Second Law of Entanglement Manipulation with Entanglement Battery

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Second law and reversibility

Second law of thermodynamics

$\Delta S \geq 0$

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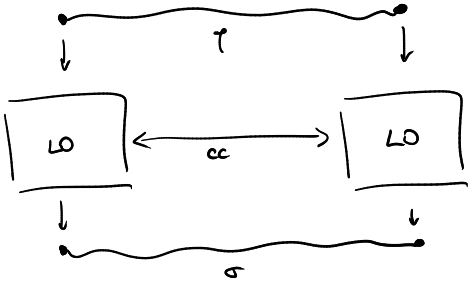
Carnot's theorem

An engine runs at the optimal efficiency iff. it is reversible

Entanglement



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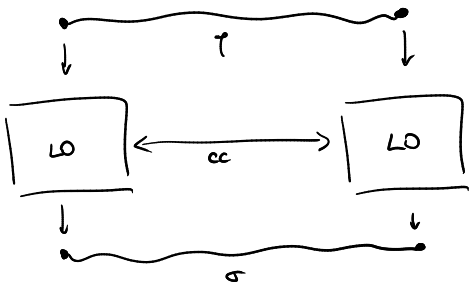

1-way LOCC

$\Lambda(\rho) = \sum_{ij} (A_i \otimes B_{ij})\rho(A_i \otimes B_{ij})^\dagger$ with $\sum_i A_i^\dagger A_i = \mathbf{1} = \sum_j B_{ij}^\dagger B_{ij}$.

Separable states

$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$.

Reversibility

Asymptotic transformation rate

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Is entanglement reversible?

No, because of bound entanglement [Horodecki Phys Lett A 1997, Horodecki[3] PRL 1998, Vidal Cirac PRL 2001]

$R(\rho \to \Phi) = 0$, but $R(\Phi \to \rho) < \infty$

Can we make entanglement reversible?

What is known

- Brandão-Plenio connected this to hypothesis testing through generalized Stein's lemma [Brandão Plenio Nat Phys 2008, Brandão Plenio CMP 2010, Berta et al Quantum 2023]

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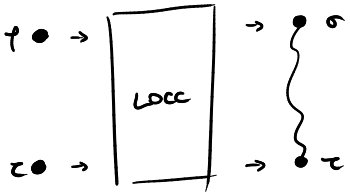
Any physical, non-probabilistic setting?

Clue: catalysis



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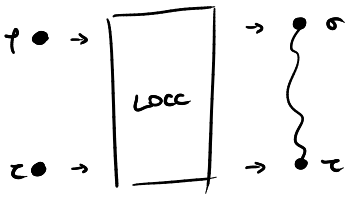

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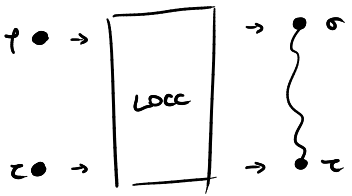

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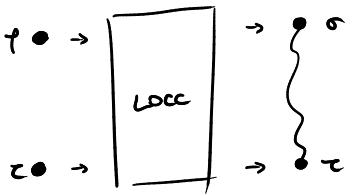

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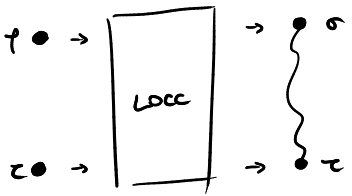

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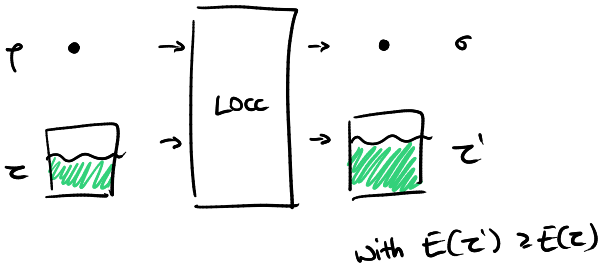
Is it reversible?

No. PPT bound entangled states are still bound entangled [Lami et al PRA 2024]

Main idea: battery



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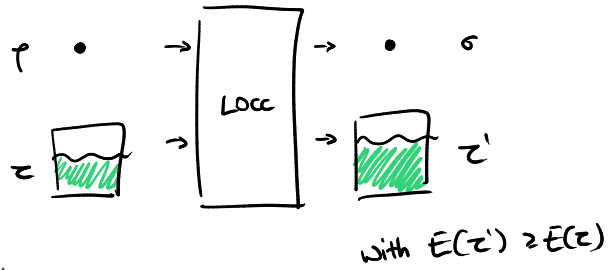

If E is additive, then

$$E(\rho) + E(\tau) = E(\rho \otimes \tau) \geq E(\sigma \otimes \tau') = E(\sigma) + E(\tau'),$$

so $E(\rho) \geq E(\sigma)$ if E is finite

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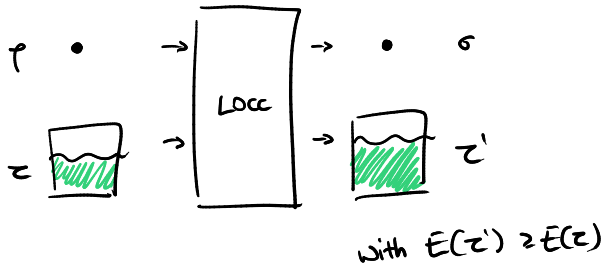

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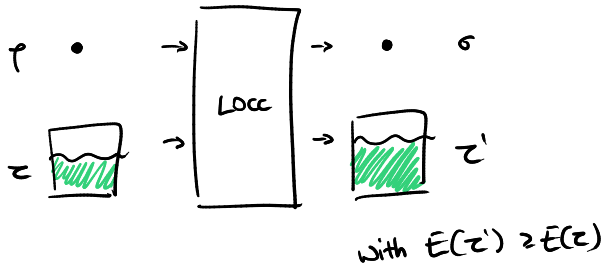

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Squashed entanglement works [Christandl Winter JMP 2004, Li Winter CMP 2014, Alicki Fannes J Phys A 2004]

Results

Theorem 1 (single-copy)

Choose E as a finite and additive entanglement measure.

Then, ρ can be transformed to σ with a battery iff. $E(\rho) \geq E(\sigma)$

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Choose E as a finite and additive entanglement measure.

Then, ρ can be transformed to σ with a battery iff. $E(\rho) \geq E(\sigma)$

Proof:

($\Rightarrow$) use properties of E

($\Leftarrow$) prepare σ in the battery, then swap by LOCC

Theorem 2 (reversibility)

Choose E as a finite, additive, and asymptotically continuous entanglement measure.

Then

$$R(\rho \to \sigma) = \frac{E(\rho)}{E(\sigma)}$$

Problem

How to get a second law in quantum thermodynamics, in a single shot setting?

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Theorem 3

ρ can be transformed to σ with a battery iff. $F(\rho) \geq F(\sigma)$, where $F(\rho) = k_B T(S(\rho\|\gamma) - \log Z)$

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- Does it prove generalized quantum Stein's lemma?
  No, but reversibility is a separate question
  Retracted proof [Yamasaki Kuroiwa arxiv 2024]
  Two more recent proofs [Hayashi Yamasaki arxiv 2024, Lami arxiv 2024]

Closing notes

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Thank you!
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