MATH5824 weekly exercises

Dr Stuart Barber

April 2022

These questions are intended to be worked on alongside reading the notes. To help keep track of where we are up to, they are separated by week they were set.

Week 2

1. In section 2.1 of the notes, what are $f^{(p)}(t_i-)$ and $f^{(p)}(t_i+)$ in terms of the coefficients $\{a_{ij}\}$? What would happen if we specified additional constraints $f^{(p)}(t_i-) = f^{(p)}(t_i+)$ for $i = 1, \ldots, n$?

Week 4

2. Give a self-contained definition of a natural spline.

3. Let
$$f(t) = 3 + 2t + 4|t|^3 + |t-1|^3.$$
Write f as a cubic polynomial in each of the intervals $(-\infty, 0)$, $(0, 1)$ and $(1, \infty)$. Verify that f and its first two derivatives are continuous at the knots.

   Is f a spline? Is f a natural spline?

4. Let
$$f(t) = 3 + |t| - |t-2|.$$
Show by direct calculation that
$$\int_{-\infty}^{\infty} \{f'(t)\}^2 \, dt = 8.$$
Show that this integral can also be written in the form $-2\mathbf{b}^T K \mathbf{b}$, where you should define $\mathbf{b}$ and K.

Week 5

5. Check that equation (28) in the notes can recover the coefficients of the linear spline in the R script from lecture 2. That is, if a natural linear spline $f(t)$ takes values $4, -1, 1$ at knots $1, 2, 3$, show that $f(t) = 2.5 - 2.5|t-1| + 3.5|t-2| - |t-3|$.

6. (Harder: Use R try to find natural linear and cubic splines which interpolate the beetle data.)

Week 8

7. Consider the 'Old Faithful' data set on geyser eruptions available in R. Use the following commands to learn more about and visualise the data:

   ```
   data(faithful)
   help(faithful)
   plot(faithful)
   ```

   Fit a cubic smoothing spline to these data, using a range of values for the smoothing parameter λ. By eye, suggest a suitable value for λ.