Lab 2 - Digitolling Time

Xin Gao

1 Abstract

We take input signals and digitally down convert them into low bandpass filtered outputs. A given input signal is sampled in an analog-to-digital converter to digitize it and then mixed with an oscillation signal to get a desired sideband, whose waveform is filtered in an form-malleable FIR filter to create a passband of allowed low passbands. Such process is of great convenience to radio astronomy for analyzing radio signals.

2 Introduction

Whereas an analog signal sends information in the original, continuous waveform, a digital signal sends information at discrete ranges. Many electronic devices today, including computer processors, use digitized signals because these signals provide exact values for the range at specific times or frequencies. Waveforms in time domain is converted to data in frequency domain via fourier transformation. In context of radio astronomy, the goal is to process the received signals, which is accomplished by a digital-down converter. Input signals can be naturally high in frequency; for easier processing, a digital-down converter takes a digitized intermediate-frequency signal, which results from a mixer using a 'heterodyning' technique and filters out all but a low frequency band 'centered' at 0, ultimately to allow our processor to output the desired information relatively quickly.

3 Methods and Procedure

Our input to be bandpass-filtered for down conversion (in a FIR filter - to be discussed later) is a digitally mixed signal. We begin our procedure with examining the process of sampling, which is digitizing an analog signal and is a process gated by criteria. Next we would examine the results of mixing signals in a FPGA (discussed later). Finally, we would design the workings of our FIR filter to put it all together for down converting an input signal mixed with our local 'clock oscillator'.

3.1 Sampling and the Nyquist Criterion

Sampled data can be interpreted in multiple ways. Of interest to us are their waveform, especially in relation to Nyquist's Criterion (see section below), and their power spectra, which is very useful for analyzing the multiple frequency makeup for an input signal. Power Spectrum can be derived using Fourier transformation.

3.1.1 Waveform

When sampling, we take the continuous, analog input signal and measure its values at discrete time intervals to produce its digitized version. The frequency ν_s at which we sample must be high enough in order for the signal to be replicated accurately. For this examination we use **20 MHz** as ths sample frequency. The maximum signal frequency $\nu_{sig} = \nu_{max}$ that can be sampled correctly is known as the Nyquist frequency and obey the following criterion, known as the Nyquist's Criterion:

$$\nu_s = 2\nu_{max} \tag{1}$$

Plot 1 of Figure 2 shows sampled data at Nyquist frequency. Frequencies ν_{sig} above the Nyquist frequencies do not get sampled properly but gets 'aliased' down to corresponding frequencies. Aliasing is an effect when certain reconstructed signals from sampling become indistinguishable from one another. Because ν_s is not high enough to accurately reconstruct a signal at, for example, $0.8\nu_s$, the reconstructed signal will be similar to that of a lower frequency. It turns out that aliased signals are symmetric about ν_{max}; that is, the same frequency difference from ν_{max} (up to ν_s) will produce similar sampled data. For example, the sampled $0.8\nu_s$ data would look like the sampled 0.2 ν_s. Mathematically, for the purpose of sampling, it would not be a stretch to say:

$$\nu_{max} + \Delta\nu \approx \nu_{max} - \Delta\nu, \Delta\nu < \nu_{max} \tag{2}$$

Figure 1 shows the result of sampling at different frequences below ν_s not at ν_{max}. Setting $\nu_{max} = \nu_s$ and sampling would produce unfruitful results since the signal would get sampled once per wavelength, creating a 'direct current' looking output. Plot 2 of Figure 2 shows this result. Similar nonsensical results are obtained when $\nu_s << \nu_{max}$ (see Figure 3).

