Lab 1: Rainy Analog Days

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1 Abstract

Construction of FM D demodulator (radio receiver) for receiving radio signals and of an amplifier for increasing such signals begin with certain circuit - RC filters (high and low pass), RLC bandpass filter, transistor, and rectifier are used for the constructions. The FM demodulator produced expected results of only certain frequencies levels at the output. The amplifier produced gains at the output. Mathematical modeling of the thermal noise within the receiver-amplifier are used in the context of internal thermal noise. Its seemingly random distribution approach a Gaussian behavior at large samples, which also lead to decreasing mean standard deviation.

2 Introduction

Applying foundational knowledge of electronics and mathematical physics, one can, from the breadboard up in the lab, create devices such a frequency-modulated (FM) radio receiver or a speaker amplifier. Simple systems such as filters and transistors can combine to make more complex systems that are used in many areas in the technical world. Through these simple experiments, the intricacies of the physical workings of electronic devices are explored. More fundamentally, the underlying math behind statistical phenomena in physics such as random noise distributions are explored and explained. In the end, from one wire to the next, everything seems connected.

3 Methods and Procedure

Experiments and observations are carried out primarily in three distinct parts: the construction of the FM radio receiver and its associated circuits; the construction of the speaker amplifier; and the measurement on the noise figure of the amplifier.

3.1 Important Equations and Terms

- Voltage (V): usually the output, as a function of frequency, of a signal. It is often associated with the amplitude of the signal.

- Impedance (Z): replaces resistance in the generalized Ohm's Law. While resistors have resistance, capacitors and inductors have reactance. They are given by

$$Z_C = \frac{1}{i\omega C} \qquad Z_L = i\omega L \tag{1}$$

  Here, i is the imaginary number and $\omega = 2\pi f$.

- Ohm's Law: $V = IZ$ where I is the current.

- The ratio of output voltage to input voltage can be generalized to

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} \tag{2}$$

  This equation arise from measuring the output voltage of a voltage divider circuit. Z_1 and Z_2 are anything with impedance in series.

3.2 FM Demodulation

The FM radio receiver is broken down into several multi-unit core components: A resistor-inductor-capacitor(RLC) filter; a voltage biaser; a high-pass filter; and coupling and decoupling capacitors. See figure 1.

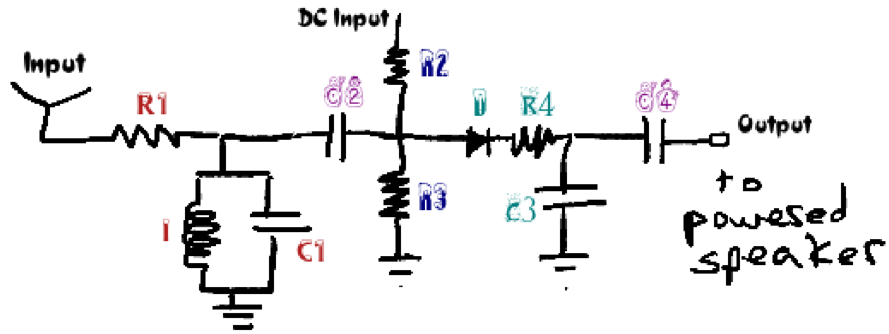

Figure 1: FM Demodulator with its resistors(R), capacitors(C), diode(D) and inductor(L) labelel and colored according to its function in the circuit. Red is the LC circuit; violet are the coupling or decoupling capacitors; torqoise is the high pass filter; and blue is the biaser.

