RADIO INTERFEROMETRY AT X BAND
March 3, 2014

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THESIS **17**

The purpose of this lab is, broadly speaking, to learn radio interferometry—the basis for much of modern radio astronomy. We'll cover the basic principles, the interferometric fringe, the response to a point source, and the response to an extended source (which is the basis of high-angular resolution mapping with interferometry). We will employ least-squares fitting and Fourier transforms to measure accurate positions for a few sources and also accurate angular diameters for the Sun and Moon. And because this is your first foray into observations, you need to learn about astronomical coordinate systems, topocentric coordinate systems, converting coordinates with rotation matrices, systems of civil time, sidereal time. This lab runs for four weeks and clearly there's a lot of new material!

1. GOALS

- Learn how diffraction theory applies to a real radio interferometer. Fringes, their amplitudes and phases. The Fourier transform relationship between interferometer response and sky brightness.
- Learn about all those things observers need to know: time, coordinates, conversions.
- Obtain horizon-to-horizon data on our various astronomical sources—point sources, and the Sun and Moon.
- Learn about linear and nonlinear least-squares fitting and use it to tease out accurate source positions.
- Use least-squares fitting to obtain accurate diameters for the Sun and Moon.

2. SCHEDULE

1. *Week 1*: Each *person* uses rotation matrices to calculate when objects of interest are up. Each *group* observes the Sun for a short time to confirm that you see fringes. Each *group* does a horizon-to-horizon observation of a point source, which requires writing an observing script to run automatically.

   Each *person* calculates Fourier power spectra of the Sun data and also the point-source data. She/He calculates the range of expected local fringe frequencies (equation 11) and compares the observed spectra with these expectations. Be ready for show and tell!

2. *Week 2*: Each *group* observes horizon-to-horizon the Sun and, if possible, the Moon with the interferometer. Each *person* derives accurate declinations for the point-source data using least-squares fitting as outlined in §5.

3. *Week 3*: Finish the Moon observations. Each *person* derives accurate angular diameters for the Sun and Moon using least-squares fitting as outlined in §6.

4. *Week 4*: Each *person* finishes all calculations and writes (and hands in!) the lab report.

3. OUR INTERFEROMETER

With our interferometer, which operates at about 11 GHz, our attention focuses on small sources. Our interferometer has a relatively short baseline and the fringe spacing is larger than the size of all sources (except the Sun and Moon). So for our interferometer all of these sources look like "point sources", for which all the radiation appears to come from a single point—just like a star in optical astronomy. The output of the interferometer is a sinusoidal-like signal called the "fringe". All of the information resides in the frequency, amplitude and phase of the fringe.

If you know the baseline, the fringe properties are direct indicators of the point-source declination. With our 10-m baseline B we can get a fringe spacing $\frac{\lambda}{B} \sim 10'$, and with a horizon-to-horizon measurement we can measure the declination 100 times more accurately. In addition, we can achieve partial angular resolution on the Sun and Moon and measure their diameters to a fraction of a percent—this turns out to be more interesting than it sounds!

We have a *multiplying* interferometer: the signals from the two telescopes (that's E_1 and E_2) are multiplied together, using one of the mixers you used in the previous lab, producing the product E_1E_2.[1] We repeatedly measure and record the time average $\langle E_1E_2\rangle$ over a suitable interval (a few seconds). Using the cross product is important because its average value is zero unless there's a source, so any detected signal has to come from the sky instead of the instrument. So when we look at a source, the fringe amplitude has no zero offset and the amplitude is directly proportional to the source flux (if it's a point source).

4. CONTINUUM SOURCES

For the first part of the lab we will concentrate on measuring source positions. Our telescopes are small, so there are only a few sources that are powerful enough for us. The strongest continuum sources are listed in the table below. You'll need to precess the coordinates to the current equinox (see §4.2). If you don't, the incorrect RA's will affect your fits.

Three of these sources are double (they are marked with the footnote *a*). The two componnts are an HII region (a hot young star ionizes gas in its vicinity; the ionized gas produces radio continuum) and (2) a supernova remnant (a hot young star lived fast and died young); both stars

[1] This multiplication is equivalent to adding the signals, squaring $[(E_1 + E_2)^2 = E_1^2 + E_2^2 + 2E_1E_2]$, and then subtracting the two self-products E_1^2 and E_2^2.

were formed coincidentally in a cluster). The position you derive is a weighted average of the two sources.

Not all of these sources are suitable for accurate declination measurement. To understand why, look at equation 9 and think about it!

name	**r.a.(2000)**	**dec(2000)**	$\mathbf{S}_{Jy}$
W3[a]	$02^h27^m04.10s$	$+61°52'27.1'$	~ 105
3C144 (Crab Nebula)	$05^h34^m31.95s$	$+22°00'52.1''$	~ 496
Orion Nebula	$05^h35^m17.3s$	$-05°23'28''$	~ 340
3C274 (Virgo A)	$12^h30^m49.423s$	$+12°23'28.04''$	~ 34
M17	18^h20^m26s	$-16°10.6'$	~ 500
W43	$18^h47^m58.0s$	$-01°56'43''$	~ 200
W49[a]	19^h10^m17s	$+09°06.0'$	~ 80
W51[a]	$19^h23^m42.0s$	$14°30'33''$	~ 116
3C405 (Cygnus A)	$19^h59^m28.357s$	$+40°44'02.10''$	~ 120
3C461 (Cas A)	23^h23^m24s	$+58°48.9'$	~ 320
SUN	varies	varies	
MOON	varies	varies	

