VSA, Analogy, and Dynamic Similarity

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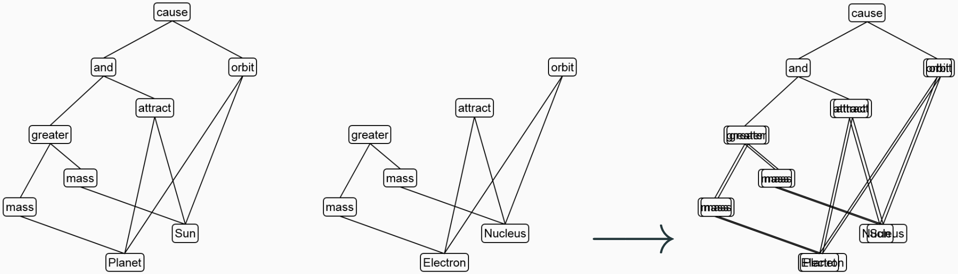
Analogy as structure mapping

- Arguable that analogy is the core of cognition (Blokpoel, Wareham, Haselager, Toni, & van Rooij, 2018; Gust, Krumnack, Kuhnberger, & Schwering, 2008)
- Analogy commonly construed as *structure mapping* between *source* and *target* (Gentner, 1983)
  - Find maximal subgraph isomorphisms between *source* and *target* graphs
  - Find a mapping between *source* and *target* that makes maximal subpgraphs *identical*



Similarity and generalistation

- Cognitive objective to generalise as widely and rapidly as possible
- Generalisation usually invokes/induces a concept of similarity
- Typical statistical/ML models use *literal* similarity
- Relational similarity typically encodes relations as literals
- Analogical similarity is based on relational structure (relations between relations)
    - Relational structures tend to reflect real causes in the environment (?)
    - Relational structures reduce reliance on “good” encoding of literals (?)
- Analogy includes literal similarity as a special case

Static similarity in VSA

- Similarity (angle between vectors) is central to VSA (Kanerva, 2009)
- Typically a fixed mapping from “things” to vectors (representations)
- Emphasis on encodings that yield useful similarity structure (Purdy, 2016; Sahlgren, 2005)
- Representations may be learned …
    - E.g. Random Indexing (Sahlgren, 2005), vector embedding (Pennington, Socher, & Manning, 2014)
- but, are effectively fixed at time of use

Human dynamic similarity

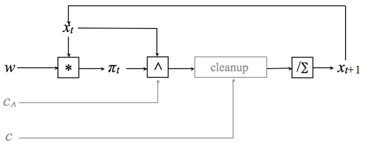
- Human similarity judgments known to be context-dependent (Cheng, 1990)
- Arguable that similarity and analogy are based on the same processes (Gentner & Markman, 1997)
- Arguable that representations are created on-the-fly in response to task demands (Chalmers, French, & Hofstadter, 1992)
- Doesn't necessarily imply that base representations are context-dependent
- Could have dynamic *working* representations derived from the static base representations by context-dependent transforms

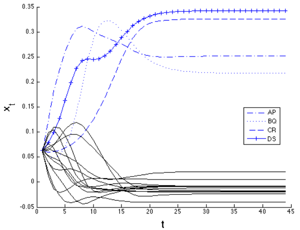
Substitution as a dynamic transformation

- An obvious candidate for a dynamic transformation function in VSA is substitution by binding
    - The substitution mapping can be specified as a vector
    - The substitution mapping vector can be dynamically generated (Kanerva, 2009)
- This implies an internal degree of freedom (a register to hold the substitution vector while it evolves) …
- and a recurrent VSA circuit to provide the dynamics to evolve the substitution vector

Maximal subgraph isomorphism circuit

- Maximal subgraph isomorphism circuit (Gayler & Levy, 2009)
  - Finds the maximal subgraph isomorphism between two graphs represented as vectors
  - Implemented as a recurrent VSA circuit with a register containing a vertex substitution vector that evolves and settles over the course of the computation
  - Final state of substitution vector represents the set of vertex substitutions that best transforms each static graph into the other graph





Connections

- The subgraph isomorphism circuit can be interpreted as related to the recently developed Resonator Circuits for factorisation of VSA representations (Kent, Frady, Sommer, & Olshausen, 2019)
    - They have internal degrees of freedom for each of the factors to be calculated
    - Recurrent VSA dynamics that settles on the factorisation
- Subgraph isomorphism circuit can be interpreted as finding the square root of the product of the two graphs
- Alternatively, find a factor (the substitution vector) such that the product of that factor with each of the graphs is a good approximation to the other graph
    - This links the VSA representation factorisation back to statistical modelling
    - Long history of approximating matrices/tensors as the product of simpler factors (Kolda & Bader, 2009).

Future work

Lots to do:

- Subgraph isomorphism circuit takes two graphs, analogical memory takes a query graph and a memory stocked with *many* graphs
- Hypergraphs rather than graphs (suggestion that cognitive relations are rank up to 4 or 5)
- Make *thoroughly* distributed (avoid localist components)

Resources

The presentation is archived on Zenodo at

`https://doi.org/10.5281/zenodo.3700836`

The source code of this presentation is publicly accessible on GitHub at

`https://github.com/rgayler/VSA_2020_presentation`

The extended abstract of this presentation is publicly accessible on GitHub at

`https://github.com/rgayler/VSA_2020_presentation/raw/master/VSA2020_Gayler_abstract.pdf`

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