

Supplementary Tables for Clark and Barr (2017): Biases of Relative Risk Estimators in Simulation Study when $n = 100$

Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the $Y = 1$ instances occur at higher values of a covariate than all of the $Y = 0$ instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.

Table 1: Biases of relative risk (RR) estimators for $x_1 = 1$ vs $x_1 = -1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim \text{binary}$ (values ± 1 with equal probability), $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.072	0.059	0.061	0.061	0.0
0.3	1.500	0.118	0.086	0.092	0.089	0.1
0.3	3.000	0.501	0.346	0.363	0.357	0.3
0.5	1.200	0.041	0.017	0.030	0.025	0.4
0.5	1.500	0.060	0.004	0.030	0.021	0.9
0.5	3.000	0.280	0.082	0.118	0.143	3.0

Table 2: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0, 1)$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.047	-0.001	0.022	0.021	0.2
0.3	1.500	0.085	-0.100	-0.020	-0.024	1.3
0.3	3.000	0.534	-1.366	-0.873	-0.545	22.7
0.5	1.200	0.026	-0.052	0.002	0.003	0.5
0.5	1.500	0.060	-0.216	-0.061	-0.031	5.4
0.5	3.000	0.416	-1.606	-1.071	-0.169	32.9

Table 3: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0, 1)$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.005	0.026	0.049	0.048	0.2
0.3	1.332	-0.001	0.068	0.148	0.131	1.3
0.3	1.346	-0.011	0.288	0.781	0.196	22.7
0.5	1.098	0.000	0.296	0.832	-0.027	32.9
0.5	1.143	-0.008	0.005	0.059	0.047	0.5
0.5	1.206	-0.009	0.078	0.233	0.104	5.4

Table 4: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim N(0, 1)$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.077	-0.025	-0.002	-0.002	0.2
0.3	1.634	0.145	-0.234	-0.154	-0.159	1.3
0.3	4.583	1.186	-2.949	-2.456	-2.128	22.7
0.5	1.260	0.071	-0.112	-0.058	-0.057	0.5
0.5	1.865	0.201	-0.582	-0.427	-0.397	5.4
0.5	8.200	3.373	-6.806	-6.271	-5.332	32.9

Table 5: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.047	-0.030	0.008	0.015	0.2
0.3	1.500	0.099	-0.171	-0.065	-0.025	1.1
0.3	3.000	0.527	-1.497	-1.079	-0.508	14.6
0.5	1.200	0.027	-0.083	-0.011	0.001	0.7
0.5	1.500	0.067	-0.284	-0.110	-0.028	4.1
0.5	3.000	0.482	-1.692	-1.232	-0.103	26.7

Table 6: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	0.002	-0.003	0.035	0.040	0.2
0.3	1.332	-0.003	-0.003	0.103	0.117	1.1
0.3	1.346	-0.012	0.157	0.575	0.157	14.6
0.5	1.098	-0.005	0.210	0.670	-0.036	26.7
0.5	1.143	-0.011	-0.026	0.046	0.039	0.7
0.5	1.206	-0.012	0.009	0.183	0.082	4.1

Table 7: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim t_4/\sqrt{2}$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.081	-0.054	-0.016	-0.008	0.2
0.3	1.634	0.176	-0.305	-0.199	-0.159	1.1
0.3	4.583	1.249	-3.080	-2.662	-2.091	14.6
0.5	1.260	0.080	-0.143	-0.071	-0.058	0.7
0.5	1.865	0.256	-0.650	-0.476	-0.393	4.1
0.5	8.200	4.374	-6.892	-6.432	-5.249	26.7

Table 8: Biases of relative risk (RR) estimators for $x_1 = 0.5$ vs $x_1 = -0.5$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.200	0.032	0.009	0.017	0.016	0.1
0.3	1.500	0.065	-0.028	0.002	-0.007	0.1
0.3	3.000	0.500	-0.934	-0.580	-0.557	20.4
0.5	1.200	0.019	-0.026	0.003	-0.001	0.6
0.5	1.500	0.044	-0.114	-0.031	-0.034	4.4
0.5	3.000	0.392	-1.324	-0.811	-0.162	38.3

Table 9: Biases of relative risk (RR) estimators for $x_1 = 2$ vs $x_1 = 1$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.173	-0.002	0.036	0.044	0.042	0.1
0.3	1.332	-0.002	0.139	0.170	0.148	0.1
0.3	1.346	-0.002	0.720	1.075	0.251	20.4
0.5	1.098	-0.002	0.579	1.092	-0.017	38.3
0.5	1.143	-0.009	0.031	0.060	0.047	0.6
0.5	1.206	-0.008	0.180	0.263	0.119	4.4

Table 10: Biases of relative risk (RR) estimators for $x_1 = -1$ vs $x_1 = -2$ when $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$. Sample size is 1000. $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$, $x_2 \sim N(0, 1)$. *Median prob* refers to the probability that $Y = 1$ when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability (%) of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject (%)
		logit	log	poisson	log-logit	
0.3	1.224	0.058	-0.015	-0.007	-0.008	0.1
0.3	1.634	0.112	-0.163	-0.132	-0.141	0.1
0.3	4.583	1.013	-2.517	-2.163	-2.141	20.4
0.5	1.260	0.054	-0.085	-0.057	-0.061	0.6
0.5	1.865	0.156	-0.479	-0.396	-0.399	4.4
0.5	8.200	2.862	-6.524	-6.011	-5.336	38.3