————(a) This source has multicomponent angular structure

4.1. Time and Space

Time is complicated, and people have made matters worse. If you want to make accurate astronomical observations, you need to both know where you are on the Earth and what time it is. That way, you know which direction you are facing. For our purposes, latitude and longitude do a decent job of encoding where on Earth you are (although if you ever try to accurately survey something, you'll discover lat/long aren't as easy as they look either — the Earth isn't perfectly spherical). Time, though, is a different story. We tune our clocks to different times depending on which time zone (and what time of year!) we are in. Time zones are political constructs, and are of no use to us. We also have a nasty habit of counting time in solar days.

To help simplify matters, decent time systems do away with time zones and synchronize to the solar day at 0° longitude, which happens to have been defined to run through the British Royal Observatory. Time and astronomers go way back, you see. Such systems are said to be based on Greenwich Mean Time (GMT). The next thing is to count seconds. You might think to count days instead, but days are complicated. Since the Earth is moving around the Sun while it is spinning, the Earth must actually rotate a little more than 2π radians in order to bring the Sun around to the same position each day. So instead of spinning 2π radians every 24 hours (86400 s) as you might think, it actually takes 86164.0908 s. That's called a *sidereal day*, and you'll notice it's awfully close to 364/365 of a solar day.

For every day of the year except one, sidereal days and solar days are out of sync. On Jan 1., they overlap. So if you are going to count time, you really ought to count from Jan. 1. But what time on Jan. 1 (and what year) should we count from? If you are an astronomer, you count in (solar) days from noon, Jan. 1, 4713 BC. You can thank Ptolemy and Joseph Scaliger for that one. If you are a computer, you count seconds from midnight at the start of Jan. 1, 1970. This is called "Unix Time", and is pretty close to Coordinated Universal Time (UTC), which is derived from atomic clocks. Just to keep us on our toes, there have been a few revisions to UTC, with a several leap seconds added in here and there. Couple that with the fact that the rotation of the Earth is slowly slowing down, and you see how time can get mind-bendingly complicated.

But now it's time to simplify. Using Network Timing Protocol (NTP) and/or the Global Positioning System (GPS), computers and cell phones routinely synchronize their clocks to atomic standards with millisecond precision. (NTP and GPS, by the way, are *not* simple). We all have UTC sitting at our fingertips these days, and that makes it relatively easy to do the rest. Here are the steps:

- Use UTC to figure out what sidereal time it is in England (longitude 0°) in hours, minutes, seconds. Because right ascension (RA) is measured in units of time, this tells you what part of the sky is overhead there.
- Use your longitude to figure out what time offset you should add to get your *local sideral time* (LST). This tells you what part of the sky is overhead here.
- Subtract your LST from the RA of your source to get the hour angle (HA) it appears at.
- Use your latitude, and the HA and declination of your source to figure out where to point.

Oh, and UCB is at latitude 37.8732N, longitude -122.2573 (in degrees).

4.1.1. PyEphem: a Useful Module

In this class, we'll use a helpful module called PyEphem to help us calculate ephemeris (positioning).

```
>>> import ephem, time
>>> print ephem.now()
2014/3/4 01:39:23   # current time in England
>>> print float(ephem.now())
41700.5700926       # PyEphem's internal time representation is in
                    # solar days since 1899/12/31 12:00.  DON'T
```

```
                    # USE IT WITH ANYTHING ELSE!
>>> print time.time()
1393897394.41       # This is UTC (seconds since 1970)
>>> print ephem.julian_date()
2456720.57226       # Good astronomers use Julian Dates
>>> obs = ephem.Observer()
>>> obs.lat = ephem.degrees(37.8732)    # set your latitude
>>> obs.long = ephem.degrees(-122.2573) # set your longitude
>>> obs.date = ephem.now()              # set the time of your observation
>>> print obs.sidereal_time()
1:36:37.80          # LST!
>>> print float(obs.sidereal_time())
0.421627770357      # radians
```

4.1.2. Sun, Moon, and Other Procedures

Now things get exciting!

```
>>> sun = ephem.Sun()
>>> sun.compute(obs)  # compute the Sun's position given the time stored in obs
>>> print sun.ra, float(sun.ra)
22:58:49.07 6.01622781701   # Sun's right ascension as a string, and radians
>>> print sun.dec, float(sun.dec)
-6:31:24.0 -0.113853520397  # And declination as well
>>> print sun.az, sun.altA
248:48:31.0 47:23:44.3      # And in topocentric coordinates!
```

There's lots more in there, so poke around a bit (there's good documentation online). One potential gotcha: make sure you update `obs.date` to keep advancing to the current time when you are observing.

