VSA, Analogy, and Dynamic Similarity

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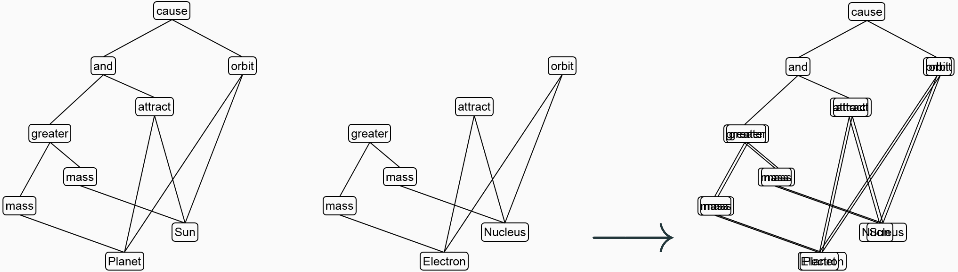
Analogy as structure mapping

- Arguable that analogy is the core of cognition (Blokpoel, Wareham, Haselager, Toni, & Rooij, 2018; Gust, Krumnack, Kuhnberger, & Schwering, 2008)
- Analogy commonly construed as *structure mapping* between *source* and *target* (Gentner, 1983)
  - Find maximal subgraph isomorphisms between *source* and *target* graphs
  - Find a mapping between *source* and *target* that makes maximal subpgraphs *identical*



Similarity and generalistation

- Cognitive objective to generalise as widely and rapidly as possible
- Generalisation usually invokes/induces a concept of similarity
- Typical statistical/ML models use *literal* similarity
- Relational similarity typically encodes relations as literals
- Analogical similarity is based on relational structure (relations between relations)
    - Relational structures tend to reflect real causes in the environment (?)
    - Relational structures reduce reliance on "good" encoding of literals (?)
- Analogy includes literal similarity as a special case

Static similarity in VSA

- Similarity (angle between vectors) is central to VSA (Kanerva, 2009)
- Typically a fixed mapping from “things” to vectors (representations)
- Emphasis on encodings that yield useful similarity structure (Purdy, 2016; Sahlgren, 2005)
- Representations may be learned …
  - E.g. Random Indexing (Sahlgren, 2005), vector embedding (Pennington, Socher, & Manning, 2014)
- but, are effectively fixed at time of use

Human dynamic similarity

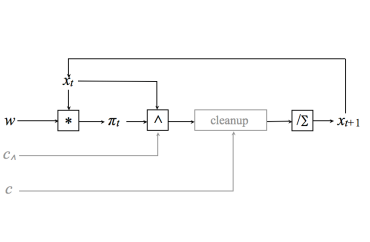
- Human similarity judgments known to be context-dependent (Cheng, 1990)
- Arguable that similarity and analogy are based on the same processes (Gentner & Markman, 1997)
- Arguable that representations are created on-the-fly in response to task demands (Chalmers, French, & Hofstadter, 1992)
- Doesn't necessarily imply that base representations are context-dependent
- Could have dynamic *working* representations derived from the static base representations by context-dependent transforms

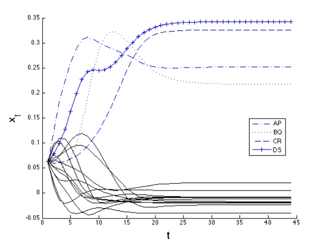
Substitution as a dynamic transformation

- An obvious candidate for a dynamic transformation function in VSA is substitution by binding
    - The substitution mapping can be specified as a vector
    - The substitution mapping vector can be dynamically generated (Kanerva, 2009)
- This implies an internal degree of freedom (a register to hold the substitution vector while it evolves) …
- and a recurrent VSA circuit to provide the dynamics to evolve the substitution vector

Maximal subgraph isomorphism circuit

- Maximal subgraph isomorphism circuit (Gayler & Levy, 2009)
  - Finds the maximal subgraph isomorphism between two graphs represented as vectors
  - Implemented as a recurrent VSA circuit with a register containing a vertex substitution vector that evolves and settles over the course of the computation
  - Final state of substitution vector represents the set of vertex substitutions that best transforms each static graph into the other graph





Connections

- The subgraph isomorphism circuit can be interpreted as related to the recently developed Resonator Circuits for factorisation of VSA representations (Kent, Frady, Sommer, & Olshausen, 2019)
    - They have internal degrees of freedom for each of the factors to be calculated
    - Recurrent VSA dynamics that settles on the factorisation
- Subgraph isomorphism circuit can be interpreted as finding the square root of the product of the two graphs
- Alternatively, find a factor (the substitution vector) such that the product of that factor with each of the graphs is a good approximation to the other graph
    - This links the VSA representation factorisation back to statistical modelling
    - Long history of approximating matrices/tensors as the product of simpler factors (Kolda & Bader, 2009).