3.1.2 Power Spectrum

The waveform we obtain from the signals is a time domain; that is, the value (voltage in our case) at each point is a function of time. We fourier transform this function, as defined by:

$$\hat{f}(\omega) = \int f(t)e^{-i\omega t}dt \tag{3}$$

Here ω is frequency. Taking the fourier transform of a time domain function outputs a frequency domain function. Since our single input signal is of primarily one frequency value, we expect $f(\omega)$ to contain a delta function, spiking at the input frequency. There are actually two spikes at positive and negative ν_{sig}. This occurence arises from the complex nature of taking the fourier transform. This makes sense mathematically taking into account the following identities:

$$e^{-i\omega t} = cos(-i\omega t) + isin(-i\omega t) \tag{4}$$

$$cos(-i\omega t) = \frac{e^{-i\omega t} + e^{i\omega t}}{2} \tag{5}$$

$$isin(-i\omega t) = \frac{e^{-i\omega t} - e^{i\omega t}}{2} \tag{6}$$

Since cosine is an even function, $cos(i\omega t = cos(-i\omega t)$; the transformation due to the complex cosine component mirrors the data at 0 frequency and spawns the negative frequencies. The plots of Figure 4 show this phenomenon. These plots are called the power spectra of the data from Figure 1. A power spectrum P can be obtained by taking the square of the absolute value of the fourier transformed values:

$$S = |\hat{f}(\omega)|^2 \tag{7}$$

The power spectrum is power as a function of frequency; it describes the amount of energy or power a given frequency has in our signal. As shown in Figure 6 and plot 2 of Figure 5, ν_{sig} at or greater than or equal to ν_s produce 0-valued power spectra, which are physically unimportant. Plot 1 of Figure 5 shows the power spectrum for ν_{max}. It peaks at its expected frequency, but at and only at the negative side, interestingly.

3.2 Mixing Frequencies - Heterodyning

Heterodyning is the process of combining two input signals to form output signals. Typically, inputs of ν_1 and ν_2 will produce both sum and difference outputs $\nu_1 \pm \nu_2$. Signals mix by an operation called convolution, or multiplication in the fourier-transformed domain:

$$f(t)g(t) = f(t) * g(t) = \int \hat{f}\omega\hat{g}\omega e^{i\omega t} d\omega \tag{8}$$

Convolution is essentially the sliding of a signal over another one.The two signals can be convolve analytically in an analog mixer such as the ZAD-1 mini circuit or digitally in the ROACH (Reconfigurable Open Architecture for Computer Hardware), which includes a FPGA (Field Programmable Gate Array), an analog-to-digital converter (in the FPGA), and a processor.

3.2.1 Mixing in the ZAD-1 and Fourier FIltering

We start our mixing experiments with analog mixing using the ZAD-1 mini circuit, which takes two input signals from the oscillators and outputs the data as array into our programs. For the frequencies of the two signals, we use $\nu_{lo} = 1MHz and \nu_{sig} = \nu_{lo} \pm .05\nu_{lo}$. For the sake of sampling the Nyquist frequency, the sample frequency chosen for each case is different. For $\nu_{sig} = \nu_{lo} - .05\nu_{lo}$, $\nu_{sum} = 1.95MHz$ so the sample frequency is **3.9 MHz**. For $\nu_{sig} = \nu_{lo} + .05\nu_{lo}$, $\nu_{sum} = 2.05MHz$ so the sample frequency is **4.1 MHz**. We use these values for our signals and then take the power spectra of the output data from the mixer. The power spectra for these two cases are plotted in Figure 7. Now, let's say, we would like only one sideband so we can see the waveform due to only one real peak value for the frequency; we would have to filter out the other one. This can be accomplished by invoking the inverse fourier transform:

$$f(t) = \frac{1}{2\pi} \int f(\omega) e^{i\omega t} d\omega \tag{9}$$

For relevance to radio astronomy, let's say we would like to keep the lower sideband. We use one of our mixer outputs as the signal we'd like to filter. Let's use $\nu_{sum} = 1.95MHz$. Its waveform and fourier transformed (not its power spectrum) data values are plotted in Figure 8. We see four peaks as expected and we want to filter out all frequencies values except those centered at the immediate positive and negative lower sideband peaks. Since the fourier transform is a complex operator, our data is a two-dimensional array so we must set both the real and imaginary values to zero using our programs. After this is accomplished, we finally take the inverse fourier transformed data, which is plotted in Figure 9.