4.2. The Point Sources

Our goal is to measure the absolute declinations as accurately as possible. Declinations can be measured on an absolute basis with interferometry by least-squares fitting the fringe phase to the hour angle; see §5 and equation 9 below. With our fringe spacing of $\sim 10'$ we might be able to measure declinations to an accuracy $\sim 20''$—maybe better. We can then compare the derived values to those listed above. Note that to compare them, you'll need to precess all coordinates to the current equinox. If you don't, the incorrect RA's will affect your fits. To do the precession, use PyEphem:

```
>>> src = ephem.FixedBody()
>>> src._ra, src._dec = '0:00', '45:00'
>>> src._epoch = ephem.J2000
>>> src.compute(obs)
>>> print src.ra, src.dec
0:00:43.02 45:04:47.0   # RA,DEC are now precessed from J2000 to current
```

Optionally—if you've done everything else and it was so easy you're bored—you can try to measure the difference in right ascension between two sources. We can't measure the *absolute* right ascensions because the right ascension coordinate has no naturally-determined zero point; rather, its zero point is defined arbitrarily by convention[2] (and it changes with time, too). In contrast, declination has a naturally-determined zero point: the equator. But we can measure the *relative* right ascension of one source with respect to another with high accuracy. At least we can do this in principle; in practice it's harder than measuring only the declinations.

We'll want *each group* to pick a source and obtain a horizon-to-horizon observation of the fringes. If you have time, groups can compare results to see if you can get the differences in right ascensions (we've never successfully done this before!). And we'll want *each person* to measure the source's declination by least-squares fitting the horizon-to-horizon track of fringe phase and amplitude. Compare your results with other members of your group. Help each other out, but *each person should write her/his own software.*

There are some considerations in picking sources, and we will save you some frustration by telling you about them beforehand:

1. The fringe frequency depends on $\cos\delta$. This means that you don't measure δ directly, but rather $\cos\delta$. Note our discussion in §5.2 about the desireability of solving for $\left[\frac{B_y}{\lambda}\cos\delta\right]$ instead of $[\cos\delta]$.
2. Southern sources present a minor problem for horizon-to-horizon tracks because of the presence of the Campanile—you have to discard some data..
3. Our telescope pointing for Northern sources may be inaccurate, leading to loss of signal/noise.
4. Our observing frequency is in the TV satellite band. Geostationary TV satellites sit in the southerly skies and generate strong signals that can enter the dish sidelobes and produce fringes.

[2]What's the convention? The zero point is defined to be the position of the Sun when it crosses the equator on March 22. This moves by about 1 arcmin per year.

4.3. The Sun and Moon

The Sun and Moon are special cases, for two reasons. First, their positions change from day to day—and for the Moon, the change in just an hour is significant[3]. Second, they are both extended sources, not point sources. This causes their fringe amplitudes and phases to change with time, in a manner that depends on their brightness distribution[4]; this makes the determination of accurate positions a bit tricky, but we'll ignore that detail for now.

You can get the Sun and Moon positions from PyEphem. For the Moon, you should be aware that it is nearby so that you have to correct for your location on the Earth's surface. The parallax effect is far greater for the Moon than the Sun—so large that unless you correct for it, the Moon will probably lie outside the telescope main beam. Fortunately, PyEphem takes care of this automatically, along with crazy things like the time-of-flight of photons, which actually matter for observing planets!

Given these difficulties, why bother with the Sun and Moon? Because they are *bright.* We include them because they provide huge signals, which is ideal for testing your observing setup and your reduction software. In particular, the Sun is so bright that you'll get a huge signal/noise and you should be able to estimate an accurate declination in the first few minutes. And if you don't see the Sun, you *know* you're doing something wrong!

4.4. Some Astrophysics

Here's a bit of physical information. M17 ("M" for Messier), W3 ("W" for Westerhout), W43, W49, W51, and Orion are HII regions—places where hot stars have produced warm ($T \sim 10^4$ K) ionized gas, where the electrons flying past the protons get deflected and produce *free-free (bremsstrahlung)* radiation. The Sun also emits free-free radiation, just like the HII regions.

The other sources, and sources paired with some of the HII regions, radiate in *synchrotron* radiation—relativistic electrons gyrating in a magnetic field. For these sources, the source designation 3CXYZ designate source number XYZ from the third Cambridge (England) catalog; in the early 1960's, Cambridge radioastronomers produced the first reliable comprehensive catalog of strong radio sources in the Northern hemisphere. The Crab Nebula (also called Taurus A) is powered by the Crab pulsar, and is a $\sim$ 1000 yr-old supernova remnant in the Galaxy about 1 kpc distant. Cas A is another supernova remnant, *not* powered by a pulsar; rather, the relativistic electrons are produced behind the fast shock wave produced by the explosion. Cas A is $\sim$ 300 yr old and about 2.5 kpc distant. Both of these supernova remnants are expanding rapidly, as

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[3]Use what you already know to make an order-of-magnitude estimate of how far the Sun and Moon move in one day!

[4]These changes in fringe properties are exactly what's necessary to map the sources!

befits their young ages, and Cas A (in which a pulsar does not constantly replenish the electrons) is gradually getting dimmer.

