Analog Lab: Impedance, Filters, Amplifiers, and Noise

January 20, 2014

The purpose of this lab is to learn some basic principles of electronic components and circuits, as well as some laboratory methods. The principles of impedance, filtering, amplification, and noise are not only central to the operation of radio telescopes, they are behind almost every modern technology you can think of: radio, wifi, cell phones, speakers, ...you name it. By the end of this lab, you should be moderately comfortable using a multimeter, measuring RF signals, building circuits, reading circuit diagrams, and poking around under the hood of electronic gadgets. You should also be able to describe mathematically how impedances combine to make low-pass and high-pass filters, how random numbers add together, and how noise is produced and passed along through a signal chain. This lab runs for 3 weeks and covers a lot of ground. Get ahead early.

1 Week 1

Prerequisites

Reading: Horowitz & Hill, Ch. 1

- Ohm's Law
- Thévenin Equivalent Resistance
- Capacitance and Inductance
- RC Filters
- Diodes

Materials

- breadboard
- misc R, C, L components
- potentiometer ("pot")
- power supply
- function generator w/ external FM reference
- oscilloscope w/ probes
- voltimeter
- speaker w/ amplifier

Some Thoughts

Breadboards

Breadboards are convenient for constructing circuts for this lab. Take some time to get to know yours using a ohmmeter. In particular, figure out what is connected, and what is not.

Grounding

Ground means different things in different contexts (e.g. earth ground, signal ground, chassis ground). For our purposes in this lab, ground simply provides a return path for current. We define it to be 0V (i.e. it is the reference from which we measure voltage differences). The ground of your circuit should be tied to the ground of your power supply. It is good practice to consistently use a color (black, blue, and green are good choices) to indicate ground lines in your circuits.

Capacitor Orientation

Some capacitors, particularly electrolytics, care about which direction voltage is applied across them. Often, one of the leads is longer, and may have a '+' near it on the body of the capacitor. Always connect this lead to the higher voltage. Failure to do so can, for higher voltages, cause them to explode.

Oscilloscopes

'Scopes can be intimidating. The solution is to press buttons until they submit. Seriously, don't be afraid to push buttons. In order to show a static display of a repeating signal, scopes need to trigger on some edge of an input signal. This is always configurable, and you'll need to figure out how. You can adjust the time (X axis) and voltage (Y axis) ranges, and should do so regularly. Many scopes have an "auto" button, which is a totally useful cop-out. Probes that connect to the scope often amplify (10:1), and scopes have a setting to undo this for the display. Finally, there is a difference between attaching a probe and attaching a cable. This difference has to do with termination. We haven't covered yet what termination is (it's next week), but suffice it to say that in order for your voltage scale to mean something, you should select 50Ω if you have connected a cable (BNC, SMA, etc.), and 1MΩ if you are using a probe.

FM Radio

Frequency modulation (FM, see Figure 1) is a clever scheme for encoding a signal. Amplitude modulation (AM) radio directly mixes a carrier tone with a signal, so that the amplitude of the signal creates an envelope inside of which the carrier tone oscillates. FM instead translates the amplitude of the signal into a change in the frequency of the carrier tone (see above). FM radio stations typically transmit between 88 and 108 MHz in the US. However, for the purpose of this lab, owing to lack of radio reception in the lab, and the difficulty in using discrete components at higher frequencies, we will create our own radio station at 1.045 MHz, instead of 104.5 MHz. Typically, FM radio stations are spaced every 200 kHz, and have a frequency deviation of ±75 kHz at maximum scale input.