3.2.2 Mixing in the ROACH and the SSB

Whereas the previous experiments deals with sampling an mixed analog output, this experiment deals with mixing digitized signals. We compare by sending similar signals to the ones used for the analog mixer to the analog-to-digital converter in the ROACH. The ROACH is connected with a oscillator 'clock' with an internal sample frequency of $\nu_{clock} = 200$ MHz. For this part we use 10 MHz for ν_{lo} and 9.5 MHz for ν_{sig}. The output data is stored in a BRAM (Block Random Access Memory) file in the FPGA. We copy the data to our program and then mercilessly murder the file from the FPGA so other processess can start running on it. The power spectrum of the digitally mixed output is plotted in Figure 10; it is similar to the result achieved from the analog mixing experiment. So far, our power spectra are all double sidebanded (DSB); that is, they contain all four peaks of the upper and lower sidebands. It is possible, however, to obtain a single sidebanded power spectrum without using filtering methods. We begin another mixing process with a signal of $\nu_{sig} = 1MHz$ and an oscillation from the clock. We use $\nu_{lo} = \nu_{clock} * (64/256)$ as our frequency modulation. It is important to have a factor that is a multiple of two in the numerator 2π is divided into 256 parts for each wavelength; if not for a factor of two, our oscillator signal would be inaccurate and we would get a direct current as the output. The sine and cosine components of ν_{lo} are each mixed with ν_{sig}. The power spectra of these two cases are plotted in the first two plots of Figure 11. Finally, we add the components together (by adding the array values in our program) and the power spectrum result is shown in plot 3 of Figure 11. Unlike all the other spectra, there are only two peaks! This is a single sideband power spectrum.

3.3 FIR Filter

Our ultimate goal is to experiment with digital down conversion. We want our final output to be a low bandpass filtered equivalent of our input mixed signal. Our filter, the finite impulse response filter, ideally, creates a passband centered at 0 and 'abruptly' ends at a frequency cutoff, above which no frequency is outputted. For this experiment we would like to create a 5/8 bandpass filter; that is only 5/8 of our input frequency, centered at 0, gets through. Let's take our input to run from -10 MHz to 10 MHz. For a 5/8 filtered result, we want the output to run from -6.25 to 62.5 MHz. This would ideally appear as a rectangular graph. To do so, we assign coefficients, which determines what the job of the filter is and, to the filter. Since multiplying in frequency domain is equivalent to convolving in time domain, to get our filtered and convolved output signal, we simply multiply, in frequency domain, our passband values by 1 and our stopband values by 0. We first segment out frequency domain input into 8 parts. We operate on the FPGA and set the coefficients that corresponds to each segment. Since the counter starts at 0 frequency towards the positive end and then jump to the negative end, the values chosen for those segments are are shown in Table 1. To get the time domain coefficients, we use the inverse fourier transform on our data.

4 Data and Results

Here are the plots from our experiments.