3.2.1 RLC Circuit

After the signal enters the circuit via the input, it first encounters the RLC circuit, composed of R_1, L, and C_1. The purpose of the RLC filter is to allow only 1 certain band of frequency to pass through. At frequency f_0 set by the signal and with specific values for R_1, L, and C_1, the output has a peak value or near 1 output to input voltage ratio with a narrow band of voltage spike around it. The equation relating the variables is:

$$\frac{f_0}{(\Delta f_{-3dB})} = \omega_0 RC \tag{3}$$

Here f_0 is the resonant frequency; it is where the output (V_{out}/V_{in}) peaks. It equals $1/2\pi sqrtLC$. ω_0 is equivalent to $1/sqrtLC$. The width of the band is approximately Δf_{-3dB} with dB meaning decibels. This expression means "the difference in values of frequencies at which the output voltage to input voltage returns -3 decibels." The decibels equation is logarithmic: dB $= 10\log_{10} V_{out}/V_{in}$. At input signal frequency of 1.045 MHz and given the band width of 200 kHz, the values chosen for R_1, C_1 and I are 234 Ω, 1×10^{-8} F, and 2×10^{-8} H respectively. Using the voltage divider equation and manipulate it for this case, the resulting equation relating V_{out}/V_{in} to frequency is:

$$\frac{V_{out}}{V_{in}} = \frac{\omega L}{R\|\omega^2 LC - 1\| + \omega L} \tag{4}$$

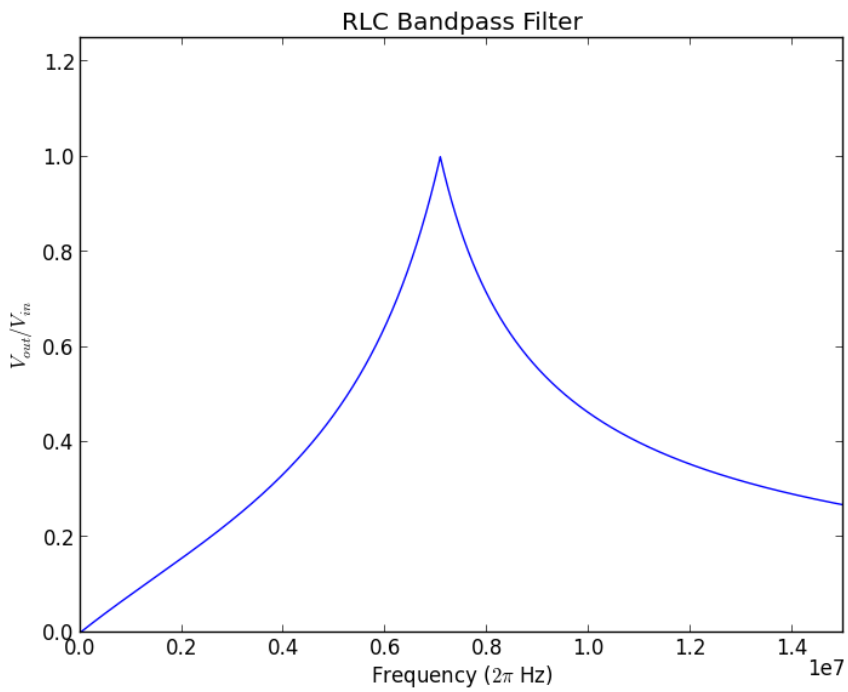

Figure 2: Output to input voltage ratio shown as a function of angular frequency ω ($2\pi f$). At the peak, ω is approximately 7 MHz (so f is approximately 1 MHz).

We measure at a frequency slightly higher than the peak frequency to avoid a measurement that can possible blow up at the peak frequency. Measuring it in the band allows us to still see a significant increase in the ratio while avoiding unnecessary complications.

3.2.2 Voltage Biaser

] R_2 and R_3 are the resistors manipulating, or biasing, the voltage coming into or going out of the circuit. These two resistors are in series so their combined value influence the amount of current flowing into the diode from the DC voltage source, which is used to increase the voltage in the circuit after the voltage has been reduced by the RLC circuit. A current in the mA range should produce a forward voltage drop (from anode to cathode). A total resistor value of about 2000 Ω should provide a good current amount (0.7 25 mA) for the diode, resulting in a 0.7 voltage drop across the diode. Before going into the diode, the incoming signal is a superposition of the RLC-filtered AC signal and the DC signal.