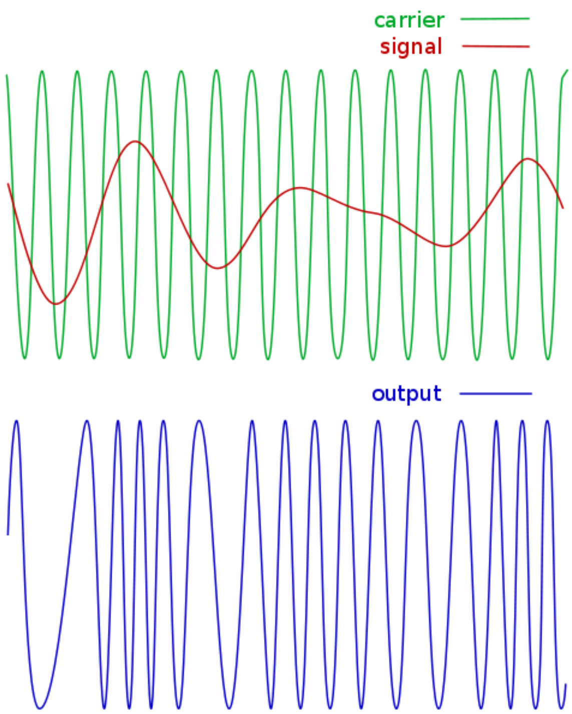

Figure 1: Frequency modulation

1.1 Resistive Voltage Divider

1.1.1 Useful Equations

$$V = IR \tag{1}$$

$$P = IV \tag{2}$$

1.1.2 Activities

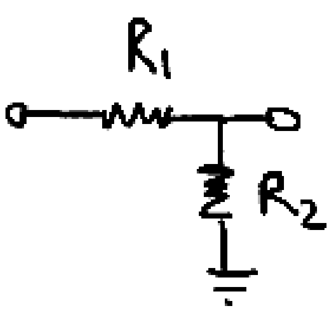

Figure 2: A voltage divider

- build voltage divider (Figure 2) out of two 1k resistors, connect input to +5V DC voltage source
- measure voltage at output
- measure current (think carefully about where before burning out the multimeter fuse!)
- apply 1 MHz sine wave, view input and output on oscilloscope
- append a second voltage divider to the output of the first (choose resistor values carefully)
- for a voltage divider, is it better to have high impedances or low? what considerations might drive you in either direction?
- from the perspective of the second voltage divider, what is the Thèvenin equivalent resistance of the first divider, as seen from its output?

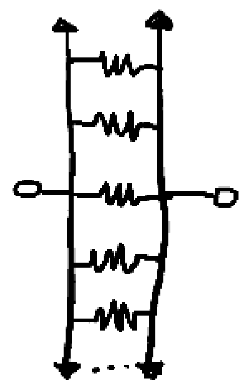

Figure 3: An infinite series of resistors

- if each resistor in Figure 3 is the same value, what is the equivalent resistance between the two terminals? (BTW, I still haven't solved http://xkcd.com/356, but don't get nerd sniped while doing this lab!)

1.2 Capacitive Voltage Divider

1.2.1 Useful Equations

$$I = C\frac{dV}{dt} \tag{3}$$

$$Z_C = \frac{1}{j\omega C} \tag{4}$$

1.2.2 Activities

- build a voltage divider out of two 1μF capacitors, connect input to 1 MHz sine wave
- compare input and output on oscilloscope. is it what you expected? what is the DC level at the output? resistor?
- add a 1k resistor to ground at the output, and then for your modified circuit, graph the expected V_{out}/V_{in} versus frequency

1.3 RC filter

1.3.1 Activities

- design a high-pass filter with cutoff (-3dB) at 100 kHz
- design a low-pass filter with the same cutoff
- plot the expected responses of each. spot-check with oscilliscope measurements.
- what could you do to get steeper transitions in RC filters such as these?

1.4 LC filter

1.4.1 Useful Equations

$$Z_L = j\omega L \tag{5}$$

$$Q = \omega_0 RC = \frac{f_0}{\Delta f_{3dB}} \tag{6}$$

1.4.2 Activities

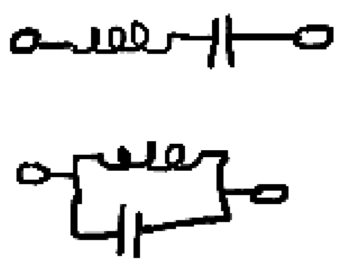

Figure 4: An inductor in series and in parallel with a capacitor

- plot the theoretical impedance versus frequency of a 1 μH inductor in parallel with and in series with a 1 μF capacitor (Figure 4).