4.1 Nyquist Criterion - Sampling Examples

Figure 1: Selected Sampled Signals for $f_s = 20MHz$

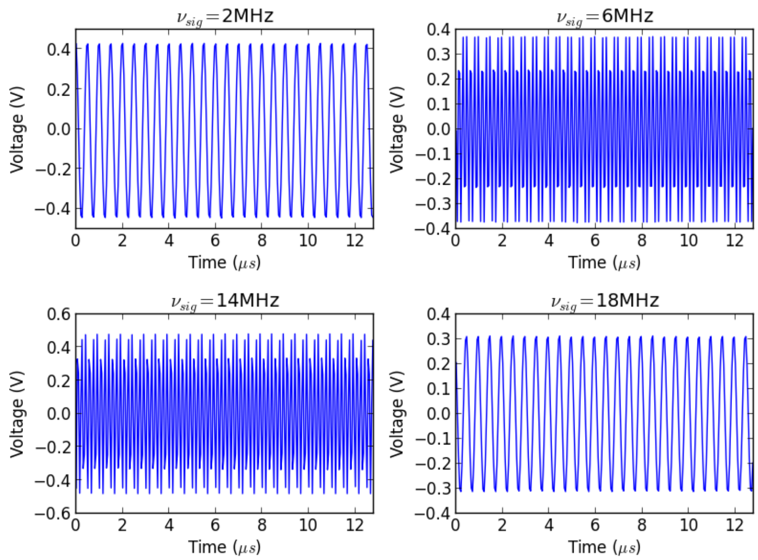


Figure 2: Sampling at Nyquist Frequency and Sampling Frequency

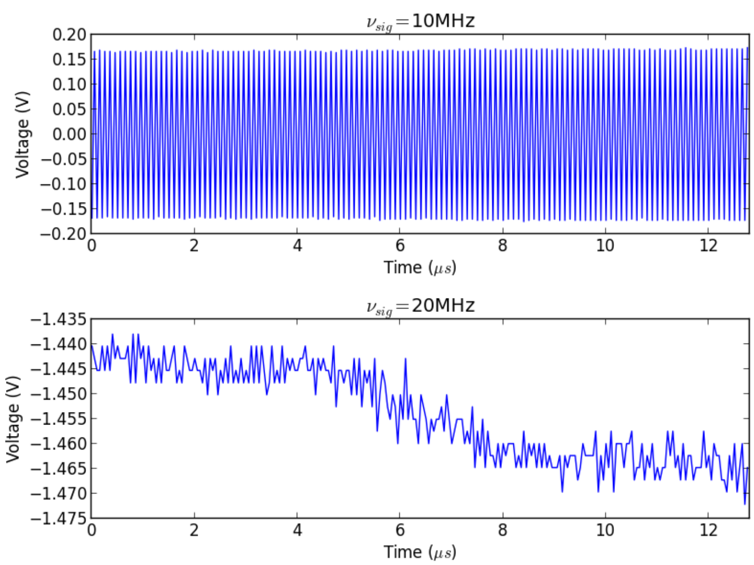


Figure 3: The Nyquist Criterion Ignored