In the external galaxies, the ultimate source of the electrons involves acceleration of electrons near the black hole at the center; the electrons are then spewed out to extragalactic space in narrow collimated jets, and produce large "emission lobes" at the end of the jets. Virgo A is a "peculiar" elliptical galaxy about 11 Mpc distant, while Cygnus A is a giant elliptical galaxy 220 Mpc distant. Cyg A is a powerful radio source—it's 10^5 times further than Cas A and just as bright! The study of the mechanism by which this enormous power is generated, which implies enormous energies, has led to the current awareness of and interest in *high-energy astrophysics.* For information on both types of source and some beautiful pictures, see chapters 10 and 13 in *Galactic and Extragalactic Radio Astronomy, second edition* (1988, ed. G.L. Verschuur and K.I. Kellermann).

The Moon is a completely different story. Contrary to what you might expect, at radio wavelengths it *doesn't* shine by reflected sunlight. Rather, its emission is blackbody radiation from its solid surface. Its surface is heated by sunlight, and at short wavelengths (but not at long ones) there's a big difference between the temperature of the sunlit and dark parts of the Lunar surface. You can tell a lot about its surface properties from the polarization of the radiation and also from its time variability as the surface heats up from sunlight and cools off from darkness—just like the Sahara.

5. MEASURING ACCURATE DECLINATIONS

5.1. General Description

We measure declinations from the fringe frequency, which depends on the baseline orientation, baseline length, declination, and hour angle (all these go into the *projected baseline*). If we observe a source horizon-to-horizon, the projected baseline changes a lot, and so does the fringe frequency. We take those data and do a least-squares fit to derive the most accurate declination from our data.

FIRST WEEK: Before doing the weak sources in the Table, do the Sun for a much shorter time, say an hour. This will give you confidence that the system works (or so we hope). There should be an easily-recognizable signal that you can look at visually, think about, and derive the approximate declination with pencil and paper. Then later you can write software to do the same, and make sure you get the right answer. Also, during this first week, do the horizon-to-horizon track of one of the sources from the Table.

Below we'll distill the formulae given in the appendix of the recommended reference to the nice, straightforward case of an east-west baseline of length B, for which the only nonzero baseline component is B_y. Our interferometer has an east-west baseline (approximately), so our distilled formulae will be applicable (approximately) to your measurements.

5.2. The Details

The two interferometer telescopes have different distances from the source. The difference can range from zero (if the source is overhead) to nearly the the full baseline (if the source is near the horizon). This distance difference is tiny compared to the distance to the source, but it's important!

It's convenient to think of the different distances in terms of relative path delay in *time* units for the two telescopes; we call this the *geometrical* path delay τ_g. But don't forget! The signals travel through a lot of electronics before they get multiplied and the two paths aren't of equal length, so there is an additional relative delay from the difference in cable length τ_c. The total relative delay is the sum of the two,

$$\tau_{tot} = \tau_g(h_s) + \tau_c \ . \tag{1}$$

We explicitly include the fact that τ_g is a function of time—that is, the hour angle of the source h_s. In contrast, τ_c is independent of time (unless somebody changes the cable setup. . .).

We don't know τ_c (but the least-squares process can tell us what it is). However, we do know τ_g because it's just geometry—the geometry discussed in the reading. For the east-west baseline, we have

$$\tau_g(h_s) = \left[\frac{B_y}{c}\cos\delta\right]\sin h_s \ . \tag{2}$$

The output of the interferometer is the product of what the two telescopes see. If they are looking at a monochromatic source then the voltages for the two telescopes are

$$E_1(t) = \cos(2\pi\nu t) \tag{3}$$

$$E_2(t) = \cos(2\pi\nu[t + \tau_{tot}]) \ . \tag{4}$$

and the product is the interferometer fringe output

$$F(t) = \cos(2\pi\nu t)\cos(2\pi\nu[t + \tau_{tot}]) \ . \tag{5}$$

There's a trigonometric identity that allows us to write this in terms of the sum and difference of the two arguments[5]. The sum term varies rapidly with time and averages to zero; it's the difference term we want, so if we exclude the sum term (and forget about the factor $\frac{1}{2}$) we get

[5] $\cos(A)\cos(B) = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

$$F(h_s) = \cos(2\pi\nu[\tau_g(h_s) + \tau_c]) \ . \tag{6}$$

We want to use a least-squares fit to find the values of quantities that comprise the argument of the cosine. If you have done least-squares fitting before (and if you remember anything about it!) you'll realize that the straightforward least-squares fitting technique won't work on this type of problem. We can simplify things for the fitting process by using another trig identity[6] and writing this as the sum of two trig functions

$$F(h_s) = \cos(2\pi\nu\tau_c)\cos\left[2\pi\nu\left(\frac{B_y}{c}\cos\delta\right)\sin h_s\right] - \sin(2\pi\nu\tau_c)\sin\left[2\pi\nu\left(\frac{B_y}{c}\cos\delta\right)\sin h_s\right] \ . \tag{7}$$

This may not look simpler! But it is, because for the purposes of least-squares it involves only a *single* variable in the trig function arguments—the combination of variables $\left(\frac{B_y}{c}\cos\delta\right)$ (we are assuming that we know the right ascension well enough to get a good value for h_s).

