

Supplementary Tables for Clark and Barr (2017): Root Mean Squared Errors of Relative Risk Estimators in Simulation Study when $n = 100$

Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the $Y = 1$ instances occur at higher values of a covariate than all of the $Y = 0$ instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.

Table 1: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 1$ vs $x_1 = -1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim \text{binary}$ (values ± 1 with equal probability), $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.432	0.411	0.413	0.413	0.0
0.3	1.500	0.595	0.558	0.561	0.562	0.1
0.3	3.000	1.720	1.544	1.551	1.547	0.3
0.5	1.200	0.272	0.251	0.258	0.259	0.4
0.5	1.500	0.366	0.335	0.340	0.341	0.9
0.5	3.000	1.058	0.898	0.898	0.933	3.0

Table 2: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0, 1)$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.245	0.177	0.201	0.202	0.2
0.3	1.500	0.352	0.207	0.224	0.237	1.3
0.3	3.000	1.489	1.379	0.920	0.831	22.7
0.5	1.200	0.154	0.112	0.124	0.134	0.5
0.5	1.500	0.247	0.239	0.153	0.194	5.4
0.5	3.000	1.480	1.610	1.089	0.860	32.9

Table 3: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0, 1)$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.151	0.179	0.206	0.206	0.2
0.3	1.332	0.116	0.194	0.268	0.251	1.3
0.3	1.346	0.091	0.345	0.833	0.290	22.7
0.5	1.098	0.044	0.317	0.855	0.075	32.9
0.5	1.143	0.069	0.100	0.137	0.123	0.5
0.5	1.206	0.042	0.128	0.272	0.137	5.4

Table 4: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0, 1)$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.317	0.179	0.200	0.201	0.2
0.3	1.634	0.528	0.297	0.272	0.284	1.3
0.3	4.583	3.229	2.955	2.473	2.219	22.7
0.5	1.260	0.271	0.150	0.137	0.145	0.5
0.5	1.865	0.668	0.590	0.449	0.441	5.4
0.5	8.200	17.074	6.807	6.274	5.422	32.9

Table 5: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.247	0.167	0.193	0.206	0.2
0.3	1.500	0.390	0.245	0.227	0.266	1.1
0.3	3.000	1.688	1.517	1.134	0.881	14.6
0.5	1.200	0.164	0.122	0.125	0.145	0.7
0.5	1.500	0.296	0.304	0.184	0.234	4.1
0.5	3.000	1.411	1.698	1.264	0.976	26.7

Table 6: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.153	0.165	0.196	0.204	0.2
0.3	1.332	0.121	0.176	0.240	0.247	1.1
0.3	1.346	0.098	0.292	0.674	0.282	14.6
0.5	1.098	0.047	0.254	0.727	0.080	26.7
0.5	1.143	0.073	0.094	0.133	0.121	0.7
0.5	1.206	0.045	0.107	0.235	0.127	4.1

Table 7: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.328	0.173	0.193	0.205	0.2
0.3	1.634	0.628	0.352	0.295	0.309	1.1
0.3	4.583	3.765	3.090	2.685	2.211	14.6
0.5	1.260	0.314	0.169	0.143	0.156	0.7
0.5	1.865	0.957	0.658	0.498	0.457	4.1
0.5	8.200	14.363	6.894	6.438	5.368	26.7

Table 8: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, \sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.220	0.192	0.197	0.200	0.1
0.3	1.500	0.315	0.231	0.242	0.241	0.1
0.3	3.000	1.555	0.965	0.690	0.780	20.4
0.5	1.200	0.141	0.117	0.125	0.127	0.6
0.5	1.500	0.226	0.175	0.161	0.181	4.4
0.5	3.000	1.356	1.329	0.843	0.830	38.3

Table 9: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, \sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.145	0.195	0.201	0.202	0.1
0.3	1.332	0.115	0.268	0.296	0.264	0.1
0.3	1.346	0.093	0.760	1.138	0.325	20.4
0.5	1.098	0.043	0.591	1.116	0.080	38.3
0.5	1.143	0.067	0.118	0.138	0.124	0.6
0.5	1.206	0.045	0.224	0.307	0.153	4.4

Table 10: Root mean squared errors (RMSEs) of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	rmse of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.283	0.192	0.197	0.199	0.1
0.3	1.634	0.469	0.281	0.276	0.279	0.1
0.3	4.583	2.965	2.529	2.195	2.209	20.4
0.5	1.260	0.241	0.142	0.137	0.141	0.6
0.5	1.865	0.589	0.497	0.427	0.437	4.4
0.5	8.200	9.081	6.525	6.015	5.408	38.3