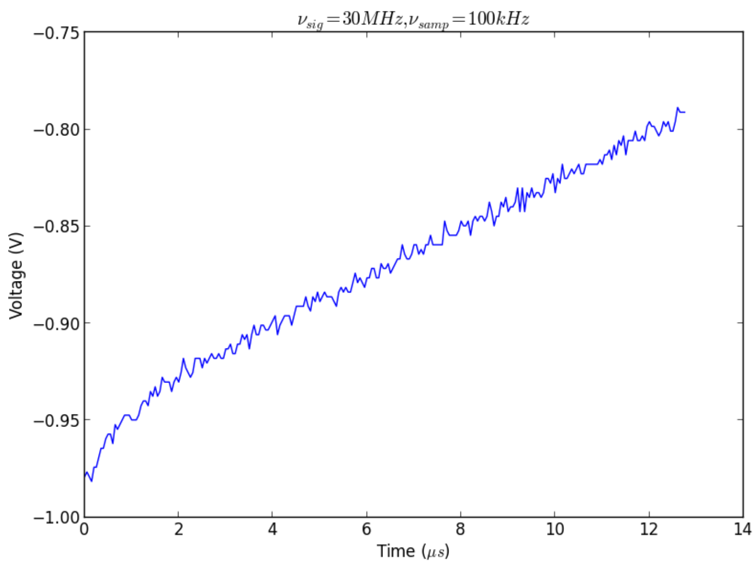


Figure 4: Power Spectra Corresponding to Data from Figure 1

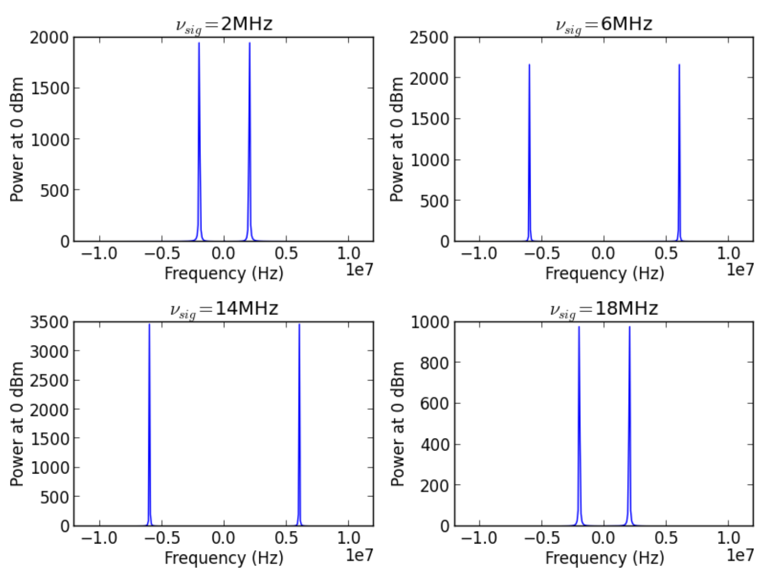


Figure 5: Power Spectra Corresponding to Data from Figure 2

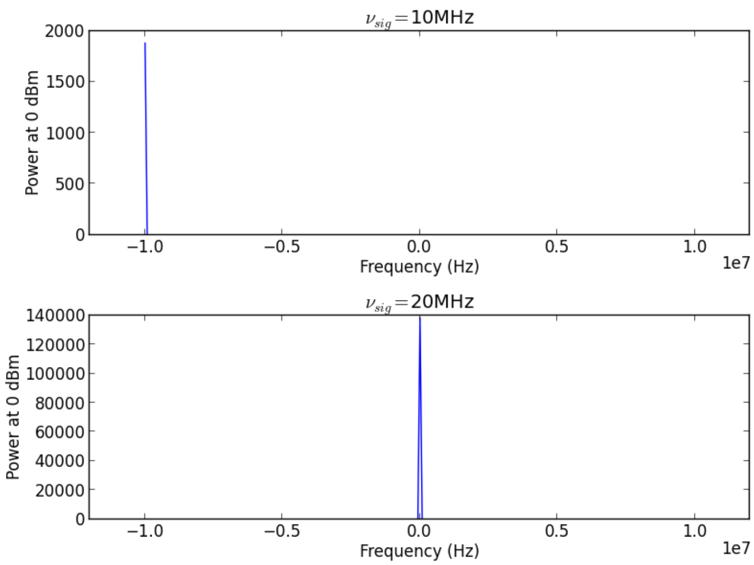
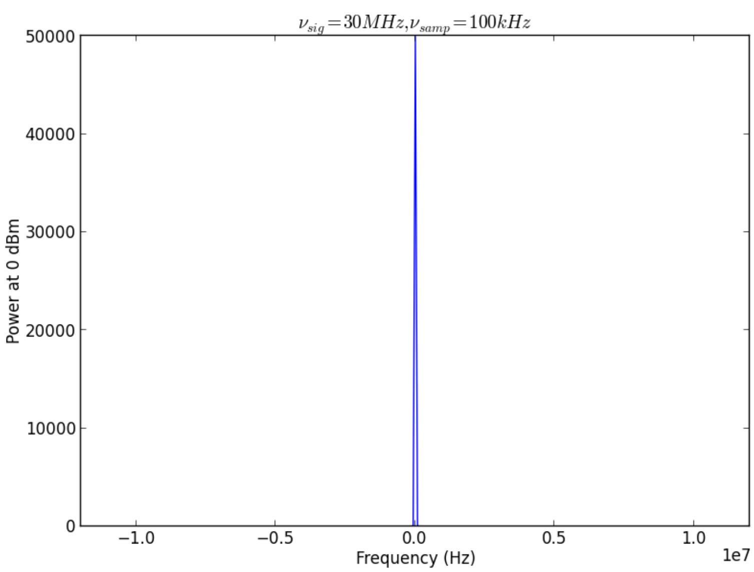