3.2.3 Low-Pass Filter with Diode

D is the diode while R_3 and C_3 make up the low-pass filter. The diode acts as a rectifier; that is, its goal is to convert AC signals to DC signals so that the low pass filter with have an easy time interpreting the incoming voltage values (an AC signal will be difficult to decipher as it alternates to negative values). The low pass filter allows low frequencies to pass while inhibiting high frequencies. The cutoff frequency is given by:

$$f = \frac{1}{2\pi RC} \tag{5}$$

For f = 100 kHz, below which would typically be considered low frequencies for such purposes, resistor and capacitor values of about 160 Ω and 1×10^{-8} should give the approriate impedance.

3.2.4 Coupling and Decoupling Capacitors

The purpose of C_2 is to decouple the signal: it removes the carrier signal from the original source and leaves up with only our filtered signal. C_4 is a coupling capacitor. Now that our signal has been filtered by a low pass filter as a DC signal, we would want an AC output so that we can read it on the oscilloscope. The couping capacitor return the original AC signal.

3.3 Speaker Amplifer

It is possible to amplify an incoming signal using an amplifier circuit. A BJT amplifer consists of a high pass filter, a transistor, and several resistors.

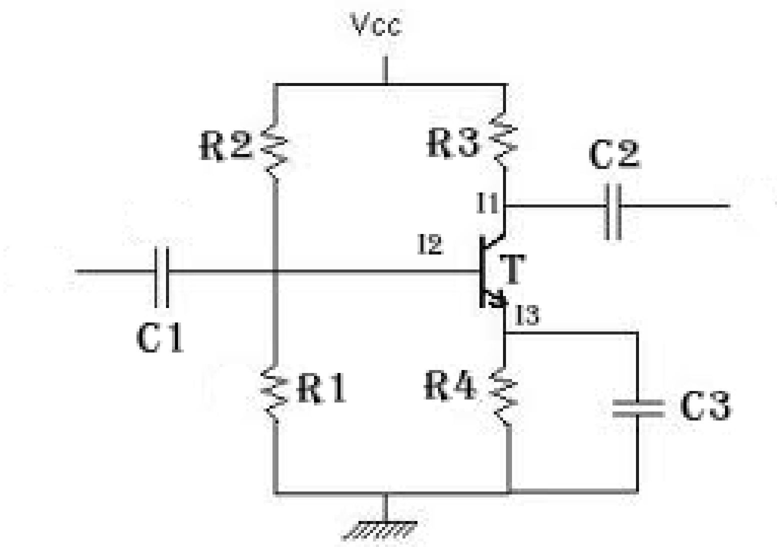

Figure 3: An amplifier. "I" denotes the current and T marks the transistor. The input comes from the left and V_{CC} is an additional voltage source. I_C is to be equal to I_E.

Here C_1 and R_2 together functions as a high pass filter. R_1 and R_2 are the resistors used to set a bias voltage into the base of the transistor. R_3 And R_4 are there to crease a bias current. The remaining capacitor are used for additional coupling purposes.

3.3.1 Transistor

The transistor consists of three parts: The base, emitter, and collector. The purposes of a transistor is to act as a "switch" that determines when a current should continue to flow through the circuit. The base receives incoming current and a voltage is applied across the transistor to the emitter, V_{BE}. V_{BE} must be around 0.6 V or higher for our transistor to continue to allow current to flow. If V_{BE} is less than 0.6 V, meaning that the outgoing voltage from the transistor is approaching the incoming value, then the transistor will cease to allow currents to pass. If V_{BE} is significantly above 0.6 V, then the collector will send currents to increase the outgoing voltage. Typically, at least $V_{CE} = 0.2V$ is required for such a reaction to occur. The transistor thus acts to hold the voltage constant at its separation point of the circuit.

