

# Differential invariant signatures for planar Lie group transformations of images

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## 1 Introduction

### 1.1 Invariants, hey!

literature (Cartan, Olver, Russians, Draisma), motivation, etc,

### 1.2 Invariants for images

Colour and the lack of independence. Fractal dimension

Calculating derivatives (discussion on desirability thereof). Euclidean-invariant smoothing is a thing, other groups not so much.

### 1.3 Introducing the cast

Define each group, mesh image of what transformations can look like

### 1.4 Technical wank

Completeness, degeneracy, moving frames as concept, table of numbers of derivatives, all that jazz, maybe scaling technique for dealing with invariants of non-unit weight

## 2 The invariants

One subsection for each group. Mathematical details/theorem/derivation, no examples

### 2.1 $SA(2)$ , $E(2)$ , $SE(2)$ , and $Sim(2)$ , or some better title

**Theorem 1.**  *$SA(2)$  invariants up to second order are:  $I_0$ ,  $I_1$ ,  $I_2$  from Jupyter document*

Corollary for  $E(2)$ ,  $SE(2)$ ,  $R_2$  and  $Sim(2)$

Commentary on geometric interpretation of these things.

## 2.2 $A(2)$

**Theorem 2.** *General affine invariants using transvectants*

Relative invariants and weights go here? probably

## 2.3 Möbius and Projective (finite dimensional)

**Theorem 3.** *Moving frame invariants for Möbius*

**Theorem 4.** *Moving frame invariants for Projective*

## 2.4 Infinite dimensional diff groups (diff con, diff vol, diff)

# 3 Computational examples

General idea of what we've done. Experiment to compare e.g.  $SA(2)$  and  $Sim(2)$ . Contrast between intuitive notion of complexity (described by number of parameters in group) versus reality.

## 3.1 Examples of all of 'em

Big matrix of pictures for comparison purposes

## 3.2 Hat tip to practicality: smoothing for $E(2)$ and subgroups thereof

Allows these groups to be used on real (i.e. not particularly differentiable) images

## 3.3 colour

# 4 Discussion and Conclusion or whatever

Why are we using invariants? This idea is extremely local, can global information be better utilised? Relation to using more computationally tractable e.g.  $SA(2)$  not  $E(2)$  Comparison with other invariant forms - e.g. Fourier, integral, etc.

## 4.1 Future work

Can we do better with the weights? Can we do better with derivatives? Projection onto Riemann sphere? Noise. Ugh.