Figure 6: Power Spectra Corresponding to Data from Figure 3



4.2 Analog Mixing

Figure 7: Power Spectra of Analytically Mixed Signals

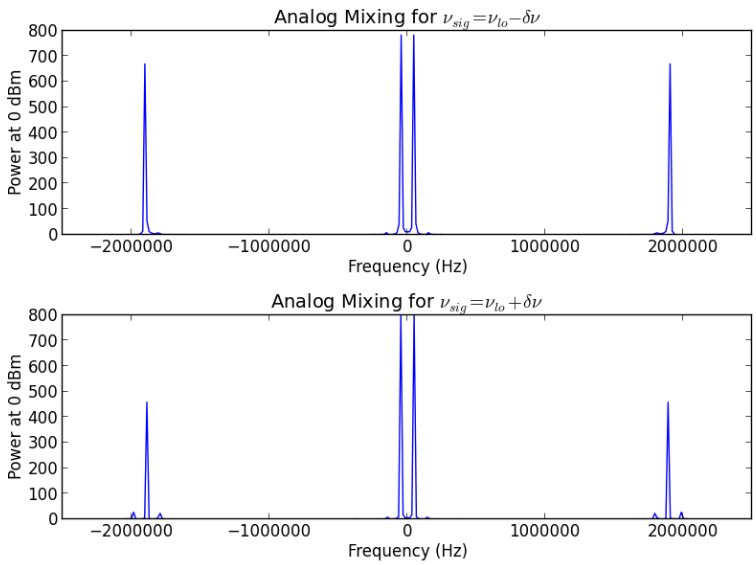


Figure 8: Waveform and Fourier Transform of 1 ZAD-mixed Output

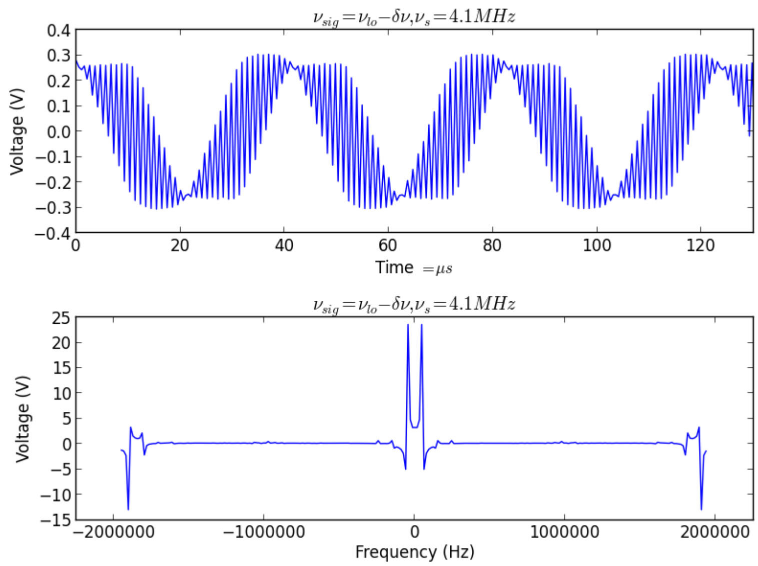
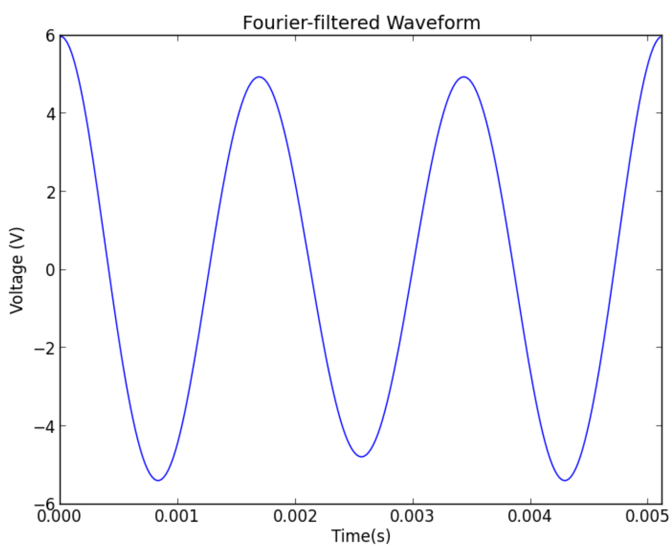