For our purposes, let the desired voltage at $V_{out} = 2.5$ V.

3.3.2 Bias Current and Gain

The gain is the ratio of the desired amplification of the output voltage. It is also the ratio of the resistance values of R_3 and $R4$ since these resistors determine the amount of gain that will occur at the output. Taking the voltage at V_{out} at 2.5, and the incoming additional voltage at 5 V, we use Ohm's Law at R_3 to determine I_C. With $R_3 = 1000\ \Omega$, we get $I_C = 8.3$ mA. Since $I_C = I_E$, to maintain a gain of 5, the resistor values have to maintain that ratio. We thus choose R_4 to be 200 Ω. With these value, the voltage at the emitter is approximately 0.3 V.

3.3.3 High Pass Filter

Our high pass filter consists of C_1 and R_2. Using the equation $f = 1/2\pi RC$ and the given signal frequency value of 10 kHz, we use a $1 * 10^{-6}$ F capacitor with a 16 Ω resistor. This filter allows most frequencies above 10 kHz to pass while denying most of lower frequencies. This circuit allows for many different bands of frequencies to be used for the signal. Since frequency is related to the impedance values chosen here, adjusting the values of this amplifer circuit will allow different frequency ranges to pass through it.

3.3.4 Bias Voltage

R_1 and R_2 constitute the part of that circuit that sets a voltage bias at the base. Since the voltage the emitter is around 0.3 V, we want our voltage at the base V_B around 1 V, a 4 V drop from $V_{CC} = 5$. We determine the value of R_1 by using this method:

$$\frac{V_B}{V_{CC}} = \frac{1}{5} = \frac{R_2}{R_1 + R_2} \tag{6}$$

We see that the value R_1 is four times that of R_2. Since we have R_2 as 16 Ω, we use 64 Ω for R_1. Now the voltage difference between the base and emitter is $V_{BE} = 0.7$.

3.3.5 Input and Output Impedance

The input impedance can be obtained when the transistor is "switched off'." When V_{BE} is quite below 0.7, the transistor stops the current flow so the path the input signal takes is through R_2 and R_4. The input impedance Z_{in} is then $16\Omega + 200\Omega = 216\Omega$. The output impedance is obtained when the switch is on. In this case, the current passes through all 4 resistors so Z_{out} is $200 + 1000 + 64 + 16 = 1280\ \Omega$, which is bigger than Z_{in} by about a factor of 6.

3.3.6 Termination

It's possible that the amplifier is connected to a speaker via long cable cord. Let's assume that the speaker inherently has 8 Ω resistance. In order for your output signal to reach the speaker smoothly, your cable should also have a resistance of 8 Ω; otherwise, part of the signal will reflect back. With matching impedances, the signal will think the speaker is a continuation of the wire and will proceed into the speaker.

3.4 Noise

Noise is usually unintended or undesired current due to the random motion of electrons in objects such as a resistor. Because electrons are omnipresent, nearly everything has some inherent noise that must be accounted for when making precise measurements. The resulting motion of the electrons also creates additional temperature within the object.

3.4.1 Determining the Noise Figure

A noise figure is a measure of the ratio of output signal power to the noise power generated internally by thermal noise. To determine the noise figure of a system (in our case a signal receiver), first calculate the temperature of a resistor by using an input voltage; specifically, we are looking for a power value, which can be obtained with P = IV. The relationship between power and resistance is given by the Stefan-Boltzmann Law:

$$P = A\sigma\epsilon T^4) \tag{7}$$

Here T is temperature, A is the cross-sectional area of the object in question, σ is the stefan-boltzmann constant, and ϵ is the emissivity constant which has a maximum value of 1, depending on how much of a blackbody the object is, thus setting a lower-bound limit of the value of temperature given power. For our purposes, we input a power value of 2 watts into a 50 Ω

resistor. Since other terms are constants, we can obtain the temperature of the resistor. For convenience, we can obtain the gain required to view the noise level of the resistor by amplyfing the noise signal until an output becomes decipherable on an oscilloscope. With the resistor temperature in mind, we now proceed to calculating our noise figure. Back to our receiver, we measure its output power. Plugging that value into the Stefan-Boltzmann Law returns the temperature of the system. Accounting for our resistor temperature, we now have the receiver temperature.

