

# Supplementary Tables for Clark and Barr (2017): Root Mean Squared Errors of Relative Risk Estimators in Simulation Study when $n = 500$

*Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the  $Y = 1$  instances occur at higher values of a covariate than all of the  $Y = 0$  instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.*

Table 1: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 1$  vs  $x_1 = -1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim \text{binary}$  (values  $\pm 1$  with equal probability),  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.168	0.163	0.164	0.163	0.0
0.3	1.500	0.212	0.204	0.205	0.204	0.0
0.3	3.000	0.503	0.475	0.476	0.475	0.9
0.5	1.200	0.112	0.105	0.107	0.105	2.8
0.5	1.500	0.146	0.140	0.139	0.139	9.3
0.5	3.000	0.361	0.348	0.337	0.344	28.9

Table 2: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.086	0.072	0.079	0.078	0.4
0.3	1.500	0.124	0.164	0.104	0.117	11.5
0.3	3.000	0.431	1.529	0.967	0.811	86.4
0.5	1.200	0.060	0.084	0.053	0.054	9.2
0.5	1.500	0.091	0.259	0.101	0.111	53.8
0.5	3.000	0.389	1.706	1.129	0.528	93.4

Table 3: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.062	0.071	0.082	0.080	0.4
0.3	1.332	0.049	0.077	0.147	0.128	11.5
0.3	1.346	0.040	0.154	0.703	0.265	86.4
0.5	1.098	0.020	0.203	0.781	0.047	93.4
0.5	1.143	0.028	0.043	0.069	0.062	9.2
0.5	1.206	0.018	0.060	0.218	0.140	53.8

Table 4: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.107	0.082	0.084	0.084	0.4
0.3	1.634	0.182	0.290	0.207	0.228	11.5
0.3	4.583	0.876	3.111	2.546	2.378	86.4
0.5	1.260	0.097	0.139	0.088	0.095	9.2
0.5	1.865	0.215	0.622	0.452	0.457	53.8
0.5	8.200	2.048	6.905	6.326	5.654	93.4

Table 5: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.093	0.083	0.079	0.081	0.2
0.3	1.500	0.129	0.279	0.147	0.139	15.6
0.3	3.000	0.445	1.721	1.276	0.797	76.3
0.5	1.200	0.063	0.126	0.057	0.060	11.1
0.5	1.500	0.101	0.365	0.168	0.123	51.0
0.5	3.000	0.382	1.826	1.381	0.537	92.7

Table 6: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.066	0.068	0.079	0.080	0.2
0.3	1.332	0.050	0.128	0.107	0.115	15.6
0.3	1.346	0.042	0.119	0.441	0.245	76.3
0.5	1.098	0.021	0.098	0.553	0.051	92.7
0.5	1.143	0.030	0.073	0.061	0.063	11.1
0.5	1.206	0.018	0.088	0.161	0.130	51.0

Table 7: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.115	0.101	0.087	0.089	0.2
0.3	1.634	0.190	0.409	0.265	0.255	15.6
0.3	4.583	0.944	3.302	2.851	2.358	76.3
0.5	1.260	0.102	0.183	0.098	0.100	11.1
0.5	1.865	0.239	0.728	0.521	0.465	51.0
0.5	8.200	2.154	7.025	6.573	5.670	92.7

Table 8: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.087	0.081	0.082	0.081	0.0
0.3	1.500	0.117	0.102	0.101	0.102	2.8
0.3	3.000	0.395	0.961	0.668	0.749	84.3
0.5	1.200	0.057	0.056	0.053	0.053	8.6
0.5	1.500	0.085	0.135	0.082	0.096	46.7
0.5	3.000	0.368	1.348	0.850	0.511	96.0

Table 9: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.062	0.083	0.085	0.084	0.0
0.3	1.332	0.046	0.154	0.173	0.161	2.8
0.3	1.346	0.038	0.706	1.015	0.279	84.3
0.5	1.098	0.019	0.557	1.062	0.041	96.0
0.5	1.143	0.027	0.054	0.073	0.066	8.6
0.5	1.206	0.017	0.180	0.254	0.147	46.7

Table 10: Root mean squared errors (RMSEs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.107	0.086	0.086	0.086	0.0
0.3	1.634	0.171	0.201	0.187	0.196	2.8
0.3	4.583	0.791	2.540	2.239	2.319	84.3
0.5	1.260	0.093	0.102	0.084	0.090	8.6
0.5	1.865	0.203	0.492	0.419	0.440	46.7
0.5	8.200	1.941	6.548	6.045	5.646	96.0