

# Supplementary Tables for Clark and Barr (2017): Standard Deviations of Relative Risk Estimators in Simulation Study when $n = 100$

*Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the  $Y = 1$  instances occur at higher values of a covariate than all of the  $Y = 0$  instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.*

Table 1: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 1$  vs  $x_1 = -1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim \text{binary}$  (values  $\pm 1$  with equal probability),  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.427	0.407	0.409	0.409	0.0
0.3	1.500	0.583	0.552	0.553	0.555	0.1
0.3	3.000	1.646	1.505	1.509	1.506	0.3
0.5	1.200	0.269	0.251	0.256	0.258	0.4
0.5	1.500	0.362	0.335	0.338	0.341	0.9
0.5	3.000	1.021	0.895	0.890	0.922	3.0

Table 2: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.240	0.178	0.200	0.201	0.2
0.3	1.500	0.342	0.182	0.224	0.236	1.3
0.3	3.000	1.391	0.190	0.290	0.628	22.7
0.5	1.200	0.151	0.100	0.124	0.134	0.5
0.5	1.500	0.240	0.101	0.141	0.192	5.4
0.5	3.000	1.421	0.111	0.200	0.843	32.9

Table 3: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.151	0.178	0.200	0.201	0.2
0.3	1.332	0.116	0.182	0.224	0.214	1.3
0.3	1.346	0.091	0.190	0.290	0.215	22.7
0.5	1.098	0.044	0.111	0.200	0.070	32.9
0.5	1.143	0.069	0.100	0.124	0.114	0.5
0.5	1.206	0.041	0.101	0.141	0.089	5.4

Table 4: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.307	0.178	0.200	0.201	0.2
0.3	1.634	0.508	0.182	0.224	0.236	1.3
0.3	4.583	3.005	0.190	0.290	0.628	22.7
0.5	1.260	0.262	0.100	0.124	0.134	0.5
0.5	1.865	0.637	0.101	0.141	0.192	5.4
0.5	8.200	16.746	0.111	0.200	0.989	32.9

Table 5: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.243	0.165	0.193	0.205	0.2
0.3	1.500	0.378	0.176	0.217	0.265	1.1
0.3	3.000	1.604	0.246	0.351	0.721	14.6
0.5	1.200	0.162	0.090	0.124	0.145	0.7
0.5	1.500	0.288	0.107	0.147	0.233	4.1
0.5	3.000	1.327	0.144	0.281	0.971	26.7

Table 6: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.153	0.165	0.193	0.200	0.2
0.3	1.332	0.121	0.176	0.217	0.218	1.1
0.3	1.346	0.097	0.246	0.351	0.234	14.6
0.5	1.098	0.046	0.144	0.281	0.071	26.7
0.5	1.143	0.072	0.090	0.124	0.115	0.7
0.5	1.206	0.043	0.107	0.147	0.097	4.1

Table 7: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.318	0.165	0.193	0.205	0.2
0.3	1.634	0.603	0.176	0.217	0.265	1.1
0.3	4.583	3.553	0.246	0.351	0.721	14.6
0.5	1.260	0.304	0.090	0.124	0.145	0.7
0.5	1.865	0.923	0.107	0.147	0.233	4.1
0.5	8.200	13.687	0.144	0.281	1.125	26.7

Table 8: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.218	0.192	0.197	0.199	0.1
0.3	1.500	0.309	0.229	0.242	0.241	0.1
0.3	3.000	1.473	0.243	0.375	0.546	20.4
0.5	1.200	0.140	0.114	0.125	0.127	0.6
0.5	1.500	0.222	0.133	0.158	0.178	4.4
0.5	3.000	1.298	0.119	0.232	0.814	38.3

Table 9: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.145	0.192	0.197	0.198	0.1
0.3	1.332	0.115	0.229	0.242	0.219	0.1
0.3	1.346	0.093	0.243	0.375	0.206	20.4
0.5	1.098	0.043	0.119	0.232	0.078	38.3
0.5	1.143	0.066	0.114	0.125	0.114	0.6
0.5	1.206	0.045	0.133	0.158	0.097	4.4

Table 10: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	sd of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.277	0.192	0.197	0.199	0.1
0.3	1.634	0.455	0.229	0.242	0.241	0.1
0.3	4.583	2.788	0.243	0.375	0.546	20.4
0.5	1.260	0.236	0.114	0.125	0.127	0.6
0.5	1.865	0.568	0.133	0.158	0.178	4.4
0.5	8.200	8.623	0.119	0.232	0.882	38.3