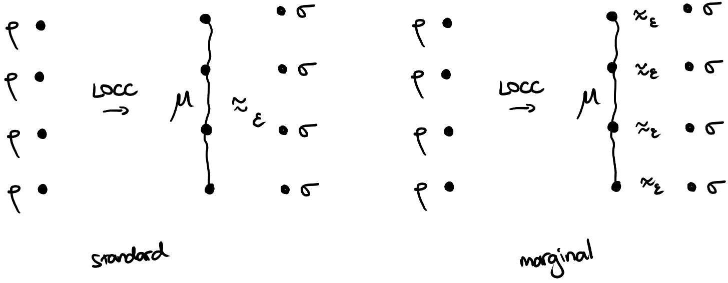

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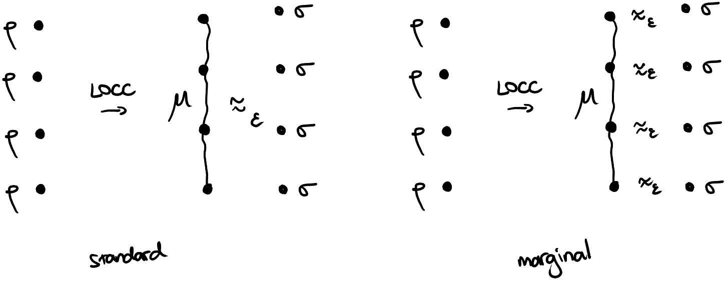
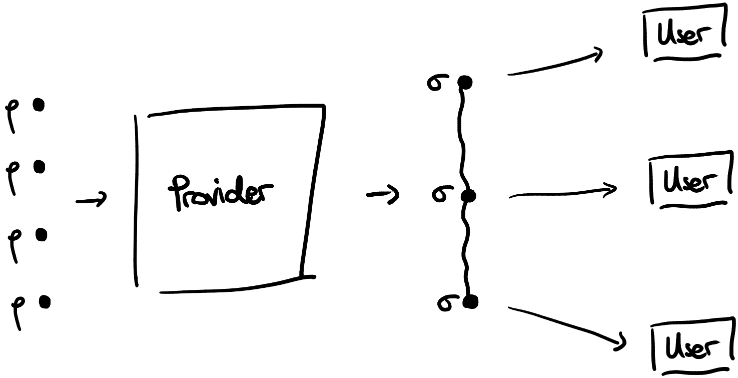


For bipartite pure states, $\psi \xrightarrow{a} \phi$ iff. $\psi \xrightarrow{ma} \phi$

Marginal asymptotics



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Also works in the multipartite setting

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Lami (arxiv 2023): There exist bound entangled states that cannot be distilled under catalysis

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