Figure 9: Fourier-Filtered Waveform of Signal from Figure 8



4.3 Digital Mixing: SSB vs. DSB

Figure 10: Power Spectra of Digitally Mixed Signals

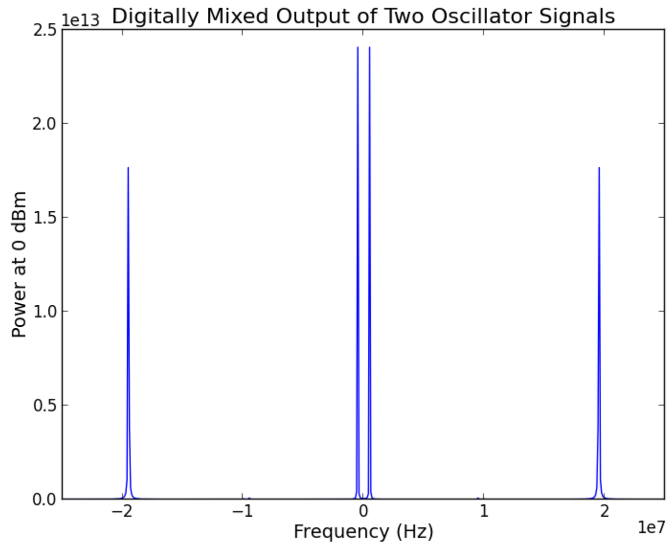
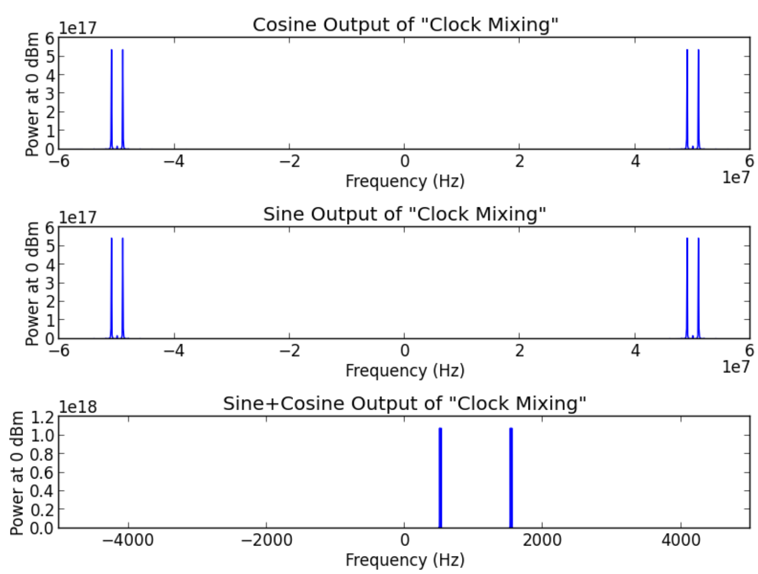


Figure 11: Power Spectra of Digitally Mixed Signals, SSB vs DSB Edition



4.4 FIR Filter Coefficients

Table 1: Coefficients for FIR Filter

N	coeff,realN	coeff,imagN
0	1	1
1	1	1
2	1	1
3	0	0
4	0	0
5	0	0
6	1	1
7	1	1

5 Analysis and Discussion: The message of the Data

What the plots convey is of paramount importance to the electronic world surrounding us. Here we interpret that information that the our data provide us with.

5.1 Allies of Alias

Figures 1 and 2 show the selected waveforms of $(N/10)\nu_s$ where N is an integer from 1 to 10. Plot 1 of Figure 2 shows the highest frequency because it is the waveform with Nyquist frequency ν_{max}. The waveform of this frequency is fully digitally represented by the sampling frequency. The same goes for the lower frequencies shown in the first two plots of Figure 1. For ν higher than ν_{max}, the sample frequency cannot accurately reconstruct the signal. The reconstructed signal looks like one of a lower frequency; in fact, the reconstructed waveforms' frequencies are symmetric about ν_{max}, as long as N is an integer between 0 and 10! This is the aliasing phenomenon. A high ν_{sig} will 'alias down' to a lower corresponding frequency. For frequencies at or above the sample frequency, the reconstruction becomes very inaccurate and almost seems noise-like. The result of approaching very relatively high frequencies for ν_{sig} sseems to behave like a function of the first order, as shown in Figure 3.

