# VSA, Analogy, and Dynamic Similarity

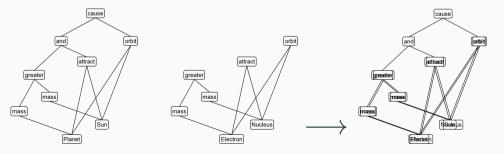
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# Analogy as structure mapping

- Arguable that analogy is the core of cognition (Blokpoel, Wareham, Haselager, Toni, & Rooij, 2018; Gust, Krumnack, Kuhnberger, & Schwering, 2008)
- Analogy commonly construed as structure mapping between source and target (Gentner, 1983)
  - · Find maximal subgraph isomorphisms between source and target graphs
  - Find a mapping between source and target that makes maximal subpgraphs identical



## Similarity and generalistation

- · Cognitive objective to generalise as widely and rapidly as possible
- · Generalisation usually invokes/induces a concept of similarity
- Typical statistical/ML models use literal similarity
- Relational similarity typically encodes relations as literals
- · Analogical similarity is based on relational structure (relations between relations)
  - · Relational structures tend to reflect real causes in the environment (?)
  - · Relational structures reduce reliance on "good" encoding of literals (?)
- · Analogy includes literal similarity as a special case

### Static similarity in VSA

- · Similarity (angle between vectors) is central to VSA (Kanerva, 2009)
- Typically a <u>fixed</u> mapping from "things" to vectors (representations)
- Emphasis on encodings that yield useful similarity structure (Purdy, 2016; Sahlgren, 2005)
- · Representations may be learned ...
  - E.g. Random Indexing (Sahlgren, 2005), vector embedding (Pennington, Socher, & Manning, 2014)
- but, are effectively fixed at time of use

## Human dynamic similarity

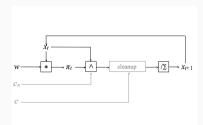
- · Human similarity judgments known to be context-dependent (Cheng, 1990)
- Arguable that similarity and analogy are based on the same processes (Gentner & Markman, 1997)
- Arguable that representations are created on-the-fly in response to task demands (Chalmers, French, & Hofstadter, 1992)
- Doesn't necessarily imply that base representations are context-dependent
- Could have dynamic working representations derived from the static base representations by context-dependent transforms

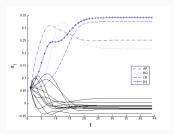
### Substitution as a dynamic transformation

- An obvious candidate for a dynamic transformation function in VSA is substitution by binding
  - The substitution mapping can be specified as a vector
  - The substitution mapping vector can be dynamically generated (Kanerva, 2009)
- This implies an internal degree of freedom (a register to hold the substitution vector while it evolves) ...
- and a recurrent VSA circuit to provide the dynamics to evolve the substitution vector

## Maximal subgraph isomorphism circuit

- Maximal subgraph isomorphism circuit (Gayler & Levy, 2009)
  - Finds the maximal subgraph isomorphism between two graphs represented as vectors
  - Implemented as a recurrent VSA circuit with a register containing a vertex substitution vector that evolves and settles over the course of the computation
  - Final state of substitution vector represents the set of vertex substitutions that best transforms each static graph into the other graph





#### Connections

- The subgraph isomorphism circuit can be interpreted as related to the recently developed Resonator Circuits for factorisation of VSA representations (Kent, Frady, Sommer, & Olshausen, 2019)
  - They have internal degrees of freedom for each of the factors to be calculated
  - · Recurrent VSA dynamics that settles on the factorisation
- Subgraph isomorphism circuit can be interpreted as finding the square root of the product of the two graphs
- Alternatively, find a factor (the substitution vector) such that the product of that factor with each of the graphs is a good approximation to the other graph
  - This links the VSA representation factorisation back to statistical modelling
  - Long history of approximating matrices/tensors as the product of simpler factors (Kolda & Bader, 2009).

#### Future work

#### Lots to do:

- Subgraph isomorphism circuit takes two graphs, analogical memory takes a query graph and a memory stocked with *many* graphs
- Hypergraphs rather than graphs (suggestion that cognitive relations are rank up to 4 or 5)
- Make thoroughly distributed (avoid localist components)

#### Resources

The presentation is archived on Zenodo at

https://doi.org/10.5281/zenodo.3700836

The source code of this presentation is publicly accessible on GitHub at

https://github.com/rgayler/VSA\_2020\_presentation

The extended abstract of this presentation is publicly accessible on GitHub at

https://github.com/rgayler/VSA\_2020\_presentation/raw/master/

VSA2020\_Gayler\_abstract.pdf



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