

MATH3823 - Solutions to Chapter 7 Exercises

Exercise 7.1 Let **status** ($i = 1, 2$) and **age** ($j = 1, \dots, m$) be factors, and let **y** store the entries in the contingency table. Also, let **agen** be a quantitative or numeric variable containing the same numbers as **age**. This model can be fitted in *R* by the command

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glm(y ~ status + age + status:agen, poisson)
```

with parameters α_i , β_j and γ_i , $i = 1, 2$; $j = 1, \dots, m$. The aliasing constraints are $\alpha_1 = 0$, $\beta_1 = 0$ and $\gamma_1 = 0$ (since the effects of γ_{1j} can be absorbed into μ and the β_j 's).

Conditioning on y_{+j} , we see that the probability function of y_{1j} is given by

$$\begin{aligned} P(y_{1j} = k | y_{+j} = n) &= P(y_{1j} = k \text{ and } y_{2j} = n - k | y_{+j} = n) \\ &= \frac{e^{-\lambda_{1j}} (\lambda_{1j})^k e^{-\lambda_{2j}} (\lambda_{2j})^{n-k} / (k!(n-k)!)}{e^{-\lambda_{+j}} (\lambda_{+j})^n / n!} \\ &= \binom{n}{k} \pi_j^k (1 - \pi_j)^{n-k}, \quad 0 \leq k \leq n \end{aligned}$$

where $\pi_j = \lambda_{1j} / \lambda_{+j}$. That is $y_{1j} \sim B(y_{+j}, \pi_j)$, where

$$\begin{aligned} \text{logit } \pi_j &= \log \lambda_{1j} - \log \lambda_{2j} = \alpha_1 - \alpha_2 + (\gamma_1 - \gamma_2)j \\ &= -\alpha_2 - \gamma_2 j. \end{aligned}$$

Further, the $\{y_{ij}, j = 1, \dots, m \text{ given } y_{+j}, j = 1, \dots, m\}$ are independent. Thus the binomial data follow a logistic regression with intercept $-\alpha_2$ and slope $-\gamma_2$, which are the identifiable parameters.

End of Solutions to Chapter 7 Exercises