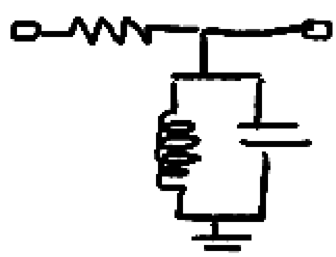

Figure 5: A bandpass RLC filter

- design and build an LC band-pass filter (Figure 5) tuned to 1 MHz

- choose an R to select a quality factor appropriate for $\Delta f_{3dB} = 200kHz$.

1.5 Diodes

- measure the voltage drop over a conducting diode (important: limit current flow with a resistor!)
- use different resistor values (or a pot) and measure the voltage drop as a function of current
- use a large (± 1V) oscillating signal into the diode and view the original and rectified signal on an oscilloscope
- apply a low-pass filter with time constant of order the input frequency, and view the output on an oscilloscope. generate a plot of what is going on here.

1.6 FM demodulation

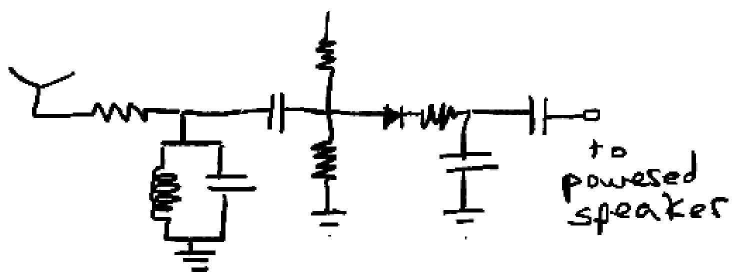

Figure 6: An FM demodulation circuit

- use your LC circuit to convert a frequency-modulated signal into an amplitude modulated signal
- use your diode circuit as an "envelope detector" that filters out the carrier frequency and extracts the amplitude modulation. Use a 100 kHz low-pass filter in the envelope detection. (Humans can only hear up to 40 kHz or so). You should have something like Figure 6.
- connect in our faked FM station at 1.045 MHz
- connect the output through a coupling capacitor (to remove the DC bias) to amplifying audio speakers
- enjoy listening to the music

1.7 For Your Report

- Present a circuit diagram of an FM radio receiver, highlighting blocks of components that act as a unit to perform some action (filtering, biasing, etc.)
- Describe in the text the overall design of the FM receiver, with subsections for each functional block, listing salient features of each (e.g. the -3dB point of a filter, the voltage biased to, etc.). Present it as if you invented it, and were writing the seminal paper that will enable others to use this design.
- Using your new understanding of impedances, plot the expected output of the LC filter in your FM receiver as a function of frequency, $f = \omega/2\pi$. Mark the location, in frequency, of

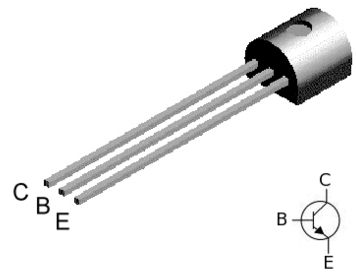

Figure 7: Pinout for a generic NPN transistor (e.g. 2N3904)

the transmission band used in the lab, and explain the rational behind its placement on the LC response curve (e.g. if our station is at 1.045 MHz, why is your LC tuned to 1.0 MHz?)

- Finally, also plot, as a function of frequency, the bandpass of the final filter that defines the band of the output audio signal. Describe (and maybe plot) the rationale for selecting the filter that you did.

2 Week 2

Prerequisites

Reading: Horowitz & Hill, Ch. 2

- Transmission Lines
- Transistors
- Amplifier Circuits

Materials

- rope and string of various thicknesses (pretty long)
- breadboard
- misc R, C, transistor components
- power supply
- function generator w/ external FM reference
- oscilloscope w/ probes
- voltimeter
- speaker w/o amplifier
- long cable (as long as possible, longer than 100m would be great)

Some Thoughts

Transistors

We'll be using an NPN bipolar junction transistor in this lab (see Figure 7). With the flat face to you, pins typically read, from left to right, emitter, base, collector. Now you know.

