MATH3823 - Solutions to Chapter 7 Exercises

Exercise 7.1 Let `status` ($i = 1, 2$) and `age` ($j = 1, ..., m$) be factors, and let `y` store the entries in the contingency table. Also, let `agen` be a quantitative or numeric variable containing the same numbers as `age`. This model can be fitted in R by the command

$$\texttt{glm(y} \sim \texttt{status} + \texttt{age} + \texttt{status:agen}, \texttt{poisson})$$

with parameters $\alpha_i,\ \beta_j$ and $\gamma_i,\ i = 1, 2;\ j = 1, \ldots, m$. The aliasing constraints are $\alpha_1 = 0,\ \beta_1 = 0$ and $\gamma_1 = 0$ (since the effects of γ_{1j} can be absorbed into μ and the β_j's).

Conditioning on y_{+j}, we see that the probability function of y_{1j} is given by

$$\begin{aligned}
P(y_{1j} = k | y_{+j} = n) &= P(y_{1j} = k \text{ and } y_{2j} = n - k | y_{+j} = n) \\
&= \frac{e^{-\lambda_{ij}} (\lambda_{ij})^k e^{-\lambda_{2j}} (\lambda_{2j})^{n-k} / (k!(n-k)!)}{e^{-\lambda_{+j}} (\lambda_{+j})^n / n!} \\
&= \binom{n}{k} \pi_j^k (1 - \pi_j)^{n-k}, \quad 0 \le k \le n
\end{aligned}$$

where $\pi_j = \lambda_{1j} / \lambda_{+j}$. That is $y_{1j} \sim B(y_{+j}, \pi_j)$, where

$$\begin{aligned}
\operatorname{logit} \pi_j = \log \lambda_{1j} - \log \lambda_{2j} &= \alpha_1 - \alpha_2 + (\gamma_1 - \gamma_2) j \\
&= -\alpha_2 - \gamma_2 j.
\end{aligned}$$

Further, the $\{y_{ij},\ j = 1, \ldots, m$ given $y_{+j},\ j = 1, \ldots, m\}$ are independent. Thus the binomial data follow a logistic regression with intercept $-\alpha_2$ and slope $-\gamma_2$, which are the identifiable parameters.

Exercise 7.2 Equation (6.4) states that:

$$y_{++} = e^{\hat{\mu}} \left(\sum_i e^{\hat{\alpha}_i} \right) \left(\sum_j e^{\hat{\beta}_j} \right).$$

and Equation (6.6) that:

$$y_{+j} = e^{\widehat{\mu}} e^{\widehat{\beta}_j} \left(\sum_i e^{\widehat{\alpha}_i} \right).$$

Taking the second divided by the first, after cancellations, yields the required result.

End of Solutions to Chapter 7 Exercises