To proceed with least-squares, replace $\cos(2\pi\nu\tau_c)$ and $\sin(2\pi\nu\tau_c)$ by two "unknown constants" A and B, respectively; assume that they are unrelated and solve for them using the standard least-squares process. Also, it's convenient and intuitive to make the substitution

$$\nu\left(\frac{B_y}{c}\cos\delta\right) = \left(\frac{B_y}{\lambda}\cos\delta\right) \tag{8}$$

which expresses the delay in units of wavelength, and thus the "number of turns" or phase. These substitutions give the *Fringe Amplitude for a point source*

$$\boxed{F(h_s) = A\cos\left[2\pi\left(\frac{B_y}{\lambda}\cos\delta\right)\sin h_s\right] - B\sin\left[2\pi\left(\frac{B_y}{\lambda}\cos\delta\right)\sin h_s\right] \ .} \tag{9}$$

Let's take a moment and reflect on this complicated-looking equation, focusing on just the first term (because the second is identical except it's a sine instead of a cosine). We want to develop the concept of a *local fringe frequency* f_f. The argument of the cosine is a constant $C = \left[2\pi\left(\frac{B_y}{\lambda}\cos\delta\right)\right]$ multiplied by $\sin(h_s)$. Now h_s is the hour angle and increases monotonically with time, so we can regard it as time[7]. Now C is multiplied not by h_s itself, but rather by $\sin(h_s)$, which is a nonlinear function of time. The product $C\sin(h_s)$ is the argument of the cosine term and makes it oscillate back and forth, but at a frequency that depends on time as h_s changes.

---

[6] $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

[7] Except that its units are radians. This is no problem: 24 hours is 2π radians.

This leads to the concept of the local fringe frequency. To see this, expand the hour angle term $\sin(h_s)$ into a Taylor series centered on the current hour angle of the source h_s:

$$\sin h = \sin(h_s) + \Delta h \left. \frac{d\sin(h)}{dh} \right|_{h_s} = \sin(h_s) + \Delta h \cos(h_s) \tag{10}$$

The local fringe frequency is contained in the second term because, for a small region around h_s in equation 9, $F(h)$ varies as $f_f\ \Delta h$, where the local fringe frequency f_f is

$$f_f = \frac{C}{2\pi}\cos(h_s) = \left(\frac{B_y}{\lambda}\cos\delta\right)\cos(h_s)\ . \tag{11}$$

This is the local fringe frequency in cycles per radian on the sky. If you want to turn it into cycles per hour coming out of the interferometer's multiplier, multiply by $\frac{dh_s}{dt} = \frac{2\pi}{24}$; or cycles per minute, multiply by $\frac{2\pi}{60\times24}$; etc. At the meridian ($h_s = 0$), the fringe frequency is $[f_f = \left(\frac{B_y}{\lambda}\cos\delta\right) \approx 0.029\cos\delta]$ cycles per second and the period is $\frac{1}{f_f} = \frac{35}{\cos\delta}$ seconds. (This assumes B_y =10 m and $\lambda = 2.5$ cm; *check these numbers* to make sure you understand this calculation!)

Now let's return to the least-squares solution of equation 9. Make a guess at the proper value of $\left[\frac{B_y}{\lambda}\cos\delta\right]$ and, also, *adopt* a value for the right ascension α (required so that you can compute h from the local sidereal time LST). With this, you know the arguments of the trig functions and this means that you can solve for A and B using the standard least-squares process; be sure and *save the sum of the squares of the residuals.* Then change the guessed-at value of $\left[\frac{B_y}{\lambda}\cos\delta\right]$ and do it again. Do this a number of times and plot the sum-of-squares versus the guessed-at value of $\left[\frac{B_y}{\lambda}\cos\delta\right]$. The best value of δ is where the sum-of-squares is a minimum. What is this? *It's [brute-force] least-squares!* You might try doing the same kind of iteration with the right ascension, but my guess is that it won't make much difference. Then use A and B to calculate the cable length difference τ_c.

Notice!!! The parameter of interest is the declination δ. In contrast, *we suggest solving for the combination* $\left[\frac{B_y}{\lambda}\cos\delta\right]$. Why is this? When you do the least-squares fitting you need to know the baseline length, the distance between the telescopes. You have to measure this! And if you do it wrong, then... well, suppose your measurement of the baseline is too small—by a factor of 100 (because you measured the length in cm instead of m!). Look at the function you fit: it has the baseline multiplied by $\cos(\delta)$. *cos*(δ) can never be larger than one. So if you use a baseline that is too small in your solution, the best value for $\cos(\delta)$ might be larger than unity—meaning that *no* declination will give a satisfactory solution! The most straightforward way around this is to fit not for the declination, but for $\left[\frac{B_y}{\lambda}\cos\delta\right]$, and then after deriving this parameter divide by the baseline B_y to find the best value of $\cos(\delta)$. This allows $\cos(\delta)$ to be greater than 1 in your trial fits—which might be especially important for sources near the equator.

If you do this least-squares reduction for a bunch of sources, you will get all of their declinations.

However, you will find different cable lengths for each source. Clearly, this is a problem because nobody has changed the setup during your measurements (you hope). The different cable lengths mean that the *adopted* right ascensions are not all mutually consistent. You need to change the relative right ascensions in such a way that the fits all give the same cable delay. This will require comparing results among groups and doing some iteration. This is a challenge: never before in this lab have the relative right ascensions been reliably determined. Have fun!