Speakers

Last week, we just stuck our signal into an amplifying speaker and didn't think much about it. This weak, since we are building our own amplifier, you might be wondering what's left that makes a "speaker." Speakers aren't that complicated. At their simplest, they are solenoids that push/pull on a small magnet attached to a diaphram. Depending on which direction current is made to flow through the solenoid by an alternating voltage signal, a magnetic field is set up that attracts or repels the magnet on the diaphram. The diaphram translates the resulting motion into pressure waves that propagate through the air.

Termination

At the end of this lab, we'll be playing with some 50Ω (and maybe some 75Ω) cable, examining how waves are reflected and transmitted. Of course, now you also have a better idea of why the oscilloscope has selectable termination, and why it might be different depending on whether you connect as scope probe or a BNC cable to the input. Now that you know why and how, you have no excuse not to exercise good termination practices!

2.1 Impedance Mismatches on Rope

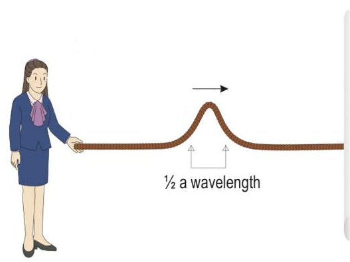

Figure 8: Sending a pulse down a rope

- Get several lengths of ropes of different thicknesses. Tie them together and tie the end of the thinner rope to a doorknob or some other stable structure. Hold the untied end, pull the rope relatively taut, and then, with a quick up-down movement of the hand, send a pulse down the rope. Observe what happens at the interface between the ropes, and at the doorknob.
- Switch the rope around so that the thicker end is now tied to the doorknob. Repeat the above. Any difference?
- What would happen, do you think, if the far end of the rope, instead of being tied to a fixed point, was tied to a ring that could move up and down on a pole?
- Any idea how to stop the wave from reflecting off the far end of the rope?
- If you have two thick ropes joined by a very short thin piece (small with respect to the wavelength of the signal), what happens at that interface?
- What about vice versa (thin ropes joined by a short thick piece)?

2.2 Impedance Mismatches in a Transmission Line

- Get a function generator that can output square waves. Get a long stretch of cable and connect the function generator output to one end of the cable. Leave a point that you can probe with an oscilloscope.
- Use an oscilloscope to try to detect the reflected wave. Draw the superposition of transmitting and reflecting waveforms that produces your output.
- How long is your cable? How fast did your pulse travel? (BTW, it can be helpful to know that, round-about, light travels 1 ft in 1 ns, and that waves on a cable can be up to a factor of 2 slower than that.)
- Use a 10Ω resistor to terminate the far end of the cable. Observe the change.
- Now properly terminate the far end, and recover your waveform! What was the impedance of the cable?

2.3 Building an Follower

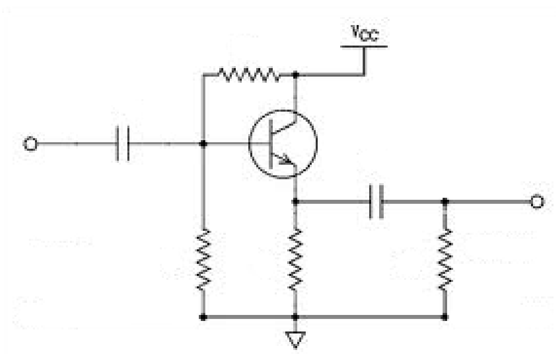

Figure 9: An emitter follower that uses a biasing circuit.

- Build the emitter-follower circuit shown in Figure 9, choosing approprite values for resistors and capacitors. Use a 2N3904 transistor, or a similar substitute NPN BJT. You will power the circuit with $V_{cc} = +5V$, and will want to amplify signals in the range 100Hz to 50kHz. Mind your R's and C's!
- Measure the bias voltage V_{BE}.
- What is the purpose of the emitter resistor (that is, the resistor most immediately connected to the emitter)?
- Input a 10 kHz sine wave with 1V amplitude. What is the relationship between the input and output voltages?
- Predict first, and then measure, the maximum sine amplitude for which this circuit correctly operates.
- Measure the bias voltage at the base. How small can you make the emitter resistor before it changes? You might need to make the resistors in your bias circuit large so that you don't burn up the emitter resistor.