3.4.2 Central Limit Demonstration

Access my repository:

- https://github.com/rgao/lab_analog

The central limit theorem states that as the number of random samples approach infinity, then the distribution becomes Gaussian with a 0 mean and 1 variance. This leads to two results:

- The variance of the sum is the sum of the variance of random samples.
- Normal (Gaussian) distributions occur everywhere in nature.

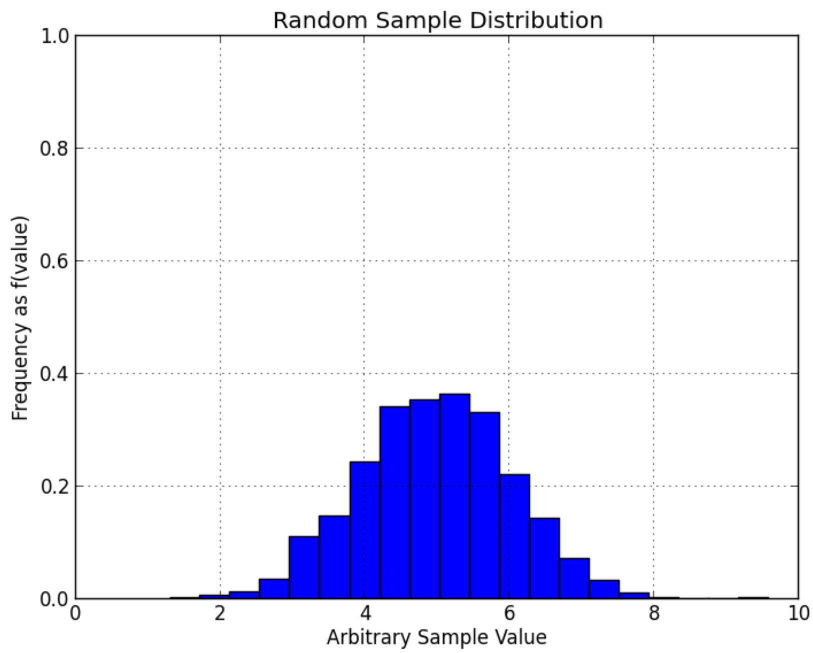

Figure 4: Random sampling converges to a Gaussian distribution as N increases; N = 1000.

Additionally, the standard deviation of the mean of the mean of N random samples over many trials decrease with $sqrtN$. Each N value is trialed 1000 times.