Future work

Lots to do:

- Subgraph isomorphism circuit takes two graphs, analogical memory takes a query graph and a memory stocked with *many* graphs
- Hypergraphs rather than graphs (suggestion that cognitive relations are rank up to 4 or 5)
- Make *thoroughly* distributed (avoid localist components)

Resources

The source code of this presentation is publicly accessible on GitHub at

`https://github.com/rgayler/VSA_2020_presentation`

The presentation is archived on Zenodo at

`https://doi.org/10.5281/zenodo.3381641` FIXME

The extended abstract of this presentation is archived on Zenodo at

`https://doi.org/10.5281/zenodo.3381641` FIXME

References

Blokpoel, M., Wareham, T., Haselager, P., Toni, I., & Rooij, I. van. (2018). Deep Analogical Inference as the Origin of Hypotheses. *The Journal of Problem Solving*, *11*(1), 1–24. https://doi.org/10.7771/1932-6246.1197

Chalmers, D. J., French, R. M., & Hofstadter, D. R. (1992). High-level perception, representation, and analogy: A critique of artificial intelligence methodology. *Journal of Experimental & Theoretical Artificial Intelligence*, *4*(3), 185–211. https://doi.org/10.1080/09528139208953747

Cheng, Y. (1990). Context-Dependent Similarity. *Proceedings of the Sixth Annual Conference on Uncertainty in Artificial Intelligence (UAI'90)*, 27–30. Retrieved from http://arxiv.org/abs/1304.1084

Gayler, R. W., & Levy, S. D. (2009). A distributed basis for analogical mapping. In B. Kokinov, K. J. Holyoak, & D. Gentner (Eds.), *New Frontiers in Analogy Research, Proceedings of the Second International Conference on Analogy, ANALOGY-2009* (pp. 165–174). Sofia, Bulgaria: New Bulgarian University.

Gentner, D. (1983). Structure-Mapping: A Theoretical Framework for Analogy. *Cognitive Science*, *7*(2), 155–170. https://doi.org/10.1207/s15516709cog0702_3

Gentner, D., & Markman, A. B. (1997). Structure mapping in analogy and similarity. *American Psychologist*, *52*(1), 45–56. https://doi.org/10.1037/0003-066X.52.1.45

Gust, H., Krumnack, U., Kuhnberger, K.-U., & Schwering, A. (2008). Analogical Reasoning: A Core of Cognition. *KI - Künstliche Intelligenz*, *1*(8), 8–12. Retrieved from http://ifgi.uni-muenster.de/~schwering/gust_KIThemenheft.pdf

Kanerva, P. (2009). Hyperdimensional Computing: An Introduction to Computing in Distributed Representation with High-Dimensional Random Vectors. *Cognitive Computation*, *1*, 139–159. https://doi.org/10.1007/s12559-009-9009-8

Kent, S. J., Frady, E. P., Sommer, F. T., & Olshausen, B. A. (2019). Resonator Circuits for factoring high-dimensional vectors. *arXiv:1906.11684 [Cs, Stat]*. Retrieved from http://arxiv.org/abs/1906.11684

Kolda, T. G., & Bader, B. W. (2009). Tensor Decompositions and Applications. *SIAM Review*, *51*(3), 455–500. https://doi.org/10.1137/07070111X

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global Vectors for Word Representation. *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, 1532–1543. https://doi.org/10.3115/v1/D14-1162

Purdy, S. (2016). Encoding Data for HTM Systems. *arXiv Preprint*. Retrieved from http://arxiv.org/abs/1602.05925

Sahlgren, M. (2005). An Introduction to Random Indexing. *Methods and Applications of Semantic Indexing Workshop at the 7th International Conference on Terminology and Knowledge Engineering, TKE 2005*, 1–9. Copenhagen, Denmark.