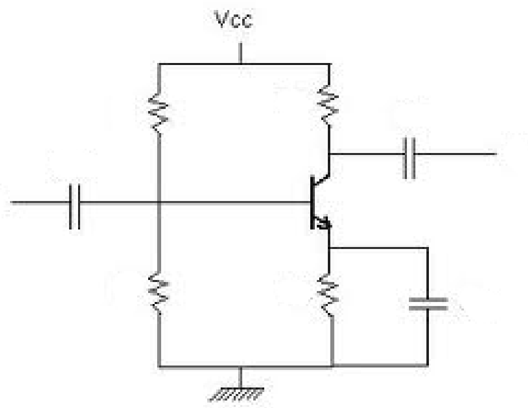

Figure 10: An amplifier based on an NPN BJT.

2.4 Building an Amplifier for our FM radio

- Build the amplifier circuit shown in Figure 10, choosing appropriate values for resistors and capacitors, using the same constraints as for the follower circuit above, and choosing a DC gain of 2. For the time being, omit the emitter capacitor.
- What is the voltage at the collector?
- Input a 10 kHz sine wave with 100mV amplitude. What is the relationship between the input and output voltages? If you did not achieve a gain of 2, why not?
- Predict first, and then measure, the maximum sine amplitude for which this circuit correctly operates.
- Now add in the emitter capacitor to select for a gain of 10 at 10kHz. Again, mind your R's and C's!
- Input a 10 kHz sine wave with 100mV amplitude. What is the relationship between the input and output voltages? If you did not achieve a gain of 10, why not?
- Predict first, and then measure, the gain at 20 kHz.
- Can you think of a way to make this circuit have the same gain across our whole band of interest?
- Finally, attach a passive speaker to the output, your passive FM demodulator to the input, and bask in the sound of your success. You'll need to pay attention to the impedance of the speaker, and plan accordingly.

2.5 For Your Report

- Add a section describing your speaker amplifier, including a circuit diagram, and describing how it functions (and what the various functional parts are). I'll be looking for a description of how the base and collector voltages are set, and why they needed to be set where they are.
- Specify the input and output impedance of your circuit at audio ($\sim$10 kHz) frequencies.
- Suppose your speaker amplifier connects to a speaker at the end of a long run of wire. Given the amplifier circuit that you built, what impedance cable to you recommend, and how would you terminate it (incorporating an 8-Ohm speaker) to avoid reflections? For now, don't worry, unless you want to, about maximizing the current through the speaker (to make it LOUD).

- Define the operating band of your amplifier. Over what frequency ranges does it operate, and what specifically determines the bounds on this range?

3 Week 3

Prerequisites

- Central Limit Theorem
- Johnson-Nyquist Noise
- Receiver Temperature
- Radiometer Equation

Materials

- 2W 50Ω resistor
- power supply
- oscilloscope w/ probes
- minicircuits amplifiers
- voltimeter
- infrared (remote) thermometer

Some Thoughts

Minicircuits Amplifiers

Now that you've built your own amplifier, you have a feel for what goes on inside an amplifier. We're now going to use pre-packaged amplifiers that are commercially available (Minicircuits is a typical vendor). These will be SMA in, SMA out, with two leads for V_{cc} (+15V, for the parts we're using) and ground. These amplifiers specify their gain in dB, and you do have to be a bit careful about signal levels. As a general rule, the output of amplifiers should be connected (to a 50Ω load) before they are powered on, so that they don't have to dissipate their output power internally.

Infrared Thermometer

If you assume that objects radiate heat as a blackbodies (which we'll discuss in later labs), you can measure their temperature remotely just by their infrared radiation. The infrared thermometer we will use in this lab is an infrared telescope (with a well-defined beam size) that is calibrated to output a voltage in mV that is the temperature in Fahrenheit. Use it wisely.