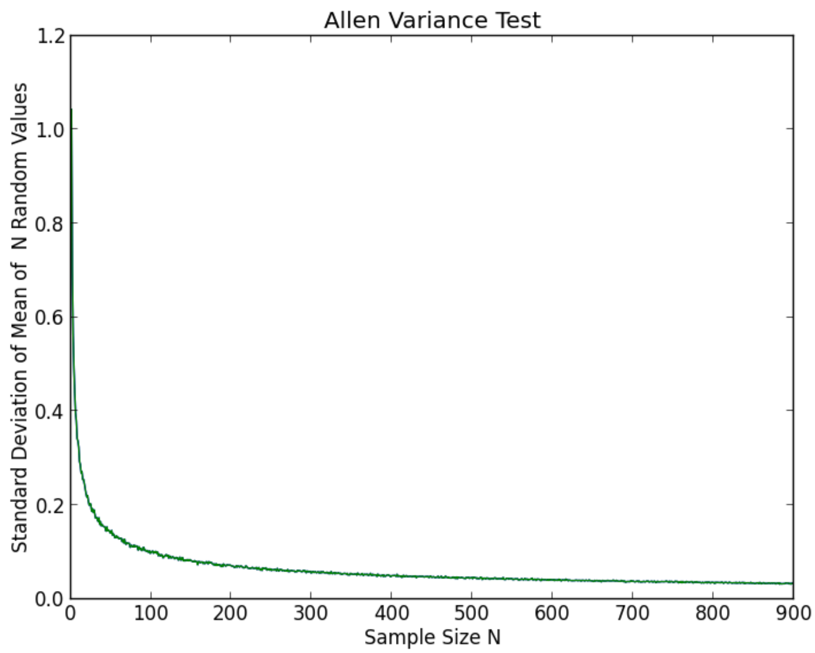

Figure 5: The standard deviation of the mean of the mean over many trials of N random samples is plotted against N. There seems to be a $sqrtN$ relationship.

4 Data and Results

The results of the experiments are collected in this section.

4.1 FM Radio Receiver

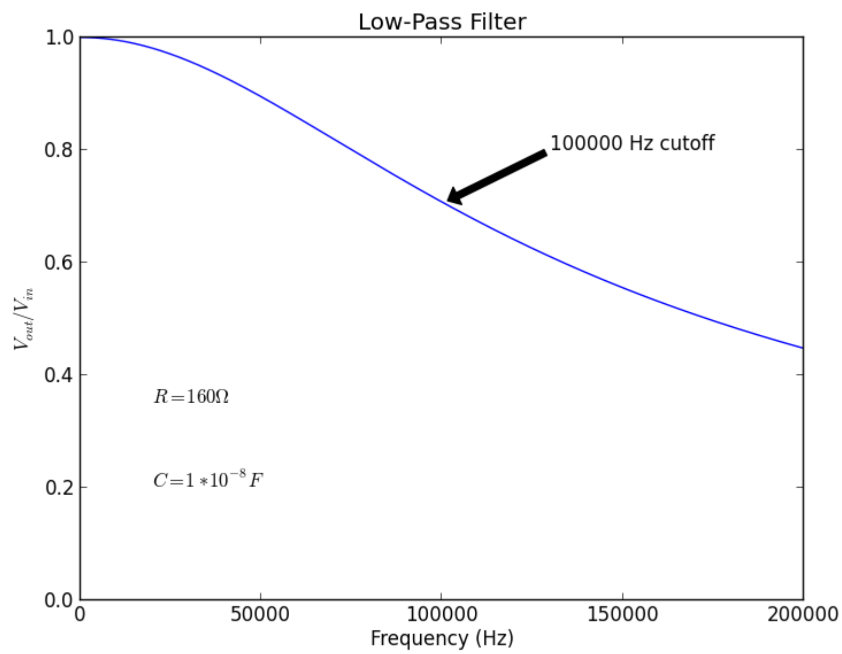

Figure 6: The output of the radio receiver after FM Demodulation plotted against frequency.

For the given signal of 100 kHz, the values chosen for the resistor and capacitor are 160Ω and $1 * 10^{-8}$ to match the frequency. As expected, the drop in voltage is steepest at the given cutoff frequency.

5 Noise and Central Limit

The thermal noise distribution follows the same math principles of random distributions. The standard deviation of the mean of the mean of N samples against N is given by Figure 5. For just the standard deviation of the mean of N sample size, it is given by this figure:

