Supplementary Tables for Clark and Barr (2017): Biases of Relative Risk Estimators in Simulation Study when $n = 500$

Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the $Y = 1$ instances occur at higher values of a covariate than all of the $Y = 0$ instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.

Table 1: Biases of relative risk (RR) estimators for $x_1 = 1$ vs $x_1 = -1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim$binary (values ± 1 with equal probability), $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.006	-0.001	0.001	0.000	0.0
0.3	1.500	0.012	-0.005	-0.001	-0.005	0.0
0.3	3.000	0.071	-0.011	0.003	-0.010	0.9
0.5	1.200	0.005	-0.013	-0.002	-0.009	2.8
0.5	1.500	0.012	-0.032	-0.009	-0.024	9.3
0.5	3.000	0.038	-0.105	-0.069	-0.078	28.9

Table 2: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0,1)$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.005	-0.018	-0.005	-0.008	0.4
0.3	1.500	0.013	-0.146	-0.052	-0.076	11.5
0.3	3.000	0.077	-1.526	-0.960	-0.787	86.4
0.5	1.200	0.003	-0.074	-0.011	-0.020	9.2
0.5	1.500	0.008	-0.255	-0.083	-0.087	53.8
0.5	3.000	0.065	-1.705	-1.126	-0.448	93.4

Table 3: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0,1)$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	-0.001	0.009	0.022	0.019	0.4
0.3	1.332	0.000	0.022	0.116	0.092	11.5
0.3	1.346	-0.002	0.128	0.694	0.246	86.4
0.5	1.098	-0.001	0.197	0.777	-0.037	93.4
0.5	1.143	-0.002	-0.017	0.046	0.036	9.2
0.5	1.206	-0.002	0.039	0.211	0.135	53.8

Table 4: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0,1)$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.009	-0.042	-0.029	-0.032	0.4
0.3	1.634	0.022	-0.280	-0.186	-0.210	11.5
0.3	4.583	0.169	-3.109	-2.543	-2.370	86.4
0.5	1.260	0.010	-0.134	-0.071	-0.080	9.2
0.5	1.865	0.030	-0.620	-0.449	-0.452	53.8
0.5	8.200	0.448	-6.905	-6.326	-5.647	93.4

Table 5: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR logit	log	poisson	log-logit	prob.reject (%)
0.3	1.200	0.006	-0.055	-0.015	-0.017	0.2
0.3	1.500	0.012	-0.266	-0.114	-0.102	15.6
0.3	3.000	0.083	-1.718	-1.261	-0.763	76.3
0.5	1.200	0.004	-0.120	-0.024	-0.024	11.1
0.5	1.500	0.008	-0.360	-0.151	-0.093	51.0
0.5	3.000	0.070	-1.825	-1.372	-0.463	92.7

Table 6: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR logit	log	poisson	log-logit	prob.reject (%)
0.3	1.173	-0.001	-0.028	0.012	0.011	0.2
0.3	1.332	-0.001	-0.098	0.054	0.066	15.6
0.3	1.346	0.000	-0.063	0.394	0.217	76.3
0.5	1.098	0.000	0.077	0.531	-0.043	92.7
0.5	1.143	-0.002	-0.063	0.033	0.032	11.1
0.5	1.206	-0.002	-0.066	0.143	0.124	51.0

Table 7: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0,1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR logit	log	poisson	log-logit	prob.reject (%)
0.3	1.224	0.011	-0.079	-0.039	-0.040	0.2
0.3	1.634	0.021	-0.400	-0.248	-0.236	15.6
0.3	4.583	0.182	-3.301	-2.844	-2.347	76.3
0.5	1.260	0.012	-0.179	-0.083	-0.084	11.1
0.5	1.865	0.029	-0.726	-0.516	-0.458	51.0
0.5	8.200	0.477	-7.025	-6.572	-5.663	92.7

Table 8: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.003	-0.006	-0.004	-0.006	0.0
0.3	1.500	0.013	-0.045	-0.025	-0.038	2.8
0.3	3.000	0.075	-0.955	-0.651	-0.729	84.3
0.5	1.200	0.004	-0.030	-0.006	-0.014	8.6
0.5	1.500	0.007	-0.123	-0.048	-0.069	46.7
0.5	3.000	0.059	-1.348	-0.845	-0.440	96.0

Table 9: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	-0.002	0.021	0.023	0.021	0.0
0.3	1.332	0.000	0.123	0.143	0.130	2.8
0.3	1.346	-0.002	0.699	1.004	0.266	84.3
0.5	1.098	-0.002	0.555	1.058	-0.026	96.0
0.5	1.143	-0.002	0.027	0.050	0.042	8.6
0.5	1.206	-0.002	0.171	0.246	0.143	46.7

Table 10: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when logit$(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit, log, poisson and log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.008	-0.030	-0.028	-0.030	0.0
0.3	1.634	0.022	-0.179	-0.159	-0.172	2.8
0.3	4.583	0.162	-2.538	-2.234	-2.312	84.3
0.5	1.260	0.012	-0.090	-0.066	-0.074	8.6
0.5	1.865	0.027	-0.489	-0.414	-0.435	46.7
0.5	8.200	0.469	-6.547	-6.045	-5.640	96.0