Invariant signatures

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1 Introduction

1.1 Notation

Let G be a planar transformation group that acts on k-colour images $f\colon \mathbb{R}^2 \to \mathbb{R}^k$ by $\varphi \cdot f \coloneqq f \circ \varphi^{-1}$, where $\varphi \in G$.

When referring to coordinates, with $x \in \mathbb{R}^2$, we write $\hat{x} = \varphi(x)$ and define the image post-transformation in the new coordinates as

$$\hat{f}(\hat{x}) = f(x). \tag{1}$$

We derive relationships between the derivatives before and after transformation by the chain rule:

$$f_{,i} = \hat{f}_{,k}\varphi_{k,i}$$

etc.

2 The Stamp Collection I: The Affine group and its subgroups

In this section we consider all the groups whose transformation is of the form. For these groups, the change of variables formulae are, in tensor, and matrix notations,

$$f_{,i} = \hat{f}_{,k}a_{ki} \qquad \nabla f = A^T \nabla \hat{f} \tag{2}$$

$$f_{,ij} = \hat{f}_{,kl}a_{ki}a_{lj} \qquad \nabla^2 f = A^T \nabla^2 \hat{f} A \tag{3}$$

$\varphi(x) = Ax + b$ for some matrix A and vector b.

2.1 SE(2)

In this case we have $A^T A = I$ and $\det A = 1$.