5.3. Least-squares fitting: Commentary

Above, we discussed the technique least-squares fitting for the declination. This is not a straightforward problem because $\frac{dF(h_s)}{d\delta}$ is not equal to a constant—rather, it depends on time (i.e., hour angle h). This means you have to do what's called a *nonlinear least-squares fit*. The general technique for this—which is not what we recommended—is: you guess a value for the declination, expand the equation for $F(h)$ in a one-term Taylor series about this guess, and solve for the correction. If you're lucky the process converges rapidly. If you have time, you might try this!

We suggested a different technique above, one I call the "brute force technique". It simply does the least-squares fitting process by iteration and inspection: you minimize the sum-of-squares of the residuals yourself instead of by a mathematical procedure. This minimization is exactly what the least-squares fit does! If there's only a *single* variable involved, then it's straightforward to use the brute-force technique. But if there's more than one, then things get complicated rapidly. This is why we rewrote the equations above so that there was only a single variable involved in the nonlinear fit.

6. MEASURING 1-D BRIGHTNESS DISTRIBUTIONS: THE FIRST STEP OF MAKING MAPS

When we think of a time-variable signal, we think of frequency as being cycles per second—and its inverse, the period, is in seconds, the number of seconds that separates adjacent peaks of the sine wave.

The interferometer projects a giant sine wave on the sky. Its frequency, which changes with position, is measured in cycles per radian—and its inverse, the period, is the angular separation of adjacent peaks, measured in the angular units of radians. You can, of course, also think of frequency in terms of cycles per degree or cycles per arcminute, with the corresponding periods ("fringe separation") in units of degrees or arcminutes.

When we observe with a range of baseline lengths and orientations, the giant sine waves in the sky have corresponding ranges of frequencies and orientations. We sample brightness of the sky in *Fourier* space. The fringes at each baseline length and orientation have amplitudes and phases. To recover the brightness of the sky in *real, angular* space, we measure as many Fourier components as we can and take their Fourier transform. If we had complete sampling in Fourier space, we would recover the true brightness distribution. In real life, we have *in*complete sampling, so we recover a distorted representation of the true distribution. There is a whole literature of techniques for minimizing this distortion, the most prominent being "cleaning" and "maximum entropy". Full-fledged research arrays, such as the Very Large Array (VLA) in New Mexico, rely on these techniques to map the sky.

In our case we have just two dishes along an east-west line. The effective baseline length changes as the source rises higher in the sky, and if the source is away from declination $\delta = 0°$ the orientation of the baseline also changes, at least to some degree. We will map the Sun and Moon, which never get very far from $\delta = 0°$, so effectively we have only a 1-d sampling of the source with a range of baseline lengths.

Figure 1 illustrates this 1-d concept. It presents a generic circular source of uniform brightness in the sky, which we call the MUN—a bastardization of the MOON and the SUN[8]. The top panel shows the MUN in the sky as it really is: the fringes cover the 2-d object. At the bottom, we integrate along vertical strips to get the 1-d brightness distribution—the vertically-integrated 1-d equivalent, in which both the brightness distribution and the fringes depend on only one coordinate.

This one coordinate is the horizontal direction in the bottom panel of the Figure. In real life this is hour angle because we have an east-west interferometer and our source is at low declination—meaning that the baseline projected on the sky is mainly east-west, the direction of hour angle. Thus we denote this direction by the letter h.

As the Earth rotates, the source moves through the fringe pattern to give the fringe response $R(h_s)$. For a point source, $R(h_s) = F(h_s)$ (see equation 9); for an extended source, we have to integrate over the extent of the source. Let $I(h - h_s)$ be the 1-d intensity distribution in the sky. The source center is the intensity-weighted mean of the position, i.e.

$$h_s = \frac{\int I(h) h dh}{\int I(h) dh} \ . \tag{12}$$

That is, on the bottom panel of Figure 1, $I(h - h_s)$ is the intensity of the source (vertical direction) and $\Delta h = h - h_s$ the horizontal coordinate—the hour angle h relative to the hour angle of the source center h_s.

We can express the interferometer response $R(h_s)$ using equation 9, which is for a point source; for an extended source, we imagine $I(\Delta h)$ as being composed of little slices in hour angle, with each little slice of the source characterized by its position offset Δh and its intensity $I(\Delta h)$, so we

[8]In truth, it represents neither, because neither the Sun nor the Moon has uniform surface brightness.