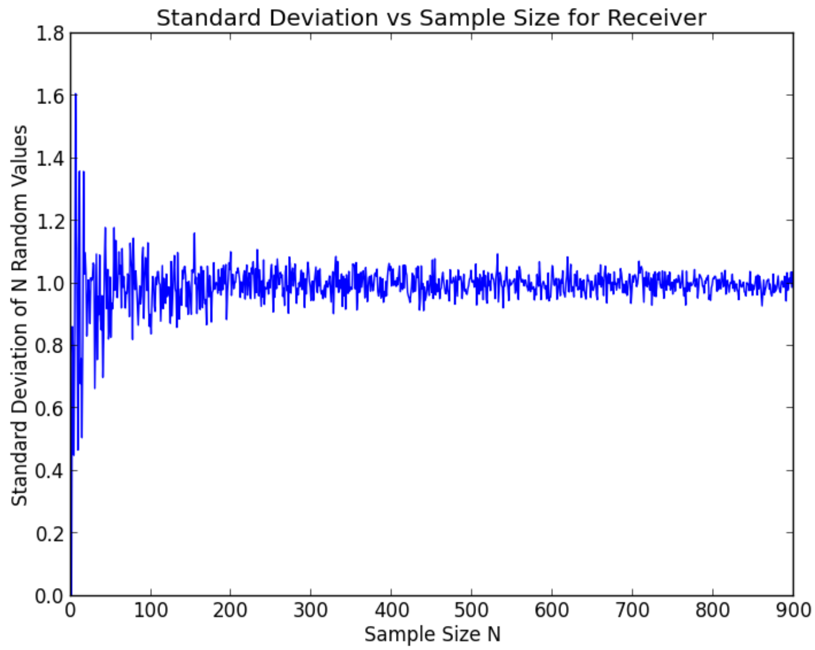

Figure 7: The standard deviation to the mean of N random samples is calculated just once for each N and is plotted against N

As expected, the graph follows some downward trend with increasing N but it's not as clean as Figure 5 because each N is trialed only once.

6 Discussion and Analysis

The results and methods presented in the previous sections and discussed, analyzed, and explained here.

6.1 FM Radio Receiver

As seen from the graph, 100000 Hz is the cutoff value of the filter once the appropriate capacitor and resistor values are used. At frequencies lower than 100000 Hz, the ratio of V_{out} to V_{in} is high; this filter is to let low frequencies pass while filtering out high frequencies. The ratio gets much lower past the cutoff frequency.

6.2 Amplifier

The amplifier is a complex circuit relative to the demodulator. The values you choose for one part of the circuit can affect another part since several values are connected by ratios to ensure that V_{BE} remains constant at 0.7 V. Fortunately, you can select which values to begin with - whether that's the signal frequency or one of the resistances. In the end, as long as the ratios are correct, the signal should be correctly amplified.

6.3 Noise

Noise is everywhere. Although samples of noise phenomena can seem like individual occurences, the Law of Large Numbers start to take effect and present the true nature of the electrons. Noise can be undesired or innocuous, depending on the nature of the experiment. In astronomy, noise can come into the form of frustrating atmostpheric interference or in the form of internal receiver signals of a telescope. Noise shows how the universe disperses its seeds and the similar behaviors, on the fundamental level, among many substances.

6.4 Central Limit Theorem

The central limit theorem makes powerful statements on the nature of our universe. It shows that, fundamentally, math can model a wide range of phenomena, even when you don't expect it to. Noise distribution is just one of many examples of submission to this aspect of nature. Systems and occurances sometimes behave statistically and it's not obvious - it's through experimental observation that these concepts come into light. The converging nature of the data really represents powerful mathematical statements.

7 Conclusion

The FM Demodulator and the amplifier served their purposes: they received certain frequency signals and amplified incoming signals, respectively. These constructions are basic models of the versions used by companies for real-life products, but they present the underlying principles in designing electronics and in manipulating electromagnetic waves for specialized usage. The sensitive nature of electronics can be challenging since precision is key; however, it is due to this nature that electronics and circuits can be very diverse and unique. At their deepest core, these systems still behave according to the mathematical laws of the universe, as shown by the presence of Gaussian distributions in even thermal noise. Ultimately, these systems converge to fundamental, big picture concepts on both the small and large scales. In radio astronomy, it would not be a surprise to encounter applicable mathematical models during real-time observations and appliance usage.