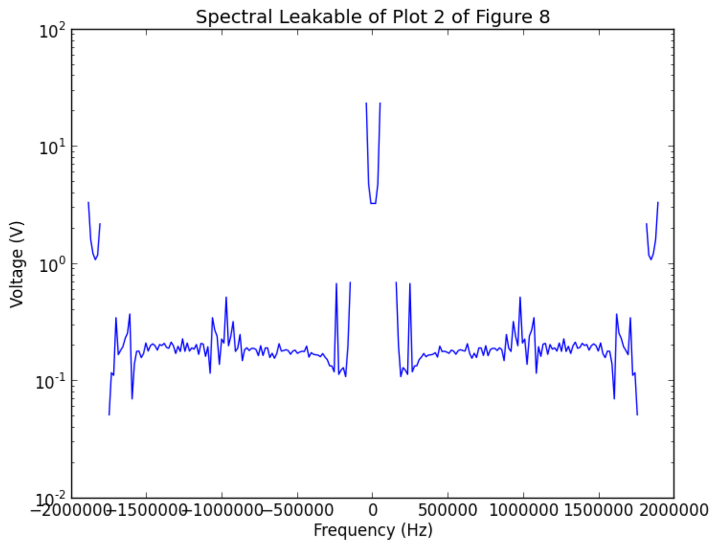
5.2 Spectra of Power

Figure 4 shows the peaks of the plots on either side of the symmetry spreading farther apart (towards the value of the peak to ν_{max}) as ν_{sig} gets closer to ν_{max}. Because of aliasing, we expect the power spectrum of plot 4 to look like that of plot 1 and the spectrum of plot 3 to look like that of plot 2, and they do. The power spectrum of the Nyquist frequency has only 1 peak. This is caused by the way the program plots Nyquist frequency samples: it begins taking data at 0 and moves to positive values before jumping to the most negative value and back to 0. Because sampling at the Nyquist frequency outputs two peaks at the same frequency value corresponding to the positive and negative 'edges', the program deems the two indistinguishable; so, when it reaches the positive end, it jumps and creates the peak value at the negative end. The power spectra for $\nu_{sig} \geq \nu_s$ are 0 because the haphazard sampling for relatively high frequencies produce meaningless reconstructions.

5.3 The Mixed Signals

There seems to be four peaks total for each case in the analog mixing experiment. They correspond to the sum and difference frequencies and the complex mirror about 0. The sum frequency is known as the upper sideband while the difference frequency is known as the lower sideband. In radio signal analysis, the lower sideband is of special importance for analysis. The digitally mixed result looks similar to our analog result, as it should be. The digital result does look 'smoother,' however. Mixing already digitized signals would create less imperfections for the power spectrum because discrete mixing bypasses many minor offsets cause my continuous mixing. There's also less leakage, which arises from the nature of transforming into the frequency domain from time domain. The waveform in time domain is primarily of singular frequency, but there are also other, low-power and less apparently frequencies mixed in. Frequency domain plots hence have fluctuations at low power or voltage levels and this fluctuation can be seen by taking the log scale of the power or voltage. This is not a very good representation of spectral leakage since this is merely a voltage leakage from the fourier-transformed plot rather than a power spectrum, but nonetheless presents the issue. Ideally, we would like our graphs to have no leakage, but that is not what nature intends. For our single sideband experiment, the sine and cosine components combine to form a single sidebanded result. This arises from that our pro-

Figure 12: Logarithmic Scale View in Frequency Domain - Leakage



gram cannot normally distinguish positive and negative frequency values for representing them on a plot, but for this case, because sine and cosine have a phase difference, adding their arrays directly will not 'mirror' the frequencies about 0. There are advantages to both single and double sidebanded data. It comes down to what we would want: to view both sidebands at once or only one, but a more noise-free one, at a time? This can be considered a quality versus quantity argument.

5.4 FIR Filter Shape

The FIR filter shape is dependent on the number of coefficients. For 8 coefficients, we expect a rather rectangular filter. As the number gets greater, the filter is less 'digitized' and will deviate from a rigid shape, displaying waveform like properties.

6 Conclusion

An input signal can be manipulated and analyzed in many ways to get desired results. The goal of down converting a signal is significant in our study of radio astronomy as it allows us to filter out unnecessary high frequency components and analyze the low frequency band equivalent of our signal. Methods of fourier transforming, sampling, and bandpass filtering allows us to achieve our goal. From our input signal, the information we extract includes how much power each present frequency contributed to the signal - a very powerful (pun!) analysis.