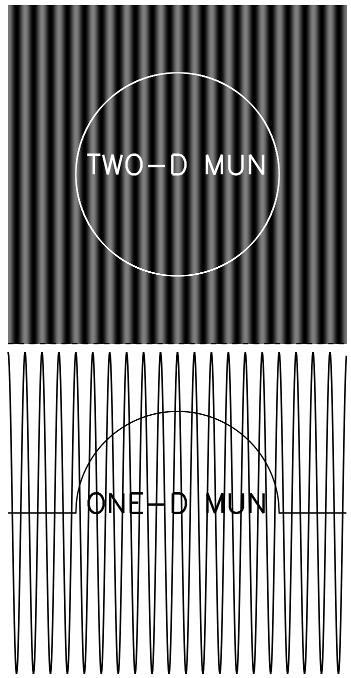

Fig. 1.— The 2-d and 1-d MUN. At the top, we see the situation in the sky as it really is: the fringes cover the 2-d object. At the bottom, we integrate along vertical strips to get the 1-d brightness distribution and, also, the fringe amplitude (which goes from –1 to +1).

just integrate:

$$R(h_s) = A \int I(\Delta h) \cos\left[2\pi \left(\frac{B}{\lambda} \cos\delta\right) \sin h\right] d\Delta h + B \int I(\Delta h) \sin\left[2\pi \left(\frac{B}{\lambda} \cos\delta\right) \sin h\right] d\Delta h \quad (13)$$

Now express $\cos\left[2\pi\left(\frac{B}{\lambda}\cos\delta\right)\sin h\right]$ in terms of the local fringe frequency (equations 10 and 11), which replaces $\cos\left[2\pi\left(\frac{B}{\lambda}\cos\delta\right)\sin h\right]$ by a sum of two terms, which we temporarily denote α and β[9]. Now, as usual, we use trig identities $[\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ and $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)]$. The first (cosine) term of equation 13 becomes

$$R^{\cos}(h_s) = A\ \cos(\alpha) \int I(\Delta h) \cos(2\pi f_f \Delta h)\ d\Delta h - A\ \sin(\alpha) \int I(\Delta h) \sin(2\pi f_f \Delta h)\ d\Delta h \quad (14)$$

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[9]where $\alpha = 2\pi\left(\frac{B}{\lambda}\cos\delta\right)\sin h_s$ and $\beta = 2\pi f_f \Delta h$.

with an equivalent, similar expression for $R^{\sin}(h_s)$.

For our source (the MUN), *we assume that $I(\Delta h)$ is symmetric* (This also retains our algebraic sanity.). This means that in the above equation the second term, which is antisymmetric, integrates to zero. Similarly, the antisymmetric term in the equivalent equation $R^{\sin}(h_s)$ also integrates to zero, so we end up with $R(h_s) = R^{\cos}(h_s) + R^{\sin}(h_s)$, or

$$R(h_s) = \underbrace{F(h_s)}_{Point-source\ Fringe} \times \underbrace{\int I(\Delta h)\cos(2\pi f_f \Delta h)\ d\Delta h}_{Fringe\ Modulator} \tag{15}$$

Note the structure of equation 15. It consists of two factors. The first "Point-source Fringe" term is identical to equation 9—it's the response to a point source located at $\Delta h = 0$. The other modulates (multiplies) this function.

Generally, *the modulating function is the Fourier transform of the source intensity distribution on the sky*. Here, we assumed a one-dimensional symmetric source, which means that the sine portion of the Fourier transform is zero; this is why equation 15 is only a cosine Fourier transform instead of a full one. More generally, the Fringe Modulator depends on the two-dimensional map of intensity on the sky, so it's a double integral instead of a single one.

Figure 2 (top panel) shows two examples of 1-d brightness distributions, a flat and a cosine distribution. The bottom panel shows the Fourier transforms. Both of the modulating functions are trig functions. In particular, for the flat distribution the modulating function is $\frac{\sin(2\pi f_f R)}{2\pi f_f R}$. It can (and does!) go through zero. *The locations of these zero points provide crucial information about the source structure.* The zeros occur for $f_f = \frac{n}{2R}$. It's more intuitive to express the zeros in terms of fringe *period* (equal to $\frac{1}{f_f}$): the zeros occur at $Period = \frac{2R}{n}$. There's a zero whenever there's an integral number of fringe periods over the source width. This makes perfect sense, because then the source contributes equally to the negative and positive portions of the fringe and the net integral is zero.

7. MEASURING THE DIAMETER OF A CIRCULAR SOURCE

7.1. Theory and Math

Our goal is to measure and compare the diameters of the Sun and Moon. We'll make the assumption that the sources are uniformly-bright disks of radius R, which means

$$I(\Delta h) = \frac{(R^2 - \Delta h^2)^{1/2}}{R} \tag{16}$$

To obtain the theoretical modulating function MF_{theory}, you use the integral in equation 15, which