Interference

In this lab, we're going to attempt to make a relatively sensitive measurement of the Johnson noise on a 50Ω resistor. It turns out that lots of things can produce signals at this level that can interfere with your measurement. For one, if you make a loop that electrons can flow around (which, after all, is what a circuit is), then a changing magnetic flux through that loop produces an EMF that drives a current that sets up a voltage across the resistor. This may be the very first radio antenna we build in this class. Unfortunately, that's not what we were going for, so we need ways to avoid this. Since the area inside the loop matters, you can try to make your circuits physically narrow. You might also try making a Faraday cage.

Writing Code for Lab

This is the first lab where a significant portion of the lab is writing code. So the first thing you should do set up a github account and start a repository that will contain all the code you turn in for this class (and while you're at it, why not your LaTeXreports too?). Next, clone your repo onto your computer. (If you haven't yet, you should get Git, Python, Numpy, and Pylab installed). Inside your repository, add a directory "lab_analog", and create the file: central_limit.py. Commit, and push your changes back up to your github account. Inside your lab report, make sure you link to your repository in your "Methods" section, along with the hash tag that indicates which commit was the one used to produce your results.

3.1 Calculating the Noise Figure of a Reciever

Determine Amount of Gain Needed

- Calculate how much amplification is needed to bring the Johnson noise on a 50Ω resistor up to at least the ∼1mV RMS level that you can measure on a scope. You'll have to pick an appropriate bandwidth, and you might want to check that you can get a filter that supplies that bandwidth.
- Looking at the data sheets for the minicircuits amplifiers, choose a number of gain stages.
- Connect the gain stages (and the filter at the end), and power them at +15V. IMPORTANT: Most amplifiers want to have the output connected first, so that the output power has some place to go.

Estimate and Measure the Resistor Temperature

$$P = A\epsilon\sigma T^4 \tag{7}$$

- Set a power supply to the appropriate voltage to dissipate ∼2W in a 50Ω resistor (make sure it is a beefy resistor rated to 2W, and do not exceed that threshold, or meltdown will ensue).
- Using the Stefan-Boltzmann Law, calculate what temperature you think the resistor will hit.
- Use an infrared thermometer to measure the temperature. Were you close?

Measure the Noise Figure

- Determine the power out of the receiver for the input resistor at room temperature. Ideally, you'd do this with a power meter or a spectrum analyzer, but today, we'll do the poor man's method of eyeballing one standard deviation from the mean on an oscilloscope. This isn't easy, and I apologize for that. To keep your sanity, you should estimate errorbars on this measurement, so you can see the effect of error on your bottom line.
- As in the previous step, set the resistor dissipating 2W, and measure the power out of the receiver. IMPORTANT: Use a blocking cap to prevent a large DC voltage from entering the amplifier chain. Otherwise, you could damage your amplifiers. When in doubt, ask.
- Since you already know the temperature the resistor hits when dissipating 2W, you should be able to solve for the gain and the receiver temperature (and if you want, you can calculate

your Y factor). This would be a good time to use your error bars to determine your confidence interval on your measurement of the gain and receiver temperature. Make a plot and draw some lines!

- Convert your receiver temperature to a noise figure.

3.2 Demonstrate the Central Limit Theorem

- Write a program (in the central_limit.py file that you created earlier) that shows that, in the large-N limit, adding samples drawn from non-Gaussian random distributions converges to a Gaussian distribution.
- Also show that the standard deviation of the mean of N Gaussian-random samples decreases as $\sqrt{N}$. If you are trying to characterize samples of an unknown character to show that they are noise-like, this is called an Allen variance test.

3.3 For Your Report

- Pretend that the amplifier whose noise figure you were determining was the one you built in §2. Describe the methodology for determining the noise figure, including the relevant equations used.
- Report the final noise figure you measured. Give error bars. What are your sources of error?
- One last charade. Let's say that you also wanted to test how the noise output by your amplifier integrates down, to be sure that there are no systematics present. Pretend you placed a known-good noise source in front of your amplifier and digitized the output of your amplifier, and you get the numbers output by your central_limit.py program. Perform an Allen variance test to show that the noise from your amplifier behaves as you'd expect. Produce a plot of variance (or standard deviation, either one) versus number of samples averaged, along with the slope of the line you're shooting for.