Supplementary Tables for Clark and Barr (2017): Biases of Relative Risk Estimators in Simulation Study when $n = 1000$

Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the $Y = 1$ instances occur at higher values of a covariate than all of the $Y = 0$ instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.

Table 1: Biases of relative risk (RR) estimators for $x_1 = 1$ vs $x_1 = -1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim$binary (values ± 1 with equal probability), $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.005	-0.001	0.001	-0.001	0.1
0.3	1.500	0.010	-0.007	-0.002	-0.007	0.4
0.3	3.000	0.038	-0.039	-0.025	-0.038	5.2
0.5	1.200	0.001	-0.017	-0.006	-0.013	7.9
0.5	1.500	0.000	-0.041	-0.020	-0.034	20.8
0.5	3.000	0.025	-0.110	-0.076	-0.088	56.5

Table 2: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0,1)$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.003	-0.017	-0.006	-0.009	0.2
0.3	1.500	0.008	-0.160	-0.053	-0.078	24.8
0.3	3.000	0.042	-1.570	-0.965	-0.805	97.1
0.5	1.200	0.002	-0.083	-0.012	-0.022	24.7
0.5	1.500	0.005	-0.266	-0.084	-0.090	82.1
0.5	3.000	0.038	-1.731	-1.129	-0.482	99.8

Table 3: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0,1)$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.000	0.010	0.021	0.018	0.2
0.3	1.332	0.001	0.008	0.115	0.090	24.8
0.3	1.346	0.001	0.084	0.690	0.255	97.1
0.5	1.098	0.000	0.172	0.774	-0.037	99.8
0.5	1.143	-0.001	-0.026	0.045	0.035	24.7
0.5	1.206	0.000	0.028	0.210	0.141	82.1

Table 4: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0,1)$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.005	-0.041	-0.030	-0.033	0.2
0.3	1.634	0.012	-0.294	-0.187	-0.212	24.8
0.3	4.583	0.083	-3.153	-2.548	-2.388	97.1
0.5	1.260	0.005	-0.142	-0.072	-0.081	24.7
0.5	1.865	0.016	-0.631	-0.449	-0.456	82.1
0.5	8.200	0.220	-6.931	-6.329	-5.682	99.8

Table 5: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.004	-0.069	-0.017	-0.020	2.5
0.3	1.500	0.009	-0.308	-0.126	-0.107	39.1
0.3	3.000	0.030	-1.777	-1.323	-0.789	96.2
0.5	1.200	0.005	-0.136	-0.025	-0.025	29.4
0.5	1.500	0.008	-0.387	-0.161	-0.093	77.5
0.5	3.000	0.032	-1.860	-1.417	-0.488	99.4

Table 6: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.000	-0.042	0.010	0.007	2.5
0.3	1.332	0.000	-0.140	0.042	0.060	39.1
0.3	1.346	-0.003	-0.123	0.331	0.215	96.2
0.5	1.098	-0.001	0.043	0.486	-0.045	99.4
0.5	1.143	0.000	-0.080	0.032	0.032	29.4
0.5	1.206	-0.001	-0.093	0.133	0.131	77.5

Table 7: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.006	-0.093	-0.041	-0.044	2.5
0.3	1.634	0.016	-0.442	-0.261	-0.242	39.1
0.3	4.583	0.083	-3.360	-2.906	-2.372	96.2
0.5	1.260	0.010	-0.196	-0.084	-0.085	29.4
0.5	1.865	0.024	-0.752	-0.527	-0.458	77.5
0.5	8.200	0.252	-7.060	-6.616	-5.688	99.4

Table 8: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.003	-0.006	-0.004	-0.006	0.4
0.3	1.500	0.008	-0.045	-0.027	-0.040	10.0
0.3	3.000	0.034	-0.955	-0.660	-0.750	98.1
0.5	1.200	0.000	-0.032	-0.010	-0.018	22.3
0.5	1.500	0.003	-0.122	-0.050	-0.073	76.6
0.5	3.000	0.028	-1.348	-0.848	-0.481	99.8

Table 9: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.000	0.021	0.024	0.021	0.4
0.3	1.332	0.001	0.123	0.141	0.128	10.0
0.3	1.346	0.000	0.699	0.994	0.272	98.1
0.5	1.098	0.000	0.554	1.055	-0.025	99.8
0.5	1.143	-0.002	0.025	0.047	0.039	22.3
0.5	1.206	-0.001	0.172	0.244	0.148	76.6

Table 10: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.005	-0.030	-0.027	-0.029	0.4
0.3	1.634	0.013	-0.179	-0.161	-0.175	10.0
0.3	4.583	0.072	-2.539	-2.243	-2.334	98.1
0.5	1.260	0.002	-0.092	-0.069	-0.078	22.3
0.5	1.865	0.012	-0.488	-0.415	-0.438	76.6
0.5	8.200	0.195	-6.548	-6.048	-5.681	99.8