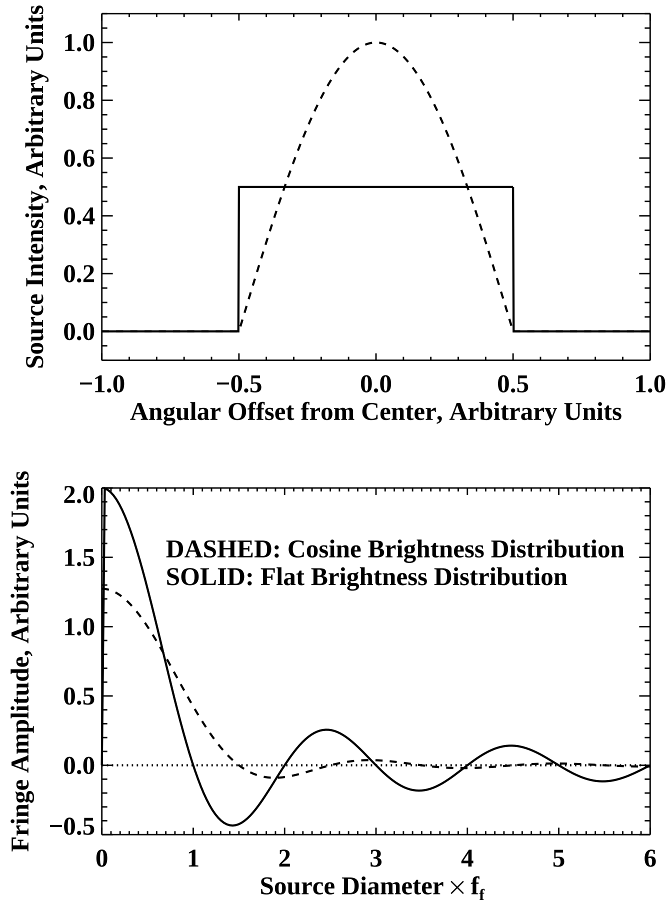

Fig. 2.— Examples of 1-d brightness distributions and their Fourier transforms. Top panel: the brightness distributions. Bottom: the Fourier transforms (Fringe Amplitude vs. $\frac{Source\ Diameter}{FringePeriod}$). In both, panels, the solid line is for a flat brightness distribution and the dashed one for a cosine distribution.

is

$$MF_{theory} = \frac{1}{R}\int_{-R}^{R}(R^2 - \Delta h^2)^{1/2}\cos(2\pi f_f \Delta h)d\Delta h \tag{17}$$

If you want, you can do this analytically by substituting $\Delta(R\cos(\theta)$ for Δh; you end up with a Bessel function.

We are running a lab class, not a math class, so let's proceed by doing the integral *numerically*!

To accomplish this, split $I(\Delta h)$ into $2N+1$ tiny little slices (the total number is odd, which makes the slices symmetric about $\Delta h = 0$). Each slice has width $\delta h = \frac{R}{N}$, and $\Delta h_n = n\delta h$, where n runs from $-N$ to $+N$. Then the integral becomes a sum:

$$MF_{theory} \approx \frac{1}{R} \sum_{n=-N}^{n=+N} [R^2 - (n\delta h)^2]^{1/2} \cos(2\pi f_f n\delta h)\delta h \tag{18}$$

which we rewrite as

$$MF_{theory} \approx \delta h \sum_{n=-N}^{n=+N} \left[1 - \left(\frac{n}{N}\right)^2\right]^{1/2} \cos\left(\frac{2\pi f_f R n}{N}\right) \tag{19}$$

7.2. Important Mathematical and ¡¡¡PRACTICAL!!! Point!

Note the *first important point* that $MF_{theory} \ldots$

- is a function *only* of the combination $f_f R$, and
- in particular, has zero crossings that occur at specific values of $f_f R$.

Note the *second important point* that for $MF_{observed} \ldots$

- The zero crossings occur at specific measured values of f_f.

Thus, by comparing the zero crossing numbers for MF_{theory} and $MF_{observed}$, you get the radius R.

8. REFERENCE READING on INTERFEROMETRY AND APERTURE SYNTHESIS

The appropriate reference for our purposes is the article *Interferometry and Aperture Synthesis*, which is chapter 10 of the book *Galactic and Extragalactic Radio Astronomy, First Edition.* The authors are Fomalont and Wright; Melvyn Wright is a research scientist in our radio lab here at Berkeley and is a real expert. This chapter is excellent, providing the basics without excessive detail (although it has more than we need). If you want more depth than we provide here, we suggest the following sections of this chapter: **(1)** §10.1.3, which describes a two-element interferometer; section e of this chapter is on polarization and you can skip it; **(2)** §10.2.1 and §10.2.2, which describe "a working interferometer"; and **(3)** Appendix II, which describes the geometrical details.

There is a scaling mistake in their equation for the fringe frequency ν_f: their equation needs to be multiplied by the rotation rate of the Earth in radians per second. For example, for an east-west baseline of 343.8 wavelengths looking at declination zero on the meridian, the fringe frequency on the sky is 343.8 fringes per radian. This means that the fringe spacing on the sky is $\frac{1}{343.8}$ radians or 10 arcmin; it takes the Earth 40 seconds of time to turn through 10 arcmin, so the fringe period in this case is 40 seconds and $\nu_f = .025$ Hz. More generally, for this east-west interferometer the fringe frequency is $\nu_f = .025 \cos\delta \cos h$ Hz, where δ is the declination and h the hour angle.

If you want to know really everything and in complete mathematical detail, read the book *Interferometry and Synthesis in Radio Astronomy* by Thompson, Moran, and Swenson. But such detail is more than we want and can be overwhelming. Our main interest is the geometry—how the baseline projects on the $u - v$ plane. This is in chapter 4, and the most important sections for us are chapter 4.2 and 4.4.

With the longer baselines of research-class interferometers/arrays comes increased angular resolution, and all of our sources become finite in angular size, which means that the telescope arrays can map the sources using Fourier techniques. Some types of source are so small that mapping them requires interferometers with baseline lengths comparable to the Earth's diameter, a technique called "Very Long Baseline Interferometry" (VLBI). And some sources, such as pulsars, are even too small for VLBI!