Coherence manipulation in asymmetry and thermodynamics

arxiv:2308.12814

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2023.12.14

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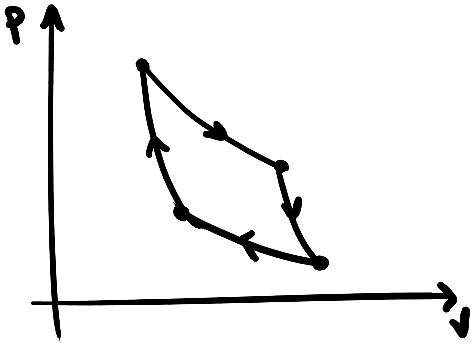
Joint work with Tulja Varun Kondra and Alexander Streltsov

What is thermodynamics?



study of state transformations

Resource theory of thermodynamics

states: (ρ, H)

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free state: Gibbs state

free operations: thermal operations

$$\Lambda(\rho_S) = \mathrm{Tr}_E \left[U(\rho_S \otimes \gamma_E) U^\dagger \right]$$

with $[U, H_S + H_E] = 0$

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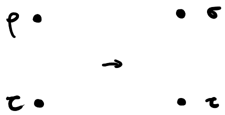
state transformations under thermal operations? hard

Catalytic transformations

definition (exact catalysis)

$\rho \underset{ec}{\longrightarrow} \sigma$ if $\exists \tau, \Lambda$ s.t. $\Lambda(\rho \otimes \tau) = \sigma \otimes \tau$



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$$\begin{array}{ccc} \rho \bullet & & \bullet\ \sigma \\ & \to & \\ \tau \bullet & & \bullet\ \tau \end{array}$$

(Brandão, PNAS 2015) characterization

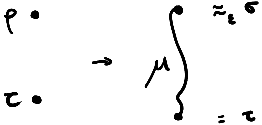
For ρ, σ incoherent, $\rho \underset{ec}{\longrightarrow} \sigma$ iff. $S_\alpha(\rho \| \gamma) \geq S_\alpha(\sigma \| \gamma)$

Correlated catalysis

definition (correlated catalysis)

$\rho \underset{cc}{\longrightarrow} \sigma$ if $\forall \epsilon > 0$, $\exists \tau, \Lambda$ s.t.

$$\mu_{SC} = \Lambda(\rho_S \otimes \tau_C)$$
$$\|\mu_S - \sigma\|_1 \leq \epsilon,\ \mu_C = \tau$$



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$$\begin{matrix} \rho\ \bullet \\ \\ \tau\ \bullet \end{matrix} \quad \rightarrow \quad \mu \left\{ \begin{matrix} \bullet & \approx_\epsilon \sigma \\ \\ \bullet & = \tau \end{matrix} \right.$$

(Müller, PRX 2018) characterization

For ρ, σ incoherent states, $\rho \underset{cc}{\longrightarrow} \sigma$ iff. $S(\rho\|\gamma) \geq S(\sigma\|\gamma)$

What about transformations with coherence?

Covariant operations

Thermal operations

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with $[U, H_S + H_E] = 0$

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(Lostaglio, Nat Comms 2015): they are covariant!

$$\Lambda\left(e^{-iH_S t} \rho e^{iH_S t}\right) = e^{-iH_S t} \Lambda(\rho) e^{iH_S t}$$

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Covariant operations

$$\Lambda(\rho) = \mathrm{Tr}_E\left[U(\rho_S \otimes \sigma_E)U^\dagger\right]$$

with $[U, H_S + H_E] = 0$ and σ incoherent

Asymmetry

symmetry: $\{U_g \,|\, g \in \mathcal{G}\}$

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symmetry: e^{-iHt}

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$$\sigma = \sum_i p_i |i\rangle\langle i|$$

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captures coherence limitations in thermodynamics

Result

definition (available/reachable coherence)

$$\mathcal{I}(\rho) = \{\Delta_{ij} = E_i - E_j | \langle i| \rho |j\rangle\}$$

$$\mathcal{J}(\rho) = \bigcup_n \mathcal{I}(\rho^{\otimes n}) = \left\{ \sum_{\Delta_{ij} \in \mathcal{I}(\rho)} m_{ij}\Delta_{ij} \right\}$$

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consequences

- coherence amplification
- zero or infinite distillation rate

Proof sketch

What is known (Takagi, PRL 2022)

1. Given enough copies of $|+_{ij}\rangle$ for all $\Delta_{ij} \in \mathcal{I}(\rho)$, we can create ρ by covariant operations

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Just need to show that we can merge this sequence into one

Proof sketch

What we showed

We can merge the sequence into one big catalytic transformation

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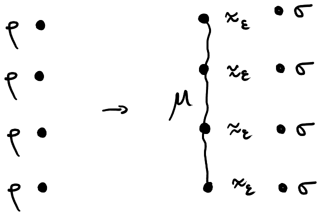
tool: marginal reducibility



talk more in entanglement session

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- marginal reducibility is transitive

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2. There exists sequence of correlated catalytic transformations that create arbitrarily many copies of $|+_{ij}\rangle$ if $\Delta_{ij} \in \mathcal{J}(\rho)$

We can merge this sequence into one big catalytic transformation!

Back to thermodynamics

conjecture

$$\rho \underset{cc}{\longrightarrow} \sigma \text{ iff. } S(\rho\|\gamma) \geq S(\sigma\|\gamma) \text{ and } \mathcal{J}(\rho) \supseteq \mathcal{J}(\sigma)$$

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- For Gibbs-preserving operations, $\rho \underset{cc}{\longrightarrow} \sigma$ iff. $S(\rho\|\gamma) \geq S(\sigma\|\gamma)$ (Shiraishi, PRL 2021)
- For covariant operations, $\rho \underset{cc}{\longrightarrow} \sigma$ iff. $\mathcal{J}(\rho) \supseteq \mathcal{J}(\sigma)$
- Thermal operations = Gibbs-preserving operations $\cap$ covariant operations

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Thanks!

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