

# Supplementary Tables for Clark and Barr (2017): Standard Deviations of Relative Risk Estimators in Simulation Study when $n = 1000$

*Note that there are some very large root mean squared errors in some tables, for the logistic and other models, when the true relative risk is large. This is because the parameter estimates are effectively infinite in some samples. This can occur in binary regression when all of the  $Y = 1$  instances occur at higher values of a covariate than all of the  $Y = 0$  instances (Hosmer et. al 2013, section 4.4). This typically occurs only when the sample size is small and the covariates have strong predictive power.*

Table 1: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 1$  vs  $x_1 = -1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim \text{binary}$  (values  $\pm 1$  with equal probability),  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.116	0.113	0.113	0.112	0.1
0.3	1.500	0.147	0.142	0.143	0.142	0.4
0.3	3.000	0.343	0.328	0.328	0.328	5.2
0.5	1.200	0.078	0.073	0.075	0.074	7.9
0.5	1.500	0.101	0.095	0.096	0.096	20.8
0.5	3.000	0.254	0.236	0.236	0.239	56.5

Table 2: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.061	0.050	0.057	0.056	0.2
0.3	1.500	0.086	0.056	0.065	0.063	24.8
0.3	3.000	0.281	0.062	0.080	0.132	97.1
0.5	1.200	0.042	0.029	0.037	0.036	24.7
0.5	1.500	0.063	0.035	0.042	0.050	82.1
0.5	3.000	0.270	0.036	0.061	0.190	99.8

Table 3: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.045	0.050	0.057	0.056	0.2
0.3	1.332	0.035	0.056	0.065	0.063	24.8
0.3	1.346	0.029	0.062	0.080	0.069	97.1
0.5	1.098	0.014	0.036	0.061	0.020	99.8
0.5	1.143	0.020	0.029	0.037	0.036	24.7
0.5	1.206	0.013	0.035	0.042	0.026	82.1

Table 4: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim N(0, 1)$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.075	0.050	0.057	0.056	0.2
0.3	1.634	0.125	0.056	0.065	0.063	24.8
0.3	4.583	0.571	0.062	0.080	0.132	97.1
0.5	1.260	0.066	0.029	0.037	0.036	24.7
0.5	1.865	0.147	0.035	0.042	0.050	82.1
0.5	8.200	1.356	0.036	0.061	0.190	99.8

Table 5: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.063	0.044	0.053	0.053	2.5
0.3	1.500	0.090	0.067	0.071	0.067	39.1
0.3	3.000	0.301	0.074	0.163	0.161	96.2
0.5	1.200	0.044	0.028	0.036	0.038	29.4
0.5	1.500	0.070	0.045	0.058	0.059	77.5
0.5	3.000	0.275	0.045	0.131	0.196	99.4

Table 6: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.173	0.045	0.044	0.053	0.053	2.5
0.3	1.332	0.035	0.067	0.071	0.067	39.1
0.3	1.346	0.029	0.074	0.163	0.080	96.2
0.5	1.098	0.015	0.045	0.131	0.018	99.4
0.5	1.143	0.020	0.028	0.036	0.038	29.4
0.5	1.206	0.011	0.045	0.058	0.025	77.5

Table 7: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim t_4/\sqrt{2}$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.224	0.078	0.044	0.053	0.053	2.5
0.3	1.634	0.132	0.067	0.071	0.067	39.1
0.3	4.583	0.652	0.074	0.163	0.161	96.2
0.5	1.260	0.070	0.028	0.036	0.038	29.4
0.5	1.865	0.169	0.045	0.058	0.059	77.5
0.5	8.200	1.498	0.045	0.131	0.196	99.4

Table 8: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 0.5$  vs  $x_1 = -0.5$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

median prob.	true RR	bias of estimated RR				prob.reject
		logit	log	poisson	log-logit	
0.3	1.200	0.059	0.055	0.056	0.056	0.4
0.3	1.500	0.082	0.066	0.070	0.067	10.0
0.3	3.000	0.264	0.075	0.106	0.118	98.1
0.5	1.200	0.041	0.034	0.038	0.037	22.3
0.5	1.500	0.061	0.041	0.047	0.048	76.6
0.5	3.000	0.239	0.035	0.069	0.169	99.8

Table 9: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = 2$  vs  $x_1 = 1$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

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		logit	log	poisson	log-logit	
0.3	1.173	0.043	0.055	0.056	0.056	0.4
0.3	1.332	0.033	0.066	0.070	0.067	10.0
0.3	1.346	0.028	0.075	0.106	0.060	98.1
0.5	1.098	0.014	0.035	0.069	0.023	99.8
0.5	1.143	0.020	0.034	0.038	0.037	22.3
0.5	1.206	0.012	0.041	0.047	0.025	76.6

Table 10: Standard deviations (SDs) of relative risk (RR) estimators for  $x_1 = -1$  vs  $x_1 = -2$  when  $\text{logit}(P[Y = 1]) = \alpha + \beta_1 x_1 + \beta_2 x_2$ . Sample size is 1000.  $x_1 \sim U(-\sqrt{3}, -\sqrt{3})$ ,  $x_2 \sim N(0, 1)$ . *Median prob* refers to the probability that  $Y = 1$  when both covariates are at their median value. *Logit*, *log*, *poisson* and *log-logit* refer to binary regression with logit link (logistic regression) and log link, Poisson regression with log link and binary regression with blended log-logit link, respectively. *Prob.reject* is the simulation probability of rejecting the null that the cutover probability is 0.8 in the blended log-logit model.

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		logit	log	poisson	log-logit	
0.3	1.224	0.073	0.055	0.056	0.056	0.4
0.3	1.634	0.119	0.066	0.070	0.067	10.0
0.3	4.583	0.525	0.075	0.106	0.118	98.1
0.5	1.260	0.065	0.034	0.038	0.037	22.3
0.5	1.865	0.141	0.041	0.047	0.048	76.6
0.5	8.200	1.185	0.035	0.069	0.169	99.8